Macroeconomics and Reality: Revolutionizing Macroeconometrics

Ignacio Moreira Lara

Ruhrmetrics Reading Group

December 12, 2024

Outline

Introduction

- *Macroeconomics and Reality* by Christopher Sims (1980) revolutionized empirical macroeconometrics.
- Objective: Shift from theory-driven to data-driven models.
- Contribution: Introduction of Vector Autoregressions (VARs).

Before 1980: Structural Econometric Models (SEMs)

- Dominance of large scale structural economic models, based on economic theory
- Characteristics:
  - Strong assumptions about causality
  - Over-identifying restrictions
  - Heavily parametrized
  - Simultaneous equations
  - Exogeneous variables

Example Macro model : Consumption Function

$$C_t = \alpha_0 + \alpha_1 Y_t + \alpha_2 R_t + \epsilon_{C,t},$$

where:

- C_t: Consumption
- Y_t: Disposable income
- R_t: Interest rate
- $\epsilon_{C,t}$: Error term

Investment function

$$I_t = \beta_0 + \beta_1 Y_t + \beta_2 R_t + \epsilon_{I,t},$$

where:

- I_t: Investment
- R_t: Interest rate (cost of capital)
- $\epsilon_{I,t}$: Error term

Example: interest rate parity

$$M_t = \gamma_0 + \gamma_1 P_t + \gamma_2 Y_t - \gamma_3 R_t + \epsilon_{M,t},$$

where:

- M_t: Money supply
- P_t: Price level
- Y_t: Output
- R_t: Interest rate
- $\epsilon_{M,t}$ Error term

IS Curve (Aggregate Demand)

$$Y_t = C_t + I_t + G_t,$$

where:

- G_t: Government spending.

Phillips Curve (Aggregate Supply)

$$\pi_t = \delta_0 + \delta_1(Y_t - \bar{Y}) + \epsilon_{\pi,t},$$

where:

- π_t: Inflation,
- $\bar{Y}$: Potential output,
- $\epsilon_{\pi,t}$: Error term.

Monetary Policy Rule

$$R_t = \phi_0 + \phi_1 \pi_t + \phi_2 (Y_t - \bar{Y}) + \epsilon_{R,t},$$

where:

- R_t: Central bank's policy interest rate,
- π_t: Inflation,
- $\epsilon_{R,t}$: Error term.

Main concerns

Sims mentions three points that discourage the use of this type of models.

- Incredible restrictions
- Dynamics - Assumptions about lag lengths and the choice of endogenous and exogenous variables.
- Expectations: Agents expected to have full information, to be heterogenous, set a structure about how to model their expectations, endogeneity through expectations

Some additional concerns:

- Poor empirical validation.
- Inflexibility to economic shocks.
- Condition of market clearing needed for obtaining results.

Sims (1980)

"...what "economic theory" tells us about them is mainly that any variable which appears on the right-hand-side of one of these equations belongs in principle on the right-hand-side of all of them"

Introduction of VARs

- Key innovation by Sims: Treating all variables as endogenous
- Quote: "The use of incredible identifying restrictions in macroeconomic models should be avoided"
- Data-driven approach: Let the data speak

VAR Model Equation

General VAR Equation:

$$\mathbf{Y}_t = \mathbf{A}_1 \mathbf{Y}_{t-1} + \mathbf{A}_2 \mathbf{Y}_{t-2} + \cdots + \mathbf{A}_p \mathbf{Y}_{t-p} + \epsilon_t$$

- $\mathbf{Y}_t$: $n \times 1$ vector of endogenous variables
- $\mathbf{A}_i$: $n \times n$ coefficient matrices, with $i = 1, \ldots, p$
- ϵ_t: $n \times 1$ Vector of shocks (innovations)

Estimation: Equation-by-equation OLS, Bayesian methods :)

Applications

Estimate the model on quarterly data from the U.S. and West Germany, for the period 1949-75 and 1958-1976, respectively. The series are money (M1), real GDP, unemployment, wages, price level and import prices. Applications:

