Math 218: Elementary Number Theory

HOMEWORK 10 : DUE OCTOBER 30

2.4 #2. Prove that $\phi(n)$ is even if $n > 2$.

2.4 #4. Prove that the sum of the elements of a reduced residue system mod m is congruent to 0 mod m if $m > 2$.

2.4 #8. Suppose p is prime.

(a) Prove that if $p^2 \mid n$ then $\phi(n) = p\phi(n/p)$.

(b) Prove that if $p \mid n$ but $p^2 \nmid n$ then $\phi(n) = (p-1)\phi(n/p)$.

(c) Use (a) and (b) to prove that if n is odd then $\phi(2n) = \phi(n)$ and if n is even then $\phi(2n) = 2\phi(n)$.

2.4 #10/ #12 For both parts of this problem, you should prove that your answer is correct.

(a) Determine all n so that $\phi(3n) = 3\phi(n)$.

(b) Determine all integers n so that $\phi(n) = 4$.

2.5 #5. For p prime, prove that $\sigma(p) + \phi(p) = p\tau(p)$.