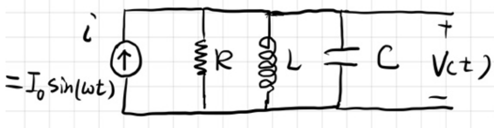
A: Write your name and 學號 on answer sheet, on problem sheet, and on formular sheet.

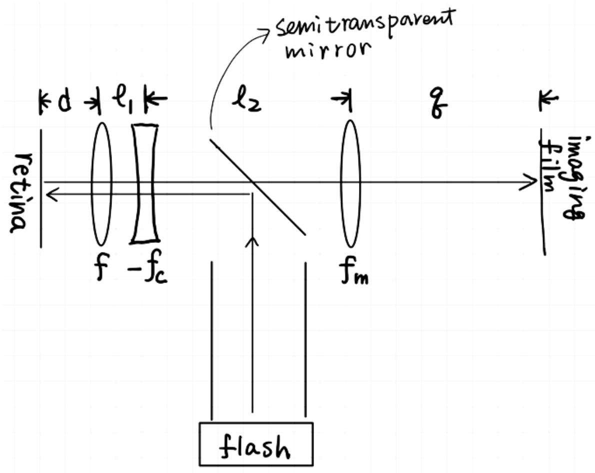
B: Above the 記分欄, draw 7 boxes, in which the TA can mark the scores for each problem.

C: Indicate, outside the vertical line of 記分欄 on the left edge of the answer sheet, the problem number .

1. Calculate, **<u>only by using the phasor diagram</u>**, the voltage $V(t)$ as a function of t (5pts).



2. The eye can be simplified as an eye lens and the retina. The focal length of the eye lens is f and the distance between the lens and the retina is d, which is the diameter of the eye. Now we try to do the **ophthalmoscopic photograph (眼底攝影)** with this simple model. The ophthalmoscopic photograph system consists of a concave(defocusing) lens of focal length $-f_c$, a semi-transparent mirror, an imaging lens of focal length f_m, an imaging film, and a flash. They are arranged as in the figure.

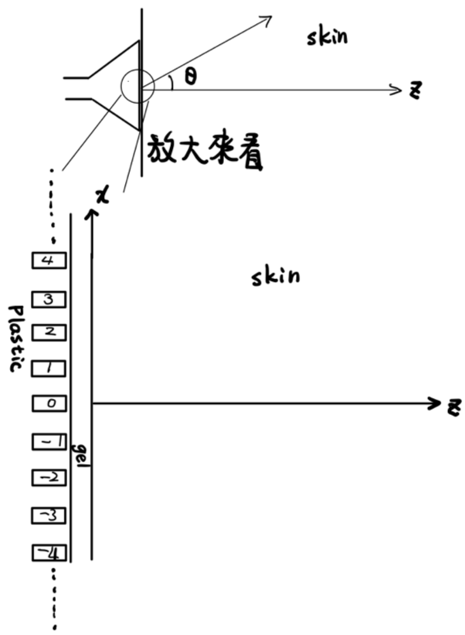


(a) If we place the system very close to the eye such that $\ell_1 \approx 0$ and want the flash to illuminate the retina relatively uniformly, what should the value of f_c be? (1 pt)

(b) (following a) What is the ABCD matrix from the retina to the imaging film (2 pts)?

(c) (following b) Where should the imaging film be placed (i.e. what is q in terms of other parameters)? What is the magnification ratio of the image of the neuron pattern on the imaging film as compared to that on the retina? (4pts)

3.The Ultrasonic sonograph transducer is made of a linear array of ultrasound generators buried in the plastic. The distance between any two neighboring ultrasound generators is d. When the transducer is attached to the skin for medical use, a layer of gel is applied on the skin to reduce the friction, and more importantly, to increase the penetration and the reception of the sound wave in and out of the skin. Sound speed in the plastic is v_p, in the gel v_g, in the skin v_s. $v_p > v_s > v_g$. The ultrasonic waves inside the plastic are of wavelength λ, and each ultrasound generator of index i transmits a wave of $\sin(\omega t + \phi_i)$

