Universität Duisburg-Essen
Fachbereich VWL
Lehrstuhl Ökonometrie

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Multivariate Time Series Analysis
Exercise Sheet 4

Exercise 1: Implied Models for Components

Consider the VAR(1) model $z_t = \phi_0 + \phi_1 z_{t-1} + a_t$ from the Exercise Sheet 3 again:

$$\phi_0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \phi_1 = \begin{pmatrix} 0.75 & 0 \\ -0.25 & 0.5 \end{pmatrix}, \quad \Sigma_a = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

a) Write down the model using lag operator notation. Then rearrange the equation such that all parts based on z_t are on the left-hand side and the remainder is on the right-hand side.

b) By factoring out z_t on the left, we obtain the lag polynomial $\phi(L)$. Compute its adjoint matrix by hand.
Hint: Treat the lag operator as some scalar. The adjoint matrix can be computed like the inverse matrix but ***without*** *the scaling by* $\frac{1}{det(\phi(L))}$.

c) Pre-multiply the model equation you got in part a) with the adjoint matrix you computed in part b).
Hint: You are supposed to end up with a diagonal matrix.

d) The result of part c) should be a collection of two univariate ARMA(p,q) models. What is the lag order of both models?

e) Simulate a trajectory with $T = 1000$ of the original VAR(1)model.
Hint: 'VARMAsim' on Slide 2–6

f) Fit a VAR(1) model to the data, store the results as a variable and estimate the predictions' mean squared error for each variable in z_t.

g) Repeat the task by fitting the two ARMA(p,q) models from b) to the data. Again compute the mean squared error for $z_{1,t}$ and $z_{2,t}$ each.
Hint: 'arima'

h) Compare the MSEs of the VAR(1) estimates and the ARMA(p,q) estimates. Did the VAR(1) and the univariate ARMA(p,q) models perform similarly? If not, provide an intuition why.

i) How can you manipulate the Σ_a matrix to equalise the MSEs of both the VAR(1) and the ARMA(p,q) models?

Exercise 2: Least Squares Estimation

a) Again use your simulated time series from exercise 1. Regress $z_{1,t}$ on $z_{1,t-1}$ and $z_{2,t-1}$, then repeat with $z_{2,t}$ as dependent variable (meaning you estimate each row of the VAR(1) specification separately). How similar are the coefficients to those obtained from the VAR(1) regression?

b) Show that you can generally estimate a VAR(p) by row-wise separate regressions using the derivations starting from Slide 3–3.
Hint: Make sure you understand how the trick in equation (3.3) works.

Exercise 3: Maximum Likelihood Estimation

a) Let $\epsilon_1, \ldots, \epsilon_T$ be an i.i.d. sample from a normal distribution with unknown mean μ and variance σ^2. Find maximum likelihood estimators for μ and σ^2.

b) Prove equation (3.12) in the lecture slides.

This exercise sheet will be discussed in the tutorial on Wednesday, 13 November 2019