Math 172
Assignment 2
Tuesday, January 30, 2018

7.4 (10, 19) 7.5 (R3) 8.2 (2, 3) 9.1 (2, 4)

7.4.10 Assume R is commutative. Prove that if P is a prime ideal of R and P contains no zero divisors, then R is an integral domain.

■

7.4.19 Let R be a finite commutative ring with identity. Prove that every prime ideal of R is a maximal ideal.

$\blacksquare$

READ ONLY 7.5.3 Let F be a field. Prove that F contains a unique smallest subfield F_0 and that F_0 is isomorphic to either $\mathbb{Q}$ or $\mathbb{Z}/p\mathbb{Z}$ for some prime p (F_0 is called the *prime subfield* of F). [See Exercise 26, Section 3.]

8.2.2 Prove that any two nonzero elements of a PID have a least common multiple (cf. Exercise 11, Section 1).

■

8.2.3 Prove that a quotient of a PID by a prime ideal is again a PID.

■

9.1.2 *Problem 9.1.2 says to repeat problem 9.1.1 under different assumptions. The following is problem 9.1.1 under those assumptions - just be aware that this is not the actual 9.1.2 problem statement.*

Let $p(x,y,z) = 2x^2y - 3xy^3z + 4y^2z^5$ and $q(x,y,z) = 7x^2 + 5x^2y^3z^4 - 3x^2z^3$ be polynomials in $\mathbb{Z}/3\mathbb{Z}[x,y,z]$.

(a) Write each of p and q as a polynomial in x with coefficients in $\mathbb{Z}[y,z]$.

(b) Find the degree of each p and q.

(c) Find the degree of p and q in each of the three variables x, y, and z.

(d) Compute pq and find the degree of pq in each of the three variables x, y, and z.

(e) Write pq as a polynomial in the variable z with coefficients in $\mathbb{Z}[x,y]$.

■

9.1.4 Prove that the ideals (x) and (x, y) are prime ideals in $\mathbb{Q}[x, y]$ but only the latter is a maximal ideal.

$\blacksquare$