

Math 321 Fall 2016
Homework 4
Due: September 23, 2016

You are welcome to work together but everyone needs to write up **distinct** solutions. If you use any books outside of our textbook or other people, please make sure to give them credit. Make sure your solutions are complete. If your handwriting is atrocious, I am happy to give you a basic introduction to L^AT_EX.

1. Let $H < G$ where $H \neq G$. Prove that the set $S = G - H$ (the complement of H relative to G) is a set of generators of G .
2. # 7.1 f,h-j
3. # 7.7
4. As always, be sure to carefully justify your results.
 - (a) # 8.11
 - (b) # 8.12
5. # 8.27
6. Prove that for $n \geq 3$, the group A_n can be generated by 3-cycles.
7. (a) Let α be the 12-cycle $(1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12)$. For which positive integers i is α^i also a 12-cycle?
 - (c) If γ is an m -cycle, m a positive integer, for which positive integers i is α^i also an m -cycle?
8. Let $\alpha \in S_n$ with $\alpha = \alpha_1\alpha_2 \cdots \alpha_r$ where the α_i are disjoint cycles. Prove that

$$o(\alpha) = \text{lcm}(o(\alpha_1), o(\alpha_2), \dots, o(\alpha_r)).$$

9. If a permutation α is odd, prove that α^{-1} is odd.