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COINTEGRATION TESTS OF PRESENT VALUE MODELS WITH A TIME-VARYING DISCOUNT FACTOR

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SUMMARY

The paper analyses the impact of persistence and volatility in the discount rate in present-value models on cointegration tests in levels and in logarithms. In simulations we find that the probability of not rejecting the null of no cointegration depends on the persistence of the discount rate process and can be very high when the expected returns process is highly persistent. In contrast, the cointegration tests are very robust with respect to the level of volatility in the discount rate. We discuss the relevance of our findings for the US stock market where standard ADF tests do not reject the null of no cointegration between stock prices and dividends. Based on estimates of persistence in four asset pricing models, we find that a model which links expected returns to the dividend yield is sufficiently persistent to explain the failure of rejecting the null that stock prices and dividends are not cointegrated.

1. INTRODUCTION

It has become standard in the applied econometrics literature to test linear rational expectations models by means of cointegration.[1] Present-value relations are perhaps the most frequently used models because they arise in numerous economic areas. Examples are the permanent income hypothesis, models on yield differentials between long and short bonds, models of stock prices, and monetary models of inflation. In this paper we will focus on the stock market but our findings apply to present-value models in general.

Although present-value relations follow from basic economic theory, this theory usually does not imply a constant discount factor. Present-value models arise from agents' intertemporal allocation of funds for consumption or investment. Combining the 'stylized fact' of trend reversion in aggregate income with the permanent income theory, expected returns on savings can be predicted to vary cyclically. In a recession current incomes are low relative to expected permanent incomes and the permanent income hypothesis predicts that savings will also be low. Suppose that investments are less responsive to cyclical variation in income than savings. To ensure clearing of the financial markets the expected return on savings must increase to induce an increase in the funds allocated to investment. The opposite result holds if investments are more sensitive with respect to business cycle variation in aggregate income than savings. Unless savings and investments display identical sensitivities with respect to changes in aggregate income, expected returns must also vary over the business cycle to clear the financial markets.

[1] See Campbell and Shiller (1987), Hall *et al.* (1991), Johansen and Juselius (1990), and Phillips and Ouliaris (1988).

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Two approaches have been suggested in the applied econometrics literature to test present-value models using cointegration methods: Tests applied to the ratio of the endogenous to the forcing variable (Craine, 19891; Cochrane 1992) or to the log-difference between these variables (Campbell and Shiller, 1988) and, second, cointegration tests using levels of the endogenous and forcing variable (Campbell and Shiller, 1987; Engle and Granger 1987). In this paper we show that when the expected rate of return varies over time the present-value model does not generally imply the existence of a stationary relationship between the integrated forcing variable and the endogenous variable in levels. As the residuals from the cointegrating regression in levels are the product of a stationary and an integrated variable, the variance of the residuals will be trending. On the other hand, plausible assumptions imply that the cointegration test based on the log-difference between the endogenous and the forcing variable will be valid in the presence of variation in the discount rate.

Practitioners' choice of test may ultimately depend on how robust these procedures are with respect to variation in discount rates. In a simulation study we find that the null of no cointegration between the endogenous and the forcing variable is not rejected in a high proportion of the simulations when the persistence in the expected rate of return process is high and the sample is small (50–100 observations). This holds both for the cointegration test in logs and for the test in levels. This conclusion is surprisingly robust with respect to the relative variance of the innovation in the processes that generate the endogenous variable and expected returns. Since the simulated series satisfy the present-value relation by construction, failure to reject the null should not be interpreted as a rejection of the underlying model. We also find that the cointegration test in logs tend to reject the null of no cointegration more frequently than the cointegration test in levels, the only exceptions arising when expected returns are highly persistent and the sample size is small.

These conclusions are applied in a study of the US stock market. We investigate four different models for expected returns on US stocks and find that only a returns process based on dividend yields generates enough persistence to explain the failure of rejecting the null of no cointegration between stock prices and dividends reported in the applied literature.

The plan of the paper is as follows. Section 2 compares the two cointegration tests of the present-value model when expected returns vary over time. Section 3 presents results from the simulation study and compares them to four models of expected returns. Section 4 concludes.

2. COINTEGRATION IN PRESENT-VALUE MODELS WITH A TIME-VARYING DISCOUNT FACTOR

The present-value model is usually written

$$P_t = \sum_{i=1}^{\infty} \beta^i E_t D_{t+i} \tag{1}$$

where P_t is the endogenous variable measured at the end of period t, D_t is the forcing variable during period t, E_t is the expectations operator conditional on the information set at the end of period t, Ω_t, which at a minimum contains $\{D_{t-i}, i \geqslant 0\}$. β is the constant discount factor ($\beta = 1/1+r$) and r is the expected rate of return. We shall assume that D_t is integrated of order one (I(1)). The result that this present-value model imposes cointegration between P_t and D_t under the assumption that ΔD_t is weakly stationary can be derived by exploiting the martingale difference approach of Broze *et al.* (1985) for solution of linear rational expectations models. Denote by $\varepsilon^P_{t+1} = P_{t+1} - E_t P_{t+1}$, $\varepsilon^D_{t+1} = D_{t+1} - E_t D_{t+1}$, and write equation (1) as

$$P_t = \beta E_t(P_{t+1} + D_{t+1}) = \beta P_{t+1} + \beta D_{t+1} - \beta\varepsilon^P_{t+1} - \beta\varepsilon^D_{t+1}$$

so

$$(1-\beta)P_{t+1} = \beta D_{t+1} + \Delta P_{t+1} - \beta\varepsilon^P_{t+1} - \beta\varepsilon^D_{t+1} \tag{2}$$

Under the assumption of rational expectations, ε^P_{t+1} and ε^D_{t+1} are martingale difference sequences. Provided that ΔD_t is weakly stationary then these sequences will also be weakly stationary. It follows that P_t and D_t are CI(1,1) with a cointegrating vector of

$$\left(1, \frac{-\beta}{1-\beta}\right) = \left(1, -\frac{1}{r}\right)$$

The result that the present-value model implies a cointegrating relationship between the endogenous and the forcing variable when the forcing variable is first-difference stationary and the discount factor is constant does not generally carry over to the present-value model which allows for variation across time in expected returns. To make this point let $\gamma_{t+i} = (D_{t+i})/(D_{t+i-1})$ and $\beta_{t+i} = (1/1 + r_{r+i})$, so that the present-value model with a time-varying discount factor can be written

$$P_t = a_t D_t \tag{3}$$

where

$$a_t = E_t\left\{\sum_{k=1}^{\infty}\prod_{j=1}^{k}\beta_{t+j}\gamma_{t+j}\right\}$$

Thus

$$P_t = \mu_a D_t + u_t \tag{4}$$

where $u_t = (a_t - \mu_a)D_t$. Suppose that a_t is stationary with first and second moments, and for simplicity assume that at a_t and D_t are statistically independent. Then it follows that the residuals from the cointegrating regression in levels (4) will have a trending variance and hence P_t and D_t will not be cointegrated. However, the ratio (P_t/D_t) is stationary, and hence a cointegrating regression applied to the log-difference $(\log(P_t/D_t))$ provides a suitable framework for testing the present value model. Craine (1989) and Cochrane (1992) were among the first to suggest using ratio-based cointegration tests of present value models, and Campbell and Shiller (1988) proceeded with a cointegration test of the present-value model based on log-differences. In practice, these approaches are likely to give very similar results since one looks at a_t and the other looks at $\log(a_t)$, and we refer to them as cointegration tests in logs. Hence there are two principal ways of proceeding with cointegration tests of the present-value model: by applying first, the cointegration test in levels and, second a cointegration test to the log-difference of the variables.

