Math 172
Assignment 11
Tuesday, April 24, 2018

Problems 1 - 6 on Sakai

1. If W and V are algebraic sets, show that $W \cap V$ is an algebraic set.

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2. If W and V are algebraic sets, show that $W \cup V$ is an algebraic set.

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3. Show that the set $D = \{(x, x) : x \in \mathbb{R} \text{ but } x \neq 1\}$ is NOT an algebraic set. (Hint: if $f \in K[x, y]$ vanishes on D, what can be said about $f(1, 1)$?)

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4. Given any monomial order, show that for nonzero polynomials f and g, that multidegree(fg)=multidegree(f)+multidegree(g). Also, multidegree($f+g$) $\leq$ max multidegree(f), multidegree(g) with equality if multidegree(f) is not equal multidegree(g).

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5. The grevlex monomial order is defined by comparing by the total degree first, then comparing exponents of the last indeterminate x_n but reversing the outcome (so the monomial with smaller exponent is larger in the ordering), followed (in case of a tie) by a similar comparison of x_{n-1}, and so forth ending with x1. Show that graded lex order and grevlex order are the same for monomials with 1 and 2 indeterminates, but find an example of two monomials (involving up to 3 indeterminates) for which lex and grevlex order those monomials differently.

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6. Perform polynomial division on $(x^3y + y)$ by $f = (xy + 2x)$ and $g = (x^3 + 1)$ to get a remainder term, and show that the remainder depends on the order of f and g in the polynomial division.