NetID: ms591

Mariam Sulakian
Professor Sorin
CS 250: Homework 2

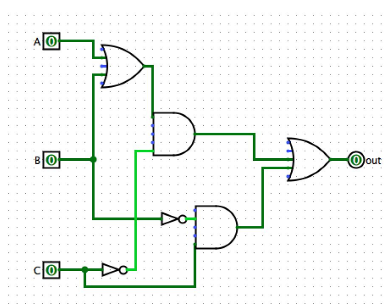
DIGITAL LOGIC DESIGN

Q1: Algebra & Combinational Logic

1. Write out a truth table: *out* = ¬B · C + (A + B) · ¬C

A	B	C	A+B	¬C	(A+B) · ¬C	¬B	¬B·C	¬B · C + (A + B) · ¬C
0	0	0	0	1	0	1	0	0
0	0	1	0	0	0	1	1	1
0	1	0	1	1	1	0	1	1
0	1	1	1	0	0	0	0	0
1	0	0	1	1	1	1	0	1
1	0	1	1	0	0	1	1	1
1	1	0	1	1	1	0	0	1
1	1	1	1	0	0	0	0	0

2. See Sakai: *circuit1.circ*



3. Write out product of sums:

Output 1: ¬A·¬B·¬C + ¬A·¬B·C + ¬A·B·¬C

= ¬A·¬B + ¬A·B·¬C = ¬B + B·¬C

Output 2: ¬A·¬B·¬C + ¬A·B·C + A·¬B·C

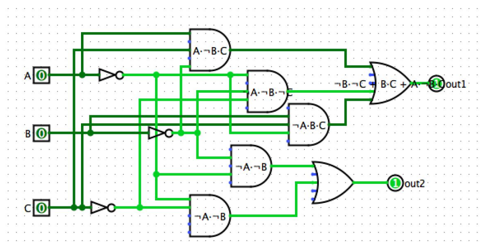
= ¬A·¬C + ¬A·¬B

K-Map for output 2 optimization:

C	AB	AB	AB
	00	01	10
0	**1**	**1**	0
1	**1**	0	0

Output: ¬A·¬C (since B changes) + ¬A·¬B (since C changes)

