

Winter Term 2019/2020

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## Multivariate Time Series Analysis

### Solution Exercise Sheet 3

#### 1 Exercise 1: VAR(1) Moments and Stationarity

Take the VAR(1) model  $z_t = \phi_0 + \phi_1 z_{t-1} + a_t$  with the following parameterisation:

$$\phi_0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \phi_1 = \begin{pmatrix} 0.75 & 0 \\ -0.25 & 0.5 \end{pmatrix}, \quad \Sigma_a = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

- a) Compute the mean of the process.

*Solution:*

$$\begin{aligned} E(\cdot) \underbrace{\mathbb{E}(z_t)}_{\mu} &= \mathbb{E} \left( \underbrace{\phi_0}_{\phi_0} + \underbrace{\phi_1 z_{t-1}}_{\phi_1 \cdot \mu} + \underbrace{a_t}_0 \right) \\ &= [\text{Key assumption? } \mathbb{E}(z_t) = \mathbb{E}(z_{t-1})] \end{aligned}$$

- b) Show that the process is stationary.
- c) Derive the Yule-Walker equations for the lag  $l = \{0, 1, 2\}$  and show that the solution for  $\Gamma_0$  coincides with equation (2.3) on slide 2-15.
- d) Compute  $\Gamma_0$  and  $\Gamma_1$  by hand based on your results from c).

#### 2 Exercise 2: Stationarity of VAR(p) Processes

Using the notation of Slide 2-27, prove that  $|I_{kp} - \Phi_1 z| = |I_k - \phi_1 z - \dots - \phi_p z^p|$ . Recall that  $|A|$  denotes the determinant of the matrix  $A$ .

*Hint: Derive  $\Phi_1$  and keep it mind that adding multiples of columns/rows to other columns/rows does not affect the determinate! The plan is to end up with a special matrix.*

### 3 Exercise 3: VAR(2) Moments and Stationarity

Consider the following VAR(2) model with i.i.d. innovations:

$$\phi_0 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \quad \phi_1 = \begin{pmatrix} 0.5 & 0.1 \\ 0.4 & 0.5 \end{pmatrix}, \quad \phi_2 = \begin{pmatrix} 0 & 0 \\ 0.25 & 0 \end{pmatrix}, \quad \Sigma_a = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

- a) Show that the process is stationary.
- b) Determine the mean vector.
- c) Derive the Yule-Walker equations for the lags  $l = \{0, 1, 2\}$  for a general VAR(2) process.
- d) Suppose we only have  $\Gamma_0, \Gamma_1$  and  $\Gamma_2$  - how can we estimate  $\phi_1$  and  $\phi_2$  from it?
- e) Write the process as a VAR(1) and calculate the mean vector again.
- f) Compute  $\Gamma_0$  based on the VAR(1) formulation.

*Hint: You can use R for the calculations.*