Math 172
Assignment 6
Tuesday, February 20, 2018

13.4.4, Problems B, C, D

13.4.4 Determine the splitting field and its degree over $\mathbb{Q}$ for $x^6 - 4$.

∎

B. Let p be a prime number and F the field of integers mod p, and let $p(x)$ and $q(x)$ be any two irreducible polynomials of degree 2 over F. Show that the fields $F[x]/(p(x))$ and $F[x]/(q(x))$ are isomorphic by constructing an explicit isomorphism.

$\blacksquare$

C. Find a real number u such that $Q(\sqrt{3}, \sqrt{5}) = Q(u)$.

$\blacksquare$

D. Suppose K is an extension of F, and $\phi : K \to K$ is an isomorphism that leaves every element of F fixed. Show that any polynomial in $F[x]$ that has a root r in K also has $\phi(r)$ as a root.

■