Number	Functional form	Range of γ
1	$p = \mathrm{poly}(t, \gamma) + (1/k)$	$\gamma \in \mathbb{Z}\,[3, 10]$
2	$p = \mathrm{poly}(t, \gamma) + (1/k) + \mathrm{poly}(t, \gamma) * 1/k$	$\gamma \in \mathbb{Z}\,[3, 10]$
3	$p = \mathrm{poly}(t, \gamma) + \log(k) + \mathrm{poly}(k, \gamma) * \log(k)$	$\gamma \in \mathbb{Z}\,[3, 10]$
4	$p = \mathrm{poly}(t, \gamma) + k + (1/k)$	$\gamma \in \mathbb{Z}\,[3, 10]$
5	$p = \mathrm{poly}(\log(t), \gamma) + \log(k)$	$\gamma \in \mathbb{Z}\,[3, 10]$
6	$p = \mathrm{poly}(\log(t), \gamma) * \log(k)$	$\gamma \in \mathbb{Z}\,[3, 10]$
7	$p = \mathrm{poly}(\log(t), \gamma) + k$	$\gamma \in \mathbb{Z}\,[3, 10]$
8	$p = \mathrm{poly}(\log(t), \gamma) * k$	$\gamma \in \mathbb{Z}\,[3, 10]$
9	$p = \mathrm{poly}(\log(t), \gamma) * k + 1/k$	$\gamma \in \mathbb{Z}\,[3, 10]$
10	$p = \mathrm{poly}(\log(t), \gamma) * \log(k)$	$\gamma \in \mathbb{Z}\,[3, 10]$
11	$p = \mathrm{poly}(\log(t), \gamma) * \log(k) + 1/k$	$\gamma \in \mathbb{Z}\,[3, 10]$
12	$p = \mathrm{poly}(\mathrm{bc}(t), \gamma) * \log(k) + \mathrm{poly}(\mathrm{bc}(t), \gamma) * 1/k$	$\gamma \in \mathbb{Z}\,[3, 10]$
13	$p = \mathrm{poly}(\mathrm{bc}(t), \gamma) * \log(k) + \mathrm{poly}(\mathrm{bc}(t), \gamma) * \sqrt{k}$	$\gamma \in \mathbb{Z}\,[3, 10]$
14	$p = \mathrm{poly}(\mathrm{bc}(t), \gamma) * \log(k) + \mathrm{poly}(\mathrm{bc}(t), \gamma) * 1/k + \sqrt{k}$	$\gamma \in \mathbb{Z}\,[3, 10]$
15	$\mathrm{bc}(p) = \mathrm{poly}(\mathrm{bc}(t), \gamma) + \log(k)$	$\gamma \in \mathbb{Z}\,[3, 10]$
16	$\mathrm{bc}(p) = \mathrm{poly}(\mathrm{bc}(t), \gamma) * \log(k)$	$\gamma \in \mathbb{Z}\,[3, 10]$
17	$\mathrm{bc}(p) = \mathrm{poly}(\mathrm{bc}(t), \gamma) * \log(k) + 1/k$	$\gamma \in \mathbb{Z}\,[3, 10]$
18	$\mathrm{bc}(p) = \mathrm{poly}(\mathrm{bc}(t), \gamma) * \log(k) + \mathrm{poly}(\mathrm{bc}(t), \gamma) * 1/k$	$\gamma \in \mathbb{Z}\,[3, 10]$
19	$\mathrm{bc}(p) = \mathrm{poly}(\mathrm{bc}(t), \gamma) * \log(k) + \mathrm{poly}(\mathrm{bc}(t), \gamma) * \sqrt{k}$	$\gamma \in \mathbb{Z}\,[3, 10]$
20	$\log(p) = \mathrm{poly}(\mathrm{bc}(t), \gamma) * \log(k)$	$\gamma \in \mathbb{Z}\,[3, 10]$
21	$\log(p) = \mathrm{poly}(\mathrm{bc}(t), \gamma) * \log(k) + 1/k$	$\gamma \in \mathbb{Z}\,[3, 10]$
22	$\log(p) = \mathrm{poly}(\mathrm{bc}(t), \gamma) * \log(k) + \mathrm{poly}(\mathrm{bc}(t), \gamma) * \sqrt{k}$	$\gamma \in \mathbb{Z}\,[3, 10]$
23	$\log(p) = \mathrm{poly}(\mathrm{bc}(t), \gamma) * \log(k) + \mathrm{poly}(\mathrm{bc}(t), \gamma) * 1/k$	$\gamma \in \mathbb{Z}\,[3, 10]$
24	$\log(p) = \mathrm{poly}(\mathrm{bc}(t), \gamma) * \log(k) + \mathrm{poly}(\mathrm{bc}(t), \gamma) * 1/k + \sqrt{k}$	$\gamma \in \mathbb{Z}\,[3, 10]$

