Bayes Theorem

Formula

$P(A \mid B) = \frac{P(B|A)\,P(A)}{P(B)}$

$P(Theory \mid Data) = \frac{P(Data|Theory)\,P(Theory)}{P(Data)}$

$Posterior : P(Theory \mid Data) \quad Likelihood : P(Data \mid Theory) \quad Prior : P(Theory) \quad Evidenz : P(Data)$

Examples

Diagnostics of AIDS

1 out of thousands has AIDS.

Test:

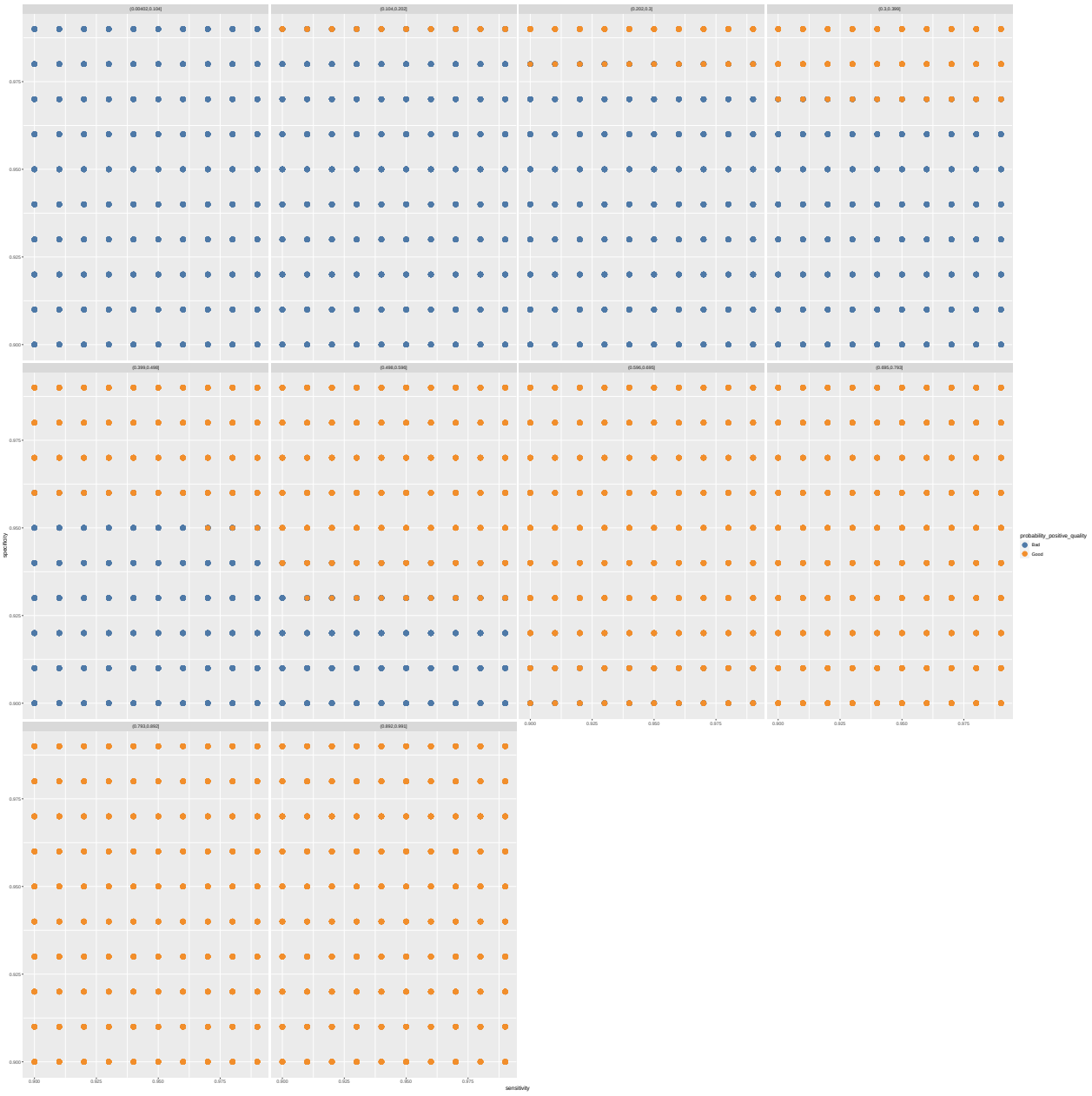
Sensitivity: P(+|AIDS) = 0.98

Specificity: P(-|noAIDS) = 0.97

$P(+|AIDS) = 0.98 \quad P(-|AIDS) = 0.02 \quad P(+|noAIDS) = 0.03 \quad P(-|noAIDS) = 0.97$

$P(AIDS \mid +) = \frac{P(+|AIDS)\,P(AIDS)}{P(+|AIDS)P(AIDS)+P(+|noAIDS)P(noAIDS)} = \frac{0.98 \times 0.001}{0.98 \times 0.001 + 0.03 \times 0.999} = 0.032$

```
## # A tibble: 19,800 x 4
##    infection_rate sensitivity specificity probability_positive
##             <dbl>       <dbl>       <dbl>                <dbl>
##  1          0.005         0.9         0.9                0.0433
##  2          0.005         0.9         0.91               0.0478
##  3          0.005         0.9         0.92               0.0535
##  4          0.005         0.9         0.93               0.0607
##  5          0.005         0.9         0.94               0.0701
##  6          0.005         0.9         0.95               0.0829
##  7          0.005         0.9         0.96               0.102
##  8          0.005         0.9         0.97               0.131
##  9          0.005         0.9         0.98               0.184
## 10          0.005         0.9         0.99               0.311
## # ... with 19,790 more rows
```


(0.00402,0.104]
(0.104,0.202]
(0.202,0.3]
(0.3,0.399]
(0.399,0.498]
(0.498,0.596]
(0.596,0.695]
(0.695,0.793]
(0.793,0.892]
(0.892,0.991]
specificity
sensitivity
probability_positive_quality
Bad
Good