MATH 60, Sec.
HW 2
9 September 2016

Colley Chapter 2.3.{33, 38, 42}, 2.4.{16, 22, 29a}

2.3.33 Find the matrix $D\mathbf{f}(\mathbf{a})$ of partial derivatives where $\mathbf{f}$ and $\mathbf{a}$ are as indicated:

$$\mathbf{f}(s,t) = (s^2, st, t^2), \mathbf{a} = (-1,1)$$

∎

$\boxed{\textbf{2.3.38}\ \text{Find an equation for the plane tangent to the graph of } z = 4\cos xy \text{ at the point } (\pi/3, 1, 2).}$

$\blacksquare$

2.3.42 Suppose that you have the following information concerning a differentiable function f:

$$f(2,3) = 12, f(1.98,3) = 12.1, f(2,3.01 = 12.2)$$

(a) Give an approximate equation for the plane tangent to the graph of f at (2,3,12).

(b) Use the result of part (a) to estimate $f(1.98, 2.98)$.

■

2.4.16 Determine all second-order partial derivatives (including mixed partials).

$$f(x,y) = ln\left(\frac{x}{y}\right)$$

■

2.4.22 Consider the function $F(x, y, z) = 2x^3y + xz^2 + y^3z^5 - 7xyz$.

(a) Find F_{xx}, F_{yy}, and F_{zz}.

(b) Calculate the mixed second-order partials $F_{xy}, F_{yx}, F_{xz}, F_{zx}, F_{yz}$, and F_{zy}, and verify Theorem 4.3.

■

2.4.29a The three-dimensional **heat equation** is the partial differential equation

$$k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)=\frac{\partial T}{\partial t}$$

where k is a positive constant. It models the temperature $T(x,y,z,t)$ at the point (x,y,z) and time t of a body in space.

(a) We examine a simplified version of the heat equation. Consider a straight wire "coordinatized" by x. Then the temperature $T(x,t)$ at time t and position x along the wire is modeled by the one-dimensional heat equation

$$k\frac{\partial^2 T}{\partial x^2}=\frac{\partial T}{\partial t}.$$

Show that the function $T(x,t)=e^{-kt}\cos x$ satisfies this equation. Note that if t is held constant at value t_0, then $T(x,t_0)$ shows how the temperature varies along the wire at time t_0. Graph the curves $z=T(x,t_0)$ for $t_0=0,1,10$, and use them to understand the graph of the surface $z=T(x,t)$ for $t\geq 0$. Explain what happens to the temperature of the wire after a long period of time.

■