CS161 – Fall 2016 — Homework1

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R-1.3, R-1.4, R-1.7, R-1.10

R-1.3

Algorithm A uses $10n \log n$ operations, while algorithm B uses n^2 operations. Determine the value n_0 such that A is better than B for $n \geq n_0$.

Answer:

$$\begin{aligned} 10n \log n &= n^2 \\ n &= 10 \log n \\ n &\approx 1.07755 \quad \text{or} \quad 58.7701 \end{aligned}$$

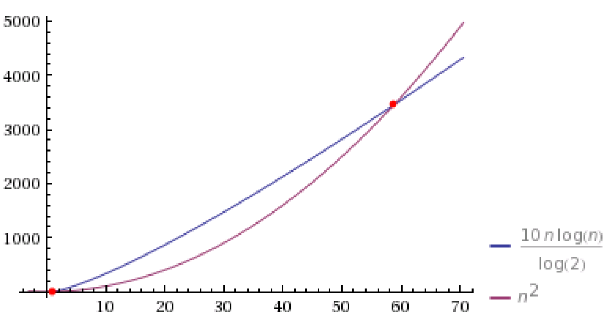

Figure 1: Plot generated by WolframAlpha for $10n \log n$ and n^2

from the graph we can see that $n_0 = 59$ such that A is better than B for $n \geq n_0$.

R-1.4

Repeat the previous problem assuming B uses $n\sqrt{n}$ operations.
Answer:

$$\begin{aligned} 10n\log n &= n\sqrt{n} \\ 10n\log n &= n^{3/2} \\ n &\approx 1.07449 \quad \text{or} \quad 20519.8 \end{aligned}$$

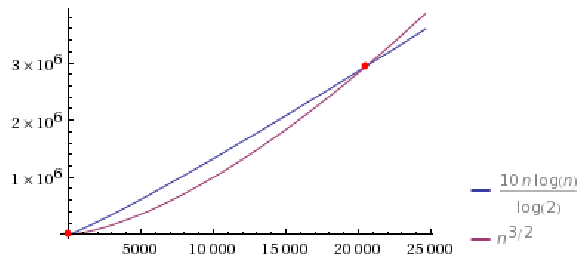

Figure 2: Plot generated by WolframAlpha for $10n\log n$ and $\sqrt{n}$

from the graph we can see that $n_0 = 20520$ such that A is better than B for $n \geq n_0$.

R-1.7

Order the following list of functions by the big-Oh notation. Group together (for example, by underlining) those functions that are big-Theta of one another.

$$\begin{array}{ccccc}
6n\log n & 2^{100} & \log\log n & \log^2 n & 2^{\log n} \\
2^{2^n} & \lceil\sqrt{n}\rceil & n^{0.01} & 1/n & 4n^{3/2} \\
3n^{0.5} & 5n & \lfloor 2n\log^2(n)\rfloor & 2^n & n\log_4 n \\
4^n & n^3 & n^2\log n & 4^{\log n} & \sqrt{\log n}
\end{array}$$

Answer:

1. $1/n$
2. 2^{100}
3. $\log\log n$
4. $\sqrt{\log n}$
5. $\log^2 n$
6. $n^{0.01}$
7. $\lceil\sqrt{n}\rceil, 3n^{0.5}$
8. $5n, 2^{\log n}$
9. $6n\log n, n\log_4 n$
10. $\lfloor 2n\log^2(n)\rfloor$
11. $4n^{3/2}$
12. $4^{\log n}$
13. $n^2\log n$
14. n^3
15. 2^n
16. 4^n
17. 2^{2^n}

R-1.10

Consider the following recurrence equation, defining $T(n)$, as

$$T(n) = \begin{cases} 4 & \text{if } n = 1 \\ T(n-1) + 4 & \text{otherwise.} \end{cases}$$

Show, by induction, that $T(n) = 4n$

Proof:

(a) base case: (n =1). $T(1) = 4 = 4 * 1$

(b) Induction step: $(n \neq 1)$.
Suppose for $n = k, T(k) = 4k$ is true
for $n = k + 1$,

$$\begin{aligned} T(k+1) &= T(k+1-1) + 4 \\ &= T(k) + 4 \\ &= 4k + 4 \\ &= 4(k+1) \end{aligned}$$

Because T(k+1) = 4(k+1), $T(n) = 4n$ is true