Math 172
Assignment 4
Tuesday, February 13, 2018

9.4 (1bcd, 2bc, R5, 6ac, 7, R11) 13.1 (R1, 3, R5) 13.2 (R1, 3)

9.4.1bcd Determine whether the following polynomials are irreducible in the rings indicated. For those that are reducible, determine their factorization into irreducibles. The notation $\mathbb{F}_p$ denotes the finite field $\mathbb{Z}/p\mathbb{Z}$, p a prime.

(b) $x^3 + x + 1$ in $\mathbb{F}_3[x]$

(c) $x^4 + 1$ in $\mathbb{F}_5[x]$

(d) $x^4 + 10x^2 + 1$ in $\mathbb{Z}[x]$

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9.4.2bc Prove that the following polynomials are irreducible in $\mathbb{Z}[x]$:

(b) $x^6 + 30x^5 - 15x^3 + 6x - 120$

(c) $x^4 + 4x^3 + 6x^2 + 2x + 1$ [Substitute $x - 1$ for x.]

$\blacksquare$

READ ONLY 9.4.5 Find all the monic irreducible polynomials of degree ≤ 3 in $\mathbb{F}_2[x]$, and the same in $\mathbb{F}_3[x]$.

9.4.6ac Construct fields of each of the following orders: (a) 9, (c) 8 (you may exhibit these as $F[x]/(f(x))$ for some F and f). [Use Exercise 2 and 3 in Section 2.]

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$\boxed{\textbf{9.4.7}\quad \text{Prove that } \mathbb{R}[x]/(x^2+1) \text{ is a field which is isomorphic to the complex numbers.}}$

$\blacksquare$

READ ONLY 9.4.11 Prove that $x^2 + y^2 - 1$ is irreducible in $\mathbb{Q}[x, y]$.

READ ONLY 13.1.1 Show that $p(x) = x^3 + 9x + 6$ is irreducible in $\mathbb{Q}[x]$. Let θ be a root of $p(x)$. Find the inverse of $1 + \theta$ in $\mathbb{Q}(\theta)$.

13.1.3 Show that $x^3 + x + 1$ is irreducible over $\mathbb{F}_2$ and let θ be a root. Compute the powers of θ in $\mathbb{F}_2(\theta)$.

$\blacksquare$

READ ONLY 13.1.5 Suppose α is a rational root of a monic polynomial in $\mathbb{Z}[x]$. Prove that α is an integer.

READ ONLY 13.2.1 Let $\mathbb{F}$ be a finite field of characteristic p. Prove that $|\mathbb{F}| = p^n$ for some positive integer n.

13.2.3 Determine the minimal polynomial over $\mathbb{Q}$ for the element $1 + i$.

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