Exercise Sheet 5

1 From Theorem 4.4:

$$C(\ell) = \log(\hat{\Sigma}_a(\ell)) + \boxed{\frac{\ell}{T} \cdot c_T}$$

i) $\lim_{T \to \infty} c_T \to \infty$

ii) $\lim_{T \to \infty} \frac{c_T}{T} \to 0$

Iff i) and ii) hold, $C(\ell)$ chooses the optimal/correct model.

AIC: $c_T = 2K^2$

$$\lim_{T \to \infty} c_T = 2K^2 \not\to \infty \quad \text{⨍}$$

BIC: $c_T = \log(T) \cdot K^2$

$$\lim_{T \to \infty} c_T = \lim_{T \to \infty} \log(T) \cdot K^2 \to \infty \quad ✓$$

$$\lim_{T \to \infty} \frac{c_T}{T} = \lim_{T \to \infty} \frac{\log(T)}{T} \cdot K^2 \to 0 \quad ✓$$

$\Rightarrow$ consistent

HQ: $c_T = 2 \cdot \log(\log(T)) K^2$

$$\lim_{T \to \infty} c_T \to \infty \quad ✓$$

$$\lim_{T \to \infty} \frac{c_T}{T} \to 0 \quad ✓$$

Note: $\ell \cdot K^2$ is the number of freely estimated parameters. Restrictions can change that number even holding the lag order constant.

(except: intercepts)

$\boxed{2}$ a) $\to R$ + Exercise Sheet 4, Task 2

b) Yes. The test rejects even for large 'm', so there must be a dynamic pattern for the VAR to model.

c) # coefficients

$$= \underbrace{K^2}_{\phi_i \ \forall i \in \{1, \dots, p\}} \cdot p + \underbrace{K}_{\text{intercept}}$$

data points (observations times dimension)

$$= T \cdot K$$

$$\Rightarrow K \cdot T \geq K^2 \cdot p + K$$

$$\Leftrightarrow \quad \frac{\cancel{K} \cdot (T-1)}{K^{\cancel{2}}} \geq p$$

d) – AIC and HQ are flat around the minima → no distinct optimum visible.

– Conceivable reasons: Persistence, omitted variables, wrong functional form. → The VAR may just work as an approximation.

– BIC is the most conservative IC.

Minima at $p = \begin{cases} 1 & BIC \\ 2 & HQ \\ 4 & AIC \end{cases}$

e) Adjust with $K^2 \cdot p$ degrees of freedom! Adjustment for the intercept is not necessary, since Z_t needs to be demeaned anyways $(\Gamma_0 = (Z_t - \mu)(Z_t - \mu)')$!

The Ljung-Box test has $m \cdot K^2$ degrees of freedom (K^2 per lagged CCM), so after adjustments, we have $(m-p)K^2$ degrees of freedom. Since the test rejects everywhere else, the VARs do not explain the dynamics entirely.

f) i) $\rightarrow$ R $(K^2 \cdot p + K)$

ii) - Residuals' (co-)variances are higher for the VAR(1)

$\Rightarrow$ VAR(4) predicts better (in-sample)

iii) No, it is exactly the other way around. Estimating more coefficients with the same information leads to less information per coefficient.

g) Again: $K^2 \cdot p + K$

+h) i) $= 2^2 \cdot \begin{Bmatrix} 1 \\ 4 \end{Bmatrix} + 2 = \begin{Bmatrix} 6 \\ 18 \end{Bmatrix}$

h) ii+iii) Same pattern as in f), but not that pronounced this time. $\frac{\#\text{coefficients}}{\#\text{data points}}$ is now larger and does increase less sharply if $K=2$ compared to $K=5$ if $p\uparrow$. Furthermore, there are less coefficients to model the same magnitude of correlation.

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a) There seems to be few correlation across time except lags 20 to 24. Hence there is not much for a VAR to exploit.

b) 0. The criteria support the Ljung-Box test.

c) No important difference. There is almost no variation taken away by the VAR(1).

d) No. (c) has shown that nothing has changed at all.

e) - Plenty of lagged cross-/auto correlations.
- Criteria suggest high lag orders
$\Rightarrow$ Fitting a VAR appears sensible.

f) Finding: e) implies conditional!

heteroscedasticity.

$\Rightarrow$ Can z_t still be weakly stationary?

$\Rightarrow a_t = \sigma_t \cdot \varepsilon_t$, $\varepsilon_t \overset{iid}{\sim} (0, \sigma_\varepsilon^2)$

and σ_t dependent on $\varphi(L)\sigma_t$.

$\Rightarrow \mathbb{E}(\sigma_t^2 \mid \sigma_{t-1}^2, \sigma_{t-2}^2, \dots)$ does not

necessarily equal $\mathbb{E}(\sigma_t^2)$!

BUT: For stationarity, only

the unconditional moments matter!

If $\varphi(L)$ is a stable process and $\varepsilon_t \overset{iid}{\sim} (0, \sigma_\varepsilon^2)$ (hence no unconditional heteroscedasticity), the time series z_t is still weakly stationary.

(analogous to a VAR(p) process where $\mathbb{E}(z_t | z_{t-1}, z_{t-2}, \ldots, z_1)$ does not necessarily equal $\mu_z = \mathbb{E}(z_t)$! Still, we haved proved those models to be stationary under some conditions)