- Specification testing
- Structural analysis
- Granger Causality tests

Specification testing

- Set up likelihood ratio tests. The tests under the null are $\chi^2(k)$ distributed, with k as the number of variables per equation
- Select the lag length
- Tests differences between two sample periods

Impulse response functions

Take the normal VAR equation and rewrite it:

$$\mathbf{A}(L)\mathbf{Y}_t = \boldsymbol{\epsilon}_t,$$

where $\mathbf{A}(L) = \mathbf{I}_K - \mathbf{A}_1 L - \ldots - \mathbf{A} L^p$ is the lag polynomial and L is the lag operator. Impulse response functions (IRFs) are obtained by inverting the lag polynomial:

$$\mathbf{Y}_t = \mathbf{A}(L)^{-1}\boldsymbol{\epsilon}_t = \boldsymbol{\Psi}(L)\boldsymbol{\epsilon}_t = \sum_{h=0}^{\infty} \boldsymbol{\Psi}_h \boldsymbol{\epsilon}_{t-h}$$

where $\boldsymbol{\Psi}_h$ is the IRF for time period h.

Structural VARs (SVARs)

Structural VAR Equation:

$$\mathbf{B}_0\mathbf{Y}_t = \mathbf{B}_1\mathbf{Y}_{t-1} + \cdots + \mathbf{B}_p\mathbf{Y}_{t-p} + \mathbf{u}_t$$

- $\mathbf{B}_0$: Matrix of contemporaneous relationships
- $\mathbf{u}_t$: Structural shocks, with $\text{Cov}(\mathbf{u}_t) = \mathbf{I}_n$

Identification

Requires additional assumptions, e.g.:

- Cholesky decomposition. We assume that $\text{Cov}(\epsilon_t) = \boldsymbol{\Sigma}\epsilon$, then the assumption of a recursive decomposition says that:

$$\boldsymbol{\Sigma}_\epsilon = \mathbb{E}[\epsilon_t \epsilon_t'] = \mathbf{B}_0(\mathbf{B}_0)',$$

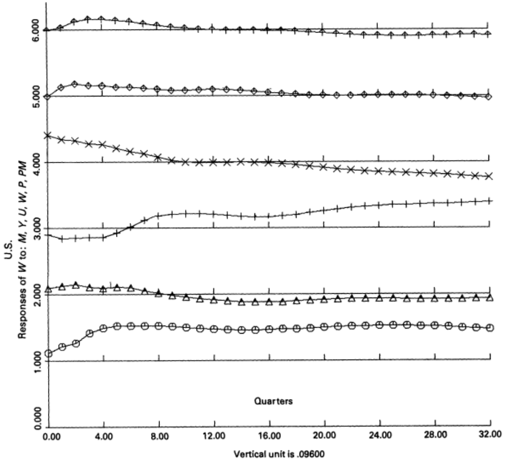
  for the structure in Sims' empirical application with $n = 6$ structure applying the Cholesky decomposition we obtain a matrix:

$$\mathbf{B}_0 = \begin{bmatrix} b_{11} & 0 & 0 & 0 & 0 & 0 \\ b_{21} & b_{22} & 0 & 0 & 0 & 0 \\ b_{31} & b_{32} & b_{33} & 0 & 0 & 0 \\ b_{41} & b_{42} & b_{43} & b_{44} & 0 & 0 \\ b_{51} & b_{52} & b_{53} & b_{54} & b_{55} & 0 \\ b_{61} & b_{62} & b_{63} & b_{64} & b_{65} & b_{66} \end{bmatrix}.$$

- Sign restrictions, impose restrictions on B_0 directly and obtain orthogonalized shocks.

