HW6 Report

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May 2021

Problems 1 and 2:

Both problem 1 and problem 2 are theoretical and are appended in a handwritten format to the end of this report.

Problem 3:

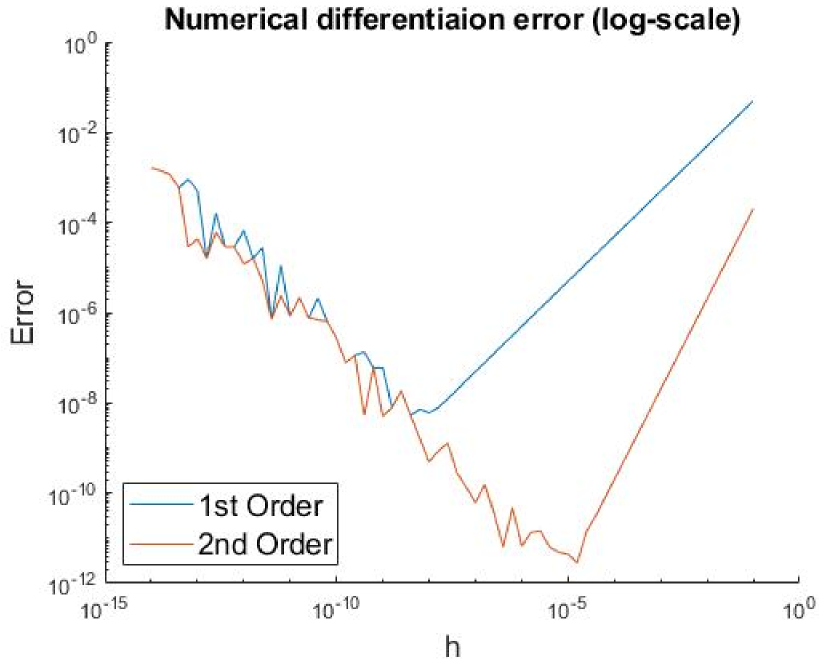
In this problem, we use numerical differentiation (first and second order) to approximate

$$\frac{d}{dx}\sin(x), \quad x = 1.45.$$

The purpose of this is to illustrate that there is an optimal value of h to achieve minimum error, and that a higher order method results in less error. We compare our numerical derivatives against the values of the actual derivative, $\cos(x)$. Below, we provide a loglog plot of both

$$E_{T,1}(h) = |q_1(h) - \cos(x)|, \quad E_{T,2}(h) = |q_2(h) - \cos(x)|,$$

versus h, where $q_1(h)$ and $q_2(h)$ are the first and second order approximations of the derivative, respectively.



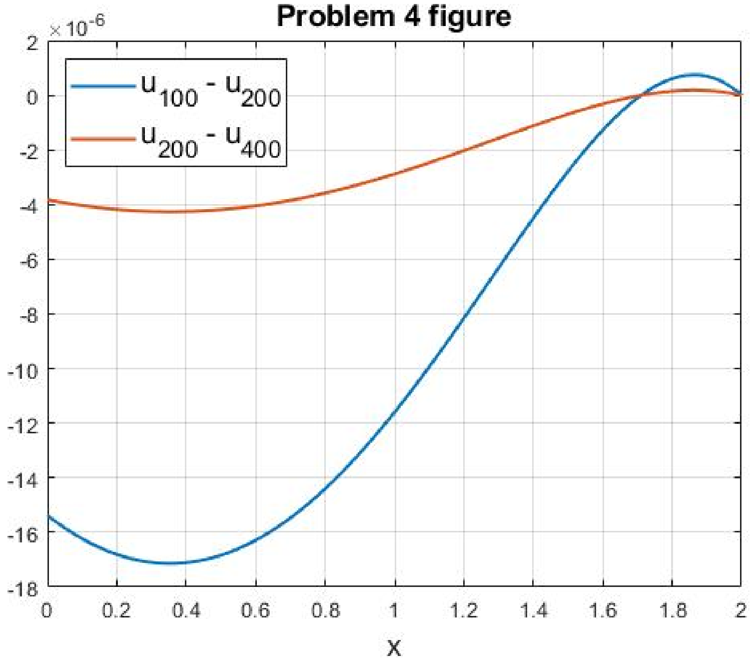
Problem 4:

In this problem, we continue with the same IBVP from problem 6 in assignment 5,

$$\begin{cases} u_t = u_{xx}, \quad x \in (0, L), \;\; t > 0 \\ u(x,0) = p(x), \quad x \in (0, L) \\ u_x(0,t) - \alpha u(0,t) = 0., \quad u(L,t) = q(t) \end{cases}$$

with $L = 2,\ \alpha = 0.4,\ p(x) = (1 - 0.5x)^2,\ q(t) = 2\sin^2(t)$.