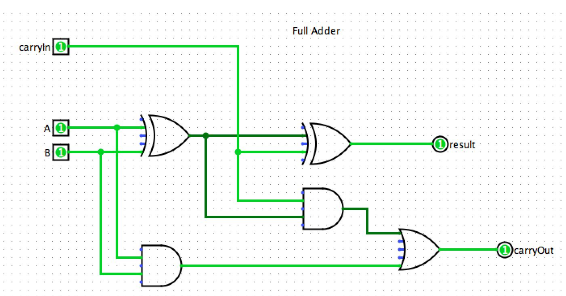
4. See Sakai: *circuit2.circ*

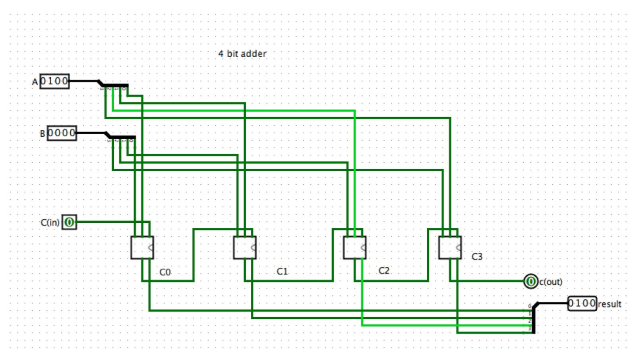


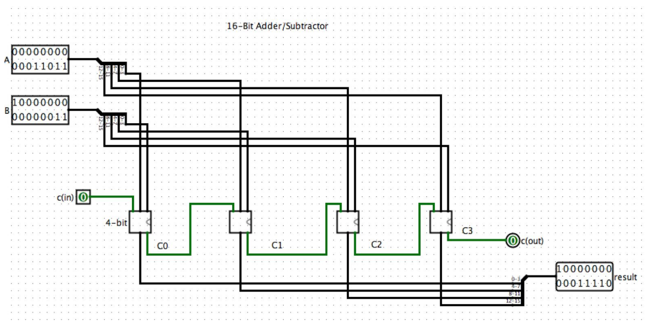
Q2: Adder / Subtractor Unit Design

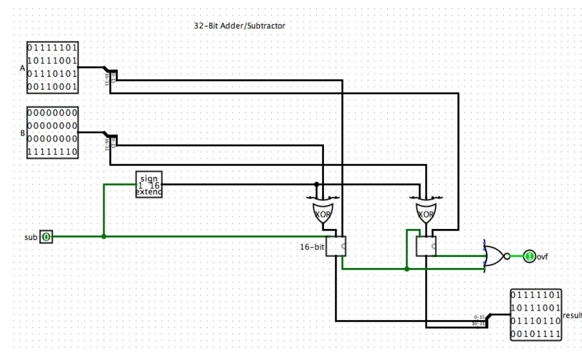
Ripple-carry adder / subtractor that performs (signed) 32-bit addition and subtraction.

