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COINTEGRATION AND CHANGES IN REGIME: THE JAPANESE CONSUMPTION FUNCTION

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SUMMARY

In this paper we examine a model of cointegration where long-run parameters are subject to switching between several different cointegrating regimes. These shifts are allowed to be governed by the outcome of an unobserved Markov chain with unknown transition probabilities. We illustrate this approach using Japanese data on consumption and disposable income, and find that the data favour a Markov-switching long-run relationship over a standard temporally stable formulation. © 1997 by John Wiley & Sons, Ltd. J. appl. econom. 12: 151–168, 1997.

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1. INTRODUCTION

In recent years the idea of cointegration and the related issue of the existence of long-run equilibrium relationships among economic variables with stochastic trends have become important facets of the analysis of multivariate time series. A multitude of statistical procedures of testing for the presence of cointegration are now available in the literature, along with a wide menu of methods of estimating models that link cointegrated variables (see Banerjee *et al.*, 1993, for a survey). However, despite the fact that economic data often come from processes with time-dependent parameters, the majority of available results rely heavily on the assumptions of time series with no structural changes and of long-run relationships that are temporally stable.

The objective of this paper is to explore the possibility of treating the cointegrating vector for a set of variables as changing over time, a formulation which includes the conventional stable linear model as a special case. The motivation behind such a notion of cointegration derives from the economically plausible situation where long-run components of variables obey equilibrium constraints, but these constraints are time-varying as a result of changes in taste, technology, or economic policy (see also Granger and Lee, 1991, and Gregory and Hansen, 1996, for analyses along similar lines). We argue that a promising approach is to allow the parameters of the cointegrating relationship to undergo occasional discrete changes, as in the Markov-switching models of Goldfeld and Quandt (1973), Cosslett and Lee (1985) and Hamilton (1989). The advantage of such a formulation is that it is flexible enough to allow for a variety of behaviour, including permanent regime shifts and short-lived changes.

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We illustrate our approach by focusing on the relationship between total consumption and disposable income in Japan. The properties of this bivariate system were previously analysed in Engle *et al.* (1993) (henceforth EGHL), where only weak support for cointegration at the annual frequency was found, It is argued here that, while the standard hypothesis of zero-frequency cointegration, including its implicit assumption of temporal stability of the cointegrating relationship, is rejected by the data, there is significant evidence in favour of a model of cointegration subject to regime changes.

The plan of the paper is as follows. Section 2 re-examines the univariate time-series characteristics of the Japanese consumption and income data, focusing attention of the possibility of changes in the structure of the series. Section 3 reports the results of an analysis of the temporal stability properties of several cointegrating relationships between consumption and income. Section 4 investigates the possibility of cointegration within a Markov-switching framework, from both a theoretical and an empirical standpoint. Section 5 summarizes and concludes.

2. UNIVARIATE CHARACTERISTICS OF THE DATA

In what follows, $\{c_t\}_{t=1}^{T}$ and $\{y_t\}_{t=1}^{T}$ respectively denote the time series of the natural logarithms of real total private consumption and real disposable income (at 1980 prices) in Japan. The data (the same as used in EGHL) are quarterly, covering the period from 1961 : 1 to 1987 : 4 ($T = 108$). Figure 1 plots $\{c_t\}$ and $\{y_t\}$ and reveals a strong upward trend in both series as well as a clear seasonal pattern. It is also apparent that both series exhibit a slowdown in their underlying growth pattern after the 1973 oil-price crisis.[1]

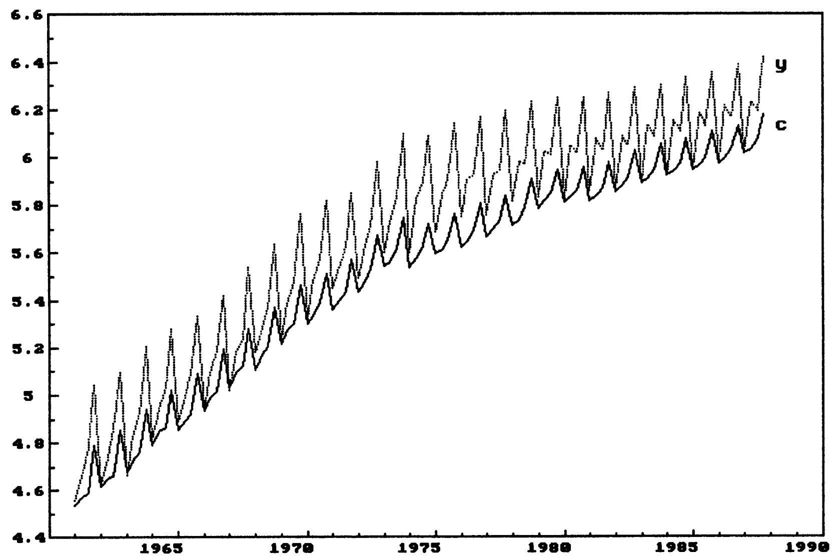

Figure 1. Logarithms of real total consumption (c) and real disposable income (y) in Japan over 1961 : 1–1987 : 4

[1] A change in the mean and seasonal pattern is also evident in the consumption-income ratio $\{c_t - y_t\}$.

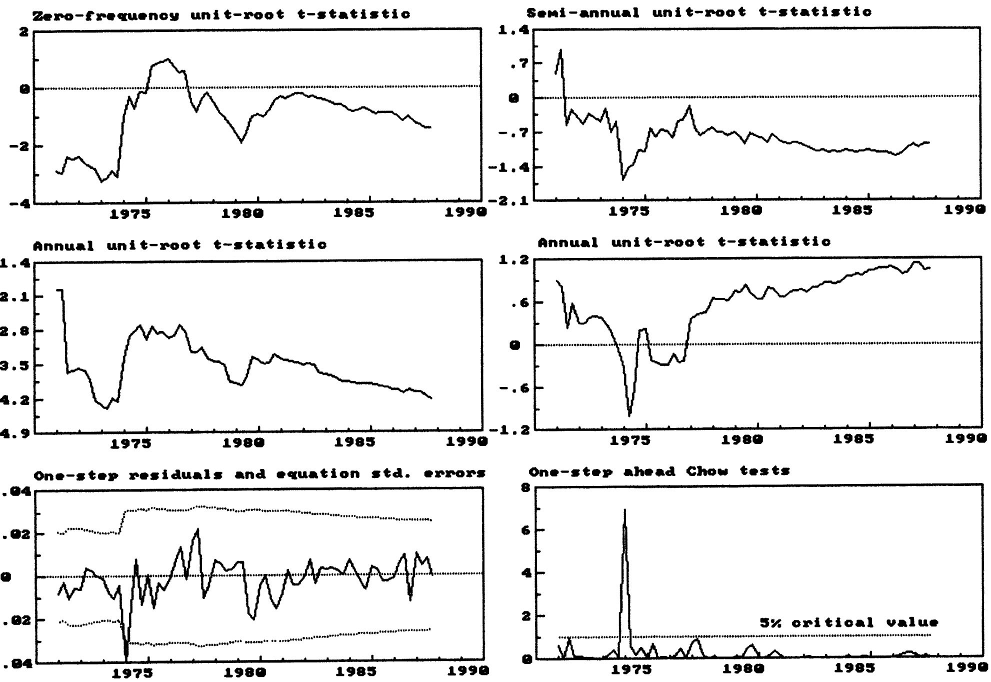

Figure 2. Recursive graphical statistics for the HEGY regression for c_t

Tests for seasonal unit roots reported in EGHL suggest that $\{c_t\}$ is integrated of order 1 at both the long-run frequency and at the seasonal frequencies, while $\{y_t\}$ is integrated of order 1 only at the frequencies 0 and π.[2] Our objective in this section is to examine whether such a characterization of non-stationarily is robust to potential structural breaks in the deterministic components of the series.