In this problem, we solve the IBVP with differing spatial grids, namely when $N = 100,\ N = 200,$ and $N = 400$. The method we use is the FTCS method, and we solve to $T = 3$ with $\Delta t = 10^{-5}$. We then use the spline function in MATLAB to map the solution for each value of N to the grid $x = [0 : 0.002 : 1]^{*}L$. Below we plot $(u_{\{N=100\}} - u_{\{N=200\}})$ vs x and then $(u_{\{N=200\}} - u_{\{N=400\}})$ vs x.



Problem 5:

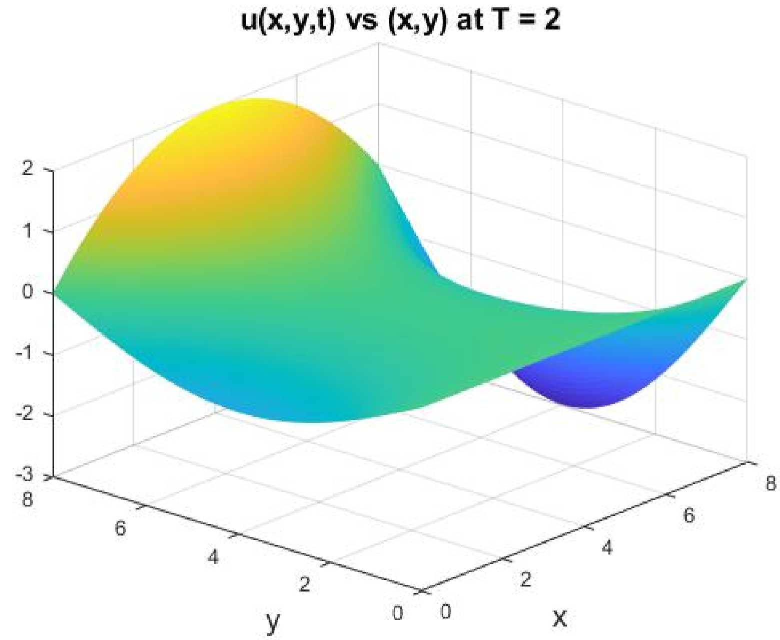
In this problem, we solve the 2D IBVP problem

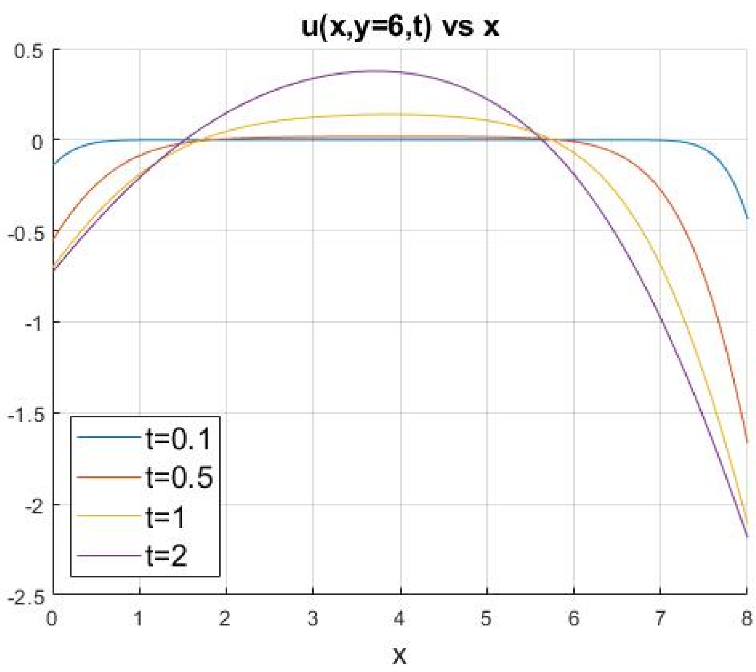
$$\begin{cases} u_t = u_{xx} + u_{yy}, \quad (x, y) \in (0, 8) \times (0, 8), \ \ t > 0 \\ u(x, y, 0) = f(x, y) \\ u(0, y, t) = g_L(y, t), \quad u(8, y, t) = g_R(y, t) \\ u(x, 0, t) = g_B(x, t), \quad u(x, 8, t) = g_T(x, t) \end{cases}$$

where

$$f(x, y) = 0$$
$$g_L(y, t) = -\sin(\pi y/8)\tanh(2t), \quad g_R(y, t) = -3\sin(\pi y/8)\tanh(2t)$$
$$g_B(x, t) = 0, g_T(x, t) = x(1 - x/8)\tanh(2t).$$

We solve to $T = 2$ using the 2D FTCS method, with $\Delta x = \Delta y = 0.08$ and $\Delta t = 1.25 \times 10^{-3}$. Below we plot $u(x, 6)$ vs x at $t = 0.1$, $0.5\,0.1\,2$ in one figure, and $u(x, y, t)$ vs (x, y) at $T = 2$ as a surface in another figure.


