MATH 232 Discrete Math
Homework 2

Basic Information
This assignment is due in Gradescope by 10 PM on the dates below.

Make sure you understand MHC honor code and have carefully read and understood the additional information on the class syllabus and the grading rubric. I am happy to discuss any questions or concerns you have!

You are always welcome to ask me for small hints or suggestions on problems.

Problems

Reading Problem 2M (Due: Sunday, February 8)

Explain in an English sentence what the sets $A = \{6x : x \in \mathbb{Z}\}$ and $B = \{3x : x \in \mathbb{Z}\}$ represent. Then explain using the "for all" definition of subsets why $A \subseteq B$. (The reading for Monday will give you some guidelines on proving that one set is a subset of another.)

Wednesday Problems HW2 (Due: Wednesday, February 11)

For each problem below don't forget to explain your answer in words even if the problem doesn't explicitly say to do so.

1. Determine if the following sets A are subsets of the set B. Explain why you think your answer is correct. (An explanation in words is fine, we don't yet need a formal proof.)
(a) $A = \{(x, y) \in \mathbb{R}^2 : x^2 - x = 0\}$ and $B = \{(x, y) \in \mathbb{R}^2 : x - 1 = 0\}$
(b) $A = X - R$ and $B = X - S$ where S and R are sets so that $S \subseteq R$.

2. Let A and B be sets, and $|A| = m$ and $|B| = n$ where m and n are positive integers.
(a) What is $|\mathcal{P}(A \times \mathcal{P}(B))|$?
(b) What about $|\{X \in \mathcal{P}(A) : |X| \leq 1\}|$?

3. For each $n \in \mathbb{N}$ let $A_n = \{-2n, 0, 2n\}$.
(a) What is $\bigcup_{i \in \mathbb{N}} A_i$?
(b) What is $\bigcap_{i \in \mathbb{N}} A_i$?

4. (8 pg 41 MR). Each of the following statements is true.

- *Laura is in the seventh grade.*
- *Laura got an A on the math test or Sarah got an A on the math test*
- *If Sarah got an A on the math test, then Laura is not in the 7th grade.*

If possible, determine the truth value of each of the following statements based on the 3 statements above. Carefully explain your reasoning.
(a) Laura got an A on the math test.
(b) Sarah got an A on the math test
(c) Either Laura or Sarah did not get an A on the math test.

5. Negate the following statements. You do not need to prove or disprove the statements, simply write the negation so that no "not" appears.

(a) For all $x \in \mathbb{R}$, we have $x^3 \geq x^2$.
(b) There exists an $x \in \mathbb{Z}$ such that for all $y \in \mathbb{Z}$ we have $2x + 3y = 1$.

6. Prove or disprove the following statement:

There exists an $n \in \mathbb{N}$ *such that* $n^2 - 1 = 0$.

Reading Problem 2F (Due: Thursday, February 12)

Prove the following statement:

If $a \mid b$ *and* $b \mid a$, *then* $a = \pm b$.