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# P-Approximation

Seminar in Econometrics

Term Paper

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from:

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## List of Abbreviations

# 1 Introduction

Meta tests have been shown to be a powerful tool when testing for the null of non-cointegration. The distribution of their test statistic, however, is mostly not available in closed form. This might pose difficulties when implementing the meta tests in econometric software packages, as one has to include the full null distribution for each combination of the underlying tests. Software package size limitations are therefore quickly exceeded.

In this paper we propose supervised Machine Learning Algorithms to approximate the p-values of the meta test by Bayer and Hanck (2012) which tests for the null of non-cointegration. This approach might reduce the size of associated software packages considerably. The algorithms are trained on simulated data for various specifications of the aforementioned test.

Ergebnis der Models (1-2 Sätze)

Inhalt Paper

## 2 Bayer Hanck Test

The choice as to which of the available cointegration tests to use is a recurrent issue in econometric time series analysis. Bayer and Hanck (2012) propose powerful meta tests which provide unambiguous test decisions. They combine several residual- and system-based tests in the manner of Fisher's (1932) Chi-squared test.

Bayer and Hanck build their paper on previous work from Pesavento (2004), who defines the underlying model as  $z'_t = [x'_t, y_t]$ , with  $x_t$  being an  $n_1 \times 1$  vector and  $y_t$  a scalar, which displays the cointegration relation. They can be written as

$$\Delta x_t = \tau_1 + v_{1t} \tag{2.1}$$

$$y_t = (\mu_2 - \gamma' \mu_1) + (\tau_2 - \gamma' \tau_1)t + \gamma' x_t + u_t, \tag{2.2}$$

$$u_t = \rho u_{t-1} + v_{2t}. \tag{2.3}$$

$\Delta x_t$  presents the regressor dynamics.  $\mu_1$ ,  $\mu_2$ ,  $\tau_1$  and  $\tau_2$  are the deterministic parts of the model. They are subject to the following restrictions: (i)  $\mu_2 - \gamma' \mu_1$  and  $\tau = 0$  which translates to no deterministics, (ii)  $\tau = 0$  which corresponds to a constant in the cointegrating vector, (iii)  $\tau_2 - \gamma' \tau_1 = 0$ , a constant plus trend.

$v_t = [v'_{1t} v_{2t}]'$  with  $\Omega$  the long-run covariance matrix of  $v_t$ . For derivation of  $v_t$  see Pesavento (2004). Pesavento shows that  $\{v_t\}$  satisfies an FCLT, i.e.  $T^{-1/2} \sum_{t=1}^{[T]} v_t \Rightarrow \Omega^{1/2} W(\cdot)$ . It is further assumed that the  $x_t$  are not cointegrated.

It clearly follows from (2.3) that  $z_t$  is cointegrated if  $\rho < 1$ . Hence the null hypothesis of no cointegration is  $H_0 : \rho = 1$ .

Furthermore, Pesavento introduces two other parameters. First,  $R^2$  measures the squared correlation of  $v_{1t}$  and  $v_{2t}$ . It can be interpreted as the influence of the right-hand side variables in (2.2). It ranks between zero and one. When there is no long-run correlation between those variables and the errors from the cointegration regression,  $R^2$  equals zero. Secondly, the number of lags is approximated by a finite number  $k$ .

### Assumptions (BH S. 84)?

Bayer and Hanck's (2012) meta test considers the test statistics of up to four stand-alone tests. Namely, these are the tests of Engle and Granger (1987), Johansen (1988), Boswijk (1994) and Banerjee, Dolado, and Mestre (1998). For the sake of brevity the detailed derivation of the underlying tests has been deliberately omitted here.

