

Quiz 3 review by Jenny Zeng

Propositional Logic

logical Agents

basic concepts:

1. **Propositional logic:** concrete statements that are either true or false. e.g. John is married to Sue.
2. **Predicate logic (first order logic/ first order predicate calculus):** allows statements to contain variables, functions, and quantifiers. e.g. For all X, Y: If X is married to Y then Y is married to X.
3. **Probability:** statements that are possibly true; the chance I win the lottery?
4. **Fuzzy logic:** vague statements; paint is slightly grey; sky is very cloudy.
5. **Modal logic:** is a class of various logics that introduce modalities:
 - **Temporal logic:** statements about time; John was a student at UCI for four years
 - **Belief and knowledge:** Mary knows that John is married to Sue
 - **Possibility and Necessity:** what might happen (possibility) and must happen (necessity);
 - **Obligation and Permission:** It is obligatory that students study for their tests; it is permissible that I go fishing when I am on vacation.
6. **Induction:** Reason from facts to the general law.
7. **Abduction:** Reason from facts to the best explanation.
8. **Analogy:** Reason that a new situation is like an old one.
9. **Schematic perspective:** If KB is true in the real world, then any sentence α entailed by KB is also true in the real world.

important concepts:

1. **Logics:** are formal languages for representing information such as conclusions can be drawn from formal inference patterns.
2. **Syntax:** Specifies all the sentences in a language that are well formed.
3. **Semantics:** Defines truth of each sentence with respect to each possible world.
4. **Entailment:** The idea that a sentence follows logically from other sentences.
5. **inference:** deriving sentences from other sentences
6. **model:** Possible world that assigns TRUE or FALSE to each proposition.
7. **soundness:** derivations produce only entailed sentences

8. **KB**: knowledge base - a set of sentences or facts
9. **valid**: sentence is true in every model (a tautology)
10. **satisfiable**: A sentence is satisfiable if it is true in some model
11. **unsatisfiable**: A sentence is unsatisfiable if it is false in all model.

logical equivalence

Two sentences are **logically equivalent** iff they are true in same models: $\alpha \equiv \beta$ iff $\alpha \models \beta$ and $\beta \models \alpha$

$$\begin{aligned}
 (\alpha \wedge \beta) &\equiv (\beta \wedge \alpha) && \text{commutativity of } \wedge \\
 (\alpha \vee \beta) &\equiv (\beta \vee \alpha) && \text{commutativity of } \vee \\
 ((\alpha \wedge \beta) \wedge \gamma) &\equiv (\alpha \wedge (\beta \wedge \gamma)) && \text{associativity of } \wedge \\
 ((\alpha \vee \beta) \vee \gamma) &\equiv (\alpha \vee (\beta \vee \gamma)) && \text{associativity of } \vee \\
 \neg(\neg\alpha) &\equiv \alpha && \text{double-negation elimination} \\
 (\alpha \Rightarrow \beta) &\equiv (\neg\beta \Rightarrow \neg\alpha) && \text{contraposition} \\
 (\alpha \Rightarrow \beta) &\equiv (\neg\alpha \vee \beta) && \text{implication elimination} \\
 (\alpha \Leftrightarrow \beta) &\equiv ((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)) && \text{biconditional elimination} \\
 \neg(\alpha \wedge \beta) &\equiv (\neg\alpha \vee \neg\beta) && \text{de Morgan} \\
 \neg(\alpha \vee \beta) &\equiv (\neg\alpha \wedge \neg\beta) && \text{de Morgan} \\
 (\alpha \wedge (\beta \vee \gamma)) &\equiv ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma)) && \text{distributivity of } \wedge \text{ over } \vee \\
 (\alpha \vee (\beta \wedge \gamma)) &\equiv ((\alpha \vee \beta) \wedge (\alpha \vee \gamma)) && \text{distributivity of } \vee \text{ over } \wedge
 \end{aligned}$$

You need to know these !

truth table

| P | Q | $\neg P$ | $P \wedge Q$ | $P \vee Q$ | $P \Rightarrow Q$ | $P \Leftrightarrow Q$ |
|-------|-------|----------|--------------|------------|-------------------|-----------------------|
| false | false | true | false | false | true | true |
| false | true | true | false | true | true | false |
| true | false | false | false | true | false | false |
| true | true | false | true | true | true | true |

Methods of Proof

1. **Sound**: An inference procedure that derives only entailed sentences.
2. **Complete**: An inference procedure that derives all entailed sentences.
3. **Propositional Symbol**: Stands for a proposition that can be true or false.
4. **Proof**: Chain of inference rule conclusions leading to a desired sentence.
5. **Conjunctive Normal Form**: Describes a sentence that is true in some model.

