Modeling Exchange Rates using the GARCH Model

John Dukich Kyung Yong Kim Huan-Hsun Lin

December 6, 2010

Abstract

In this paper we evaluate the performance of the generalized autoregressive conditional heteroskedastic (GARCH) model for modeling daily changes in logarithmic exchange rates (LPRs). In particular, we consider the LPR of three exchange rate sequences; the British pound / U.S. dollar; the Japanese yen / U.S. dollar; and the Euro / U.S. dollar. For each LPR sequence we fit three GARCH models, with varying numbers of parameters, and attempt to replicate the empirical LPR sequence via simulation. Assessing the fit of each model, we conclude that the family of GARCH models does not adequately reflect the empirical nature of the LPR sequences.

1 Introduction

Generalized autoregressive conditional heteroskedastic (GARCH) models have received ample attention in recent years, especially with regard to financial applications. This class of models, introduced by Bollerslev (1986), has been used to forecast fluctuations in commodities, securities and exchange rates. The aim in this paper is to assess the empirical adequacy of this class of models. To accomplish this, we consider three exchange rate sequences and evaluate how well the GARCH model replicates the empirical nature of these sequences.

Specifically, we consider the British pound / U.S. dollar (GBP/USD) exchange rate; the Japanese yen / U.S. dollar (JPY/USD) exchange rate; and the Euro / U.S. dollar (EUR/USD) exchange rate.[1] In particular, we observe over 2700 trading days between January 4, 1999 and January 4, 2010. The first trading day of 1999 (January 4th) was choosen as the first observation due to it being the first day in which the Euro was traded on currency markets. For each sequence, we consider changes in the daily logarthmic exchange rates. That is, if X_t is the exchange rate at time t, we transform the sequence of exchange rates as follows:

$$Y_t = \ln(\frac{X_t}{X_{t-1}}) = \ln(X_t) - \ln(X_{t-1}),$$

where Y_t is known as the log price relative (LPR) at time t.

The GARCH model is frequently used to model changes in the variance of the LPR of some financial instrument, and as such, it is for that purpose for which we will model the LPR of the daily exchange rate sequences. Moreover, generalized autoregressive conditional heteroskedastic models, are appropriate for modeling time series in which there is nonconstant variance, and in which the variance at one time period is dependent on the variance at a previous time period. Thus the GARCH model is appropriate for modeling time series that exhibit a heavy-tailed distribution, and furthermore, that display some degree of serial correleation among the observations. The GARCH model can be expressed as:

$$\sigma_t^2 = \omega + \beta(L)\sigma_{t-k}^2 + \alpha(L)\eta_t^2,$$

where σ_t^2 is the conditional variance, $\beta(L)$ and $\alpha(L)$ are polynomials of the lag operators, and $\eta_t^2 = Y_t - \mu$ is known as the innovation (Campbell, Lo, and MacKinlay 1997, p. 483). Note that the variance at time t is a function of the variance at time $t-i$ and the innovation at time t.

[1] Data were obtained from http://www.federalreserve.gov/releases/h10/hist/

Before assessing the fit of the GARCH model to each LPR sequenace, we must first assess the assumptions underlying the GARCH model. Namely, we must verify that each exchange rate sequence is in fact heavy-tailed, and does indeed exhibit serial correlation.

2 Empirical LPR Sequences

It is well-known that the distribution of the LPR, for a variety financial assets, securities and exchange rates, is heavy-tailed. Moreover, such sequences often exhibit serial correlation. However, before fitting GARCH models for the various exchange rate sequences, we must first assess the assumptions underlying the GARCH model. For each sequence we assess characteristics of the distribution as well as characteristics of the correlation among observations. The GARCH model requires that the time series under consideration must exhibit serial correlation as well as long-term dependence. As such, it is these attributes of the exchange rate sequences we will investigate. Recall that, for a sample of size N, the autocorrelation function (ACF) can be used to test for serial correlation, and is defined as:

$$\hat{\rho_k} = \frac{\sum_{t=k+1}^{N}(Y_t - \bar{Y})(Y_{t-k} - \bar{Y})}{\sum_{t=1}^{N}(Y_t - \bar{Y})^2}, \qquad 0 \leq k \leq N-1,$$

where k is the lag-time of the sample autocorrelation and $\bar{Y}$ is the sample average (Tsay 2005, p. 26).

To assess long-term dependence, we use the partial autocorrelation function (PACF). The PACF is the autocorrelation between an observations at time t and $t-k$, excluding all intermediate observations. Thus, the PACF removes autocorrelations at lag-times smaller than k. Specifically, we assess the PACF for the square of the LPR sequences. The GARCH model assumes that there is long-term dependence among the observations.

2.1 GBP/USD Exchange Rate

Fig 1 displays the time series and histogram of the LPR of the British pound / U.S. dollar (GBP/USD) exchange rate. Note that the time series exhibits nonconstant variance. This feature is especially noticable near observation 2500 – the sudden increase in variability corresponds to the financial crisis, beginning in 2008. Also note that the histogram, although slightly skewed left ($\gamma = -0.315$), exhibits heavy tails, with a kurtosis of $\kappa = 6.143$.

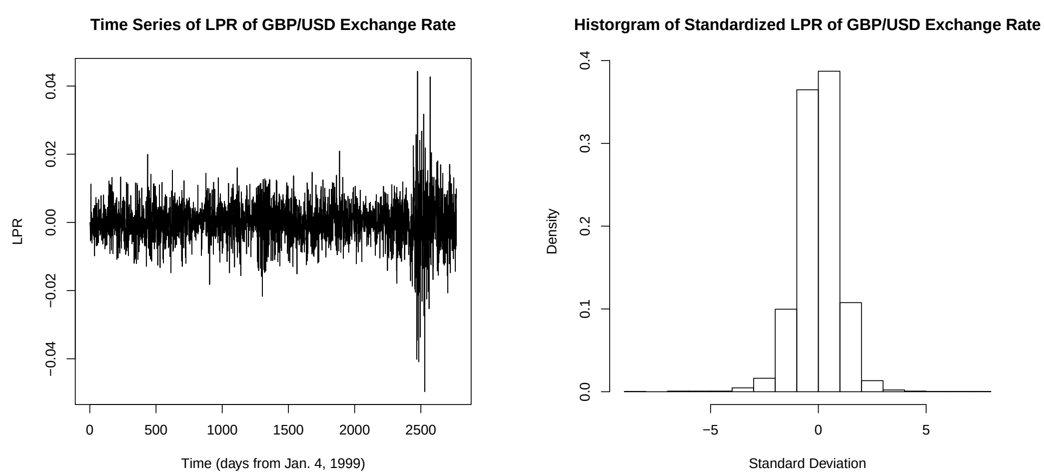

Figure 1: Time Series of LPR and Histogram of Standardized LPR of GBP/USD Exchange Rate

Moreover, the below plot of the autocorrelation function (ACF) provides evidence of serial correlation among the observations. Furthermore, the plot of the partial autocorrelation function (PACF) indicates long-term dependence among the observations. These characteristics of the GBP/USD exchange rate sequence, in addition to the heavy-tailed nature of the LPR distribution, suggest that the GARCH model is perhaps appropriate for fitting these data.

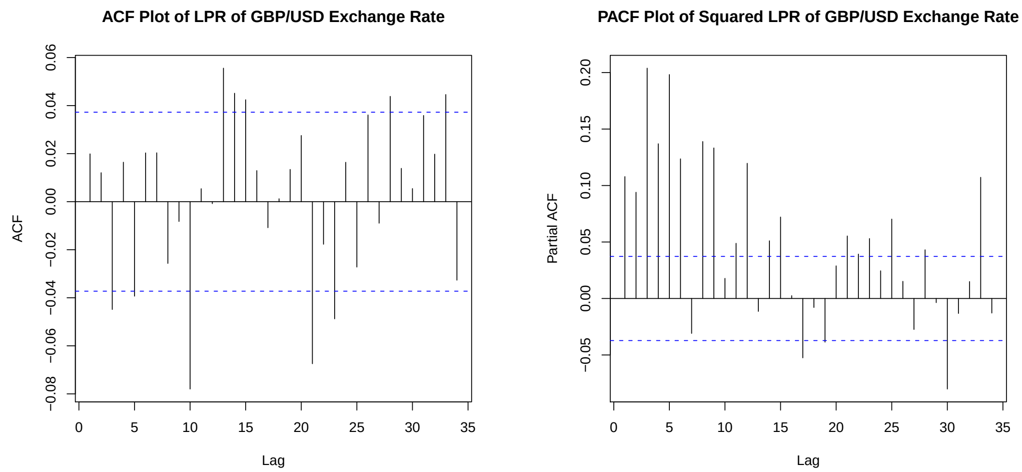

Figure 2: ACF Plot of LPR and PACF Plot of Squared LPR of GBP/USD Exchange Rate

2.2 JPY/USD Exchange Rate

The time series and histogram of the LPR of the Japanese yen / U.S. dollar (JPY/USD) exchange rate are displayed as Fig 3. Note the wide swings in variability, especially around the onset of the recent financial crisis. This type of behavior is similar to what we see in the GBP/USD exchange rate. Although skewed slightly to the left ($\gamma = -0.50$), the distribution is heavy-tailed, with a kurtosis value, $\kappa = 3.81$.

