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1 Introduction

Meta tests have been shown to be a powerful tool when testing for the null of non-cointegration. The distribution of their test statistic, however, is mostly not available in closed form. This might pose difficulties when implementing the meta tests in econometric software packages, as one has to include the full null distribution for each combination of the underlying tests. Software package size limitations are therefore quickly exceeded.

In this paper we propose supervised Machine Learning Algorithms to approximate the p-values of the meta test by Bayer and Hanck (2012) which tests for the null of non-cointegration. This approach might reduce the size of associated software packages considerably. The algorithms are trained on simulated data for various specifications of the aforementioned test.

Ergebnis der Models (1-2 Sätze)

Inhalt Paper

2 Bayer Hanck Test

The choice as to which of the available cointegration tests to use is a recurrent issue in econometric time series analysis. Bayer and Hanck (2012) propose powerful meta tests which provide unambiguous test decisions. They combine several residual- and system-based tests in the manner of Fisher's (1932) Chi-squared test.

Bayer and Hanck build their paper on previous work from Pesavento (2004), who defines the underlying model as $z_t' = [x_t', y_t]$, with x_t being an $n_1 \times 1$ vector and y_t a scalar, which displays the cointegration relation. They can be written as \begin{subequations}

$$\Delta x_t = \tau_1 + v_{1t} \tag{2.1}$$

$$y_t = (\mu_2 - \gamma'\mu_1) + (\tau_2 - \gamma'\tau_1)t + \gamma' x_t + u_t, \tag{2.2}$$

$$u_t = \rho u_{t-1} + v_{2t}. \tag{2.3}$$

\end{subequation} Δx_t presents the regressor dynamics. μ_1, μ_2, τ_1 and τ_2 are the deterministic parts of the model. They are subject to the following restrictions: (i) $\mu_2 - \gamma'\mu_1$ and $\tau = 0$ which translates to no deterministics, (ii) $\tau = 0$ which corresponds to a constant in the cointegrating vector, (iii) $\tau_2 - \gamma'\tau_1 = 0$, a constant plus trend.

$v_t = [v_{1t}' v_{2t}]'$ with Ω the long-run covariance matrix of v_t. For derivation of v_t see Pesavento (2004). Pesavento shows that $\{v_t\}$ satisfies an FCLT, i.e. $T^{-1/2}\sum_{t=1}^{[T\cdot]} v_t \Rightarrow \Omega^{1/2}W(\cdot)$. It is further assumed that the x_t are not cointegrated.

It clearly follows from (2.3) that z_t is cointegrated if $\rho < 1$. Hence the null hypothesis of no cointegration is $H_0 : p = 1$. Furthermore, Pesavento introduces two other parameters. First, R^2 measures the squared correlation of v_{1t} and v_{2t}. It can be interpreted as the influence of the right-hand side variables in (2.2). It ranks between zero and one. When there is no long-run correlation between those variables and the errors from the cointegration regression, R^2 equals zero. Secondly, the number of lags is approximated by a finite number k.

Assumptions (BH S. 84)?

Bayer and Hanck's (2012) meta test considers the test statistics of up to four stand-alone tests. Namely, these are the tests of Engle and Granger (1987), Johansen (1988), Boswijk (1994) and Banerjee, Dolado, and Mestre (1998). For the sake of brevity the detailed derivation of the underlying tests has been deliberately omitted here.

Engle and Granger (1987) propose a two-step procedure to test the null hypothesis of no cointegration against the alternative of at least one cointegrating vector. First, the long-run relationship between y_t and $\mathbf{x}_t$ is estimated by least squares regression. The obtained residuals $\hat{u}_t$ are then tested for a unit root. For this, Engle and Granger suggest the use of the t-statistic t_γ^{ADF} in the Augmented Dickey-Fuller (ADF) regression:

$$\Delta\hat{u}_t = \gamma\hat{u}_{t-1} + \sum_{i=1}^{k}\pi_i\Delta\hat{u}_{t-i} + \varepsilon_t. \tag{2.4}$$

The rejection of a unit root points to a cointegration relationship.

