University of Duisburg-Essen
Faculty of Business Administration and Economics
Chair of Econometrics

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Multivariate Time Series Analysis

Solution Exercise Sheet 4

1 Exercise 1: Information Criteria

Prove Corollary 4.5 from Slide 4-7.

Solution:

From Theorem 4.4:

$$C(l) = \log\left(\hat{\Sigma}_a(l)\right) + \frac{l}{T} \cdot c_T$$

i) $\lim\limits_{T\to\infty} c_T \longrightarrow \infty$

ii) $\lim\limits_{T\to\infty} \frac{c_T}{T} \longrightarrow 0$

If i) and ii) hold, $C(l)$ chooses the optimal/correct model.

- AIC: $c_T = 2K^2$

$$\lim_{T\to\infty} c_T = 2\ K^2 \ \not\Longrightarrow\ \infty$$

$\Rightarrow$ not consistent

- BIC: $c_T = \log(T) \cdot K^2$

$$\lim_{T\to\infty} c_T = \log(T) \cdot K^2 \ \Longrightarrow\ \infty \lim_{T\to\infty} \frac{c_T}{T} \frac{\log(T)}{T} K^2 \ \Longrightarrow\ 0$$

$\Rightarrow$ consistent

- HQ: $c_T = 2\ \log(\log(T))\ K^2$

$$\lim_{T\to\infty} c_T = 2\ \log(\log(T))\ K^2 \ \Longrightarrow\ \infty \lim_{T\to\infty} \frac{c_T}{T} \frac{2\ \log(\log(T))\ K^2}{T} \ \Longrightarrow\ 0$$

$\Rightarrow$ consistent

2 Exercise 2: VAR(p): Data application

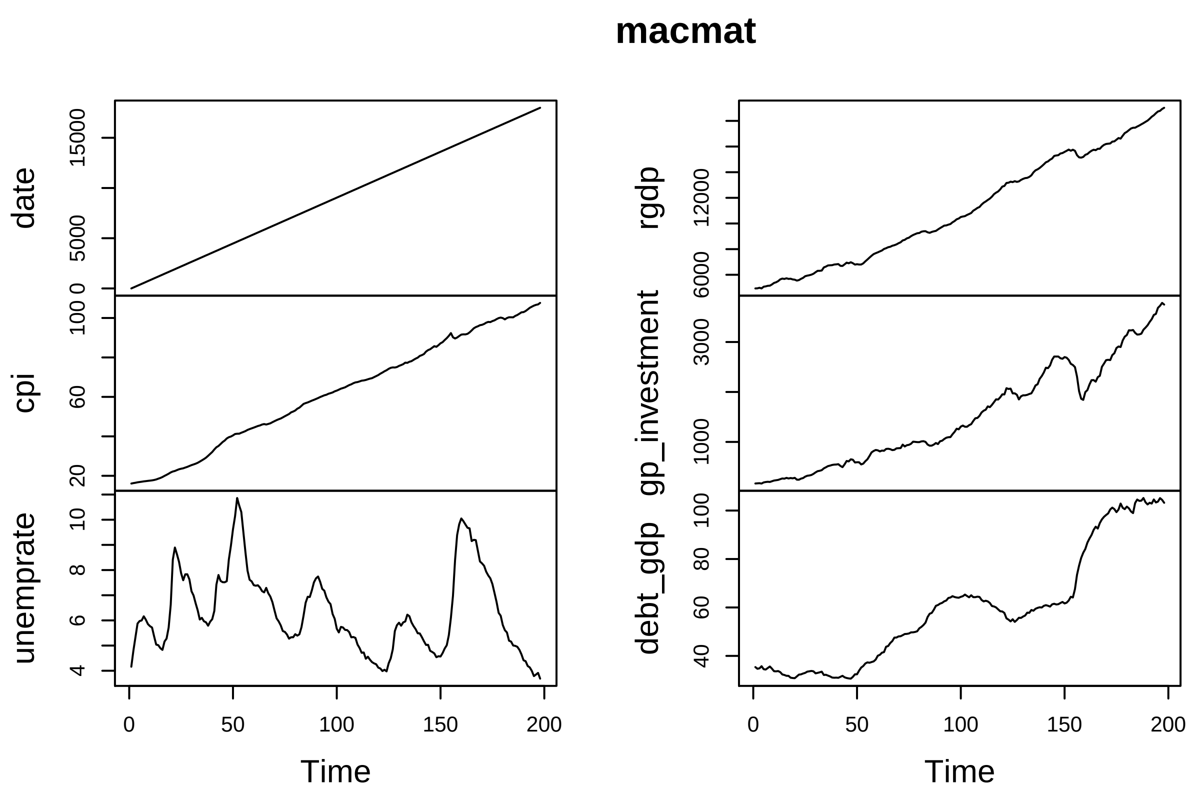
This exercise is concerned with finding an appropriate VAR(p) model for US macroeconomic data. You can find the dataset `us_macrodata.Rda` attached to this exercise sheet in the Moodle folder for this tutorial. Please use the `load` command to import the dataset from your directory into R. There are 5 variables – CPI, Real GDP, the unemployment rate, general private investment and the debt-to-GDP ratio. All series have been sampled quarterly and were seasonally adjusted before downloaded from FRED.

```
# loading data
load(file = here::here("exercise_MTSA/00_data/us_macrodata.Rda"))
# loading the MTS package
library(MTS)
```