Engle and Granger (1987) propose a two-step procedure to test the null hypothesis of no cointegration against the alternative of at least one cointegrating vector. First, the long-run relationship between  $y_t$  and  $\mathbf{x}_t$  is estimated by least squares regression. The obtained residuals  $\hat{u}_t$  are then tested for a unit root. For this, Engle and Granger suggest the use of the  $t$ -statistic  $t_\gamma^{\text{ADF}}$  in the Augmented Dickey-Fuller (ADF) regression:

$$\Delta \hat{u}_t = \gamma \hat{u}_{t-1} + \sum_{i=1}^k \pi_i \Delta \hat{u}_{t-i} + \varepsilon_t. \quad (2.4)$$

The rejection of a unit root points to a cointegration relationship.

Johansen's (1988) maximum eigenvalue test is a system-based test that allows for several cointegration relationships. Take the vector error correction model (VECM)<sup>1</sup>

$$\Delta \mathbf{z}_t = \Pi \mathbf{z}_{t-1} + \sum_{i=1}^k \Gamma_i \Delta \mathbf{z}_{t-i} + \mathbf{d}_t + \varepsilon_t. \quad (2.5)$$

blabla Johansen test statistic

Banerjee and Boswijk

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<sup>1</sup>Due to practical reasons we omit the derivation of the VECM which is presumed to be known.

To combine the results from the underlying tests Bayer and Hanck draw upon Fisher’s combined probability test (Fisher, 1932). It merges the tests using the formula

$$\tilde{\chi}_{\mathcal{I}}^2 := -2 \sum_{i \in \mathcal{I}} \ln(p_i). \quad (2.6)$$

Let  $t_i$  be the  $i^{th}$  test statistic. If test  $i$  rejects for large values, take  $\xi_i := t_i$ . If test  $i$  rejects for small values, take  $-\xi_i := t_i$ . With  $\Xi_i(x) := \Pr_{\mathcal{H}_i}(\xi_i \geq x)$  the p-value of the  $i^{th}$  test is  $p_i := \Xi_i(\xi_i)$ .

Fisher (1932) shows that under the assumption of independence the null distribution of  $\tilde{\chi}_{\mathcal{I}}^2$  follows a chi-squared distribution with  $2\mathcal{I}$  degrees of freedom. If this assumption is violated the null distribution is less evident. Here, the latter case occurs, as the  $\xi_i$  are not independent. The  $\tilde{\chi}_{\mathcal{I}}^2$ , however, have well-defined asymptotic null distributions  $F_{\mathcal{F}_{\mathcal{I}}}$ , as  $\tilde{\chi}_{\mathcal{I}}^2 \rightarrow_d \mathcal{F}_{\mathcal{I}}$  under  $\mathcal{H}_0$  if  $T \rightarrow \infty$ , with  $\mathcal{F}_{\mathcal{I}}$  some random variable. It is therefore feasible to simulate the joint null distribution of the  $\xi_i$  to obtain the distribution  $F_{\mathcal{F}_{\mathcal{I}}}$  of (2.6). The  $F_{\mathcal{F}_{\mathcal{I}}}$  depend on which and how many tests are combined. The distributions of the  $\xi_i$  depend on  $K - 1$  and the deterministic case.

### 3 Simulation

In this section, we describe the simulation of the null distribution of the Bayer Hanck meta test. The objective is to obtain data for training machine learning algorithms on approximating the p-values of the aforementioned test. In consideration of the different forms of the meta test we generated six data sets. These vary according to the specific combinations of the underlying tests and also account for the above-mentioned restrictions on the deterministic parts of the model.

This simulation relies largely on previous work by Pesavento (2004). We consider  $R^2 \in \{0, 0.05, 0.1, \dots, 0.95\}$ ,  $k = 11$  and  $c = 0^2$  and set the number of repetitions to 1,000,000. **N? c vielleicht mal definieren**

To calculate the Bayer Hanck test statistic we first simulate the null distributions of the underlying test statistics. It can be shown that asymptotically these are non-standard but a function of standard Brownian motions. The latter is approximated by step functions using Gaussian random walk with

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<sup>2</sup>Since we solely aim at simulating the distribution of the null of no cointegration we will not consider any further values of  $c$  here.

$N = 1000$  observations. Referenz Theorem? OU Prozess? Nochmal auf Transformation je nach case eingehen? p values durch cdf, Fisherstat + pvalues berechnen.