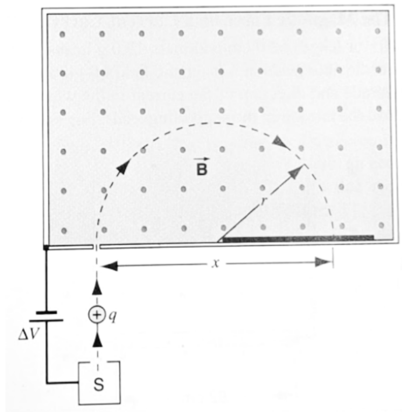


(a) If we want the constructed plane wave in the skin to propagate in the direction with an angle θ above the z-axis, what should ϕ_i be? (3pts)

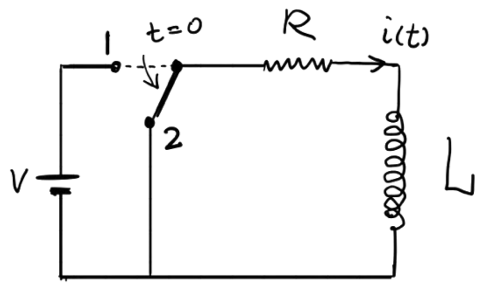
(b) If we want as much as possible the energy of the ultrasonic wave to go from the transducer into the skin, what is the optimal thickness of the gel layer? (3pts)

4. Describe the method how, by a camera outside the water, you can take a photograph of a fish in the water when very strong sun light is reflected from the surface of the water. Why does this method work? (5 pts)

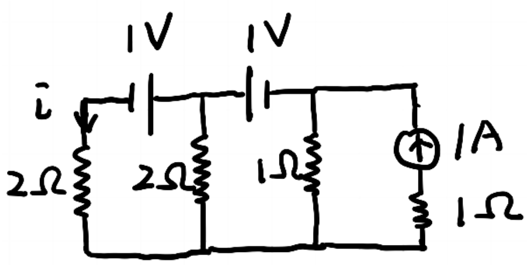
5. The figure shows an arrangement used to measure the masses of ions. An ion of mass m and charge $+q$ is produced essentially at rest in source S, a chamber in which a gas discharge is taking place. The ion is accelerated by potential difference ΔV and allowed to enter a magnetic field B. In the field it moves in a semicircle, striking a photographic plate at distance x from the entry slit. What is the mass m? (5 pts)



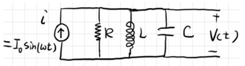
6. The switch is connected to point 1 for $t < 0$ and connected to point 2 at $t \geq 0$. Calculate the current $i(t)$ for $t > 0$. (6 pts)



7. Calculate the current i. (6 pts)

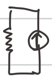


1. Calculate, **only by using the phasor diagram**, the voltage $V(t)$ as a function of t (5pts).



1. i 加起来要 $= I_0 \sin \omega t$

2. V 相同



$i = i_0 \sin \omega t$

$V_R = \underline{i_0 R} \sin \omega t$

$i = i_0 \sin \omega t$

$L\frac{di}{dt} = V_L$

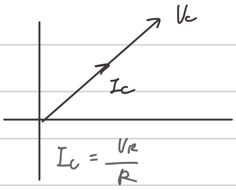
$V_L = L \cdot \omega \cdot i_0 \cos \omega t$

$= L \cdot \omega \cdot i_0 \sin(\omega t + 90^\circ)$

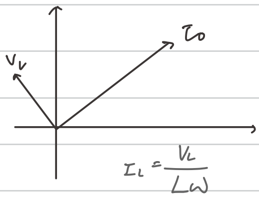
$i = i_0 \sin \omega t$

$q = \int_0^t i\,dt = -\frac{i_0}{\omega}\cos \omega t = \frac{i_0}{\omega}\sin(\omega_d t - 90^\circ)$

