Math 215 Fall 2018
Problem Set 4
Due: September 17, 2018

(22 points) Make sure you are familiar with the Academic Honesty policies for this class, as detailed on the syllabus. All work is due on the given day by the time lecture starts.

1. (5 points) For this problem we will do a double containment proof to show the two sets $A = \{3x + 1 : x \in \mathbb{Z}\}$ and $B = \{3x - 2 : x \in \mathbb{Z}\}$ are equal. Like on the last homework, some blanks have been left. Fill in each of the blank spaces to make the proof correct. **Please underline or color differently the filled in blanks on your submitted answer.**

   We prove this by double containment. We first show $A \subseteq B$. Take __________________________. Then $a = 3x+1$ for some $x \in \mathbb{Z}$. By __________________________. Since __________________________ is an integer, we have that a is in B. Since __________________________, we have proven that $A \subseteq B$.

   Now we show __________________________. Take an arbitrary __________________________. Then, $b =$ __________________________ for some $x \in \mathbb{Z}$. Again, __________________________. Since __________________________ is an integer, we have that $b \in A$. Since b was arbitrary, we have proven that $B \subseteq A$. Since __________________________, we can conclude that $A = B$.

2. (6 points) Suppose A and B are subsets of some set U. Use the proof technique for showing one set is a subset of another and the definitions of union and intersection to prove the following.

   (a) $A \cap B \subseteq A$

   (b) $A \subseteq A \cup B$

3. (5 points) Suppose A and B subsets of some set U. Prove or disprove that the sets $A \cap B$ and $A \backslash B$ are disjoint.

4. (6 points) The following claim is false:

   Let A, B, and C be subsets of some set U. If $A \not\subseteq B$ and $B \not\subseteq C$, then $A \not\subseteq C$.

   (a) Here is a wrong "Proof". Describe precisely where the logic fails in this proof.

   We assume that A, B, and C are subsets of U and that $A \not\subseteq B$ and $B \not\subseteq C$. This means that there exists an element $x \in A$ that is not in B and there exists an element x that is in B and not in C. Therefore, $x \in A$ and $x \notin C$, and we have proved that $A \not\subseteq C$.

   (b) Suppose U is the integers, $\mathbb{Z}$. Come up with an explicit example for sets A, B, and C where the claim stated above fails. If you work with others to come up with ideas for this part, you should each have different final answers here.