Versuch V: Aufschwungsteuerung des Doppelpendels

Andreas Jentsch, Ali Kerem Sacakli Praktikumsbericht – Praktikum Matlab/Simulink II 04. Juli 2017





4.8 Berechnung der Trajektorien

In diesem Versuch geht es um den Entwurf und der Simulation einer Aufschwungsteuerung für den Doppelpendel. Hierzu werden zunächst Codes generiert, um die Trajektorien zu generieren. Im nächsten Kapitel folgen die Simulationsergebnisse.

Die relevanten Codes sind wie folgt:

Listing 4.1: Quellcode der Funktion RandwertproblemDGL

```
function dxdt = RandwertproblemDGL(t, x, stPendel, Q, R)
      % t:
                       Zeit
      % x:
                       Zustände x(1) und x(2)
      % Pendeldaten: Pendeldaten
      % Q:
                       Gewichtungsmatrix für Zustände
      % R:
                      Gewichtung für Stellgröße
      % 11,12,m1,m2,g,Rp1,Rp2,q1,q2,q3,q4,sym_R
      11 = stPendel.11;
      12 = stPendel.12;
11
      g = stPendel.g;
      m1 = stPendel.m1;
      m2 = stPendel.m2;
      Rp1 = stPendel.Rp1;
      Rp2 = stPendel.Rp2;
16
      q1 = Q(1,1);
      q2 = Q(2,2);
      q3 = Q(3,3);
      q4 = Q(4,4);
      sym_R = R;
      dxdt = 0;
      % Differentialgleichung 8ter Ordnung,
      % x1-x4: Originale Zustände,
      % x5-x8: Lagrange-Multiplikatoren
      phi1 = x(1);
      dphi1 = x(2);
      phi2 = x(3);
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      dphi2 = x(4);
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lambda1 = x(5);
                                       lambda2 = x(6);
                                       lambda3 = x(7);
                                       lambda4 = x(8);
36
                                       dx1 = dphi1;
                                       dx2 = -(3*(4*12*((12*lambda2)/(11^2*sym_R*(4*m1 + 12*m2 - 9*m2*)
                                                        \leftarrowcos(phi1 - phi2)^2)) - (18*lambda4*cos(phi1 - phi2))/(11*l2*\rightarrow
                                                        \leftarrow \text{sym}_R * (4 * \text{m1} + 12 * \text{m2} - 9 * \text{m2} * \cos(\text{phi1} - \text{phi2})^2))) + 4 * \text{Rp1} * \rightarrow
                                                        ←dphi1*12 + 6*Rp2*dphi1*l1*cos(phi1 - phi2) - 6*Rp2*dphi2*l1*→
                                                        \leftarrowcos(phi1 - phi2) + 2*dphi2^2*11*12^2*m2*sin(phi1 - phi2) + \rightarrow
                                                       \leftarrow 2 \cdot g \cdot 11 \cdot 12 \cdot m1 \cdot sin(phi1) + 4 \cdot g \cdot 11 \cdot 12 \cdot m2 \cdot sin(phi1) + 3 \cdot dphi1 \cdot 2 \cdot \rightarrow 2 \cdot g \cdot 11 \cdot 12 \cdot m1 \cdot sin(phi1) + 3 \cdot dphi1 \cdot 2 \cdot 3 \cdot dphi1 \cdot 2 \cdot
                                                        \leftarrow 11^2*12*m2*cos(phi1 - phi2)*sin(phi1 - phi2) - 3*g*11*12*m2*\rightarrow 3*g*11*12*m2*
                                                        \leftarrowcos(phi1 - phi2)*sin(phi2)))/(11^2*12*(4*m1 + 12*m2 - 9*m2*\rightarrow
                                                        ←cos(phi1 - phi2)^2));
41
                                      dx3 = dphi2;
                                       dx4 = (3*(6*12*m2*cos(phi1 - phi2)*((12*lambda2)/(11^2*sym_R*(4*)))
                                                        \leftarrowm1 + 12*m2 - 9*m2*cos(phi1 - phi2)^2)) - (18*lambda4*cos(\rightarrow
                                                        \epsilonphi1 - phi2))/(11*12*sym_R*(4*m1 + 12*m2 - 9*m2*cos(phi1 - \rightarrow