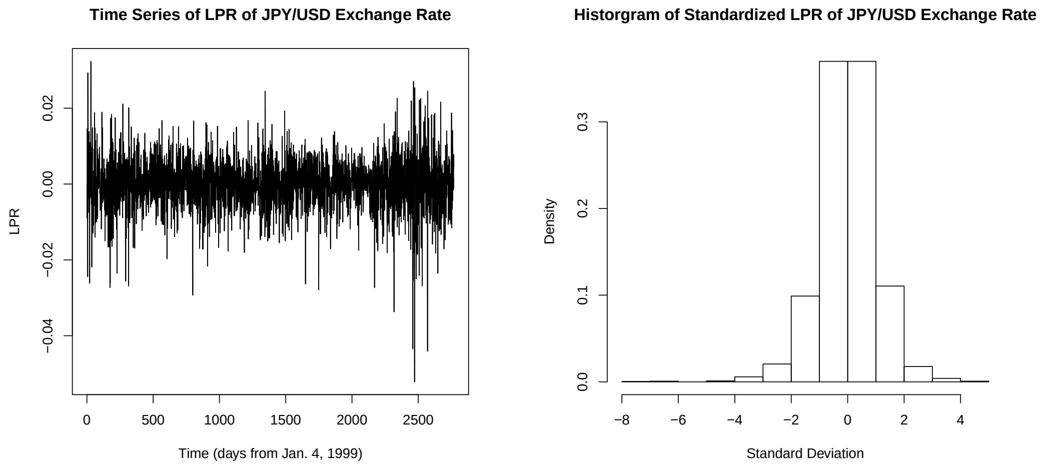

Figure 3: Time Series of LPR and Histogram of Standardized LPR of JPY/USD Exchange Rate

Also note that, according to the ACF plot, there appears to be serial correlation among the LPR values. Moreover, the PACF plot indicates long-term dependence. Thus, like the GBP/USD exchange rate, the JPY/USD exchange rate also satisfies the assumptions underlying the GARCH model.

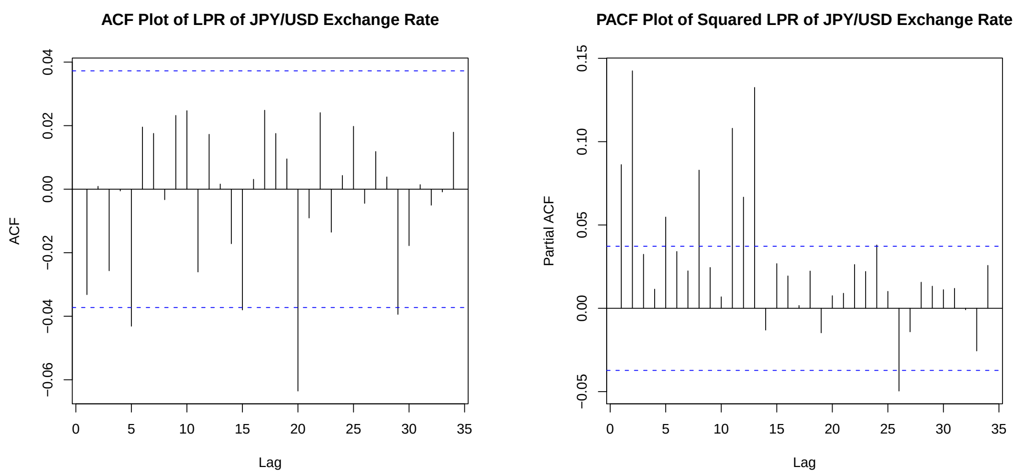

Figure 4: ACF Plot of LPR and PACF Plot of Squared LPR of JPY/USD Exchange Rate

2.3 EUR/USD Exchange Rate

Finally, consider the LPR of the Euro / U.S. dollar (EUR/USD) exchange rate. The time series and histogram are displayed as Fig 5. Again, note the sudden increase in variability associated with the recent financial crisis. The distribution of the LPR of the EUR/USD exchange rate is more symmetric ($\gamma = 0.17$) than the other two exchange rate series considered here, but like the other exchange rate series, is characterized by heavy tails ($\kappa = 2.51$).

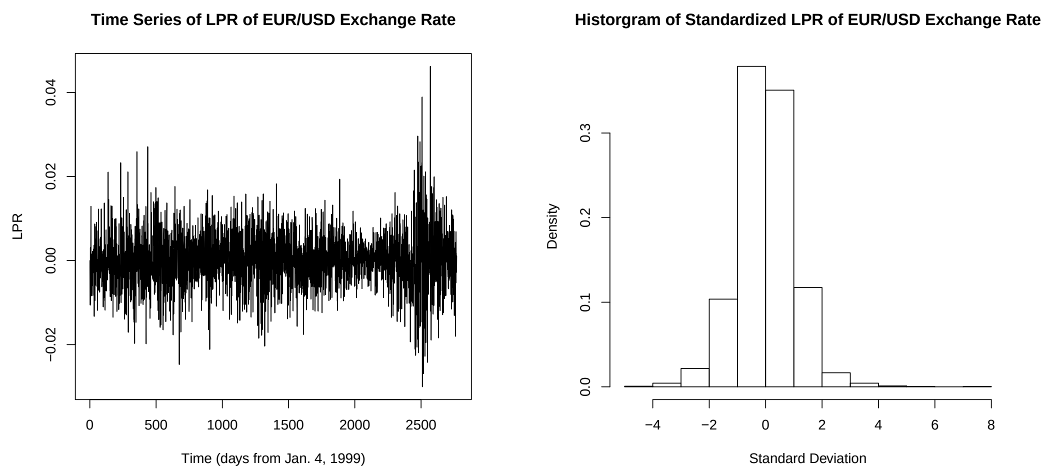

Figure 5: Time Series of LPR and Histogram of Standardized LPR of EUR/USD Exchange Rate

As we have just discussed, the LPR sequences of the various exchange rates satisfy the assumptions underlying the GARCH model. In the following section we attempt to fit various GARCH models to the different LPR sequences. In doing so, the aim is to replicate the empirical LPR sequences with the GARCH model, via simulation.

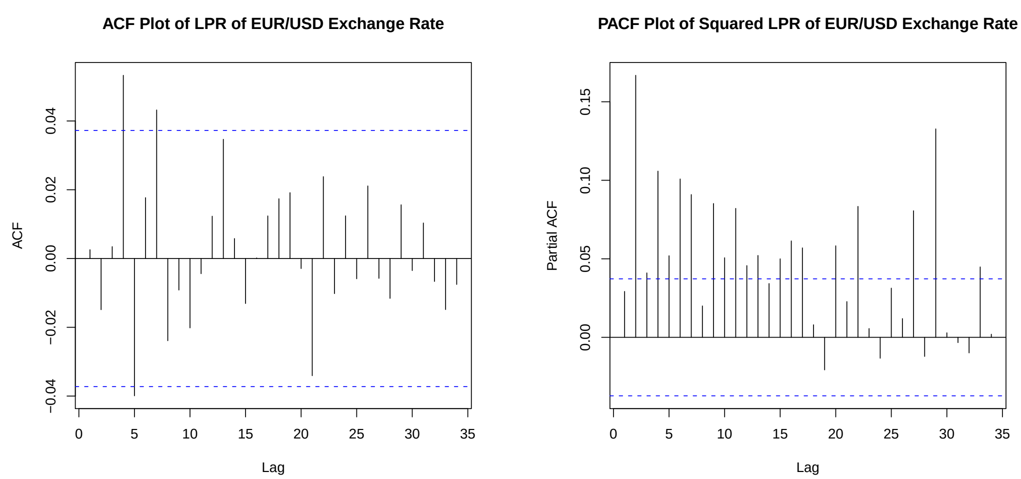

Figure 6: ACF Plot of LPR and PACF Plot of Squared LPR of EUR/USD Exchange Rate

3 The GARCH Model

In this section, we fit GARCH models to the various LPR sequences. Recall that the GARCH model can be expressed as:

$$\sigma_t^2 = \omega + \beta(L)\sigma_{t-k}^2 + \alpha(L)\eta_t^2,$$

where σ_t^2 is the conditional variance, $\beta(L)$ and $\alpha(L)$ are polynomials of the lag operators, and $\eta_t^2 = Y_t - \mu$ is known as the innovation (Campbell, Lo, and MacKinlay 1997, p. 483). Note that a GARCH model with lag parameters β_p and α_q are often referred to as GARCH(p, q). Alternatively, the GARCH model can be written as (Tsay 2005, p. 114):

