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Winter Term 2019/2020

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Multivariate Time Series Analysis

Solution Exercise Sheet 8

1 Exercise 1: Forecast Intervals and Distributional Assumptions

a) Which key assumption about the innovations a_t is made in the lecture to derive the distribution of the forecast errors $e_T(h)$?

Solution:

$$a_t \overset{\text{i.i.d.}}{\sim} N(0, \Sigma_a)$$

- i.i.d $\Rightarrow$ no autocorrelation
- N $\Rightarrow$ normal distributed
- Σ_a $\Rightarrow$ heteroskedasticity

b) Assume we knew all parameters / coefficients and let Σ_a be the identity matrix $I_{3\times 3}$. Based on the assumption from a), derive the distribution of $e_T(1)$ for any stationary VAR(p).

Solution:

$$\begin{aligned} e_T(1) &= z_{T+1} - z_T(1) \\ &= a_{T+1} \end{aligned}$$

holds for any VAR(p) since

$$z_T(1) = \mathbb{E}\left(z_{T+1} | z_T, \dots, z_0\right)$$

$$\Rightarrow e_t(1) = a_{T+1} \sim N\left(0, I_{3\times 3}\right)$$

c) Derive the confidence ellipsoid associated to b) for $\alpha = 5\%$. What is the fraction forecast errors that lie inside the ellposid?

Solution:

llipsoid:

$$\left\{ z \in \mathbb{R}^3 : (z_T(1) - z)' \, \Sigma_e^{-1}(1) \, (z_T(1) - z) \leq \chi^2_{3,1-\alpha} \right\}$$

By defining $z_T(1) - z =: \epsilon$ and using that $\Sigma_e(1) = \Sigma_a = I_{3\times 3}$ the ellipsoid is: $\left\{ \epsilon \in \mathbb{R}^3 : \epsilon' \, \epsilon \leq \chi^2_{3,1-\alpha} \right\}$. For $\alpha = 5\%$, 95% of the observed forecast errors are expected to fall inside the confidence ellipsoid.

d) Run a simulation in 'R ': Draw the forecast error $e_T(1)$ defined in a) and b) with $K = 3$. Check if it is located inside or outside the confidence ellipsoid derived in c). Use $N = 10000$ repetitions in total and conclude whether the confidence ellipsoid is appropriate.

Solution:

Just check if $e_T(1)' e_T(1) \leq \chi^2_{3,0.95}$ and compute $\frac{1}{N} \sum_{i=1}^{N} \mathbb{1} \left(e_T(1)' e_T(1) \leq \chi^2_{3,0.95} \right)$.

```
N <- 10000 # number of repetitions
K <- 3 # dimension of VAR
 # drawing a_t from iid N(0,I)
gauss <- mvrnorm(n = N, mu = c(0,0,0), Sigma = diag(K))
# computing e'e for all draws in one take equals diag(EE')
msfe_gauss <- diag(gauss %*% t(gauss))
# this is only a one-sided test since we have squared each error!
limit <- qchisq(p = 0.95, df=3, lower.tail=TRUE)
sum(msfe_gauss < limit) / N
```

```
## [1] 0.9499
```

e) Repeat the simulation from above but this time assume a_t to be drawn from a uniform distribution. $\Sigma_a = I_{3\times 3}$ remains. How reliable is the confidence ellipsoid in this case? *Hint: Set* $\pm\frac{\sqrt{12}}{2}$ *as lower / upper bound for unit variance.*

Solution:

```r
# variance = 1 again, Kurtosis is < 3 for this one
unif <- matrix(data = runif(n = N * 3, min = -sqrt(12)/2,
                            max = sqrt(12)/2), nrow = N, ncol = 3)
msfe_unif <- rowSums(unif^2)
sum(msfe_unif < limit) / N
```

```
## [1] 0.9987
```

It is to conservative.

f) Repeat the simulation drawing innovations from a t-distribution with 2 degrees of freedom and conclude.

Solution:

```r
# variance = 1 by default, kurtosis > 3 and this hurts a lot
t2 <- matrix(data = rt(n = N * 3, df = 2), nrow = N, ncol = 3)
msfe_t2 <- rowSums(t2^2)
sum(msfe_t2 < limit) / N
```

```
## [1] 0.6424
```

Too liberal, the ellipsoid is not appropriate.