**AM213B Assignment #0**

Problem 1

In Matlab, learn how to plot multiple curves in one figure and how to add a legend to denote individual curves.

Write a Matlab code to produce a figure like the one below. Make sure that

- you use fonts of suitable size for title, labels, and legend,
- you use lines of suitable line width to plots curves, and
- you use symbols of distinct shapes and colors.
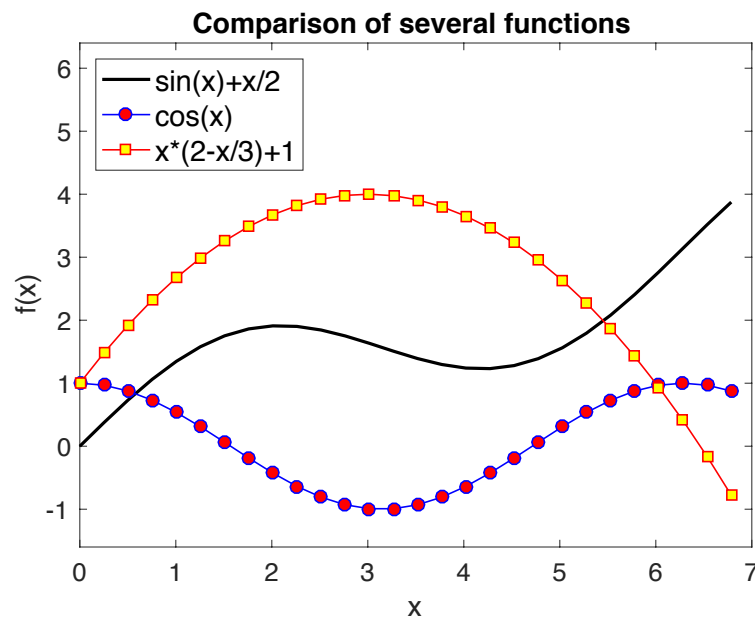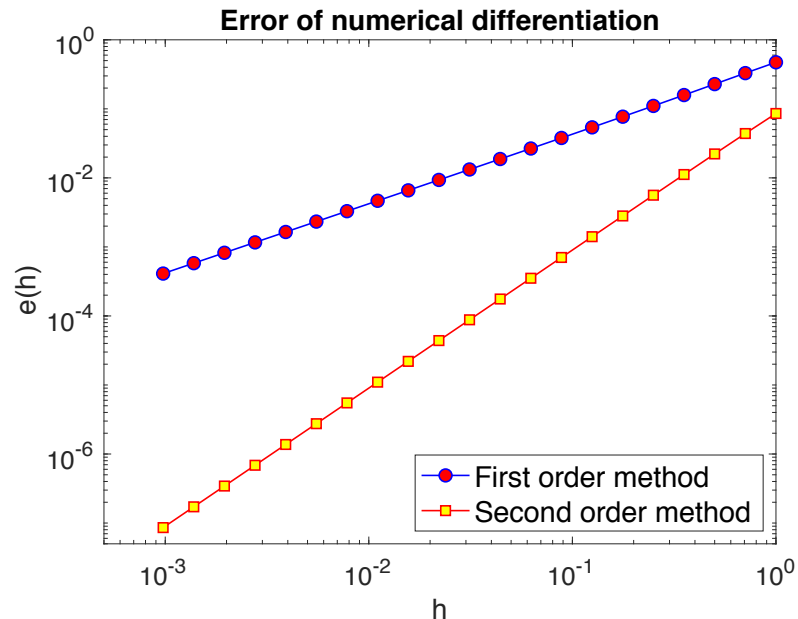

Problem 2

Use the first and the second order numerical methods to differentiate $f(x) = \sin(x)$ at $x = 1$. In this simple problem, the exact solution is known: $f'(x) = \cos(x)$.

The exact errors of the two methods are

$$er_1(h) = \left| \frac{f(x+h) - f(x)}{h} - f'(x) \right|, \qquad x = 1$$

$$er_2(h) = \left| \frac{f(x+h) - f(x-h)}{2h} - f'(x) \right|, \qquad x = 1$$

Calculate $er_1(h)$ and $er_2(h)$ for h=2.^(–[0:0.5:10]). Plot $er_1(h)$ vs h and $er_2(h)$ vs h in one figure. Use log-log plot to accommodate the wide ranges of h and errors. Add a legend. You should get a figure like the one below.

Problem 3

Review the summation of geometric series. Work out

- $$\sum_{n=0}^{N-1} r^n = \frac{r^N - 1}{r-1}$$

- $$\sum_{n=0}^{N-1} r^{n+m} = r^m \sum_{n=0}^{N-1} r^n = r^m \cdot \frac{r^N - 1}{r-1}$$

Problem 4

Show that $1+\alpha \leq \exp(\alpha)$ for all α.

Problem 5

Consider function $f(x) \equiv \dfrac{\exp(x)-1}{x}$. Show that

   i) $x = 0$ is a removable discontinuity of $f(x)$ and

   ii) $f(x)$ is an increasing function of x.

Problem 6

Review the technique of Taylor expansion.

Consider the second order numerical differentiation for approximating $f''(x)$

$$\underbrace{\frac{f(x+h)-2f(x)+f(x-h)}{h^2}}_{\text{Numerical approximation}} = \underbrace{f''(x)}_{\text{Exact}} + \underbrace{e(h)}_{\substack{\text{Discretization} \\ \text{error}}}$$

Use the Taylor expansion to show that

$$e(h) = f^{(4)}(x)\frac{h^2}{12}+\cdots = C \cdot h^2 + \cdots = O(h^2)$$