Math 172
Assignment 7
Tuesday, March 27, 2018

14.1 (1a, 3, 7), 14.2 (3, 13, 14)

14.1.1a Show that if a field K is generated over F by the elements $\alpha_1, \ldots, \alpha_n$ then an automorphism σ of K fixing F is uniquely determined by $\sigma(\alpha_1), \ldots, \sigma(\alpha_n)$. In particular show that an automorphism fixes K if and only if it fixes a set of generators for K.

$\blacksquare$

14.1.3 Determine the fixed field of complex conjugation on $\mathbb{C}$.

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14.1.7 This exercise determines $\mathrm{Aut}(\mathbb{R}/\mathbb{Q})$.

(a) Prove that any $\sigma \in \mathrm{Aut}(\mathbb{R}/\mathbb{Q})$ takes squares to squares and takes positive reals to positive reals. Conclude that $a < b$ implies $\sigma a < \sigma b$ for every $a, b \in \mathbb{R}$.

(b) Prove that $-\frac{1}{m} < a - b < \frac{1}{m}$ implies $-\frac{1}{m} < \sigma a - \sigma b < \frac{1}{m}$ for every positive integer m. Conclude that σ is a continuous map on $\mathbb{R}$.

(c) Prove that any continuous map on $\mathbb{R}$ which is the identity on $\mathbb{Q}$ is the identity map, hence $\mathrm{Aut}(\mathbb{R}/\mathbb{Q}) = 1$.

$\blacksquare$

14.2.3 Determine the Galois group of $(x^2 - 2)(x^2 - 3)(x^2 - 5)$. Determine *all* the subfields of the splitting field of this polynomial.

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14.2.13 Prove that if the Galois group of the splitting field of a cubic over $\mathbb{Q}$ is the cyclic group of order 3 then all the roots of the cubic are real.

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14.2.14 Show that $\mathbb{Q}(\sqrt{2+\sqrt{2}})$ is a cyclic quartic field, i.e., is a Galois group of degree 4 with cyclic Galois group.

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