Math 172
Assignment 2
Tuesday, January 30, 2016

7.4 (10, 19) 7.5 (R3) 8.2 (2, 3) 9.1 (2, 4)

7.4.10 Assume R is commutative. Prove that if P is a prime ideal of R and P contains no zero divisors, then R is an integral domain.

■

7.4.19 Let R be a finite commutative ring with identity. Prove that every prime ideal of R is a maximal ideal.

$\blacksquare$

READ ONLY 7.5.3 Let F be a field. Prove that F contains a unique smallest subfield F_0 and that F_0 is isomorphic to either $\mathbb{Q}$ or $\mathbb{Z}/p\mathbb{Z}$ for some prime p (F_0 is called the *prime subfield* of F). [See Exercise 26, Section 3.]

8.2.2 Prove that any two nonzero elements of a PID have a least common multiple (cf. Exercise 11, Section 1).

∎

8.2.3 Prove that a quotient of a PID by a prime ideal is again a PID.

∎

9.1.3 If R is a commutative ring and $x_1, x_2, \ldots, x_n$ are independent variables over R, prove that $R[x_{\pi(1)}, x_{\pi(2)}, \ldots, x_{\pi(n)}]$ is isomorphic to $R[x_1, x_2, \ldots, x_n]$ for any permutation π of $\{1, 2, \ldots, n\}$.

$\blacksquare$

9.1.4 Prove that the ideals (x) and (x, y) are prime ideals in $\mathbb{Q}[x, y]$ but only the latter is a maximal ideal.

■