See Sakai: *adder.circ*



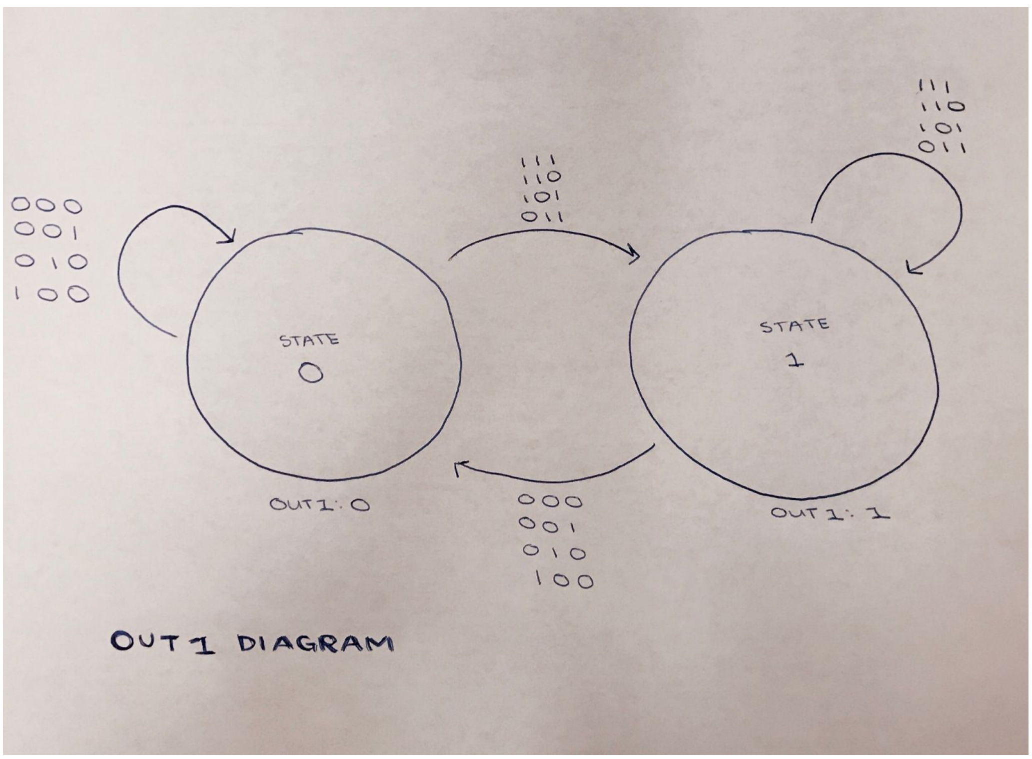


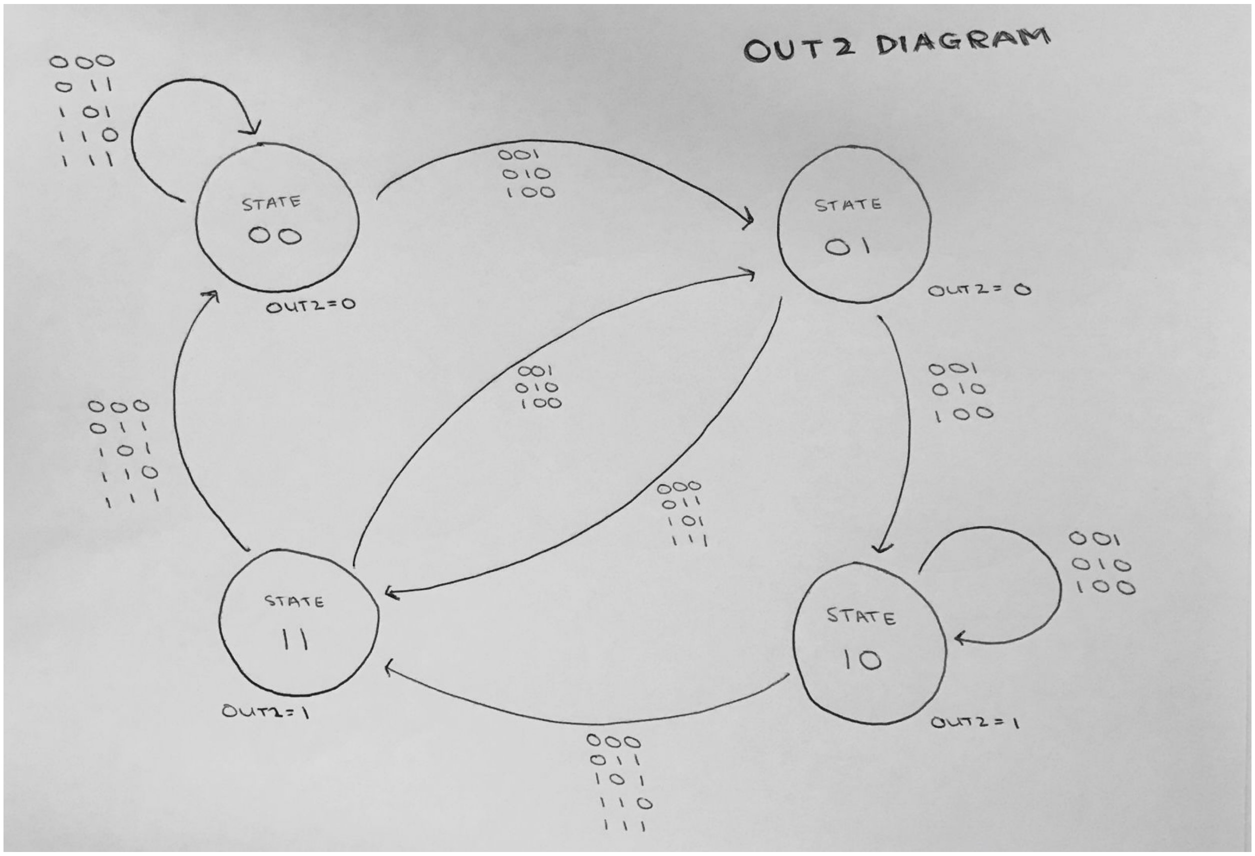




Q3: Finite State Machines

1. Transition State Diagrams





2. Truth Tables (product of sums work at end of diagram)

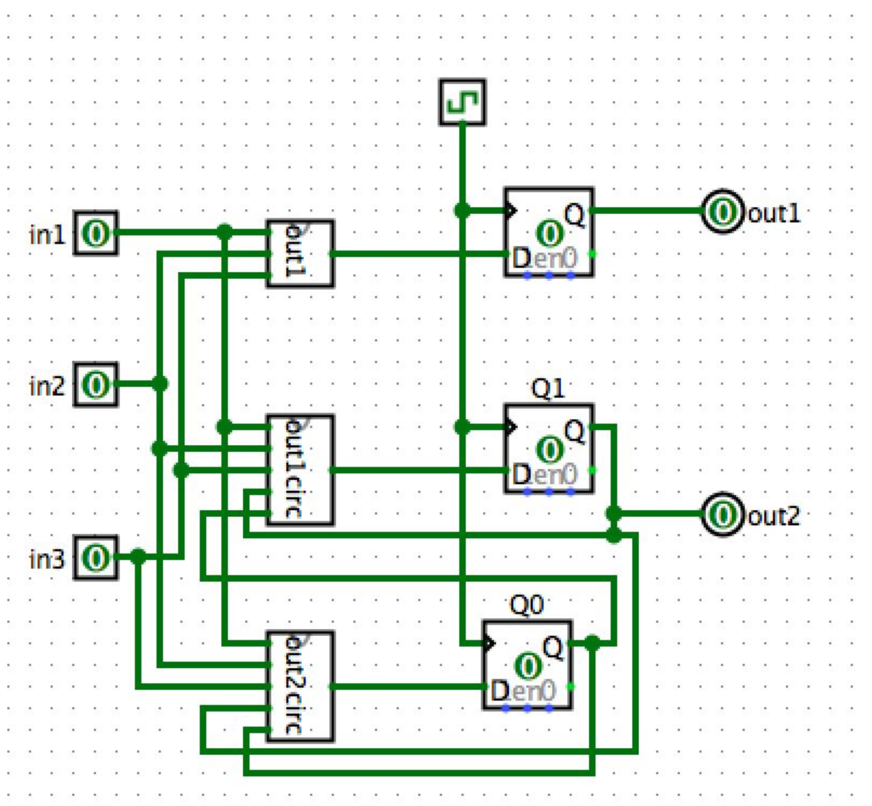
Truth Table for Output 1							
Current State		out1	in1	in2	in3	Next State	
Q1	Q0					D1	D0
0	0	0	0	0	0	0	0
			0	0	1		
			0	1	0		
			1	0	0		
0	0	0	1	1	1	0	1
			1	1	0		
			1	0	1		
			0	1	1		
0	1	1	1	1	1	0	1
			1	1	0		

			1	0	1		
			0	1	1		
0	1	1	0	0	0	0	0
			0	0	1		
			0	1	0		
			1	0	0		

Truth Table for Output 2							
Current State		out2	in1	in2	in3	Next State	
Q1	Q0					D1	D0
0	0	0	0	0	0	0	0
			0	1	1		
			1	0	1		
			1	1	0		
			1	1	1		
0	0	0	0	0	1	0	1
