Math 321 Fall 2011
Homework 2
Due: September 9, 2011

You are welcome to work together but everyone needs to write up **distinct** solutions. If you use any books outside of our textbook or other people, please make sure to give them credit. Make sure your solutions are complete. If your handwriting is atrocious, I am happy to give you a basic introduction to LaTeX.

1. pg 43 # 1. If a and b are both nonzero and $b \mid a$ then $|b| \leq |a|$. Hence if a and b are both positive and $b \mid a$ then $b \leq a$.

2. pg 45 #2-4. Assume a and b are both positive integers and $a \nmid b$ and $b \nmid a$.

   (a) Define q_i, $r_i \in \mathbb{Z}$ by

$$a = q_1 b + r_1 \qquad 0 \leq r < b$$

$$b = q_2 r_1 + r_2 \qquad 0 \leq r_2 < r_1$$

$$r_i = q_{i+2} r_{i+1} + r_{i+2} \qquad i \in \mathbb{N}, i \geq 1.$$

   Prove there exists a positive integer n such that $r_{n+1} = 0$

   (b) Prove that $\gcd(a, b) = \gcd(b, r_1) = \gcd(r_i, r_{i+1})$ for $1 \leq i \leq m - 1$, hence that $r_m = \gcd(a, b)$.

   (c) Use (a) and (b) to find $\gcd(991, 236)$.

3. pg. 69 # 2. Let $\mathbb{Q}_0 = \mathbb{Q} - \{0\}$.
   (a) Prove that $\mathbb{Q}_0$ is a group with respect to multiplication.
   (b) Prove that the only elements of finite order in $\mathbb{Q}_0$ are 1 and -1. What is $o(1)$? $o(-1)$?

4. pg 71 # 1. Let z be an element in a group G and let $o(z) = mn$. Prove that there exist elements a, $b \in G$ such that $ab = ba$ and $o(a) = n$, $o(b) = m$. (Hint: Let $a = z^m$ and $b = z^n$.)

5. pg 74. #1. Prove that every cyclic group is abelian.

6. Suppose a and b are integers that divide the integer c. If a and b are relatively prime, show that ab divides c. Show, by example, that if a and b are not relatively prime, then ab need not divide c.

7. Prove that every prime greater than 3 can be written in the form $6n + 1$ or $6n + 5$.

8. Prove that there are infinitely many primes. (Hint: Let p_1, p_2, …, p_n be primes. Show that $p_1 p_2 \cdots p_n + 1$ is divisible by none of these primes.)