# Math 218: Elementary Number Theory

Homework 2 : Due September 11

§1.3 #4. Prove that 3 divides $2^{2n} - 1$ for every positive integer $n$.

§1.3 # 7. If $x \neq 1$ prove that for every positive integer $n$

$$\frac{1 - x^n}{1 - x} = 1 + x + \cdots + x^{n-1}.$$

§1.3 # 10. Prove that for any positive integers $k$ there exist sequences of $k$ consecutive composite integers. For example, when $k = 3$, the sequence $14, 15, 16$ is 3 consecutive composite integers.

§1.4 #2. If $p$ is a prime greater than 4, prove that $p$ has the form $4k + r$ where $r = 1$ or $r = 3$.

1. Prove by induction on $n \geq 3$ that $n^2 \geq 3n$.

2. A chocolate bar consists of $n$ squares arranged in a rectangular pattern. You split the bar into small squares, always breaking along the lines between the squares. Use induction to prove that no matter how you break the bar, it takes $n-1$ breaks to split it into the $n$ smaller squares.

   Comment: Chocolate bars are not necessarily one long line of rectangles. When $n = 6$ the bar could consist of 6 small squares in a row, or it could consist of two rows of 3 squares each.

   Here is a picture of a chocolate bar, and some physics on why they typically break at the seams: `http://physics.stackexchange.com/questions/238202/why-do-chocolate-bars-usually-break-at-the-cleavages`