We start by analysing the sensitivity of the EGHL results with respect to changes in the sample period. This is done by estimating the EGHL test regressions (including an intercept, seasonal dummies, and a linear time trend) recursively using increasing sub-samples of the data. The graphs in Figures 2 and 3 summarize much of the relevant information by showing: (1) the time series of recursively computed Hylleberg *et al.* (1990) (henceforth HEGY) unit-root t-statistics;[3] (2) one-step-ahead residuals together with $0 \pm 2\hat{\sigma}_t$, where $\hat{\sigma}_t$ is the estimated equation standard error at sample size t; (3) a sequence of one-step-ahead Chow tests, scaled by their single-test 5% critical values at each possible change point. The unit-root t-statistics are considerably volatile over the sample period, such that both reject and non-reject outcomes occur for many of the tests. The effect of the 1973 oil-price shock is also apparent in these plots and is further reflected by the one-step residuals and the associated parameter constancy tests. Such evidence, paired with the tendency of unit-root tests to overestimate the order of

[2] An integrated of order 1 series is defined as a series with an autoregressive component which has a root of modulus one corresponding to a particular frequency. This implies that the spectrum of the series is unbounded above at the frequency in question.

[3] These are the t-ratios on π_1, π_2, π_3, and π_4 in equation (3.8) of HEGY.

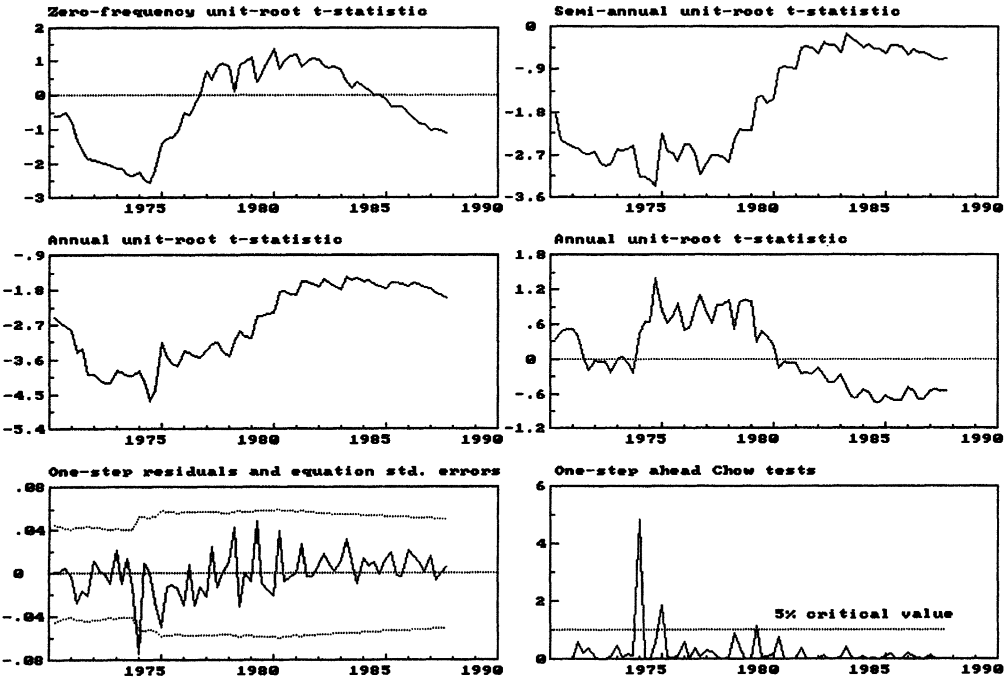

Figure 3. Recursive graphical statistics for the HEGY regression for y_t

integration of a series when structural shifts are not accounted for (cf. Perron, 1989; Hall and Scott, 1990; Chysels, 1991, *inter alia*), brings the reliability of the EGHL inferences into question.

One way to avoid the troubling effects of unaccounted structural breaks is to test the null hypothesis of a unit root at the long-run and seasonal frequencies against a (stochastically) stationary alternative which allows for a segmented trend and seasonal pattern. For a given time series of data $\{x_t\}$, such tests can be based on the auxiliary ordinary least-squares (OLS) regression:

$$(1 - B^4)x_t = \pi_1 x_{1,t-1} + \pi_2 x_{2,t-1} + \pi_3 x_{3,t-2} + \pi_4 x_{3,t-1} + \mu + \sum_{j=1}^{3} \alpha_j D_t^j$$

$$+ \gamma t + \mu_d d_t + \sum_{j=1}^{3} \alpha_j^d D_t^j d_t + \gamma_d t d_t + \sum_{j=1}^{k} \psi_j (1 - B^4) x_{t-j} + e_t \qquad (1)$$

where B is the lag operator such that $B^j x_t = x_{t-j}$, $\{e_t\}$ is a white noise, D_t^j is a standard zero/one seasonal dummy variable corresponding to quarter j, d_t is a zero/one step dummy with step in 1974 : 1, $x_{1t} = (1 + B + B^2 + B^3)x_t$, $x_{2t} = -(1 - B + B^2 - B^3)x_t$, and $x_{3t} = -(1 - B^2)x_t$. The existence of a unit root at the zero frequency, the semi-annual frequency (π), and the annual frequency ($\pi/2$, $3\pi/2$) implies that $\pi_1 = 0$, $\pi_2 = 0$, and $\pi_3 = \pi_4 = 0$, respectively. Hence, a zero-frequency unit root can be tested against the stationary alternative $\pi_1 < 0$ using the t-ratio for π_1. Similarly, a test for a semi-annual unit root against $\pi_2 < 0$ can be based on the t-ratio for π_2. For

Table I. HEGY tests for seasonal unit roots

		Frequency					
		0	π	$\pi/2$			π and $\pi/2$
Series	Lags	π_1	π_2	π_3	π_4	$\pi_3 \cap \pi_4$	$\pi_2 \cap \pi_3 \cap \pi_4$
$\{c_t\}$	1,4,5,6, 8,10,12	−0·937 (−4·07) [−3·79]	0·706 (−3·10) [−2·80]	−5·049 (−5·01) [−4·66]	1·782 (−2·30, 2·37) [−1·92, 1·98]	15·747 (13·6) [12·1]	10·515 (10·8) [9·50]
$\{y_t\}$	0	−2·949 (−4·12) [−3·82]	−5·246 (−3·09) [−2·80]	−6·087 (−4·98) [−4·66]	0·294 (−2·34, 2·40) [−2·04, 1·99]	18·580 (13·5) [12·0]	16·770 (10·6) [9·48]

Note: Numbers in parentheses (square brackets) are 5% (10%) finite-sample critical values. They were obtained via Monte Carlo simulation, using 5000 realizations of a seasonally integrated process with independently and identically distributed (i.i.d.) $N(0, 1)$ innovations.

the annual frequency, a unit-root test can be based on an F-statistic for $\pi_3 = \pi_4 = 0$; alternatively, a two-step procedure may be employed, based on a t-type test of $\pi_4 = 0$ against $\pi_4 \neq 0$, followed by a t-type test of $\pi_3 = 0$ against $\pi_3 < 0$. An F-statistic for $\pi_2 = \pi_3 = \pi_4 = 0$ provides a test of the null of a unit root at all seasonal frequencies simultaneously (Ghysels *et al.*, 1994).