Identification

Figure: SIRFs



Identification

TABLE III

PROPORTIONS OF FORECAST ERROR *k* QUARTERS AHEAD PRODUCED BY EACH INNOVATION: U.S. 1949–1975[a]

Forecast error in:	*k*	Triangularized innovation in: *M*	*Y/P*	*U*	*W*	*P*	*PM*
M	1	1.00	0	0	0	0	0
	3	.96	0	.03	0	0	0
	9	.73	0	.24	.02	0	0
	33	.54	0	.27	.09	0	.09
Y/P	1	.15	.85	0	0	0	0
	3	.35	.59	.04	.01	.01	0
	9	.30	.18	.37	.13	.00	.02
	33	.28	.15	.33	.16	.02	.06
U	1	.02	.35	.63	0	0	0
	3	.14	.49	.32	0	.03	0
	9	.26	.20	.41	.09	.02	.02
	33	.34	.14	.34	.13	.03	.03
W	1	.08	.05	.04	.84	0	0
	3	.17	.06	.07	.55	.09	.06
	9	.45	.02	.05	.25	.08	.16
	33	.64	.02	.19	.07	.02	.07
P	1	0	.04	.15	.24	.56	0
	3	.04	.01	.14	.36	.33	.12
	9	.14	.02	.12	.25	.11	.36
	33	.60	.02	.20	.07	.02	.09
PM	1	0	0	.06	.05	.08	.81
	3	.01	.01	.02	.13	.10	.75
	9	.06	.02	.13	.08	.03	.68
	33	.54	.03	.20	.04	.01	.18

[a] The moving average representation on which this table was based was computed from a system estimate in which the *PM* equation was estimated by generalized least squares in two steps. An initial estimate by ordinary least squares was used to construct an estimate of the ratio of residual variance in *PM* during 1949–71 to the residual variance in 1971–75, and this ratio was used (as if error-free) to re-estimate the equation by generalized least squares. This procedure is not in fact efficient, since once the break in residual variance in the *PM* equation is admitted, the usual asymptotic equivalence of single-equation and multiple-equation autoregression estimates breaks down.

Granger causality tests

- Testing causation between groups of variables
- Main hypothesis: monetary policy has no direct effect on real variables
- No causation of past values of m, the monetary policy, on real variables y

Granger causality tests

Let's check an example for a VAR(2) model, where we have:

$$\begin{bmatrix} y_t \\ m_t \end{bmatrix} = \begin{bmatrix} a_{11}^{(1)} & a_{12}^{(1)} \\ a_{21}^{(1)} & a_{22}^{(1)} \end{bmatrix} \begin{bmatrix} y_{t-1} \\ m_{t-1} \end{bmatrix} + \begin{bmatrix} a_{11}^{(2)} & a_{12}^{(2)} \\ a_{21}^{(2)} & a_{22}^{(2)} \end{bmatrix} \begin{bmatrix} y_{t-2} \\ m_{t-2} \end{bmatrix} + \begin{bmatrix} u_{y,t} \\ u_{m,t} \end{bmatrix}.$$

The hypothesis of Granger Causality of m_t on y_t can be tested in the following manner:

$$H_0: \ a_{12}^{(1)} = a_{12}^{(2)} = 0 \quad H_1: a_{12}^{(1)} \neq 0, \ a_{12}^{(2)} \neq 0$$

Granger Causality: results

- Block exogeneity of real variables rejected for both Germany and the U.S
- Granger causality found of money on real variables
- Similar finding for price on real variables

Comparison: Before and After Sims

Before Sims:

- Theory-driven SEMs.
- Over-reliance on assumptions

After Sims:

- Data-driven VAR models.
- Focus on understanding data dynamics

State of the methodology

- Forecasting with VAR models is a valid approach
- Shrinkage models in frequentist and Bayesian settings
- Identification techniques such as: stochastic volatility, sign restrictions, instrumental variables (IV) and higher moments
- Mixed-frequencies, time varying models and panel data VARs.

References

- Sims, C. A. (1980). *Macroeconomics and Reality*. Econometrica, 48(1), 1-48.
- Additional references on VARs and macroeconometrics.