The conclusion on the distribution of cointegration test in logs relied on the assumption that a_t has two moments, which cannot always be guaranteed to hold. In the more general case suppose that the processes generating the discount rate r_{t+j} and the growth rate γ_{t+j} are strongly stationary. Then it follows that the ratio P_t/D_t (being a measurable function of strongly stationary variables) is strongly stationary as long as the sum defining the random variable a_t

converges in distribution.[2] However, it does not follow that the ratio of the endogenous to the forcing variable is covariance stationary since this ratio may not have finite moments. Hence the distributional properties of the two cointegration tests are difficult to analyse explicitly since the residuals adopted in the levels test are the product of a strongly stationary and a first-difference stationary process while the logarithmic test statistic is only guaranteed to be strongly stationary. Since cointegration tests of present-value models are widely used in applied work it is important to analyse the properties of the two alternatives. This is the topic of the next section.

3. A SIMULATION STUDY OF THE US STOCK MARKET

In a widely quoted study Campbell and Shiller (1987) obtained mixed results in tests of the hypothesis of cointegration between stock prices and dividends for a US portfolio over the period 1871–1986. Using a DF test based on the residuals from the cointegrating regression between levels of real stock prices and dividends they rejected the null of no cointegration at the 5 per cent level. However, when lagged values of the change in the residuals were included in an ADF test, the null of no cointegration could not be rejected at the 10 per cent level. Similarly, Phillips and Ouliaris (1988), using principal components methods, came to the conclusion that the null of no cointegration between dividends and stock prices could not be rejected at the 10 per cent level. These tests assumed expected returns were constant. The evidence of a unit root in the price-dividend ratio is equally ambiguous. Using the same data set as in the above studies (printed in Chapter 26 in Shiller, 1989), the values of the DF test applied to this ratio over the period 1871–1986 were −4·16 (without trend) and −4·36 (with trend), while the values of the ADF test with four lags were −2·63 (without trend) and −2·69 (with trend). Hence the DF test rejects the null of a unit root at the 1 per cent level while the ADF(4) test does not reject the null of a unit root in the price–dividend ratio at the 10 per cent level.[3] These results are potentially very damaging to the efficient markets hypothesis. If even the basic relationship between stock prices and dividends implied by the present value model does not hold, most economists would not be inclined to think that stock markets are reasonably efficient. Here we attempt to answer the question: Can models of expected returns with reasonable persistence and volatility explain the apparent rejection of the present value model for the US stock market?

Initially we tested for a unit root in real stock prices and real dividends. Using a Bartlett window with a truncation point of twelve observations we computed Phillips-Perron (1987) unit root statistics for real stock prices and real dividends over the period 1871–1986. The values of the $Z(t_{\hat{\alpha}})$ statistic applied to real stock prices and real dividends were −1·59 and −1·02, respectively, and the corresponding values for the logarithm of real stock prices and real dividends were −1·77 and −1·63. None of these values are statistically significant at the 5 per cent level so we can maintain the null hypothesis of a unit root in the series. Nor did the DF and ADF(4) statistics for real dividends (−1·21 and −0·99, respectively) and real stock prices

[2] Let

$$X_{nt} = \sum_{k=1}^{n} \prod_{j=1}^{k} \beta_{t+j}\gamma_{t+j}$$

and $X_{\infty t}$ be the almost certain limit of X_{nt} as n goes to infinity. Given that X_{nt} is strongly stationary for fixed n and that, for any finite k and any sequence $(t_1, t_2, .., t_k)$, $(X_{nt_1}, X_{nt_2}, ..., X_{nt_n})$ converges in distribution to $(X_{\infty t_1}, X_{\infty t_2}, ..., X_{\infty t_n})$, then it follows that $X_{\infty t}$ will be strongly stationary as well.

[3] While there was no significant serial correlation in the residuals from the DF regression, there was strong evidence of residual serial correlation when one lag of the first-differenced residuals was added in an ADF(1) test. For the ADF(4) test there was again no evidence of serial correlation in the residuals.

($-1{\cdot}45$ and $-1{\cdot}55$) or the logarithm of real dividends ($-1{\cdot}83$ and $-1{\cdot}34$) and real stock prices ($-1{\cdot}76$ and $-1{\cdot}69$) indicate that the series are stationary.[4]

An approximation to the present-value model with time-varying expected returns can be derived by linearizing equation (3) around $\bar{r}$ inside the expectation operator (cf. Poterba and Summers. 1986) to obtain the first-order Taylor approximation

$$P_t \approx E_t\left\{\sum_{j=1}^{\infty}\left(\frac{1}{1+\bar{r}}\right)^j D_{t+j} + \sum_{j=1}^{\infty}\frac{\partial P_t}{\partial r_{t+j\bar{r}}}\,|\,(r_{t+j}-\bar{r})\right\} \tag{5}$$

where

$$\frac{\partial P_t}{\partial r_{t+j\bar{r}}}\,|= E_t\left\{\frac{1}{(1+\bar{r})^j}\left\{D_{t+j}+\frac{D_{t+j+1}}{1+\bar{r}}+\frac{D_{t+j+2}}{(1+\bar{r})^2}+\cdots\right\}\frac{-1}{(1+\bar{r})^2}\right\}$$