                                                        ←phi2)^2))) + 4*Rp2*dphi1*11*m1 + 12*Rp2*dphi1*11*m2 - 4*Rp2*→
                                                        \epsilondphi2*11*m1 - 12*Rp2*dphi2*11*m2 + 6*dphi1^2*11^2*12*m2^2*\pm
                                                        \leftarrow \sin(\text{phi1} - \text{phi2}) - 6 \cdot \text{g} \cdot 11 \cdot 12 \cdot \text{m2} \cdot 2 \cdot \sin(\text{phi2}) + 6 \cdot \text{Rp1} \cdot \text{dphi1} \cdot 12 \rightarrow
                                                        \leftarrow *m2*cos(phi1 - phi2) + 3*dphi2^2*l1*l2^2*m2^2*cos(phi1 - \rightarrow
                                                        \epsilonphi2)*sin(phi1 - phi2) + 2*dphi1^2*11^2*12*m1*m2*sin(phi1 - \epsilon
                                                        \leftarrowphi2) + 6*g*11*12*m2^2*cos(phi1 - phi2)*sin(phi1) - 2*g*11*\rightarrow
                                                        \leftarrow 12 \times m1 \times m2 \times sin(phi2) + 3 \times g \times 11 \times 12 \times m1 \times m2 \times cos(phi1 - phi2) \times sin(\rightarrow ph
                                                        (+phi1))/(11*12^2*m2*(4*m1 + 12*m2 - 9*m2*cos(phi1 - phi2)^2)
                                                        (→
                                       dx5 = lambda2*((3*(4*12*((18*lambda4*sin(phi1 - phi2))/(l1*12*)
                                                        ←sym_R*...
                                                               (4*m1 + 12*m2 - 9*m2*\cos(phi1 - phi2)^2)) - (216*lambda2*m2*\rightarrow
                                                                                ←cos(phi1 - phi2)...
                                                              *\sin(\phi) (11^2*sym_R*(4*m1 + 12*m2 - 9*m2*cos(phi1→
                                                                                ← - phi2)^2)...
                                                              + (324*lambda4*m2*cos(phi1 - phi2)^2*sin(phi1 - phi2))/(11*\rightarrow
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                                                                                ←12*sym_R*...
                                                               (4*m1 + 12*m2 - 9*m2*\cos(phi1 - phi2)^2)^2) - 6*Rp2*dphi1* \rightarrow
                                                                                ←11*sin(phi1 - phi2) ...
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+ 6*Rp2*dphi2*l1*sin(phi1 - phi2) + 3*dphi1^2*l1^2*l2*m2*cos→
   ←(phi1 - phi2)^2 ...
-3*dphi1^2*11^2*12*m2*sin(phi1 - phi2)^2 + 2*dphi2^2*11*12 \rightarrow
    \leftarrow 2*m2*cos(phi1 - phi2) ...
+ 2*g*11*12*m1*cos(phi1) + 4*g*11*12*m2*cos(phi1) + 3*g*11*\rightarrow
    \leftarrow 12*m2*sin(phi1 - phi2)...
*\sin(\text{phi2}))/(11^2*12*(4*m1 + 12*m2 - 9*m2*\cos(\text{phi1 - phi2})→
    \leftarrow^2)) - (54*m2*cos(phi1 - phi2)*sin(phi1 - phi2)*(4*12\rightarrow
   \leftarrow*((12*lambda2)/(11^2*sym_R*(4*m1 + 12*m2 - 9*m2*cos(phi1\rightarrow
   \leftarrow - phi2)^2)) - (18*lambda4*cos(phi1 - phi2))/(11*12*<math>\rightarrow
   \epsilon sym_R*(4*m1 + 12*m2 - 9*m2*cos(phi1 - phi2)^2)) + 4*Rp1 \rightarrow
    \leftarrow*dphi1*12 + 6*Rp2*dphi1*11*\cos(phi1 - phi2) - 6*Rp2*\rightarrow
   \leftarrowdphi2*l1*cos(phi1 - phi2) + 2*dphi2^2*l1*l2^2*m2*sin(\rightarrow
   \leftarrowphi1 - phi2) + 2*g*11*12*m1*sin(phi1) + 4*g*11*12*m2*sin\rightarrow
   \leftarrow(phi1) + 3*dphi1^2*11^2*12*m2*cos(phi1 - phi2)*sin(phi1 \rightarrow
