# Clustering Aggregation

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Some terminology:

**Clustering**: A group of clusters output by a clustering algorithm

**Cluster**: A group of points

# Clustering Aggregation

**Goals**:

1. Compare clusterings
2. Combine the information from multiple clusterings to create a new clustering

# Comparing Clusterings

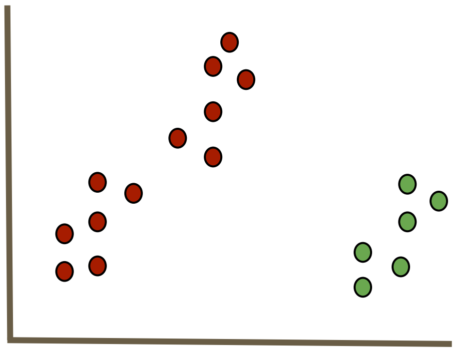
The many methods / cost functions make comparing clusterings difficult.

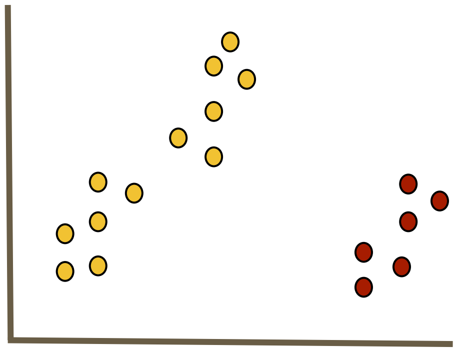
Need to compare clusterings by looking at their assignment of points to clusters.

If many points were assigned to the same clusters in both clustering C and clustering P, then C and P should have a small distance.

But identifying which clusters are the same in P and C is not easy. Why?

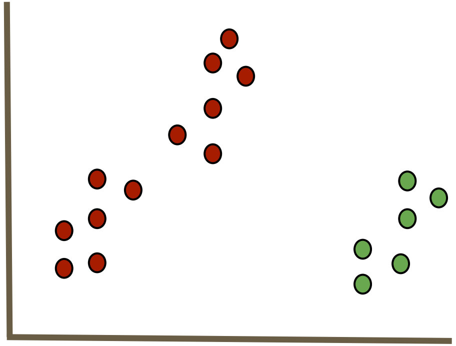
# Comparing Clusterings

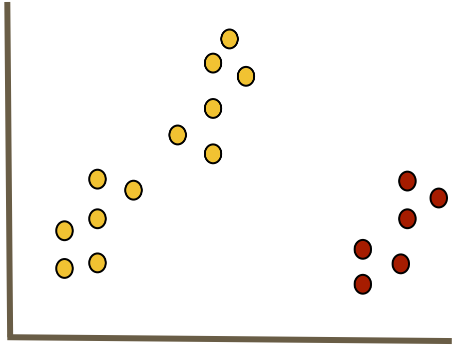




Clearly these clusterings are the same. Yet the assignments / labels are inconsistent.

# Comparing Clusterings





Asking “is x in cluster “red”” in the left clustering is equivalent to asking “is x in cluster “yellow”” on the right clustering but we cannot know this conversion up front unless there is a known set of conventions.

# Comparing Clusterings

Let's not limit ourselves with such a set of convention and instead ask a different question:

Are x and y clustered together in both P and C?

# Disagreement Distance

Given 2 clusterings P and C

$$D(P, C) = \sum_{x,y} \mathbb{I}_{P,C}(x, y)$$

where

$$\mathbb{I}_{P,C}(x, y) = \begin{cases} 1 & \text{if P \& C disagree on which clusters x \& y belong to} \\ 0 & \end{cases}$$

# Disagreement Distance

| | P | C |
|---|---|---|
| $x_1$ | 1 | 1 |
| $x_2$ | 1 | 2 |
| $x_3$ | 2 | 1 |
| $x_4$ | 3 | 3 |
| $x_5$ | 3 | 4 |

What is the disagreement distance between P and C?

# Disagreement Distance

| | P | C |
|---|---|---|
| $x_1$ | 1 | 1 |
| $x_2$ | 1 | 2 |
| $x_3$ | 2 | 1 |
| $x_4$ | 3 | 3 |
| $x_5$ | 3 | 4 |

| | | |
|---|---|---|
| $x_2$ | $x_1$ | 1 |
| $x_3$ | $x_1$ | 1 |
| $x_4$ | $x_1$ | 0 |
| $x_5$ | $x_1$ | 0 |
| $x_3$ | $x_2$ | 0 |
| $x_4$ | $x_2$ | 0 |
| $x_5$ | $x_2$ | 0 |
| $x_4$ | $x_3$ | 0 |
| $x_5$ | $x_3$ | 0 |
| $x_4$ | $x_5$ | 1 |

# Disagreement Distance

Is D(P, C) a distance function?

1. D(C, P) = 0 iff C = P
2. D(C, P) = D(P, C)
3. Triangle Inequality:

$$\mathbb{I}_{C_1,C_3}(x,y) \leq \mathbb{I}_{C_1,C_2}(x,y) + \mathbb{I}_{C_2,C_3}(x,y)$$

Since $\mathbf{I_{C,P}}$ can only be 0 or 1, the above can only be violated if

$\mathbf{I_{x,y}(C_1,C_3) = 1}$ **,** $\mathbf{I_{x,y}(C_1,C_2) = 0}$ **,** $\mathbf{I_{x,y}(C_2,C_3) = 0}$ is this possible?

# Aggregate Clustering

**Goal**: From a set of clusterings $\mathbf{C_1, \ldots, C_m}$ , generate a clustering $\mathbf{C^*}$ that minimizes:

$$\sum_{i=1}^{m} D(C^*, C_i)$$

The problem is equivalent to clustering categorical data

# Aggregate Clustering

| | City | Profession | Nationality |
|---|---|---|---|
| $\mathbf{x}_1$ | NY | Doctor | US |
| $\mathbf{x}_2$ | NY | Teacher | French |
| $\mathbf{x}_3$ | Boston | Lawyer | Canada |
| $\mathbf{x}_4$ | Boston | Doctor | US |
| $\mathbf{x}_5$ | LA | Lawyer | Canda |
| $\mathbf{x}_6$ | LA | Actor | French |

# Aggregate Clustering

Benefits:

1. Can identify the best number of clusters (optimization function does not make any assumptions on the number of clusters)
2. Can handle / detect outliers (points where there is no consensus)
3. Improve robustness of the clustering algorithms - combining clusterings can produce a better result
4. Privacy preserving clustering (can compute aggregate clustering without sharing the data, need only share the assignments)

# Aggregate Clustering

But… The problem is NP-Hard.

Often use approximations and heuristics to solve this problem.

What about the majority rule?

This only works **if** it produces a clustering

Possible to have a majority saying:

1. $x_1$ & $x_2$ together
2. $x_2$ & $x_3$ together
3. $x_1$ & $x_3$ separate

# Aggregate Clustering