University of Duisburg-Essen
Faculty of Business Administration and Economics
Chair of Econometrics

Winter Term 2019/2020

Dr. Yannick Hoga Thilo Reinschlüssel

Multivariate Time Series Analysis

Solution Exercise Sheet 3

1 Exercise 1: VAR(1) Moments and Stationarity

Take the VAR(1) model $z_t = \phi_0 + \phi_1 z_{t-1} + a_t$ with the following parameterisation:

$$\phi_0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \phi_1 = \begin{pmatrix} 0.75 & 0 \\ -0.25 & 0.5 \end{pmatrix}, \quad \Sigma_a = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

a) Compute the mean of the process.

Solution:

$$\underset{\Rightarrow}{E(\cdot)} \underbrace{\mathbb{E}(z_t)}_{\mu} = \mathbb{E}\left(\underbrace{\phi_0}_{\phi_0} + \underbrace{\phi_1 z_{t-1}}_{\phi_1 \cdot \mu} + \underbrace{a_t}_{0} \right)$$

[Key assumption? $\mathbb{E}(z_t) = \mathbb{E}(z_{t-1})$]

$$\Leftrightarrow (I - \phi_1) \cdot \mu = \phi_0$$

b) Show that the process is stationary.

c) Derive the Yule-Walker equations for the lage $l = \{0, 1, 2\}$ and show that the solution for Γ_0 coincides with equation (2.3) on slide 2-15.

d) Compute Γ_0 and Γ_1 by hand based on your results from c).

2 Exercise 2: Stationarity of VAR(p) Processes

Using the notation of Slide 2-27, prove that $|I_{kp} - \mathbf{\Phi_1} z| = |I_k - \phi_1 z - \ldots - \phi_p z^p|$. Recall that $|A|$ denotes the determinant od the matrix A

Hint: Derive $\mathbf{\Phi}_1$ and keep it mind that adding multiplies of columns/rows to other columns/rows does not affect the determinate! The plan is to end up with a special matrix.

3 Exercise 3: VAR(2) Moments and Stationarity

Consider the following VAR(2) model with i.i.d. innovations:

$$\phi_0 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \quad \phi_1 = \begin{pmatrix} 0.5 & 0.1 \\ 0.4 & 0.5 \end{pmatrix}, \quad \phi_2 = \begin{pmatrix} 0 & 0 \\ 0.25 & 0 \end{pmatrix}, \quad \Sigma_a = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

a) Show that the process is stationary.

b) Determine the mean vector.

c) Derive the Yule-Walker equations for the lage $l = \{0, 1, 2\}$ for a general VAR(2) process.

d) Suppose we only Γ_0, Γ_1 and Γ_2 - how can we estimate ϕ_1 and ϕ_2 from it?

e) Write the process as a VAR(1) and calculate the mean vector again.

f) Compute Γ_0 based on the VAR(1) formulation.

Hint: You can use R for the calculations.