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \eta_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2.$$

For example, a GARCH(1, 1) model can be expressed as:

$$\sigma_t^2 = \omega + \alpha_1 \eta_0 + \beta_1 \sigma_{t-1}^2.$$

Here, we fit three GARCH models to each LPR sequence: GARCH(1,1), GARCH(1,2) and GARCH(2,1). The GARCH(1,1) model was selected due to its widespread application in finance. We also consider the more complicated GARCH models, GARCH(1,2) and GARCH(2,1), as extensions of the GARCH(1,1) model. In each case, we obtain estimates of the model parameters, and use them to simulate a data set from the specified GARCH model. For each GARCH model, we assess how well the model replicates the empirical LPR sequence.

The parameters of the GARCH model are estimated by fitting a GARCH model (using the R function, garch(), within the tseries and fseries packages), with the specified number of parameters, to each empirical LPR sequence. The parameter estimates are then used to simulate a GARCH sequence (using the garch.sim function in the TSA package). The adequacy of the simulated GARCH sequences are then assessed with respect to the associated empirical LPR sequence. Moreover, we perform other diagnostic checks of the model, such as assessing both the serial correlation and independence among residuals. In particular, we consider the Ljung-Box test, which tests for independence of residuals. The Ljung-Box test statistic is defined as:

$$Q = n(n+2) \sum_{k=1}^{h} \frac{\hat{\rho}_k^2}{N-k},$$

where N is the sample size, $\hat{\rho}_k$ is the sample correlation at lag k, and h is the number of lags being tested. For significance level α, the critical region for rejection of the hypothesis of independence is:

$$Q > \chi^2_{1-\alpha,h}$$

where $\chi^2_{1-\alpha,h}$ is the α-quantile of the chi-square distribution with h degrees of freedom. Note that the Ljung-Box test is applied to the residuals of a fitted GARCH model, not the original data. For each LPR series, only those models for which the test statistic is significant will be plotted.

3.1 GBP/USD Exchange Rate

For the GBP/USD exchange rate, we fit a GARCH(1,1), GARCH(2,1), and GARCH(1,2) model, obtaining the following:

1. GARCH(1,1): $\sigma_t^2 = 0.0000002887 + 0.04493a_{t-1}^2 + 0.9467\sigma_{t-1}^2$

2. GARCH(2,1): $\sigma_t^2 = 0.0000003117 + 0.02778a_{t-1}^2 + 0.0185a_{t-2}^2 + 0.9445\sigma_{t-1}^2$

3. GARCH(1,2): $\sigma_t^2 = 0.0000014621 + 0.08345a_{t-1}^2 + 0.1208\sigma_{t-1}^2 + 0.7449\sigma_{t-2}^2$

We first assess the residuals of the above models before determining which model most closely reflects the empirical nature of the LPR sequence. In assessing the residuals, we consider ACF plots of the residuals and squared residuals. We also consider normal probability plots of the residuals and assess each model with respect to the Ljung-Box statistic.

3.1.1 ACF Plots of the Residuals and Squared Residuals of the Three GARCH Models

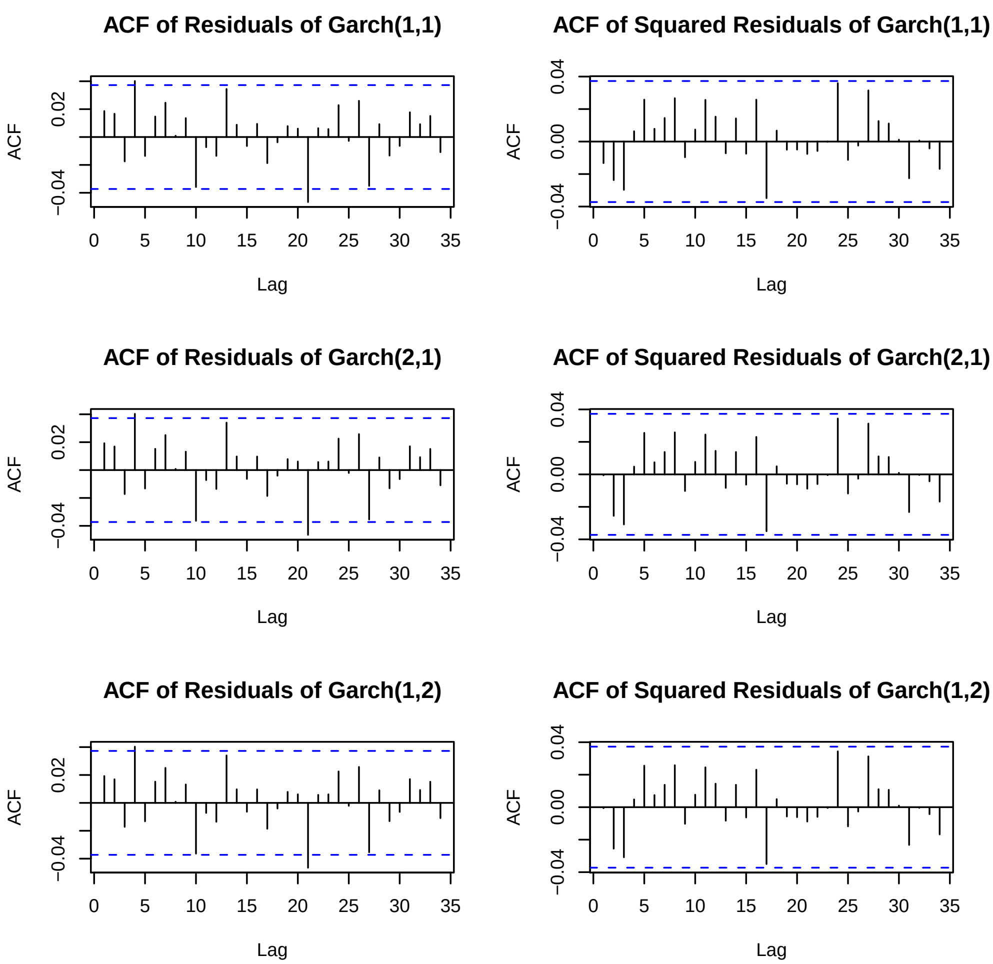

Figure 7: ACF Plots of Residuals and Squared Residuals of Three GARCH Models

The ACF plots of the residuals indicate that, with the exception of two values (at lag times 4 and 21), all autocorrelation values are within two standard deviations of the sample autocorrelation. Since majority of the residuals are within two standard deviations of the sample autocorrelation, we adopt the assumption of independent errors. However, we also consider the distribution of the residuals and assess each sequence with respect to the Ljung-Box statistic.

3.1.2 Checking Normality of the Errors

We use the normal probability plot and histogram to check the normality assumption of the errors. It is known that if the errors follow a normal distribution, the residuals also follow an approximately normal distribution.

