Math 321 Fall 2016
Homework 7
Due: October 28, 2016

You are welcome to work together but everyone needs to write up **distinct** solutions. If you use any books outside of our textbook or other people, please make sure to give them credit. Make sure your solutions are complete. If your handwriting is atrocious, I am happy to give you a basic introduction to LaTeX.

1. # 11.29
2. Let N be a normal subgroup of G and let H be any subgroup of G. Prove that $NH = \{n \cdot h \mid n \in N, h \in H\}$ is a subgroup of G. Give an example to show that NH need not be a subgroup of G if neither N nor H is.
3. Consider the additive quotient group $\mathbb{Q}/\mathbb{Z}$. Show that every coset of $\mathbb{Z}$ in $\mathbb{Q}$ contains exactly one respresntative $q \in \mathbb{Q}$ in the range $0 \leq q < 1$.
4. # 12.1 (a), (b), (d)
5. # 12.7
6. # 12.13 (This completes a piece of a proofs from class.)
7. # 12.19 (This completes a piece of a proofs from class.)
8. Define $\phi : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ by sending (x, y) to $x + y$.
   (a) Prove that ϕ is a surjective homomorphism.
   (b) What is the kernel of this mapping?
9. # 13.5