Universität Duisburg-Essen
Fachbereich VWL
Lehrstuhl Ökonometrie

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Multivariate Time Series Analysis
Exercise Sheet 3

Exercise 1: VAR(1) Moments and Stationarity

Take the VAR(1) model $z_t = \phi_0 + \phi_1 z_{t-1} + a_t$ with the following parameterisation:

$$\phi_0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \phi_1 = \begin{pmatrix} 0.75 & 0 \\ -0.25 & 0.5 \end{pmatrix}, \quad \Sigma_a = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

a) Compute the mean of the process.
b) Show that the process is stationary.
c) Derive the Yule-Walker equations for the lags $l = \{0, 1, 2\}$ and show that the solution for Γ_0 coincides with equation (2.3) on Slide 2-15.
d) Compute Γ_0 and Γ_1 by hand based on your results from c).

Exercise 2: Stationarity of VAR(p) Processes

Using the notation of Slide 2-27, prove that $|I_{kp} - \Phi_1 z| = |I_k - \phi_1 z - \cdots - \phi_p z^p|$. Recall that $|A|$ denotes the determinant of the matrix A.
Hint: Derive Φ_1 and keep it mind that adding multiples of columns/rows to other columns/rows does not affect the determinant! The plan is to end up with a special matrix.

Exercise 3: VAR(2) Moments and Stationarity

Consider the following VAR(2) model with iid innovations:

$$\phi_0 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \quad \phi_1 = \begin{pmatrix} 0.5 & 0.1 \\ 0.4 & 0.5 \end{pmatrix}, \quad \phi_2 = \begin{pmatrix} 0 & 0 \\ 0.25 & 0 \end{pmatrix}, \quad \Sigma_a = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

a) Show that the process is stationary.
b) Determine the mean vector.
c) Derive the Yule-Walker equations with $l = \{0, 1, 2\}$ for a general VAR(2) process.
d) Suppose we only knew Γ_0, Γ_1 and Γ_2 – how can we estimate ϕ_1 and ϕ_2 from it?
e) Write the process as a VAR(1) and calculate the mean vector again.
f) Compute Γ_0 based on the VAR(1) formulation.
Hint: You can use R for the calculations.

This exercise sheet will be discussed in the exercise course on Wednesday, 6 November 2019