Exercise Sheet 8

1

a) $a_t \overset{iid}{\sim} N(0, \Sigma_a)$

no auto-correlation — distributional assumption — homoscedastic

b) $e_T(1) = z_{T+1} - z_T(1)$

$= a_{T+1}$

holds for any VAR(p) since

$z_T(1) = \mathbb{E}(z_{T+1} | z_T, \dots, z_0)$

$$\Rightarrow e_T(1) = a_{T+1} \sim N(0, I_{3\times 3})$$

c) Ellipsoid:

$$\left\{ z \in \mathbb{R}^3 : (z_T(1) - z)' \Sigma_e^{-1}(1) (z_T(1) - z) \leq \chi^2_{3,1-\alpha} \right\}$$

By defining $z_T(1) - z =: \varepsilon$ and using that $\Sigma_e(1) = \Sigma_a = I_{3\times 3}$ the ellipsoid is: $\left\{ \varepsilon \in \mathbb{R}^3 : \varepsilon' \varepsilon \leq \chi^2_{3,1-\alpha} \right\}$

For $\alpha = 5\%$, 95% of the observed forecast errors are expected to fall inside the confidence ellipsoid.

d) Just check if

$$e_T(1)' e_T(1) \leq \chi^2_{3,0.95}$$ and

compute $\frac{1}{N}\sum_{i=1}^{N} \mathbb{I}\left(e_T^{(i)}(1)' e_T^{(i)}(1) \leq \chi^2_{3,0.95}\right).$

$\mathbb{I}$ = indicator function

→ R

✓

e) → R

It's too conservative.

f) $\to R$

Too liberal, the ellipsoid is not appropriate.