Knowledge engineering process

1. Identify the task.

2. Assemble the relevant knowledge.
3. Decide on a vocabulary of predicates, functions, and constants.
4. Encode general knowledge about the domain.
5. Encode a description of the specific problem instance.
6. Pose queries to the inference procedure and get answers.
7. Debug the knowledge base.

First-order Logic Syntax

Syntax of FOL:

- A **Predicate** is a list of m-tuples making the predicate true.
 - E.g., PrimeFactorOf = { <2,4>, <2,6>, <3,6>, <2,8>, <3,9>, ... }
- A **Property** lists the m-tuples that have the property.
 - E.g., IsRed = { < Ball-5 >, < Toy-7 >, < Car-11 > }
- A **Function** CAN BE represented as an m-ary relation
 - the first (m-1) objects are the arguments and the mth is the value.
- An **Object** CAN BE represented as a function of zero arguments that returns the object.
- **Term** = logical expression that refers to an object
 - **Constant Symbols** stand for (or name) objects
 - **Function Symbols** map tuples of objects to an object
- **Atomic Sentences** state facts (logical truth values).

possible quiz questions

1. All persons are mortal.

"All persons are mortal."

[Use: Person(x), Mortal (x)]

$$\forall x \text{ Person}(x) \Rightarrow \text{Mortal}(x)$$

$$\forall x \neg \text{Person}(x) \vee \text{Mortal}(x)$$

Common Mistakes:

$$\forall x \text{ Person}(x) \wedge \text{Mortal}(x)$$

2. Fifi has a sister who is a cat.

"Fifi has a sister who is a cat."

[Use: Sister(Fifi, x), Cat(x)]

$$\exists x \text{ Sister}(\text{Fifi}, x) \wedge \text{Cat}(x)$$

Common Mistakes:

$$\exists x \text{ Sister}(\text{Fifi}, x) \Rightarrow \text{Cat}(x)$$

3. For every food, there is a person who eats that food.

"For every food, there is a person who eats that food."

[Use: Food(x), Person(y), Eats(y, x)]

$$\forall x \exists y \text{ Food}(x) \Rightarrow [\text{Person}(y) \wedge \text{Eats}(y, x)]$$

$$\forall x \text{ Food}(x) \Rightarrow \exists y [\text{Person}(y) \wedge \text{Eats}(y, x)]$$

$$\forall x \exists y \neg \text{Food}(x) \vee [\text{Person}(y) \wedge \text{Eats}(y, x)]$$

$$\forall x \exists y [\neg \text{Food}(x) \vee \text{Person}(y)] \wedge [\neg \text{Food}(x) \vee \text{Eats}(y, x)]$$

$$\forall x \exists y [\text{Food}(x) \Rightarrow \text{Person}(y)] \wedge [\text{Food}(x) \Rightarrow \text{Eats}(y, x)]$$

Common Mistakes:

$$\forall x \exists y [\text{Food}(x) \wedge \text{Person}(y)] \Rightarrow \text{Eats}(y, x)$$

$$\forall x \exists y \text{ Food}(x) \wedge \text{Person}(y) \wedge \text{Eats}(y, x)$$

4. every person eats every food.

"Every person eats every food."

[Use: Person (x), Food (y), Eats(x, y)]

$$\forall x \forall y [\text{Person}(x) \wedge \text{Food}(y)] \Rightarrow \text{Eats}(x, y)$$

$$\forall x \forall y \neg \text{Person}(x) \vee \neg \text{Food}(y) \vee \text{Eats}(x, y)$$

$$\forall x \forall y \text{ Person}(x) \Rightarrow [\text{Food}(y) \Rightarrow \text{Eats}(x, y)]$$

$$\forall x \forall y \text{ Person}(x) \Rightarrow [\neg \text{Food}(y) \vee \text{Eats}(x, y)]$$

$$\forall x \forall y \neg \text{Person}(x) \vee [\text{Food}(y) \Rightarrow \text{Eats}(x, y)]$$

Common Mistakes:

$$\forall x \forall y \text{ Person}(x) \Rightarrow [\text{Food}(y) \wedge \text{Eats}(x, y)]$$

$$\forall x \forall y \text{ Person}(x) \wedge \text{Food}(y) \wedge \text{Eats}(x, y)$$

5. all greedy kings are evil.