Furthermore, assume that real dividends follow a geometric random walk with Gaussian increments

$$\log(D_t)=\log(D_{t-1})+\varepsilon_t, \qquad \varepsilon_t \sim \mathrm{IIN}(\mu, \sigma_d^2) \tag{6}$$

so $E_t(D_{t+k})=\exp(k(\mu+\sigma_d^2/2)$. Suppose the expected rate of return follows a stationary AR(1) process around its mean, $\bar{r}$:

$$r_{t+j}-\bar{r}=\rho(r_{t+j-1}-\bar{r})+\varepsilon_t^r, \qquad \varepsilon_t^r \sim \mathrm{IIN}(0, \sigma_r^2) \tag{7}$$

such that $E_t(r_{t+j}-\bar{r})=\rho^j(r_t-\bar{r})$. This assumption has the advantage that it allows us to derive a solution for the approximate present-value stock price and it is easy to measure persistence and volatility by the parameters ρ and σ_r^2. We also assume that ε_t^r and ε_t are independent, an assumption that may not apply to many models of the expected rate of return on shares. Using equation (5)–(7) we obtain after some algebra

$$P_t=\left(\frac{1+g}{\bar{r}-g}-\frac{\rho(1+g)(r_t-\bar{r})}{(\bar{r}-g)(1+\bar{r})(1+\bar{r}-\rho(1+g))}\right)D_t \tag{8}$$

where $g=\exp(\mu+\sigma_d^2/2)-1$. The standard formula for the stock price in a model with a constant expected rate of return can be derived by setting $r_t=\bar{r}$. Equations (6)–(8) form the basis of the Monte Carlo experiment. We need to assign values to r_1 and D_1 (the initial values) and the parameters $\bar{r}$, ρ, σ_d^2, μ, and σ_r^2 as well as the sample size T. Using data on dividends per share over the period 1871–1986 for the Standard & Poor's 500 portfolio, we obtained estimates of $\hat{\sigma}_d=0{\cdot}1318$ and $\hat{\mu}=0{\cdot}0132$. This gives an estimate of g of 2·2 per cent per annum. D_1 was fixed at its historical value in 1871 and we set $r_1=\bar{r}$, where $\bar{r}$ was fixed to ensure that the average dividend yield of the model $((\bar{r}-g/1+g))$ equals the mean yield of the US data (0·05).

A thousand samples of length T of dividends and expected returns were generated at random for models (6) and (7) and stock prices were computed using model (8). For each of these samples the two-stage Engle–Granger (1987) cointegration test proceeds by estimating an equation

$$P_t=\alpha+\beta D_t+u_t \tag{9}$$

[4] These statistics include a drift but not a time trend. Only for logged dividends was there some evidence that the null of a unit root might be rejected for low orders of the ADF test once a time trend was included.

where the estimated residuals, $\hat{u}_t$, are used to obtain the parameter $\hat{\phi}$ from the regression

$$\Delta\hat{u}_t = -\phi\hat{u}_{t-1} + b_1\Delta\hat{u}_{t-1} + b_2\Delta\hat{u}_{t-2} + b_3\Delta\hat{u}_{t-3} + b_4\Delta\hat{u}_{t-4} + e_t \tag{10}$$

On the basis of $\hat{\phi}$ we computed the t-statistic $\hat{\tau}_\phi$ and compared it to the critical values for the ADF(4) statistic implied by the response surface estimates of critical values for integrated series reported in Table 1 in MacKinnon(1991). We also computed the DF statistic by leaving out the terms $\Delta\hat{u}_{t-1}, \ldots, \Delta\hat{u}_{t-4}$ in equation (10).[5] This is the appropriate test to use in the present context since the model used to generate data implies that there is no serial correlation in the residuals.

In the case of the cointegration test in logs we simply computed the DF and ADF(4) test statistic with an intercept ($\hat{\tau}_\mu$) for the variable

$$\log(P_t) - \log(D_t) = \log\left(\frac{(1+g)((1+\bar{r})(1+\bar{r}-\rho(1+g)) - \rho(r_t-\bar{r}))}{(1+\bar{r})(\bar{r}-g)(1+\bar{r}-\rho(1+g))}\right) \tag{11}$$

The simulated values were compared to the critical values computed by using Table 1 in MacKinnon (1991). These critical values are very similar to the ones provided in Table 8.5.2 in Fuller (1976).

We varied ρ from 0·01 to 0·95 and σ_r from 0·012 to 0·12.[6] These values of σ_r were chosen in order to ensure that the probability that stock prices in model (8) become negative is negligible and cannot be said to be overly high since the largest value of σ_r is 0·013.[7] Initially we fixed the sample size (T) at 100 observations which is similar to the sample size used in the studies quoted above. Table I reports sample statistics for the Engle–Granger residuals-based unit root statistics $\hat{\tau}_\phi$ and $\hat{\tau}_\mu$ for the grid of values of ρ and for $\sigma_r = 0{\cdot}1\sigma_d$ and $\sigma_r = 0{\cdot}01\sigma_d$. First, consider the cointegration test in levels. In a large proportion of cases the null of no cointegration is not rejected when the discount factor varies across time and is sufficiently persistent. It is also clear that the distribution of the statistic $\tilde{\tau}_\phi$ is remarkably robust with respect to the value of the ratio σ_d/σ_r. The crucial parameter which determines the proportion of rejections of the null of no cointegration is evidently the degree of persistence, ρ, in the expected returns process. This impression is enforced by Table II, which presents the mean and variance of the DF and ADF statistics as a function of ρ and σ_r.

When interpreting Table I it is important to recall that the data-generating process implies that it is right to apply a cointegration test in logs without a trend since the residuals by construction are strongly stationary when $|\rho| < 1$. In fact, when we applied a cointegration test in logs which included a drift and a trend, this test led to a less frequent rejection of the null as compared to the cointegration test which only includes a drift: The increase in the critical values of the cointegration test which includes a trend (Table 8.5.2 in Fuller, 1976) was larger than the increase in the average value of the test statistic which, in our simulations, does not contain a deterministic trend.

[5] The critical values for the Engle–Granger DF statistic implied by the MacKinnon study are very similar to the critical values given in Table 2 in Engle and Granger (1987). However, whereas the critical values for the ADF test reported in Engle and Granger's Table 2 are 0·2 and 0·3 smaller than those of the DF statistic, there is no such difference in the two sets of critical values implied by the MacKinnon study. Since Phillips (1987) has shown that the critical values of the DF and the ADF test are asymptotically identical we used the critical values in MacKinnon for the ADF test.

[6] Since we do not introduce an arbitrary noise component in the present value relation, P_t and D_t will be perfectly linearly correlated when $\rho = 0$ (cf model (8)). Thus cointegration tests only make sense in our setup for a non-zero value of ρ.

[7] When σ_r took a very large value (e.g. $\sigma_r = \sigma_d$) the simulation results were very similar to those reported in Tables I and II.