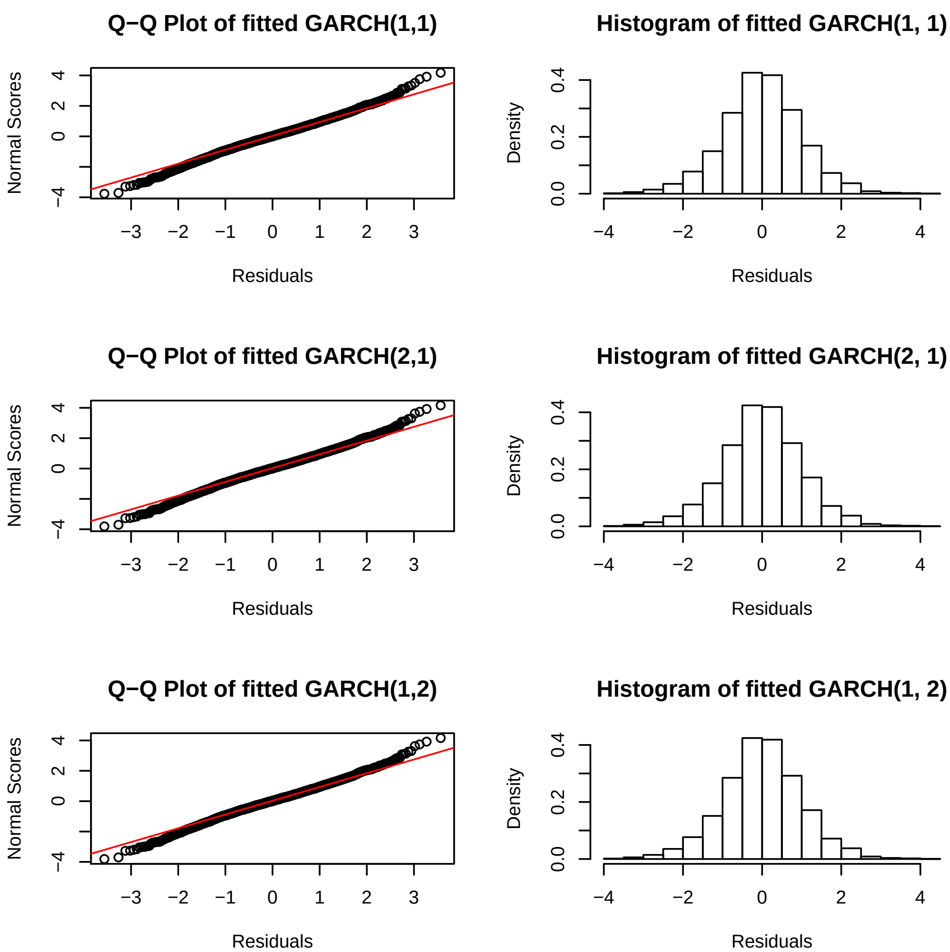

Figure 8: Normal Probability Plots and Histograms for the Fitted GARCH Models

Model	Skewness	Kurtosis
Data	-0.315	6.137
Garch (1,1)	-0.063	0.602
Garch (2,1)	-0.060	0.601
Garch (1,2)	-0.067	0.750

Table 1: Skewness and Kurtosis of the Three Models for GBP/USD

According to the normal probability plots and histograms of the fitted GARCH models, the distribution of the errors of the three different GARCH models are clearly normal. Also note that the skewness and kurtosis values of the residuals indicate that the associated distributions are reasonably symmetric with tails that are not too much heavier than the normal distribution. This indicates that all three GARCH models for the LPR of the GBP/USD exchange rate all reasonably satisfy the normality assumption of the errors.

3.1.3 Checking Independence of the Errors

Next, we assess independence of the residuals using the Ljung-Box statistic. Recall that the null hypothesis of this test is that the residuals are independent. Below are the results for the three different GARCH models.

Model	p-value	α	Test Results
GARCH(1,1)	0.4852	0.05	H_0
GARCH(2,1)	0.9746	0.05	H_0
GARCH(1,2)	0.0199	0.05	H_1

Table 2: Results of the Ljung-Box test

The p-values of GARCH(1,1) and GARCH(1,2) models are greater than any reasonable level of significance (α) which indicates that the null hypothesis cannot be rejected. However, the p-value of GARCH(2,1) model is 0.0199, which is very small compared to the other two models. Thus we can reject the null hypothesis at the significance level $\alpha = 0.05$. We conclude that only GARCH(1,1) and GARCH(1,2) models for the LPR of the GBP/USD exchange rate satisfy the model assumption that the errors are independent.

3.1.4 Plots of Simulated and Original Observations

Although only the GARCH(1,1) and GARCH(2,1) models satisfy the model assumptions that the errors are normally distributed and independent, for purposes of comparison, we simulate data from all three GARCH models and evaluate the simulated data with respect to the empirical LPR sequences.

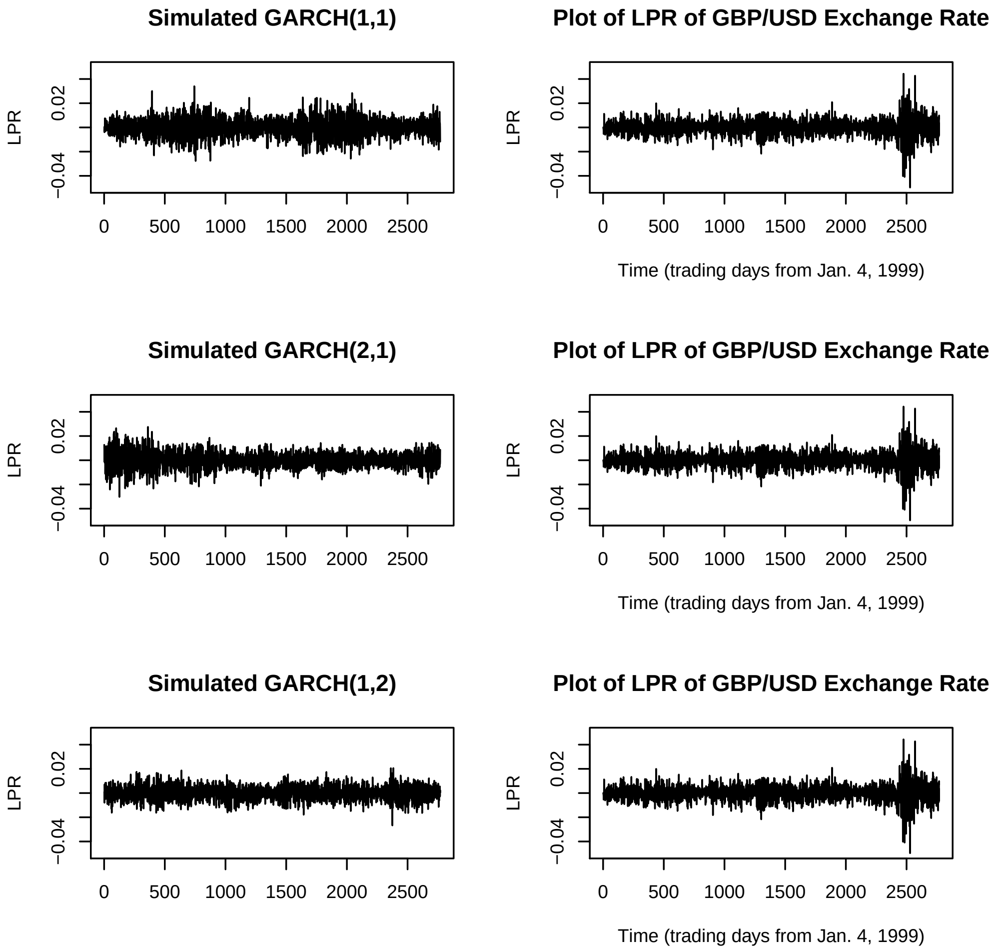

Figure 9: Plots of Simulated GARCH and Empirical GBP/USD LPR Sequences

Although none of the GARCH models considered here fit the empirical data particularly well, the GARCH(1,2) model appears, visually, to best reflect the empirical LPR sequence of the GBP/USD exchange rate. Interestingly, even though the Ljung-Box statistic indicates that the residuals of the GARCH(1,2) are dependent, the GARCH(1,2) model nonetheless most closely replicates the empirical LPR sequence. This suggests that perhaps the GARCH model is not an approriate model of the GBP/USD LPR sequence; the GARCH model does not capture certain salient features of the empirical data.

3.2 JPY/USD Exchange Rate

For the JPY/USD exchange rate, we fit a GARCH(1,1), GARCH(2,1), and GARCH(1,2) model, and obtain the following:

1. GARCH(1,1): $\sigma_t^2 = 0.0000005587 + 0.03542a_{t-1}^2 + 0.9525\sigma_{t-1}^2$
2. GARCH(2,1): $\sigma_t^2 = 0.0000005730 + 0.03092a_{t-1}^2 + 0.0054a_{t-2}^2 + 0.9513\sigma_{t-1}^2$
3. GARCH(1,2): $\sigma_t^2 = 0.0000005821 + 0.03601a_{t-1}^2 + 0.9513\sigma_{t-1}^2 + 0.00000004\sigma_{t-2}^2$

Next, we assess the residuals of each model, and in turn assess the empirical adequacy of the associated GARCH models.