Johansen's (1988) maximum eigenvalue test is a system-based test that allows for several cointegration relationships. Take the vector error correction model (VECM)[1]

$$\Delta\mathbf{z}_t = \mathbf{\Pi}\mathbf{z}_{t-1} + \sum_{i=1}^{k}\mathbf{\Gamma}_p\Delta\mathbf{z}_{t-p} + \mathbf{d}_t + \varepsilon_t. \tag{2.5}$$

blabla Johansen test statistic

Banerjee and Boswijk

[1]Due to practical reasons we omit the derivation of the VECM which is presumed to be known.

To combine the results from the underlying tests Bayer and Hanck draw upon Fisher's combined probability test (Fisher, 1932). It merges the tests using the formula

$$\tilde{\chi}^2_{\mathcal{I}} := -2 \sum_{i \in \mathcal{I}} \ln (p_i). \tag{2.6}$$

Let t_i be the i^{th} test statistic. If test i rejects for large values, take $\xi_i := t_i$. If test i rejects for small values, take $-\xi_i := t_i$. With $\Xi_i(x) := \Pr_{\mathcal{H}'}(\xi_i \geq x)$ the p-value of the i^{th} test is $p_i := \Xi_i(\xi_i)$.

Fisher (1932) shows that under the assumption of independence the null distribution of $\tilde{\chi}^2_{\mathcal{I}}$ follows a chi-squared distribution with $2\mathcal{I}$ degrees of freedom. If this assumption is violated the null distribution is less evident. Here, the latter case occurs, as the ξ_i are not independent. The $\tilde{\chi}^2_{\mathcal{I}}$, however, have well-defined asymptotic null distributions $F_{\mathcal{F}_{\mathcal{I}}}$, as $\tilde{\chi}^2_{\mathcal{I}} \rightarrow_d \mathcal{F}_{\mathcal{I}}$ under $\mathcal{H}_0$ if $T \rightarrow \infty$, with $\mathcal{F}_{\mathcal{I}}$ some random variable. It is therefore feasible to simulate the joint null distribution of the ξ_i to obtain the distribution $F_{\mathcal{F}_{\mathcal{I}}}$ of (2.6). The $F_{\mathcal{F}_{\mathcal{I}}}$ depend on which and how many tests are combined. The distributions of the ξ_i depend on $K-1$ and the deterministic case.

3 Simulation

In this section, we describe the simulation of the null distribution of the Bayer Hanck meta test. The objective is to obtain data for training machine learning algorithms on approximating the p-values of the aforementioned test. In consideration of the different forms of the meta test we generate six data sets. These vary according to the specific combinations of the underlying tests and also account for the above-mentioned restrictions on the deterministic parts of the model.

The following approach relies largely on previous work by Pesavento (2004). For calculating the Bayer Hanck test statistic we require the p-values of the underlying tests. For this, we simulate their null distributions. It can be shown that asymptotically these are functions of standard Brownian motions. Here, the latter are constructed by step functions using Gaussian random walk with $N = 1000$ observations. The number of repetitions is set to 1,000,000. Furthermore, we consider $\mathrm{R}^2 \in \{0, 0.05, 0.1, ..., 0.95\}$, $k = 11$ and $c = 0$[2] (c mal definieren).

[2]Since we solely aim at simulating the distribution of the null of no cointegration we

From the mass of test statistics we build the cumulative distribution function of each underlying test and calculate the respective p-values. These are inserted into (2.6) to eventually obtain the Bayer Hanck test statistics. Analogous to the previous approach, we deduce the associated null distribution and the p-values.

4 Models

5 Package

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will not consider any further values of c here.

References

Banerjee, A., Dolado, J., & Mestre, R. (1998). Error-correction mechanism tests for cointegration in a single-equation framework. *Journal of Time Series Analysis*, *19*(3), 267–283. Retrieved from https://EconPapers.repec.org/RePEc:bla:jtsera:v:19:y:1998:i:3:p:267-283

Bayer, C., & Hanck, C. (2012). Combining non-cointegration tests. *Journal of Time Series Analysis*.

