Math 218: Elementary Number Theory

Homework 7 : Due October 9

1.11 #1. Calculate $\sigma(72)$, $\sigma(250)$ and $\sigma(8000)$. You can use minimal words on this problem.

1.11 #7. For what values of n does $\sigma(n) = b$ if:

(a) $b = 14$ (b) $b = 15$

(c) $b = 16$ (d) $b = 18$

2.1 #7. Prove that if $a \mid b$ then $M_a + \{r\} \supseteq M_b + \{r\}$.

2.1 #9. Prove that if $M_m + \{r_1\} \neq M_m + \{r_2\}$ then they are disjoint (i.e. $M_m + \{r_1\} \cap M_m + \{r_2\}$ is empty). This problem shows that the residue classes form a *partition* of the integers.

2.2 #1. If $a \equiv b \bmod m$ and $c \equiv d \bmod m$, prove that $ax + cy \equiv bx + dy \bmod m$ for any integers x and y.

1. Determine whether each of the following relations is reflexive, symmetric, and transitive (you should check each individual property, not all three at once). If a certain property fails, you should give a specific counterexample. (This problem will be worth 6 points.)

   a. $S = \mathbb{Z}$ where $a \sim b$ means $a - b \neq 1$.

   b. $S = \mathbb{Z}$ where $a \sim b$ means that both a and b are even.

   c. $S = \mathbb{Z}$ where $a \sim b$ means $a \mid b$.

2. Let r and s be real numbers. Define $r \sim s$ whenever $r - s$ is an integer. Prove that this is an equivalence relation and describe what the equivalence classes look like.

3. A friend tries to convince you that the reflexive property is redundant in the definition of an equivalence relation because they claim that symmetry and transitivity imply it. Here is the argument they propose:

   "If $a \sim b$, then $b \sim a$ by symmetry, so $a \sim a$ by transitivity. This gives the reflexive property."

   Now you know that their argument must be wrong because one of the examples in Problem 1 is symmetric and transitive but not reflexive. Pinpoint the error in your friend's argument. Be as explicit and descriptive as you can.