Universität Duisburg-Essen
Fachbereich VWL
Lehrstuhl Ökonometrie

Winter 2019/20

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Multivariate Time Series Analysis
Exercise Sheet 11

Exercise 1: Unit Roots

Consider the following two separate univariate processes. Both innovation sequences a_t and b_t are white noise and mutually independent.

$$x_t = 0.7x_{t-1} + 0.3x_{t-2} + a_t, \quad t = 1, \dots, T, \tag{1}$$

$$y_t = 2y_{t-1} - y_{t-2} + b_t, \quad t = 1, \dots, T. \tag{2}$$

a) Check whether the processes are stationary. If not, how many unit roots are there?

b) Determine the order of integration d for both processes.

c) Show that $\Delta x_t = (1 - L)\, x_t$ is stationary.

d) Simulate both processes for $T = 200$ using 'arima.sim'. Then plot both trajectories.
Hint: You need the stationary (differenced) time series and the order of integration d.

e) Regress x_t on Δy_t and inspect the t-statistics of the associated coefficients. Are the test statistics valid?
Hint: Plot the residuals.

f) Run a Monte Carlo simulation with $N = 1000$: Within each iteration, generate x_t and Δy_t for $t = 1, \dots, T$. Then regress x_t on Δy_t and store the results. Report the mean of the regression coefficients and the fraction of p-values below 5%.
Start with $T = 25$ and repeat the simulation study with $T = 100$ and $T = 400$. What do you observe?

Exercise 2: Integrated Process

Assume the process $x_t = 1.5x_{t-1} - 0.5x_{t-2} + a_t$ with $a_t \sim [0, \sigma^2]$.

a) Show that x_t is an $I(1)$ process and compute the roots of the characteristic polynomial by hand.

b) Derive Δx_t and find the roots of the characteristic polynomial by hand. Is Δx_t $I(0)$?

Exercise 3: VAR(1) Application

Consider the following VAR(1) process with the innovation sequence $a_t \overset{iid}{\sim} [\mathbf{O}, \Sigma_a]$.

$$z_t = \begin{pmatrix} 1.1 & -0.2 \\ -0.2 & 1.4 \end{pmatrix} z_{t-1} + a_t, \quad t = 1, \dots, T,$$

a) Determine the roots of the characteristic polynomial. Are there unit roots? If yes, how many?

b) Derive the corresponding univariate ARMA(p,q) processes and show that both univariate time series entail a unit root.
Hint: Implied Models for Components.

c) Do the response functions for z_t of an impulse in $t = 0$ still converge to zero as $t \to \infty$?

This exercise sheet will be discussed in the tutorial on Wednesday, 15 January 2020