Boswijk, H. P. (1994). Testing for an unstable root in conditional and structural error correction models. *Journal of Econometrics*, *63*(1), 37–60. Retrieved from https://EconPapers.repec.org/RePEc:eee:econom:v:63:y:1994:i:1:p:37-60

Engle, R., & Granger, C. W. (1987). Co-integration and error correction: Representation, estimation and testing. *Econometrica*, *55*, 251–276.

Fisher, R. A. (1932). *Statistical methods for research workers*. Oliver, Boyd, Edinburgh, and London.

Johansen, S. (1988). Statistical analysis of cointegration vectors. *Journal of Economic Dynamics and Control*, *12*(2), 231–254. doi:https://doi.org/10.1016/0165-1889(88)90041-3

Pesavento, E. (2004). Analytical evaluation of the power of tests for the absence of cointegration. *Journal of Econometrics*, *122*(2), 349–384.

Software-References

Breiman, L., Cutler, A., Liaw, A., & Wiener., M. (2018). *Randomforest: Breiman and cutler's random forests for classification and regression.* R package version 4.6-14. Retrieved from https://CRAN.R-project.org/package=randomForest

Croissant, Y., Millo, G., & Tappe, K. (2019). *Plm: Linear models for panel data.* R package version 2.1-0. Retrieved from https://CRAN.R-project.org/package=plm

Friedman, J., Hastie, T., Tibshirani, R., Simon, N., Narasimhan, B., & Qian, J. (2019). *Glmnet: Lasso and elastic-net regularized generalized linear models.* R package version 2.0-18. Retrieved from https://CRAN.R-project.org/package=glmnet

Greenwell, B., Boehmke, B., Cunningham, J., & Developers, G. (2019). *Gbm: Generalized boosted regression models.* R package version 2.1.5. Retrieved from https://CRAN.R-project.org/package=gbm

Henry, L., & Wickham, H. (2019). *Purrr: Functional programming tools.* R package version 0.3.2. Retrieved from https://CRAN.R-project.org/package=purrr

Hlavac, M. (2018). *Stargazer: Well-formatted regression and summary statistics tables.* R package version 5.2.2. Retrieved from https://CRAN.R-project.org/package=stargazer

Izrailev, S. (2014). *Tictoc: Functions for timing r scripts, as well as implementations of stack and list structures.* R package version 1.0. Retrieved from https://CRAN.R-project.org/package=tictoc

Kuhn, M., Wing, J., Weston, S., Williams, A., Keefer, C., Engelhardt, A., … Hunt., T. (2019). *Caret: Classification and regression training.* R package version 6.0-84. Retrieved from https://CRAN.R-project.org/package=caret

Lumley, T., & Miller, A. (2017). *Leaps: Regression subset selection.* R package version 3.0. Retrieved from https://CRAN.R-project.org/package=leaps

Mevik, B.-H., Wehrens, R., & Liland, K. H. (2019). *Pls: Partial least squares and principal component regression.* R package version 2.7-1. Retrieved from https://CRAN.R-project.org/package=pls

Milborrow, S. (2019a). *Plotmo: Plot a model's residuals, response, and partial dependence plots*. R package version 3.5.5. Retrieved from https://CRAN.R-project.org/package=plotmo

Milborrow, S. (2019b). *Rpart.plot: Plot 'rpart' models: An enhanced version of 'plot.rpart'*. R package version 3.0.7. Retrieved from https://CRAN.R-project.org/package=rpart.plot

R Core Team. (2019). *R: A language and environment for statistical computing*. R Foundation for Statistical Computing. Vienna, Austria. Retrieved from https://www.R-project.org/

Ripley, B. (2019a). *Class: Functions for classification*. R package version 7.3-15. Retrieved from https://CRAN.R-project.org/package=class