    \leftarrow phi2) - 3*g*11*12*m2*cos(phi1 - phi2)*sin(phi2)))/(11\rightarrow
    \leftarrow 2 * 12 * (4 * m1 + 12 * m2 - 9 * m2 * cos(phi1 - phi2) ^ 2) ^ 2) - \rightarrow
    ←phi1*q1 - lambda4*((3*(6*12*m2*cos(phi1 - phi2)...
((18*lambda4*sin(phi1 - phi2))/(l1*l2*sym_R*(4*m1 + 12*m2 \rightarrow 4*sin(phi1 - phi2)))
    \leftarrow 9*m2*cos(phi1 - phi2)^2)) - (216*lambda2*m2*cos(phi1 - \rightarrow
    \epsilonphi2)*sin(phi1 - phi2))/(11^2*sym_R*(4*m1 + 12*m2 - 9*m2\rightarrow
    \leftarrow*cos(phi1 - phi2)^2)^2) + (324*lambda4*m2*cos(phi1 - \rightarrow
   \epsilonphi2)^2*sin(phi1 - phi2))/(11*12*sym_R*(4*m1 + 12*m2 - \rightarrow
   \leftarrow 9 \times 2 \times \cos(\text{phi1} - \text{phi2})^2)^2) - 6 \times 12 \times 2 \times \sin(\text{phi1} - \text{phi2})^2
   \leftarrow*((12*lambda2)/(l1^2*sym_R*(4*m1 + 12*m2 - 9*m2*cos(phi1)
    \leftarrow - phi2)^2)) - (18*lambda4*\cos(phi1 - phi2))/(l1*l2*\rightarrow
   \leftarrow \text{sym}_R * (4*m1 + 12*m2 - 9*m2*\cos(\text{phi1} - \text{phi2})^2))) - 6*Rp1 \rightarrow
    \leftarrow*dphi1*12*m2*sin(phi1 - phi2) + 6*dphi1^2*11^2*m2^2*\rightarrow
    \leftarrowcos(phi1 - phi2) + 3*dphi2^2*l1*l2^2*m2^2*cos(phi1 - \rightarrow
    ←phi2)^2 - 3*dphi2^2*l1*l2^2*m2^2*sin(phi1 - phi2)^2 ...
+ 2*dphi1^2*11^2*m1*m2*cos(phi1 - phi2) + 6*g*11*12*m2^2* \rightarrow
    \leftarrowcos(phi1 - phi2)*cos(phi1) - 6*g*l1*l2*m2^2*sin(phi1 - \rightarrow
    \epsilonphi2)*sin(phi1) + 3*g*l1*l2*m1*m2*cos(phi1 - phi2)*cos(\epsilon
    \epsilonphi1) - 3*g*l1*l2*m1*m2*sin(phi1 - phi2)*sin(phi1)))/(l1+
    \leftarrow (54*cos(phi1 - phi2)*sin(phi1 - phi2)*(6*12*m2*cos(phi1 \rightarrow
   \leftarrow phi2)*((12*lambda2)/(l1^2*sym_R*(4*m1 + 12*m2 - 9*m2*\rightarrow
   \leftarrowcos(phi1 - phi2)^2)) - (18*lambda4*cos(phi1 - phi2))/(11<math>\rightarrow
   \leftarrow*12*sym_R*(4*m1 + 12*m2 - 9*m2*cos(phi1 - phi2)^2))) + \rightarrow

←4*Rp2*dphi1*11*m1 + 12*Rp2*dphi1*11*m2 - 4*Rp2*dphi2*11*→
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\leftarrowm1 - 12*Rp2*dphi2*l1*m2 + 6*dphi1^2*l1^2*l2*m2^2*sin(\rightarrow
                                              \leftarrowphi1 - phi2) - 6*g*11*12*m2^2*sin(phi2) ...
                                   + 6*Rp1*dphi1*l2*m2*cos(phi1 - phi2) + 3*dphi2^2*l1*l2^2*m2→
                                              \leftarrow^2*cos(phi1 - phi2)*sin(phi1 - phi2) + 2*dphi1^2*11^2*12\rightarrow
                                              \leftarrow m1*m2*sin(phi1 - phi2) + 6*g*11*12*m2^2*cos(phi1 - phi2)
                                              \leftarrow)*sin(phi1) - 2*g*l1*l2*m1*m2*sin(phi2) + 3*g*l1*l2*m1*\rightarrow
                                              \leftarrow m2*\cos(phi1 - phi2)*\sin(phi1))/(11*12^2*(4*m1 + 12*m2 \rightarrow phi2)*\sin(phi1))
                                              \leftarrow 9*m2*cos(phi1 - phi2)^2));
                      dx6 = (3*lambda2*(6*dphi1*l2*m2*cos(phi1 - phi2)*sin(phi1 - phi2)
                                \leftarrow)*11*2 + 6*Rp2*cos(phi1 - phi2)*11 + 4*Rp1*12))/(11*2*12*(4*\rightarrow
                                \epsilonm1 + 12*m2 - 9*m2*cos(phi1 - phi2)^2)) - dphi1*q2 - lambda1 \rightarrow
                                ←- (3*lambda4*(12*dphi1*l2*sin(phi1 - phi2)*l1^2*m2^2 + 4*\rightarrow
                                \leftarrow dphi1*12*m1*sin(phi1 - phi2)*11^2*m2 + 12*Rp2*11*m2 + 4*Rp2* \rightarrow 12*Rp2*11*m2 + 4*Rp2*11*m2 + 4*Rp2* \rightarrow 12*Rp2*11*m2 + 4*Rp2* + 4*Rp2*11*m2 + 4*Rp2* 