Table I. Percentage of rejections of the null of no-cointegration in the present-value model with a time varying expected rate of return. (Sample size 100)

		Cointegration test in levels						Cointegration test in logs (no trend)					
		EG–ADF(4) test			EG–DF test			ADF(4) test			DF test		
	ρ	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
$\sigma_d/\sigma_r = 10$	0·01	74·1	91·9	95·7	100·0	100·0	100·0	91·9	99·1	99·8	100·0	100·0	100·0
	0·1	71·1	91·0	95·5	100·0	100·0	100·0	91·7	99·0	99·8	100·0	100·0	100·0
	0·2	66·9	88·8	93·6	100·0	100·0	100·0	88·2	99·1	99·9	100·0	100·0	100·0
	0·3	65·2	88·6	93·4	99·9	100·0	100·0	83·6	98·3	99·4	100·0	100·0	100·0
	0·4	57·1	81·2	91·6	99·6	99·9	100·0	77·4	97·3	99·6	100·0	100·0	100·0
	0·5	46·9	75·3	85·9	99·5	99·9	99·9	67·8	92·0	97·2	100·0	100·0	100·0
	0·6	38·1	68·4	80·2	93·8	97·6	98·5	55·4	87·5	95·3	99·5	100·0	100·0
	0·7	30·0	58·6	71·4	76·3	94·1	96·6	37·4	74·3	87·6	93·4	99·8	100·0
	0·8	15·1	39·0	55·4	44·0	72·7	84·0	17·6	53·1	73·1	52·8	87·6	95·8
	0·9	7·8	20·5	32·5	13·7	33·7	48·6	5·5	19·9	36·1	9·2	33·8	54·4
	0·95	3·1	9·9	18·7	5·2	17·0	26·5	2·1	12·1	24·4	3·1	18·2	30·5
$\sigma_d/\sigma_r = 100$	0·01	77·2	93·6	96·6	100·0	100·0	100·0	92·8	99·5	99·9	100·0	100·0	100·0
	0·1	72·9	91·8	95·8	99·8	99·9	99·9	91·2	99·8	100·0	100·0	100·0	100·0
	0·2	65·7	89·0	94·3	100·0	100·0	100·0	87·6	99·2	100·0	100·0	100·0	100·0
	0·3	64·4	84·5	93·1	99·7	100·0	100·0	82·9	98·1	99·5	100·0	100·0	100·0
	0·4	54·6	84·2	90·6	99·6	99·9	99·9	78·4	97·1	99·4	100·0	100·0	100·0
	0·5	48·6	76·4	87·5	98·3	99·7	99·9	67·1	93·4	97·8	100·0	100·0	100·0
	0·6	38·0	69·1	81·3	95·7	99·1	99·2	54·8	88·0	96·2	100·0	100·0	100·0
	0·7	25·9	57·4	70·9	74·3	91·8	95·0	36·9	74·8	87·3	92·5	99·5	100·0
	0·8	17·3	39·6	54·9	41·6	70·3	82·7	19·5	52·9	72·2	51·1	87·6	96·0
	0·9	7·0	18·8	31·2	11·9	32·5	47·3	5·3	21·7	37·3	9·4	32·1	52·8
	0·95	3·5	10·5	17·9	5·9	16·3	26·5	1·2	9·8	18·4	2·6	12·2	21·4

Note: The simulations were based on models (6)–(8) in the text. The EG–ADF(4) test gives the value of the augmented Dickey–Fuller statistic with 4 lags suggested by Engle and Granger while EG–DF gives the value of the Dickey–Fuller statistic suggested in that study. DF and ADF give the corresponding unit root tests suggested in Fuller (1976). ρ is the persistence parameter in the AR(1) process for expected returns, σ_d is the standard deviation of the innovation in the dividend process and σ_r is the standard deviation of the innovation in the expected returns process.

For both cointegration tests the percentage of rejections steadily declines as ρ increases.[8] The percentage of rejections tends to be higher for the test in logs than for the test in levels, although for high values of the persistence parameter this is not always the case. Thus, although we showed in Section 2 that it is not correct to apply a cointegration test in levels to the present-value model when the discount rate varies over time, we arrive at the somewhat surprising conclusion that the cointegration test in levels may lead a researcher to reject the null with a higher probability than the cointegration test in logs provided the discount rate is strongly persistent.[9]

To study the dependence of the proportion of rejections of the null of no cointegration with

[8] This finding is similar to results obtained by Craine (1989).

[9] The reported simulations assume that shocks to the dividend process (ε_t) are uncorrelated with shocks to the expected returns series (ε_t^r). To investigate the importance of this assumption, we repeated the simulations, but now assuming that $\varepsilon_t^r = \sigma_r \rho_{d,r} + u_t \sqrt{\sigma_r^2(1-\rho_{d,r}^2)}$, where $u_t \sim \mathrm{IIN}(0,1)$ is uncorrelated with ε_t, and $\rho_{d,r}^2$ is the squared correlation between ε_t *and* $\varepsilon_t{}^r$. We found that the simulation results were very robust with respect to the independence assumption even for values of $\rho_{d,r}^2$ as large as 0·9. Only when $\rho_{d,r}^2$ took on a rather extreme value ($\rho_{d,r}^2 = 0{\cdot}99$) did the power of the cointegration test in levels decrease significantly, while the power of the logs test was unaffected.

Table II. Mean and standard deviation of Dickey–Fuller and augmented Dickey–Fuller test statistics