Ripley, B. (2019b). *Mass: Support functions and datasets for venables and ripley's mass*. R package version 7.3-51.4. Retrieved from https://CRAN.R-project.org/package=MASS

Ripley, B. (2019c). *Tree: Classification and regression trees*. R package version 1.0-40. Retrieved from https://CRAN.R-project.org/package=tree

RStudio Team. (2019). *Rstudio: Integrated development environment for r*. Version 1.2.1541. RStudio, Inc. Boston, MA. Retrieved from http://www.rstudio.com/

Rushworth, A. (2019). *Inspectdf: Inspection, comparison and visualisation of data frames*. R package version 0.0.4. Retrieved from https://CRAN.R-project.org/package=inspectdf

Sievert, C., Parmer, C., Hocking, T., Chamberlain, S., Ram, K., Corvellec, M., & Despouy, P. (2019). *Plotly: Create interactive web graphics via 'plotly.js'*. R package version 4.9.0. Retrieved from https://CRAN.R-project.org/package=plotly

Therneau, T., & Atkinson, B. (2019). *Rpart: Recursive partitioning and regression trees*. R package version 4.1-15. Retrieved from https://CRAN.R-project.org/package=rpart

Ushey, K., Allaire, J., Wickham, H., & Ritchie, G. (2019). *Rstudioapi: Safely access the rstudio api*. R package version 0.10. Retrieved from https://CRAN.R-project.org/package=rstudioapi

Wickham, H. (2019). *Stringr: Simple, consistent wrappers for common string operations*. R package version 1.4.0. Retrieved from https://CRAN.R-project.org/package=stringr

Wickham, H., François, R., Henry, L., & Müller, K. (2019). *Dplyr: A grammar of data manipulation*. R package version 0.8.0.1. Retrieved from https://CRAN.R-project.org/package=dplyr

Wickham, H., & Henry, L. (2019). *Tidyr: Easily tidy data with 'spread()' and 'gather()' functions*. R package version 0.8.3. Retrieved from https://CRAN.R-project.org/package=tidyr

Xie, Y. (2019). *Knitr: A general-purpose package for dynamic report generation in r*. R package version 1.23. Retrieved from https://CRAN.R-project.org/package=knitr