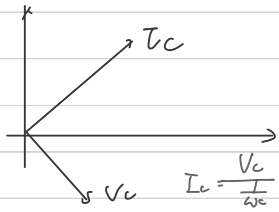
$V_c = \frac{q}{C} = \frac{i_0}{\omega C}\sin(\omega_d t - 90^\circ)$



$I_c = \frac{V_R}{R}$

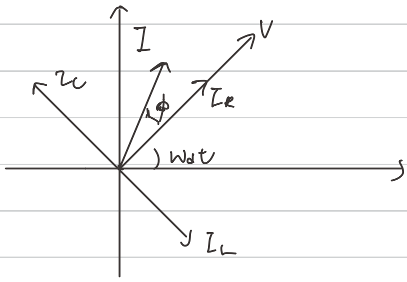


$I_L = \frac{V_L}{L\omega}$



$I_c = \frac{V_c}{\frac{1}{\omega C}}$

3. V 相同



$$I^2 = (I_C - I_L)^2 + I_R^2$$

$$= \left(\frac{V}{\frac{1}{\omega C}} - \frac{V}{\omega L}\right)^2 + \left(\frac{V_R}{R}\right)^2 = V^2 \times \left[\left(\omega C - \frac{1}{\omega L}\right)^2 + \frac{1}{R^2}\right]$$

$$V = \frac{I}{\sqrt{\left(\omega C - \frac{1}{\omega L}\right)^2 + \frac{1}{R^2}}} = \varepsilon$$

$$\tan\phi = \frac{I_C - I_L}{I_R} = \frac{\omega C - \frac{1}{\omega L}}{\frac{1}{R}} = R\left(\omega C - \frac{1}{L\omega}\right)$$

$$V = V_0 \sin(\omega_d t + \phi) = \frac{I_0}{\sqrt{\left(\omega C - \frac{1}{\omega L}\right) + \frac{1}{R^2}}} \sin\left\{ \omega_d t + \tan^{-1}\left[R\left(\omega C - \frac{1}{\omega L}\right)\right]\right\}$$

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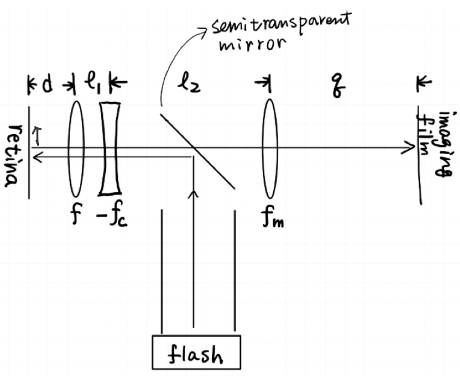

(a) If we place the system very close to the eye such that $\ell_1 \approx 0$ and want the flash to illuminate the retina relatively uniformly, what should the value of f_c be? (1 pt)

(b) (following a) What is the ABCD matrix from the retina to the imaging film (2 pts)?

(c) (following b) Where should the imaging film be placed (i.e. what is q in terms of other parameters)? What is the magnification ratio of the image of the neuron pattern on the imaging film as compared to that on the retina? (4pts)

ⓐ 讓系統變成無焦 (將光源視為一整片的平行光用相同焦距的凹透鏡抵銷水晶體的作用 ⇒ 維持平行)

$$\begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ \frac{1}{f_c} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad -\frac{1}{f}+\frac{1}{f_c}=0 \qquad f_c = f$$

ⓑ
$$\begin{bmatrix} 1 & q \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -\frac{1}{f_m} & 1 \end{bmatrix}\begin{bmatrix} 1 & \ell_2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ \frac{1}{f_c} & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix}\begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & q \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -\frac{1}{f_m} & 1 \end{bmatrix}\begin{bmatrix} 1 & \ell_2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & q \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -\frac{1}{f_m} & 1 \end{bmatrix}\begin{bmatrix} 1 & \ell_2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & q \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -\frac{1}{f_m} & 1 \end{bmatrix}\begin{bmatrix} 1 & d+\ell_2 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & q \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & d+\ell_2 \\ -\frac{1}{f_m} & \frac{d+\ell_2}{-f_m}+1 \end{bmatrix} = \begin{bmatrix} 1-\frac{q}{f_m} & d+\ell_2+\left(\frac{-(d+\ell_2)}{f_m}\right)q+q \\ -\frac{1}{f_m} & \frac{d+\ell_2}{-f_m}+1 \end{bmatrix}$$