σ_d/σ_r	10		20		30		40		50		60		70		80		90		100	
ρ	mean	s.d.	mean	s.d.	mean	s.d.	mean	s.d.	mean	s.d.	mean	s.d.	mean	s.d.	mean	s.d.	mean	s.d.	mean	s.d.
Part I: Test in levels: Engle–Granger DF statistic																				
0·01	10·09	1·44	10·13	1·52	10·13	1·50	10·07	1·49	10·13	1·55	10·15	1·61	10·08	1·43	10·22	1·56	10·17	1·56	10·10	1·58
0·1	9·33	1·39	9·20	1·38	9·25	1·44	9·27	1·42	9·25	1·34	9·27	1·44	9·24	1·37	9·24	1·37	9·22	1·37	9·31	1·40
0·2	8·40	1·28	8·38	1·25	8·39	1·27	8·50	1·30	8·31	1·26	8·38	1·26	8·41	1·27	8·43	1·33	8·34	1·31	8·46	1·37
0·3	7·65	1·25	7·60	1·18	7·57	1·21	7·67	1·19	7·70	1·20	7·72	1·25	7·61	1·19	7·68	1·28	7·60	1·16	7·60	1·18
0·4	6·95	1·13	6·88	1·13	6·79	1·10	6·84	1·16	6·82	1·12	6·91	1·11	6·82	1·07	6·91	1·21	6·88	1·12	6·81	1·08
0·5	6·22	1·09	6·13	1·04	6·10	1·04	6·12	1·06	6·15	1·07	6·17	1·12	6·23	1·07	6·19	1·04	6·14	1·09	6·14	1·02
0·6	5·43	1·00	5·45	0·97	5·34	0·98	5·37	1·04	5·41	1·01	5·36	0·99	5·37	1·01	5·41	1·02	5·41	1·03	5·36	1·00
0·7	4·65	0·93	4·68	0·98	4·64	0·93	4·70	0·95	4·64	0·92	4·64	0·90	4·66	0·91	4·66	0·92	4·67	0·97	4·63	0·91
0·8	3·92	0·93	3·94	0·93	3·89	0·93	3·94	0·92	3·87	0·94	3·88	0·92	3·86	0·93	3·91	0·92	3·86	0·91	3·93	0·92
0·9	3·09	0·91	2·99	0·91	3·03	0·88	3·04	0·91	3·03	0·96	3·07	0·91	3·07	0·91	2·99	0·92	3·03	0·89	3·07	0·93
0·95	2·61	0·89	2·59	0·93	2·63	0·96	2·60	0·95	2·66	0·93	2·59	0·94	2·60	0·96	2·59	0·95	2·62	0·94	2·59	0·93
Part II: Test in levels: augmented Engle–Granger ADF statistic (EG–ADF(4)).																				
0·01	4·59	0·95	4·63	0·95	4·63	0·92	4·62	0·95	4·63	0·95	4·59	0·92	4·53	0·92	4·60	0·94	4·63	0·97	4·64	0·96
0·1	4·47	0·90	4·57	0·92	4·55	0·99	4·56	0·91	4·53	0·90	4·60	0·99	4·55	0·90	4·49	0·96	4·53	0·90	4·50	0·91
0·2	4·44	0·89	4·42	0·94	4·50	0·95	4·43	0·92	4·43	0·93	4·44	0·88	4·46	0·92	4·42	0·93	4·47	0·99	4·46	0·94
0·3	4·35	0·91	4·34	0·89	4·32	0·95	4·30	0·94	4·37	0·90	4·40	0·92	4·35	0·97	0·97	4·32	0·94	0·94	4·29	0·94
0·4	4·16	0·91	4·24	0·91	4·17	0·94	4·19	0·95	4·15	0·89	4·23	0·91	4·22	0·90	4·21	0·91	4·16	0·89	4·22	0·92
0·5	4·07	0·87	3·99	0·90	4·04	0·91	3·99	0·92	4·03	0·92	4·05	0·93	4·07	0·92	4·05	0·90	4·01	0·86	4·01	0·88
0·6	3·84	0·87	3·84	0·90	3·79	0·88	3·82	0·90	3·79	0·83	3·83	0·94	3·80	0·90	3·81	0·88	3·84	0·88	3·79	0·87
0·7	3·59	0·90	3·59	0·88	3·58	0·90	3·59	0·92	3·55	0·88	3·57	0·88	3·59	0·90	3·54	0·88	3·55	0·90	3·56	0·86
0·8	3·23	0·88	3·21	0·82	3·23	0·85	3·24	0·84	3·20	0·87	3·24	0·84	3·23	0·86	3·26	0·89	3·18	0·86	3·22	0·87
0·9	2·77	0·87	2·72	0·83	2·73	0·82	2·73	0·87	2·69	0·85	2·71	0·82	2·75	0·87	2·72	0·90	2·73	0·89	2·76	0·90
0·95	2·43	0·83	2·38	0·85	2·44	0·90	2·41	0·87	2·42	0·85	2·44	0·91	2·42	0·88	2·38	0·85	2·45	0·91	2·39	0·86
Part III: Test in logs: Dickey–Fuller statistic																				
0·01	9·96	0·96	10·00	0·99	10·00	0·99	9·99	0·98	9·94	1·02	9·99	1·05	9·96	0·95	9·99	1·00	9·99	1·04	9·93	1·04
0·1	9·14	0·88	9·08	0·92	9·10	0·96	9·13	0·95	9·04	0·89	9·12	0·92	9·09	0·92	9·15	0·90	9·10	0·90	9·10	0·95
0·2	8·24	0·87	8·21	0·84	8·24	0·87	8·28	0·86	8·19	0·83	8·24	0·88	8·23	0·86	8·23	0·85	8·21	0·87	8·26	0·86
0·3	7·42	0·82	7·42	0·79	7·45	0·82	7·48	0·76	7·45	0·79	7·50	0·78	7·42	0·76	7·46	0·78	7·43	0·77	7·45	0·77
0·4	6·69	0·76	6·67	0·76	6·63	0·74	6·64	0·74	6·63	0·71	6·67	0·74	6·62	0·70	6·67	0·75	6·69	0·74	6·61	0·70
0·5	5·94	0·74	5·89	0·70	5·91	0·73	5·90	0·70	5·89	0·69	5·92	0·75	5·95	0·70	5·94	0·72	5·89	0·72	5·88	0·69
0·6	5·16	0·65	5·19	0·64	5·10	0·66	5·12	0·66	5·17	0·65	5·13	0·67	5·13	0·61	5·13	0·69	5·18	0·69	5·13	0·65
0·7	4·37	0·64	4·38	0·62	4·41	0·64	4·41	0·60	4·38	0·64	4·37	0·61	4·37	0·61	4·39	0·61	4·37	0·64	4·38	0·61
0·8	3·60	0·65	3·57	0·63	3·58	0·66	3·58	0·62	3·54	0·62	3·53	0·61	3·55	0·59	3·61	0·65	3·54	0·59	3·58	0·59
0·9	2·70	0·65	2·62	0·64	2·65	0·61	2·64	0·62	2·64	0·65	2·65	0·62	2·65	0·63	2·64	0·60	2·61	0·65	2·65	0·63
0·95	2·32	0·62	2·15	0·67	2·16	0·72	2·13	0·66	2·13	0·66	2·11	0·68	2·12	0·66	2·10	0·68	2·12	0·64	2·10	0·67
Part IV: Test in logs: augmented Dickey–Fuller statistic (ADF(4))																				
0·01	4·44	0·68	4·43	0·69	4·45	0·67	4·46	0·68	4·45	0·68	4·45	0·69	4·38	0·68	4·46	0·69	4·45	0·71	4·45	0·68
0·1	4·32	0·67	4·36	0·67	4·36	0·68	4·37	0·68	4·35	0·65	4·39	0·68	4·39	0·69	4·33	0·71	4·37	0·68	4·35	0·70
0·2	4·27	0·67	4·22	0·67	4·29	0·68	4·25	0·67	4·25	0·69	4·29	0·66	4·23	0·68	4·22	0·68	4·27	0·66	4·28	0·69
0·3	4·16	0·82	4·16	0·68	4·14	0·68	4·12	0·69	4·17	0·69	4·18	0·70	4·16	0·66	4·13	0·68	4·15	0·66	4·09	0·67
0·4	4·00	0·65	4·02	0·68	3·98	0·68	3·99	0·67	3·98	0·65	4·02	0·69	4·04	0·66	4·03	0·67	3·96	0·64	4·00	0·68
0·5	3·85	0·64	3·80	0·65	3·84	0·655	3·82	0·67	3·83	0·66	3·82	0·65	0·38	0·67	3·84	0·65	3·80	0·65	3·80	0·63
0·6	3·60	0·65	3·62	0·64	3·58	0·66	3·58	0·65	3·57	0·62	3·60	0·67	3·59	0·65	3·59	0·66	3·62	0·63	3·60	0·65
0·7	3·34	0·63	3·35	0·66	3·34	0·65	3·35	0·66	3·31	0·65	3·34	0·66	3·35	0·66	3·32	6·33	3·20	6·53	3·30	6·60
0·8	2·94	0·65	2·94	0·65	2·94	0·64	2·96	0·63	2·93	0·64	2·92	0·62	2·95	0·66	3·00	0·65	2·93	0·63	2·92	0·63
0·9	2·42	0·67	3·39	0·64	2·40	6·04	2·38	0·65	2·37	0·62	2·38	0·64	2·41	0·67	2·40	0·67	2·39	0·70	2·40	0·68
0·95	2·08	0·72	2·02	0·71	2·03	0·75	1·98	0·66	2·01	0·70	2·01	0·72	2·01	0·68	1·97	0·69	2·01	0·68	1·98	0·65