A Appendices

Number	Functional form	Range of γ
1	$p = \mathrm{poly}(t, \gamma) + (1/k)$	$\gamma \in \mathbb{Z}\,[3, 10]$
2	$p = \mathrm{poly}(t, \gamma) + (1/k) + \mathrm{poly}(t, \gamma) * 1/k$	$\gamma \in \mathbb{Z}\,[3, 10]$
3	$p = \mathrm{poly}(t, \gamma) + \log(k) + \mathrm{poly}(k, \gamma) * \log(k)$	$\gamma \in \mathbb{Z}\,[3, 10]$
4	$p = \mathrm{poly}(t, \gamma) + k + (1/k)$	$\gamma \in \mathbb{Z}\,[3, 10]$
5	$p = \mathrm{poly}(\log(t), \gamma) + \log(k)$	$\gamma \in \mathbb{Z}\,[3, 10]$
6	$p = \mathrm{poly}(\log(t), \gamma) * \log(k)$	$\gamma \in \mathbb{Z}\,[3, 10]$
7	$p = \mathrm{poly}(\log(t), \gamma) + k$	$\gamma \in \mathbb{Z}\,[3, 10]$
8	$p = \mathrm{poly}(\log(t), \gamma) * k$	$\gamma \in \mathbb{Z}\,[3, 10]$
9	$p = \mathrm{poly}(\log(t), \gamma) * k + 1/k$	$\gamma \in \mathbb{Z}\,[3, 10]$
10	$p = \mathrm{poly}(\log(t), \gamma) * \log(k)$	$\gamma \in \mathbb{Z}\,[3, 10]$
11	$p = \mathrm{poly}(\log(t), \gamma) * \log(k) + 1/k$	$\gamma \in \mathbb{Z}\,[3, 10]$
12	$p = \mathrm{poly}(\mathrm{bc}(t), \gamma) * \log(k) + \mathrm{poly}(\mathrm{bc}(t), \gamma) * 1/k$	$\gamma \in \mathbb{Z}\,[3, 10]$
13	$p = \mathrm{poly}(\mathrm{bc}(t), \gamma) * \log(k) + \mathrm{poly}(\mathrm{bc}(t), \gamma) * \sqrt{k}$	$\gamma \in \mathbb{Z}\,[3, 10]$
14	$p = \mathrm{poly}(\mathrm{bc}(t), \gamma) * \log(k) + \mathrm{poly}(\mathrm{bc}(t), \gamma) * 1/k + \sqrt{k}$	$\gamma \in \mathbb{Z}\,[3, 10]$
15	$\mathrm{bc}(p) = \mathrm{poly}(\mathrm{bc}(t), \gamma) + \log(k)$	$\gamma \in \mathbb{Z}\,[3, 10]$
16	$\mathrm{bc}(p) = \mathrm{poly}(\mathrm{bc}(t), \gamma) * \log(k)$	$\gamma \in \mathbb{Z}\,[3, 10]$
17	$\mathrm{bc}(p) = \mathrm{poly}(\mathrm{bc}(t), \gamma) * \log(k) + 1/k$	$\gamma \in \mathbb{Z}\,[3, 10]$
18	$\mathrm{bc}(p) = \mathrm{poly}(\mathrm{bc}(t), \gamma) * \log(k) + \mathrm{poly}(\mathrm{bc}(t), \gamma) * 1/k$	$\gamma \in \mathbb{Z}\,[3, 10]$
19	$\mathrm{bc}(p) = \mathrm{poly}(\mathrm{bc}(t), \gamma) * \log(k) + \mathrm{poly}(\mathrm{bc}(t), \gamma) * \sqrt{k}$	$\gamma \in \mathbb{Z}\,[3, 10]$
20	$\log(p) = \mathrm{poly}(\mathrm{bc}(t), \gamma) * \log(k)$	$\gamma \in \mathbb{Z}\,[3, 10]$
21	$\log(p) = \mathrm{poly}(\mathrm{bc}(t), \gamma) * \log(k) + 1/k$	$\gamma \in \mathbb{Z}\,[3, 10]$
22	$\log(p) = \mathrm{poly}(\mathrm{bc}(t), \gamma) * \log(k) + \mathrm{poly}(\mathrm{bc}(t), \gamma) * \sqrt{k}$	$\gamma \in \mathbb{Z}\,[3, 10]$
23	$\log(p) = \mathrm{poly}(\mathrm{bc}(t), \gamma) * \log(k) + \mathrm{poly}(\mathrm{bc}(t), \gamma) * 1/k$	$\gamma \in \mathbb{Z}\,[3, 10]$
24	$\log(p) = \mathrm{poly}(\mathrm{bc}(t), \gamma) * \log(k) + \mathrm{poly}(\mathrm{bc}(t), \gamma) * 1/k + \sqrt{k}$	$\gamma \in \mathbb{Z}\,[3, 10]$

Table A1: Description of all tested models....

