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## Multivariate Time Series Analysis

### Exercise Sheet 8

#### Exercise 1: Forecast Intervals and Distributional Assumptions

- a) Which key assumption about the innovations  $a_t$  is made in the lecture to derive the distribution of the forecast errors  $e_T(h)$ ?
- b) Assume we knew all parameters / coefficients and let  $\Sigma_a$  be the identity matrix  $I_{3 \times 3}$ . Based on the assumptions from (a), derive the distribution of  $e_T(1)$  for any stationary VAR(p).
- c) Derive the confidence ellipsoid associated to b) for  $\alpha = 5\%$ . What is the fraction of forecast errors that lie inside the ellipsoid?
- d) Run a simulation in R: Draw the forecast error  $e_T(1)$  defined in a) and b) with  $K = 3$ . Check if it is located inside or outside the confidence ellipsoid derived in c). Use  $N = 10000$  repetitions in total and conclude whether the confidence ellipsoid is appropriate.
- e) Repeat the simulation from above but this time assume  $a_t$  to be drawn from a uniform distribution.  $\Sigma_a = I_{3 \times 3}$  remains. How reliable is the confidence ellipsoid in this case?  
*Hint: Set  $\pm \frac{\sqrt{12}}{2}$  as lower / upper bound for unit variance.*
- f) Repeat the simulation drawing innovations from a t-distribution with 2 degrees of freedom and conclude.

*This exercise sheet will be discussed in the tutorial on Wednesday, 11 December 2019*