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Multivariate Time Series Analysis
Exercise Sheet 5

Exercise 1: Information Criteria

Prove Corollary 4.5 from Slide 4-7.

Exercise 2: VAR(p): Data application

This exercise is concerned with finding an appropriate VAR(p) model for US macroeconomic data. You can find the dataset 'us_macrodata.Rda' attached to this exercise sheet in the Moodle folder for this tutorial. Please use the load command to import the dataset from your directory into R. There are 5 variables – CPI, Real GDP, the unemployment rate, general private investment and the debt-to-GDP ratio. All series have been sampled quarterly and were seasonally adjusted before downloaded from *FRED*.

a) Plot all time series and judge which time series seem non-stationary. Proceed to compute growth rates of the non-stationary variables.

b) Perform a Ljung-Box test on the dataset. Does it look worthwhile to estimate a VAR(p) model here?

c) Determine the length of the time series. How many coefficients can be estimated and what does it mean for K and p?

d) Consult the AIC, BIC and HQ to determine the optimal lag order for a VAR(p) model for the whole dataset. Plot the values of the three criteria for the lag orders p from 1 to 5 in one plot.
   Hint: 'VARorder'

e) Fit VAR(p) models incorporating all variables using the optimal lag order(s) p suggested by each of the information criteria. Apply the Ljung-Box test to inspect the residuals' properties. For which models does the test reject the null hypothesis on one of the first ten lags?
   Hint: Do not forget that residuals are obtained by estimating coefficients/parameters. This comes at a price and you need to account for that! (Slide 4-15)

f) Now take a VAR(1) and a VAR(4) model with all variables included and an intercept specified.

   i) How many coefficients are estimated in each case?
   ii) Look at both estimates of Σ_a – are there major differences?
   iii) Compare the standard errors associated with the ϕ_1 matrices of the VAR(1) and VAR(4) from above. Do you see the same pattern regarding Σ_a?

g) Repeat the task from above with CPI and the debt-to-gdp ratio as the only variables (hence $K = 2$). How many coefficients are estimated in this case?

h) At last, go back to VAR(1) and VAR(4) models from task f). Use the standard error matrices to compute t-statistics for each coefficient with the null hypothesis $H_0 : \phi_{p,jk} = 0$. How often is the null hypothesis rejected at the 5% level in each of the two models?

Exercise 3:

This exercise is concerned with predicting growth rates of exchange rates. Please download the file 'quandl_fx_download.R' from the Moodle and install the package 'Quandl'. Executing the script will then download and prepare the two time series we are interested in.

a) Apply the Ljung-Box test on the multivariate time series and comment.

b) Do the usual information criteria support the finding of the Ljung-Box test?

c) Regardless of a) and b), fit a VAR(1) to the time series. Compare Σ_a with Γ_0.

d) Apply the Ljung-Box test on the residuals. Do the results surprise you?

e) Repeat the Ljung-Box test but this time with the squared residuals. Also have a look at the information criteria.

f) Can you rule out weak stationarity for the growth rates of the exchange rates only based on your findings up to this point?

This exercise sheet will be discussed in the tutorial on Wednesday, 20 November 2019