3.2.1 ACF Plots of the Residuals and Squared Residuals of the Three GARCH Models

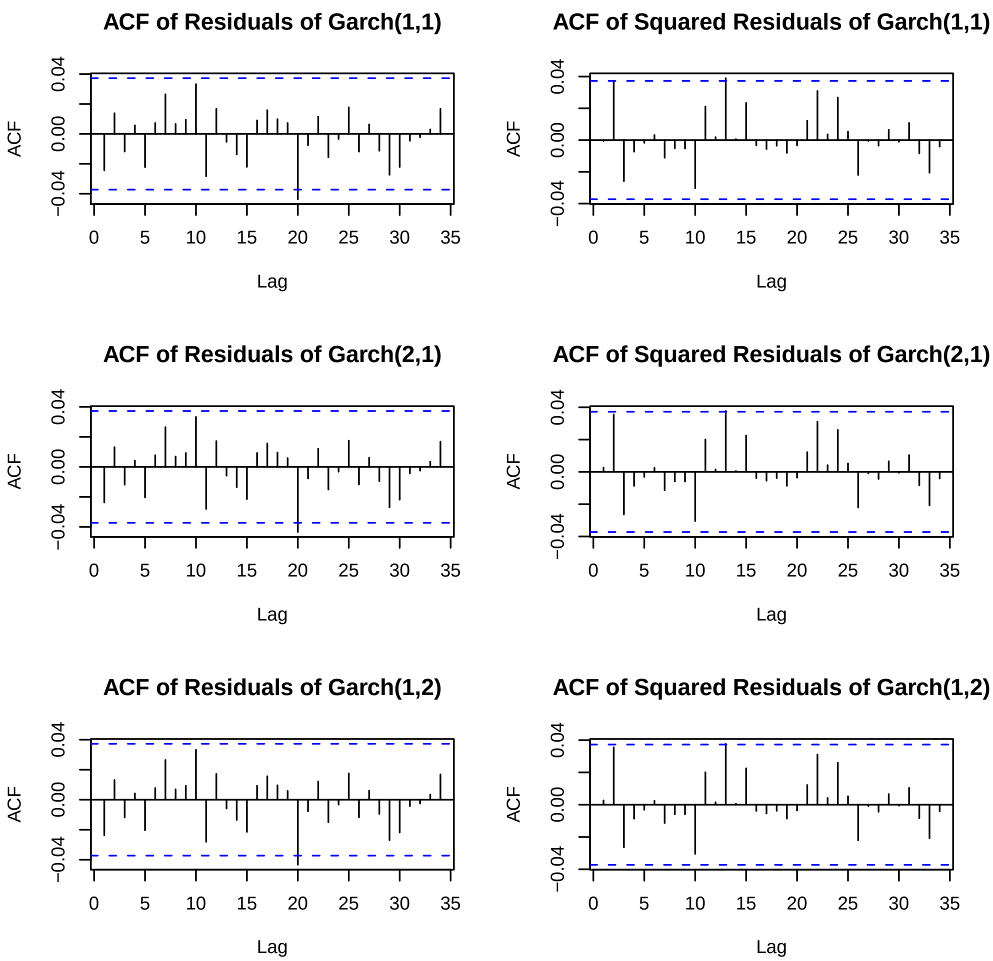

Figure 10: ACF Plots of Residuals and Squared Residuals of Three GARCH Models

ACF plots of the residuals and squared residuals do not provide evidence of serial correlation among the observations. Yet, we must also assess the distribution of residuals and the independence of the residuals.

3.2.2 Checking Normality of the Errors

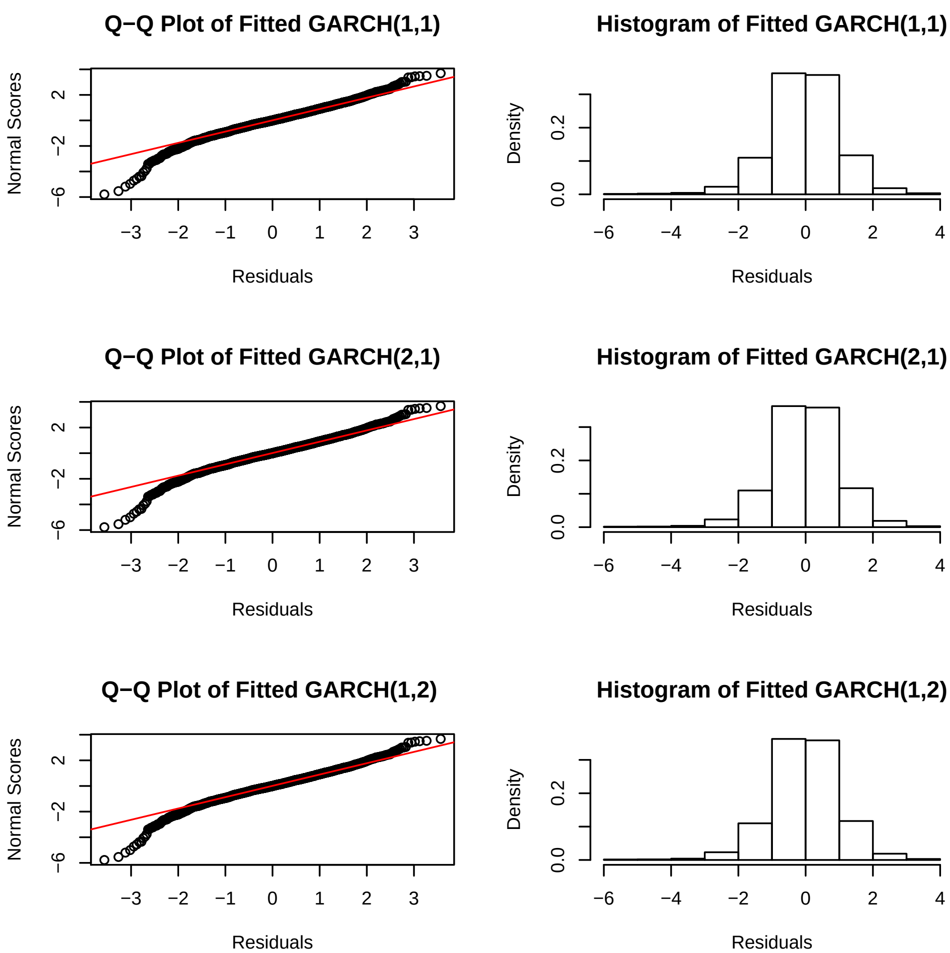

Figure 11: Normal Probability Plots and Histograms for the Fitted GARCH Models

Model	Skewness	Kurtosis
Data	-0.496	3.814
Garch (1,1)	-0.435	2.113
Garch (2,1)	-0.433	2.114
Garch (1,2)	-0.435	2.113

Table 3: Skewness and Kurtosis of the Three Models for JPY/USD

Normal probability plots and histograms of the fitted GARCH models for the LPR of the JPY/USD exchange rate indicate that the distribution of the errors of the three different GARCH models are normal.Closer examination of the skewness and kurtosis of each distribution indicates that each distribution is reasonably symmetric, and perhaps slightly skewed left, with tails heavier than those found in the normal distribution.

Finally, we assess the independence of the residuals.

3.2.3 Checking Independence of the Errors

Model	p-value	α	Test Results
GARCH(1,1)	0.9762	0.05	H_0
GARCH(2,1)	0.89	0.05	H_0
GARCH(1,2)	0.9285	0.05	H_0

Table 4: Results of the Ljung-Box test

The p-values of GARCH(1,1), GARCH(1,2), and GARCH(2,1) models for the LPR of the JPY/USD exchange rate are greater than any reasonable level of significance (α) which indicates that the null hypothesis of independence between errors cannot be rejected. Thus, we can conclude that all three GARCH models satisfy the model assumption that the errors are independent.

3.2.4 Plots of Simulated and Original Observations

We compare the empirical LPR sequence with the simulated data from the GARCH(1,1), GARCH(1,2), and GARCH(2,1) models.

