

# P-Approximation

## Seminar in Econometrics

### Term Paper

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# 1 Introduction

Meta tests have been shown to be a powerful tool when testing for the null of non-cointegration. The distribution of their test statistic, however, is mostly not available in closed form. This might pose difficulties when implementing the meta tests in econometric software packages, as one has to include tables of critical values and p-values for each combination of the underlying tests. Software package size limitations are therefore quickly exceeded.

In this paper we propose supervised Machine Learning Algorithms to approximate the p-values of the meta test by Bayer and Hanck (2012) which tests for the null of non-cointegration. This approach might reduce the size of associated software packages considerably. The algorithms are trained on simulated data for various specifications of the aforementioned test.

Ergebnis der Models (1-2 Sätze)

Inhalt Paper

## 2 Bayer Hanck Test

The choice as to which of the available cointegration tests to use is a recurrent issue in econometric time series analysis. Bayer and Hanck (2012) propose powerful meta tests which provide unambiguous test decisions. They combine several residual- and system-based tests in the manner of Fisher's (1932) Chi-squared test.

Bayer and Hanck build their work on results from Pesavento (2004), who defines the underlying model as  $z'_t = [x'_t, y_t]$ .  $x_t$ , an  $n_1 \times 1$  vector, describes the regressor dynamics, while  $y_t$  is a scalar which defines the cointegrating relation. They can be written as

$$\Delta x_t = \tau_1 + v_{1t}, \quad (2.1)$$

$$y_t = (\mu_2 - \gamma' \mu_1) + (\tau_2 - \gamma' \tau_1)t + \gamma' x_t + u_t, \quad (2.2)$$

$$u_t = \rho u_{t-1} + v_{2t}. \quad (2.3)$$

$\mu_1$ ,  $\mu_2$ ,  $\tau_1$  and  $\tau_2$  are the deterministic parts of the model. They are subject to the following restrictions: (i)  $\mu_2 - \gamma' \mu_1$  and  $\tau = 0$  which translates to no deterministics, (ii)  $\tau = 0$  which corresponds to a constant in the cointegrating vector, (iii)  $\tau_2 - \gamma' \tau_1 = 0$ , a constant plus trend.

$v_t = [v'_{1t} v_{2t}]'$  with  $\Omega$  the long-run covariance matrix of  $v_t$ . For derivation of  $v_t$  see Pesavento (2004). Pesavento shows that  $\{v_t\}$  satisfies an FCLT, i.e.  $T^{-1/2} \sum_{t=1}^{\lfloor T \cdot \rfloor} v_t \Rightarrow \Omega^{1/2} W(\cdot)$ . It is further assumed that the  $x_t$  are not cointegrated.

It clearly follows from (2.3) that  $z_t$  is cointegrated if  $\rho < 1$ . Hence the null hypothesis of no cointegration is  $H_0 : \rho = 1$ .

Furthermore, Pesavento introduces two other parameters. First,  $R^2$  measures the squared correlation of  $v_{1t}$  and  $v_{2t}$ . It can be interpreted as the influence of the right-hand side variables in (2.2). It ranks between zero and one. When there is no long-run correlation between those variables and the errors from the cointegration regression,  $R^2$  equals zero. Secondly, the number of lags is approximated by a finite number  $k$ .

**Assumptions (BH S. 84)?**

Bayer and Hanck's (2012) meta test combines the test statistics of four stand-alone tests. Namely, these are the tests of Engle and Granger (1987), Johansen (1988), Boswijk (1994) and Banerjee, Dolado, and Mestre (1998).

**Engle-Granger**

**Johansen**

**Banerjee and Boswijk**

"the test statistics are shown to converge under a local alternative to random variables whose distributions are functions of Brownian Motions and Ornstein–Uhlenbeck processes and of a single nuisance parameter"

To combine the results from the underlying tests Bayer and Hanck draw upon Fisher's combined probability test (Fisher, 1932). It merges the tests using the formula

$$\tilde{\chi}_{\mathcal{I}}^2 := -2 \sum_{i \in \mathcal{I}} \ln(p_i). \quad (2.4)$$

Let  $t_i$  be the  $i^{th}$  test statistic. If test  $i$  rejects for large values, take  $\xi_i := t_i$ . If test  $i$  rejects for small values, take  $-\xi_i := t_i$ . With  $\Xi_i(x) := \Pr_{\mathcal{H}_i}(\xi_i \geq x)$  the p-value of the  $i^{th}$  test is  $p_i := \Xi_i(\xi_i)$ .

**3    Simulation**

**4    Models**

**5    Package**

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# A Appendices

Better sorting of the Appendix

### **Eidesstattliche Versicherung**

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Essen, den \_\_\_\_\_

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