Assessing Forecast Performance in a Cointegrated System

Dennis L. Hoffman; Robert H. Rasche

Journal of Applied Econometrics, Vol. 11, No. 5, Special Issue: Econometric Forecasting. (Sep. - Oct., 1996), pp. 495-517.

Stable URL:
http://links.jstor.org/sici?sici=0883-7252%28199609%2F10%2911%3A5%3C495%3AAFPIAC%3E2.0.CO%3B2-Q

Journal of Applied Econometrics is currently published by John Wiley & Sons.

Your use of the JSTOR archive indicates your acceptance of JSTOR's Terms and Conditions of Use, available at http://www.jstor.org/about/terms.html. JSTOR's Terms and Conditions of Use provides, in part, that unless you have obtained prior permission, you may not download an entire issue of a journal or multiple copies of articles, and you may use content in the JSTOR archive only for your personal, non-commercial use.

Please contact the publisher regarding any further use of this work. Publisher contact information may be obtained at http://www.jstor.org/journals/jwiley.html.

Each copy of any part of a JSTOR transmission must contain the same copyright notice that appears on the screen or printed page of such transmission.

The JSTOR Archive is a trusted digital repository providing for long-term preservation and access to leading academic journals and scholarly literature from around the world. The Archive is supported by libraries, scholarly societies, publishers, and foundations. It is an initiative of JSTOR, a not-for-profit organization with a mission to help the scholarly community take advantage of advances in technology. For more information regarding JSTOR, please contact support@jstor.org.

http://www.jstor.org
Wed Mar 19 06:17:54 2008

JOURNAL OF APPLIED ECONOMETRICS, VOL. 11, 495–517 (1996)

ASSESSING FORECAST PERFORMANCE IN A COINTEGRATED SYSTEM

DENNIS L. HOFFMAN
Department of Economics, Arizona State University, Tempe, Arizona, 85287, USA

ROBERT H. RASCHE
Department of Economics, Michigan State University, East Lansing, Michigan 48824, USA

SUMMARY

This paper examines the forecast performance of a cointegrated system relative to the forecast performance of a comparable VAR that fails to recognize that the system is characterized by cointegration. The cointegrated system we examine is composed of three vectors, a money demand representation, a Fisher equation, and a risk premium captured by an interest rate differential. The forecasts produced by the vector error correction model (VECM) associated with this system are compared with those obtained from a corresponding differenced vector autoregression, (DVAR) as well as a vector autoregression based upon the levels of the data (LVAR). Forecast evaluation is conducted using both the 'full-system' criterion proposed by Clements and Hendry (1993) and by comparing forecast performance for specific variables. Overall our findings suggest that selective forecast performance improvement (especially at long forecast horizons) may be observed by incorporating knowledge of cointegration rank. Our general conclusion is that when the advantage of incorporating cointegration appears, it is generally at longer forecast horizons. This is consistent with the predictions of Engle and Yoo (1987). But we also find, consistent with Clements and Hendry (1995) that relative gain in forecast performance clearly depends upon the chosen data transformation.

1. INTRODUCTION

Cointegration in a vector time series (Engle and Granger, 1987) has a number of implications for work in empirical macroeconomics. One of the purported advantages of recognizing cointegration rank in an integrated vector process is that it will result in improved forecast performance. Engle and Yoo (1987) illustrate that forecasts taken from cointegrated systems are 'tied together' because the cointegrating relations must 'hold exactly in the long-run'. They demonstrate in a series of Monte Carlo experiments that incorporating cointegration into the forecasting model, can reduce mean squared forecast errors by up to 40% at medium to long forecast horizons. In a recent application Clements and Hendry (1995) re-examine this issue, concluding that incorporating knowledge of cointegration rank results in significant MSFE reduction only in models formed from relatively small samples. Moreover, the relative gains(or losses) can depend upon the particular representation of data (e.g. levels, differences, linear combinations, etc.) in which one is interested. Clements and Hendry base their conclusions on both Monte Carlo experiments and a simple bivariate application, using a single cointegrating vector, that employs UK data. In the Monte Carlo work, they measure system forecast performance with the Generalized Forecast Error Second Moment (GFESM) that provides a

CCC 0883–7252/96/050495–23
© 1996 by John Wiley & Sons, Ltd.

Received May 1995
Revised April 1996

transformation invariant measure of the relative performance of competing 'systems based' forecasts. The GFESM is discussed in Clements and Hendry (1993).

Our exercise in forecast evaluation is designed to reveal the importance of accounting for multiple cointegrating vectors in a system comprised of five relevant macro aggregates. We chose a set of cointegrating vectors that have been well documented in previous applied work and cast them in a setting that clearly reveals whether knowledge of cointegration rank can improve dynamic forecasts of the macro time series. Forecasts taken from the vector autoregressive representation (*VECM*) that corresponds to this system are compared with simple VAR models based on differences (*DVAR*) and levels (*LVAR*) of the data respectively. Clearly, *DVAR* is misspecified if cointegration prevails in the system. The levels specification, *LVAR*, contains all the long-run information but may result in inferior forecast performance since the specification fails to explicitly recognize the long-run anchors in the data. Forecast evaluation is conducted using various portions of a 32-period out-sample that spans 1987:1—1994:4. We devote considerable effort to demonstrate that the conclusions we draw from this specification are robust to alternative data definitions used to quantify the system. Unlike the Monte Carlo work conducted by Clements and Hendry our system is *not* characterized by weak exogeneity, and the cointegrated system is considerably more complex. In contrast to Engle and Yoo we conduct the relative comparison on a variable-by-variable basis and make use of the Clements and Hendry system metric. The results we obtain offer an interesting extension of the contrasting implications of Engle and Yoo and Clements and Hendry, reaffirming some of the basic conclusions of the later paper, namely we do not uniformly observe the forecasting advantages predicted by Engle and Yoo. Clearly, forecast performance depends upon choice of data transformation used to assess performance and whether evaluation focuses on system evaluation or is conducted on a variable-by-variable basis. At the same time, we do observe advantages to accounting for knowledge of the long-run relations for certain variables—especially at medium- to longer-term forecast horizons. We speculate on the reasons for observed differences in performance, by variable, in the text.

2. TESTING FOR COINTEGRATION

In recent years tests of cointegration have revealed that various linear combinations of individually integrated processes such as real money balances, real income, inflation, and nominal interest rate series are in fact linked by linear combinations that are stationary. Hoffman and Rasche (1991), Johansen and Juselius (1990), Baba, Hendry, and Starr (1992), and Stock and Watson (1993), among others present evidence on the stationarity of money demand relations. This is sometimes estimated as a velocity relation that links income velocity to movements in a measure of nominal interest rates as in Hoffman, Rasche, and Tieslau (1995). Mishkin (1992), and Crowder and Hoffman (1996) present evidence of a Fisher equation, while Stock and Watson (1992) and Friedman and Kuttner (1993) have examined the relation between risk-free and risky returns on securities of similar maturities; an interest rate differential.

We examined a five-dimensional vector process that allows us to test whether there is evidence that distinct money demand, Fisher, and interest rate differential relations prevail in the data. The variables used in the analysis include a measure of real M1 money balances ($m1p$), a measure of inflation (ifl), commercial paper rates (cpr), real income (gdp) and treasury bill rates (tbr). The primary data used in this paper span 54 : 1 to 94 : 4 and are taken from the Citibase data set. A consistent series for M1 over the full sample is obtained from Rasche (1987). In addition to the data described above we explore the robustness of our conclusions using interest rate series that are adjusted for the 'own rate' on money balances as well as data on GDP and nominal money balances that are not seasonally adjusted. The money series not seasonally adjusted are from Rasche (1987) before 1959 and the Board of Governors since 1958. Not seasonally adjusted nominal GDP data are from the US Department of Commerce National Income and Product Accounts.[1]

Real money balances are obtained by deflating the nominal series by the GDP deflator. Inflation is measured as the percent change (log difference) in the GDP deflator expressed as an annual rate and both real balances and real GDP are expressed as natural logarithms. The primary data used in the empirical analysis are depicted in Figure 1.[2]

Ordering the variables under investigation as $x_t' = [m1p_t\ \ ifl_t\ \ cpr_t\ \ gdp_t\ \ tbr_t]$, a strict interpretation of the cointegration space that reflects all three hypothetical long-run relations is

$$\beta' = \begin{bmatrix} 1 & 0 & 0 & -1 & \beta_i \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 \end{bmatrix} \qquad (1)$$

where the first row of equation (1) reflects the 'money demand' relation with unitary income elasticity and interest elasticity defined by β_i.[3] The second row of equation (1) defines the Fisher relation that links movements in annual inflation rates (as a proxy for expected inflation) to nominal interest rates and the third row defines the interest rate differential that prevails between risky and risk-free short-term interest rate measures, commercial paper rates, and treasury bill rates.

Table I depicts the results of applying standard cointegration tests (Johansen, 1988) to the five-variable system described above. The vector autoregressions used to calculate the tests for cointegration using lags of 4, 5, and 6 were augmented with dummies that allow for possible breaks in deterministic trends at three separate points in the sample. The 1967:4 dummy coincides with the beginning of the Viet Nam inflation spiral while the 1979:4 and 1982:1 dummies are designed to differentiate the Fed's experiment with New Operating Procedures. Results below suggest that only the inflation series is significantly influenced by these 'breaks'.[4]

The trace statistics obtained in our application can be compared with critical values that are simulated with and without allowing for the possibility of breaks in the deterministic trends. When compared with the break adjusted critical values, there is some evidence that the system exhibits cointegration rank three and when compared with conventional critical values the case for cointegration rank three is considerably weaker.

We used an autoregression lag length of $k = 5$, maintained the hypothesis of cointegration rank three, and applied tests described in Johansen and Juselius to examine whether the cointegration space observed in the system spans the theoretical space in equation (1). The

[1] The money data used in this study are posted on the Internet homepage maintained by Robert H. Rasche. The page is located at www.msu.edu/user/rasche

[2] The degree of integration maintained by these series has been widely discussed throughout the literature. We are operating under the assumption that each series either maintains a single unit root or is well approximated by the assumption that it follows an $I(1)$ process.

[3] We adopt the unitary income elasticity assumption that was shown to be consistent with the data in Hoffman *et al.* (1995) and in Hoffman and Rasche (1996). At the same time, we recognize that there are a number of both theoretically and empirically plausible values for the income elasticity. Cursory investigation on our part reveals that the basic conclusions of this paper are not significantly influenced by this assumption.