Table 1: Description of all tested models....

Functional form	Full distribution		Lower tail ($p < 0.2$)	
	RMSE	cRMSE	RMSE	cRMSE
$\mathrm{bc}(p) = \mathrm{poly}(\mathrm{bc}(t), 10) * \log(k) + \mathrm{poly}(\mathrm{bc}(t), 10) * \sqrt{k}$	$4.97 \cdot 10^{-4}$	$4.69 \cdot 10^{-4}$	$8.05 \cdot 10^{-4}$	$7.16 \cdot 10^{-4}$
$\mathrm{bc}(p) = \mathrm{poly}(\mathrm{bc}(t), 10) * \log(k) + \mathrm{poly}(\mathrm{bc}(t), 10) * 1/k$	$5.39 \cdot 10^{-4}$	$5.11 \cdot 10^{-4}$	$8.54 \cdot 10^{-4}$	$7.61 \cdot 10^{-4}$
$p = \mathrm{poly}(\mathrm{bc}(t), 10) * \log(k) + \mathrm{poly}(\mathrm{bc}(t), 10) * \sqrt{k}$	$7.68 \cdot 10^{-4}$	$6.91 \cdot 10^{-4}$	$1.01 \cdot 10^{-3}$	$8.97 \cdot 10^{-4}$
$p = \mathrm{poly}(\mathrm{bc}(t), 10) * \log(k) + \mathrm{poly}(\mathrm{bc}(t), 10) * 1/k + \sqrt{k}$	$7.79 \cdot 10^{-4}$	$7.04 \cdot 10^{-4}$	$1.05 \cdot 10^{-3}$	$9.31 \cdot 10^{-4}$
$p = \mathrm{poly}(\mathrm{bc}(t), 10) * \log(k) + \mathrm{poly}(\mathrm{bc}(t), 10) * 1/k$	$7.82 \cdot 10^{-4}$	$7.07 \cdot 10^{-4}$	$1.06 \cdot 10^{-3}$	$9.41 \cdot 10^{-4}$

Table 2: The 5 best models for the combined Non-Cointegration test of Bayer and Hanck, where *all* underlying test are included and case 1.

Functional form	Full distribution		Lower tail ($p < 0.2$)	
	RMSE	cRMSE	RMSE	cRMSE
$\log(p) = \mathrm{poly}(\mathrm{bc}(t), 10) * \log(k)$	$1.27 \cdot 10^{-3}$	$1.25 \cdot 10^{-3}$	$1.05 \cdot 10^{-3}$	$9.52 \cdot 10^{-4}$
$\log(p) = \mathrm{poly}(\mathrm{bc}(t), 10) * \log(k) + \mathrm{poly}(\mathrm{bc}(t), 10) * \sqrt{k}$	$6.82 \cdot 10^{-4}$	$6.22 \cdot 10^{-4}$	$1.28 \cdot 10^{-3}$	$1.12 \cdot 10^{-3}$
$\log(p) = \mathrm{poly}(\mathrm{bc}(t), 10) * \log(k) + \mathrm{poly}(\mathrm{bc}(t), 10) * 1/k$	$7.32 \cdot 10^{-4}$	$6.63 \cdot 10^{-4}$	$1.39 \cdot 10^{-3}$	$1.20 \cdot 10^{-3}$
$\log(p) = \mathrm{poly}(\mathrm{bc}(t), 10) * \log(k) + \mathrm{poly}(\mathrm{bc}(t), 10) * 1/k + \sqrt{k}$	$8.38 \cdot 10^{-4}$	$7.78 \cdot 10^{-4}$	$1.48 \cdot 10^{-3}$	$1.31 \cdot 10^{-3}$
$\mathrm{bc}(p) = \mathrm{poly}(\mathrm{bc}(t), 10) * \log(k) + \mathrm{poly}(\mathrm{bc}(t), 10) * 1/k$	$9.08 \cdot 10^{-4}$	$8.42 \cdot 10^{-4}$	$1.69 \cdot 10^{-3}$	$1.50 \cdot 10^{-3}$

Table 3: The 5 best models for the combined Non-Cointegration test of Bayer and Hanck, where *all* underlying test are included and case 2.

