How many linearly separable 3-dimensional Boolean functions are there?

To answer this question we derive the number of separable functions for the functions mapping exactly k ($k = 0, 1, ..., 8$) patterns to 1 separately. We simplify the calculations by acknowledging that there is a symmetry between the functions for $k = 0$ and $k = 8$, $k = 1$ and $k = 7$, $k = 2$ and $k = 6$, $k = 3$ and $k = 5$, in the sense that they are each others negation. More specifically, their patterns are inversions of each others. So in this case, if a Boolean function is linearly separable, then so is the negation of that function. This means that the functions that have this symmetry have the same number of linearly separable functions. Hence we only have to derive the linearly separable functions for $k = 0, 1, 2, 3, 4$.

Before diving in to each case we also note that for two dimensions, XOR and XNOR are the linearly inseparable functions. This also holds for three dimensions, meaning that if either of the six sides of the "cube"(made up of all possible patterns) display any of these two problems, the function is linearly inseparable. In three dimensions we also have the case when patterns in opposite corners have target 1, which also implies inseparability for $k \leq 4$. Examples of these linearly inseparable functions are illustrated in Figure 1.

Taking the above notes into account we find that there is only one symmetry (here meaning cubes that can be mapped onto each other by rotation and/or reflection) for $k = 0, 1, 2, 3$ and two symmetries for $k = 4$ that are linearly separable. These symmetries are illustrated in Figure 2. Now we simply count the number of functions for each symmetry.

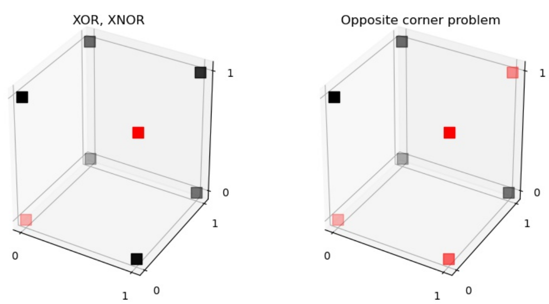

Figure 1: *Illustration of functions that are inseparable.*

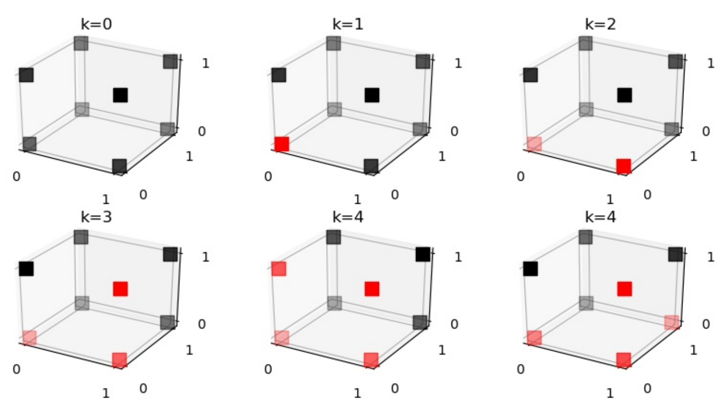

Figure 2: *Linearly separable symmetries for k = 0, 1, 2, 3, 4. Red squares represents target 1 and black squares represents target 0.*

In the table below we have the number of linearly separable functions for each of the symmetries represented in Figure 2. To obtain the total number (for $k = 0, 1, 2, 3, 4, 5, 6, 7, 8$) of linearly separable functions we calculate

$$(1 + 8 + 12 + 24) * 2 + 6 + 8 = 104.$$

k	0	1	2	3	4(1)	4(2)
Linearly separable functions	1	8	12	24	6	8