See Table I for notes.

respect to sample size we repeated our simulations for a relatively small sample of 50 observations (Table III) and a larger sample of 200 observations (Table IV). Table III clearly shows a significant drop in the percentage of rejections of the null when the sample size goes from 100 to 50 observations. Variation in expected returns with very little persistence still leads to a failure of rejecting the null at the 5 per cent critical level in around 45 per cent of the simulations for the ADF test in levels and in 40 per cent of the simulations for the ADF test in logs. The DF test rejects the null in a much larger proportion of the simulations: even when $\rho = 0{\cdot}5$, the proportion of rejections of the null at the 5 per cent critical level is 90 per cent or higher. For a sample size of 50 the ADF test in levels tends to reject the null more frequently than the test in logs at the 1 per cent critical level and also at the 5 per cent and 10 per cent critical levels when ρ is high.

Increasing the sample size from 100 to 200 observations, the proportion of rejections of the null of no cointegration rises significantly. Thus when ρ is as high as 0·9 the proportion of simulations where the ADF test rejects the null at the 5 per cent critical level is higher than 60 per cent for the test in logs (without a trend) and the test in levels. For this sample size the proportion of rejections of the null tends to be higher for the cointegration test in logs than for

Table III. Percentage of rejections of the null of no-cointegration in the present-value model with a time-varying expected rate of return. (Sample size 50)

		Cointegration test in levels						Cointegration test in logs (no trend)					
		EG–ADF(4) test			EG–DF test			ADF(4) test			DF test		
	ρ	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
$\sigma_d \mid \sigma_r = 10$	0·01	14·4	38·0	55·9	99·5	99·9	99·9	25·4	60·0	76·1	100·0	100·0	100·0
	0·1	12·2	37·6	53·3	98·9	99·9	100·0	21·6	55·7	73·6	100·0	100·0	100·0
	0·2	12·5	35·5	50·7	97·7	99·5	99·9	22·5	55·0	73·6	99·7	100·0	100·0
	0·3	9·5	31·4	47·4	93·3	98·9	99·7	17·8	49·9	66·8	99·4	100·0	100·0
	0·4	8·5	25·1	43·1	82·0	96·2	98·4	13·9	44·1	62·9	96·1	99·8	100·0
	0·5	8·0	23·9	39·4	62·7	89·4	95·0	11·3	37·0	54·8	83·3	98·1	99·5
	0·6	4·5	17·9	31·1	44·3	74·8	87·5	8·7	30·3	47·9	59·3	91·7	97·6
	0·7	4·8	17·1	27·3	23·6	52·3	69·4	6·7	23·9	40·6	30·5	69·0	85·1
	0·8	2·8	9·4	16·9	9·9	26·7	42·6	2·9	13·4	26·1	10·1	33·4	51·9
	0·9	1·1	5·5	12·2	3·3	13·2	22·3	1·7	8·7	15·6	3·4	12·4	22·7
	0·95	0·9	3·6	7·4	2·4	7·8	13·2	1·2	4·9	10·8	1·5	6·5	15·0
$\sigma_d \mid \sigma_r = 100$	0·01	13·8	37·6	55·5	100·0	100·0	100·0	24·1	58·0	77·2	100·0	100·0	100·0
	0·1	13·8	36·9	53·5	99·5	99·8	99·9	23·4	57·9	75·0	100·0	100·0	100·0
	0·2	12·7	33·6	50·4	98·3	99·6	99·7	22·1	54·1	73·5	100·0	100·0	100·0
	0·3	11·8	32·8	48·1	93·0	99·0	99·6	19·4	49·2	66·3	99·0	99·8	100·0
	0·4	10·2	29·1	43·3	83·7	97·1	98·6	16·2	43·8	61·6	96·5	99·8	100·0
	0·5	7·9	24·4	37·5	63·8	90·3	96·2	11·6	39·0	57·3	84·7	98·9	99·6
	0·6	5·8	19·7	32·4	41·9	75·5	85·4	8·8	30·1	44·8	57·6	89·9	96·9
	0·7	4·5	14·6	25·7	21·0	50·1	68·3	5·5	21·6	38·9	29·9	68·0	82·1
	0·8	2·5	9·9	18·3	7·5	23·9	39·3	3·4	13·4	27·3	8·9	30·9	49·9
	0·9	2·3	6·6	11·5	2·8	12·5	21·7	2·3	8·0	15·6	2·9	12·5	23·2
	0·95	1·4	4·8	9·3	2·8	8·8	16·2	1·0	6·3	11·8	1·3	7·4	15·1

See Table I for Notes.

Table IV. Percentage of rejections of the null of no-cointegration in the present-value model with a time-varying expected rate of return. (Sample size 200)