Functional form	Full distribution		Lower tail ($p < 0.2$)	
	RMSE	cRMSE	RMSE	cRMSE
$\log(p) = \mathrm{poly}(\mathrm{bc}(t), 10) * \log(k) + \mathrm{poly}(\mathrm{bc}(t), 10) * \sqrt{k}$	$4.58 \cdot 10^{-4}$	$4.55 \cdot 10^{-4}$	$3.37 \cdot 10^{-4}$	$3.16 \cdot 10^{-4}$
$\log(p) = \mathrm{poly}(\mathrm{bc}(t), 10) * \log(k) + \mathrm{poly}(\mathrm{bc}(t), 10) * 1/k$	$5.17 \cdot 10^{-4}$	$5.14 \cdot 10^{-4}$	$3.90 \cdot 10^{-4}$	$3.73 \cdot 10^{-4}$
$\log(p) = \mathrm{poly}(\mathrm{bc}(t), 10) * \log(k)$	$1.04 \cdot 10^{-3}$	$1.04 \cdot 10^{-3}$	$6.76 \cdot 10^{-4}$	$6.50 \cdot 10^{-4}$
$\log(p) = \mathrm{poly}(\mathrm{bc}(t), 10) * \log(k) + \mathrm{poly}(\mathrm{bc}(t), 10) * 1/k + \sqrt{k}$	$1.18 \cdot 10^{-3}$	$1.17 \cdot 10^{-3}$	$2.06 \cdot 10^{-3}$	$2.05 \cdot 10^{-3}$
$\mathrm{bc}(p) = \mathrm{poly}(\mathrm{bc}(t), 10) * \log(k) + \mathrm{poly}(\mathrm{bc}(t), 10) * 1/k$	$1.16 \cdot 10^{-3}$	$1.06 \cdot 10^{-3}$	$2.08 \cdot 10^{-3}$	$1.80 \cdot 10^{-3}$

Table 4: The 5 best models for the combined Non-Cointegration test of Bayer and Hanck, where *all* underlying test are included and case 3.

Functional form	Full distribution		Lower tail ($p < 0.2$)	
	RMSE	cRMSE	RMSE	cRMSE
$\log(p) = \mathrm{poly}(\mathrm{bc}(t), 10) * \log(k) + \mathrm{poly}(\mathrm{bc}(t), 10) * 1/k$	$4.75 \cdot 10^{-4}$	$4.44 \cdot 10^{-4}$	$7.81 \cdot 10^{-4}$	$6.84 \cdot 10^{-4}$
$\log(p) = \mathrm{poly}(\mathrm{bc}(t), 10) * \log(k)$	$6.54 \cdot 10^{-4}$	$5.87 \cdot 10^{-4}$	$1.01 \cdot 10^{-3}$	$7.81 \cdot 10^{-4}$
$\log(p) = \mathrm{poly}(\mathrm{bc}(t), 10) * \log(k) + \mathrm{poly}(\mathrm{bc}(t), 10) * \sqrt{k}$	$7.60 \cdot 10^{-4}$	$6.13 \cdot 10^{-4}$	$1.46 \cdot 10^{-3}$	$1.06 \cdot 10^{-3}$
$\log(p) = \mathrm{poly}(\mathrm{bc}(t), 10) * \log(k) + \mathrm{poly}(\mathrm{bc}(t), 10) * 1/k + \sqrt{k}$	$7.64 \cdot 10^{-4}$	$7.45 \cdot 10^{-4}$	$1.29 \cdot 10^{-3}$	$1.23 \cdot 10^{-3}$
$\mathrm{bc}(p) = \mathrm{poly}(\mathrm{bc}(t), 10) * \log(k) + \mathrm{poly}(\mathrm{bc}(t), 10) * 1/k$	$1.01 \cdot 10^{-3}$	$9.17 \cdot 10^{-4}$	$1.89 \cdot 10^{-3}$	$1.65 \cdot 10^{-3}$

Table 5: The 5 best models for the combined Non-Cointegration test of Bayer and Hanck, where *EG-J* underlying test are included and case 1.