The results of the tests for $\{c_t\}$ and $\{y_t\}$ are reported in Table I.[4] The annual-frequency unit-root hypothesis can be rejected at the 5% level for both series, and so can the hypothesis of a semi-annual unit root in $\{y_t\}$. Moreover, the joint test for seasonal unit roots in $\{c_t\}$ rejects the null hypothesis at the 10% level.[5]

As a heuristic check against low test power, we also test the null hypothesis of deterministic seasonality (subject to an one-time change in 1973 : 4) against unit-root non-stationarities at the seasonal frequencies using the Lagrange multiplier (LM) test statistics of Canova and Hansen (1995). These are based on a regression model of the form:

$$(1 - B)x_t = \mu + \sum_{j=1}^{3} \alpha_j D_t^j + \mu_d d_t + \sum_{j=1}^{3} \alpha_j^d D_t^j d_t + \varepsilon_t \tag{2}$$

where ε_t is a random disturbance term. Contrary to Canova and Hansen's (1995) suggestion, $(1 - B)x_{t-1}$ is not included as a regressor in (2) since this affects adversely the performance of tests against a unit root at the semi-annual frequency (see Hylleberg, 1995). Also, as the disturbances $\{\varepsilon_t\}$ are typically heteroscedastic and serially correlated, test statistics are constructed using robust kernel estimates of variance–covariance parameters.[6]

Table II reports the outcome of individual LM tests against non-stationarity at the semi-annual and annual frequencies, as well as the outcome of a joint test against unit roots at both

[4] Since the inclusion of redundant deterministic terms in equation (1) is likely to have adverse effects on the power of the tests, we have imposed the restrictions $\alpha_1^d = \alpha_3^d = 0$ (on the basis of evidence provided by significance tests). For the same reason, only lags of $(1 - B^4)x_t$ with statistically significant coefficients have been included in the auxiliary regressions.
[5] Psaradakis (1996) presents evidence favouring joint tests based on F-statistics over individual t-type tests.
[6] Throughout the paper, kernel estimators of covariance matrices are constructed using a quadratic-spectral kernel, an autoregressive prewhitening procedure, and automatic data-dependent bandwidth selection based on univariate autoregressive approximating models (for details see Andrews, 1991; Andrews and Monahan, 1992).

Table II. LM tests of deterministic versus non-stationary stochastic seasonality

	Frequency		
Series	π	$\pi/2$	π and $\pi/2$
$\{(1-B)c_t\}$	0·335	0·683	0·830
$\{(1-B)y_t\}$	0·208	0·561	0·692
	(0·470)	(0·749)	(1·010)
	[0·353]	[0·610]	[0·846]

Note: Numbers in parentheses (square brackets) are 5% (10%) asymptotic critical values.

seasonal frequencies. The evidence against seasonal unit roots is overwhelming, with all three tests failing to reject the null hypothesis of deterministic seasonality at the 5% level of significance.[7] Interestingly, similar results are obtained if 1979 : 3 (second oil-price shock) is chosen as the location of an one-off change in seasonality.

To summarize, once structural breaks in the deterministic components of $\{c_t\}$ and $\{y_t\}$ are accounted for, the evidence emerging from the data is not supportive of a non-stationary stochastic characterization of seasonality. Apart from its apparent statistical significance, such evidence is also appealing from an economic point of view as it precludes the rather extreme possibility of no long-run links between different seasons that is implied by a unit-root specification (cf. Osborn, 1993). Furthermore, the segmented seasonals are consistent with the non-trivial interactions between seasonal movements and business-cycle conditions implied by many dynamic macroeconomic models and empirically documented by Ghysels (1991) and Canova and Ghysels (1994).

3. COINTEGRATION AND PARAMETER CONSTANCY

Let us now turn our attention to possible cointegrating relations for the Japanese consumption and income data and examine their temporal stability. A cointegrating vector which is likely to remove all unit roots from the resulting series is the theoretically appealing $[1, -1]$ vector. Yet, EGHL found $\{c_t - y_t\}$ to be integrated of order 1 at the long-run frequency and all seasonal frequencies. Furthermore, tests based on the residuals from a cointegrating regression between c_{1t} and y_{1t} also failed to reject non-cointegration at the zero frequency, and a similar result was obtained when the cointegrating vector was fixed at $[1, -1]$.

It must be borne in mind, however, that such inferences are heavily dependent upon the (untested) underlying assumptions of time-invariance of the cointegrating relationship and of time-series processes with no changes in structure. Even so, regime changes and structural breaks are both economically and empirically relevant, and can severely affect the properties of inferential procedures. Specifically, just as univariate unit-root tests are biased towards non-stationarity for series with structural changes, the power of conventional residual-based tests for cointegration can fall sharply in the presence of unaccounted regime shifts (see Gregory and Nason, 1991; Gregory and Hansen, 1996; Campos *et al.*, 1996).

[7] In agreement with the HEGY-type tests reported in EGHL, LM tests strongly favour the unit-root hypothesis for both series when a one-off change in the seasonal pattern in 1973:4 is not allowed for.

Table III. Temporal stability tests for cointegrating regressions

Regressand	Regressor y_t	Regressor y_{1t}	Deterministic terms	*supLM*	*meanLM*	L_c
c_t	0·5834 (0·0339)	—	INT, TR	27·239 [0·010]	19·204 [0·010]	1·636 [0·010]
c_t	0·9512 (0·0146)	—	INT	23·086 [0·010]	10·766 [0·010]	1·144 [0·010]
$c_t^{(*)}$	0·7814 (0·0270)	—	INT, TR	19·449 [0·010]	10·215 [0·010]	0·901 [0·010]
$c_t^{(*)}$	0·9587 (0·0132)	—	INT	25·653 [0·010]	13·180 [0·010]	1·093 [0·010]
c_{1t}	—	0·8118 (0·0507)	INT, TR	492·08 [0·010]	127·01 [0·010]	0·148 [>0·20]
c_{1t}	—	1·1184 (0·0414)	INT	14·306 [0·027]	9·354 [0·010]	0·643 [0·016]

Notes: An asterisk indicates that deterministic seasonality has been removed from the series prior to estimating the cointegrating parameters. INT and TR denote an intercept and a linear trend, respectively. Numbers in parentheses are (fully modified) standard errors. Numbers in square brackets are approximate asymptotic p-values, calculated as in Hansen (1992).

To test for parameter constancy in the cointegrating regressions for Japan, while treating the change point as unknown, the sequential LM-type tests of Hansen (1992) are employed. This entails computing the LM test statistic for a subset of possible change points, and then base a test upon the maximum of these statistics (denoted here by *supLM*). The average of the LM statistics (denoted by *meanLM*) provides an alternative test of the null of temporal stability which is most powerful against hypotheses involving stochastic parameter variation of a martingale form. Similar optimality properties are also shared by Hansen's (1992) L_c test, which is an LM-type test against the alternative hypothesis of martingale parameters. The test statistics are computed using semiparametric estimates of the cointegrating vector (Phillips and Hansen, 1990), while covariance parameters are estimated by means of a robust kernel estimator. The *supLM* and *meanLM* statistics are calculated over change points within the central 70% observations of the sample.

Table III reports the Phillips–Hansen fully modified OLS estimates of the long-run parameters, along with tests for parameter constancy. Notice that the estimate of the long-run income elasticity is sensitive to the inclusion of a time trend in the regression—when the trend is present, estimates are considerably smaller than the economically more plausible value of 0.95. The parameter constancy tests suggest clearly that all relationships between c_t and y_t considered are temporally unstable. The sequences of LM statistics generally cross the 5% *supLM* critical value several times during the period 1973–81. Similar results are also obtained for the zero-frequency cointegrating regression between c_{1t} and y_{1t}: with the exception of the L_c statistic for the model with a trend, the tests firmly reject the existence of a constant-parameter long-run relationship.[8]

[8] It is interesting to note that when the sample period is split at either 1973:4 or 1979:3, non-cointegration within each sub-sample is rejected by regression-based tests like those used in Section 4.3.

4. COINTEGRATION AND STOCHASTIC MODELS OF CHANGES IN REGIME

In this section we are concerned with the possibility of a more general type of cointegration where the cointegrating vector is allowed to undergo occasional discrete changes during the sample period. Such shifts may be the result of sudden radical changes that often occur in policy, technology, and economic institutions. To motivate the analysis, we begin by presenting a simple example of a dynamic economic mechanism that could give rise to a Markov-switching long-run relationship.

