
Math 218: Elementary Number Theory

HOMEWORK 8 SUPPLEMENTAL PROBLEM

Blue problems below should be done without consulting your classmates. You are still encouraged to ask Hung or I about them, though!

1. Suppose that you are creating a password from 26 letters, 10 numbers, and 15 special characters. How many such 10-character passwords are possible if they must have exactly 6 letters, 2 numbers, and 2 special characters, and they must consist of 10 distinct symbols in the password (so *P1ssw0rd!!* is not a legitimate password because it contains two *s*'s and two *!*'s)?

2. (a) Prove that

$$3^n = \sum_{k=0}^n \binom{n}{k} 2^k.$$

- (b) Generalize part (a) to find the sum

$$\sum_{k=0}^n \binom{n}{k} r^k$$

for any real number r .

3. For $n \geq 1$, prove

$$\binom{n}{0} - \binom{n}{1} + \cdots + (-1)^n \binom{n}{n} = 0.$$

4. By integrating the equation in the binomial theorem (and setting $y = 1$), prove that, for a positive integer n ,

$$1 + \frac{1}{2} \binom{n}{1} + \frac{1}{3} \binom{n}{2} + \cdots + \frac{1}{n+1} \binom{n}{n} = \frac{2^{n+1} - 1}{n+1}.$$