

Math 321 Fall 2016
Homework 8
Due: November 4, 2016

You are welcome to work together but everyone needs to write up **distinct** solutions. If you use any books outside of our textbook or other people, please make sure to give them credit. Make sure your solutions are complete. If your handwriting is atrocious, I am happy to give you a basic introduction to L^AT_EX.

1. Let $G = \left\{ \begin{bmatrix} a & 2b \\ b & a \end{bmatrix} \mid a, b \in \mathbb{Q} \right\}$ and let $H = \{a + b\sqrt{2} \mid a, b \in \mathbb{Q}\}$
 - (a) Prove that G and H are isomorphic under addition.
 - (b) Notice that G and H are closed under multiplication (you can just check this for yourself). Does your isomorphism preserve multiplication as well as addition?
2. 13.8
3. If g and a are elements of a group, prove that $Z(a)$ is isomorphic to $Z(gag^{-1})$ (where $Z(a) = \{g \in G \mid gag^{-1} = a\}$ is the *centralizer* of the element a and not to be confused with the *center* of a group, see page 93).
4. 13.18 (We needed this to prove the Correspondence Theorem.)
5. 16.1
6. 16.9 (Part (a) should look familiar!)
7. 16.11
8. 16.15
9. 16.23
10. 16.24 (The mathematical concepts in this problem are some of the concepts we will study in more depth next semester in MAT-324.)