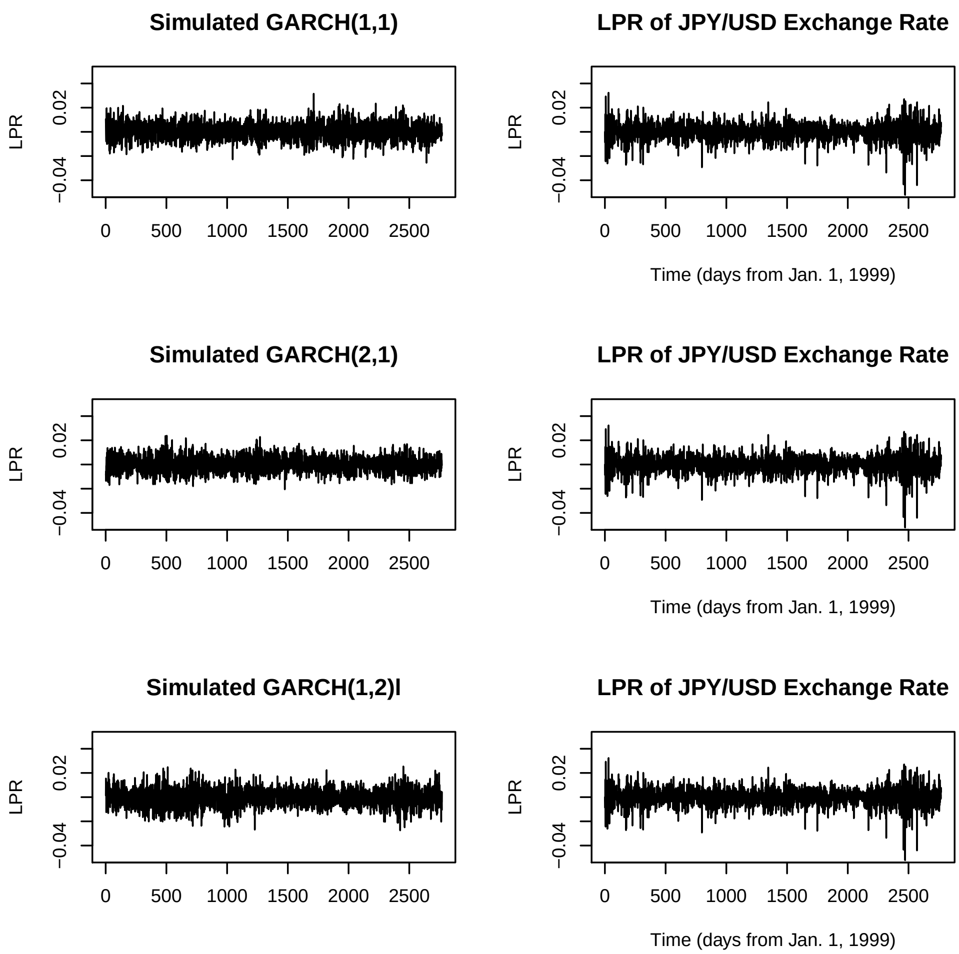

Figure 12: Plots of Simulated GARCH and Empirical JPY/USD LPR Sequences

Like the LPR of the GBP/USD exchange rate, none of the models considered here fit the empirical LPR sequence particularly well. However, visually, of the three GARCH models considered here, the GARCH(1,2) model appears to most closely reflect the empirical nature of the LPR sequence. The simulated data suggest, however, that the GARCH family of models is not appropriate for modeling the LPR of the JPY/USD exchange rate. Noticably, none of the models are able to replicate the sudden increase in variability associated with the financial crisis. A crisis of that magnitude cannot be ignored, and must be reflected in any adequate econometric model

3.3 EUR/USD Exchange Rate

For the EUR/USD exchange rate, we fit a GARCH(1,1), GARCH(2,1), and GARCH(1,2) model, and obtain the following:

1. GARCH(1,1): $\sigma_t^2 = 0.0000001132 + 0.02897a_{t-1}^2 + 0.9687\sigma_{t-1}^2$
2. GARCH(2,1): $\sigma_t^2 = 0.0000005335 + 0.0104a_{t-2}^2 + 0.8959\sigma_{t-1}^2$
3. GARCH(1,2): $\sigma_t^2 = 0.0000040853 + 0.20797a_{t-1}^2 + 0.7360\sigma_{t-1}^2$

We now turn to assessing the above models.

3.3.1 ACF Plots of the Residuals and Squared Residuals of the Three GARCH Models

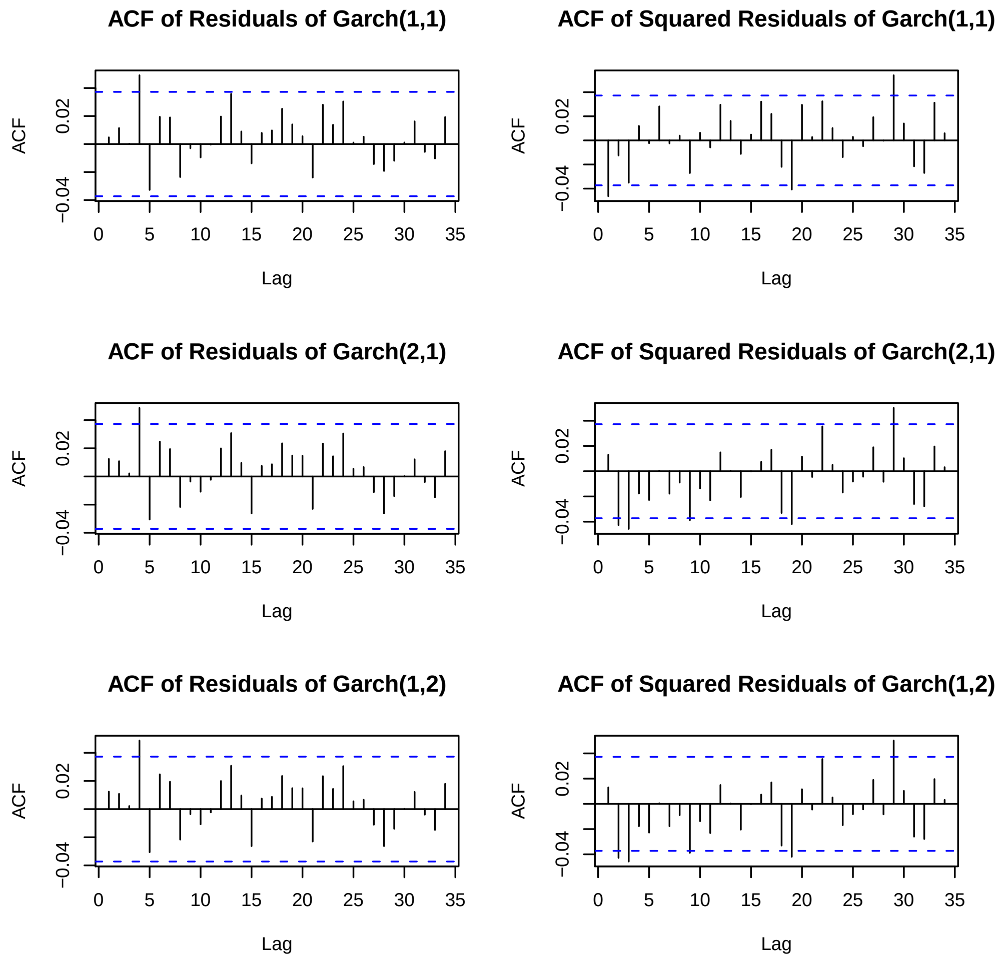

Figure 13: ACF Plots of Residuals and Squared Residuals of Three GARCH Models

The above ACF plots of the residuals and squared residuals indicate that there is some short- and long-term serial correlation among the residuals for each of the GARCH models. However, since the majority of the lag times fall within two standard deviations of the sample autocorrelation, we assume independence of errors.