		Cointegration test in levels						Cointegration test in logs (no trend)					
		EG–ADF(4) test			EG–DF test			ADF(4) test			DF test		
	ρ	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
$\sigma_d/\sigma_r = 10$	0·01	99·2	99·5	99·9	100·0	100·0	100·0	100·0	100·0	100·0	100·0	100·0	100·0
	0·1	98·9	99·7	99·9	100·0	100·0	100·0	100·0	100·0	100·0	100·0	100·0	100·0
	0·2	98·0	99·4	99·7	100·0	100·0	100·0	100·0	100·0	100·0	100·0	100·0	100·0
	0·3	98·0	99·2	99·5	100·0	100·0	100·0	100·0	100·0	100·0	100·0	100·0	100·0
	0·4	96·1	98·6	98·8	100·0	100·0	100·0	100·0	100·0	100·0	100·0	100·0	100·0
	0·5	95·3	98·8	99·1	100·0	100·0	100·0	100·0	100·0	100·0	100·0	100·0	100·0
	0·6	89·3	96·4	97·7	99·5	99·6	99·8	99·7	99·9	100·0	100·0	100·0	100·0
	0·7	84·8	93·9	96·2	98·3	99·0	99·2	97·5	99·7	100·0	100·0	100·0	100·0
	0·8	65·8	84·0	90·4	90·3	96·3	97·8	83·5	98·2	99·8	99·7	100·0	100·0
	0·9	34·0	56·1	69·4	50·1	73·3	83·0	31·8	68·4	85·1	51·7	88·5	96·3
	0·95	15·6	32·0	44·5	21·6	41·5	54·9	9·3	29·7	51·1	15·7	52·2	74·5
$\sigma_d/\sigma_r = 100$	0·01	98·6	99·7	99·9	100·0	100·0	100·0	100·0	100·0	100·0	100·0	100·0	100·0
	0·1	98·4	99·7	99·9	100·0	100·0	100·0	100·0	100·0	100·0	100·0	100·0	100·0
	0·2	98·0	98·9	99·5	100·0	100·0	100·0	100·0	100·0	100·0	100·0	100·0	100·0
	0·3	97·0	99·0	99·7	100·0	100·0	100·0	100·0	100·0	100·0	100·0	100·0	100·0
	0·4	95·9	98·7	99·3	100·0	100·0	100·0	100·0	100·0	100·0	100·0	100·0	100·0
	0·5	94·3	98·6	99·0	100·0	100·0	100·0	100·0	100·0	100·0	100·0	100·0	100·0
	0·6	90·2	97·1	98·5	99·6	100·0	100·0	99·8	10·0	100·0	100·0	100·0	100·0
	0·7	85·0	94·8	96·9	98·3	99·4	99·6	98·1	99·9	100·0	100·0	100·0	100·0
	0·8	66·9	86·3	92·0	91·7	96·8	97·8	82·1	98·2	99·7	100·0	100·0	100·0
	0·9	32·5	57·1	68·7	49·3	73·0	83·1	28·9	64·6	82·8	47·0	84·6	95·8
	0·95	15·3	30·0	43·1	21·6	41·5	55·0	7·2	25·8	43·9	8·2	32·4	52·5

See Table I for Notes

the cointegration test in levels—even when excess returns are highly persistent. The only exception to this is at the 1 per cent critical level when $\rho = 0{\cdot}9$ or $0{\cdot}95$ and the cointegration test in levels rejects the null more frequently than the test in logs does.

What explains the 'power'[10] of the cointegration test in levels and the test in logs? From equation (3) the cointegration test in logs takes the form of a unit root test for a_t while the cointegration test in levels is a unit root test for $u_t = (a_t - E(a_t))D_t$. From previous simulation studies (e.g. Dickey and Fuller, 1981) we know that the power to reject the null of a unit root in a series is closely linked to the persistence of the series: the more persistent the series, the lower is the power of the unit root test. Thus when ρ is small, a_t is not very persistent and the null of a unit root in a_t is easily rejected. When ρ is high the cointegration test in logs often fails to reject the null since a_t is also highly persistent. In this case the cointegration test in levels may work better because u_t may be less persistent than a_t. To see if this intuition is valid we present the estimated first-order autocorrelation coefficients for a_t and u_t in Table V. It is clear that the decline in the proportion of rejections of the null as ρ goes up is explained by the increase in the serial correlation of both a_t and u_t. Also, the first-order order autocorrelation estimates tend to

[10] The percentage of rejections of the null of no cointegration should not really be interpreted as the power of the tests since the null of no cointegration is true for the model based on levels of the variables.

Table V. Persistence estimates of simulated stock prices ($\sigma_d/\sigma_r = 100$)

	Sample size = 50		Sample size = 100		Sample size = 200	
	Estimate of first-order autocorrelation		Estimate of first-order autocorrelation		Estimate of first-order autocorrelation	
ρ	Levels	Logs	Levels	Logs	Levels	Logs
0·1	0·036	0·071	0·060	0·089	0·074	0·097
0·2	0·137	0·176	0·151	0·182	0·159	0·191
0·3	0·222	0·264	0·248	0·278	0·257	0·291
0·4	0·305	0·350	0·345	0·383	0·346	0·387
0·5	0·384	0·442	0·430	0·477	0·450	0·487
0·6	0·483	0·540	0·526	0·574	0·537	0·586
0·7	0·558	0·623	0·618	0·668	0·635	0·684
0·8	0·645	0·721	0·705	0·761	0·727	0·782
0·9	0·729	0·804	0·795	0·854	0·824	0·878
0·95	0·767	0·848	0·885	0·949	0·873	0·926

Note: The Estimate of the first order autocorrelation is based on the average autocorrelation coefficient over the 1000 simulations. The cointegration test in logs included a constant but not a time trend.

be around 0·02 to 0·08 higher for the residuals from the cointegration test in logs compared to the residuals from the test in levels. This helps to explain the overall higher rate of rejections of the null of the test in logs.

3.1. Persistence and Variability of Some Expected Returns Processes for the US Stock Market

In the light of the simulation results the question is whether reasonable models for expected returns display sufficient persistence to explain the finding that stock prices and dividends may not be cointegrated. To find out, we investigated four models of expected returns which have been suggested in various studies. The first, and simplest, model assumes that the expected rate of return equals the risk-free rate of return (r_t^f) plus a constant risk premium (e.g. Merton, 1980). Although this specification attracted some early interest, it is too simple. It is not clear, for instance, that a risk-free real interest rate exists. In our study we used data on a nominal 6-month (annualized) commercial paper rate obtained from Shiller (1989, Table 26.1, Series 4) and deducted the annual rate of inflation to obtain a real interest rate. The second model is based on work by Breeden (1979) and assumes that the following Euler condition holds:

$$P_t = E_t\left\{\frac{U'(C_{t+1})}{U'(C_t)}(P_{t+1} + D_{t+1})\right\} \tag{12}$$

where $U(.)$ is the utility function of a representative agent and C_t is the agent's real consumption in period t. In our computations we will assume that the agent has a constant relative risk-aversion utility function of the type $U(C_t) = C_t^{1-A}/(1-A)$, where $0 < A < \infty$, such that

$$\frac{1}{1+r_t} = \left(\frac{C_t}{C_{t+1}}\right)^A$$

and

$$r_t = \left(\frac{C_{t+1}}{C_t}\right)^A - 1$$

This specification was adopted by Grossman and Shiller (1981) and is standard in the financial econometrics literature. We used the real consumption data in Shiller (1989, Table 26.2, Series 9) to obtain estimates of the persistence parameter of this process.

