1 Problem:

Minimize

$$g(m) = \frac{1}{m} - m \tag{1}$$

where m is a complex number.

2 Solution:

Choose to represent m in vector notation where $m = M\underline{/\theta}$:

$$g(M\underline{/\theta}) = \frac{1}{M}\underline{/-\theta} - M\underline{/\theta} \tag{2}$$

where both M and θ are real numbers.

Expand this into vector notation into rectangular form:

$$\begin{aligned}
&= \frac{1}{M}[\cos(-\theta) + j\sin(-\theta)] - M[\cos(\theta) + j\sin(\theta)] \\
&= \frac{1}{M}\cos\theta - j\frac{1}{M}\sin\theta - M\cos\theta - jM\sin\theta \\
&= (\frac{1}{M} - M)\cos\theta - j(\frac{1}{M} + M)\sin\theta
\end{aligned}$$

Express the magnitude of the complex number $g(M\underline{/\theta})$:

$$|g(M\underline{/\theta})| = \sqrt{\left(\left(\frac{1}{M} - M\right)\cos\theta\right)^2 + \left(\left(\frac{1}{M} + M\right)\sin\theta\right)^2} \tag{3}$$

Minimizing the square of the magnitude is the same as minimizing the magnitude itself, and it is more convenient:

$$
\begin{aligned}
|g(M\underline{/\theta})|^2 &= \left(\left(\frac{1}{M} - M\right)\cos\theta\right)^2 + \left(\left(\frac{1}{M} + M\right)\sin\theta\right)^2 \\
&= \left(\frac{1}{M} - M\right)^2 \cos^2\theta + \left(\frac{1}{M} + M\right)^2 \sin^2\theta \\
&= \left(\frac{1}{M^2} - 2 + M^2\right)\cos^2\theta + \left(\frac{1}{M^2} + 2 + M^2\right)\sin^2\theta \\
&= \frac{\cos^2\theta}{M^2} - 2\cos^2\theta + M^2\cos^2\theta + \frac{\sin^2\theta}{M^2} + 2\sin^2\theta + M^2\sin^2\theta \qquad =
\end{aligned}
$$