University of Duisburg-Essen
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Multivariate Time Series Analysis

Solution Exercise Sheet 4

1 Exercise 1: Implied Models for Components

Consider the VAR(1) model $z_t = \phi_0 + \phi_1 z_{t-1} + a_t$ from the Exercise Sheet 3 again:

$$\phi_0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \phi_1 = \begin{pmatrix} 0.75 & 0 \\ -0.25 & 0.5 \end{pmatrix}, \quad \Sigma_a = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} .$$

a) Write down the model using lag operator notation. Then rearrange the equation such that all parts based on z_t are on the left-hand side and the remainder is on the right-hand side.

Solution:

$$\text{Model:} z_t = \phi_0 + \phi_1 z_{t-1} + a_t$$
$$\text{Lag notation: } z_t = \phi_0 + \phi_1 \,\mathrm{L}\, z_t + a_t$$

$$\Leftrightarrow z_t - \phi_1 \,\mathrm{L}\, Z_t = \phi_0 + a_t$$

b) By factoring out z_t on the left, we obtain the lag polynomial $\phi(\mathrm{L})$. Compute its adjoint matrix by hand.

*Hint: Treat the lag operator as some scalar. The adjoint matrix can be computed like the inverse matrix but **without** the scaling by* $\dfrac{1}{\det(\phi(\mathrm{L}))}$.

Solution:

$$
\begin{aligned}
\underbrace{(I - \phi_1\,\mathrm{L})}_{=:\phi\,\mathrm{L}}\, z_t &= \phi_0 + a_t \\
\Leftrightarrow \phi(\mathrm{L}) &= \begin{pmatrix} 1 - 0.75\,\mathrm{L} & 0 \\ -0.25\,\mathrm{L} & 1 - 0.5\,\mathrm{L} \end{pmatrix} \\
\Leftrightarrow \phi^{\mathrm{adj}} &= \begin{pmatrix} 1 - 0.5\,\mathrm{L} & 0 \\ 0.25\,\mathrm{L} & 1 - 0.75\,\mathrm{L} \end{pmatrix}
\end{aligned}
$$

c) Pre-multiply the model equation you got in part a) with the adjoint matrix you computed in part b).

Hint: You are supposed to end up with a diagonal matrix.

Solution:

$$
\begin{aligned}
\begin{pmatrix} 1 - 0.5\,\mathrm{L} & 0 \\ 0.25\,\mathrm{L} & 1 - 0.75\,\mathrm{L} \end{pmatrix} \begin{pmatrix} 1 - 0.75\,\mathrm{L} & 0 \\ -0.25\,\mathrm{L} & 1 - 0.5\,\mathrm{L} \end{pmatrix} z_t &= \begin{pmatrix} 1 - 0.75\,\mathrm{L} & 0 \\ 0.25\,\mathrm{L} & 1 - 0.75\,\mathrm{L} \end{pmatrix} \cdot (\phi_0 + a_t) \\
\Leftrightarrow \begin{pmatrix} (1 - 0.5\,\mathrm{L})(1 - 0.75\,\mathrm{L}) & 0 \\ (0.25\,\mathrm{L})(1 - 0.75\,\mathrm{L}) + (1 - 0.75\,\mathrm{L})(-0.25\,\mathrm{L}) & (1 - 0.75\,\mathrm{L})(1 - 0.5\,\mathrm{L}) \end{pmatrix} z_t &= \begin{pmatrix} (1 - 0.5\,\mathrm{L}) \cdot 1 \\ 0.25\,\mathrm{L} \cdot 1 + (1 - 0.75\,\mathrm{L}) \cdot 0 \end{pmatrix} + \begin{pmatrix} (1 - 0.5\,\mathrm{L}) \cdot a_{1,t} \\ (0.25\,\mathrm{L}) a_{1,t} + 1(1 - 0.75\,\mathrm{L}) a_{2,t} \end{pmatrix} \\
\Leftrightarrow \begin{pmatrix} z_{1,t} - 1.25 z_{1,t-1} + 0.375 z_{1,t-2} \\ z_{2,t} - 1.25 z_{2,t-1} + 0.375 z_{2,t-2} \end{pmatrix} &= \begin{pmatrix} 0.5 \\ 0.25 \end{pmatrix} + \begin{pmatrix} a_{1,t} - 0.5 a_{1,t-1} \\ 0 + 0.25 a_{1,t-1} + a_{2,t} - 0.75 a_{2,t-1} \end{pmatrix}
\end{aligned}
$$

d) The result of part c) should be a collection of two univariate ARMA(p,q) models. What is the lag order of both models?

Solution: