Universität Duisburg-Essen
Fachbereich VWL
Lehrstuhl Ökonometrie

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Multivariate Time Series Analysis
Exercise Sheet 7

Exercise 1: The optimal forecast

a) Show that the stationary VAR(1) process $z_t = \phi z_{t-1} + a_t$ with a_t a standard white noise has the following causal representation:

$$z_t = \sum_{i=0}^{\infty} \theta_i \, a_{t-i}.$$

b) Assume the linear forecasting model $y_T(h) = \Psi \, y_T$ and show that $\Psi = \phi^h$ minimises the MSE of $y_T(h)$ given that y_t is a VAR(1) process.

Exercise 2: Properties of forecast errors

a) Show that for a general VAR(p) process

$$z_{T+h} - z_T(h) = e_T(h) = \sum_{i=0}^{h-1} \theta_i a_{T+h-i},$$

where $z_T(h)$ is assumed to be the optimal forecast.
Hint: (5.9)

b) Assume that $a_t \sim N(0, \Sigma_a)$. Derive the distribution of $e_T(h)$.

c) Prove that $Cov\left(e_T(h)\right) \to \Gamma_0$ as $h \to \infty$.

Exercise 3: Forecast intervals

Derive the confidence ellipsoid for $e_T(h)$ (see slide 5-14) from (5.9) based on your results in Exercise 2.

Exercise 4: Delta Method

For this task, assume both y_t and x_t to be $K \times 1$ vectors and $x_t \overset{iid}{\sim} [\mu_x, \Sigma_x]$.

a) Let $y_t = f(x_t) = \phi_1 x_t$. Compute the mean and variance of y_t.

b) Derive the distribution of $\sqrt{T}\,(\overline{y_T} - E(y))$ from your results in a).

c) Now let $f(\cdot)$ be some function $f(x) : \mathbb{R}^k \mapsto \mathbb{R}^k$. Derive the first order Taylor expansion for $f(x)$ at μ_x and write it down in detail.
Hint: You need the Jacobian matrix.

d) Based on the expression obtained in c), show that a CLT applies for $\sqrt{T}\,(f\,(\overline{x_T}) - f\,(\mu_x))$, and derive the distribution.
Hint: Factor out deterministic parts. Since $f(\cdot)$ is deterministic, $f(c)$ is deterministic if c is.

e) Lastly, assume the variable x_t to be known (meaning it is not stochastic). We want to predict y_t using $y_t = \phi_1 x_t$. Unfortunately, we only have $\widehat{\phi}_1$ which is stochastic with $\sqrt{T}\left(\widehat{\phi}_1 - \phi_1\right) \xrightarrow{d} N\left(0, \Sigma_{\phi_1}\right)$. Can we say something about the distribution of the prediction error $\widehat{y}_t - y_t$?

This exercise sheet will be discussed in the tutorial on Wednesday, 4 December 2019