a Plot all time series and judge which time series seem non-stationary. Proceed to compute growth rates of the non-stationary variables.

Solution:

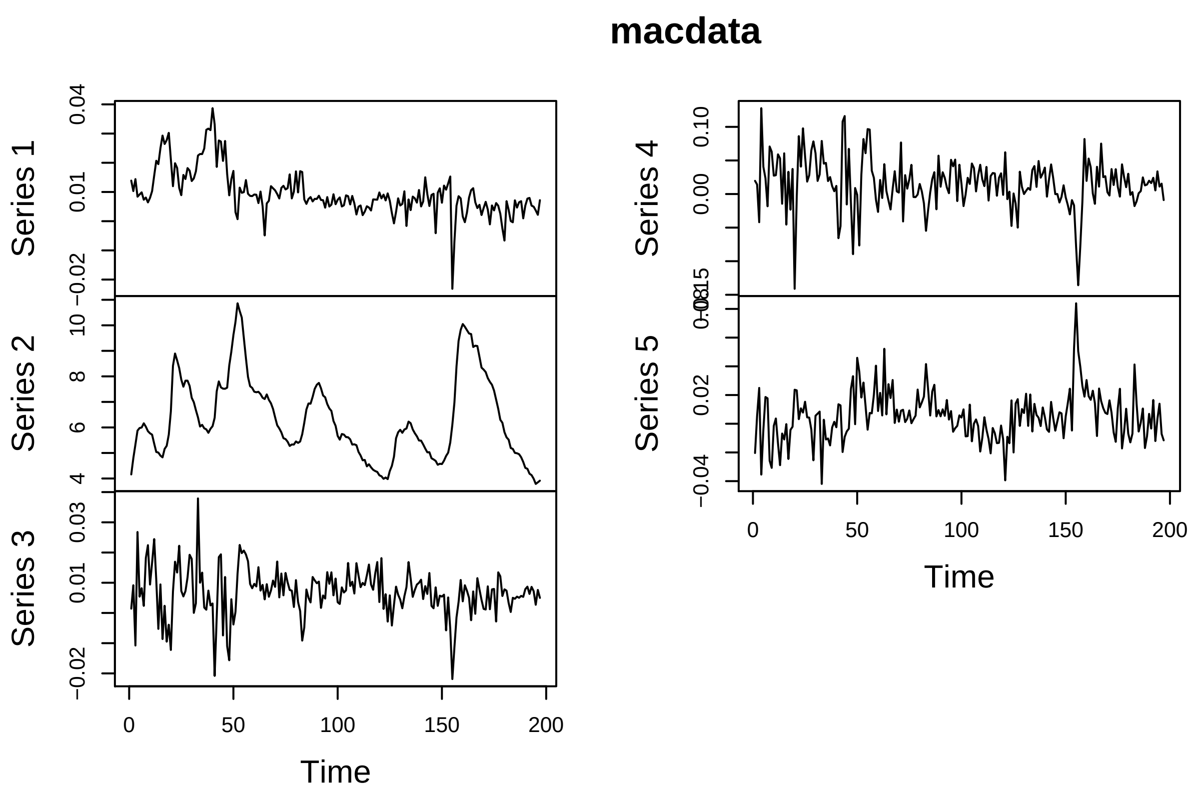
```
macmat <- data.matrix(us.macro_series)
plot.ts(macmat)
```



Every series except unemployment looks non-stationary. Regarding the debt-to-gdp ratio, this is surprising, but we better difference it as well.

```
macdata <- cbind(diff(log(us.macro_series$cpi)),
                 us.macro_series$unemprate[-(nrow(macmat))],
                 diff(log(us.macro_series$rgdp)),
                 diff(log(us.macro_series$gp_investment)),
                 diff(log(us.macro_series$debt_gdp)))

plot.ts(macdata)
```



Note that the last observation of "unemp" was dropped for conformable length. Its last and not first due to the date information: measurements are always from the first day of a quarter.