ⓒ
$$\begin{bmatrix} 1-\frac{q}{f_m} & d+\ell_2+q\left(1-\frac{d+\ell_2}{f_m}\right) \\ -\frac{1}{f_m} & \frac{d+\ell_2}{-f_m}+1 \end{bmatrix}\begin{bmatrix} r_1 \\ 0 \end{bmatrix} = \begin{bmatrix} r_2 \\ r_2' \end{bmatrix} = \begin{bmatrix} \left(1-\frac{q}{f_m}\right)r_1 \\ \frac{r_1}{-f_m} \end{bmatrix}$$

$\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ → 成像 $B=0$

$$\begin{bmatrix} 1-\frac{q}{f_m} & d+\ell_2+q\left(1-\frac{d+\ell_2}{f_m}\right) \\ -\frac{1}{f_m} & \frac{d+\ell_2}{-f_m}+1 \end{bmatrix}\begin{bmatrix} r_1 \\ \frac{-r_1}{d} \end{bmatrix} = \begin{bmatrix} r_2 \\ r_2'' \end{bmatrix} = \begin{bmatrix} \left(1-\frac{q}{f_m}\right)r_1 + \\ \frac{r_1}{-f_m} + \end{bmatrix}$$

$$\frac{-r_1}{d}\left(d+\ell_2+q\left(1-\frac{d+\ell_2}{f_m}\right)\right)=0 \Rightarrow d+\ell_2+q-\frac{(d+\ell_2)q}{f_m}=0 \qquad d+\ell_2 = \frac{d+\ell_2\, q}{f_m}-q$$

設 $\frac{r_1}{d} \neq 0$ ⇒ 兩條光線

$$f_m(d+\ell_2) = d+\ell_2 \cdot q - q\cdot f_m$$
$$f_m(d+\ell_2) = q(d+\ell_2-f_m) \qquad q = \frac{(d+\ell_2)f_m}{d+\ell_2-f_m}$$

放大率: $$\frac{r_2}{r_1} = 1-\frac{q}{f_m} = 1-\frac{\frac{(d+\ell_2)\cancel{f_m}}{d+\ell_2-f_m}}{\cancel{f_m}} = 1-\left(\frac{d+\ell_2}{d+\ell_2-f_m}\right) = \frac{-f_m}{d+\ell_2-f_m}$$

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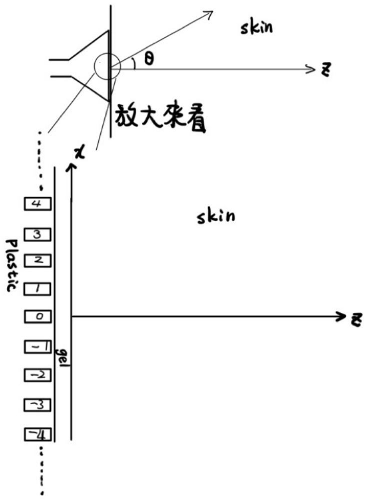
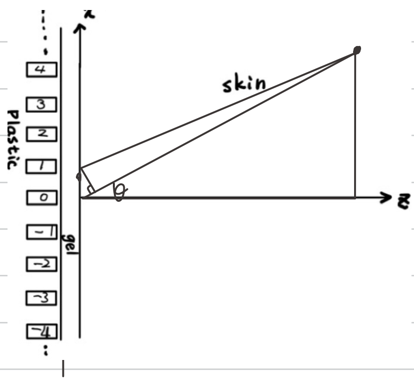


