
Math 218: Elementary Number Theory

HOMEWORK 3 : DUE SEPTEMBER 16

§1.4 #3. If $a = 4q_1 + 3$ and $b = 4q_2 + 3$ prove that $ab = 4q_3 + 1$ where q_1 , q_2 , and q_3 represent integers.

§1.4 # 9. Prove that 3, 7, 11 is the only set of three consecutive *primes* of the form c , $c + 4$, $c + 8$.

§1.4 # 12. If a product of primes is of the form $4q + 3$ prove that at least one of the primes must have this form. *Hint*: Use problem §1.4 # 2 from the last assignment.

§1.5 #2. (a) Write several elements of the set $M_4 + M_{10}$. You do not need to include words with this answer.

(b) What is the least positive element of the set? Explain how you know your answer is correct.

(c) Write your answer from (b) in the form $4x + 10y$. You do not need to include words with this answer.

(d) Is the set $M_4 + M_{10}$ closed under multiplication and subtraction? WHY or why not?

§1.5 # 3. Prove that $M_{12} \subseteq M_6$.

§1.5 # 4. (a) Prove that $M_4 \subseteq M_4 + M_{10}$.

(b) Prove that M_4 and $M_4 + \{1\}$ have no elements in common.

§1.5 # 6. Assume $a \mid b$.

(a) Prove that $M_b \subseteq M_a$.

(b) Prove that $[a, b] = |b|$.