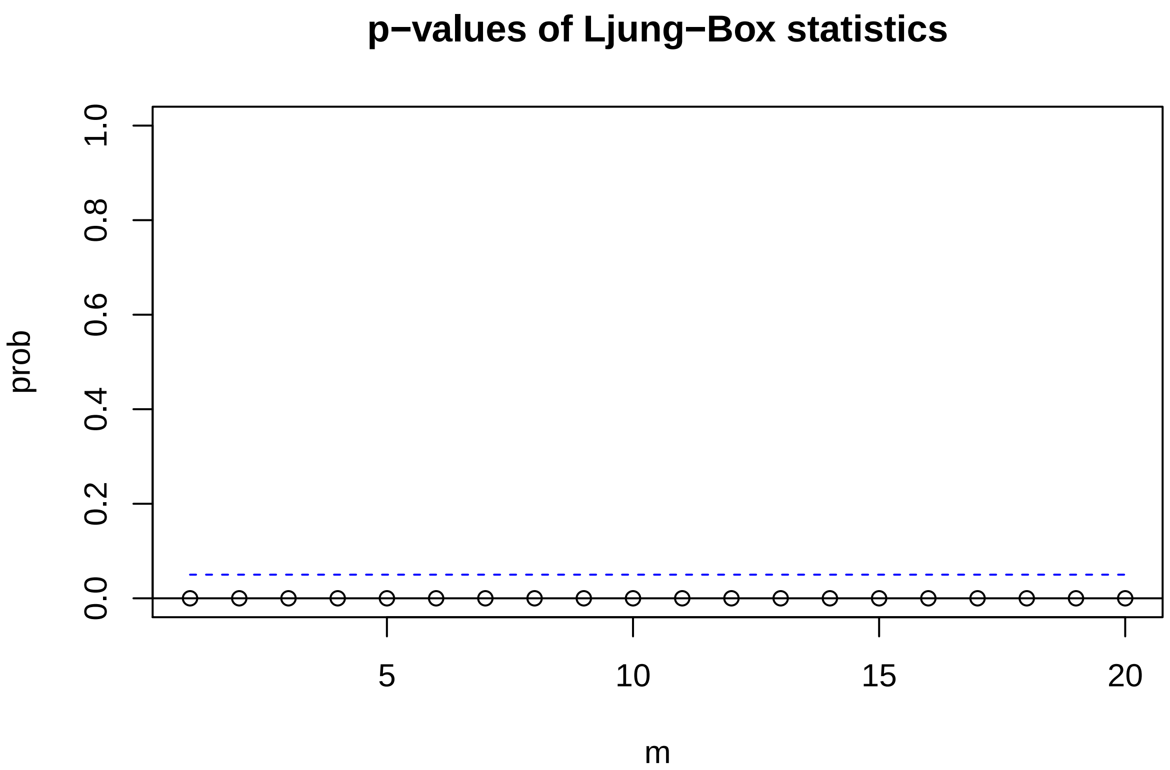
b Perform a Ljung-Box test on the dataset. Does it look worthwhile to estimate a VAR(p)

