Math 215 Fall 2018
Problem Set 6
Due: September 24, 2018

(17 points) Make sure you are familiar with the Academic Honesty policies for this class, as detailed on the syllabus. All work is due on the given day by the time lecture starts.

1. (4 points) In the previous assignment, you showed that $g : \mathbb{Z} \to \mathbb{Z}$ satisfying $g(m) = 2m+1$ is not surjective. Here we will prove that $h : \mathbb{R} \to \mathbb{R}$ satisfying $h(x) = 2x+1$ is surjective. I have set up the outline of the proof. You should either fill in the blanks or you may write your own proof from scratch, but it should look very similar to my outline. **If you fill in the blanks, please underline or color differently the filled in blanks on your submitted answer.**

   We need to prove that for all $y \in \mathbb{R}$ there is an $x \in \mathbb{R}$ so that $h(x) = y$. We fix an arbitrary $y \in \mathbb{R}$. Let $x =$ ________________________ . Then $h(x) =$ ________________________ $= y$. So we have found an x so that $h(x) = y$. Hence the function is surjective.

2. Suppose that $f : A \to B$ and $g : B \to C$ are both surjective functions.

   (a) (1 point) Write the precise definition for the fact that g is surjective.

   (b) (1 point) Write the precise definition for the fact that f is surjective.

   (c) (3 points) Prove the following statement:

   If f and g are both surjective functions, then $g \circ f$ is a surjective function.

   I encourage you to consult the previous problem for the structure of proving a function is surjective. Also, this is part (c) for a reason. You will need to use the fact that g and f are surjective, in that order.

3. (4 points) Prove the following two sets are equal by showing that $A \subseteq B$ and that $B \subseteq A$. You should explicitly take an element in A and show it is in B and then take an element of B and show it is in A.

$$A = \left\{ \begin{pmatrix} 3 \\ 5 \end{pmatrix} + c \begin{pmatrix} 4 \\ 2 \end{pmatrix} : c \in \mathbb{R} \right\} \text{ and } B = \left\{ \begin{pmatrix} -1 \\ 3 \end{pmatrix} + c \begin{pmatrix} 4 \\ 2 \end{pmatrix} : c \in \mathbb{R} \right\}$$

   If you aren't sure where to start, take a specific element in A and play around with it to get it to look like an element in B. Then can you generalize your idea?

4. (4 points) (a) Is $\binom{2}{1}$ a linear combination of the vectors $\binom{1}{1}$ and $\binom{1}{2}$? Explain why or why not using the definition of a linear combination.

   (b) Is $\binom{0}{1}$ a linear combination of the vectors $\binom{1}{1}$ and $\binom{1}{2}$? Again, explain why or why not using the definition.