ProblemSet2_3

Name: Zhen Qin, Uniqname: qinzhen, UMID: 48800866, Dept: Statistics

a.

Write a function to generate n iid samples from the square $\{(x_1, x_2) : |x_1| \leq 1, |x_2| \leq 1\}$.

```
## Generate n iid samples from the square
genmc=function(x){
  x1=runif(x,-1,1)
  x2=runif(x,-1,1)
  return(as.data.frame(cbind(x1,x2)))
}
```

b.

Let X be a Bernoulli random variable. $X = 1$ when the point is in the unit circle, otherwise $X = 0$, so $X \sim Ber(\frac{\pi}{4}), EX = \frac{\pi}{4}, VarX = \frac{\pi}{4}(1 - \frac{\pi}{4})$. Denote $Y = 4X$. $EY = 4EX = \pi, VarY = 16(EX^2 - (EX)^2) = \pi(4 - \pi)$. According to the theory, $\sqrt{n}(\overline{\theta}_n - \theta) \rightarrow_d N(0, \sigma^2)$. I estimated π because $\overline{\theta}_n \rightarrow \theta, \theta = EY = \pi, \overline{\theta}_n = \sum_n Y_i/n = 4\sum_n X_i/n = 4\overline{x}$. Then I estimate the constant π.

```
## Estimate the constant Pi
set.seed(1)
mcdf=genmc(100000)
mn =sum(mcdf$x1^2+mcdf$x2^2<=1)*1.0/100000
cat("The estimator of pi is",mn*4,".\n")
```

```
## The estimator of pi is 3.13648 .
```

c.

According to theory, the confidence interval is $\overline{\theta}_n - \frac{1}{\sqrt{n}}\hat{\sigma}qnorm \leq \theta \leq \overline{\theta}_n + \frac{1}{\sqrt{n}}\hat{\sigma}qnorm, \hat{\sigma}^2 = 16(\overline{x} - \overline{x}^2)$. The interval cover the true value.

```
## Compute a 95% confidence interval
m = qnorm(1-{1-.95}/2)
se = sqrt(mn*(1-mn)/100000)
lcb = mn - m*se
ucb = mn + m*se
cat("The confidence interval is (",4*lcb,",",4*ucb,").\n")
```

```
## The confidence interval is ( 3.12628 , 3.14668 ).
```

d.

To estimate two significant digits accurately with 99% confidence, make sure that $0.05 \geq qnorm \times \frac{\hat{\sigma}}{\sqrt{n}}$ at level 0.99. So $\sqrt{n} \geq qnorm \times \frac{\hat{\sigma}}{0.05}$. Since $\hat{\sigma} = 4\sqrt{E(X)(1 - E(X))} \leq 2$, we need $n \geq 10616$. Then the 99% CI is accurate to two significant digits.

```
## Estimate two significant digits accurately with 99% confidence
m1 = qnorm(1-{1-.99}/2)
n1 = ceiling((4*m1*0.5/0.05)^2)
cat("The sample size is",n1,".\n")
```

```
## The sample size is 10616 .
```

```r
set.seed(100000)
mcdf1 = genmc(n1)
mn1 =sum(mcdf1$x1^2+mcdf1$x2^2<=1)*1.0/n1
se1 = sqrt(mn1*(1-mn1)/n1)
lcb1 = mn1 - m1*se1
ucb1 = mn1 + m1*se1
cat("The estimator of pi is",mn1*4,".\n")
```

```
## The estimator of pi is 3.120573 .
```

```r
cat("The confidence interval is (",4*lcb1,",",4*ucb1,").\n")
```

```
## The confidence interval is ( 3.079158 , 3.161987 ).
```

e.

Monte Carlo estimate to get an estimator of the same constant is the same. No need to modify the variance because X is the same.

```r
## Repeat this exercise using the smaller square
set.seed(200000)
mcdf2=abs(mcdf)
mn2=sum(mcdf2$x1^2+mcdf2$x2^2<=1)*1.0/100000
cat("The estimator of pi is",mn2*4,".\n")
```

```
## The estimator of pi is 3.13648 .
```

```r
se2= sqrt(mn2*(1-mn2)/100000)
lcb2 = mn2 - m*se2
ucb2 = mn2 + m*se2
cat("The confidence interval is (",4*lcb2,",",4*ucb2,").\n")
```

```
## The confidence interval is ( 3.12628 , 3.14668 ).
```