$f\lambda = v_p$ $\quad \frac{v_p}{\lambda} = f$ $\quad v_s = f \cdot \lambda_s = \frac{v_p}{\lambda} \cdot \lambda_s$ $\quad \frac{v_s}{v_p} \lambda$

(a) If we want the constructed plane wave in the skin to propagate in the direction with an angle θ above the z-axis, what should ϕ_i be? (3pts)

(b) If we want as much as possible the energy of the ultrasonic wave to go from the transducer into the skin, what is the optimal thickness of the gel layer? (3pts)



相差 $\frac{d\sin\theta}{\lambda_s} \times 2\pi$ 個相位 ⇒ 相鄰2個.

$n \times \frac{d\sin\theta}{\lambda_s} \times 2\pi$ 相距n個之間的相位差

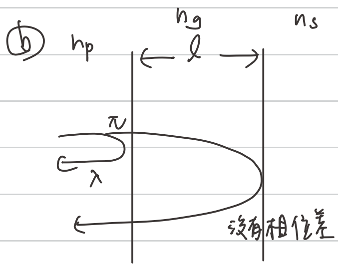
建設性干涉 $\frac{i d\sin\theta}{\lambda_s} \cdot 2\pi + (\phi_0 - \phi_i) = 2\pi \cdot n$

let $\phi_0 = 0$

$\phi_i = \frac{n d\sin\theta}{\lambda_s} 2\pi - 2\pi n$ 對相位沒有影響

$= 2\pi n d\sin\theta / \lambda_s$ $\quad$ λ_s 要用 $\frac{v_s}{v_p}\lambda$ 代入

第 i 個偵測器和第 0 個偵測器 相差的相位



減少反射. 在 np & ng 之間的反射 有 π 的相位差

⇒ 讓 ng & ns 之間的反射 和 np & ng 之間的反射 destructive 干涉

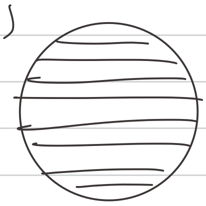
$2 \times l = n\lambda$ ⇒ 破壞性干涉, $\quad l = \frac{n\lambda}{2}$ #

$(n+\frac{1}{2})\lambda$ ⇒ 建設性

4. Describe the method how, by a camera outside the water, you can take a photograph of a fish in the water when very strong sun light is reflected from the surface of the water. Why does this method work? (5 pts)

利用偏振片來抵銷掉反射光的能量.

1. 在偏振片上鍍有平行條狀，高分子或是金屬導體.
(如下圖)

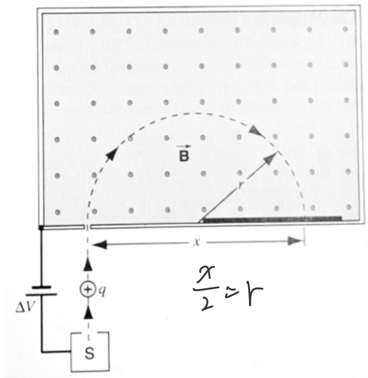


當光線平行於金屬導體時. 電磁波的電場會讓金屬條上的負電荷一起來回. 消耗能量
最終使得能量消耗光. 沒有光通過
而當電磁波方向垂直金屬條時，負電荷無法上下移動. 因此不受影響

2. 當電磁波打到材料時，電場振動方向平行入射面時. 會深入材料產生折射，反射效率較差產生偏振
. 而折射角度可能會使電場振動方向和視覺平行. 從而產生完全偏振. 而此時的入射角即為 Brewster's Angle

布魯斯特角: Brewster's Angle: 當入射角在特定角度時. 反射光會是單一方向的完全偏振光

5. The figure shows an arrangement used to measure the masses of ions. An ion of mass m and charge $+q$ is produced essentially at rest in source S, a chamber in which a gas discharge is taking place. The ion is accelerated by potential difference ΔV and allowed to enter a magnetic field B. In the field it moves in a semicircle, striking a photographic plate at distance x from the entry slit. What is the mass m? (5 pts)

