Math 218: Elementary Number Theory

HOMEWORK 4 : DUE SEPTEMBER 20

§1.5 #7. If $k > 0$ prove that $[ka, kb] = k[a, b]$.

§1.6 #3. For all integers x, prove that $(15x + 17, 10x + 11) = 1$.

§1.6 #4. If $(a, b) = 1$ and $d' \mid a$ and $d'' \mid b$, prove that $(d', d'') = 1$.

§1.6 #5. If $(a, b) = d$ and $(a, c) = f$ and $(b, c) = 1$, prove that $(d, f) = 1$.

§1.6 #7. (a) If $a \mid c$ and $b \mid c$ and $(a, b) = 1$, prove that $ab \mid c$.

(b) Given an example to show that the statement in (a) need not be true if $(a, b) \neq 1$.

§1.6 #8. (a) If $(a, b) = d$ and $(a, c) = f$ and $(b, c) = 1$, prove that $(a, bc) = df$.

(Hint: Let $(a, bc) = k$. Use problems 5 and 7, plus characterization (2) of the greatest common divisor to prove that $k \mid df$ and $df \mid k$.)

(b) Give an example to show that (a) need not be true if $(b, c) \neq 1$.