HW 2 Report

Jensen Davies

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Note that this report primarily includes results and commentary on Part 1 of the homework. The written solutions for the theory questions of the homework, Part 2, are in a PDF in the folder "Part2". Part 2 references are included at the bottom of this PDF.

Part 1: Programming Problems

In this portion of the homework, we're asked to program some basic Fortran routines (warm-up), and then program Gaussian Elimination with Partial Pivoting, and LU decomposition with Partial Pivoting. Note that there are only problems 2,3,4 and 5 (problem 1 doesn't exist in the HW PDF). Another note: All programs are run in the same driver file, and the output of the driver is stored in the textfile "output.txt". It didn't make sense to create multiple driver programs for this assignment.

Problem 2

This problem asks us write multiple subroutines and test them in a driver program. The first subroutine, which I named printMat, must print a matrix its dimensions. The second, stdNorm, takes a vector and its dimension, and outputs the Euclidean norm of that vector. Thus, for vector v with n entries, stdNorm(v, n, normVal) gives us

$$\text{normVal} = \sqrt{v_1^2 + v_2^2 + \cdots + v_n^2}.$$

The third subroutine, traceMat, outputs the trace of a given matrix. These three subroutines were tested with the matrix provided in the file "Amat.dat". The matrix is

$$A = \begin{bmatrix} 2 & 1 & 1 & 0 \\ 4 & 3 & 3 & 1 \\ 8 & 7 & 9 & 5 \\ 6 & 7 & 9 & 8 \end{bmatrix}.$$

Clearly the trace of the matrix is $2+3+9+8=22$, which is what traceMat returns. The 2-norm values of the columns of A are also correct, which were verified via MATLAB.

Problem 3

In this problem, the code written performs Gaussian Elimination on the matrix provided in the file "Amat.dat", and solves the system $AX=B$ utilizing backsubstitution and the provided "Bmat.dat" file. The algorithm was coded via the provided Gaussian Elimination with partial pivoting and backsubtituion pseudocode provided in lecture. The solution from solving the system $AX=B$ was verified through MATLAB, as well as through the norms of our error matrix $E=AX-B$. The solution matrix X is (leaving off a lot of precision for formats sake):

$$X = \begin{bmatrix} 0 & 3.5 & 0.25 & -2.01 & 360.27 & 10.63 \\ 1 & -6 & -0.5 & 33.4 & -1234.36 & -22.33 \\ -3 & 5 & -1.5 & 3.69 & 223.04 & 7.125 \end{bmatrix}.$$

The error matrix E is very close to a machine accuracy zero matrix, with its largest entries of order 10^{-13}, the same of which is true for the 2-norm values of each column of E. To see a double precision solution matrix X, open textfile "output.txt".

Problem 4

In this problem, the code written performs an LU decomposition on matrix A. It then solves the system $LUX=B$. It does this by first solving $LY=B$, and then $UX=Y$. The algorithm was coded via the provided LU decomposition with partial pivoting backsubtituion pseudocode provided in lecture. The solution matrix X from solving $AX=B$ via the LU decomposition for A yields the same X from Problem 3. The decomposition of A was checked via MATLAB. The error matrix $E=AX-B$ is again nearly a machine accuracy zero matrix, with largest entries on the order of 10^{-13}. The same holds for the 2-norm values of the columns of E. The LU decomposition of A is (leaving off a lot of precision for formats sake):

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.75 & 1 & 0 & 0 \\ 0.5 & -0.286 & 1 & 0 \\ 0.25 & -0.429 & 0.33 & 1 \end{bmatrix}, \quad U = \begin{bmatrix} 8 & 7 & 9 & 5 \\ 0 & 1.75 & 2.25 & 4.25 \\ 0 & 0 & -0.857 & -0.2857 \\ 0 & 0 & 0 & 0.66 \end{bmatrix}.$$

For full precision matrices, refer to the textfile "output.txt".

Problem 5

In this problem, the code solves for the normal coefficients a, b, c of the plane that contains the points

$$A=(x_1,y_1,z_1)=(1,2,3), \quad B=(x_2,y_2,z_2)=(-3,2,5), \quad C=(x_3,y_3,z_3)=(\pi,e,-\sqrt{2}).$$

Recall that the equation of a plane is $ax + by + cz + d = 0$, where the vector $[a, b, c]^T$ is the direction (normal) of the plane. Since we have three points (which aren't collinear) that all lie in this plane, we know that

$$ax_1 + by_1 + cz_1 + d = 0 \tag{1}$$
$$ax_2 + by_2 + cz_2 + d = 0 \tag{2}$$
$$ax_3 + by_3 + cz_3 + d = 0 \tag{3}$$

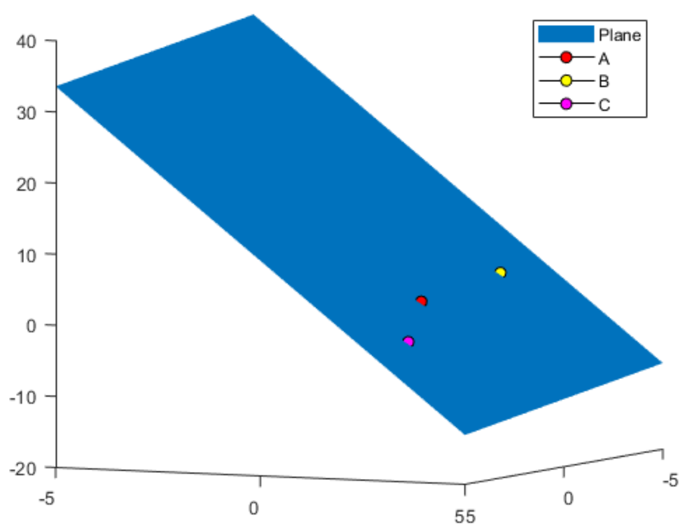
Assuming that the plane doesn't pass through the origin (i.e. d is nonzero), for convenience we can simply let $d = -1$. Then, our system becomes

$$\begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$$

Our code solves for these coefficients a, b, c by applying Gaussian Elimination to the above system, giving us

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0.00390335731207141351 \\ 0.36338249407750051 \\ 0.00780671462414282563 \end{bmatrix}$$

Below is the requested figure of the plane, with points A, B, and C clearly labeled.



Part 2: Theory References

Note: these problems are submitted as a handwritten PDF. But, any references found to help understand/construct solutions will be listed below.

Problem 5 References

I managed the first three inequalities on my own, and couldn't see the next step. I used this stackexchange article to understand the next inequality: https://math.stackexchange.com/questions/2485574/strictly-column-diagonally-dominant-matrices-and-gaussian-elimination-with-parti