

14.1 (1a, 3, 7), 14.2 (3, 13, 14)

14.1.1a Show that if a field K is generated over F by the elements $\alpha_1, \dots, \alpha_n$ then an automorphism σ of K fixing F is uniquely determined by $\sigma(\alpha_1), \dots, \sigma(\alpha_n)$. In particular show that an automorphism fixes K if and only if it fixes a set of generators for K .

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14.1.3 Determine the fixed field of complex conjugation on \mathbb{C} .

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14.1.7 This exercise determines $\text{Aut}(\mathbb{R}/\mathbb{Q})$.

- (a) Prove that any $\sigma \in \text{Aut}(\mathbb{R}/\mathbb{Q})$ takes squares to squares and takes positive reals to positive reals. Conclude that $a < b$ implies $\sigma a < \sigma b$ for every $a, b \in \mathbb{R}$.
- (b) Prove that $-\frac{1}{m}a - b < \frac{1}{m}$ implies $-\frac{1}{m}\sigma a - \sigma b < \frac{1}{m}$ for every positive integer m . Conclude that σ is a continuous map on \mathbb{R} .
- (c) Prove that any continuous map on \mathbb{R} which is the identity on \mathbb{Q} is the identity map, hence $\text{Aut}(\mathbb{R}/\mathbb{Q}) = 1$.

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14.2.3 Determine the Galois group of $(x^2 - 2)(x^2 - 3)(x^2 - 5)$. Determine *all* the subfields of the splitting field of this polynomial.

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14.2.13 Prove that if the Galois group of the splitting field of a cubic over \mathbb{Q} is the cyclic group of order 3 then all the roots of the cubic are real.

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14.2.14 Show that $\mathbb{Q}(\sqrt{2 + \sqrt{2}})$ is a cyclic quartic field, i.e., is a Galois group of degree 4 with cyclic Galois group.

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