ProblemSet2_4

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Generate functions to get `m` Monte Carlo samples. The original sample size is `n` and distribution is `dis`. Use bootstrap to estimate coverage probability and width of confidence interval, and then use robust estimator to estimate coverage probability and width of confidence interval.

```
## Generate functions to get m Monte Carlo samples
mcsamp=function(n,dis,m){
  if(dis=='t'){
    samp=rt(m*n,df=10)
    z=qt(1-{1-.95}/2,df=10)
    qm=qt(0.5,df=10)
  }
  else if(dis=='normal'){
    samp=rnorm(m*n)
    z=qnorm(1-{1-.95}/2)
    qm=qnorm(0.5)
  }
  else if(dis=='uniform'){
    samp=runif(m*n)
    z=qunif(1-{1-.95}/2)
    qm=qunif(0.5)
  }
  else if(dis=='exponential'){
    samp=rexp(m*n)
    z=qexp(.95)
    qm=qexp(0.5)
  }
  else
    return('Use another distribution')
  return(list(bsample=samp,bquantile=z,bmedian=qm,bnumber=n,mcnumber=m))
}
## Use bootstrap to estimate coverage probability and width of confidence interval
bootstrap_est=function(samp,z,qm,n,m){
  dim(samp)=c(m,n)
  interval=c()
  for(i in 1:m){
    bsamp=sample(samp[i,],1000*n,replace = TRUE)
    dim(bsamp)=c(1000,n)
    me=apply(bsamp,2,median)
    interval = rbind(interval,quantile(me,c(0.025,0.975)))
  }
  width=mean(interval[,2]-interval[,1])
  coverage = sum(qm>=interval[,1]&qm<=interval[,2])/m
  return(c(coverage,width))
}
## Use robust estimator to estimate coverage probability and width of confidence interval
robust_est=function(samp,z,qm,n,m){
  dim(samp)=c(m,n)
  me=apply(samp,1,median)
  ma=apply(samp,1,mad)
```

```
  se=z*1.49*ma/sqrt(n)
  width =2*mean(se)
  coverage = sum(qm>=me-se&qm<=me+se)/m
  return(c(coverage,width))
}
```

For sample size = 500, 750, 1000, calculate coverage probability and width of confidence interval.

```
## For sample sizes, calculate coverage probability and width of confidence interval
for(i in c(500,750,1000)){
  for(j in c('normal','t','uniform','exponential')){
    cat("sample:",i,"distribution:",j,"\n")
    sampdata=mcsamp(i,j,1000)
    re.boot=bootstrap_est(sampdata$bsample,sampdata$bquantile,sampdata$bmedian,\
                          sampdata$bnumber,sampdata$mcnumber)
    cat("method: bootstrap, coverage probability:",re.boot[1],", width:",re.boot[2],"\n")
    re.robust=robust_est(sampdata$bsample,sampdata$bquantile,sampdata$bmedian,\
                         sampdata$bnumber,sampdata$mcnumber)
    cat("method: robust, coverage probability:",re.robust[1],", width:",re.robust[2],"\n")
    cat("\n")
  }
}
```

Hide the result and reformat it to the tables below.

Coverage Probability Table(bootstrap/robust)

Distribution&Size	500	750	1000
N(0, 1)	0.819 / 0.981	0.91 / 0.981	0.945 / 0.978
t(10)	0.829 / 0.992	0.913 / 0.992	0.955 / 0.99
U(0, 1)	0.819 / 0.706	0.888 / 0.706	0.957 / 0.733
exp(1)	0.842 / 0.997	0.924 / 0.999	0.951 / 0.998

Width Table(bootstrap/robust)

Distribution&Size	500	750	1000
N(0, 1)	0.1527747 / 0.2607659	0.1544315 / 0.2132925	0.1549248 / 0.1841783
t(10)	0.1564531 / 0.3078745	0.1582663 / 0.250752	0.1580319 / 0.2175079
U(0, 1)	0.06089743 / 0.04797614	0.06120196 / 0.03923175	0.06152766 / 0.03400253
exp(1)	0.1231505 / 0.2839198	0.1238471 / 0.2317532	0.12322 / 0.2011469

We can get some rules from tables:

- By both methods I can get satisfactory results that are big coverage probability and small width of CI.
- Bigger coverage probability means bigger width of CI for the same sample size and distribution.
- Increase of samples results in increase of coverage probability by bootstrap but nothing by robust.
- Increase of samples results in decrease of width of CIs by robust but nothing by bootstrap.
- Data varies a lot for different distributions. Uniform distribution has smaller width by both methods and smaller coverage probability by robust.