                                \leftarrowm1*11 + 6*Rp1*12*cos(phi1 - phi2)*m2))/(11*12*2*m2*(4*m1 + \rightarrow
                                +12*m2 - 9*m2*cos(phi1 - phi2)^2);
                      dx7 = lambda4*((3*(6*l2*m2*cos(phi1 - phi2)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*lambda4*sin()*)*((18*
56
                                \epsilon phi1 - phi2))/(11*12*sym_R...
                                    *(4*m1 + 12*m2 - 9*m2*\cos(phi1 - phi2)^2)) - (216*lambda2*m2 \rightarrow (4*m1 + 12*m2 - 9*m2*\cos(phi1 - phi2)^2))
                                              ←*cos(phi1 - phi2)*...
                                   \sin(\text{phi1} - \text{phi2}))/(11^2*\text{sym}_R*(4*\text{m1} + 12*\text{m2} - 9*\text{m2}*\cos(\text{phi1} \rightarrow
                                              ←- phi2)^2)^2) + (324*lambda4*m2...
                                    *\cos(\text{phi1} - \text{phi2})^2*\sin(\text{phi1} - \text{phi2}))/(11*12*sym_R*(4*m1 + \rightarrow
                                              \leftarrow 12 \text{ m2} - 9 \text{ m2} \text{ cos}(\text{phi1} - \text{phi2})^2) ...
                                    \leftarrow 12*m2 - 9*m2*cos(phi1 - phi2)^2) ...
                                    -(18*lambda4*cos(phi1 - phi2))/(11*l2*sym_R*(4*m1 + 12*m2 \rightarrow 
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                                              ← 9*m2*cos(phi1 - phi2)^2))) ...
                                    - 6*Rp1*dphi1*12*m2*sin(phi1 - phi2) + 6*dphi1^2*11^2*m2→
                                              \leftarrow 2*cos(phi1 - phi2) + 6*g*11*12*m2^2*cos(phi2) + 3*dphi2\rightarrow
                                              \leftarrow 2*11*12^2*m2^2*cos(phi1 - phi2)^2 - 3*dphi2^2*11*12^2* \rightarrow
                                              +m2^2*sin(phi1 - phi2)^2 + 2*dphi1^2*11^2*12*m1*m2*cos(+)
                                              \epsilonphi1 - phi2) + 2*g*l1*l2*m1*m2*cos(phi2) - 6*g*l1*l2*m2+
                                              \leftarrow 2*sin(phi1 - phi2)*sin(phi1) - 3*g*l1*l2*m1*m2*sin(phi1\rightarrow
                                              \leftarrow - phi2)*sin(phi1)))/(11*12^2*m2*(4*m1 + 12*m2 - 9*m2*\rightarrow
                                              \leftarrowcos(phi1 - phi2)^2)) - (54*cos(phi1 - phi2)*sin(phi1 - \rightarrow
                                              \epsilonphi2)*(6*12*m2*cos(phi1 - phi2)*((12*lambda2)/(11^2*)
                                              \epsilon sym_R*(4*m1 + 12*m2 - 9*m2*cos(phi1 - phi2)^2)) - (18* \rightarrow 8*m2*cos(phi1 - phi2)^2)
                                              \leftarrowlambda4*cos(phi1 - phi2))/(l1*12*sym_R*(4*m1 + 12*m2 - \rightarrow
                                              \leftarrow 9 \times m2 \times cos(phi1 - phi2) \times 2)) ...
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+ 4*Rp2*dphi1*l1*m1 + 12*Rp2*dphi1*l1*m2 - 4*Rp2*dphi2*l1*m1 + 4*Rp2*dphi2*l1*m1 + 12*Rp2*dphi1*l1*m2 - 4*Rp2*dphi2*l1*m1 + 12*Rp2*dphi2*l1*m1 + 12*Rp2*dphi2*l1*m1 + 12*Rp2*dphi2*l1*m1 + 12*Rp2*dphi2*l1*m1 + 12*Rp2*dphi2*l1*m1 + 12*Rp2*dphi2*l1*m1 + 12*Rp2*dphi2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*l1*m2*
                              \leftarrow - 12*Rp2*dphi2*l1*m2 + 6*dphi1^2*l1*2*l2*m2^2*sin(phi1 \rightarrow
                               \leftarrow phi2) - 6*g*11*12*m2^2*sin(phi2) + 6*Rp1*dphi1*12*m2*\rightarrow
                              \leftarrowcos(phi1 - phi2) + 3*dphi2^2*11*12^2*m2^2*cos(phi1 - \rightarrow