3.3.2 Checking Normality of the Errors

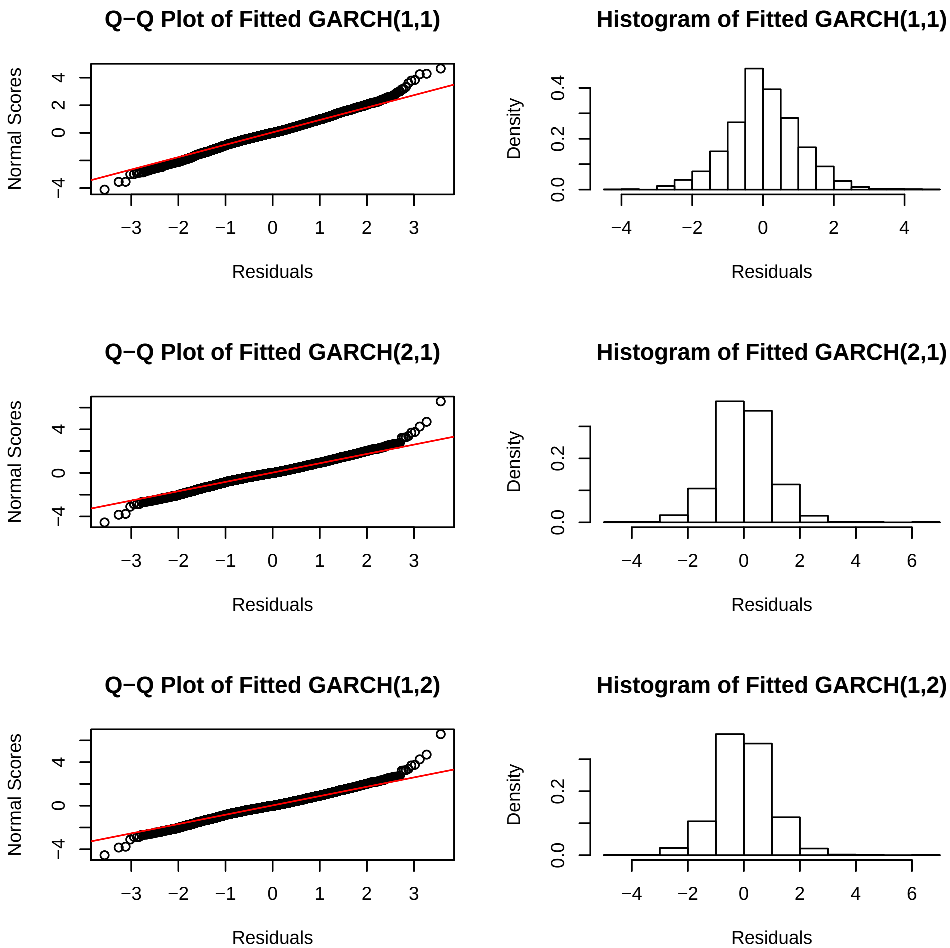

Figure 14: Normal Probability Plots and Histograms for the Fitted GARCH Models

Model	Skewness	Kurtosis
Data	0.172	2.514
Garch (1,1)	0.055	0.742
Garch (2,1)	0.128	1.524
Garch (1,2)	0.170	1.568

Table 5: Skewness and Kurtosis of the Three Models for EUR/USD

The normal probability plots and histograms of the fitted GARCH models for the LPR of the EUR/USD exchange rate show that the distribution of the errors of the three different GARCH models follow a normal distribution. However, more careful examination of the skewness and kurtosis of the distributions of residuals suggest that the residuals of the GARCH(1,1) model most closely follow are normal distribution.

GARCH(2,1) and GARCH(1,2) models appear reasonably symmetric with heavier tails than those found in both the normal distribution and distribution of residuals of the GARCH(1,1) model.

3.3.3 Checking Independence of the Errors

Model	p-value	α	Test Results
GARCH(1,1)	0.01472	0.05	H_1
GARCH(2,1)	0.4921	0.05	H_0
GARCH(1,2)	$2.097x10^{-5}$	0.05	H_1

Table 6: Results of the Ljung-Box test

The p-value of GARCH(2,1) model are greater than any reasonable level of significance (α) which indicates that the null hypothesis of independence between the errors cannot be rejected. However, since p-values of the GARCH(1,1) and GARCH(1,2) models are less than significance level 0.05, we can reject the null hypothesis. Hence we can conclude that only the GARCH(2,1) model satisfies the model assumption that the errors are independent. For purposes of comparison, however, we perform and plot results from simulations of all three GARCH models.

3.3.4 Plots of Simulated and Original Observations

Among GARCH(1,1), GARCH(2,1), and GARCH(1,2) models, the only model that satisfies the model assumptions is the GARCH(2,1) model. However, for purposes of comparison, we display simulation results for all three models.

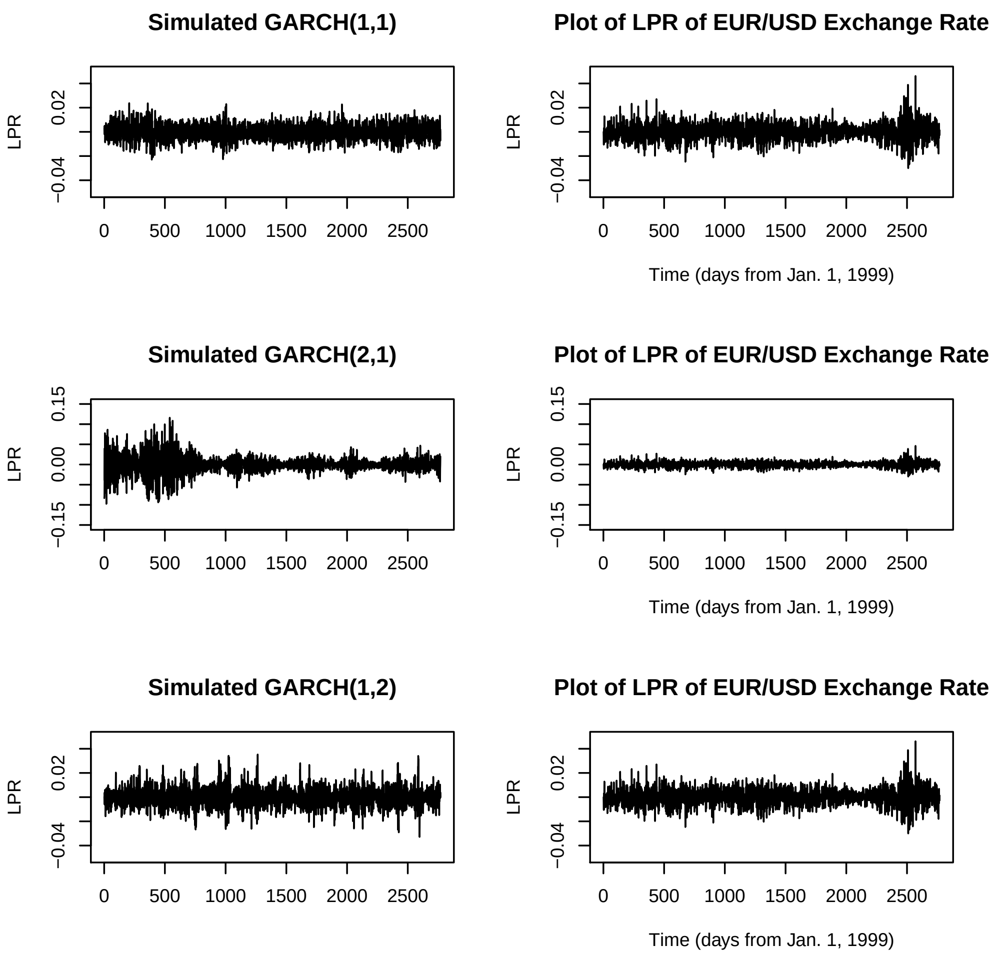

Figure 15: Plots of Simulated GARCH and Empirical EUR/USD LPR Sequences

In this case, none of the GARCH models considered here adequately reflect the empirical nature of the LPR sequence. Noticably, the GARCH(1,2) model performs especially poorly, as there is excessive variability at the start of the simulated sequence. Although the variability settles down after (about) 750 observations, it continues to exceed that observed in the empirical LPR sequence. Of the other two models, the GARCH(1,1) model appears to most closely reflect the empirical LPR sequence. The GARCH(2,1) model is too punctuated by changes in variability to adequately resemble the empirical LPR sequence.

4 Conclusion

We considered daily differences in logarithmic prices (LPR) for three exchange rate series – the British pound / U.S. dollar (GBP/USD), the Japanese yen / U.S. dollar (JPY/USD) and the Euro / U.S. dollar (EUR/USD) – and attempted to fit each series with various generalized autoregressive heteroskedastic (GARCH) models. In particular, to each sequence, we fit a GARCH(1,1), GARCH(1,2) and GARCH(2,1) model, and assessed the adequacy of each model. Tests of model adequacy were performed by simulating each GARCH model and comparing it to the corresponding empirical LPR sequence.