"All greedy kings are evil."

[Use: King(x), Greedy(x), Evil(x)]

$$\forall x [\text{Greedy}(x) \wedge \text{King}(x)] \Rightarrow \text{Evil}(x)$$

$$\forall x \neg \text{Greedy}(x) \vee \neg \text{King}(x) \vee \text{Evil}(x)$$

$$\forall x \text{ Greedy}(x) \Rightarrow [\text{King}(x) \Rightarrow \text{Evil}(x)]$$

Common Mistakes:

$$\forall x \text{ Greedy}(x) \wedge \text{King}(x) \wedge \text{Evil}(x)$$

6. everyone has a favorite food.

"Everyone has a favorite food."

[Use: Person(x), Food(y), Favorite(y, x)]

$$\forall x \exists y \text{ Person}(x) \Rightarrow [\text{Food}(y) \wedge \text{Favorite}(y, x)]$$

$$\forall x \text{ Person}(x) \Rightarrow \exists y [\text{Food}(y) \wedge \text{Favorite}(y, x)]$$

$$\forall x \exists y \neg \text{Person}(x) \vee [\text{Food}(y) \wedge \text{Favorite}(y, x)]$$

$$\forall x \exists y [\neg \text{Person}(x) \vee \text{Food}(y)] \wedge [\neg \text{Person}(x) \vee \text{Favorite}(y, x)]$$

$$\forall x \exists y [\text{Person}(x) \Rightarrow \text{Food}(y)] \wedge [\text{Person}(x) \Rightarrow \text{Favorite}(y, x)]$$

Common Mistakes:

$$\forall x \exists y [\text{Person}(x) \wedge \text{Food}(y)] \Rightarrow \text{Favorite}(y, x)$$

$$\forall x \exists y \text{ Person}(x) \wedge \text{Food}(y) \wedge \text{Favorite}(y, x)$$

7. There is someone at UCI who is smart.

"There is someone at UCI who is smart."

[Use: Person(x), At(x, UCI), Smart(x)]

$\exists x \text{ Person}(x) \wedge \text{At}(x, \text{UCI}) \wedge \text{Smart}(x)$

Common Mistakes:

$\exists x [\text{Person}(x) \wedge \text{At}(x, \text{UCI})] \Rightarrow \text{Smart}(x)$

8. Every one at UCI is smart.

"Everyone at UCI is smart."

[Use: Person(x), At(x, UCI), Smart(x)]

$\forall x [\text{Person}(x) \wedge \text{At}(x, \text{UCI})] \Rightarrow \text{Smart}(x)$

$\forall x \neg [\text{Person}(x) \wedge \text{At}(x, \text{UCI})] \vee \text{Smart}(x)$

$\forall x \neg \text{Person}(x) \vee \neg \text{At}(x, \text{UCI}) \vee \text{Smart}(x)$

Common Mistakes:

$\forall x \text{ Person}(x) \wedge \text{At}(x, \text{UCI}) \wedge \text{Smart}(x)$

$\forall x \text{ Person}(x) \Rightarrow [\text{At}(x, \text{UCI}) \wedge \text{Smart}(x)]$

9. Every person eats some food.

"Every person eats some food."

[Use: Person (x), Food (y), Eats(x, y)]

$\forall x \exists y \text{ Person}(x) \Rightarrow [\text{Food}(y) \wedge \text{Eats}(x, y)]$

$\forall x \text{ Person}(x) \Rightarrow \exists y [\text{Food}(y) \wedge \text{Eats}(x, y)]$

$\forall x \exists y \neg \text{Person}(x) \vee [\text{Food}(y) \wedge \text{Eats}(x, y)]$

$\forall x \exists y [\neg \text{Person}(x) \vee \text{Food}(y)] \wedge [\neg \text{Person}(x) \vee \text{Eats}(x, y)]$

Common Mistakes:

$\forall x \exists y [\text{Person}(x) \wedge \text{Food}(y)] \Rightarrow \text{Eats}(x, y)$

$\forall x \exists y \text{ Person}(x) \wedge \text{Food}(y) \wedge \text{Eats}(x, y)$

10. some person eats some food.

"Some person eats some food."

[Use: Person (x), Food (y), Eats(x, y)]

$\exists x \exists y \text{ Person}(x) \wedge \text{Food}(y) \wedge \text{Eats}(x, y)$

Common Mistakes:

$\exists x \exists y [\text{Person}(x) \wedge \text{Food}(y)] \Rightarrow \text{Eats}(x, y)$