## 4 Models

## 5 Package

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## A Appendices

Number	Functional form	Range of $\gamma$
1	$p = \text{poly}(t, \gamma) + (1/k)$	$\gamma \in \mathbb{Z} [3, 10]$
2	$p = \text{poly}(t, \gamma) + (1/k) + \text{poly}(t, \gamma) * 1/k$	$\gamma \in \mathbb{Z} [3, 10]$
3	$p = \text{poly}(t, \gamma) + \log(k) + \text{poly}(k, \gamma) * \log(k)$	$\gamma \in \mathbb{Z} [3, 10]$
4	$p = \text{poly}(t, \gamma) + k + (1/k)$	$\gamma \in \mathbb{Z} [3, 10]$
5	$p = \text{poly}(\log(t), \gamma) + \log(k)$	$\gamma \in \mathbb{Z} [3, 10]$
6	$p = \text{poly}(\log(t), \gamma) * \log(k)$	$\gamma \in \mathbb{Z} [3, 10]$
7	$p = \text{poly}(\log(t), \gamma) + k$	$\gamma \in \mathbb{Z} [3, 10]$
8	$p = \text{poly}(\log(t), \gamma) * k$	$\gamma \in \mathbb{Z} [3, 10]$
9	$p = \text{poly}(\log(t), \gamma) * k + 1/k$	$\gamma \in \mathbb{Z} [3, 10]$
10	$p = \text{poly}(\log(t), \gamma) * \log(k)$	$\gamma \in \mathbb{Z} [3, 10]$
11	$p = \text{poly}(\log(t), \gamma) * \log(k) + 1/k$	$\gamma \in \mathbb{Z} [3, 10]$
12	$p = \text{poly}(\text{bc}(t), \gamma) * \log(k) + \text{poly}(\text{bc}(t), \gamma) * 1/k$	$\gamma \in \mathbb{Z} [3, 10]$
13	$p = \text{poly}(\text{bc}(t), \gamma) * \log(k) + \text{poly}(\text{bc}(t), \gamma) * \sqrt{k}$	$\gamma \in \mathbb{Z} [3, 10]$
14	$p = \text{poly}(\text{bc}(t), \gamma) * \log(k) + \text{poly}(\text{bc}(t), \gamma) * 1/k + \sqrt{k}$	$\gamma \in \mathbb{Z} [3, 10]$
15	$\text{bc}(p) = \text{poly}(\text{bc}(t), \gamma) + \log(k)$	$\gamma \in \mathbb{Z} [3, 10]$
16	$\text{bc}(p) = \text{poly}(\text{bc}(t), \gamma) * \log(k)$	$\gamma \in \mathbb{Z} [3, 10]$
17	$\text{bc}(p) = \text{poly}(\text{bc}(t), \gamma) * \log(k) + 1/k$	$\gamma \in \mathbb{Z} [3, 10]$
18	$\text{bc}(p) = \text{poly}(\text{bc}(t), \gamma) * \log(k) + \text{poly}(\text{bc}(t), \gamma) * 1/k$	$\gamma \in \mathbb{Z} [3, 10]$
19	$\text{bc}(p) = \text{poly}(\text{bc}(t), \gamma) * \log(k) + \text{poly}(\text{bc}(t), \gamma) * \sqrt{k}$	$\gamma \in \mathbb{Z} [3, 10]$
20	$\log(p) = \text{poly}(\text{bc}(t), \gamma) * \log(k)$	$\gamma \in \mathbb{Z} [3, 10]$
21	$\log(p) = \text{poly}(\text{bc}(t), \gamma) * \log(k) + 1/k$	$\gamma \in \mathbb{Z} [3, 10]$
22	$\log(p) = \text{poly}(\text{bc}(t), \gamma) * \log(k) + \text{poly}(\text{bc}(t), \gamma) * \sqrt{k}$	$\gamma \in \mathbb{Z} [3, 10]$
23	$\log(p) = \text{poly}(\text{bc}(t), \gamma) * \log(k) + \text{poly}(\text{bc}(t), \gamma) * 1/k$	$\gamma \in \mathbb{Z} [3, 10]$
24	$\log(p) = \text{poly}(\text{bc}(t), \gamma) * \log(k) + \text{poly}(\text{bc}(t), \gamma) * 1/k + \sqrt{k}$	$\gamma \in \mathbb{Z} [3, 10]$

Table A1: Description of all tested models....

