Math 321 Fall 2011
Homework 1
Due: September 2, 2011

You are welcome to work together but everyone needs to write up **distinct** solutions. If you use any books outside of our textbook or other people, please make sure to give them credit. Make sure your solutions are complete. If your handwriting is atrocious, I am happy to give you a basic introduction to LaTeX.

1. pg. 56 # 1. Let $\mathbb{Q}$ be the set of rational numbers and let $G = \mathbb{Q} - \{1\}$. Define a binary operation $\star$ on G by means of $a * b = a + b - ab$ for all $a, b \in G$. Show that the structure so defined is a group. (ADD: Why is $\star$ a binary operation?)
2. pg. 62 # 4. (TOP) Prove: If x is an element in a group G then $(x^{-1})^{-1} = x$. What is the corresponding equation if G is an additive group?
3. pg. 62 # 2. (BOTTOM) Let a, x, and y be elements in a group G. Prove $ax = ay$ if and only if $x = y$.
4. Prove that the set of all 2×2 matrices with entries from $\mathbb{R}$ and determinant $+1$ is a group under matrix multiplication. (This group is called the *special linear group*.)
5. Let G be a group with the following property: Whenever a, b, and c belong to G and $ab = ca$, then $b = c$. Prove that G is abelian.
6. Show that any finite group with 3 elements must be abelian.

Challenge

1. Show that any finite group with 4 elements must be abelian.