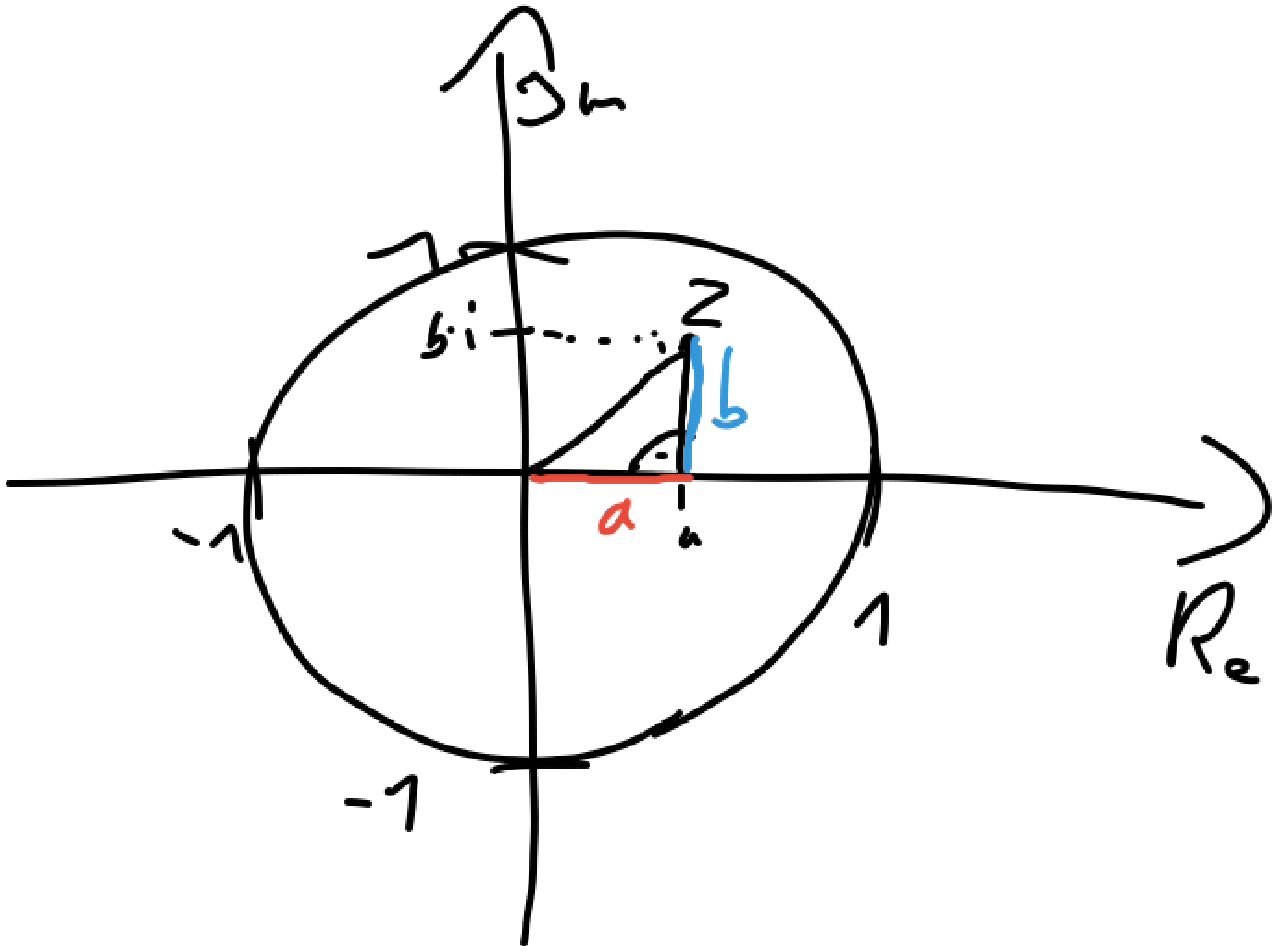
Notes on complex roots

Reminder: All roots z of the (reverse) characteristic polynomial must lie inside the complex unit circle.

Often enough, we have complex numbers '$a + bi$' with $i^2 = -1$ as solutions.

real ↑ (a), ↑ imaginary (bi)

Luckily, we can map those numbers using a standard basis, so Pythagoras' theorem helps us out.



$$= r = |z| = \sqrt{a^2+b^2}$$

And the complex part matters, since we are interested in $\lim\limits_{j\to\infty} \varphi^j$:

$$\Rightarrow \lim_{j\to\infty} y^j = \frac{1}{z^j}$$

Note that for $z = a + b\cdot i$,

$$z^j = z^2 \cdot z^{j-2}$$

and $z^2 = (a + b \cdot i)^2$

$$= a^2 + b^2 \cdot i^2 + 2a \cdot b \cdot i$$

$$= \underbrace{a^2 - b^2}_{\text{Real}} + \underbrace{2abi}_{\text{Imaginary}}$$