

Math 321 Fall 2011  
Homework 3  
Due: September 16, 2011

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You are welcome to work together but everyone needs to write up **distinct** solutions. If you use any books outside of our textbook or other people, please make sure to give them credit. Make sure your solutions are complete. If your handwriting is atrocious, I am happy to give you a basic introduction to L<sup>A</sup>T<sub>E</sub>X.

1. Given any  $\alpha \in S_n$  with  $\alpha \neq e$ , prove that there is some  $\beta \in S_n$  so that  $\alpha\beta \neq \beta\alpha$ .
2. (a) Let  $\alpha$  be the 12-cycle  $(1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12)$ . For which positive integers  $i$  is  $\alpha^i$  also a 12-cycle?  
(b) Let  $\beta$  be the 8-cycle  $(1\ 2\ 3\ 4\ 5\ 6\ 7\ 8)$ . For which positive integers is  $\beta^i$  also an 8-cycle?
3. Let  $\alpha \in S_n$  with  $\alpha = \alpha_1\alpha_2 \cdots \alpha_r$  where the  $\alpha_i$  are disjoint cycles. Prove that
$$o(\alpha) = \text{lcm}(o(\alpha_1), o(\alpha_2), \dots, o(\alpha_r)).$$
4. Find all numbers  $n$  such that  $S_5$  contains an element of order  $n$ .
5. If a permutation  $\alpha$  is odd, prove that  $\alpha^{-1}$  is odd.
6. Prove that for all  $n \geq 3$ ,  $A_n$  is generated by the set of 3-cycles.
7. (a) Write out a complete Cayley table for  $D_3$ .  
(b) Is  $D_3$  abelian?
8. For each  $D_n$ , prove that  $r^i s = s r^{-i}$  for all  $i \in \mathbb{Z}^+$ .

### Challenge

1. We showed in class that  $D_n$  is not abelian for  $n \geq 3$ . What elements in  $D_n$  commute with every other element in  $D_n$ ? Prove your assertion for all  $n \geq 3$ .