

Aufgabenblatt 6

Aufgabe 1

$$\begin{aligned} s_y^2 &= \frac{1}{n} \sum_{i=1}^n (y_i - \bar{y})^2 && \text{einsetzen} \\ &= \frac{1}{n} \sum_{i=1}^n (\alpha + \beta x_i - (\alpha + \beta \bar{x}))^2 \\ &= \frac{1}{n} \sum_{i=1}^n (\alpha + \beta x_i - \alpha - \beta \bar{x})^2 \\ &= \frac{1}{n} \sum_{i=1}^n (\beta x_i - \beta \bar{x})^2 && \text{f\u00fcr } x \\ &= \frac{1}{n} \beta^2 \sum_{i=1}^n (x_i - \bar{x})^2 && s_x^2 = \frac{1}{n} \sum_{j=1}^n (x_j - \bar{x})^2 \\ &= \beta^2 s_x^2 \end{aligned}$$

Aufgabe 2

$$\begin{aligned} s^2 &= \frac{1}{n} \sum_{i=1}^n x_i^2 - \bar{x}^2 \\ s^2 &:= \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2 && \text{Bin. Form.} \\ &= \frac{1}{n} \sum_{i=1}^n x_i^2 - \frac{1}{n} \sum_{i=1}^n 2x_i \bar{x} + \frac{1}{n} \sum_{i=1}^n \bar{x}^2 \\ &= \frac{1}{n} \sum_{i=1}^n x_i^2 - 2 \cdot \frac{1}{n} \cdot \bar{x} \cdot \sum_{i=1}^n x_i + \frac{1}{n} \cdot n \cdot \bar{x}^2 \\ &= \frac{1}{n} \sum_{i=1}^n x_i^2 - 2 \cdot \bar{x} \cdot \bar{x} + \bar{x}^2 \end{aligned}$$

$$= \frac{1}{N} \sum_{j=1}^N x_j^2 - \bar{x}^2 //$$

Aufgabe 3 Zentrierter Parameter

$$\Theta_k^r = \Theta_k^r - 3 \quad \text{rel. Kurtosisparameter}$$

$$\Theta_k^r = \frac{\Theta_{11}^r}{s^4} \quad \begin{array}{l} \text{abs. Kurtosisp} \\ s^4 = \text{quadratische Varianz} \end{array}$$

$$\Theta_k = \frac{1}{n} \sum_{j=1}^n (x_j - \bar{x})^4$$

1. Schritt $\bar{x} = 6,25$

$$\Theta_k = \frac{1}{20} \left((9 - 6,25)^4 + (7 - 6,25)^4 + \dots + (5 - 6,25)^4 \right)$$

$$= 7,682071 \approx 7,682$$

2. Schritt

$$s^2 = 1,7875; s^4 = 3,1952$$

$$\Theta_k^r = \frac{7,6820}{3,1952} = 2,4042$$

3. Schritt

$$\theta_k^N = \theta_{12}^r - 3 = 2,4042 - 3 = -0,5958$$

Aufgabe 4 $\bar{x} = 6,55$

Schritt 1

$$\begin{aligned}\sigma_k &= 0,3 (5 - 6,55)^4 + 0,25 (6 - 6,55)^4 + 0,15 (7 - 6,55)^4 \\ &\quad + 0,2 (8 - 6,55)^4 + 0,1 (9 - 6,55)^4 \\ &= 6,2477\end{aligned}$$

2. Schritt

$$s^2 = 1,8475; \quad s^4 = 3,413$$

$$\sigma_k^N = \frac{6,2477}{3,413} = 1,8304$$

3. Schritt

$$\sigma_k^N = 1,8304 - 3 = -1,1696$$