Functional form	Full distribution RMSE	Full distribution cRMSE	Lower tail ($p < 0.2$) RMSE	Lower tail ($p < 0.2$) cRMSE
$\mathrm{bc}(p) = \mathrm{poly}(\mathrm{bc}(t), 10) * \log(k) + \mathrm{poly}(\mathrm{bc}(t), 10) * \sqrt{k}$	$4.97 \cdot 10^{-4}$	$4.69 \cdot 10^{-4}$	$8.05 \cdot 10^{-4}$	$7.16 \cdot 10^{-4}$
$\mathrm{bc}(p) = \mathrm{poly}(\mathrm{bc}(t), 10) * \log(k) + \mathrm{poly}(\mathrm{bc}(t), 10) * 1/k$	$5.39 \cdot 10^{-4}$	$5.11 \cdot 10^{-4}$	$8.54 \cdot 10^{-4}$	$7.61 \cdot 10^{-4}$
$p = \mathrm{poly}(\mathrm{bc}(t), 10) * \log(k) + \mathrm{poly}(\mathrm{bc}(t), 10) * \sqrt{k}$	$7.68 \cdot 10^{-4}$	$6.91 \cdot 10^{-4}$	$1.01 \cdot 10^{-3}$	$8.97 \cdot 10^{-4}$
$p = \mathrm{poly}(\mathrm{bc}(t), 10) * \log(k) + \mathrm{poly}(\mathrm{bc}(t), 10) * 1/k + \sqrt{k}$	$7.79 \cdot 10^{-4}$	$7.04 \cdot 10^{-4}$	$1.05 \cdot 10^{-3}$	$9.31 \cdot 10^{-4}$
$p = \mathrm{poly}(\mathrm{bc}(t), 10) * \log(k) + \mathrm{poly}(\mathrm{bc}(t), 10) * 1/k$	$7.82 \cdot 10^{-4}$	$7.07 \cdot 10^{-4}$	$1.06 \cdot 10^{-3}$	$9.41 \cdot 10^{-4}$

Table A2: The 5 best models for the combined Non-Cointegration test of Bayer and Hanck, where *all* underlying test are included and case 1.

Functional form	Full distribution RMSE	Full distribution cRMSE	Lower tail ($p < 0.2$) RMSE	Lower tail ($p < 0.2$) cRMSE
$\log(p) = \mathrm{poly}(\mathrm{bc}(t), 10) * \log(k)$	$1.27 \cdot 10^{-3}$	$1.25 \cdot 10^{-3}$	$1.05 \cdot 10^{-3}$	$9.52 \cdot 10^{-4}$
$\log(p) = \mathrm{poly}(\mathrm{bc}(t), 10) * \log(k) + \mathrm{poly}(\mathrm{bc}(t), 10) * \sqrt{k}$	$6.82 \cdot 10^{-4}$	$6.22 \cdot 10^{-4}$	$1.28 \cdot 10^{-3}$	$1.12 \cdot 10^{-3}$
$\log(p) = \mathrm{poly}(\mathrm{bc}(t), 10) * \log(k) + \mathrm{poly}(\mathrm{bc}(t), 10) * 1/k$	$7.32 \cdot 10^{-4}$	$6.63 \cdot 10^{-4}$	$1.39 \cdot 10^{-3}$	$1.20 \cdot 10^{-3}$
$\log(p) = \mathrm{poly}(\mathrm{bc}(t), 10) * \log(k) + \mathrm{poly}(\mathrm{bc}(t), 10) * 1/k + \sqrt{k}$	$8.38 \cdot 10^{-4}$	$7.78 \cdot 10^{-4}$	$1.48 \cdot 10^{-3}$	$1.31 \cdot 10^{-3}$
$\mathrm{bc}(p) = \mathrm{poly}(\mathrm{bc}(t), 10) * \log(k) + \mathrm{poly}(\mathrm{bc}(t), 10) * 1/k$	$9.08 \cdot 10^{-4}$	$8.42 \cdot 10^{-4}$	$1.69 \cdot 10^{-3}$	$1.50 \cdot 10^{-3}$

Table A3: The 5 best models for the combined Non-Cointegration test of Bayer and Hanck, where *all* underlying test are included and case 2.

