Math 133: Calculus II

Final Exam Review

This is just a guide to help you study. I do not guarantee that anything will or will not be on the exam based on this guide.

1 Basics

The exam will be **Friday, December 20 2-5PM in our classroom for the 1 PM section and Wednesday, December 18, 2-5PM in Noyce 2022 for the 2:30 PM section**. No books or notes or cell phones. You may use a scientific calculator and the unit circle (provided by me). The formulae from Exam 3 will be provided.

Office Hours

I will be in my office

- Monday, December 16 from 1-4 PM
- Tuesday December 17th from 10:30 AM -12:30 PM and 3:00 PM -5:00 PM
- Thursday, December 19th from 11 AM -3 PM
- Friday, December 20th from 10:30 AM -12:30 PM

2 Suggestions

- Work lots and lots of problems, especially those on material you don't understand as well.
- When possible, ask yourself WHY you are solving a problem a certain way or WHY the result is true.
- Do not look at solutions unless you are desperate.
- Check your work!!

3 Material

Integration Techniques

We covered material from 8.1-8.4 including:

- integration by parts
- trig integrals
- trig substitution
- integration by partial fractions

Parametric/Polar Equations

We covered material from 11.1-11.3 including:

- parametric equations and graphs
- arc length
- polar coordinates and graphs

Vectors

These topics are in 13.1-13.5 and 14.1-14.2.

- vectors
- dot and cross products
- equations of lines and planes
- vector functions
- space curves
- derivative of vector function

Partial Derivatives

We covered material from 15.1 and 15.3-15.8.

- functions of several variables
- partial derivatives
- tangenet plane and linear approximation
- chain rule
- directional derivatives
- gradient vector
- maximums and minimums (local an absolute)
- Lagrange multipliers

Double and Triple Integrals

Sections covered on this material were 16.4. 16.6-16.9

- Riemann sums for multivariable functions
- iterated integrals
- double integrals over a general region
- double intergrals in polar coordinates
- triple integrals
- triple integrals in cylindrical coordinates
- Jacobian
- change of variables

Vector Fields

This material is from 17.1-17.4

- vector fields
- line integrals
- gradient vector fields and consequences
- Green's Theorem

Not on the Final

- limits and continuity of multivariable functions
- trapezoid/midpoint/Simpson's Rule
- spherical coordinates

4 Practice Problems

- See the previous exam reviews for practice problems from the material covered on the first 3 exams.
- pg. 1057 **Concept Check**: 8a, 9ab
- pg. 1058-1059 **Exercises**: 26-27, 32, 34, 47
- pg. 1142 **Concept Check**: 1,2, 4, 5, 6, 7
- pg. 1143-1144 **Exercises**: 1a, 9-17

5 Sample Problems From Chapter 16 and 17

These are questions from the material covered since the last exam. For sample problems for the material covered before that, see the previous exam review sheets.

1. Evaluate $\iiint\limits_R z$ where R is the region between $x^2 + y^2 = z$ and $z = 9$.

2. (a) Find the image of the region S defined as the triangle with coordinates $(0, 0)$, $(1, 0)$, $(1, 1)$ under the transformation $T(u, v) = (u^2 - v^2, 2uv)$.

   (b) Let D be the region found in (a). Compute $\iint\limits_D \sqrt{x^2 + y^2}\ \mathrm{d}x\ \mathrm{d}y$ using the transformation from (a).

3. Find the gradient vector field ∇f of $f(x, y) = x^2 y$ and sketch it.

4. Evaluate the line integral $\int_C \mathbf{F} \bullet\ \mathrm{d}\mathbf{r}$ where $\mathbf{F}$ is the vector field $\langle y^2, x^2 \rangle$ and C is the curve $y = x^{-1}$ for $1 \leq x \leq 2$.

5. Find the work done by the force field $\mathbf{F}(x, y) = \langle x^2, y - x \rangle$ in moving an object along the curve given by $\mathbf{r}(t) = \langle (1 + \sin t), (t - \cos t) \rangle$ from $t = 0$ to $t = \pi$.

6. Determine whether or not $\mathbf{F}$ is a conservative vector field. If it is, find a function f so that $\mathbf{F} = \nabla f$ and compute $\int_C \mathbf{F} \bullet\ d\mathbf{r}$ where C is the parametrization $x = t^2$ and $y = 1/t$ for $1 \le t \le 2$.

   (a) $\mathbf{F}(x, y) = \langle \frac{x}{y^2+1}, \frac{y}{x^2+1} \rangle$

   (b) $\mathbf{F}(x, y) = \langle 2xy + y^3, x^2 + 3xy^2 + 2y \rangle$

7. Use Green's Theorem to evaluate $\int_C \mathbf{F} \bullet\ d\mathbf{r}$ where $\mathbf{F}(x, y) = \langle x + y, x^2 - y \rangle$ and C is the boundary of the region defined by $y = x^2$ and $y = \sqrt{x}$, negatively oriented from $0 \le x \le 1$.