The third model considered in this paper builds on the empirical finding (Campbell, 1987; Fama and French, 1989) that a term premium can be used to predict stock returns. Following Fama and French, we shall assume that the term premium identifies variation in the equity premium and that r_t is a linear function of the term premium. We used the difference between a 3-month and a 1-month T-bill rate as a measure of the term premium. These series were acquired from the risk-free interest rate file of the Centre for Research in Security Prices (CRSP). Finally, a number of studies (Shiller, 1984; Fama and French 1989) have concluded that the dividend yield is capable of predicting stock returns. Rozeff (1984) argues that the yield acts as a proxy for the risk premium and hence that the predictive power of the yield over stock returns does not represent an inefficient market. Using a slightly different linearization from the one adopted in this paper, Campbell and Ammer (1993, equation 4) show that the log dividend yield has forecasting power over expected stock returns. In our model the relationship between the dividend yield and the expected return, r_t, is perhaps best seen from model (8), which implies that

$$r_t - \bar{r} = \frac{1}{\rho(1+g)}\left((1+g)(1+\bar{r})(1+\bar{r}-\rho(1+g)) - \frac{(1+\bar{r})(\bar{r}-g)(1+\bar{r}-\rho(1+g))}{D_t/P_t}\right)$$

from which expected returns can be generated. Naturally, this implies that the innovation in dividends and expected returns are not uncorrelated, but see footnote 9 on this point.

Table VI provides results from estimating simple AR(1) models for the four measures of expected returns. None of the models generated a statistically significant coefficient for a second autoregressive lag. The least persistent process appears to be the consumption-based model where there is even evidence of negative (though statistically insignificant) serial correlation for the period 1890–1985. This result is reversed for the post-war period where the serial correlation is positive and statistically significant. It is fair to say, however, that when the risk-aversion coefficient, A, takes a reasonable value (e.g. 1–4), then this model does not generate sufficient persistence to explain the failure to reject the null of no cointegration between stock prices and dividends.

The model based on the 'risk-free' real interest rate shows clearer evidence of persistence and gives an estimate of ρ around 0·4. Still, this is probably not enough persistence to explain the failure of the ADF test of rejecting the null of no cointegration between stock prices and dividends at the 10 per cent critical level. Table I shows that the probability of not rejecting the null of no cointegration at the 10 per cent critical level when $\rho = 0{\cdot}4$ is around 5 per cent. The same is true for the model based on the term premium where the estimate of the autoregressive

Table VI. Persistence estimates for expected returns models

Returns model	Persistence	Estimated period	Standard error
Consumption growth	−0·152	1890–1985	0·0342
$(C_t/C_{t-1})-1$	(0·102)		
	0·339	1948–85	0·0116
	(0·146)		
Consumption growth	−0·183	1890–1985	0·142
$(C_t/C_{t-1})^4-1$	(0·101)		
	0·341	1948–85	0·0489
	(0·147)		
Dividend yield	0·637	1873–1986	0·0114
(D_t/P_t)	(0·073)		
	0·771	1948–86	0·0081
	(0·099)		
Risk-free interest rate	0·382	1873–1986	0·0813
(r^f)	(0·088)		
	0·446	1948–86	0·0386
	(0·107)		
Term premium	0·490	1927–1986	0·248
	(0·111)		
	0·441	1948–86	0·266
	(0·146)		

Notes: Persistence is measured as the estimate ρ in a first-order autoregressive model $x_t=\rho x_{t-1}+\varepsilon_t$. Numbers in parentheses under the estimate of ρ give OLS standard errors, and the standard error of the regression is provided in the fourth column. Description of the data sources is provided in the text.

parameter is around 0·4–0·5. Dividend yields are, however, highly persistent. For this model of expected returns the estimates of ρ are 0·64 (1873–1986) and 0·77 (1948–86).[11] In the light of our simulation results these estimates appear to be sufficiently high to explain the failure of rejecting the null of no cointegration between stock prices and dividends. For $\rho=0{\cdot}6$ the ADF(4) cointegration test in levels fails to reject the null at the 10 per cent critical level in 10–15 per cent of all simulations, and the corresponding proportion when $\rho=0{\cdot}8$ is 34 per cent. When the ADF cointegration test in logs is applied at the 10 per cent critical level, the null of no cointegration is rejected in 5 per cent of the simulations for $\rho=0{\cdot}6$ and in 30 per cent of the simulations for $\rho=0{\cdot}8$. For these values of ρ the probability of rejecting the null of no cointegration in levels by applying the DF statistic at the 1 per cent critical level is around 40 per cent.

These results are interesting since the dividend yield is the variable which most frequently has been found to possess predictive power over stock returns. It is often thought that the variability of the yield is not sufficiently large to explain many of the existing puzzles in financial economics (excess volatility of stock prices, predictability of stock returns). However, our simulations indicate that as far as explaining the failure of rejecting the null of no cointegration between stock prices and dividends goes, it is not the variability of the yield but rather its

[11] Although we report sample statistics for the yield rather than for the logarithm of the yield (used in the cointegration test in logs), the evidence of first-order autocorrelation in the two series was very similar. The evidence of first-order autocorrelation in the price dividend ratio was equally strong.

persistence that matters.[12] This finding is also consistent with the recent study by Campbell and Ammer (1993):

> The key point is that changes in expected excess returns are highly persistent so that modest movements in short run expected returns are capitalized into large changes in stock prices. *The persistence of expected returns arises largely from the persistence of the dividend–price ratio, one of the main forecasting variables for excess stock returns* (my emphasis).

Our results suggest that the failure of rejecting the null of no cointegration between stock prices and dividends can be explained by a model where expected returns depend on the dividend yield.

4. CONCLUSION

A number of conclusions of importance to applied econometric work arise from this paper. The first conclusion is good news for applied econometricians: Cointegration tests of present-value models (whether in logs or in levels) seem to be fairly robust even in the presence of volatile expected returns provided that the expected returns process is not strongly persistent. Second, when expected returns are strongly persistent, the null of no cointegration is unlikely to be rejected by ADF tests for the small sample sizes often used in economics (i.e. 50–100 observations) even when the present-value model is valid. The crucial point is that this holds even when the variance in expected returns is small as compared to the variance of the forcing variable in the present-value model. Third, as the sample size increases to 200 or more observations the probability of rejecting the null of no cointegration increases rapidly and is very high at the 5 per cent critical level both for the DF test and for the ADF test. Thus one should be very cautious in applying cointegration tests to present-value models if one suspects that expected returns may be persistent and the sample size is relatively small.

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[12] In fact, the assumption about the value of σ_r in our simulations (for $\sigma_r = 0{\cdot}1\sigma_d$ we have $s_{r1} = 0{\cdot}013$) closely corresponds to the value of the standard error obtained from the yield equation in Table VI (0·011 for the period 1873—1986).

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