Fraktale Bildcodierung

Jens Ochsenmeier

Inhalt des Vortrags ist …

Recap — bisherige Themen

Metrischer Raum

Ein metrischer Raum ist eine Menge X mit einer reellwertigen Funktion $X \times X \rightarrow \mathbb{R}$. Diese misst den Abstand zwischen zwei Punkten $x, y \in X$ und muss dazu folgende Axiome erfüllen ($x, y, z \in X$):

1. **Symmetrie**: $d(x, y) = d(y, x)$

2. **Positive Definitheit**: $0 \leq d(x, y)$ und $d(x, y) = 0 \Rightarrow x = y$

3. **Dreiecksungleichung**: $d(x, z) \leq d(x, y) + d(y, z)$

Mithilfe von Cauchy-Folgen lassen sich nun vollständige metrische Räume konstruieren. Insbesondere brauchen wir kompakte metrische Räume, eine Teilmenge der vollständigen metrischen Räume.

"Raum der Fraktale"

Für eine gegebene Menge X bezeichnen wir zunächst die Menge aller kompakten Teilmengen von X mit $\mathcal{H}(X)$. Die leere Menge schließen wir hierbei aus.
  Um mit $\mathcal{H}(X)$ arbeiten zu können definieren wir passende Abstandsbegriffe:

- $d(x, A) = \min \{d(x, a) : a \in A\}$ ($x \in X$, $A \in \mathcal{H}(X)$) sei der Abstand zwischen einem Punkt $x \in X$ und einem Element A von $\mathcal{H}(X)$.

- $d(A, B) = \max \{d(x, B) : x \in A\}$ sei der Abstand zwischen zwei Mengen in $\mathcal{H}(X)$.

Mithilfe dieser Abstandsbegriffe konstruieren wir die *Hausdorff-Distanz* zwischen zwei Mengen $A, B \in \mathcal{H}(X)$ durch

$$h(A, B) := \max \{d(A, B), \ d(B, A)\}.$$

Die Hausdorff-Distanz bildet eine Metrik auf $\mathcal{H}(X)$.
  Wir erhalten den **Raum der Fraktale**

$$(\mathcal{H}(X), h).$$

Für uns ist — zumindest vorerst — jede Teilmenge von $(\mathcal{H}(X), h)$ ein Fraktal. Weiter kann gezeigt werden, dass folgender zentraler Satz über den Raum der Fraktale gilt:

> **Vollständigkeit des Raums der Fraktale**
>
> Es gilt
>
> $$(X, d) \text{ ist ein vollständiger metrischer Raum}$$
> $$\Rightarrow (\mathcal{H}(X), h) \text{ ist ein vollständiger metrischer Raum.}$$

Page Layout

Headings

This style provides A- and B-heads (that is, \section and \subsection), demonstrated above.

The Tufte-LATEX classes will emit an error if you try to use \subsubsection and smaller headings.

IN HIS LATER BOOKS,[1] Tufte starts each section with a bit of vertical space, a non-indented paragraph, and sets the first few words of the sentence in SMALL CAPS. To accomplish this using this style, use the \newthought command:

```
\newthought{In his later books}, Tufte starts...
```

[1] Edward R. Tufte. *Beautiful Evidence*. Graphics Press, LLC, first edition, May 2006. ISBN 0-9613921-7-7

Sidenotes

One of the most prominent and distinctive features of this style is the extensive use of sidenotes. There is a wide margin to provide ample room for sidenotes and small figures. Any \footnotes will automatically be converted to sidenotes.[2] If you'd like to place ancillary information in the margin without the sidenote mark (the superscript number), you can use the \marginnote command.

[2] This is a sidenote that was entered using the \footnote command.

This is a margin note. Notice that there isn't a number preceding the note, and there is no number in the main text where this note was written.

The specification of the \sidenote command is:

```
\sidenote[⟨number⟩][⟨offset⟩]{Sidenote text.}
```

Both the ⟨*number*⟩ and ⟨*offset*⟩ arguments are optional. If you provide a ⟨*number*⟩ argument, then that number will be used as the sidenote number. It will change of the number of the current sidenote only and will not affect the numbering sequence of subsequent sidenotes.