None of the GARCH models considered in our analysis captured the empirical nature of the LPR series particularly well; each model failed to adequately reproduce the sudden shift in variability associated with the recent financial crisis (near observation 2500 in the empirical sequences). Moreover, histograms of the residuals show mixed results. Residuals for each GARCH model of the GBP/USD and EUR/USD sequences are approximately normally distributed, whereas residuals for the JPY/USD sequence appear to be slightly heavier-tailed or skewed left. Thus, while each LPR sequence satisfies the assumptions underlying the GARCH model,the GARCH model does not appear to faithfully reflect the empirical nature of those sequences.

Although the GARCH model may not adequately capture the empirical nature of the LPR of the exchange rates considered above, there are other varieties of the GARCH models (such as the IGARCH, FIGARCH or EGARCH) available for data-fitting. Perhaps one of these models more faithfully represents the empirical LPR sequences. Finding a more empirically adequate model is important, as a multitude of financial decisions are made on the basis of such models. Perhaps additional work is required for understanding what gives rise to market volatility, from a psychological perspective, and incorpoate these considerations into a data-driven model of market volatility. Moreover, these considerations may lead to a revision of the statistical assumptions underlying models of financial markets.

References

[1] Bollerslev, Tim (1986). "Generalized autoregressive conditional heteroskedasticity", *Journal of Econometrics*, 31, 3, 307-327.

[2] Campbell, John Y., Andrew W. Lo and A. Craig MacKinlay (1997). *The Econometrics of Financial Markets*. Princeton, New Jersey: Princeton University Press.

[3] Ruey, Tsay S. (2005). *Analysis of Financial Time Series.*Hoboken, New Jersey: John Wiley and Sons, Inc.

5 Appendix: R Code

The below R code was used to obtain results for the GBP/USD exchange rate. With some modification, similar results can be obtained for the JPY/USD and EUR/USD exchange rates.

```
#Loading Required Packages
.libPaths( c(.libPaths(), "/group/statsoft/Rlibs64"))
library("fSeries")
library(tseries)
library(nlme)
library(TSA)
library(e1071)
library(locfit)
library(akima)

#Attach Data
gbp.usd <- read.table("gbp_usd.txt", header = T)
attach(gbp.usd)

#Time Series Plot of LPR against Time
plot(ts(lpr), ylab = "LPR", xlab = "Time (days from Jan. 1, 1999)")

#ACF of LPR and PACF of Squared LPR
par(mfrow = c(2, 1))
acf(lpr, main = "ACF Plot of LPR of GBP/USD Exchange Rate")
pacf(lpr^2, main = "PACF Plot of Squared LPR of GBP/USD Exchange Rate")


#Histogram of Standaridized LPR of GBP/USD Exchange Rate
std.lpr <- rep(NA,length(lpr))
for(i in 1:length(lpr)){
  std.lpr[i] <- (lpr[i] - mean(lpr))/sd(lpr)
  i = i + 1
}
hist(std.lpr, main = "Historgram of Standardized LPR",
xlab = "Standard Deviation", prob=TRUE)

#Fitting GARCH(1,1), GARCH(2,1) and GARCH(1,2) Model
##GARCH(1,1)
gbp.usd.garch <- garch(lpr, order = c(1, 1))
summary(gbp.usd.garch)
gbp.usd.garch.resid <- na.omit(gbp.usd.garch$residuals)

##GARCH(2,1)
gbp.usd.garch2 <- garch(lpr, order = c(1, 2))
summary(gbp.usd.garch2)
gbp.usd.garch2.resid <- na.omit(gbp.usd.garch2$residuals)

##GARCH(1,2)
gbp.usd.garch3 <- garch(lpr, order = c(2, 1))
summary(gbp.usd.garch3)
```

```
gbp.usd.garch3.resid <- na.omit(gbp.usd.garch3$residuals)

#Model Checking
## 1. ACF Plot of Residuals and Squared Residuals of the Three Models
par(mfrow = c(3, 2))
###GARCH(1,1)
acf(gbp.usd.garch.resid, main ="ACF Plot of Residuals")
acf(gbp.usd.garch.resid^2, main ="ACF Plot of Squared Residuals")

###GARCH(2,1)
acf(gbp.usd.garch2.resid, main = "ACF Plot of Residuals")
acf(gbp.usd.garch2.resid^2,
    main = "ACF Plot of Squared Residuals")

###GARCH(1,2)
acf(gbp.usd.garch3.resid, main = "ACF Plot of Residuals")
acf(gbp.usd.garch3.resid^2,
    main = "ACF Plot of Squared Residuals")

## 2. Skewness and Kurtosis of the Original Data and all Three Models
###Original Data
skewness(lpr)
kurtosis(lpr)

###GARCH(1,1)
skewness(gbp.usd.garch.resid)
kurtosis(gbp.usd.garch.resid)

###GARCH(2,1)
skewness(gbp.usd.garch2.resid)
kurtosis(gbp.usd.garch2.resid)

###GARCH(1,2)
skewness(gbp.usd.garch3.resid)
kurtosis(gbp.usd.garch3.resid)

## 3. Normal Probability Plot and Histogram of Residuals
par(mfrow = c(3, 2))
###GARCH(1,1)
qqnorm(gbp.usd.garch.resid, xlab = "Residuals", ylab = "Normal Scores",
main = "Normal Probability Plot of the Residuals")
qqline(gbp.usd.garch.resid)
hist(gbp.usd.garch.resid, prob=TRUE, main = "Histogram")

###GARCH(2,1)
qqnorm(gbp.usd.garch2.resid, xlab = "Residuals", ylab = "Normal Scores",
main = "Normal Probability Plot of the Residuals")
qqline(gbp.usd.garch2.resid)
hist(gbp.usd.garch2.resid, prob=TRUE, main = "Histogram")

###GARCH(1,2)
```

```
qqnorm(gbp.usd.garch3.resid, xlab = "Residuals", ylab = "Normal Scores",
main = "Normal Probability Plot of the Residuals")
qqline(gbp.usd.garch3.resid)
hist(gbp.usd.garch3.resid, prob=TRUE, main = "Histogram")

#Simuation of GARCH(1,1), GARCH(2,1) and GARCH(1,2) Models
##GARCH(1,1)
set.seed(1)
a0<-2.877e-07
a1<-4.493e-02
b1<-9.467e-01
n <- 2769
gbp.usd.garch.sim <- garch.sim(alpha = c(a0, a1), beta = b1, n,
ntrans = 100,  rnd = rnorm)

##GARCH(2,1)
a0<-3.117e-07
a1<-2.778e-02
a2<-1.85e-02
b1<-9.445e-01
gbp.usd.garch2.sim <- garch.sim(alpha = c(a0, a1, a2), beta = b1, n,
ntrans = 100, rnd = rnorm)

##GARCH(1,2)
a0<-1.462e-06
a1<-8.345e-02
b1<-1.208e-01
b2<-7.449e-01
gbp.usd.garch3.sim <- garch.sim(alpha = c(a0, a1), beta =  c(b1, b2), n,
ntrans = 100, rnd = rnorm)

#Plots of Simulations, Original Observations
par(mfrow = c(3, 2))
##GARCH(1,1)
plot(ts(gbp.usd.garch.sim),xlab=" ",ylab="LPR", main="GARCH(1,1)",
ylim = c(-0.05,0.05))
plot(ts(lpr), xlab = "Time (days from Jan. 4, 1999)", ylab="LPR",
main = "Plot of Exchange Rate",ylim=c(-0.05,0.05))

##GARCH(2,1)
plot(ts(gbp.usd.garch2.sim),xlab=" ",ylab="LPR", main="GARCH(2,1)",
ylim = c(-0.05,0.05))
plot(ts(lpr), xlab = "Time (days from Jan. 4, 1999)", ylab="LPR",
main = "Plot of Exchange Rate",ylim=c(-0.05,0.05))

##GARCH(1,2)
plot(ts(gbp.usd.garch3.sim),xlab=" ",ylab="LPR", main="GARCH(1,2)",
ylim = c(-0.05,0.05))
plot(ts(lpr), xlab = "Time (days from Jan. 4, 1999)", ylab="LPR",
main = "Plot of Exchange Rate",ylim=c(-0.05,0.05))
```