Notes: VAR to VECM

1

VAR: $\Phi_1 = \begin{pmatrix} 0.7 & 0 \\ 0 & 0.5 \end{pmatrix}$

$\Phi_2 = \begin{pmatrix} 0.3 & 0 \\ 0 & 0.5 \end{pmatrix}$

$$|I - \Phi_1 z - \Phi_2 z^2| = \begin{vmatrix} 1-0.7z-0.3z^2 & \\ & 1-0.5z-0.5z^2 \end{vmatrix}$$

$$= (1-0.7z-0.3z^2)(1-0.5z-0.5z^2) \stackrel{!}{=} 0$$

$$\Leftrightarrow 1-1.2z-0.45z^2+0.5z^3+0.15z^4 \stackrel{!}{=} 0$$

$$\Rightarrow z_1 = z_2 = 1, \quad z_3 = -2, \quad z_4 = 3.\overline{3}$$

$$\Pi = \Phi_1 + \Phi_2 - I = \begin{pmatrix} 0.7+0.3-1 & 0 \\ 0 & 0.5+0.5-1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$|\Pi| = 0, \quad rk(\Pi) = 0$$

$$\Rightarrow \text{Case}(I)$$ independent white not process! VECM → VAR

2

$$z_t = \begin{pmatrix} 1.5 & 0.5 \\ 0.3 & 1.3 \end{pmatrix} z_{t-1} + a_t$$

$$\Rightarrow |I - \Phi_1 z| = 1 - 2.8z + 1.8z^2 \doteq 0$$

$$\Rightarrow z_1 = 0.\overline{5}, \quad z_2 = 1$$

$$|\Pi| = \begin{vmatrix} 0.5 & 0.5 \\ 0.3 & 0.3 \end{vmatrix} = 0$$

$$rk(\Pi) = 1$$

$\Rightarrow$ VECM is suitable!

3 $$z_t = \begin{pmatrix} 0.75 & 0 \\ 0.25 & 0.5 \end{pmatrix} z_{t-1}$$

$$\Rightarrow z_1 = \frac{4}{3},\ z_2 = 2$$

$\Rightarrow$ stationary!

$$|\Pi| = \begin{vmatrix} -0.25 & 0 \\ 0.25 & -0.5 \end{vmatrix} = \frac{1}{8} \neq 0$$

$$\Rightarrow rk(\Pi) = 2 = k$$

=> VECM is not suitable!