4.1. Permanent Income Hypothesis and Regime Shifts

Consider the permanent income hypothesis of consumption behaviour, expressed as:

$$C_t = r(1+r)^{-1}\sum_{j=0}^{\infty}(1+r)^{-j}E(Y_{t+j}\mid\Omega_t) \tag{3}$$

where C_t is aggregate consumption, Y_t is real income, r is the real interest rate (assumed to be constant over time), and Ω_t is the information set that is available to economic agents at time t. Equation (3) may also be seen as representing a forward solution to the expectational difference equation:

$$C_t = r(1+r)^{-1}E(Y_t\mid\Omega_t) + (1+r)^{-1}E(C_{t+1}\mid\Omega_t) \tag{4}$$

The logarithm of income is assumed to evolve as a Gaussian random walk with drift, albeit one in which the drift and innovation variance are dependent upon the state of the economy (cf. Hamilton, 1989).[9] Thus, $\{Y_t\}$ satisfy the equation:

$$(1-B)\ln(Y_t) = b_0(1-s_t) + b_1 s_t + [\sigma_{y0}(1-s_t) + \sigma_{y1}s_t]\eta_t \tag{5}$$

where s_t is a latent, discrete-valued random variable that indicates the 'state' or 'regime' operative at time t, and $\{\eta_t\}$ is a sequence of i.i.d. $N(0,1)$ random variables independent of $\{s_m\}$ for all integer t and m. Following Hamilton (1989), the regime s_t is modelled as the outcome of a temporally homogeneous, first-order Markov chain with state space $\{0,1\}$ and transition probabilities:

$$\begin{aligned}
\Pr(s_t = 1\mid s_{t-1} = 1) &= p\\
\Pr(s_t = 0\mid s_{t-1} = 1) &= 1-p\\
\Pr(s_t = 0\mid s_{t-1} = 0) &= q\\
\Pr(s_t = 1\mid s_{t-1} = 0) &= 1-q
\end{aligned}$$

[9] Examples of economic changes that would affect the evolution of income include changes in technology and changes in fiscal policy.

If economic agents, unlike the econometrician, know the state of the economy at time t, it follows from equations (4) and (5) that $C_t = \lambda_0 Y_t$ when $s_t = 0$, and $C_t = \lambda_1 Y_t$ when $s_t = 1$, where λ_0 and λ_1 satisfy the system of equations (6) and (7):

$$\lambda_0 = (1+r)^{-1}[r + \lambda_0 q R_0 + \lambda_1(1-q)R_1] \tag{6}$$

$$\lambda_1 = (1+r)^{-1}[r + \lambda_0(1-p)R_0 + \lambda_1 p R_1] \tag{7}$$

with $R_i = \exp(b_i + \frac{1}{2}\sigma_{yi}^2)$ $(i = 0, 1)$. Consequently, the consumption–income equation may be written as:

$$\ln(C_t) = \ln(\lambda_0)(1 - s_t) + \ln(\lambda_1)s_t + \ln(Y_t) \tag{8}$$

where

$$\lambda_0 = \frac{r[1 + (1-p-q)(1+r)^{-1}R_1]}{(1 + r - pR_1 - qR_0) - (1+r)^{-1}(1-p-q)R_0R_1}$$

$$\lambda_1 = \frac{r[1 + (1-p-q)(1+r)^{-1}R_0]}{(1 + r - pR_1 - qR_0) - (1+r)^{-1}(1-p-q)R_0R_1}$$

Hence, although $\ln(C_t)$ and $\ln(Y_t)$ are linearly cointegrated under (3), the equilibrium equation will undergo discrete level shifts driven by the Markovian variable s_t (provided that $b_0 + \frac{1}{2}\sigma_{y0}^2 \neq b_1 + \frac{1}{2}\sigma_{y1}^2$).[10]

It should be noted that non-stationarities other than Markov regime-switching would also result in a time-varying long-run relationship. In fact, if the behavioural assumption expressed by equation (3) is correct, the consumption–income relationship cannot be expected to remain stable under any type of change in the structure of the process that governs the evolution of income (cf. Lucas, 1976). Moreover, income could be generated without change, but a non-constant r would give rise to long-run parameters that are not time-invariant.

4.2. Economic Specifications

In our empirical analysis, we will use the following model to describe the long-run relationship between consumption and income in Japan:

$$c_t = (\beta_0 + \beta_1 s_t)y_t + (\mu_0 + \mu_1 s_t) + \sum_{j=1}^{3}(\alpha_j^0 + \alpha_j^1 s_t)D_t^j + [\omega_0(1 - s_t) + \omega_1 s_t]u_t \tag{9}$$

where the state variable s_t is defined as before, and $\{u_t\}$ is a stationary and ergodic random sequence, independent of $\{s_m\}$ for all integer t and m, with $E(u_t) = 0$ and $\mathrm{var}(u_t) = 1$. Equation (9) may be regarded as a stochastic version of an equilibrium relationship, with the term $[\omega_0(1 - s_t) + \omega_1 s_t]u_t$ representing the extent to which the system is out of long-run equilibrium.

[10] Qualitatively similar results would be obtained if a stationary, zero-mean random disturbance term were included in equation (3) (to account, for instance, for random measurement errors). Such an approach is, indeed, followed in our empirical analysis.

Model (9) extends the linear specifications analysed in EGHL by allowing the long-run parameters to be functions of the (stochastically chosen) regime controlling the system at time t. What is more, the probability law that governs these regime changes is specified to be flexible enough to encompass a broad range of empirically interesting outcomes, allowing the data to determine the specific form of parameter instability that is coherent with the observed sample information. It also generalizes the regime-shift cointegration models of Gregory and Hansen (1996) which can be obtained as special cases of model (9) if $p = 1$ or $q = 1$ (i.e., single permanent change in regime).

A possible shortcoming of the specification of model (9) is the requirement that changes in both the long-run income elasticity and the seasonal shift factors are driven by the same Markovian regime-shift variable. However, such a restriction might not be entirely justifiable since it is likely that economic changes could affect seasonality and the propensity to consume in different ways. For this reason, we also examine a more flexible specification than (9) which allows the long-run parameters to change as a result of two separate regime-shift variables. Building on the work of K. Phillips (1991) and Ravn and Sola (1995), this switching cointegration model is parameterized as:

$$c_t = (\beta_0 + \beta_1 s_{1t})y_t + (\mu_0 + \mu_1 s_{2t}) + \sum_{j=1}^{3}(\alpha_j^0 + \alpha_j^1 s_{2t})D_t^j + \sigma_{s_t^*}u_t^* \tag{10}$$

where $\{s_{1t}\}$ and $\{s_{2t}\}$ are generated by two independent first-order Markov chains with values on $\{0, 1\}$, $\{u_t^*\}$ is a stationary and ergodic random sequence, independent of $\{s_{1m}\}$ and $\{s_{2m}\}$ for all t and m, with $E(u_t^*) = 0$ and $\mathrm{var}(u_t^*) = 1$, and $\sigma_{s_t^*}$ represents a standard-deviation shift function. There are thus two possible regimes for the long-run income elasticity and for the coefficients on the deterministic components of the cointegrating regression; the four different combinations of these are the states of the Markov chain $\{s_t^*\}$ that characterizes the regime operative at time t. The latent variable s_t^* is defined as follows:

$$\begin{aligned}
s_t^* &= 1 \text{ if } s_{1t} = 1 \text{ and } s_{2t} = 1\\
s_t^* &= 2 \text{ if } s_{1t} = 0 \text{ and } s_{2t} = 1\\
s_t^* &= 3 \text{ if } s_{1t} = 1 \text{ and } s_{2t} = 0\\
s_t^* &= 4 \text{ if } s_{1t} = 0 \text{ and } s_{2t} = 0
\end{aligned}$$

