Math 218: Elementary Number Theory

Homework 1 : Due September 6

§1.1 #6. Prove for every non-zero integer a that $a \mid \pm a$.

§1.1 # 8. Let a, b, m, and $n \in \mathbb{Z}$. Prove that if $a \mid m$ and $b \mid n$ that $ab \mid mn$.

§1.1 # 9. Prove that if there exist integers x and y such that $9x + 12y = n$ then $3 \mid n$.

§1.1 #10. Let a, b, c, and $d \in \mathbb{Z}$ and assume $a + b = c$.

(a) Prove that if d divides any two of the integers a, b, and c, then d divides all three of them. (We will use this result frequently throughout the semester.)

(b) Use (a) to prove that if $d \mid c$ then d divides both a and b or d divides neither a nor b.

(c) Give examples to show that both situations in (b) do occur.

§1.1 #11. If an integer a divides $12n + 5$ and $4n + 2$ for some $n \in \mathbb{Z}$, prove that $a = \pm 1$.

§1.1 # 14. Let a_0, a_1, and a_2 be integers.

(a) Prove that if x is an integer and $a \mid x$ that $a \mid a_1x + a_2x^2$.

(b) Use (a) to prove that if x is a nonzero integer such that $a_2x^2 + a_1x + a_0 = 0$ then $x \mid a_0$.

(c) Based on (b), what integers would you have to try if you wanted to find an integer solution to the equation $x^2 + 4x + 3 = 0$? Why only those integers?

(d) Generalize (b)-(c) for the case of a polynomial equation of degree n with a_i the integer coefficients, i.e. suppose that $a_nx^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x + a_0 = 0$. Now what can you say about potential integer solutions x?

§1.2 # 7. Consider the table on page 8 of the book. For what integers is $\tau(n)$ odd? What common property do those integers share?

§1.2 # 13. Write a definition for prime numbers which uses the function $\sigma(n)$. In particular, your definition should apply to all prime numbers and no composite numbers.