[4] See Hoffman, Rasche, and Tieslau (1995) and Hoffman and Rasche (1996) for a detailed discussion of these deterministic breaks. The qualitative conclusions of this paper are not altered by eliminating the breaks from the analysis. We discuss the specific implications of including (or excluding) the break dummies at 'known' break points below. Inference may indeed be affected if the break points were assumed unknown.

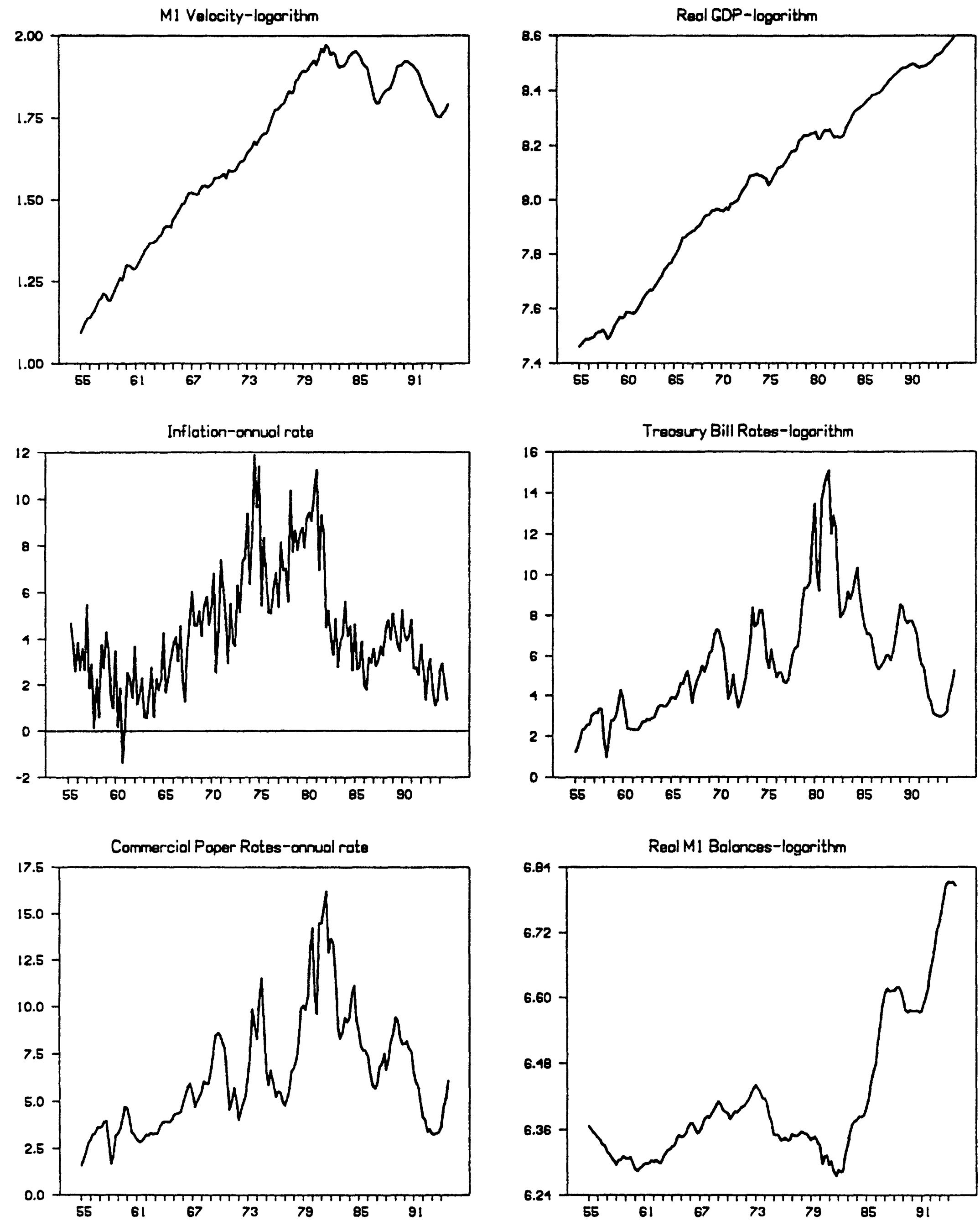

Figure 1. Time-series data, 1954:1–1994:4

Table I. Tests for the number of steady-state relations in the five-variable VAR system (*m1p*, *ifl*, *cpr*, *gdp*, *tbr*). Tests based upon seasonally adjusted data, 56:3–94:4

Likelihood ratio tests (five variables)

H_0	$r=0$	$r\leq 1$	$r\leq 2$	$r\leq 3$	$r\leq 4$
$LR_{trace} k=4$	104·71	54·55	18·68	4·18	0·08
$LR_{trace} k=5$	107·50	61·98	18·09	4·12	0·02
$LR_{trace} k=6$	123·42	69·89	25·15	6·73	0·05

Trace test critical values

95%	Break	67·2	47·5	16·9	9·0	3·8
90%		63·6	43·1	14·7	7·4	2·7
95%	Standard	68·5	47·2	29·7	15·4	3·8
90%		64·8	44·0	26·8	13·3	2·7

Tests of specific cointegration vectors

H_0: The cointegration space spans:	χ^2 *df*	*lik·.ratio*	*p*-value
Money demand, Fisher, interest rate differential	6	9·15	0·166
Unitary Income	3	7·37	0·061
Fisher, Interest rate differential	4	3·59	0·464

With unitary income elasticity restriction in place

$x'_t = [vel_t \;\; ifl_t \;\; cpr \;\; tbr_t]$

Normalized estimate of the steady-state relations

$$\tilde{\beta}' = \begin{bmatrix} 1 & 0 & 0 & -0{\cdot}181 \\ (-) & (-) & (-) & (0.021) \\ 0 & 1 & 0 & -1{\cdot}65 \\ (-) & (-) & (-) & (1{\cdot}17) \\ 0 & 0 & 1 & -1{\cdot}00 \\ (-) & (-) & (-) & (0{\cdot}10) \end{bmatrix} \qquad \tilde{\beta}' = \begin{bmatrix} 1 & 0 & 0 & -0{\cdot}110 \\ (-) & (-) & (-) & (0{\cdot}018) \\ 0 & 1 & 0 & -1{\cdot}00 \\ (-) & (-) & (-) & (-) \\ 0 & 0 & 1 & -1{\cdot}00 \\ (-) & (-) & (-) & (-) \end{bmatrix}$$

Likelihood ratio tests (velocity restricted)

H_0:	$r=0$	$r\leq 1$	$r\leq 2$	$r\leq 3$
$LR_{trace} k=5$	98·46	53·28	14·51	2·45

Wald and likelihood ratio tests under the null of no cointegration wrt certain vectors

H_0: No cointegration on the margins:	HW Test	Value	*No* break 5% CV	'*Break*' 5% CV	LR	Value	5% CV
Money demand, Fisher, interest differential	W(5, 0, 2, 1)	105·34	49·55	54·37	χ^2_{15}	92·62	25·00
Money demand	W(5, 2, 2, 1)	42·01	37·21	39·21	χ^2_5	37·35	11·07
Fisher, interest differential	W(4, 0, 2, 1)	90·62	40·76	43·94	χ^2_{10}	78·73	18·31

Note: Johansen Critical values are based on results of simulations designed to account for the possibility of breaks in the deterministic drifts of the data corresponding to intervention dummies *D*67, *D*79 and *D*82. The simulations were performed on DisCo that was supplied to us by Bent Nielsen and the experiments were designed as prescribed by Johansen and Nielsen (1993). Conventional critical values are taken from Osterwald-Lenum (1992). The standard errors associated with the normalized estimates of the steady-state relations are obtained using the Wald test prescribed by Johansen (1992). Statistical inference that is based upon this procedure does depend upon the chosen matrix normalization. The specific long-run relations are *money demand* $(vel_t - 0{\cdot}11 tbr_t)$, *unitary income*, $(vel_t - \beta_t \times tbr_t)$, *Fisher*$(ifl_t - tbr_t)$, and *interest rate differential* $(cpr_t - tbr_t)$. The Horvarth–Watson test is based on the null of no cointegration and critical values, calculated by simulations performed by HW, allow for the possibility of deterministic trends in the data. They are simulated with and without breaks as discussed in the text. Likelihood ratio tests are based upon comparison of restricted and unrestricted residual moment matrices.

unitary income elasticity restriction imposes the restrictions implied by the fourth column of the equation. This unitary income elasticity restriction may be imposed on the five dimensional system with a restriction of the form

$$H\phi = 0$$

where

$$H = \begin{bmatrix} 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

and ϕ denotes an unconstrained 5×3 cointegration space. Joint tests of all restrictions implied by equation (1) appear in the middle of Table I. The unitary income elasticity imposes single restrictions on each of the three hypothesized cointegrating vectors. The implied restrictions are not rejected at the 5% level though the marginal significance level is 0·061. The Fisher and interest rate differential hypotheses (defined by rows 2 and 3 of equation (1)) impose two restrictions on two of the vectors of the cointegration space—pinning down four of the six free parameters that remain in the normalized cointegration space. There is no evidence that these restrictions are at odds with the data.

Near the bottom of Table I we present estimates of the cointegration space obtained after imposing the unitary income elasticity restriction alone, and then adding the Fisher restriction and the interest rate differential restriction.[5] With just the unitary income elasticity in place, estimates of the interest rate coefficients in the three cointegrating vectors are respectively $(\beta_{1i}, \beta_{2i}, \beta_{3i})$ are reasonably close to the strict representation in equation (1) though the estimates are not particularly precise. When the Fisher and interest differential vectors are imposed we obtain the sharpest inference regarding the single remaining parameter, the interest elasticity in the money demand equation (β_i). Using the estimate obtained in this restricted estimation $(\hat{\beta}_i = 0{\cdot}11)$ we tested all three restrictions implied by equation (1)—thereby pinning down all six free parameters in the normalized cointegration space. Results presented at the middle of Table I provide considerable evidence that the cointegration space estimated over our sample spans the strict representation depicted in equation (1).[6]

Johansen trace tests based on the five-variable system with the unitary income elasticity restriction in place (creating a four-dimensional structure) also appear near the bottom of Table I. These tests provide considerably stronger evidence of cointegration rank three.

