University of Duisburg-Essen
Faculty of Business Administration and Economics
Chair of Econometrics

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Multivariate Time Series Analysis

Exercise Sheet 2

1 Exercise 1: Moments and Simulation of a VAR(1) Process

Take the model from Example 2.4 on Slide 2-6:

a) Derive a formula to obtain the population cross-covariance matrices for the lags 1 to 10 and compute them using R

$$z_t = \phi_1 z_{t-1} + a_t$$

$$\begin{aligned}
\Gamma_0 &= \phi_1 \; \Gamma_0 \; \phi_1^{'} + \Sigma_a \\
\Leftrightarrow \text{vec}(\Gamma_0) &= (\phi_1 \otimes \phi_1) \cdot \text{vec}(\Gamma_0) + \text{vec}(\Sigma_a) \\
\Leftrightarrow \text{vec}(\Gamma_0) &= (I_{K^2} - \phi_1 \otimes \phi_1) \cdot \text{vec}(\Sigma_a) \\
\Rrightarrow \Gamma_1 &= \phi_1 \Gamma_0 \\
\Rrightarrow \Gamma_l &= \phi_{l-1} \Gamma_{l-1} = \phi_1^l \Gamma_0
\end{aligned}$$

```r
# First define parameters/coefficients:
Phi <- matrix(data = c(0.2, -0.6, 0.3, 1.1),
              byrow = FALSE, nrow = 2) # VAR coefficients
Sigma_a <- matrix(data = c(1, 0.8, 0.8, 2.0),
                  byrow = FALSE, nrow = 2) # innovations' covariances

Phi_kron <- kronecker(X = Phi, Y = Phi) # kronecker product
# Phi %x% Phi # alternative command

Ident <- diag(ncol(Phi)^2) # identity matrix with the same dimensions as Phi_kron
```

```r
Gamma0.vec <- solve(Ident - Phi_kron) %*% c(Sigma_a)
# c() works like the "vec" operator

Gamma0.mat <- matrix(data = Gamma0.vec, nrow = 2, byrow = FALSE)
Gamma0.mat
```

```
##          [,1]     [,2]
## [1,] 2.288889 3.511111
## [2,] 3.511111 8.622222
```

Hint: A glance at the slides and a loop might save you some time

b) Based on your results, compute the cross-correlation matrices

c) Draw a corresponding innovation sequence at for 300 periods from a (multivariate) Gaussian distribution and simulate the given VAR(1) process without any further built-in functions

Hint: 'mvrnorm' and 'for' are still allowed

d) Plot the multivariate time series you have just created. Does it look stationary?

e) Estimate the sample cross-covariance and cross-correlation matrices. Compare these with the population moment matrices from task a)

2 Exercise 2: Checking VAR(1) Stationarity

Recall the conditions to check if a VAR(1) process is stationary. Now assume the VAR(1) model $z_t = \phi_1 z_{t-1} + a_t$ with a_t as asequences of $i.i.d.$ innovations:

a) Do you need to make further assumptions on the cross-correlations of a_t to ensure stationarity

b) Which of the following processes are stationary? $\phi_1 = \ldots$

$$\text{i)} \begin{pmatrix} 0.2 & 0.3 \\ -0.6 & 1.1 \end{pmatrix} \text{ ii)} \begin{pmatrix} 0.5 & 0.3 \\ 0 & -0.3 \end{pmatrix} \text{ iii)} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \text{ iv)} \begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix} \text{ v)} \begin{pmatrix} 1 & -0.5 \\ -0.5 & 0 \end{pmatrix}$$