Math 218: Elementary Number Theory

HOMEWORK 11 : DUE NOVEMBER 6

2.5 #4. (a) For an arbitrary prime power p^k, determine what the following sum is.

$$\sum_{d|p^k} \phi(d).$$

(b) Now define the function $F(n)$ as $F(n) = \sum_{d|n} \phi(d)$. We proved ϕ is multiplicative already so use that fact and the value you found in (a) to determine what $F(n)$ is.

2.5 #9. Let $F(n) = \sum_{d|n} f(d)g\left(\frac{n}{d}\right)$ where f and g are both multiplicative functions. Prove that F is also multiplicative.

2.5 #10. Prove that for any positive integer n

$$\sum_{d|n} \sigma(d)\phi\left(\frac{n}{d}\right) = n\tau(n).$$

1. Suppose you toss a 6-sided dice 10 times and record the number on the top of the dice each time. Use Inclusion-Exclusion to determine the number of ways those dice could be thrown so that each of the 6 numbers occur at least once in your list of 10 numbers. Here we assume tossing a 1 and then nine 6's is different than tossing nine 6's first and then a 1.

2. Use Inclusion-Exclusion to determine the number of permutations of the set $\{1, 2, \ldots, 9\}$ in which at least one odd integer is fixed.

   A *permutation* is an arrangement of a list of n elements, so for $n = 3$ the 6 permutations are: $1, 2, 3 \quad 1, 3, 2 \quad 2, 1, 3 \quad 2, 3, 1 \quad 3, 1, 2 \quad 3, 2, 1.$

   Here we say $1, 2, 3$ and $1, 3, 2$ both fix 1, while $1, 2, 3$ and $3, 2, 1$ both fix 2.

3. Suppose n students go to the DHall for dinner and they leave their coats outside. How many ways can the students pick up their coats after dinner so that no one gets their original coat back? Your final answer will have n in it.