The standard-deviation shift function may be accordingly parameterized as $\sigma_{s_t^*} = \sum_{j=1}^{4}\sigma_j s_{jt}^*$, where $s_{jt}^* = 1$ if $s_t^* = j$, and $s_{jt}^* = 0$ if $s_t^* \neq j$. Since $\{s_{1t}\}$ is presumed to be independent of $\{s_{2t}\}$, the transitions between the four states of $\{s_t^*\}$ are governed by the transition matrix:[11]

$$\mathbf{P} = \begin{bmatrix}
p_{11}p_{21} & p_{21}(1-p_{10}) & p_{11}(1-p_{20}) & (1-p_{10})(1-p_{20})\\
p_{21}(1-p_{11}) & p_{10}p_{21} & (1-p_{11})(1-p_{20}) & p_{10}(1-p_{20})\\
p_{11}(1-p_{21}) & (1-p_{10})(1-p_{21}) & p_{11}p_{20} & p_{20}(1-p_{10})\\
(1-p_{11})(1-p_{21}) & p_{10}(1-p_{21}) & p_{20}(1-p_{11}) & p_{10}p_{20}
\end{bmatrix}$$

[11] The (i,j)th element of $\mathbf{P}$ is the transition probability $\Pr(s_t^* = i \mid s_{t-1}^* = j)$.

where $p_{11} = \Pr(s_{1t} = 1 \mid s_{1,t-1} = 1)$, $p_{10} = \Pr(s_{1t} = 0 \mid s_{1,t-1} = 0)$, $p_{21} = \Pr(s_{2t} = 1 \mid s_{2,t-1} = 1)$, and $p_{20} = \Pr(s_{2t} = 0 \mid s_{2,t-1} = 0)$.

It is important to appreciate that models (9) and (10) are not intended as empirical implementations of the theoretical structure discussed in example of the previous section. They provide richer parameterizations which allow for both switching seasonal intercepts and time-varying long-run income elasticity, and consequently include structures like (8) as, indeed, special cases.

4.3. Likelihood-based Inference

Statistical inference in the context of Markov-switching models like (9) and (10) can proceed by making use of the discrete version of the Kalman filter algorithm described in Hamilton (1994, pp. 692–4). This gives as a by-product the sample likelihood function which can be maximized numerically with respect to the unknown parameters, subject to the constraint that the transition probabilities lie in the unit interval.

When the variables in Markov-switching models are stationary, it is typically assumed that the maximum likelihood (ML) estimator is consistent and asymptotically normal.[12] If that is the case, from the analyses of P. Phillips (1991) and Phillips and Loretan (1991) we expect that standard asymptotic results will also provide a valid basis for inference in models (9) and (10), providing that y_t is strongly exogenous for the long-run parameters and test statistics are constructed in a way that accounts for possible temporal dependence and heterogeneity in the regression disturbances.[13,14]

Finally, once the ML estimates are obtained, inferences about the unobserved regimes $\{s_t\}$ may be based on either 'filter' probabilities like $\Pr(s_t = 1 \mid \mathbf{w}_t, \mathbf{w}_{t-1} \ldots, \mathbf{w}_1)$ or 'smoothed' probabilities like $\Pr(s_t = 1 \mid \mathbf{w}_T, \mathbf{w}_{T-1}, \ldots, \mathbf{w}_1)$, where $\mathbf{w}_t = [c_t, y_t, D_t^1, D_t^2, D_t^3]$ (see Hamilton, 1994, Ch. 22).

4.4. Empirical Results

We now investigate the possibility of Markov-switching cointegration between consumption and income in Japan, by means of the models described in Section 4.2. Table IV reports ML estimates of the parameters of model (9) (based on the Gaussian likelihood), along with corresponding asymptotic standard errors. The latter have been computed using a heteroscedasticity and autocorrelation robust estimator of the relevant asymptotic covariance matrix (see Gallant and White, 1988, Ch. 6).[15] We also give the values of several model selection criteria for model (9), as well as the outcome of a likelihood ratio (LR) test of a single-regime null model against the two-regime alternative. Note, however, that since the transition probabilities are unidentified under

[12] To our knowledge, no formal proof of the asymptotic properties of the ML estimator exists for models other than switching regressions with bounded i.i.d. regressors and Gaussian i.i.d. errors (Kiefer, 1978). An investigation of the finite-sample properties of the ML estimator in stable dynamic models can be found in Psaradakis and Sola (1995).

[13] Failure of strong exogeneity may be accommodated by augmenting models (9) and (10) by lags and leads of $(1\text{-}B)y_t$, as in Saikkonen (1991) and Stock and Watson (1993).

[14] A Monte Carlo analysis of the ML estimator for Markov-switching cointegration models like those considered in Section 4.2 revealed satisfactory performance in samples of around 100 and 200 observations. In particular, the ML estimators were found to display negligible bias and high concentration probabilities, while the empirical size of tests based on conventional t-statistics and $N(0, 1)$ reference distributions did not deviate significantly from the nominal test size (the detailed results are available from the authors).

[15] Maximization of the likelihood function was carried out by means of a secant algorithm that approximates the Hessian according to the Broyden–Fletcher–Goldfarb–Shanno update computed from numerical derivatives (see, e.g., Dennis and Schnabel, 1983).

Table IV. ML estimates of parameters for two-regime Markov-switching regressions

Parameter	Regression 1 c_t on y_t Estimate	Std error	Regression 2 c_{1t} on y_1t Estimate	Std error	Regression 3 c_{1t} on y_{1t} Estimate	Std error
β_0	1·1554	0·0633	0·9774	0·0054	0·9617	0·0040
β_1	−0·2444	0·0648	−0·1247	0·0196	0·0110	0·0050
μ_0	−1·2531	0·3999	−0·0949	0·1097	—	—
μ_1	1·4626	0·4054	2·6883	0·4250	—	—
α_1^0	0·3231	0·0271	—	—	—	—
α_1^1	−0·1099	0·0294	—	—	—	—
α_2^0	0·0818	0·0161	—	—	—	—
α_2^1	0·0371	0·0186	—	—	—	—
α_3^0	0·1667	0·0169	—	—	—	—
α_3^1	−0·0802	0·0185	—	—	—	—
ω_0	0·0107	0·0024	0·0452	0·0095	0·0619	0·0300
ω_1	0·0279	0·0026	0·0586	0·0089	0·0565	0·0135
p	0·9917	0·0285	0·9706	0·0261	0·9897	0·0166
q	0·9876	0·0119	0·9861	0·0162	0·9485	0·0618
$\Pr(s_t = 0)$	0·4010		0·6790		0·1667	
$\Pr(s_t = 1)$	0·5990		0·3210		0·8333	
LR-test	138·04	[0·000]	163·72	[0·000]	140·30	[0·000]
AIC	−691·08	(−571·04)	−487·80	(−336·07)	−466·34	(−336·04)
SIC	−653·53	(−557·63)	−466·57	(−330·77)	−450·41	(−333·39)
HQ	−675·86	(−565·60)	−479·20	(−333·92)	−459·89	(−334·97)

Notes: The Akaike, Schwarz, and Hannan–Quinn model selection criteria are calculated as AIC $= -2l_m + 2g$, SIC $= -2l_m + g\ln(T)$, and HQ $= -2l_m + 2g\ln[\ln(T)]$ respectively, where l_m is the maximum value of the Gaussian log-likelihood function and g is the number of freely estimated parameters.
Numbers in parentheses are the values of the criteria for the corresponding single-regime regressions. Numbers in square brackets give an upper bound on the p-value of the LR tests.

this null hypothesis, standard asymptotic distributional results are not valid for the LR test. To overcome this difficulty, our testing procedure is based on the test principle proposed in Davies (1987).[16] The ML estimates are reasonable and significant, and the estimated transition probabilities suggest that, if the system is in either of the two regimes, it is likely to remain in that regime. Moreover, both the LR test and the model selection criteria favour the Markov-switching regression over the one with time-invariant parameters.