                               \epsilonphi2)*sin(phi1 - phi2) + 2*dphi1^2*l1^2*l2*m1*m2*sin(\epsilon
                              \epsilonphi1 - phi2) + 6*g*11*12*m2^2*cos(phi1 - phi2)*sin(phi1)\rightarrow
                               \leftarrow - 2*g*l1*l2*m1*m2*sin(phi2) + 3*g*l1*l2*m1*m2*cos(phi1 \rightarrow
                               \leftarrow phi2)*sin(phi1)))/(l1*l2^2*(4*m1 + 12*m2 - 9*m2*cos(\rightarrow
                              \epsilon phi1 - phi2^{\circ})^{\circ}) - phi2^{\circ}q3 - lambda2^{\circ}((3^{\circ}(4^{\circ}12^{\circ}((18^{\circ}) + 10^{\circ})^{\circ}))))
                              \leftarrow lambda4*sin(phi1 - phi2))/(l1*l2*sym_R*(4*m1 + 12*m2 - \rightarrow architecture))
                               \leftarrow9*m2*cos(phi1 - phi2)*2)) - (216*lambda2*m2*cos(phi1 - \rightarrow
                               ←phi2)*sin(phi1 - phi2))/(l1^2*sym_R...
                        *(4*m1 + 12*m2 - 9*m2*cos(phi1 - phi2)^2)^2) + (324*lambda4* \rightarrow 
                               \leftarrowm2*cos(phi1 - phi2)^2*sin(phi1 - phi2))/(l1*l2*sym_R*(4*\rightarrow
                              \leftarrowm1 + 12*m2 - 9*m2*cos(phi1 - phi2)^2) - 6*Rp2*dphi1*\rightarrow
                              \leftarrow 11*\sin(\text{phi1} - \text{phi2}) + 6*Rp2*dphi2*11*\sin(\text{phi1} - \text{phi2}) + \rightarrow
                               \leftarrow3*dphi1^2*11^2*12*m2*cos(phi1 - phi2)^2 - 3*dphi1^2*11\rightarrow
                               \leftarrow^2*12*m2*sin(phi1 - phi2)^2 + 2*dphi2^2*11*12^2*m2*cos(\rightarrow
                              \leftarrowphi1 - phi2) + 3*g*11*12*m2*sin(phi1 - phi2)*sin(phi2) +\rightarrow
                               \leftarrow 3*g*11*12*m2*cos(phi1 - phi2)*cos(phi2)))/(11^2*12*(4*)
                              \epsilonm1 + 12*m2 - 9*m2*cos(phi1 - phi2)^2)) - (54*m2*cos(phi1\rightarrow
                               \leftarrow - phi2)*sin(phi1 - phi2)*(4*12*((12*lambda2)/(l1^2*)
                               \leftarrow \text{sym}_R^*(4*m1 + 12*m2 - 9*m2*\cos(\text{phi1} - \text{phi2})^2)) - (18* \rightarrow 0
                               ←lambda4*cos(phi1 - phi2))...
                        /(11*12*sym_R*(4*m1 + 12*m2 - 9*m2*cos(phi1 - phi2)^2))) + \rightarrow
                              ←4*Rp1*dphi1*12 + 6*Rp2*dphi1*11*cos(phi1 - phi2) - 6*Rp2→
                               \leftarrow*dphi2*l1*cos(phi1 - phi2) + 2*dphi2^2*l1*l2^2*m2*sin(\rightarrow
                               \leftarrowphi1 - phi2) + 2*g*l1*l2*m1*sin(phi1) + 4*g*l1*l2*m2*sin→
                               \leftarrow(phi1) + 3*dphi1^2*l1^2*l2*m2*cos(phi1 - phi2)*sin(phi1 \rightarrow
                              \leftarrow phi2) - 3*g*11*12*m2*cos(phi1 - phi2)*sin(phi2)))/(11\rightarrow
                              \leftarrow^2*12*(4*m1 + 12*m2 - 9*m2*cos(phi1 - phi2)^2)^2);
               dx8 = (3*lambda4*(-6*dphi2*l1*cos(phi1 - phi2)*sin(phi1 - phi2))
66
                     \leftarrow*12^2*m2^2 + 12*Rp2*l1*m2 + 4*Rp2*l1*m1))/(11*l2^2*m2*(4*m1 \rightarrow
                      \leftarrow+ 12*m2 - 9*m2*cos(phi1 - phi2)^2)) - dphi2*q4 - (3*lambda2\rightarrow
                     \leftarrow*(- 4*dphi2*l1*m2*sin(phi1 - phi2)*l2^2 + 6*Rp2*l1*cos(phi1 \rightarrow
                      \leftarrow phi2)))/(11^2*12*(4*m1 + 12*m2 - 9*m2*cos(phi1 - phi2)^2))\rightarrow
                      ← - lambda3:
               dxdt = [dx1; dx2; dx3; dx4; dx5; dx6; dx7; dx8];
```

