Math 218 Spring 2017
Homework 13
Due: April 17

Green problems below should be done without consulting your classmates. You are still encouraged to ask Jun Taek and I about them, though!

1. Let E be the set of even integers, and define a mapping f from $\mathbb{Z}$ to E so that $f(n) = 2n$. Prove or disprove that this map f is a bijection.

2. (a) Define f to be a function from the set of *pairs* of natural numbers to the natural numbers defined as $f(m, n) = m^2 + n$. Prove that f is onto but not one-to-one.

   (b) Define g to be a function from pairs of integers to pairs of integers defined by $g(m, n) = (m + n, mn)$ (here we mean pairs of numbers (a, b), not the gcd of those numbers). Show g is not onto and is not one-to-one.

3. (a) Is the function $\sigma(n)$ injective? Why or why not?

   (b) Is the function $\tau(n)$ surjective? Why or why not?

4. Show that if p is a prime and p is at least 7, then there are always two consecutive quadratic residues of p. (Hint: First show that at least one of 2, 5, and 10 is a quadratic residue of p.)