The standard tests for cointegration presented in Table I are obtained under the assumption that none of the cointegrating vectors in the system are known a *a priori*. Horvath and Watson (1995) develop Wald tests for cointegration in the presence of either complete or limited knowledge of the cointegration space. This is particularly relevant to our application because HW tests exhibit higher power against relevant alternatives. We present Horvath and Watson tests for cointegration at the bottom of Table I. The tests are formulated so that rejections provide evidence of cointegration. We sequentially test whether all three vectors exhibit evidence of cointegration, whether the money demand relation exhibits cointegration on the margin, and finally whether the Fisher and interest rate differential relations exhibit evidence of

[5] A procedure for estimating a subset of cointegration vectors after imposing restrictions on others in outlined in Johansen and Juselius (1992).

[6] Standard tests for 'exclusion' of any members of the five variable system are uniformly rejected at the 5 per cent level.

cointegration on the margin. We compared the test statistics using critical values simulated by HW under the assumption $\alpha_{ak} = (Fisher, Interest\ Differential)$ so that the Fisher and interest differential relations are assumed known, and that the money demand relation is unknown since the interest elasticity is estimated in the analysis. As a result we compare our cointegration test statistic with the HW critical values associated with a HW (5, 0, 2, 1) simulation (see Horvath and Watson, Table I). We then used the HW (5, 2, 2, 1) critical value to assess the money demand margin and the HW (4, 0, 2, 1) for the margin associated with the two 'known' vectors, (*Fisher, Interest Differential*). In each case the null hypothesis of no marginal cointegration is strongly rejected. The tests remain significant even when compared with critical values simulated under the assumption that deterministic breaks prevail in the data.[7]

In addition to the Horvath–Watson tests, we have included standard likelihood ratio tests obtained by comparing the unrestricted likelihood (formed from the covariance matrix of unrestricted residuals) with the restricted likelihood formed analogously after excluding the indicated error correction terms from the VECM. The reported critical values for the likelihood ratio tests are based upon standard Chi-squared distributions that may not apply in this case.

2.1. How Robust is the Case for Cointegration?

We examined the robustness of our conclusions regarding cointegration rank by conducting several additional experiments. First, cointegration analysis of the five variable system reveals that the system depicted in equation (1) may actually be represented parsimoniously by a four-dimensional vector representation with income velocity replacing real money balances. This system differs from the velocity restricted results depicted in Table 1 because in the four-dimensional model, the unitary income elasticity restriction also applies to the short-run dynamics. The case for cointegration rank three based upon this four-dimensional representation for cointegration is slightly stronger than the five-variable results. Moreover, the remaining testable restrictions implied by the Fisher and Interest Differential specifications remain consistent with the data. A summary of our four-dimensional results is available on request.

Next, we took steps to ensure that the cointegration evidence was not generated by simply including the deterministic dummies in the specification. First, in examining the constancy of the VECM associated with our cointegrated system, we determined that the three deterministic dummies only played a significant role in the determination of the inflation series. In the spirit of Perron (1989) we then adjusted the inflation series alone for these possible breaks and re-examined the evidence for cointegration in the absence of the dummies. All the results portrayed in Table I are maintained in this specification. A summary of these findings is available on request.

As indicated in the preceding section, we then recalculated the Horvath–Watson critical values after allowing for breaks in the underlying DGPs but also controlling for those breaks in the estimation of the model as is our intent in including the dummies in the specification. The HW 'break-adjusted' critical values we obtained were very similar to the original set of values presented in Horvath and Watson's Table I. The statistics we obtain are highly significant regardless of whether they are compared with either the 'no-break' or 'break-adjusted' Horvath and Watson values.

In a final experiment, we simply purged the dummies from the system when conducting the HW tests. The HW test statistic is 46·80. In this case all three vectors were assumed to be 'known' since the evidence for cointegration is assembled in a system (void of dummies) that is

[7] We consider the possible effect of the break dummies on the cointegration test inference in the next section.

not identical to the system in which the vectors are estimated. The HW 5% critical values for this case (HW(5,0,3,0)) is 39·03, suggesting that there remains considerable evidence in favour of cointegration in the absence of the dummies.

We also examined several other data series that may be used to represent the five dimensional system discussed above. Baba, Hendry, and Starr (1992) suggest that the opportunity cost for holding money balances be adjusted for the 'own rate' on M1 balances. We re-examined the case for cointegration in the light of this suggestion by subtracting the own interest rate on M1 constructed by the staff of the Board of Governors from each of the interest rate series in the system. We treated the 'own rate' as a stationary variable so the same cointegration space applies. The case for cointegration is slightly stronger with this 'own rate' adjustment with the HW test statistic for two known and one unknown vectors of the form described in Table I at a highly significant 110·64.

In addition, since the forecasting performance of the systems we examine may depend upon the use of seasonally adjusted data (as discussed by Ericsson, Hendry, and Tran (1994)), we obtained seasonally unadjusted data for the M1 series and the nominal GDP series and re-examined the case for cointegration. In this case the vector autoregressions were augmented with seasonal dummies to account for the non-seasonality in the data. Again, the evidence in favour of cointegration rank three is unaltered by the use of these alternative data definitions. The HW test statistic for two known and one unknown vectors is in this case, 106·71.

We conclude on the basis of evidence from the Horvath and Watson test, the standard likelihood ratio test applied to the VECM, and the Johansen inference from the four-variable system that our system is indeed characterized by cointegration rank three. This result is robust to alternative measures of the time series that represent the fundamental macro aggregates in our study and does not depend upon the inclusion of the deterministic dummies in the specification.

The error correction terms implied by equation (1), using an interest elasticity estimate of, $\hat{\beta}_i = 0{\cdot}11$, are depicted in Figure 2. The vectors allow for the possibility that there are shifts in the constant terms associated with the vectors as the deterministic dummy regimes shift in 1967, 1979, and 1982.

We have chosen to base the primary conclusions of this investigation on the seasonally adjusted measures of GDP and M1 and on the gross interest rate measures. We believe that the case for cointegration among these variables is sufficiently compelling and that forecasts of these variables are of substantial interest. Reference to results obtained using the alternative data definitions, 'own rate' adjusted interest rates and not seasonally adjusted data, are included in the discussion and summarized in some of the forecast evaluation presented below.

2.2. Constancy

Prior to the investigation of forecast performance we conduct a series of tests designed to determine whether the equations that comprise the cointegrated system exhibit constancy over the sample. The constancy analysis is initially directed at the stability of the cointegration and error correction spaces of the *VECM* and then extended to the alternative forecasting specifications, *DVAR* and *LVAR*.

Table II contains a summary of the recursive statistics obtained using the Hansen and Johansen (1993) procedure to estimate the cointegration space recursively.[8] The results in

[8] Hansen and Johansen utilize the full sample estimates of the short-run dynamics at each recursive estimation of the cointegration space rather than reestimating all parameters in the system.

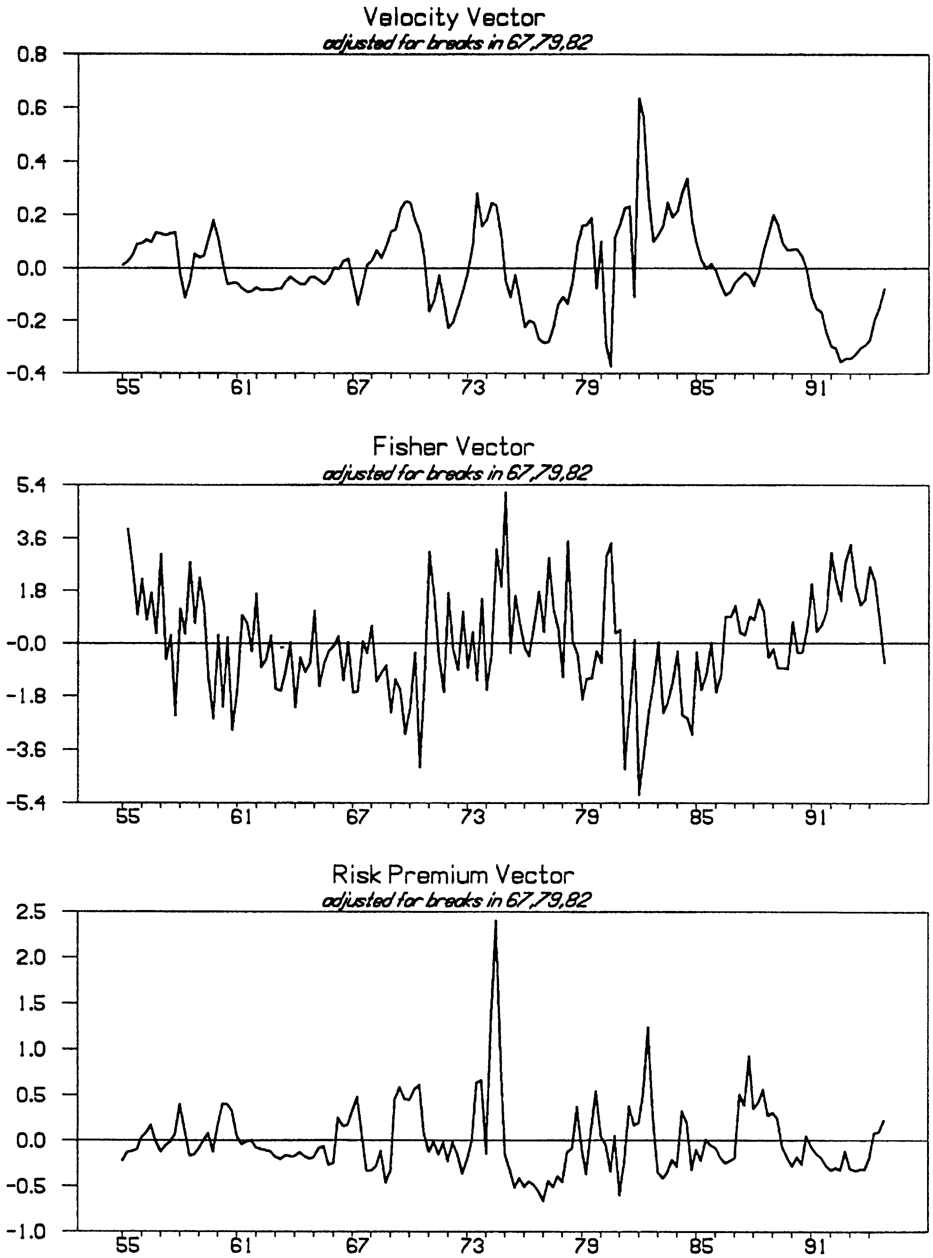

Figure 2. Error correction terms, 1954:1–1994:4

Table II are based upon the five-dimensional system with the velocity restriction imposed. The full sample estimates are contained in Table I. There remains considerable evidence of three cointegrating vectors throughout the recursive estimations. In addition, the cointegration space is relatively stable over the 1980s. For samples that end in the 1970s some instabilities emerge. Prior evidence of instabilities of money demand functions for samples that end in the 1970s is

