Math 218: Elementary Number Theory

Homework 12 : Due November 11

2.6 #1. Find the multiplicative inverse of 5 mod 16 using Euler's theorem.

2.6 #8 Let p, as always, be a prime. If $a^p \equiv b^p \bmod p$, prove that $a \equiv b \bmod p$.

2.6 #11. If $a \equiv b \bmod p$ (with p prime), prove that $a^p \equiv b^p \bmod p^2$.

2.6 #9. (a) Find the remainder when 6^{385} is divided by 16.

(b) What are the last **two** digits of the ordinary decimal form of 3^{404}?

7.1 # 3. (a) Prove that the function $\omega(n)$ is additive but not completely additive. This function was defined in class (and in example 7.1.1) as the number of distinct primes that divide n.

(b) Is the function defined as $\nu(n) = a_1 + a_2 + \cdots + a_k$ where $n = p_1^{a_1} p_2^{a_2} \cdot p_k^{a_k}$ additive? Why or why not? If it is, is it completely additive?

7.1 # 6. Use induction and the definition of an additive function to prove Theorem 7.1.1.