71 **end** % function RandwertproblemDGL

Listing 4.2: Quellcode der Funktion RandwertproblemRB

14 end

Listing 4.3: Quellcode der Funktion berechneTrajektorie

```
function [ stTraj ] = berechneTrajektorie( stPendel, Q, R, T )
  %BERECHNETRAJEKTORIE Summary of this function goes here
      Detailed explanation goes here
  RandwertproblemDGLhandle = @(t,x) RandwertproblemDGL(t,x,stPendel,Q,→
     ←R);
  intervals = 1000;
  solinit.x = linspace(0,T,intervals);
  solinit.y = [linspace(0,pi,intervals);
               zeros(1,intervals);
               linspace(0,pi,intervals);
               zeros(1, intervals);
               zeros(1,intervals);
               zeros(1,intervals);
15
               zeros(1,intervals);
               zeros(1,intervals);];
```

```
RelTol = 1e-10;
  bvp4cOptions = bvpset('RelTol', RelTol, 'Stats', 'on');
  for ii = 1:15
      sol = bvp4c(RandwertproblemDGLhandle,@RandwertproblemRB,solinit,→
          ←bvp4c0ptions);
      solinit = sol;
      if isfield(sol,'stats')
25
           break
      end
  end
  [f, h] = nonlinear_model;
  dfdu_symb = jacobian(f,sym('M'));
35
  dfdu = double(subs(dfdu_symb,{sym('phi1'),sym('phi2')},{sol.y(1,:),→
     ←sol.y(3,:)}));
  stTraj.T = T;
  stTraj.vT = sol.x;
  stTraj.vU = diag(- R^{(-1)}* dfdu' * sol.y(5:8,:))';
  % vU = diag(stTraj.vU);
  stTraj.mX = sol.y(1:4,:);
  end
```

Lösungen können für den Wertebereich von $0,65 \text{ s} \leq T \leq 1,9 \text{ s}$ gefunden werden. Die Unterschreitung von T=0,65 s führt zu numerischen Problemen. Eine Überschreitung von T=1,9 s ist physikalisch unmöglich, da das zweite Pendel eine Mindestgeschwindigkeit braucht um in die aufrechte Lage aufgeschwungen werden zu können.

Die kleinste T, bei der die Stellgrößenbeschränkung $|M|_{max}=1,5\,Nm$ noch eingehalten wird ist $T=1,1185\,\mathrm{s}$.

Nachfolgend werden Plots der Zustände $\mathbf{x}^*(\mathbf{t})$ und $\mathbf{M}^*(\mathbf{t})$ für verschiedene Übergangszeiten gezeigt.

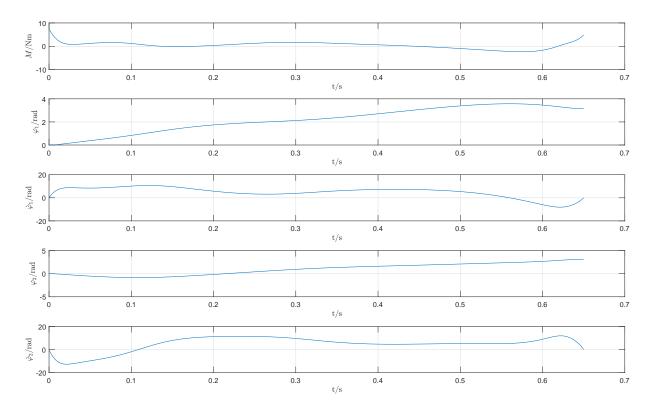


Figure 4.1: Plot der Zustände $\mathbf{x}^*(\mathbf{t})$ und $\mathbf{M}^*(\mathbf{t})$ für $T=0.65\,\mathrm{s}$

