Problem 2

Prove the following statements:

1. $\mathbb{P}(A^c) = 1 - \mathbb{P}(A)$

   By definition of set complements, we can see that A^c and A are disjoint sets. Using **Axiom 3** of the definition of probability, we see that $\mathbb{P}\Omega = \mathbb{P}(A^c \cup A) = \mathbb{P}(A^c) + \mathbb{P}(A)$. Since the probability of the sample space is 1, we can then solve for $\mathbb{P}(A^c) = 1 - \mathbb{P}(A)$.

2. $\mathbb{P}(\emptyset) = 0$

   Using 1 we know that $\mathbb{P}(\emptyset) = 1 - \mathbb{P}(\Omega) = 0$.

3. If $A \subset B$ then $\mathbb{P}(A) \leq \mathbb{P}(B)$

   Since A is a subset of B, then we must have $B = A \cup (B - A)$, a union of two disjoint sets. Therefore $\mathbb{P}(B) = \mathbb{P}(A) + \mathbb{P}(B - A)$. Since $\mathbb{P}(B - A)$ is at least zero, this means that $\mathbb{P}(B) \geq \mathbb{P}(A)$ and equality only holds if $\mathbb{P}(B - A) = 0$.