Functional form	Full distribution		Lower tail ( $p < 0.2$ )	
	RMSE	cRMSE	RMSE	cRMSE
$bc(p) = \text{poly}(bc(t), 10) * \log(k) + \text{poly}(bc(t), 10) * \sqrt{k}$	$4.97 \cdot 10^{-4}$	$4.69 \cdot 10^{-4}$	$8.05 \cdot 10^{-4}$	$7.16 \cdot 10^{-4}$
$bc(p) = \text{poly}(bc(t), 10) * \log(k) + \text{poly}(bc(t), 10) * 1/k$	$5.39 \cdot 10^{-4}$	$5.11 \cdot 10^{-4}$	$8.54 \cdot 10^{-4}$	$7.61 \cdot 10^{-4}$
$p = \text{poly}(bc(t), 10) * \log(k) + \text{poly}(bc(t), 10) * \sqrt{k}$	$7.68 \cdot 10^{-4}$	$6.91 \cdot 10^{-4}$	$1.01 \cdot 10^{-3}$	$8.97 \cdot 10^{-4}$
$p = \text{poly}(bc(t), 10) * \log(k) + \text{poly}(bc(t), 10) * 1/k + \sqrt{k}$	$7.79 \cdot 10^{-4}$	$7.04 \cdot 10^{-4}$	$1.05 \cdot 10^{-3}$	$9.31 \cdot 10^{-4}$
$p = \text{poly}(bc(t), 10) * \log(k) + \text{poly}(bc(t), 10) * 1/k$	$7.82 \cdot 10^{-4}$	$7.07 \cdot 10^{-4}$	$1.06 \cdot 10^{-3}$	$9.41 \cdot 10^{-4}$

Table A2: The 5 best models for the combined Non-Cointegration test of Bayer and Hanck, where *all* underlying test are included and case 1.

Functional form	Full distribution		Lower tail ( $p < 0.2$ )	
	RMSE	cRMSE	RMSE	cRMSE
$\log(p) = \text{poly}(bc(t), 10) * \log(k)$	$1.27 \cdot 10^{-3}$	$1.25 \cdot 10^{-3}$	$1.05 \cdot 10^{-3}$	$9.52 \cdot 10^{-4}$
$\log(p) = \text{poly}(bc(t), 10) * \log(k) + \text{poly}(bc(t), 10) * \sqrt{k}$	$6.82 \cdot 10^{-4}$	$6.22 \cdot 10^{-4}$	$1.28 \cdot 10^{-3}$	$1.12 \cdot 10^{-3}$
$\log(p) = \text{poly}(bc(t), 10) * \log(k) + \text{poly}(bc(t), 10) * 1/k$	$7.32 \cdot 10^{-4}$	$6.63 \cdot 10^{-4}$	$1.39 \cdot 10^{-3}$	$1.20 \cdot 10^{-3}$
$\log(p) = \text{poly}(bc(t), 10) * \log(k) + \text{poly}(bc(t), 10) * 1/k + \sqrt{k}$	$8.38 \cdot 10^{-4}$	$7.78 \cdot 10^{-4}$	$1.48 \cdot 10^{-3}$	$1.31 \cdot 10^{-3}$
$bc(p) = \text{poly}(bc(t), 10) * \log(k) + \text{poly}(bc(t), 10) * 1/k$	$9.08 \cdot 10^{-4}$	$8.42 \cdot 10^{-4}$	$1.69 \cdot 10^{-3}$	$1.50 \cdot 10^{-3}$

Table A3: The 5 best models for the combined Non-Cointegration test of Bayer and Hanck, where *all* underlying test are included and case 2.