Solution:

```
mq(x = macdata, lag = 20)
```

```
## Ljung-Box Statistics:
```

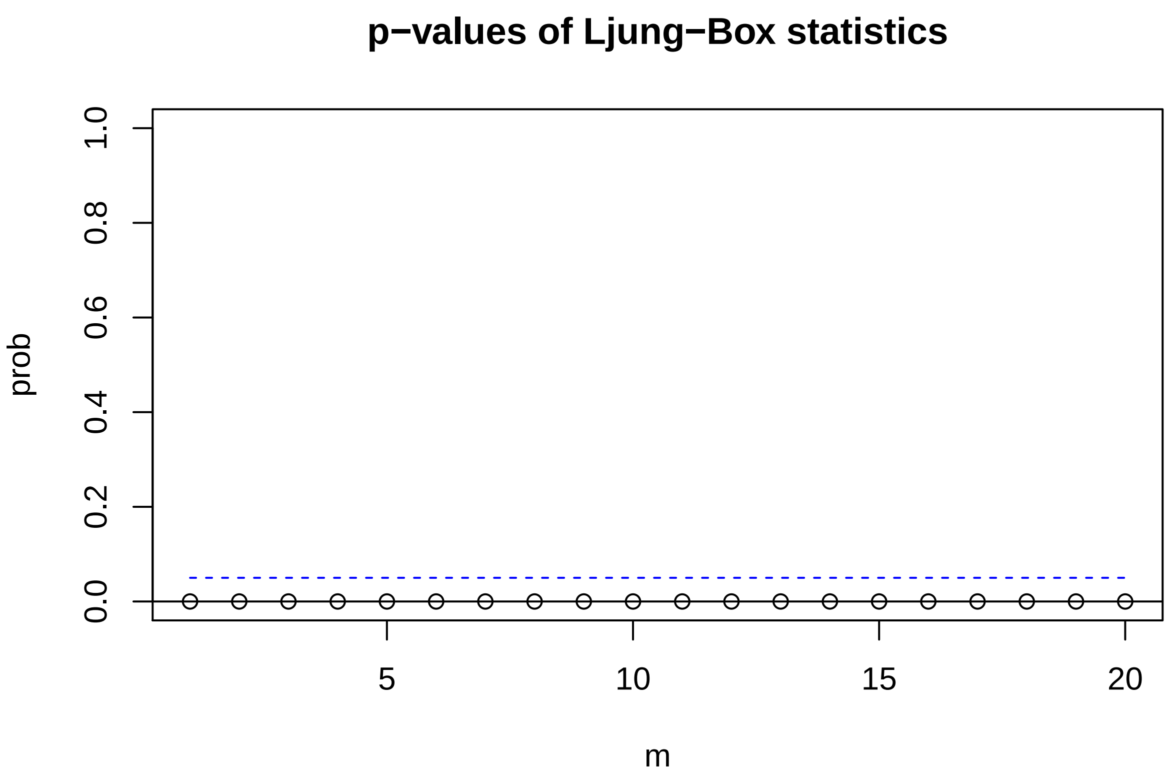
```
##          m  Q(m)  df  p-value
##  [1,]    1   369  25        0
##  [2,]    2   658  50        0
##  [3,]    3   932  75        0
##  [4,]    4  1211 100        0
##  [5,]    5  1430 125        0
##  [6,]    6  1624 150        0
##  [7,]    7  1796 175        0
##  [8,]    8  1953 200        0
##  [9,]    9  2083 225        0
## [10,]   10  2205 250        0
## [11,]   11  2313 275        0
## [12,]   12  2418 300        0
## [13,]   13  2513 325        0
## [14,]   14  2619 350        0
## [15,]   15  2702 375        0
## [16,]   16  2793 400        0
## [17,]   17  2881 425        0
## [18,]   18  2965 450        0
## [19,]   19  3031 475        0
## [20,]   20  3112 500        0
```



There is some correlation in the dataset.

```r
mq(x = macdata[,-3], lag = 20)
```

```
## Ljung-Box Statistics:
##           m       Q(m)      df    p-value
##  [1,]     1        337      16          0
##  [2,]     2        607      32          0
##  [3,]     3        866      48          0
##  [4,]     4       1121      64          0
##  [5,]     5       1325      80          0
##  [6,]     6       1502      96          0
##  [7,]     7       1650     112          0
##  [8,]     8       1781     128          0
##  [9,]     9       1894     144          0
## [10,]    10       2003     160          0
## [11,]    11       2101     176          0
## [12,]    12       2197     192          0
## [13,]    13       2278     208          0
## [14,]    14       2372     224          0
## [15,]    15       2444     240          0
## [16,]    16       2518     256          0
## [17,]    17       2589     272          0
## [18,]    18       2651     288          0
## [19,]    19       2702     304          0
## [20,]    20       2761     320          0
```



Even without unemployment, there is some correlation in the dataset.

c Determine the length of the time series. How many coefficients can be estimated and what does it mean for K and p?

```
data_dim <- dim(macdata)

Tmax <- data_dim[1] # observations
K <- data_dim[2] # variables
(max.p <- (Tmax * K - K) / K^2 )
```

```
## [1] 39.2
```

39 lags can be estimated in addition to the intercept.