Table II. Examination of constancy of the cointegrating relations (Hansen–Johansen recursive stability tests)

End date	Trace tests (5% critical values)			Recursive Estimates			
	$r=0$	$r\leqslant 1$	$r\leqslant 2$	$\tilde{\beta}_{1i}$	$\tilde{\beta}_{2i}$	$\tilde{\beta}_{3i}$	$\tilde{\beta}_{i}$
	47·2	29·7	15·4				
93:4	96·6	53·3	13·6	0·18	−1·56	−1·01	0·117
92:4	92·9	52·1	12·1	0·19	−1·64	−1·01	0·118
91:4	96·2	52·8	13·1	0·19	−1·63	−1·00	0·119
90:4	97·4	54·6	14·8	0·17	−1·50	−1·02	0·119
89:4	101·3	54·8	15·8	0·16	−1·42	−1·02	0·116
88:4	97·9	51·8	15·7	0·16	−1·43	−1·02	0·114
87:4	101·3	51·5	14·0	0·15	−1·36	−1·04	0·113
86:4	102·4	50·8	13·3	0·14	−1·29	−1·04	0·110
85:4	104·4	51·0	14·0	0·15	−1·38	−1·04	0·112
84:4	103·2	50·4	12·8	0·15	−1·41	−1·04	0·111
83:4	105·5	51·3	14·0	0·13	−1·23	−1·04	0·110
82:4	112·1	58·1	20·0	0·16	−1·43	−1·04	0·112
81:4	99·0	45·6	21·4	0·17	−1·56	−1·03	0·107
80:4	92·6	39·8	16·8	0·16	−1·45	−1·03	0·104
79:4	89·5	35·0	12·5	0·14	−1·36	−1·04	0·098
78:4	93·9	43·8	18·4	0·28	−2·49	−1·00	0·142
77:4	86·5	40·0	16·2	0·74	−6·20	−0·72	0·179
76:4	95·6	48·1	20·2	0·28	−2·41	−1·02	0·179
75:4	109·2	53·9	23·1	0·23	−2·11	−1·05	0·171
74:4	90·3	53·6	23·5	0·31	−3·05	−1·16	0·187
73:4	101·6	60·3	26·7	0·29	−2·42	−1·11	0·214
72:4	122·2	64·5	29·5	0·30	−2·51	−1·14	0·195
71:4	132·1	70·0	27·9	0·36	−3·04	−1·20	0·170

Note: The HJ recursive procedure fixes estimates of the 'Short—run' dynamics at full sample values and estimates the cointegration space at each recursive iteration using the full sample estimates of the short-run parameters. Estimates of the trace test are taken from the five-dimensional 'velocity constrained' system, effectively reducing it to a four-dimensional system. Parameters β_{1i}, β_{2i}, and β_{3i} correspond to the interest rate coefficients in the money demand, Fisher and interest rate differential cointegrating relations respectively. Parameter β_i is the interest rate coefficient obtained from the system estimated with the Fisher and interest rate differential coefficients in place.

well documented by Stock and Watson (1993). When the cointegration space is restricted to contain just a single free parameter, β_i, there is even more evidence of stability as indicated by the estimates in the last column of Table II.[9]

Table III contains a detailed summary of the corresponding *VECM* estimates obtained over the period 1956:3–1994:4. These results suggest that at least one of the error correction terms contributes significantly to the short-run movements of each variable in the system and there is little evidence of weak exogeneity.[10] Residual diagnostics suggest that the specifications purge

[9] We also conducted recursive experiments using the not seasonally adjusted data. Results are very similar to those portrayed in Table II and are available on request.

[10] Weak exogeneity of GDP does not prevail across a recursive sample that ends in the early and mid-1980s. Specifically, it does not prevail in the sample ending in 1986:4 used in our forecast evaluation experiments below.

Table III. Estimates of the error correction coefficients and significance tests for the five-variable system (56:3–94:4)

	Dependent variable				
	$\Delta m1p_t$	Δifl_t	Δcpr_t	Δgdp_t	Δtbr_t
money demand x_{t-1}	−0·0101	−5·0230	0·5330	−0·0016	0·0631
	(1·51)	(3·90)	(0·74)	(2·11)	(0·10)
$Fisher_{t-1}$	−0·0012	−0·5387	0·1509	−0·0015	0·1558
	(1·64)	(3·68)	(1·86)	(1·75)	(2·18)
interest rate	−0·0059	1·2304	−0·5545	−0·0020	−0·0158
$differential_{t-1}$	(2·07)	(2·24)	(1·81)	(0·63)	(0·06)
$F_{\Delta vel}$ (p-value)	0·002	0·470	0·000	0·634	0·001
$F_{\Delta ifl}$ (p-value)	0·682	0·001	0·002	0·787	0·001
$F_{\Delta cpr}$ (p-value)	0·517	0·325	0·012	0·010	0·595
$F_{\Delta gdp}$ (p-value)	0·681	0·444	0·376	0·828	0·061
$F_{\Delta tbr}$ (p-value)	0·105	0·602	0·056	0·019	0·118
R^2	0·70	0·51	0·51	0·41	0·50
$Q(\chi^2_{24})p$-value	0·012	0·839	0·665	0·914	0·489
$WE(\chi^2_3)$	0·018	0·001	0·012	0·078	0·005
MSE % differ	6·76	15·16	7·35	4·59	8·64

Note: Each equation contained four lags of each of the dependent variables in the system as well as deterministic break dummies *D*67, *D*79, and *D*82. Numbers in parentheses below each coefficient represent *t*-statistics. The F-statistics are formed from test of four lagged variables. The *Q* statistic is based on 24 residual autocorrelations. The *WE* test is a test for the joint significance of the three error correction coefficients and the *MSE* column measure the percentage increase in MSE observed when the error correction terms are removed from the equation.

significant serial correlation from the errors in each of the five equations except the *m1p* equation. Interestingly, significant serial correlation is absent in all equations, including *m1p*, in the estimation sample used as the basis for our forecasting experiments, 56:3–86:4 or in the full sample with the lag length set at $k = 6$. Figure 3 depicts estimates of the error correction terms estimated recursively over samples that end 82:4–94:4, using the specifications that include the three dummy variables. Again we observe considerable stability.

Table IV contains a series of structural stability tests applied to each of the equations in the *VECM* as well as the alternative forecasting specifications, *DVAR* and *LVAR*. The Chow break point tests are based upon the null of stability and are designed to test whether there is any evidence of coefficient instabilities in samples that represent approximately one third of the data. The dummies are purged from the equations in conducting these break point tests. The predictive failure tests in Table IV are based upon the estimation sample that serves as the basis for our forecasting experiments below, 56:3–1986:4. The test examines forecast stability over the period 1987:1–1994:4. Finally, the dummy variable tests are based upon the null hypothesis that the deterministic breaks have no role in the specification. In each case the *p*-values for the indicated test statistics are tabulated.

The results in Table IV suggest that there is considerable evidence of stability in each of the three possible VAR specifications. There is some indication of structural instability of the 'levels VAR' specification, *LVAR*, in the break point test though there is no significant evidence of 'predictive failure' in the post-estimation sample we use in our forecasting experiments. Interestingly, there is also very little evidence that the 'break dummies' capture significant structural shifts in any of the series other than the inflation rate.

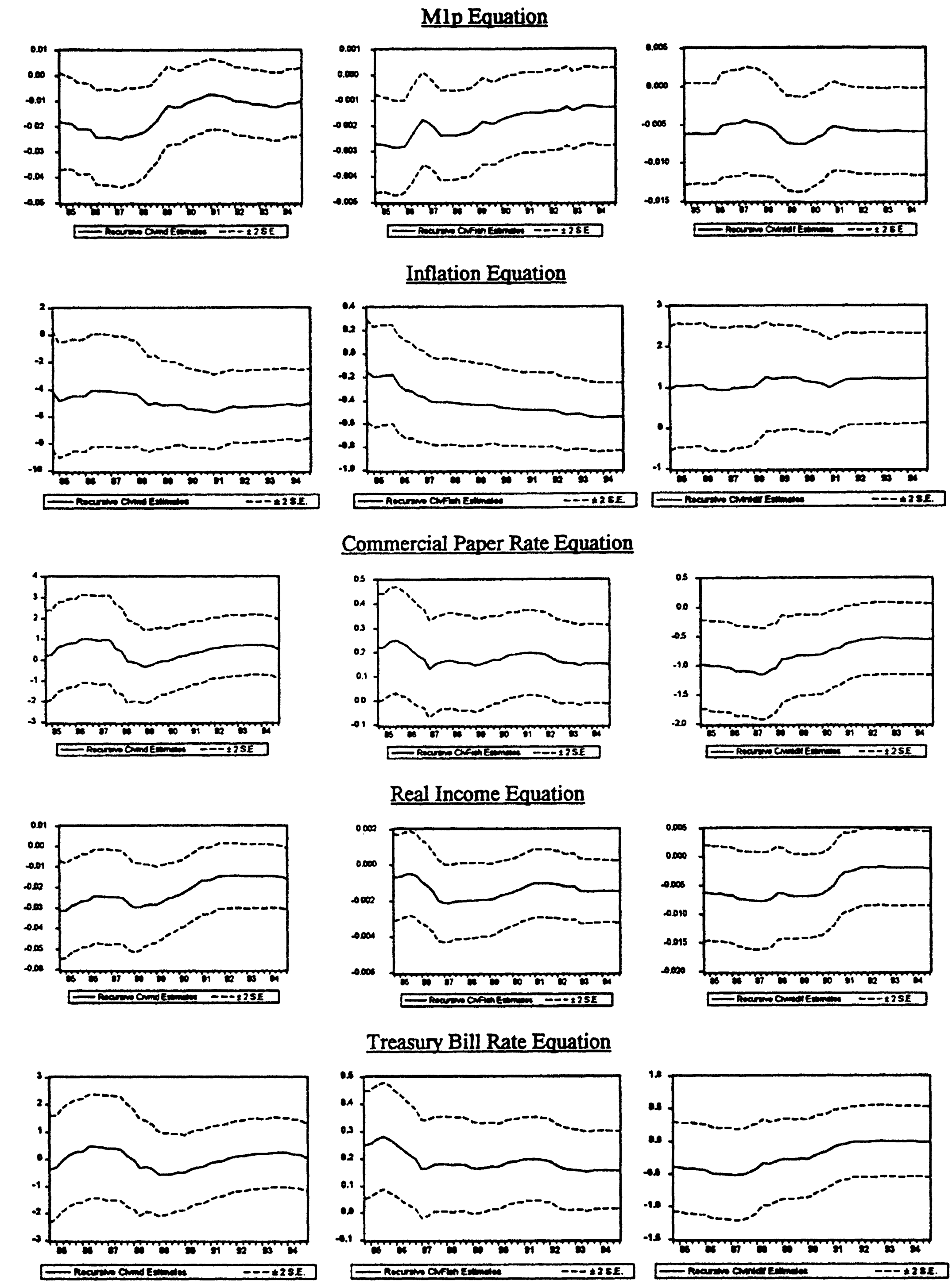

Figure 3. Recursive estimates of the error correction coefficients (models include break dummies)

