

Dr. Yannick Hoga Thilo Reinschlüssel

Multivariate Time Series Analysis

Exercise Sheet 10

Exercise 1: Computing IRFs by hand

Consider the following VAR(1) process:

$$z_t = \begin{pmatrix} 0.5 & 0 \\ 0.25 & 0.5 \end{pmatrix} z_{t-1} + a_t \quad \text{with} \quad \Sigma_a = \begin{pmatrix} 0.25 & 0.125 \\ 0.125 & 0.3125 \end{pmatrix}$$

- List all relationships of Granger causality in this process. Is there instantaneous causality?
- Compute and plot the responses of a unit impulse on $a_{1,t}$ by hand for five periods.
- Compute and plot the responses of a unit orthogonal impulse on $a_{1,t}$ by hand for five periods.
- Explain the major differences between the results of b) and c) and why this was expected due to your findings in a).
- Under which condition are the 'ordinary' IRFs and the orthogonal IRFs identical?
- Based on your recent insights, sketch the IRFs of a unit 'ordinary' impulse on $a_{2,t}$.

Exercise 2: Forecast Error Variance Decomposition

Again consider the process given in Exercise 1.

- Based on the Granger causality structure: Which $w_{ij}(h)$ (see slide 6-29) are zero for each $h > 0$?
- Compute the forecast error variance decomposition for the process using the command `FEVdec` from the 'MTS' package for $h = \{1, \dots, 5\}$. Then reproduce the results writing your own code.

Exercise 3: Summing it up

- Assume that there is no Granger causality. Can you make a guess about the p -value of a Ljung-Box test?
- Again assume that there is no Granger causality. How does the forecast error variance decomposition look like?
- Now suppose you have a new dataset, run a Ljung-Box test and the p -value is close to one. Does this result tell you anything about Granger causality in the data?
- Next, assume the opposite case of many p -values close to zero. Which implications does this have for the IRFs?

Hint: Think about Granger causality first.

e) How does instantaneous causality show up in a Ljung-Box test?

For the next questions, please have a look at the VAR(1) depicted on slide 6-7 in the lecture slides.
We focus on the coefficient matrix:

- f) Sketch the impulse responses of investment after a unit impulse on consumption and vice versa for $t = 1, 2$.
- g) Which $w_{ij}(h)$ from the forecast error variance decomposition are zero for $h = 1$ and $h = 2$ each?
- h) Which entry has to be zero such that consumption does not Granger cause investment?

This exercise sheet will be discussed in the tutorial on Wednesday, 8 January 2020