Functional form	Full distribution		Lower tail ( $p < 0.2$ )	
	RMSE	cRMSE	RMSE	cRMSE
$\log(p) = \text{poly}(bc(t), 10) * \log(k) + \text{poly}(bc(t), 10) * \sqrt{k}$	$4.58 \cdot 10^{-4}$	$4.55 \cdot 10^{-4}$	$3.37 \cdot 10^{-4}$	$3.16 \cdot 10^{-4}$
$\log(p) = \text{poly}(bc(t), 10) * \log(k) + \text{poly}(bc(t), 10) * 1/k$	$5.17 \cdot 10^{-4}$	$5.14 \cdot 10^{-4}$	$3.90 \cdot 10^{-4}$	$3.73 \cdot 10^{-4}$
$\log(p) = \text{poly}(bc(t), 10) * \log(k)$	$1.04 \cdot 10^{-3}$	$1.04 \cdot 10^{-3}$	$6.76 \cdot 10^{-4}$	$6.50 \cdot 10^{-4}$
$\log(p) = \text{poly}(bc(t), 10) * \log(k) + \text{poly}(bc(t), 10) * 1/k + \sqrt{k}$	$1.18 \cdot 10^{-3}$	$1.17 \cdot 10^{-3}$	$2.06 \cdot 10^{-3}$	$2.05 \cdot 10^{-3}$
$bc(p) = \text{poly}(bc(t), 10) * \log(k) + \text{poly}(bc(t), 10) * 1/k$	$1.16 \cdot 10^{-3}$	$1.06 \cdot 10^{-3}$	$2.08 \cdot 10^{-3}$	$1.80 \cdot 10^{-3}$

Table A4: The 5 best models for the combined Non-Cointegration test of Bayer and Hanck, where *all* underlying test are included and case 3.

Functional form	Full distribution		Lower tail ( $p < 0.2$ )	
	RMSE	cRMSE	RMSE	cRMSE
$\log(p) = \text{poly}(bc(t), 10) * \log(k) + \text{poly}(bc(t), 10) * 1/k$	$4.75 \cdot 10^{-4}$	$4.44 \cdot 10^{-4}$	$7.81 \cdot 10^{-4}$	$6.84 \cdot 10^{-4}$
$\log(p) = \text{poly}(bc(t), 10) * \log(k)$	$6.54 \cdot 10^{-4}$	$5.87 \cdot 10^{-4}$	$1.01 \cdot 10^{-3}$	$7.81 \cdot 10^{-4}$
$\log(p) = \text{poly}(bc(t), 10) * \log(k) + \text{poly}(bc(t), 10) * \sqrt{k}$	$7.60 \cdot 10^{-4}$	$6.13 \cdot 10^{-4}$	$1.46 \cdot 10^{-3}$	$1.06 \cdot 10^{-3}$
$\log(p) = \text{poly}(bc(t), 10) * \log(k) + \text{poly}(bc(t), 10) * 1/k + \sqrt{k}$	$7.64 \cdot 10^{-4}$	$7.45 \cdot 10^{-4}$	$1.29 \cdot 10^{-3}$	$1.23 \cdot 10^{-3}$
$bc(p) = \text{poly}(bc(t), 10) * \log(k) + \text{poly}(bc(t), 10) * 1/k$	$1.01 \cdot 10^{-3}$	$9.17 \cdot 10^{-4}$	$1.89 \cdot 10^{-3}$	$1.65 \cdot 10^{-3}$

Table A5: The 5 best models for the combined Non-Cointegration test of Bayer and Hanck, where *EG-J* underlying test are included and case 1.