Table IV. Structural stability tests for the alternative forecasting specifications: *DVAR*, *LVAR*, and *VECM*

Test	Equation: *m1p*	*ifl*	*cpr*	*gdp*	*tbr*
p-values: DVAR specification					
Chow break point	0·462	0·137	0·279	0·159	0·316
Predictive failure	0·336	0·889	0·730	0·990	0·799
Dummy Variables	0·476	0·132	0·889	0·122	0·558
p-values: LVAR specification					
Chow break point	0·456	0·076	0·034	0·129	0·069
Predictive failure	0·144	0·999	0·846	0·951	0·892
Dummy variables	0·973	0·007	0·894	0·984	0·266
p-values: VECM specification					
Chow break point	0·252	0·248	0·212	0·087	0·237
Predictive failure	0·186	0·999	0·496	0·888	0·767
Dummy Variables	0·450	0·001	0·537	0·052	0·075

Note: The Chow breakpoint test is an F-test obtained by removing the intervention dummies from each equation and testing for possible structural breaks at 1967:4 and 1982:1. The predictive failure F-test uses the period 56:3–86:4 for the estimation sample and 1987:1–1983:4 for the prediction sample. The dummy variable test is a simple F-test of the significance of the three dummy variables in each of the equations in each specification.

3. ASSESSING FORECAST PERFORMANCE

The simplest way of measuring the gain in forecast performance that is achieved by incorporating knowledge of cointegration rank is to measure the relative advantage maintained by the *VECM* over a *DVAR* (excluding the error correction terms) in a within-sample mean-squared error comparison. The *DVAR* specification is based upon the same lag length in the differenced autoregressions and includes the deterministic dummy variables so accurate comparisons can be made.

The in-sample MSE comparisons appear at the bottom of Table III. The MSE in the real balance equation is reduced by 6·76% when the error correction terms are included. MSE reduction in the inflation equation is 15·16%. MSE reduction of 7·35% is observed in the commercial paper rate model. The treasury bill model exhibits a 8·64% reduction and the MSE reduction in the GDP model is 4·59% with the inclusion of the error correction terms. The significance of the reduction in MSE obtained in the VECM specification is measured by the tests for weak exogeneity in Table III.

3.1. A Simple Post-estimation Sample Forecast Comparison

Our initial post-estimation experiment spans the period 87:1–94:4, a period about one third as long as the estimation sample or, equivalently, about 25% of the full sample used to establish cointegration rank. Forecasts of the levels of variables like inflation and the two nominal interest rate series would certainly be of interest, while forecast diagnostics based upon either the level or the rates of change for real money balances or for real GDP might be of interest. We produced dynamic forecasts over the out-sample period for each of the variables in our model—including both levels and growth rates of real output—using the cointegrated system based on the *VECM* specification, a simple differenced vector autoregression (*DVAR*) and a simple vector autoregression in the levels of the data (*LVAR*).

Figure 4 depicts the forecast errors observed over the 1987:1–1994:4 period for particular linear transformations for each of the variables in our system. The forecasts that provide a basis for these errors are formed dynamically from each of the three models. The errors are measured as ‘actual–predicted’. The forecast errors for *VECM*, *LVAR* and *DVAR* are solid, broken, and

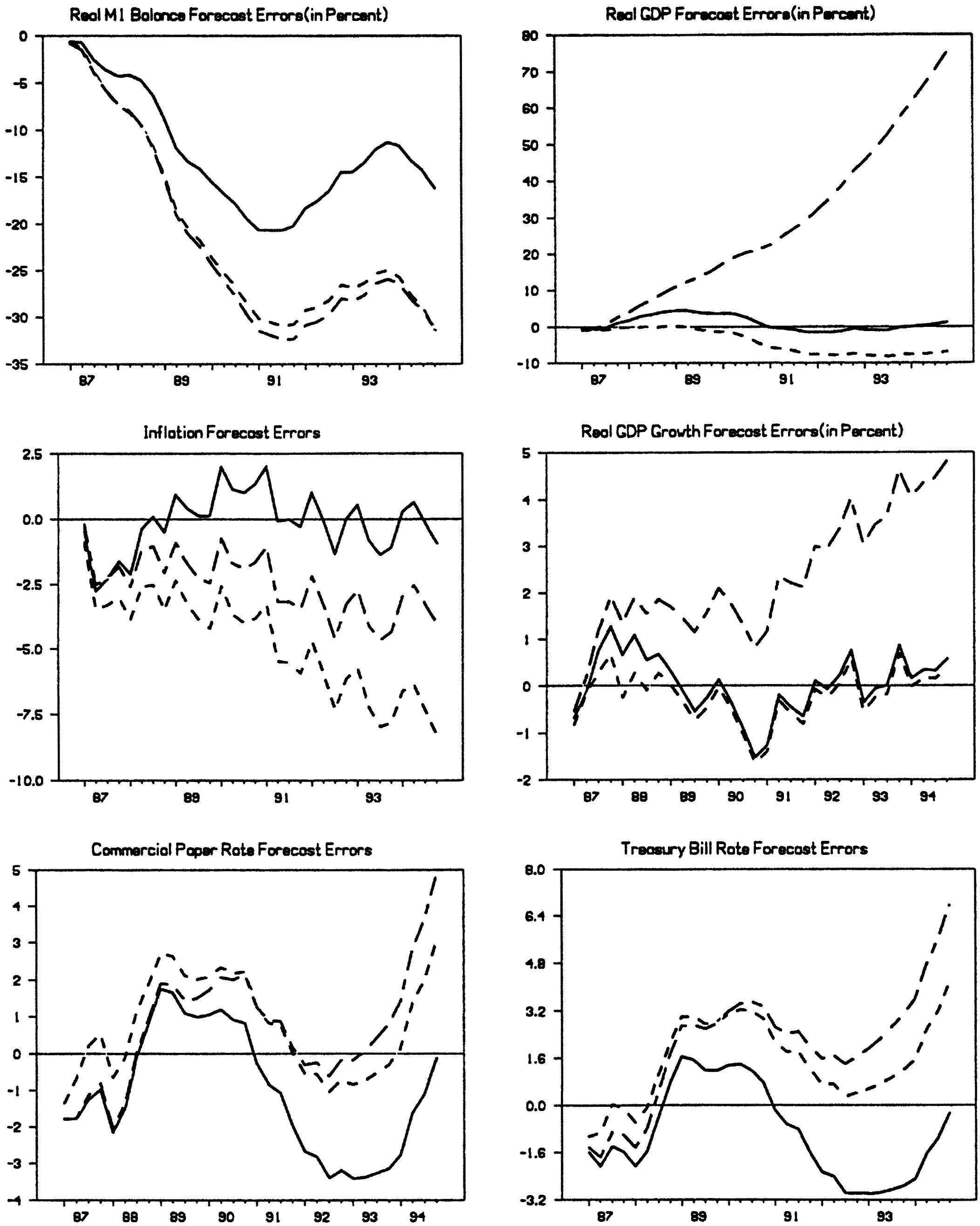

Figure 4. Forecast errors from the 1986:4 base

dashed lines respectively. The forecasted values for real money balances tend to be substantially greater than the actual values for each of the three forecasts. However, the *VECM* model (solid line) performs better in relative terms—exhibiting errors that range between 10% and 20% at the 4- to 8-year horizon while the alternative VAR models produce forecast errors of between 25% and 35% over the same period. Dynamic inflation forecast errors from the *VECM* model are markedly superior to the alternatives, with forecast errors less than 2% (200 basis points) throughout the entire eight-year post-estimation sample. In contrast, *LVAR* forecasts (broken line) exceed 4% (400 basis points) at the six-year horizon and *DVAR* inflation errors (dashed line) are 8% (800 basis points) above observed inflation rates at the six-year horizon. The commercial paper rates and treasury bill rate forecasts share a similar pattern. Early in the post estimation period (at horizons of less than two years) the *DVAR* clearly produces errors (less than 100 basis points) that are smaller than either *VECM* or *LVAR* (between 100 and 200 basis points). In the intermediate period (two to four years) the *VECM* projections are smaller (just over 100 basis points) than the two alternatives (between 200 and 300 basis points). In the 4- to 6-year interval, the *DVAR* and *LVAR* models tend to outperform the *VECM* forecasts and the *VECM* forecast errors are the smallest at the end of the post-estimation period. Forecasts of real GDP over the post-estimation period are the most diverse across the three models. The *DVAR* forecasts are superior for the first four years of the post-estimation period (the dashed forecast error line is indistinguishable from the 'zero' line of demarcation) with the *VECM* forecasts gaining the advantage for the remainder of the period. Forecast errors for the growth rates of real GDP reveal that the *VECM* forecasts of real GDP growth are too low early in the sample but later in the sample they are essentially the same as those produced by the *DVAR*. The *LVAR* yields forecasts that indicate the potential for dynamic instability in the post-estimation period. After about one year, forecasts for real GDP taken from the *LVAR* exhibit increasingly positive forecast errors, until quarterly growth rate errors are averaging over 4% by the end of the 8-year out-sample period.[11]

The basic conclusion that may be drawn from the forecast errors in Figure 4 is that incorporating knowledge of cointegration rank can result in appreciable reduction in forecast errors in a particular 'out-sample' period. At the same time Figure 4 only contains a single set of forecast error comparisons for each of 32 forecast horizons so one must be cautious in drawing general conclusions.