Sometimes a sidenote may run over the top of other text or graphics in the margin space. If this happens, you can adjust the vertical position of the sidenote by providing a dimension in the ⟨*offset*⟩ argument. Some examples of valid dimensions are:

```
1.0in    2.54cm    254mm    6\baselineskip
```

If the dimension is positive it will push the sidenote down the page; if the dimension is negative, it will move the sidenote up the page.

While both the ⟨*number*⟩ and ⟨*offset*⟩ arguments are optional, they must be provided in order. To adjust the vertical position of the sidenote while leaving the sidenote number alone, use the following syntax:

`\sidenote[][`⟨*offset*⟩`]{`*Sidenote text.*`}`

The empty brackets tell the `\sidenote` command to use the default sidenote number.

If you *only* want to change the sidenote number, however, you may completely omit the ⟨*offset*⟩ argument:

`\sidenote[`⟨*number*⟩`]{`*Sidenote text.*`}`

The `\marginnote` command has a similar *offset* argument:

`\marginnote[`⟨*offset*⟩`]{`*Margin note text.*`}`

References

References are placed alongside their citations as sidenotes, as well. This can be accomplished using the normal `\cite` command.[3]

The complete list of references may also be printed automatically by using the `\bibliography` command. (See the end of this document for an example.) If you do not want to print a bibliography at the end of your document, use the `\nobibliography` command in its place.

To enter multiple citations at one location,[4] you can provide a list of keys separated by commas and the same optional vertical offset argument: `\cite{Tufte2006,Tufte1990}`.

`\cite[`⟨*offset*⟩`]{`*bibkey1,bibkey2,...*`}`

Figures and Tables

Images and graphics play an integral role in Tufte's work. In addition to the standard figure and tabular environments, this style provides special figure and table environments for full-width floats.

Full page–width figures and tables may be placed in figure* or table* environments. To place figures or tables in the margin, use the marginfigure or margintable environments as follows (see figure 1):

```
\begin{marginfigure}
  \includegraphics{helix}
  \caption{This is a margin figure.}
\end{marginfigure}
```

The marginfigure and margintable environments accept an optional parameter ⟨*offset*⟩ that adjusts the vertical position of the figure or table. See the "Sidenotes" section above for examples. The specifications are:

```
\begin{marginfigure}[`⟨*offset*⟩`]
    ...
```

[3] The first paragraph of this document includes a citation.

[4] Edward R. Tufte. *Beautiful Evidence*. Graphics Press, LLC, first edition, May 2006. ISBN 0-9613921-7-7; and Edward R. Tufte. *Envisioning Information*. Graphics Press, Cheshire, Connecticut, 1990. ISBN 0-9613921-1-8

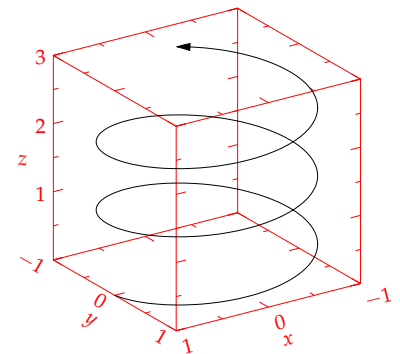

Abbildung 1: This is a margin figure. The helix is defined by $x = \cos(2\pi z)$, $y = \sin(2\pi z)$, and $z = [0,2,7]$. The figure was drawn using Asymptote (http://asymptote.sf.net/).

```
\end{marginfigure}

\begin{margintable}[⟨offset⟩]
    ...
\end{margintable}
```

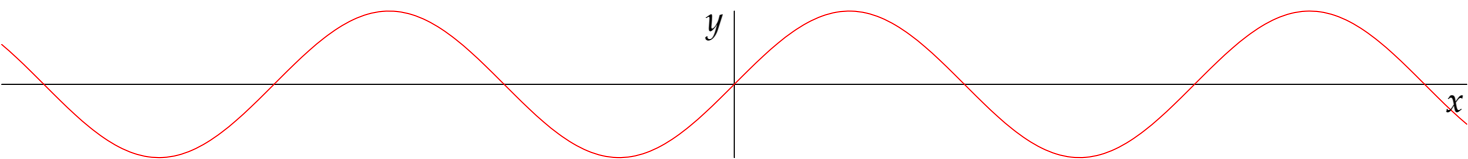Figure 2 is an example of the `figure*` environment and figure 3 is an example of the normal `figure` environment.