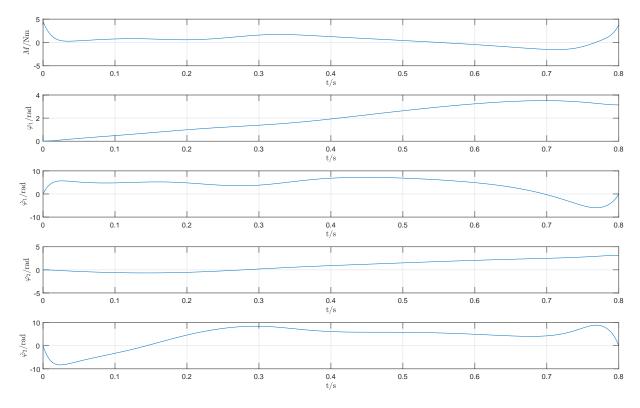


Figure 4.2: Plot der Zustände $\mathbf{x}^*(\mathbf{t})$ und $\mathbf{M}^*(\mathbf{t})$ für $T=0.8\,\mathrm{s}$

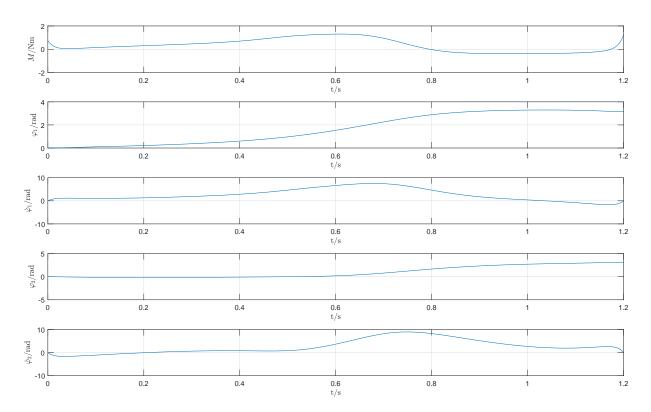


Figure 4.3: Plot der Zustände $\mathbf{x}^*(\mathbf{t})$ und $\mathbf{M}^*(\mathbf{t})$ für $T=1,2\,\mathrm{s}$

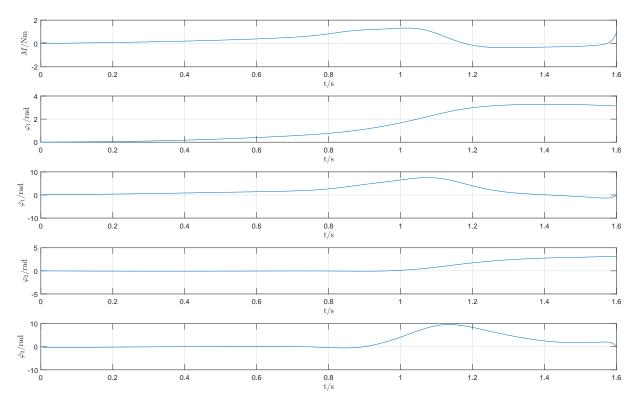


Figure 4.4: Plot der Zustände $\mathbf{x}^*(\mathbf{t})$ und $\mathbf{M}^*(\mathbf{t})$ für $T=1.6\,\mathrm{s}$

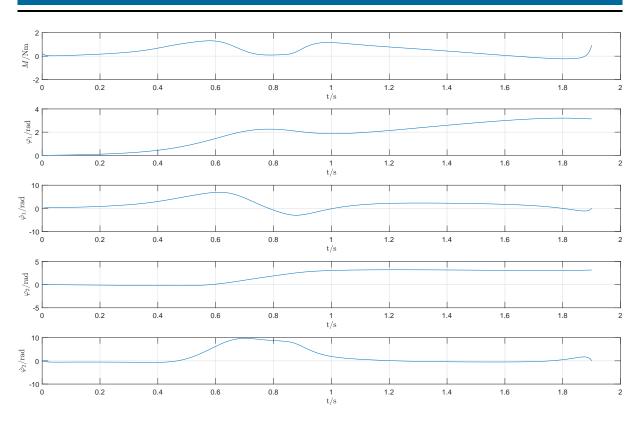


Figure 4.5: Plot der Zustände $\mathbf{x}^*(\mathbf{t})$ und $\mathbf{M}^*(\mathbf{t})$ für $T=1.9\,\mathrm{s}$

Figure 4.1 bis 4.5 zeigen, dass eine größere Übergangszeit T ein Geringeres Stellgrößenmaximum nach sich zieht.

4.9 Simulation

Das Simulinkmodell der Aufschwungsteuerung ist in Figure 4.6 dargestellt.

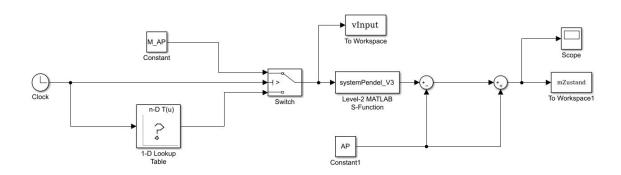


Figure 4.6: Simulink-Modell der Aufschwungsteuerung

Die folgende Abbildung veranschaulicht die Simulation und zusammen mit den vorgegebenen Trajektorien bei gegebenen Pendeldaten. Für die darauffolgenden Abbildungen wurden die Pendeldaten verändert.