3.2. System Forecast Comparisons Based upon Different Reference Points in the Out-sample Period

We are able to learn more about the relative forecast performance over the 'out-sample' by calculating forecast errors at alternative forecast error horizons using different reference points across the post-sample period. In addition, we want to determine if the conclusions we draw are robust. Hence, we examine the relative forecasting performance over four model specifications, at different forecast horizons, and, using both gross and 'own-rate' adjusted measures of interest rates as well as both seasonally adjusted and not seasonally adjusted data.

The relative measures of forecast performance are calculated over the post-sample period in two distinct exercises. First, we measure relative 'system-wide' forecast performance of each of

[11] We experimented with several Baysian prior specifications in an attempt to eliminate the dynamic instability from the LVAR forecasts. None of the 'standard prior' specifications helped control the problem at the longer horizons. However, further experimentation might prove to be fruitful. We chose to present results based on the unrestricted LVAR estimates for purposes of comparison and because dynamic instabilities do not appear to be a persistent problem in the variable forecast horizon experiments conducted below.

the three competing specifications using the GFESM proposed as the basis for predictive likelihood comparison by Clements and Hendry (1993). We then examine root mean squared forecast errors (RMSFE's) for specific representations of each variable in the system. The advantage of the system-wide comparison is that it is invariant to the choice of linear transformations used in the forecast error variance comparison and it condenses the relative forecast performance at all horizons down to a single measure of system forecast performance. The disadvantage of the GFESM comparison is that it is based upon the calculation of a sample moment matrix that is of the order $(p \cdot h) \times (p \cdot h)$ where p is the number of variables in the system and h is the maximum forecast horizon under consideration. In our application, the system is five-dimensional and we have a maximum of 32 out-sample periods, but only 28 out-sample periods if we want to conduct the exercise using not seasonally adjusted data.[12] Also, in contrast to the Monte Carlo exercises conducted by Engle and Yoo and again by Clements and Hendry we have only limited 'looks' at out-sample performance. For example, a 28-period forecast interval contains 28 one-step-ahead forecasts, 27 two-step-ahead forecasts, 25 four-step-ahead forecasts, etc.

In gauging the GFESM, Clements and Hendry (1993) recommend constructing the moment matrix

$$\phi_h = \mathrm{E}[EE]'$$

where

$$E' = [e'_{t+1}, e'_{t+2}, \ldots, e'_{t+h-1}, e'_{t+h}]$$

and e'_{t+h} denotes the h-step-ahead forecast errors for all the variables in the system. Then $\log|\hat{\phi}_h|$ forms the basis for the comparison of predicted likelihoods. Models that minimize $\log|\phi_h|$ will maximize the corresponding predictive likelihood.

We took steps to economize on data in the estimation of the sample moment matrix $\hat{\phi}_h$. First, we expressed the moment matrix as a block diagonal by assuming that the forecast errors across variables are uncorrelated. This simplification implies that the 'cross-variable' implications of system forecast ranking is lost in our application but the 'cross forecast horizon' nature of the comparison is retained.[13] Second, we estimated the 'blocks' in the matrix separately using the maximum amount of data possible to calculate the moment matrix in each component of the block diagonal rather than truncate all samples to comply with the largest forecast horizon considered.

Table V summarizes the estimates, $\log|\hat{\phi}_h|$ which determine the ranking of corresponding predictive likelihoods for each of the three system forecast specifications using each of four separate representations of the data. The summary data at the top of the table offer comparisons based upon $\log|\hat{\phi}_h|$ that are based upon models that employ gross interest rate measures. This comparison reveals that the *LVAR* and *VECM* models generally outperform the *DVAR* specification (producing smaller values for $\log|\hat{\phi}_h|$ that imply higher predictive likelihoods. In the models composed of gross interest rate measures, the *VECM* specification holds a slight advantage. When the models are estimated with the 'own-rate' adjustments in place, the *DVAR* specification is again inferior to its competitors, but the *VECM* model is clearly superior—especially at the longer forecast horizons. When judged on the basis of relative

[12] The sample of not seasonally adjusted data ends in 1993:4.

[13] We calculated $\hat{\phi}_h$ using both complete and our variable specific measures for horizons in which calculation of the full measure was actually feasible (horizon 4) and found that the ranking based upon the two measures was not significantly affected.

Table V. Generalized forecast error second moment(GFESM) rankings for each forecasting specification using several representations of the data. Estimation sample, 56:3–86:4, and forecast Sample 87:1–93:4

Models based on gross interest rate measure*							
Seasonally adjusted data				Not seasonally adjusted data			
Horizon	*DVAR*	*LVAR*	*VECM*	Horizon	*DVAR*	*LVAR*	*VECM*
1	1·72	0·41	0·34	1	1·28	-0·52	-0·32
2	3·81	2·34	2·18	2	3·14	0·18	0·47
4	11·17	6·29	5·46	4	9·32	2·74	2·80
8	25·04	13·47	11·42	8	21·24	6·74	5·86
12	37·02	21·50	18·30	12	31·22	10·74	9·52
16	46·36	29·74	26·25	16	38·14	12·62	12·68

Models based on net interest rate measure*							
Seasonally adjusted data				Not seasonally adjusted data			
Horizon	*DVAR*	*LVAR*	*VECM*	Horizon	*DVAR*	*LVAR*	*VECM*
1	1·70	0·03	-0·32	1	1·42	-0·93	-0·80
2	3·68	1·48	0·80	2	3·41	-0·70	-0·34
4	10·95	4·83	3·29	4	9·54	1·13	1·16
8	25·15	10·82	6·42	8	22·05	3·11	1·90
12	37·74	16·77	9·08	12	32·93	4·25	1·44
16	47·26	22·54	13·45	16	40·33	3·45	1·37

Note: The net interest rate results are based upon models obtained using interest rate series that are adjusted for the opportunity cost of holding M1 balances. The entries in these tables are the log $|\hat{\phi}_h|$ that determine the predictive likelihood suggested by Clements and Hendry (1993). Sample ends in 1993:4 to facilitate comparison with results obtained from not seasonally adjusted data.

system forecast performance there appears to be some advantage to incorporating knowledge of cointegration rank. Below we examine whether that advantage applies to all variables or is confined to certain elements of the system.

3.3. Relative Forecast Performance for Specific Variables

Tables VI–VIII summarize the root mean squared forecast errors (RMSFEs) observed over the out-sample period. As in the calculation of the system measure of forecast performance, the forecasts are produced dynamically from starting values as of 87:1 and updated with forecasts produced by the model, with no reliance on actual data over the post-estimation period. The reference point of the forecast is updated to provide multiple observations on 1-, 2-, 4-, 8-, 12-, and 16-step-ahead forecast errors. Moreover, all RMSFE diagnostics are taken from system representations that are based on gross interest rates and seasonally adjusted data. The conclusions we draw from a variable by variable RMSFE comparison are robust to the alternative data definitions, adjusting for the 'own rate' on money balances or the use of not seasonally adjusted data. Results based on these other measures are available on request.

The entries in Tables VI–VIII represent the RMSFEs for the forecasts obtained for the indicated forecast horizon for each variable. The entries in parentheses estimate the standard errors of these

Table VI. Empirical root mean squared forecast error (RMSFEs) at various horizons, 87:1–93:4, based upon seasonally adjusted data with variables expressed as 'First Differences' (standard errors in parentheses)

Horizon	$\Delta m1p$	Δifl	Δcpr	Δgdp	Δtbr	ΔM
DVAR						
1	0·0079	2·00	0·89	0·0069	0·77	0·0087
	(0·002)	(·34)	(0·31)	(0·002)	(0·20)	(0·003)
2	0·0113	1·31	0·87	0·0071	0·77	0·0113
	(0·003)	(0·29)	(0·31)	(0·001)	(0·18)	(0·004)
4	0·0139	1·00	0·71	0·0051	0·62	0·0120
	(0·003)	(0·21)	(0·22)	(0·001)	(0·18)	(0·004)
8	0·0165	1·02	0·52	0·0083	0·47	0·0161
	(0·005)	(0·26)	(0·18)	(0·003)	(0·15)	(0·006)
12	0·0117	1·05	0·42	0·0073	0·37	0·0141
	(0·003)	(0·30)	(0·15)	(0·003)	(0·12)	(0·004)
16	0·0102	1·06	0·45	0·0074	·38	0·0148
	(0·003)	(0·37)	(0·18)	(0·003)	(0·14)	(0·005)
LVAR						
1	0·0183	1·23	0·99	0·0088	0·90	0·0176
	(0·002)	(0·17)	(0·27)	(0·002)	(0·19)	(0·002)
2	0·0225	1·35	0·99	0·009	0·81	0·0222
	(0·003)	(0·35)	(0·25)	(0·26)	(0·18)	(0·003)
4	0·0213	1·15	1·37	0·0076	1·09	0·0232
	(0·005)	(0·27)	(0·20)	(0·002)	(0·06)	(0·004)
8	0·0161	1·13	0·70	0·0164	0·57	0·019
	(0·004)	(0·32)	(0·23)	(0·003)	(0·18)	(0·005)
12	0·0115	1·04	0·39	0·0182	0·36	0·016
	(0·003)	(0·29)	(0·11)	(0·002)	(0·07)	(0·003)
16	0·0097	1·04	0·37	0·230	0·32	0·0126
	(0·004)	(0·33)	(0·13)	(0·003)	(0·10)	(0·004)
VECM						
1	0·0108	1·26	0·97	0·0102	0·85	0·0108
	(0·002)	(0·25)	(0·27)	(0·002)	(0·17)	(0·002)
2	0·0140	1·28	1·00	0·0097	0·87	0·0130
	(0·003)	(0·32)	(0·25)	(0·002)	(0·20)	(0·003)
4	0·0153	0·99	0·94	0·0051	1·78	0·0135
	(0·003)	(0·24)	(0·18)	(0·009)	(0·37)	(0·003)
8	0·0143	1·07	0·76	0·0069	0·64	0·0123
	(0·003)	(0·33)	(0·23)	(0·002)	(0·18)	(0·004)
12	0·0109	1·04	0·50	0·0058	0·45	0·009
	(0·002)	(0·16)	(0·04)	(0·002)	(0·15)	(0·003)
16	0·0116	1·03	0·47	0·0065	0·43	0·009
	(0·003)	(0·22)	(0·19)	(0·003)	(0·17)	(0·004)

Note: entries in this table are obtained by averaging the 28 one-step-ahead, 27 two-step-ahead, 25 four-step-ahead, etc. forecast errors obtained for each horizon over the 'out-sample period'. Standard errors for the RMSFE estimates are imputed from the sampling distribution of the corresponding mean squared errors.

