INF5620 Exam, Problem 5

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Nonlinear diffusion

$$u_t = \nabla \cdot (\alpha(u)\nabla u), \quad x \in \Omega, t > 0 \tag{1}$$
$$u(x,0) = I(x), \quad x \in \Omega \tag{2}$$
$$u(x,t) = g(x), \quad x \in \partial\Omega_D,\ t > 0 \tag{3}$$
$$-\alpha(u)\frac{\partial}{\partial n}u(x,t) = h(u - T_s), \quad x \in \partial\Omega_R \tag{4}$$
$$\alpha(u) = (1 + \alpha_0 u^4) \tag{5}$$

a) Compact difference notation CN,

$$[D_t u = \nabla \cdot (\overline{\alpha(u)}^t \nabla u)]^{n+\frac{1}{2}} \tag{6}$$

Written out

$$\frac{u^{n+1} - u^n}{\Delta t} = \nabla \cdot (\overline{\alpha(u^{n+\frac{1}{2}})}^t \nabla u^{n+\frac{1}{2}}) \tag{7}$$
$$\frac{u^{n+1} - u^n}{\Delta t} = \frac{1}{4}(\nabla \cdot (\alpha(u^n) + \alpha(u^{n+1})\nabla(u^n + u^{n+1})) \tag{8}$$

Picard

$$F(u^{n+1}) = u^{n+1} - u^n - \Delta t\frac{1}{4}(\nabla \cdot (\alpha(u^n) + \alpha(u^{n+1})\nabla(u^n + u^{n+1})) \tag{9}$$

Replace u^{n+1} with $u^{n+1,k}$, where k is the picard iteration index, in all the non-linear terms

$$F(u^{n+1}) = u^{n+1} - u^n - \Delta t\frac{1}{4}(\nabla \cdot (\alpha(u^n) + \alpha(u^{n+1,k})\nabla(u^n + u^{n+1,k})) \tag{10}$$

b) Perform a Backward-Euler time discretization and derive the variational form for the resulting spatial problems. Picard iteration method to linearize

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c) Newton's method to the nonlinear variational form F = 0 in b).

$$J_{i,j}\delta u = -F_i \tag{11}$$

$$J_{i,j} = \frac{\partial F_i}{\partial u_j}$$