Abbildung 2: This graph shows $y = \sin x$ from about $x = [-10, 10]$. *Notice that this figure takes up the full page width.*
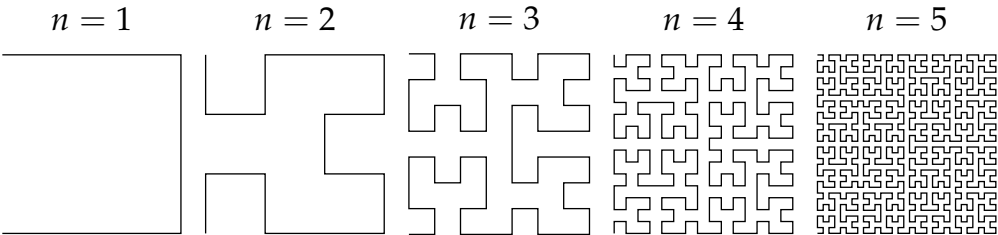

$n = 1$ $n = 2$ $n = 3$ $n = 4$ $n = 5$

Abbildung 3: Hilbert curves of various degrees n. *Notice that this figure only takes up the main textblock width.*

Table 1 shows table created with the `booktabs` package. Notice the lack of vertical rules—they serve only to clutter the table's data.

Margin	Length
Paper width	$8^{1}/_{2}$ inches
Paper height	11 inches
Textblock width	$6^{1}/_{2}$ inches
Textblock/sidenote gutter	$^{3}/_{8}$ inches
Sidenote width	2 inches

Tabelle 1: Here are the dimensions of the various margins used in the Tufte-handout class.

Full-width text blocks

In addition to the new float types, there is a `fullwidth` environment that stretches across the main text block and the sidenotes area.

```
\begin{fullwidth}
Lorem ipsum dolor sit amet...
\end{fullwidth}
```

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique

senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

Typography

Typefaces

If the Palatino, Helvetica, and `Bera Mono` typefaces are installed, this style will use them automatically. Otherwise, we'll fall back on the Computer Modern typefaces.

Letterspacing

This document class includes two new commands and some improvements on existing commands for letterspacing.

When setting strings of A L L   C A P S or sᴍᴀʟʟ ᴄᴀᴘs, the letterspacing—that is, the spacing between the letters—should be increased slightly.[5] The `\allcaps` command has proper letterspacing for strings of F U L L   C A P I T A L   L E T T E R S, and the `\smallcaps` command has letterspacing for sᴍᴀʟʟ ᴄᴀᴘɪᴛᴀʟ ʟᴇᴛᴛᴇʀs. These commands will also automatically convert the case of the text to upper- or lowercase, respectively.

The `\textsc` command has also been redefined to include letterspacing. The case of the `\textsc` argument is left as is, however. This allows one to use both uppercase and lowercase letters: Tʜᴇ Iɴɪᴛɪᴀʟ Lᴇᴛᴛᴇʀs Oғ Tʜᴇ Wᴏʀᴅs Iɴ Tʜɪs Sᴇɴᴛᴇɴᴄᴇ Aʀᴇ Cᴀᴘɪᴛᴀʟɪᴢᴇᴅ.

[5] Robert Bringhurst. *The Elements of Typography*. Hartley & Marks, 3.1 edition, 2005. ISBN 0-88179-205-5

Installation

To install the Tufte-LaTeX classes, simply drop the following files into the same directory as your `.tex` file:

```
tufte-book.cls
tufte-common.def
tufte-handout.cls
tufte.bst
```

More Documentation

For more documentation on the Tufte-LaTeX document classes (including commands not mentioned in this handout), please see the sample book.

Support

The website for the Tufte-LaTeX packages is located at https://github.com/Tufte-LaTeX/tufte-latex. On our website, you'll find links to our sᴠɴ repository, mailing lists,

bug tracker, and documentation.

Literatur

Robert Bringhurst. *The Elements of Typography*. Hartley & Marks, 3.1 edition, 2005.
    ISBN 0-88179-205-5.

Edward R. Tufte. *Envisioning Information*. Graphics Press, Cheshire, Connecticut, 1990.
    ISBN 0-9613921-1-8.

Edward R. Tufte. *Beautiful Evidence*. Graphics Press, LLC, first edition, May 2006.
    ISBN 0-9613921-7-7.