			0	1	0		
			1	0	0		
0	1	0	0	0	0	1	1
			0	1	1		
			1	0	1		
			1	1	0		
			1	1	1		
0	1	0	0	0	1	1	0

			0	1	0		
			1	0	0		
1	0	1	0	0	0	1	1
			0	1	1		
			1	0	1		
			1	1	0		
			1	1	1		
1	0	1	0	0	1	1	0
			0	1	0		
			1	0	0		
1	1	1	0	0	0	0	0
			0	1	1		
			1	0	1		
			1	1	0		
			1	1	1		
1	1	1	0	0	1	0	1
			0	1	0		
			1	0	0		

3. Logism implementation, see Sakai: *fsm.circ*



Product of Sums work:

Out: Q0 OUT 1

D0: $AB C + \bar{A}BC + A\bar{B}C + AB\bar{C}$

$= BC + A\bar{B}C + AB\bar{C}$

$= BC + A(\bar{B}C + B\bar{C})$

$= BC + A(B \oplus C)$ ↳ Q0

KEY

in1 = A
in2 = B
in3 = C

OUT 2

Out: Q1

D1: $(\overline{Q1} \cdot Q0) \cdot (ABC + \overset{\bar{A}BC}{\cancel{A\bar{B}\bar{C}}} + A\bar{B}C + AB\bar{C} + \bar{A}\bar{B}\bar{C}) +$

