Universität Duisburg-Essen
Fachbereich VWL
Lehrstuhl Ökonometrie

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Dr. Yannick Hoga Thilo Reinschlüssel

Multivariate Time Series Analysis
Exercise Sheet 9

Exercise 1: Granger Causality – Theory

Let $z_t = (x_t, y_t)'$ be a stationary time series with two dimensions. Define the forecast error as the univariate series $e_T(h) = y_{T+h} - y_T(h)$ with $y_T(h) = E(y_{T+h}|\Omega_T)$. The information set Ω_T contains all relevant variables available whereas $\Omega_T^{\setminus x} = \Omega_T \setminus \{x_t\}_{t=0}^T$ omits the variable x. entirely. (This setting is the univariate equivalent to Definition 6.1 on Slide 6-4.)

a) Prove that $E\left(e_T(h)|\Omega_T^{\setminus x}\right) = 0$.

b) Prove that $Var\left(e_T(h)|\Omega_T\right) \leq Var\left(e_T(h)|\Omega_T^{\setminus x}\right)$.

Exercise 2: Granger Causality and IRFs in Data

We return to the dataset 'fx_series.Rda' and examine Granger (Non)-Causality and the Impulse Response Functions (IRFs). Remember that this dataset contains two time series of exchange rates. For the tasks below, please use the package 'vars' with the commands *causality* and *irf*.

a) Do you find any Granger Causality in a VAR(1) model? Which zero restrictions are implied for the coefficient matrix ϕ_1?

b) Is there evidence for instantaneous causality? What are the implications regarding Σ_a?

c) Before you plot the IRFs, make a guess about their appearance based on Granger Causality. Then compute the IRFs (do not use orthogonal innovations!) for five periods and comment.

Exercise 3: Granger Non-Causality and IRFs (if time permits)

Consider a general three-dimensional VAR(1) in which the first variable $z_{1,t}$ does not Granger cause the other variables. Show that a shock $a_{1,T}$ does not affect $\{z_{2,t}\}_{t=T}^{\infty}$ and $\{z_{3,t}\}_{t=T}^{\infty}$. (This is an example for Remark 6.10.)

This exercise sheet will be discussed in the tutorial on Wednesday, 18 December 2019