Figure 4 plots the inferred probabilities that the system was in regime 1 (of lower income elasticity) at each date in the sample, based on currently available information. For comparison, we also show a time plot of the scaled residuals from the corresponding single-regime model. On the basis of the decision rule $\Pr(s_t = 1 | \mathbf{w}_t, \mathbf{w}_{t-1}, \ldots, \mathbf{w}_1) \gtrless 0{\cdot}5$, the filter probabilities locate a clear change in regime at 1979 : 3, the system remaining in regime 0 until the end of the sample

[16] On the assumption that the likelihood ratio exhibits a single peak when viewed as a function of the n nuisance parameters that are unidentified under the null hypothesis, an upper bound on the significance level of the LR test is given by $\Pr(\chi_n^2 > 2h) + 2h^{n/2}[e^h\Gamma(n/2)]^{-1}$, where $\Gamma(\cdot)$ denotes the gamma function, and h is the difference between the maximized log-likelihood under the alternative and null hypotheses.

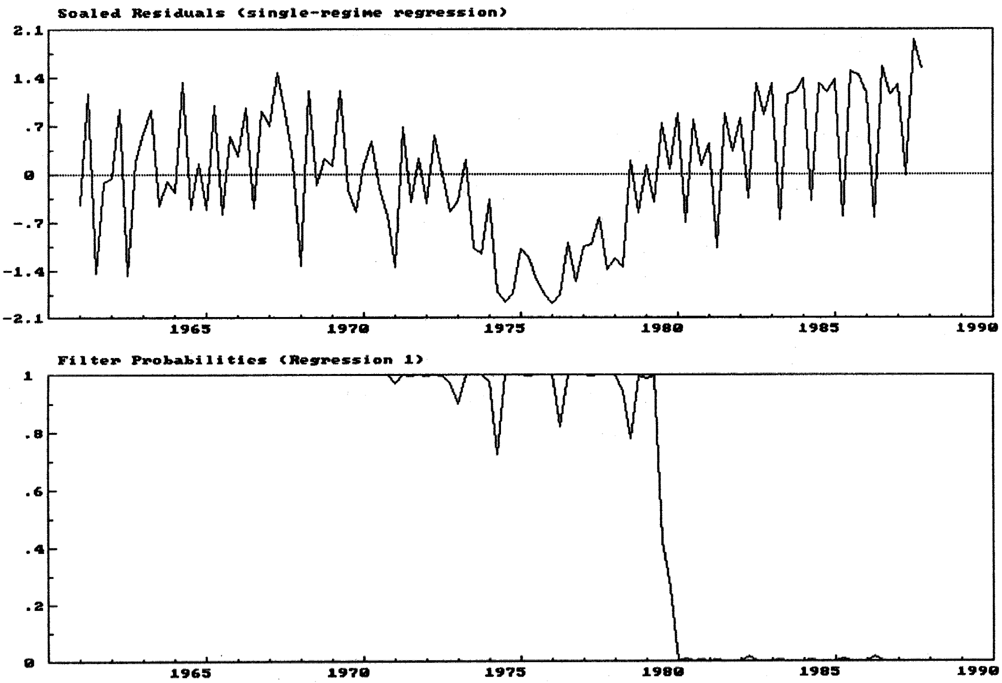

Figure 4. Residuals from single-regime regression and inferred probabilities for Regime 1

period. This dating corresponds precisely to the second oil crisis and the rapid yen depreciation and increase in the current account deficit that followed (see, e.g., Ito, 1992). Also, as might have been expected on the basis of a visual inspection of the residuals from the single-regime regression, there are brief departures from regime 1 during the period between the two oil-price shocks.

The results from estimating a two-regime switching model similar to (9) for the seasonally adjusted $\{c_{1t}\}$ and $\{y_{1t}\}$ series are also presented in Table IV. As before, there appears to be significant evidence of shifts between the two regimes. The inferred filter probabilities that the system was in regime 1 are shown in Figure 5, along with the time series of the residuals from the corresponding single-regime models. The period between the two oil crises is again identified as being associated with a shift in the consumption–income relationship. In addition, for the model with an intercept, the probabilities suggest three more abrupt changes in regime at 1967 : 3, 1969 : 3 and 1970 : 2.

The results from ML estimation of model (10) are given in Table V. Most of the model's parameters appear to change significantly between regimes, and a time-invariant representation is indeed rejected against the four-regime alternative. The inferred probabilities $\Pr(s_{1t} = 1 \mid \mathbf{w}_t, \mathbf{w}_{t-1}, \ldots, \mathbf{w}_1)$ and $\Pr(s_{2t} = 1 \mid \mathbf{w}_t, \mathbf{w}_{t-1}, \ldots, \mathbf{w}_1)$ associated with each of the two variables that govern regime shifts in model (10) are plotted in Figure 6. It is clear that the deterministic component of the cointegrating regression underwent a change in 1979:3. The filter probabilities also suggest some switching between the two regimes associated with the long-run income elasticity, but the latter does not change significantly between the two regimes. Such results are, of course, consistent with the theoretical structure discussed in Section 4.1, although the model

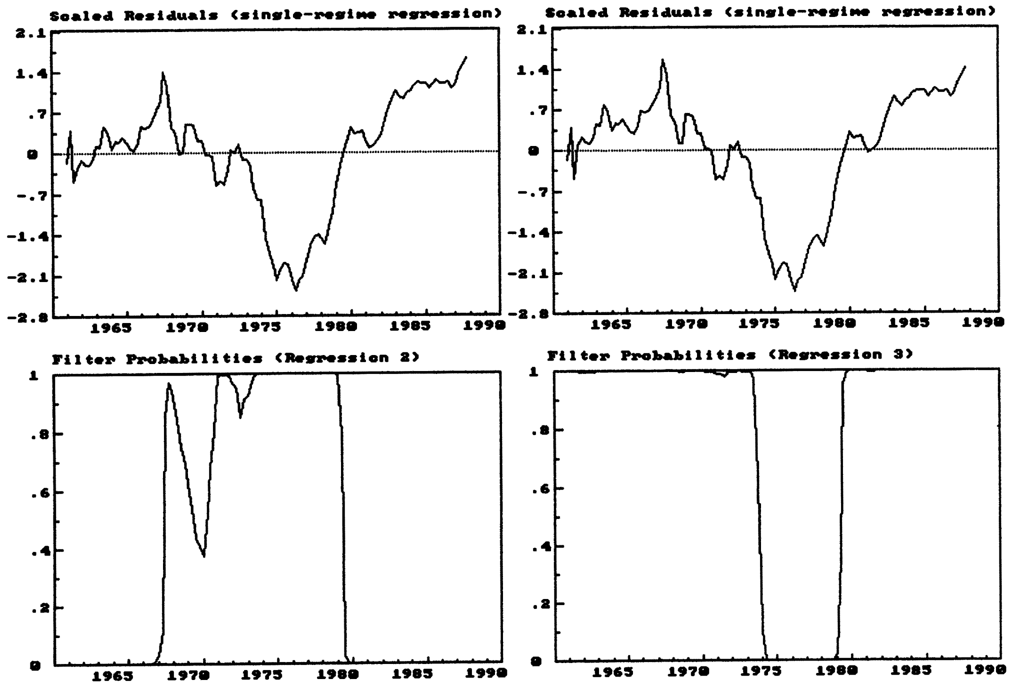

Figure 5. Residuals from single-regime regressions and inferred probabilities for Regime 1

Table V. ML estimates of parameters for the four-regime Markov-switching regression