$(\overline{Q1} \cdot Q0) \cdot (\bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C}) +$

$(Q1 \cdot \overline{Q0}) \cdot (\bar{A}\bar{B}\bar{C} + \bar{A}BC + A\bar{B}C + AB\bar{C} + ABC) +$

$(Q1 \cdot \overline{Q0}) \cdot (\bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C}) =$

$(Q1 \oplus Q0)(\underset{1}{\underline{\bar{A}\bar{B}\bar{C}}} + \bar{A}BC + A\bar{B}C + \underset{2}{\underline{AB\bar{C}}} + \underset{2}{\underline{ABC}} + \bar{A}\bar{B}C + \underset{1}{\underline{\bar{A}B\bar{C}}} + A\bar{B}\bar{C})$

$= (Q1 \oplus Q0)(\bar{A}\bar{C} + AB + \bar{A}BC + \underset{1}{\underline{A\bar{B}C}} + \underset{1}{\underline{\bar{A}\bar{B}C}} + A\bar{B}\bar{C})$

$= (Q1 \oplus Q0)(\bar{A}\bar{C} + AB + \bar{A}BC + \underset{1}{\underline{\bar{B}C}} + \underset{1}{\underline{A\cancel{\bar{B}}C}} + A\bar{B}\bar{C})$

$= \cancel{(Q1 \oplus Q0)(\bar{A}\bar{C} + AB + \bar{A}BC + \bar{B}C)}$

$= (Q1 \oplus Q0)(\bar{A}\bar{C} + AB + \bar{A}BC + \bar{B}C + A\bar{B}\bar{C})$

D0: $(\overline{Q1} \cdot \overline{Q0})(\bar{A}\bar{B}C + \bar{A}B\bar{C} + \bar{A}BC) +$

$(\overline{Q1} \cdot Q0)(\bar{A}\bar{B}\bar{C} + \bar{A}BC + A\bar{B}C + AB\bar{C} + ABC) +$

$(Q1 \cdot \overline{Q0})(\bar{A}\bar{B}\bar{C} + \bar{A}BC + A\bar{B}C + AB\bar{C} + ABC) +$

$(Q1 \cdot Q0)(\bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C}) =$

$(Q1 \oplus Q0)(\underline{\bar{A}\bar{B}\bar{C}} + \underline{\bar{A}BC} + A\bar{B}C + \underset{2}{\underline{AB\bar{C}}} + \underset{2}{\underline{ABC}}) +$

$(Q1 \mathbin{\overline{\oplus}} Q0)(\bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C})$

$= (Q1 \oplus Q0)(\bar{A}(\overbrace{\bar{B}\bar{C} + BC}^{B \overline{\oplus} C}) + A\bar{B}C + \cancel{AB\bar{C}} + AB)) +$

$(Q1 \mathbin{\overline{\oplus}} Q0)(\bar{A}(\underbrace{\bar{B}C + B\bar{C}}_{B \oplus C}) + A\bar{B}\bar{C})$

$= (Q1 \oplus Q0)(\bar{A}(B \mathbin{\overline{\oplus}} C) + A(\overbrace{\bar{B}C + B\bar{C}}^{B \oplus C}) + AB) +$

$(Q1 \mathbin{\overline{\oplus}} Q0)(\bar{A}(B \oplus C) + A\bar{B}\bar{C})$

$= (Q1 \oplus Q0)(\bar{A}(B \mathbin{\overline{\oplus}} C) + A(B \oplus C) + AB) +$

$(Q1 \mathbin{\overline{\oplus}} Q0)(\bar{A}(B \oplus C) + A\bar{B}\bar{C})$

$= (Q1 \oplus Q0)(\bar{A}(B \mathbin{\overline{\oplus}} C) + AB + A\bar{B}C) +$

$(Q1 \mathbin{\overline{\oplus}} Q0)(\bar{A}(B \oplus C) + A\bar{B}\bar{C})$

$\oplus$ XOR

$\overline{\oplus}$ XNOR