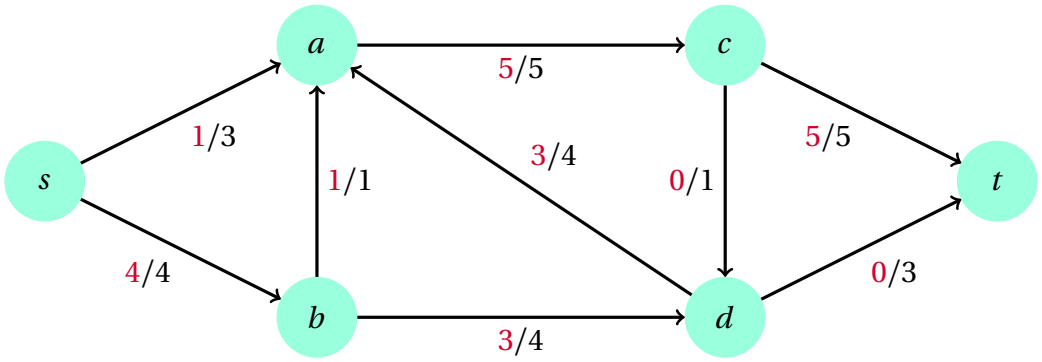
Week 9 Preparation

(Solutions)

Useful advice: The following solutions pertain to the preparation problems. You are strongly advised to attempt the problems thoroughly before looking at these solutions. Simply reading the solutions without thinking about the problems will rob you of the practice required to be able to solve complicated problems on your own. You will perform poorly on the exam if you simply attempt to memorise solutions to these problems. Thinking about a problem, even if you do not solve it will greatly increase your understanding of the underlying concepts.

Problems

Problem 1. Consider the following flow network in which the current flows are in red and the capacities of the edges are in black:



(a) Draw the corresponding residual network.

(b) Identify an augmenting path in the residual network and state its capacity.

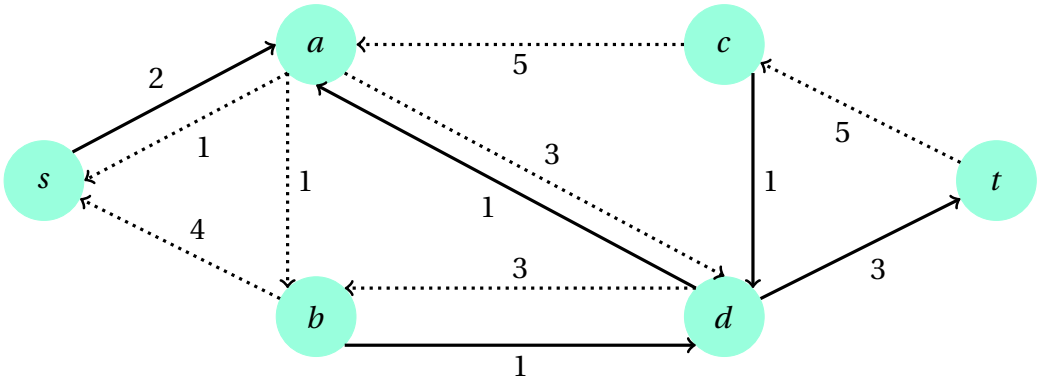
(c) Augment the flow of the network along the augmenting path, showing the resulting flow network.

(d) Complete the Ford-Fulkerson method for the network, showing the final flow network with a maximum flow.

(e) Using your solution to the max-flow problem, list the vertices in the two components of a minimum cut in the network. Identify the edges that cross the cut and verify that their capacity adds up to the value of the maximum flow.

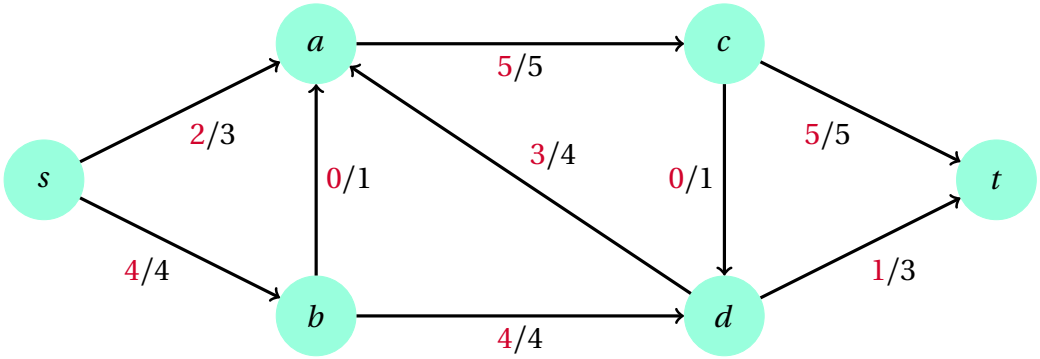
Solution

(a) The corresponding residual network:

