Universität Duisburg-Essen
Fachbereich VWL
Lehrstuhl Ökonometrie

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Multivariate Time Series Analysis
Exercise Sheet 12

Exercise 1: Cointegration Basics

Consider the variable $z_t = (z_{1,t}, z_{2,t})^\top$, where

$$z_{1,t} = \sum_{i=1}^{t} a_{3,i} + a_{1,t}$$
$$z_{2,t} = \frac{1}{2}\sum_{i=1}^{t} a_{3,i} + a_{2,t}$$

Assume the innovation sequences $a_{1,t}$, $a_{2,t}$ and $a_{3,t}$ to be white noise and mutually independent. Show that z_t is cointegrated and determine the cointegration rank.

Exercise 2: Cointegrating VAR?

Consider again the following VAR(1) process with the innovation sequence $a_t \overset{iid}{\sim} (\mathbf{0}, \mathbf{\Sigma}_a)$.

$$z_t = \begin{pmatrix} 1.1 & -0.2 \\ -0.2 & 1.4 \end{pmatrix} z_{t-1} + a_t, \quad t = 1, \dots, T.$$

Can you write the process in VECM form?

Hint: Use the results from Exercise 3 on Exercise Sheet 11.

Exercise 3: Cointegration Ranks

What is the maximum possible cointegrating rank of a three-dimensional process $z_t = (z_{1,t}, z_{2,t}, z_{3,t})^\top$,

a) if $z_{1,t}$, $z_{2,t}$ and $z_{3,t}$ are univariate stationary processes?
b) if $z_{1,t}$, $z_{2,t}$ are $I(0)$ and $z_{3,t}$ is $I(1)$?
c) if $z_{1,t}$, $z_{2,t}$ and $z_{3,t}$ are $I(1)$ but $z_{1,t}$ and $z_{2,t}$ are not cointegrated in a bivariate system?
d) if $z_{1,t}$, $z_{2,t}$ and $z_{3,t}$ are $I(1)$ but $(z_{1,t}, z_{2,t})^\top$ and $(z_{2,t}, z_{3,t})^\top$ are not cointegrated as bivariate systems?

Exercise 4: Cointegration Vectors

Consider a system of $K = 3$ integrated variables z_{1t}, z_{2t} and z_{3t}. Suppose

$$z_{1t} - z_{2t} = v_{1t} \sim I(0),$$
$$z_{2t} - z_{3t} = v_{2t} \sim I(0).$$

That is, $z_{1,t}$ and $z_{2,t}$ as well as $z_{2,t}$ and $z_{3,t}$ are cointegrated. The cointegration vectors are $\beta_1 \equiv (1,\ -1,\ 0)^\top$ and $\beta_2 \equiv (0,\ 1,\ -1)^\top$, respectively.

a) Show that the two cointegration vectors are linearly independent.

b) Show that the cointegration vectors are only defined up to scalar multiples, i.e. that $a\beta_i$ is also a cointegration vector with scalar a and $i \in \{1, 2\}$.

c) Show that any linear combination $a\beta_1 + (1-a)\beta_2$ is also a cointegration vector. That is, the cointegration vectors span a cointegration space and hence are not unique.

This exercise sheet will be discussed in the tutorial on Wednesday, 22 January 2020