Parameter	Estimate	Std error	Parameter	Estimate	Std error
β_0	0·9239	0·0197	α_3^1	0·0150	0·0107
β_1	−0·0049	0·0039	σ_1	0·0061	0·0013
μ_0	0·1477	0·1097	σ_2	0·0526	0·0208
μ_1	0·1048	0·0210	σ_3	0·0259	0·0094
α_1^0	0·2170	0·0109	σ_4	0·0207	0·0050
α_1^1	0·0087	0·0101	p_{11}	0·9410	0·0466
α_2^0	0·1247	0·0119	p_{10}	0·9495	0·0309
α_2^1	−0·0964	0·0121	p_{21}	0·9879	0·0064
α_3^0	0·0902	0·0108	p_{20}	0·9918	0·0089
$\Pr(s_t^* = 1)$	0·1862		$\Pr(s_t^* = 3)$	0·2750	
$\Pr(s_t^* = 2)$	0·2175		$\Pr(s_t^* = 4)$	0·3213	
LR-test	118·16	[0·000]			
AIC	−663·20	(−571·04)			
SIC	−614·92	(−557·63)			
HQ	−643·63	(−565·60)			

Note: See the notes to Table IV.

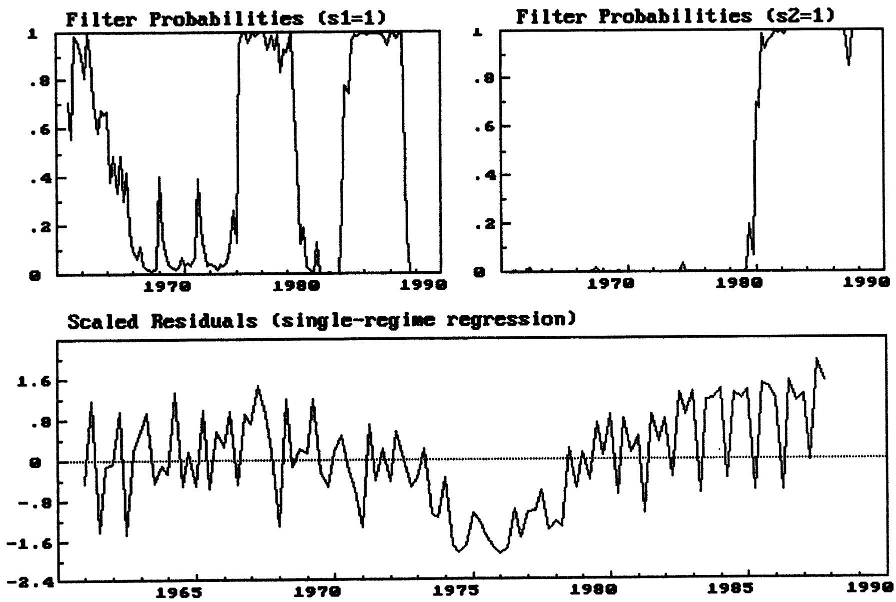

Figure 6. Inferred probabilities for Regime 1 and residuals from single-regime regression

selection criteria appear to favour the two-regime specification in Table IV over the four-regime one.

In order to test for the existence of a switching long-run relationship between consumption and income, we employ a residual-based approach based on scalar unit-root tests applied to the standardized residuals from the Markov-switching cointegrating regressions. Table VI records the outcome of several such tests, including the Augmented Dickey–Fuller (ADF) test recommended in Engle and Granger (1987), the $\hat{Z}_\alpha$ and $\hat{Z}_t$ tests of Phillips and Ouliaris (1990), and the Durbin–Hausman DHS test of Choi (1994).

Cointegration tests based on the residuals from switching regressions are, however, unlikely to share similar asymptotic or finite-sample properties with standard OLS-based tests. Thus, in order to get some idea about the distribution of the test statistics under the null hypothesis of no cointegration, we conduct a small Monte Carlo experiment based upon switching models like (9) and (10). Specifically, using the estimated models in Tables IV and V, we generate in each case 1000 independent realizations of a series $\{c_t\}$ (of length 108), with $\{y_t\}$ and $\{u_t\}$ being independent, driftless random walks with i.i.d. $N(0, 1)$ innovations. Cointegration tests are then carried out using the residuals from the corresponding Markov-switching regression. On the basis of the empirical percentiles of the simulated test statistics given in Table VI, the null hypothesis of no cointegration can be firmly rejected for the Japanese consumption and income data.

5. CONCLUSION

This paper has explored a general type of cointegration where the cointegrating vector is allowed to change over time as a result of changes in the economic environment. Following Hamilton

Table VI. Cointegration tests

	ADF	Lags	$\hat{Z}_t$	$\hat{Z}_\alpha$	DHS
Regression 1	−7·191	4	−7·597	−71·475	1020·1
	(−5·03)		(−5·21)	(−41·12)	(185·83)
	[−4·48]		[−4·50]	[−33·11]	[120·00]
Regression 2	−4·556	1,3,5,7	−3·588	−26·168	75·250
	(−2·39)		(−2·46)	(−9·27)	(23·51)
	[−1·96]		[−1·95]	[−5·79]	[12·19]
Regression 3	−1·510	1,2,4	−3·449	−24·597	68·536
	(−1·90)		(−2·03)	(−5·62)	(10·99)
	[−1·56]		[−1·65]	[−3·44]	[7·20]
Regression 4	−8·219	4	−8·824	−87·640	3966·4
	(−6·21)		(−6·46)	(−49·24)	(203·45)
	[−5·58]		[−5·59]	[−41·65]	[192·34]

Notes: The first three panels of the table refer to Regressions 1–3 in Table IV, while the last panel refers to the regression in Table V. Numbers in parentheses (square brackets) are 5% (10%) finite-sample critical values, obtained from 1000 Monte Carlo replications.

(1989), the generating mechanism of these shifts was modelled as a finite-state Markov process with unknown (but stationary) transition mechanism.

Our approach was illustrated using Japanese data for real total consumption and real disposable income. We argued that although univariate representations of both series are characterized by the presence of a stochastic trend component, seasonal patters are best approximated by a (stochastically) stationary specification with a structural break. In a multivariate framework, the existence of a linear, temporally stable long-run relationship between consumption and income was strongly rejected by the data. In contrast, we found significant evidence in favour of time-varying cointegration where the long-run parameters are allowed to switch stochastically between two different cointegrating regimes.

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REFERENCES

Andrews, D. W. K. (1991), 'Heteroskedasticity and autocorrelation consistent covariance matrix estimation', *Econometrica*, **59**, 817–58.

Andrews, D. W. K. and J. C. Monahan (1992), 'An improved heteroskedasticity and autocorrelation consistent covariance matrix estimator', *Econometrica*, **60**, 953–66.

Banerjee, A., J. Dolado, J. W. Galbraith and D. F. Hendry (1993), *Co-integration, Error-Correction, and the Econometric Analysis of Non-stationary Data*, Oxford University Press, Oxford.

Campos, J., N. R. Ericsson and D. F. Hendry (1996), 'Cointegration tests in the presence of structural breaks', *Journal of Econometrics*, **70**, 187–220.

Canova, F. and E. Chysels (1994), 'Changes in seasonal patterns: are they cyclical?' *Journal of Economic Dynamics and Control*, **18**, 1143–71.

Canova, F. and B. E. Hansen (1995), 'Are seasonal patterns constant over time? A test for seasonal stability', *Journal of Business & Economic Statistics*, **13**, 237–52.

Choi, I. (1994), 'Durbin–Hausman tests for cointegration', *Journal of Economic Dynamics and Control*, **18**, 467–80.

Cosslett, S. R. and L.-F. Lee (1985), 'Serial correlation in discrete variable models', *Journal of Econometrics*, **27**, 79–97.