Functional form	Full distribution RMSE	Full distribution cRMSE	Lower tail ($p < 0.2$) RMSE	Lower tail ($p < 0.2$) cRMSE
$\log(p) = \mathrm{poly}(\mathrm{bc}(t), 10) * \log(k) + \mathrm{poly}(\mathrm{bc}(t), 10) * \sqrt{k}$	$4.58 \cdot 10^{-4}$	$4.55 \cdot 10^{-4}$	$3.37 \cdot 10^{-4}$	$3.16 \cdot 10^{-4}$
$\log(p) = \mathrm{poly}(\mathrm{bc}(t), 10) * \log(k) + \mathrm{poly}(\mathrm{bc}(t), 10) * 1/k$	$5.17 \cdot 10^{-4}$	$5.14 \cdot 10^{-4}$	$3.90 \cdot 10^{-4}$	$3.73 \cdot 10^{-4}$
$\log(p) = \mathrm{poly}(\mathrm{bc}(t), 10) * \log(k)$	$1.04 \cdot 10^{-3}$	$1.04 \cdot 10^{-3}$	$6.76 \cdot 10^{-4}$	$6.50 \cdot 10^{-4}$
$\log(p) = \mathrm{poly}(\mathrm{bc}(t), 10) * \log(k) + \mathrm{poly}(\mathrm{bc}(t), 10) * 1/k + \sqrt{k}$	$1.18 \cdot 10^{-3}$	$1.17 \cdot 10^{-3}$	$2.06 \cdot 10^{-3}$	$2.05 \cdot 10^{-3}$
$\mathrm{bc}(p) = \mathrm{poly}(\mathrm{bc}(t), 10) * \log(k) + \mathrm{poly}(\mathrm{bc}(t), 10) * 1/k$	$1.16 \cdot 10^{-3}$	$1.06 \cdot 10^{-3}$	$2.08 \cdot 10^{-3}$	$1.80 \cdot 10^{-3}$

Table A4: The 5 best models for the combined Non-Cointegration test of Bayer and Hanck, where *all* underlying test are included and case 3.

Functional form	Full distribution RMSE	Full distribution cRMSE	Lower tail ($p < 0.2$) RMSE	Lower tail ($p < 0.2$) cRMSE
$\log(p) = \mathrm{poly}(\mathrm{bc}(t), 10) * \log(k) + \mathrm{poly}(\mathrm{bc}(t), 10) * 1/k$	$4.75 \cdot 10^{-4}$	$4.44 \cdot 10^{-4}$	$7.81 \cdot 10^{-4}$	$6.84 \cdot 10^{-4}$
$\log(p) = \mathrm{poly}(\mathrm{bc}(t), 10) * \log(k)$	$6.54 \cdot 10^{-4}$	$5.87 \cdot 10^{-4}$	$1.01 \cdot 10^{-3}$	$7.81 \cdot 10^{-4}$
$\log(p) = \mathrm{poly}(\mathrm{bc}(t), 10) * \log(k) + \mathrm{poly}(\mathrm{bc}(t), 10) * \sqrt{k}$	$7.60 \cdot 10^{-4}$	$6.13 \cdot 10^{-4}$	$1.46 \cdot 10^{-3}$	$1.06 \cdot 10^{-3}$
$\log(p) = \mathrm{poly}(\mathrm{bc}(t), 10) * \log(k) + \mathrm{poly}(\mathrm{bc}(t), 10) * 1/k + \sqrt{k}$	$7.64 \cdot 10^{-4}$	$7.45 \cdot 10^{-4}$	$1.29 \cdot 10^{-3}$	$1.23 \cdot 10^{-3}$
$\mathrm{bc}(p) = \mathrm{poly}(\mathrm{bc}(t), 10) * \log(k) + \mathrm{poly}(\mathrm{bc}(t), 10) * 1/k$	$1.01 \cdot 10^{-3}$	$9.17 \cdot 10^{-4}$	$1.89 \cdot 10^{-3}$	$1.65 \cdot 10^{-3}$

Table A5: The 5 best models for the combined Non-Cointegration test of Bayer and Hanck, where *EG-J* underlying test are included and case 1.