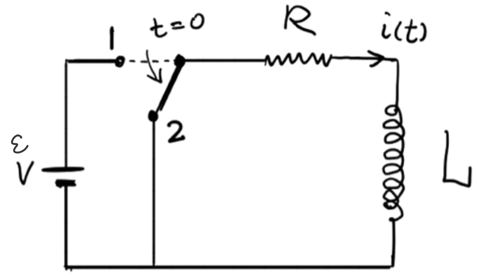


$q \cdot \Delta V = \frac{1}{2} m v^2$ $\qquad$ $v = \sqrt{\frac{2q\Delta V}{m}}$

$q \cancel{v} B = \frac{m v^{\cancel{2}}}{R}$ $\qquad$ $q \cdot B = m \times \sqrt{\frac{2q\Delta V}{m}} \times \frac{2}{x}$

$q^2 B^2 = m^{\cancel{2}} \cdot \frac{2q\Delta V}{\cancel{m}} \cdot \frac{4}{x^2}$ $\qquad$ $m = \frac{q^2 B^2 \cdot x^2}{8 \cancel{q} \Delta V} = \frac{q B^2 x^2}{8 \Delta V}$

6. The switch is connected to point 1 for $t < 0$ and connected to point 2 at $t \geq 0$. Calculate the current $i(t)$ for $t > 0$. (6 pts)



當 $t=0$ 時 $i=\frac{V}{R}$

switch, point 2 $t>0$: $L\frac{di}{dt}+iR=0$

$$\frac{L}{R}\frac{di}{dt}=-i$$

$$\frac{di}{i}=\frac{-R}{L}dt$$

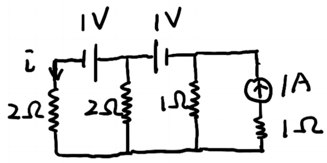
$$\int_{\frac{V}{R}}^{i(t)}\frac{1}{i}di=\int_0^t -\frac{R}{L}dt$$

$$\frac{\ln(i)}{\ln(\frac{V}{R})}=-\frac{R}{L}t$$

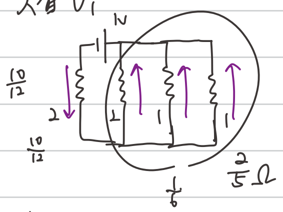
$$i\cdot\frac{R}{V}=e^{-\frac{R}{L}t}$$

$$i(t)=\frac{V}{R}e^{-\frac{R}{L}t}$$

7. Calculate the current i. (6 pts)



只看 V_1



$\frac{1}{2}+\frac{1}{1}+\frac{1}{1}=\frac{5}{2}$

$\frac{2}{5}+2=\frac{12}{5}$

Ⓐ: $\frac{5}{12}$

$i_1=\frac{5}{12}$ ↓

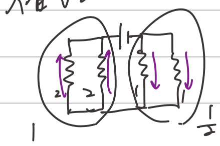
$i_2=\frac{1}{12}$ ↑

$i_3=\frac{2}{12}$ ↑

$i_4=\frac{2}{12}$ ↑

$i_1=\frac{5}{12}-\frac{1}{3}+\frac{1}{4}=\frac{5-4+3}{12}=\frac{1}{12}$?

只看 V_2



Ⓐ $\frac{2}{3}$

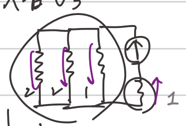
$i_1=\frac{1}{3}$ ↑

$i_2=\frac{1}{3}$ ↑

$i_3=\frac{1}{3}$ ↓

$i_4=\frac{1}{3}$ ↓

只看 V_3



$\frac{1}{2}+1=$

Ⓐ 1 Ⓞ $=\frac{2}{3}$

Ⓥ $=\frac{3}{2}$

$i_1=\frac{1}{4}$ ↓

$i_2=\frac{1}{4}$ ↓

$i_3=\frac{1}{2}$ ↓

$i_4=1$ ↑