Week 7 Preparation

Instructions to the students: The preparation problems are not assessed, but we strongly recommend that you try to solve them before your applied class this week. These preparation problems test your basic knowledge of the contents taught in the seminar of the previous week. The problems in the applied class assume that you have this basic knowledge and will build on top of it. You might find it helpful to try these problems before doing the quiz that is due this week.

Problems

Problem 1. Recall the unbounded knapsack problem shown in the Seminar and notes, where we attempt to find the maximum value that we can fit in a knapsack of capacity C.

Consider the following items with weights and values.

Item i	1	2	3	4	5
Weight w_i	2	3	5	7	10
Value v_i	120	200	250	450	750

For a knapsack with capacity $C = 15$, use the following recurrence relation to fill the memoisation table MaxValue$[c]$ for $0 \leq c \leq 15$.

$$\text{MaxValue}[c] = \begin{cases} 0 & \text{if } c < w_i \text{for all } i, \\ \max\limits_{\substack{1 \leq i \leq n \\ w_i \leq c}} (v_i + \text{MaxValue}[c - w_i]) & \text{otherwise.} \end{cases}$$

where

$$\text{MaxValue}[c] = \{\text{The maximum value that we can fit in a capacity of } c\}$$

Problem 2. Recall the coin change problem described in the Seminar.

$$\text{MinCoins}[v] = \begin{cases} 0 & \text{if } v = 0, \\ \infty & \text{if } v > 0 \text{ and } v < c[i] \text{ for all } i, \\ \min\limits_{\substack{1 \leq i \leq n \\ c[i] \leq v}} (1 + \text{MinCoins}[v - c[i]]) & \text{otherwise.} \end{cases}$$

where

$$\text{MinCoins}[v] = \{\text{The minimum number of coins that we can use to add to } \$v\}$$

The denominations available are \$1, \$3, and \$8.

Coin Type i	1	2	3
Denomination	1	3	8

Given the completed memoisation table below, determine which coins are used to make \$7 via backtracking.

Value v	0	1	2	3	4	5	6	7	8	9
MinCoins$[v]$	0	1	2	1	2	3	2	3	1	2
Decision i	-	1	1	2	1	1	2	1	3	1

Problem 3. Implement the solution to the coin change problem described in the lectures. Your solution should return the number of coins needed, along with how many of each denomination are required.

(a) Use the bottom-up strategy to compute the solutions.

(b) Use the top-down strategy to compute the solutions.

Consult the notes if you are unclear on the difference between the two approaches.