u(x,y,t) vs (x,y) at T = 2
2
1
0
-1
-2
-3
8
6
4
2
0
y
0
2
4
6
8
x


u(x,y=6,t) vs x
0.5
0
-0.5
-1
-1.5
-2
-2.5
0
1
2
3
4
5
6
7
8
x
t=0.1
t=0.5
t=1
t=2

Problem 6:

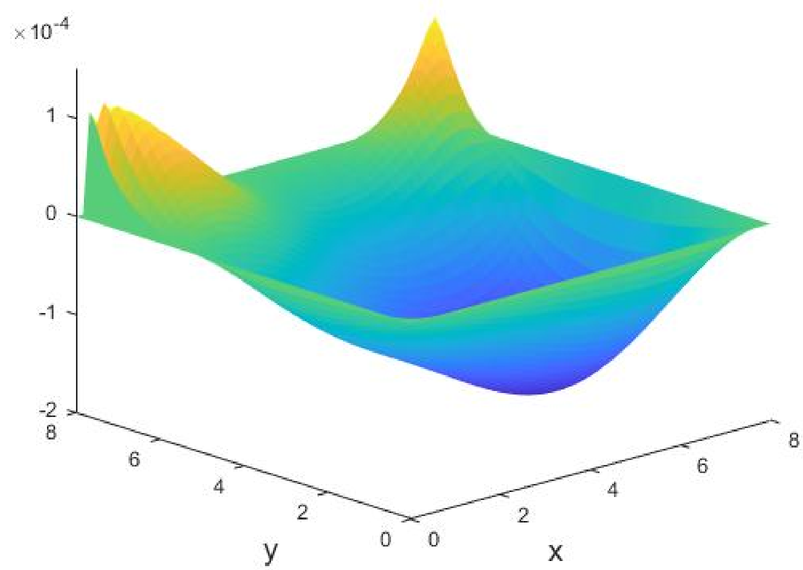
This problem is a continuation of the problem above. Using $\Delta x = \Delta y = 0.08$ and $\Delta t = 1.25 \times 10^{-3}$, we first solve the IBVP to $T = 20$. For each time level, we calculate the max value of $|u_t|$ at all of the internal points. We store the maximum at each t_n in the vector

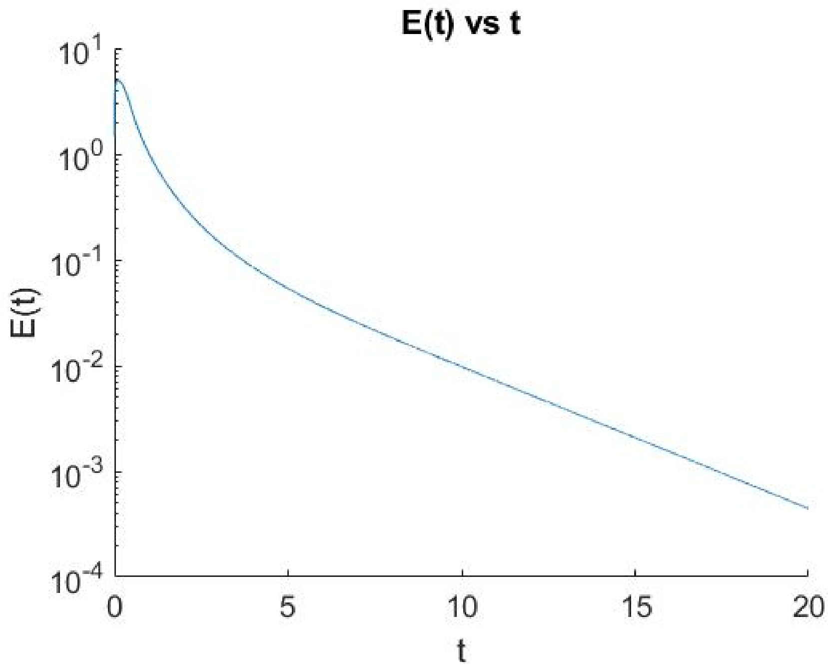
$$E(t_n) = \max \frac{|u_{i,j}^{n+1} - u_{i,j}^{n}|}{\Delta t}, \quad 1 \le i \le N_x - 1, \quad 1 \le j \le N_y - 1.$$

We then plot $E(t)$ against t, where $E(t)$ is on a logarithmic scale (t on a linear scale).

We also re-solve the IBVP using $\Delta x = \Delta y = 0.16$ and $\Delta t = 1.25 \times 10^{-3}$, using the new solution to calculate $u_{\{\Delta=0.16\}} - u_{\{\Delta=0.08\}}$ for $T = 20$, and then plot the result vs (x, y) as a surface.

$u_{\{\Delta=0.16\}} - u_{\{\Delta=0.08\}}$ at T = 20.




E(t) vs t
E(t)
t
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
0
5
10
15
20