Math 215 Fall 2018
Problem Set 3
Due: September 14, 2018

(19 points) Make sure you are familiar with the Academic Honesty policies for this class, as detailed on the syllabus. All work is due on the given day by the time lecture starts.

1. (4 points) Below is the proof of the statement

   If a and b are both odd integers then $a + b$ is an even integer.

   I have deleted some pieces of the proof. Fill in each of the blank spaces to make the proof correct. **Please underline or color differently the filled in blanks on your submitted answer.**

   Fix a and b as ______________________ odd integers. By the definition of an odd number, we know that a is ______________________ and b is ______________________ . Then $a + b =$ ______________________. Since ______________________ is an integer we conclude that $a + b$ is even by the definition of an even integer.

2. (4 points) Below is the proof of the statement

   For all $a \in \mathbb{Z}$ we have $2a^5 + 6a^3 - 4a + 3$ is odd.

   I have deleted some pieces of the proof. Fill in each of the blank spaces to make the proof correct. **Please underline or color differently the filled in blanks on your submitted answer.**

   ________________________________. Then $2a^5+6a^3-4a+3 =$ ______________________ Since ______________________ is an integer, then ______________________ by the definition of odd numbers. Since a was arbitrary, we have proven the result.

3. (6 points) Write the *contrapositive* of each of the following statements. Your final answer should not have any *not* in it. You should not discuss whether the statements are true or false as stated.

   (a) If $|x| \neq -x$ then $x \leq 0$.

   (b) For all real numbers a and b, if $a \neq 0$ and $b \neq 0$ then $ab \neq 0$.

   (c) Let a be an integer. If there exists an $m \in \mathbb{Z}$ so that $a = 4m + 1$ then a is odd. (Hint: To get rid of the $\neq$ symbol, maybe consider the number line.)

4. (5 points) Prove the following statement by (a) writing down its contrapositive and then (b) proving the contrapositive.

   Let $a \in \mathbb{Z}$. If $3a + 2$ is odd, then a is odd.