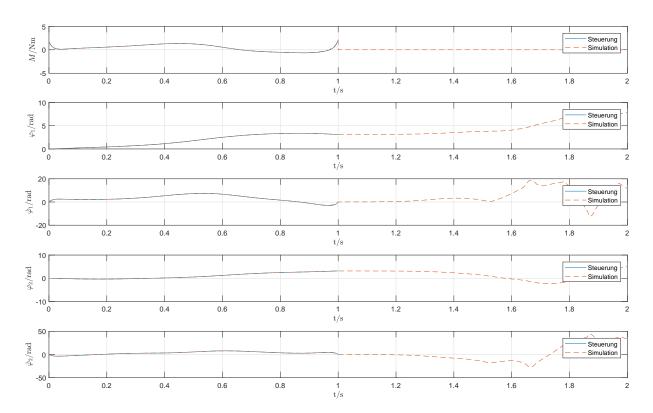


Figure 4.7: Plot der Zustände $\mathbf{x}^*(\mathbf{t})$ und $\mathbf{M}^*(\mathbf{t})$ für $T=1\,\mathrm{s}$, $T_{sim}=2\,\mathrm{s}$ mit gegebenen Pendeldaten

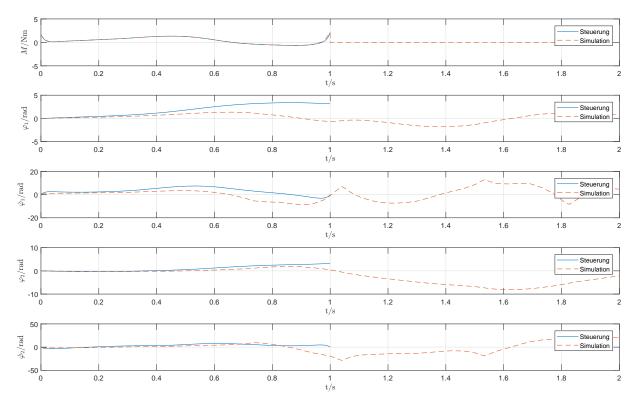


Figure 4.8: Plot der Zustände $\mathbf{x}^*(\mathbf{t})$ und $\mathbf{M}^*(\mathbf{t})$ für $T=1\,\mathrm{s}$, $T_{sim}=2\,\mathrm{s}$ ($l_1=0,3\,m$, $l_2=0,3\,m$, $R_{p1}=10^{-3}$, $R_{p2}=10^{-4}$)

4.9 Simulation

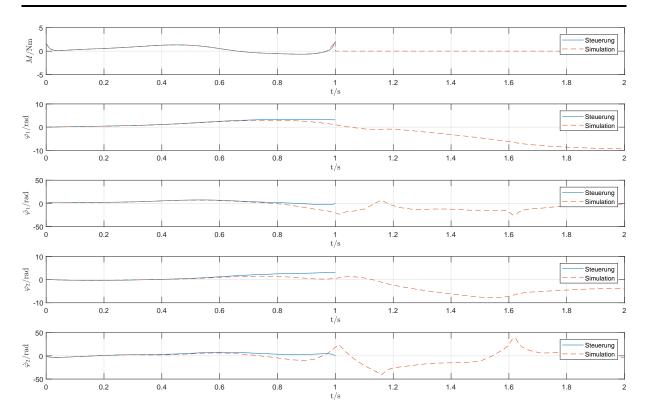


Figure 4.9: Plot der Zustände $\mathbf{x}^*(\mathbf{t})$ und $\mathbf{M}^*(\mathbf{t})$ für $T=1\,\mathrm{s}$, $T_{sim}=2\,\mathrm{s}$ (g=11N/kg)

In Figure 4.7 ist zu sehen, dass die berechneten Trajektorien exakt mit den simulierten Verläufen übereinstimmen. Das Pendel wird in die obere Ruhelage überführt, verweilt hier eine kurze zeit und verlässt sie dann wieder. Verändert man die Parameter des simulierten Pendels gegenüber denen anhand derer die Aufschwungsteuerung berechnet wurde, kann die Steuerung das Pendel in der Simulation nichtmehr in die obere Ruhelage überführen. Dies ist in Figure 4.8 und 4.9 zu sehen. Eine Reine Aufschwungsteuerung ohne Regelung ist nur für die ursprünglichen berechneten Pendelparameter gültig.