INF5620 Exam, Problem 3

Jens Kristoffer Reitan Markussen

December 16, 2013

Analysis of wave problems

a)

$$u''(t) + \omega^2 u(t) = 0, \tag{1}$$
$$u(0) = I \tag{2}$$
$$u'(0) = V \tag{3}$$

Compact differentiation notation

$$[D_t D_t u + \omega^2 u(t) = 0]^n$$

Explain how the stability and the accuracy of this scheme can be investigated via exact solutions of the discrete equations.Illustrate the numerical problems that can arise from this scheme.

...

b) Wave equation

$$u_{tt} = c^2 u_{xx} \tag{4}$$
$$u_x = 0 \tag{5}$$
$$u(0) = I \tag{6}$$
$$u_t(0) = V \tag{7}$$

Compact differentiation notation

$$[D_t D_t u = c^2 D_x D_x]_i^n$$

Explain how the stability and the accuracy of this scheme can be investigated via exact solutions of the discrete equations.

...

c) Explain how the analysis can help us to understand why a smooth initial condition gives relatively small numerical artifacts, and why a less smooth initial condition gives rise to significant numerical artifacts.

...

d) Truncation error analysis on a), correction term so the order of the scheme becomes Δt^4

...

e) Truncation error analysis on b), correction term so the order of the scheme becomes Δt^4, Δx^4

...