Math 172
Assignment 3
Tuesday, February 6, 2018

9.1 (6, 7) 9.2 (R1, 2, 3, 4) 9.3 (3)

9.1.6 Prove that (x, y) is not a principal ideal in $\mathbb{Q}[x, y]$.

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9.1.7 Let R be a commutative ring with 1. Prove that a polynomial ring in more than one variable over R is not a Principal Ideal Domain.

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READ ONLY 9.2.1 Let $f(x) \in F[x]$ be a polynomial of degree $n \geq 1$ and let bars denote passage to the quotient $F[x]/(f(x))$. Prove that for each $\overline{g(x)}$ there is a unique polynomial $g_0(x)$ of degree $\leq n-1$ such that $\overline{g_(x)} = \overline{g_0(x)}$ (equivalently, the elements $\overline{1}, \overline{x}, \ldots, \overline{x^{n-1}}$ are a *basis* of the vector space $F[x]/(f(x))$ over F – in particular, the dimension of this space is n). [Use the Division Algorithm.]

9.2.2 Let F be a finite field of order q and let $f(x)$ be a polynomial in $F[x]$ of degree $n \geq 1$. Prove that $F[x]/(f(x))$ has q^n elements (use previous exercise).

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9.2.3 Let $f(x)$ be a polynomial in $F[x]$. Prove that $F[x]/(f(x))$ is a field if and only if $f(x)$ is irreducible. [Use Proposition 7, Section 8.2.]

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9.3.3 Let F be a field. Prove that the set R of polynomials in $F[x]$ whose coefficient of x is equal to 0 is a subring of $F[x]$ and that R is not a UFD. [Show that $x^6 = (x^2)^3 = (x^3)^2$ gives two disinct factorizations of x^6 into irreducibles.]

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