Davies, R. B. (1987), 'Hypothesis testing when a nuisance parameter is present only under the alternative', *Biometrika*, **74**, 33–43.

Dennis, J. E., Jr and R. B. Schnabel (1983), *Numerical Methods of Unconstrained Optimization and Nonlinear Equations*, Prentice Hall, Engelwood Cliffs, NJ.

Engle, R. F. and C. W. J. Granger (1987), 'Co-integration and error correction: representation, estimation and testing, *Econometrica*, **55**, 251–76; reprinted in R. F. Engle and C. W. J. Granger (eds), *Long-Run Economic Relationships: Readings in Cointegration*, Oxford University Press, Oxford, 1991.

Engle, R. F., C. W. J. Granger, S. Hylleberg, and H. S. Lee (1993), 'Seasonal cointegration: the Japanese consumption function', *Journal of Econometrics*, **55**, 275–98.

Gallant, A. R. and H. White (1988), *A Unified Theory of Estimation and Inference for Nonlinear Dynamic Models*, Blackwell, Oxford.

Ghysels, E. (1991), 'On seasonal asymmetries and their implications for stochastic and deterministic models of seasonality', manuscript, Department of Economics, University of Montreal.

Ghysels, E., H. S. Lee and J. Noh (1994), 'Testing for unit roots in seasonal time series: some theoretical extensions and a Monte Carlo investigation', *Journal of Econometrics*, **62**, 415–42.

Goldfeld, S. M. and R. E. Quandt (1973), 'A Markov model of switching regressions', *Journal of Econometrics*, **1**, 3–16.

Granger, C. W. J. and H. S. Lee (1991), 'An introduction to time-varying parameter cointegration', in P. Hackl and A. H. Westlund (eds), *Economic Structural Change: Analysis and Forecasting*, Springer-Verlag, Berlin.

Gregory, A. W. and J. M. Nason (1991), 'Testing for structural breaks in cointegrated relations', Discussion Paper No. 827, Institute for Economic Research, Queen's University, Ontario.

Gregory, A. W. and B. E. Hansen (1996), 'Residual-based tests for cointegration in models with regime shifts', *Journal of Econometrics*, **70**, 99–126.

Hall, S. G. and A. Scott (1990), 'Seasonal unit roots, structural breaks and cointegration', Discussion Paper No. 24-90, Centre for Economic Forecasting, London Business School.

Hamilton, J. D. (1989), 'A new approach to the economic analysis of nonstationary time series and the business cycle', *Econometrica*, **57**, 357–84.

Hamilton, J. D. (1994), *Time Series Analysis*, Princeton University Press, Princeton, NJ.

Hansen, B. E. (1992), 'Tests for parameter instability in regressions with I(1) processes', *Journal of Business & Economic Statistics*, **10**, 321–35.

Hylleberg, S. (1995), 'Tests for seasonal unit roots: general to specific or specific to general?' *Journal of Econometrics*, **69**, 5–25.

Hylleberg, S., R. F. Engle, C. W. J. Granger and B. S. Yoo (1990), 'Seasonal integration and cointegration', *Journal of Econometrics*, **44**, 215–38; reprinted in S. Hylleberg (ed), *Modelling Seasonality*, Oxford University Press, Oxford, 1992.

Ito, T. (1992), *The Japanese Economy*, MIT Press, Cambridge, MA.

Kiefer, N. M. (1978), 'Discrete parameter variation: efficient estimation of a switching regression model', *Econometrica*, **46**, 427–34.

Lucas, R. E., Jr (1976), 'Econometric policy evaluation: a critique', in K. Brunner and A. H. Meltzer (eds), *The Phillips Curve and Labor Markets*, Carnegie-Rochester Conference Series on Public Policy, Vol. 1, North-Holland, Amsterdam.

Osborn, D. R. (1993), 'Discussion: seasonal cointegration', *Journal of Econometrics*, **55**, 299–303.

Perron, P. (1989), 'The Great Crash, the oil price shock, and the unit root hypothesis', *Econometrica*, **57**, 1361–401.

Phillips, K. L. (1991), 'A two-country model of stochastic output with changes in regime', *Journal of International Economics*, **31**, 121–42.

Phillips, P. C. B. (1991), 'Optimal inference in cointegrated systems', *Econometrica*, **59**, 283–306.
Phillips, P. C. B. and B. E. Hansen (1990), 'Statistical inference in instrumental variables regression with I(1) processes', *Review of Economic Studies*, **57**, 99–125.
Phillips, P. C. B. and M. Loretan (1991), 'Estimating long-run economic equilibria', *Review of Economic Studies*, **58**, 407–36.
Phillips, P. C. B. and S. Ouliaris (1990), 'Asymptotic properties of residual based tests for cointegration', *Econometrica*, **58**, 165–93.
Psaradakis, Z. (1996), 'Testing for unit roots in time series with nearly deterministic seasonal variation', Discussion Paper No. 2/96, Department of Economics, Birkbeck College, University of London.
Psaradakis, Z. and M. Sola (1995), 'Finite-sample properties of the maximum likelihood estimator in autoregressive models with Markov switching', Discussion Paper No. FE-8/95, Department of Economics, Birkbeck College, University of London.
Ravn, M. O. and M. Sola (1995), 'Stylized facts and regime changes—are prices procyclical?', *Journal of Monetary Economics*, forthcoming.
Saikkonen, P. (1991), 'Asymptotically efficient estimation of cointegration regressions', *Econometric Theory*, **7**, 1–21.
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http://links.jstor.org/sici?sici=0012-9682%28197803%2946%3A2%3C427%3ADPVEEO%3E2.0.CO%3B2-I

The Great Crash, the Oil Price Shock, and the Unit Root Hypothesis
Pierre Perron
Econometrica, Vol. 57, No. 6. (Nov., 1989), pp. 1361-1401.
Stable URL:
http://links.jstor.org/sici?sici=0012-9682%28198911%2957%3A6%3C1361%3ATGCTOP%3E2.0.CO%3B2-W

Optimal Inference in Cointegrated Systems
P. C. B. Phillips
Econometrica, Vol. 59, No. 2. (Mar., 1991), pp. 283-306.
Stable URL:
http://links.jstor.org/sici?sici=0012-9682%28199103%2959%3A2%3C283%3AOIICS%3E2.0.CO%3B2-5

Statistical Inference in Instrumental Variables Regression with I(1) Processes
Peter C. B. Phillips; Bruce E. Hansen
The Review of Economic Studies, Vol. 57, No. 1. (Jan., 1990), pp. 99-125.
Stable URL:
http://links.jstor.org/sici?sici=0034-6527%28199001%2957%3A1%3C99%3ASIIIVR%3E2.0.CO%3B2-6

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LINKED CITATIONS

Estimating Long-Run Economic Equilibria
Peter C. B. Phillips; Mico Loretan
The Review of Economic Studies, Vol. 58, No. 3, Special Issue: The Econometrics of Financial Markets. (May, 1991), pp. 407-436.
Stable URL:
http://links.jstor.org/sici?sici=0034-6527%28199105%2958%3A3%3C407%3AELEE%3E2.0.CO%3B2-C

Asymptotic Properties of Residual Based Tests for Cointegration
P. C. B. Phillips; S. Ouliaris
Econometrica, Vol. 58, No. 1. (Jan., 1990), pp. 165-193.
Stable URL:
http://links.jstor.org/sici?sici=0012-9682%28199001%2958%3A1%3C165%3AAPORBT%3E2.0.CO%3B2-V

A Simple Estimator of Cointegrating Vectors in Higher Order Integrated Systems
James H. Stock; Mark W. Watson
Econometrica, Vol. 61, No. 4. (Jul., 1993), pp. 783-820.
Stable URL:
http://links.jstor.org/sici?sici=0012-9682%28199307%2961%3A4%3C783%3AASEOCV%3E2.0.CO%3B2-Z