Math 218: Elementary Number Theory

HOMEWORK 15 : DUE NOVEMBER 25

1. Find all integers that give the remainders 1, 2, 3 when divided by 3, 4, 5, respectively.

3.2 #4. In the arithmetic progression $11x + 7$ for $x = 1, 2, 3, \ldots$ find three consecutive terms divisible by 2, 3, 5, respectively. You must use Theorem 3.2.2 to solve this.

3.2 #5. Find the solution of the simultaneous congruences

$$5x \equiv 2 \bmod 3$$

$$2x \equiv 4 \bmod 10$$

$$4x \equiv 7 \bmod 9$$

The last three problems are about the topics of injectivity and surjectivity. We did not cover these topics directly in class, but they are important to refresh from linear algebra. If you don't remember these definitions/concepts, come find me.

2. Let E be the set of even integers, and define a mapping f from $\mathbb{Z}$ to E so that $f(n) = 2n$. Prove or disprove that this map f is a bijection.

3. (a) Is the function $\sigma(n)$ injective? Why or why not?

   (b) Is the function $\tau(n)$ surjective? Why or why not?

4. (a) Define f to be a function from the set of *pairs* of natural numbers to the natural numbers defined as $f(m, n) = m^2 + n$. Prove that f is onto but not one-to-one.

   (b) Define g to be a function from pairs of integers to pairs of integers defined by $g(m, n) = (m + n, mn)$ (here we mean pairs of numbers (a, b), not the gcd of those numbers). Show g is not onto and is not one-to-one.