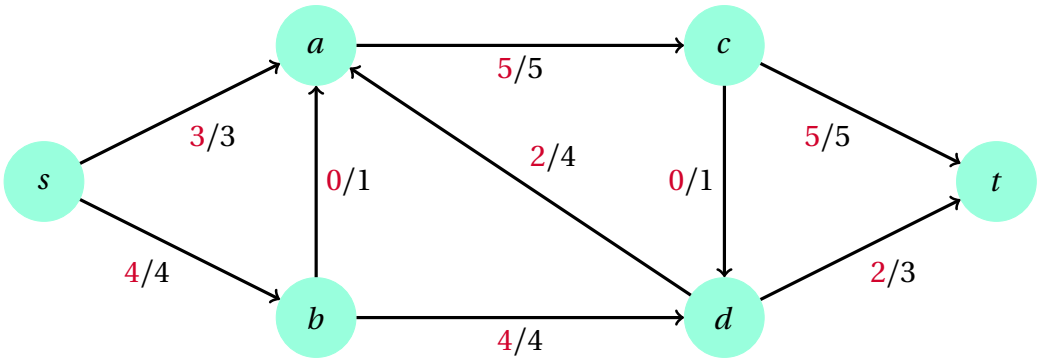


(b) One augmenting path is $s \to a \to b \to d \to t$ with capacity 1.

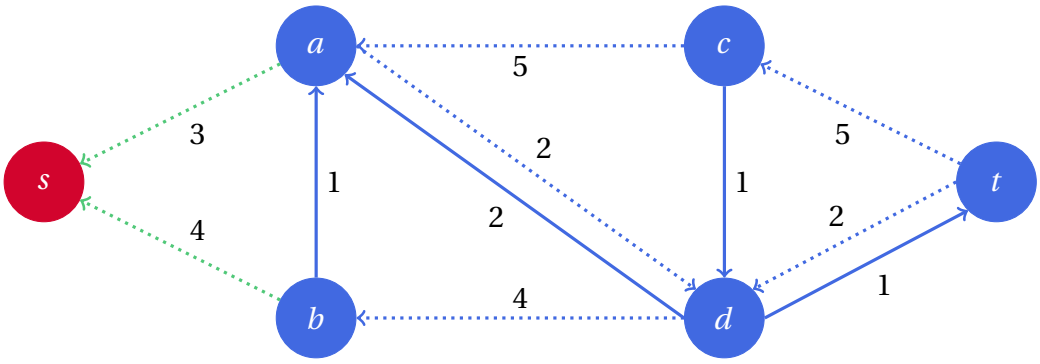
(c) Note that there are other possible augmenting paths and therefore other solutions.



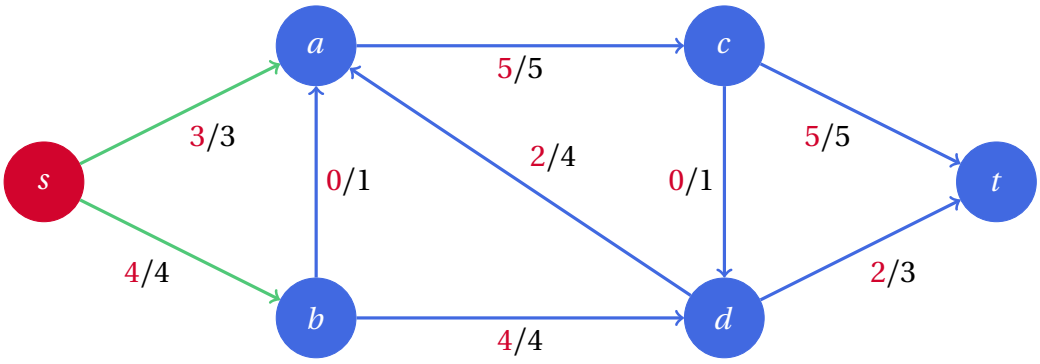
(d) One possibility for the final flow network with a maximum flow is:



(e) To find the minimum cut after running the Ford-Fulkerson algorithm, we simply identify all vertices reachable from the source in the residual graph. These vertices will be on one side of the cut, and all remaining vertices will be on the other. The corresponding residual network:



In this graph, s has no outgoing edges in the residual graph, since the two edges outgoing from s are full. So the source is on one side of the cut, and everything else is on the other. The corresponding cut on the flow network is:



Component 1: $\{s\}$
Component 2: $\{a, b, c, d, t\}$

The flow across this cut is indeed 7, as expected.