Universität Duisburg-Essen
Fachbereich VWL
Lehrstuhl Ökonometrie

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Multivariate Time Series Analysis
Exercise Sheet 13

Exercise 1: Deterministic Trends

Consider the compound model $\mathbf{z}_t = \mu_0 + \mu_1 t + \mathbf{x}_t$ where $\mathbf{x}_t$ is some zero-mean cointegrating VAR(p) with dimension $K = 2$ and standard white noise innovations. The coefficients μ_0 and μ_1 are deterministic vectors with two entries each.

a) Derive the VECM for $\mathbf{z}_t$.
b) Suppose you plot the multivariate time series $\mathbf{z}_t$ and observe that none of the two variables drifts. Which restrictions can you impose on the VECM?
c) Another scenario: You suspect at least one component of $\mathbf{z}_t$ drifts. In which ways can the VECM specified above model such a drift? Carefully specify the restrictions if there are any. How can you pick the 'right' approach by analysing the data?
d) Which restrictions need to be fulfilled to ensure that $z_{1,t}$ does not Granger-cause $z_{2,t}$ in the VECM derived in a)?

Exercise 2: Common Trends in VECMs

Simulate a VECM based on a two-dimensional cointegrated VAR(1) with deterministic components for all cases discussed in the lecture. Employ coefficients and parameters of your own choice. Plot the trajectories $\mathbf{z}_t$, the cointegrating relations $\beta'\mathbf{z}_t$ and the common stochastic trends $\alpha'_\perp \mathbf{z}_t$ for each case.

Exercise 3: Forecasting Cointegrating VAR

Suppose the following VAR(1) model with $\mathbf{a}_t \overset{iid}{\sim} N(\mathbf{0}, \mathbf{I}_2)$.

$$\mathbf{z}_t = \begin{pmatrix} z_{1,t} \\ z_{2,t} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} z_{1,t-1} \\ z_{2,t-1} \end{pmatrix} + \begin{pmatrix} a_{1,t} \\ a_{2,t} \end{pmatrix} \quad \text{with} \quad t = 1, \ldots, T$$

a) Derive the VECM representation and check whether it is appropriate.
b) Decompose the matrix Π into α and β' with the normalisation $\beta_{(1)} = 1$.
c) Show that the forecast error of the optimal forecast $\mathbf{z}_T(h)$ diverges in h.
Hint: Derive the coefficients θ of the causal representation as you did for the stationary case and find the common structure.

d) What is the optimal forecast for $\mathbf{y}_{T+h} = \beta' \mathbf{z}_{T+h}$ at origin T? Derive the associated confidence interval.
Hint: Tasks c) and d) are optional.

Exercise 4: VECM and IRF

Consider the VECM:

$$\Delta \mathbf{z}_t = \begin{pmatrix} -0.1 \\ \\ 0.1 \end{pmatrix} \begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{z}_{t-1} + \mathbf{a}_t \quad \text{with} \quad t = 1, \ldots, T \quad \text{and} \quad \mathbf{a}_t \overset{iid}{\sim} N(\mathbf{0}, \Sigma_a)$$

a) Rewrite the process in VAR form.
b) Determine the roots of the reverse characteristic polynomial.
c) Determine forecast interval for the two variables for forecast horizon h.
d) Has a forecast error impulse on $\mathbf{z}_{1,T}$ a permanent impact on $\mathbf{z}_{2,T+h}$ for $h \geq 1$? Has a forecast error impulse on $\mathbf{z}_{2,T}$ a permanent impact on $\mathbf{z}_{1,T+h}$ for $h \geq 1$?
Hint: Tasks c) and d) are optional.

This exercise sheet will be discussed in the tutorial on Wednesday, 29 January 2020