Functional form	Full distribution RMSE	cRMSE	Lower tail ($p < 0.2$) RMSE	cRMSE
$\log(p) = \text{poly}(\text{bc}(t), 10) * \log(k)$	$7.36 \cdot 10^{-4}$	$7.25 \cdot 10^{-4}$	$7.04 \cdot 10^{-4}$	$6.45 \cdot 10^{-4}$
$\log(p) = \text{poly}(\text{bc}(t), 10) * \log(k) + \text{poly}(\text{bc}(t), 10) * 1/k$	$5.53 \cdot 10^{-4}$	$5.12 \cdot 10^{-4}$	$9.75 \cdot 10^{-4}$	$8.56 \cdot 10^{-4}$
$\log(p) = \text{poly}(\text{bc}(t), 10) * \log(k) + \text{poly}(\text{bc}(t), 10) * \sqrt{k}$	$5.53 \cdot 10^{-4}$	$5.11 \cdot 10^{-4}$	$9.87 \cdot 10^{-4}$	$8.66 \cdot 10^{-4}$
$\log(p) = \text{poly}(\text{bc}(t), 10) * \log(k) + \text{poly}(\text{bc}(t), 10) * 1/k + \sqrt{k}$	$6.00 \cdot 10^{-4}$	$5.62 \cdot 10^{-4}$	$1.11 \cdot 10^{-3}$	$1.01 \cdot 10^{-3}$
$\text{bc}(p) = \text{poly}(\text{bc}(t), 10) * \log(k) + \text{poly}(\text{bc}(t), 10) * \sqrt{k}$	$1.05 \cdot 10^{-3}$	$9.54 \cdot 10^{-4}$	$2.00 \cdot 10^{-3}$	$1.75 \cdot 10^{-3}$

Table 6: The 5 best models for the combined Non-Cointegration test of Bayer and Hanck, where *EG-J* underlying test are included and case 2.

Functional form	Full distribution RMSE	cRMSE	Lower tail ($p < 0.2$) RMSE	cRMSE
$\log(p) = \text{poly}(\text{bc}(t), 10) * \log(k) + \text{poly}(\text{bc}(t), 10) * \sqrt{k}$	$3.85 \cdot 10^{-4}$	$3.73 \cdot 10^{-4}$	$5.03 \cdot 10^{-4}$	$4.58 \cdot 10^{-4}$
$\log(p) = \text{poly}(\text{bc}(t), 10) * \log(k)$	$7.55 \cdot 10^{-4}$	$7.54 \cdot 10^{-4}$	$4.85 \cdot 10^{-4}$	$4.70 \cdot 10^{-4}$
$\log(p) = \text{poly}(\text{bc}(t), 10) * \log(k) + \text{poly}(\text{bc}(t), 10) * 1/k$	$3.73 \cdot 10^{-4}$	$3.59 \cdot 10^{-4}$	$5.34 \cdot 10^{-4}$	$4.83 \cdot 10^{-4}$
$\log(p) = \text{poly}(\text{bc}(t), 10) * \log(k) + \text{poly}(\text{bc}(t), 10) * 1/k + \sqrt{k}$	$4.87 \cdot 10^{-4}$	$4.76 \cdot 10^{-4}$	$8.52 \cdot 10^{-4}$	$8.19 \cdot 10^{-4}$
$\text{bc}(p) = \text{poly}(\text{bc}(t), 10) * \log(k) + \text{poly}(\text{bc}(t), 10) * \sqrt{k}$	$1.02 \cdot 10^{-3}$	$9.35 \cdot 10^{-4}$	$1.94 \cdot 10^{-3}$	$1.70 \cdot 10^{-3}$

Table 7: The 5 best models for the combined Non-Cointegration test of Bayer and Hanck, where *EG-J* underlying test are included and case 3.