

---

# Math 218: Elementary Number Theory

HOMEWORK 6 : DUE SEPTEMBER 30

---

**Note:** *There will be no rewrite opportunities on this homework.*

1.9 #6. Is the function  $\pi(n)$  a multiplicative function? Why or why not? (Recall that  $\pi(n)$  is the number of primes less than or equal to  $n$ .)

1.9 #8. (a) If  $a$ ,  $b$ , and  $c$  are integers that are pairwise relatively prime, prove that  $\tau(abc) = \tau(a)\tau(b)\tau(c)$ .

**Warning:** In class we proved that when  $(a, b) = 1$  then  $\tau(ab) = \tau(a)\tau(b)$ . In this problem you need to carefully prove that this extends to the product of three integers. You should be careful not to already assume the result you need to carefully prove.

(b) Show by an explicit example that if we only assume  $(a, b, c) = 1$ , it may not be true that  $\tau(abc) = \tau(a)\tau(b)\tau(c)$ .

1.9 #12. Let  $a$  be a fixed integer. Define the function  $f(n) = (a, n)$ . Prove that  $f$  is multiplicative.

1.10 #8. Explain which  $n$  have  $\tau(n)$  equals the integers below. Your answer for each should explicitly describe how you know you have found all possible such  $n$ .

(a) 14

(b) 15

(c) 18

1.10 #9. Prove that  $n$  is a square if and only if  $\tau(n)$  is odd. (This was a conjecture many of you made on the first homework.)

1. A local restaurant offers a meat and cheese sandwich. You can choose one of three kinds of bread, one of four kinds of meat, and one of three kinds of cheese. How many sandwiches are possible? Assume that anyone ordering this particular sandwich wants a piece of bread and a piece of cheese and a piece of meat.

2. How many four digit numbers are there which are multiples of at least one of 2 or 5? Assume 0985 or 0027 are not four digit numbers (i.e. the thousands place should not be 0).

continued on the next page...

3. In poker, a hand consists of five cards dealt to a player. “Four of a kind” means your hand consists of four cards of the same face value. How many different “four of a kind” hands are there where we do not care what order the cards are dealt to you?

Deck of card background in case you need it. A standard deck of cards has 52 cards. There are four suits (hearts ♡, spades ♠, diamonds ◇, and clubs ♣) and each suit has 13 cards (the numbers 2 through 10 which I call the “number cards”, the Ace, and the “face cards”: king, queen, and jack). A “hand” is the set of cards you are dealt in a game like poker.