Table VII. Empirical root mean squared forecast errors (RMSEs) at various horizons, 87:1–94:4, based upon seasonally adjusted data with variables expressed as 'levels' (standard errors in parentheses)

Horizon	*m1p*	*ifl*	*cpr*	*gdp*	*tbr*	*M*
DVAR						
1	0·0079	2·00	0·89	0·0069	0·77	0·0087
	(0·002)	(0·34)	(0·31)	(0·002)	(0·20)	(0·003)
2	0·0177	2·49	1·44	0·0124	1·38	0·0184
	(0·005)	(0·46)	(0·48)	(0·003)	(0·37)	(0·006)
4	0·0412	3·01	1·94	0·0162	2·09	0·0396
	(0·009)	(0·58)	(0·60)	(0·003)	(0·78)	(0·0137)
8	0·1039	2·92	2·66	0·0213	2·96	0·0916
	(0·023)	(0·63)	(0·73)	(0·006)	(0·75)	(0·03)
12	0·1582	2·52	2·53	0·391	2·94	0·1542
	(0·038)	(0·62)	(0·88)	(0·005)	(0·91)	(0·05)
16	0·1993	2·20	1·92	0·0564	2·42	0·2160
	(0·050)	(0·59)	(0·76)	(0·005)	(0·79)	(0·082)
LVAR						
1	0·0183	1·23	0·99	0·0088	0·90	0·0176
	(0·002)	(0·17)	(0·27)	(0·002)	(0·14)	(0·002)
2	0·0399	1·61	1·77	0·0162	1·56	0·0388
	(0·0054)	(0·34)	(0·34)	(0·004)	(0·30)	(0·005)
4	0·0828	2·28	2·83	0·0207	2·25	0·0849
	(0·01)	(0·53)	(0·56)	(0·004)	(0·42)	(0·010)
8	0·1465	3·62	3·64	0·570	2·72	0·1641
	(0·03)	(0·65)	(0·81)	(0·01)	(0·52)	(0·023)
12	0·2028	3·67	2·68	0·1200	2·11	0·2434
	(0·04)	(0·60)	(0·59)	(·02)	(0·35)	(0·038)
16	0·2448	3·89	1·97	0·1982	1·62	0·3160
	(0·04)	(0·43)	(0·40)	(·01)	(1·11)	(0·037)
VECM						
1	0·0108	1·26	0·97	0·0102	0·85	0·0108
	(0·002)	(0·25)	(0·27)	(0·002)	(0·17)	(0·002)
2	0·0234	1·69	1·78	0·190	1·61	0·0222
	(0·0045)	(0·36)	(0·35)	(0·004)	(0·50)	(0·005)
4	0·0509	1·79	2·77	0·0336	2·56	0·0469
	(0·009)	(0·71)	(0·54)	(0·007)	(0·51)	(0·001)
8	0·1012	1·89	3·72	0·0498	3·34	0·0909
	(0·0179)	(0·56)	(0·84)	(0·01)	(0·75)	(0·0174)
12	0·1330	1·81	3·72	0·0604	3·39	0·1258
	(0·03)	(0·60)	(0·81)	(0·01)	(0·75)	(0·026)
16	0·1489	1·72	3·68	0·0621	3·34	0·1455
	(0·04)	(0·49)	(0·74)	(0·01)	(0·70)	(0·033)

Note: entries in this table are obtained by averaging the 28 one-step-ahead, 27 two-step-ahead, 25 four-step-ahead, etc. forecast errors obtained for each horizon over the 'out-sample-period'. Standard errors for the RMSFE estimates are imputed from the sampling distribution of the corresponding mean squared errors.

Table VIII. Empirical root mean squared forecast errors (RMSEs) at various horizons, 87:1–94:4, based upon seasonally adjusted data with variables expressed as the three cointegrating relations (standard errors in parentheses)

Horizon	*m1pciv*	*Fishciv*	*IntDifciv*
DVAR			
1	0·0828	2·06	0·33
	(0·0252)	(0·37)	(0·09)
2	0·1403	2·22	0·45
	(0·037)	(0·42)	(0·11)
4	0·2023	2·05	0·51
	(0·06)	(0·47)	(0·14)
8	0·2503	2·99	0·48
	(0·06)	(0·82)	(0·17)
12	0·2151	3·53	0·69
	(0·08)	(1·08)	(0·15)
16	0·1449	3·65	0·95
	(0·06)	(1·37)	(0·19)
LVAR			
1	0·0916	1·17	0·43
	(0·020)	(0·24)	(0·06)
2	0·1581	1·18	0·59
	(0·03)	(0·22)	(0·08)
4	0·2791	1·29	0·79
	(0·055)	(0·29)	(0·10)
8	0·3707	2·57	1·37
	(0·071)	(0·54)	(0·10)
12	0·3437	3·57	1·53
	(0·058)	(0·63)	(0·06)
16	0·3823	4·32	1·63
	(0·047)	(0·58)	(0·08)
VECM			
1	0·0911	1·18	0·32
	(0·019)	(0·25)	(0·14)
2	0·1688	1·08	0·42
	(0·0398)	(0·21)	(0·15)
4	0·2727	1·43	0·37
	(0·053)	(0·39)	(0·11)
8	0·3489	1·82	0·43
	(0·073)	(0·44)	(0·07)
12	0·3786	1·92	0·42
	(0·066)	(0·53)	(0·06)
16	0·4176	2·08	0·36
	(0·053)	(0·54)	(0·06)

Note: entries in this table are obtained by averaging the 28 one-step-ahead, 27 two-step-ahead, 25 four-step-ahead, etc. forecast errors obtained for each horizon over the 'out-sample period'. Standard errors for the RMSFE are imputed from the sampling distribution of the corresponding mean squared errors.

RMSFE estimates.[14] Table VI contains the differences of all the variables in the system, plus the growth in nominal M1 balances that can be imputed from the forecasts of the five variables in our system. Table VII contains the RMSFEs for the 'levels' of each of these variables and Table VIII contains the RMSFEs for the cointegrating combinations of the variables.

Careful examination of the RMSFEs from each of the models reveals a story that is somewhat different from the single 32-period assessment in Figure 4 and the system diagnostics in Table V. There is little difference in forecast performance for $\Delta m1p$, Δifl, Δcpr, and Δtbr as revealed by the similarity of the RMSFEs in Table VI. The only discernible advantage is held by the *VECM* specification that produces markedly lower RMSFEs for the Δgdp and ΔM series at the longer forecast horizons. The pattern of results in Table VII offers a more interesting relative comparison. The VECM produces considerably lower RMSFEs at longer horizons for the level of $m1p$, ifl and M. However, the nominal interest rate forecasts produced by the *VECM* model generally have *higher* RMSFE than either of the competitors even at the longer forecast error horizon. Table VIII contains forecasts of the cointegrating relations at the same set of horizons over the post-sample period. Interestingly, forecasts of the money demand vector are actually lowest in the *DVAR* model! *VECM* forecasts of the other two vectors, the *ex post* real rate in the case of the Fisher vector and the risk premium, the interest differential vector, produce appreciably lower RMSFEs.

The pattern of relative forecast performance across the variables in the system is quite diverse. While both Engle and Yoo and Clements and Hendry observe that the relative forecasting advantages of *VECM* specifications accrue in level representations of data, Clements and Hendry demonstrate that similar advantages are not observed in other representations (differences and cointegrating combinations). The more complex specification we examine yields conclusions in the spirit of Clements and Hendry but our results reveal that the advantage does not lie exclusively in one type of data representation. The lesson conveyed by our findings is that the advantage to incorporating knowledge of cointegration rank is not confined to any particular representation of the data.

The pattern of RMSFEs observed in Tables VI–VIII is also worthy noting on several other counts. Regardless of the chosen forecasting model, RMSFEs for the differenced representations of the data are generally invariant to the length of the forecast horizon, except for the differences of the interest rate variables where forecast errors tend to decline. According to the expectations theory of the term structure, longer horizon forecasts of short term rates may be accumulated to form forecasts of the long-term rate. The lower volatility in longer horizon forecasts of short rates may then simply be capturing the fact that long-term interest rates are generally less volatile than are short-term rates.

Perhaps the most interesting conclusion that we draw is that incorporating the Fisher equation and interest rate differential anchors into the system clearly leads to improved forecasts of inflation rates, the *ex post* real interest rate and the risk premium associated with risk-free and risky measures of short-term interest rates. In contrast, incorporating the money demand anchor into the VAR did not improve forecasts of the long-run relation linking money balance, GDP and nominal interest rates in the post sample period.[15]

[14] The measures of precision are obtained by first estimating the standard error associated with the MSFEs. Then the estimated standard error for the RMSFE was set at values that delivered exactly the same inference for the test of hypothesis that both the MSFE and the RMSFE is equal to zero.

[15] We conducted a similar experiment using in sample one-step-ahead forecast errors for the 56:3–86:4 period and evaluated the performance of the three specifications in forecasting the three cointegrating vectors. In this case *DVAR* and *LVAR* forecasts are identical and inferior to those of *VECM* for each of the three vectors. However, the MSE reduction in the forecasts for the money demand vector was considerably smaller (9·31%) than the significant reductions observed in the case of the later two vectors. (19·63% and 19·46% respectively).