Functional form	Full distribution		Lower tail ( $p < 0.2$ )	
	RMSE	cRMSE	RMSE	cRMSE
$\log(p) = \text{poly}(\text{bc}(t), 10) * \log(k)$	$7.36 \cdot 10^{-4}$	$7.25 \cdot 10^{-4}$	$7.04 \cdot 10^{-4}$	$6.45 \cdot 10^{-4}$
$\log(p) = \text{poly}(\text{bc}(t), 10) * \log(k) + \text{poly}(\text{bc}(t), 10) * 1/k$	$5.53 \cdot 10^{-4}$	$5.12 \cdot 10^{-4}$	$9.75 \cdot 10^{-4}$	$8.56 \cdot 10^{-4}$
$\log(p) = \text{poly}(\text{bc}(t), 10) * \log(k) + \text{poly}(\text{bc}(t), 10) * \sqrt{k}$	$5.53 \cdot 10^{-4}$	$5.11 \cdot 10^{-4}$	$9.87 \cdot 10^{-4}$	$8.66 \cdot 10^{-4}$
$\log(p) = \text{poly}(\text{bc}(t), 10) * \log(k) + \text{poly}(\text{bc}(t), 10) * 1/k + \sqrt{k}$	$6.00 \cdot 10^{-4}$	$5.62 \cdot 10^{-4}$	$1.11 \cdot 10^{-3}$	$1.01 \cdot 10^{-3}$
$\text{bc}(p) = \text{poly}(\text{bc}(t), 10) * \log(k) + \text{poly}(\text{bc}(t), 10) * \sqrt{k}$	$1.05 \cdot 10^{-3}$	$9.54 \cdot 10^{-4}$	$2.00 \cdot 10^{-3}$	$1.75 \cdot 10^{-3}$

Table A6: The 5 best models for the combined Non-Cointegration test of Bayer and Hanck, where  $EG-J$  underlying test are included and case 2.

Functional form	Full distribution		Lower tail ( $p < 0.2$ )	
	RMSE	cRMSE	RMSE	cRMSE
$\log(p) = \text{poly}(\text{bc}(t), 10) * \log(k) + \text{poly}(\text{bc}(t), 10) * \sqrt{k}$	$3.85 \cdot 10^{-4}$	$3.73 \cdot 10^{-4}$	$5.03 \cdot 10^{-4}$	$4.58 \cdot 10^{-4}$
$\log(p) = \text{poly}(\text{bc}(t), 10) * \log(k)$	$7.55 \cdot 10^{-4}$	$7.54 \cdot 10^{-4}$	$4.85 \cdot 10^{-4}$	$4.70 \cdot 10^{-4}$
$\log(p) = \text{poly}(\text{bc}(t), 10) * \log(k) + \text{poly}(\text{bc}(t), 10) * 1/k$	$3.73 \cdot 10^{-4}$	$3.59 \cdot 10^{-4}$	$5.34 \cdot 10^{-4}$	$4.83 \cdot 10^{-4}$
$\log(p) = \text{poly}(\text{bc}(t), 10) * \log(k) + \text{poly}(\text{bc}(t), 10) * 1/k + \sqrt{k}$	$4.87 \cdot 10^{-4}$	$4.76 \cdot 10^{-4}$	$8.52 \cdot 10^{-4}$	$8.19 \cdot 10^{-4}$
$\text{bc}(p) = \text{poly}(\text{bc}(t), 10) * \log(k) + \text{poly}(\text{bc}(t), 10) * \sqrt{k}$	$1.02 \cdot 10^{-3}$	$9.35 \cdot 10^{-4}$	$1.94 \cdot 10^{-3}$	$1.70 \cdot 10^{-3}$

Table A7: The 5 best models for the combined Non-Cointegration test of Bayer and Hanck, where  $EG-J$  underlying test are included and case 3.

### **Eidesstattliche Versicherung**

Ich versichere an Eides statt durch meine Unterschrift, dass ich die vorstehende Arbeit selbständig und ohne fremde Hilfe angefertigt und alle Stellen, die ich wörtlich oder annähernd wörtlich aus Veröffentlichungen entnommen habe, als solche kenntlich gemacht habe, mich auch keiner anderen als der angegebenen Literatur oder sonstiger Hilfsmittel bedient habe. Die Arbeit hat in dieser oder ähnlicher Form noch keiner anderen Prüfungsbehörde vorgelegen.

Essen, den \_\_\_\_\_

\_\_\_\_\_  
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