Functional form	Full distribution		Lower tail ($p < 0.2$)	
	RMSE	cRMSE	RMSE	cRMSE
$\log(p) = \text{poly}(\text{bc}(t), 10) * \log(k)$	$7.36 \cdot 10^{-4}$	$7.25 \cdot 10^{-4}$	$7.04 \cdot 10^{-4}$	$6.45 \cdot 10^{-4}$
$\log(p) = \text{poly}(\text{bc}(t), 10) * \log(k) + \text{poly}(\text{bc}(t), 10) * 1/k$	$5.53 \cdot 10^{-4}$	$5.12 \cdot 10^{-4}$	$9.75 \cdot 10^{-4}$	$8.56 \cdot 10^{-4}$
$\log(p) = \text{poly}(\text{bc}(t), 10) * \log(k) + \text{poly}(\text{bc}(t), 10) * \sqrt{k}$	$5.53 \cdot 10^{-4}$	$5.11 \cdot 10^{-4}$	$9.87 \cdot 10^{-4}$	$8.66 \cdot 10^{-4}$
$\log(p) = \text{poly}(\text{bc}(t), 10) * \log(k) + \text{poly}(\text{bc}(t), 10) * 1/k + \sqrt{k}$	$6.00 \cdot 10^{-4}$	$5.62 \cdot 10^{-4}$	$1.11 \cdot 10^{-3}$	$1.01 \cdot 10^{-3}$
$\text{bc}(p) = \text{poly}(\text{bc}(t), 10) * \log(k) + \text{poly}(\text{bc}(t), 10) * \sqrt{k}$	$1.05 \cdot 10^{-3}$	$9.54 \cdot 10^{-4}$	$2.00 \cdot 10^{-3}$	$1.75 \cdot 10^{-3}$

Table A6: The 5 best models for the combined Non-Cointegration test of Bayer and Hanck, where *EG-J* underlying test are included and case 2.

Functional form	Full distribution		Lower tail ($p < 0.2$)	
	RMSE	cRMSE	RMSE	cRMSE
$\log(p) = \text{poly}(\text{bc}(t), 10) * \log(k) + \text{poly}(\text{bc}(t), 10) * \sqrt{k}$	$3.85 \cdot 10^{-4}$	$3.73 \cdot 10^{-4}$	$5.03 \cdot 10^{-4}$	$4.58 \cdot 10^{-4}$
$\log(p) = \text{poly}(\text{bc}(t), 10) * \log(k)$	$7.55 \cdot 10^{-4}$	$7.54 \cdot 10^{-4}$	$4.85 \cdot 10^{-4}$	$4.70 \cdot 10^{-4}$
$\log(p) = \text{poly}(\text{bc}(t), 10) * \log(k) + \text{poly}(\text{bc}(t), 10) * 1/k$	$3.73 \cdot 10^{-4}$	$3.59 \cdot 10^{-4}$	$5.34 \cdot 10^{-4}$	$4.83 \cdot 10^{-4}$
$\log(p) = \text{poly}(\text{bc}(t), 10) * \log(k) + \text{poly}(\text{bc}(t), 10) * 1/k + \sqrt{k}$	$4.87 \cdot 10^{-4}$	$4.76 \cdot 10^{-4}$	$8.52 \cdot 10^{-4}$	$8.19 \cdot 10^{-4}$
$\text{bc}(p) = \text{poly}(\text{bc}(t), 10) * \log(k) + \text{poly}(\text{bc}(t), 10) * \sqrt{k}$	$1.02 \cdot 10^{-3}$	$9.35 \cdot 10^{-4}$	$1.94 \cdot 10^{-3}$	$1.70 \cdot 10^{-3}$

Table A7: The 5 best models for the combined Non-Cointegration test of Bayer and Hanck, where *EG-J* underlying test are included and case 3.

Eidesstattliche Versicherung

Ich versichere an Eides statt durch meine Unterschrift, dass ich die vorstehende Arbeit selbständig und ohne fremde Hilfe angefertigt und alle Stellen, die ich wörtlich oder annähernd wörtlich aus Veröffentlichungen entnommen habe, als solche kenntlich gemacht habe, mich auch keiner anderen als der angegebenen Literatur oder sonstiger Hilfsmittel bedient habe. Die Arbeit hat in dieser oder ähnlicher Form noch keiner anderen Prüfungsbehörde vorgelegen.

Essen, den ____________________ ________________________________

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