The relative performance of the three models in explaining the RMSFEs in the cointegrating vectors clearly stems from the fact that the DVAR simply does a better job of forecasting the level of *tbr* and the level of *gdp* at longer horizons in our sample. This result may stem from the fact that there is some evidence of weak exogeneity of *gdp* in our specification, (see Table III).[16] The *VECM* clearly performs better for those cointegrating combinations that are composed of variables that respond significantly to departures from equilibrium as in the case of inflation. An analogous result is obtained by Clements and Hendry (1993) using a simple bivariate interest rate/velocity representation where the interest rate is weakly exogenous. In this case the corresponding *VECM* yields lower RMSFEs for velocity (where the error correction term enters the system), but forecasts of the 'weakly exogenous' interest rates and the cointegrating combination are not improved by the *VECM* specification. When combined with our conclusions, these results suggest that one may expect to observe forecast error advantages in those segments of the *VECM* that exhibit the most evidence of cointegration and 'error correction'. It also implies that preliminary tests for the presence of weak exogeneity in a *VECM* may have an important role in dictating forecast performance across the variables in the system.

4. CONCLUSION

The cointegration literature suggests that forecast errors may be reduced by incorporating knowledge of cointegration rank into models characterized by cointegration. We examine a system composed of real money balances, inflation rates, commercial paper rates, real GDP, and the treasury bill rate that displays cointegration rank three.

The forecast performance of the *VECM* formed from this cointegrated system is compared with forecasts produced by simple VAR structures formed by simply differencing the system *DVAR* and representing the system in levels *LVAR*. The error correction terms offer substantial reduction in MSE observed in a sample that spans 56:3–86:4. Over the post-estimation sample period of 1987:1–1994:4, the relative forecast performance of the three models; *VECM*, *DVAR*, and *LVAR* is mixed—depending on whether forecast evaluation is conducted on a 'system-wide' basis, whether forecasts for particular variables are scrutinized, or whether a particular forecast horizon is chosen. In general, our findings suggest that the gains from incorporating knowledge of cointegration rank *can* be substantial but will typically accrue, as Engle and Yoo suggested, only at longer forecast horizons. Also, as observed in a recent paper by Clements and Hendry, forecast performance of the simple VAR models can match or even slightly outperform that of the *VECM* forecasts. It is clear that the *VECM* does not uniformly result in improved forecast performance for all the variables in our simple cointegrated system—especially at short- and intermediate-term horizons of less than three years. We also observe that the presence (or absence) of weak exogeneity can dictate whether relative forecast advantages will characterize *VECM* specifications.

REFERENCES

Baba, Y., D. F. Hendry and R. M. Starr (1992), 'The demand for M1 in the U.S.A.', *Review of Economic Studies*, **59**, No. 1, 25–61.

Clements, M. P. and D. F. Hendry (1993), 'On the limitations of comparing mean square forecast errors', *Journal of Forecasting*, 617–637.

[16] As discussed earlier, there is less evidence of weak exogeneity when the estimation sample ends in 1986:4 and this specification is used as the basis for the forecasting experiments. However, there is evidence of weak exogeneity of velocity in a four-dimensional specification that embodies the combined effects of real money balances and real GDP.

Clements, M. P. and D. F. Hendry (1995), 'Forecasting in cointegrated systems', *Journal of Applied Econometrics*, **10**, 127–146.

Crowder, W. and D. L. Hoffman (1996), 'The long-run relationship between nominal interest rates and the rate of inflation: the Fisher equation revisited', *Journal of Money Credit and Banking*, **28**, 102–118.

Engle, R. F. and C. W. J. Granger (1987), 'Co-Integration and error correction: Representation, estimation and testing', *Econometrica*, **55**, 251–276.

Engle, R. F. and B. S. Yoo (1987), 'Forecasting and testing in co-Integrated systems', *Journal of Econometrics*, **35**, 143–159.

Ericsson, N., D. F. Hendry and H.-A. Tran, (1994), 'Cointegration, seasonality, encompassing, and the demand for money in the United Kingdom', C. Hargreaves (ed.), *NonStationary Time Series Analysis and Cointegration*, 179–224, Oxford University Press, Oxford.

Friedman, B. M. and K. N. Kuttner (1993), 'Why does the paper-bill spread predict real economic activity?' in J. H. Stock, J. H. and M. W. Watson, (eds), *Business Cycles, Indicators, and Forecasting.*, University of Chicago Press, Chicago.

Hansen, H. and S. Johansen (1993), 'Recursive estimation in cointegrated models', Institute of Mathematical Statistics, preprint no.1.

Hoffman, D. L. and R. H. Rasche (1991), 'Long-run income and interest elasticities of money demand in the United States', *Review of Economics and Statistics.*, **78**, 665-674.

Hoffman, D. L. and R. H. Rasche (1996), *'Aggregate Money Demand Functions: Empirical Applications in Cointegrated Systems*, Kluwer Academic Publishers, Boston.

Hoffman, D. L., R. H. Rasche and M. A. Tieslau (1995), 'The stability of long-run money demand in five industrialized countries', *Journal of Monetary Economics*, **35**, 317–339.

Horvath, M. T. K. and M. W. Watson (1995), 'Testing for cointegration when some of the cointegrating vectors are known', *Econometric Theory*, **11**, 984–1014.

Johansen, S. (1988), 'Statistical analysis of cointegrating vectors', *Journal of Economic Dynamics and Control*, **12**, 231–254.

Johansen, S. (1992), 'Cointegration in partial systems and the efficiency of single equation analysis', *Journal of Econometrics*, **52**, 389–402.

Johansen, S. and K. Juselius (1990), 'Maximum likelihood estimation and inference on cointegration—with applications to the demand for money', *Oxford Bulletin of Economics and Statistics*, **52**, 169–210.

Johansen, S. and K. Juselius (1992), 'Testing structural hypotheses in a multivariate cointegration analysis of the PPP and the UIP for U.K.', *Journal of Econometrics*, **53**, 211–244.

Johansen, S. and B. Nielson (1993), 'Asymptotics for cointegration rank tests in the presence of intervention dummies', mimeo, Institute of Mathematical Statistics, University of Copenhagen.

Mishkin, F. S. (1992), 'Is the Fisher effect for real? A reexamination of the relationship between inflation and interest rates', *Journal of Monetary Economics*, *30*, 195–215.

Osterwald-Lenum, M. (1992), 'A note with quantiles of the asymptotic distribution of the maximum likelihood cointegration rank test statistics', *Oxford Bulletin of Economics and Statistics*, **54**, 461–471.

Perron, P. (1989), 'The great crash, the oil price shock and the unit root hypothesis', *Econometrica*, **57**, 1361–1401.

Rasche, R. H. (1987), 'M1-velocity and money demand functions:do stable relationships exist?' *Carnegie Rochester Conference Series on Public Policy*, **27**, 9–88.

Stock, J. H. and M. W. Watson (1992), 'A procedure for predicting recessions with leading indicators: econometric issues and recent performance', Working Paper No. 7, Research Department, Federal Reserve Bank of Chicago.

Stock, J. H. and M. W. Watson (1993), 'A Simple estimator of cointegrating vectors in higher order integrated systems', *Econometrica*, **61**, 783–820.

http://www.jstor.org

LINKED CITATIONS

You have printed the following article:

Assessing Forecast Performance in a Cointegrated System
Dennis L. Hoffman; Robert H. Rasche
Journal of Applied Econometrics, Vol. 11, No. 5, Special Issue: Econometric Forecasting. (Sep. - Oct., 1996), pp. 495-517.
Stable URL:
http://links.jstor.org/sici?sici=0883-7252%28199609%2F10%2911%3A5%3C495%3AAFPIAC%3E2.0.CO%3B2-Q

This article references the following linked citations. If you are trying to access articles from an off-campus location, you may be required to first logon via your library web site to access JSTOR. Please visit your library's website or contact a librarian to learn about options for remote access to JSTOR.

References

The Demand for M1 in the U.S.A., 1960-1988
Yoshihisa Baba; David F. Hendry; Ross M. Starr
The Review of Economic Studies, Vol. 59, No. 1. (Jan., 1992), pp. 25-61.
Stable URL:
http://links.jstor.org/sici?sici=0034-6527%28199201%2959%3A1%3C25%3ATDFMIT%3E2.0.CO%3B2-Z

Forecasting in Cointegrated Systems
Michael P. Clements; David F. Hendry
Journal of Applied Econometrics, Vol. 10, No. 2. (Apr. - Jun., 1995), pp. 127-146.
Stable URL:
http://links.jstor.org/sici?sici=0883-7252%28199504%2F06%2910%3A2%3C127%3AFICS%3E2.0.CO%3B2-J

The Long-Run Relationship between Nominal Interest Rates and Inflation: The Fisher Equation Revisited
William J. Crowder; Dennis L. Hoffman
Journal of Money, Credit and Banking, Vol. 28, No. 1. (Feb., 1996), pp. 102-118.
Stable URL:
http://links.jstor.org/sici?sici=0022-2879%28199602%2928%3A1%3C102%3ATLRBNI%3E2.0.CO%3B2-2

http://www.jstor.org

LINKED CITATIONS

Co-Integration and Error Correction: Representation, Estimation, and Testing
Robert F. Engle; C. W. J. Granger
Econometrica, Vol. 55, No. 2. (Mar., 1987), pp. 251-276.
Stable URL:
http://links.jstor.org/sici?sici=0012-9682%28198703%2955%3A2%3C251%3ACAECRE%3E2.0.CO%3B2-T

Long-Run Income and Interest Elasticities of Money Demand in the United States
Dennis L. Hoffman; Robert H. Rasche
The Review of Economics and Statistics, Vol. 73, No. 4. (Nov., 1991), pp. 665-674.
Stable URL:
http://links.jstor.org/sici?sici=0034-6535%28199111%2973%3A4%3C665%3ALIAIEO%3E2.0.CO%3B2-L

The Great Crash, the Oil Price Shock, and the Unit Root Hypothesis
Pierre Perron
Econometrica, Vol. 57, No. 6. (Nov., 1989), pp. 1361-1401.
Stable URL:
http://links.jstor.org/sici?sici=0012-9682%28198911%2957%3A6%3C1361%3ATGCTOP%3E2.0.CO%3B2-W

A Simple Estimator of Cointegrating Vectors in Higher Order Integrated Systems
James H. Stock; Mark W. Watson
Econometrica, Vol. 61, No. 4. (Jul., 1993), pp. 783-820.
Stable URL:
http://links.jstor.org/sici?sici=0012-9682%28199307%2961%3A4%3C783%3AASEOCV%3E2.0.CO%3B2-Z