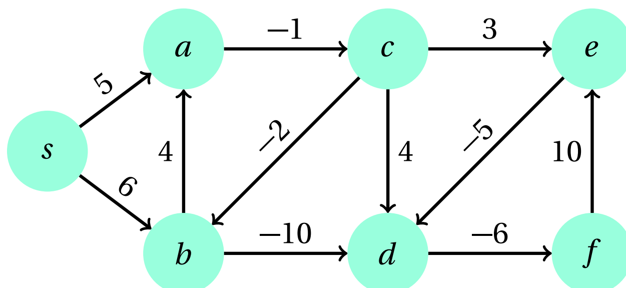
Week 8 Preparation

(Solutions)

Useful advice: The following solutions pertain to the preparation problems. You are strongly advised to attempt the problems thoroughly before looking at these solutions. Simply reading the solutions without thinking about the problems will rob you of the practice required to be able to solve complicated problems on your own. You will perform poorly on the exam if you simply attempt to memorise solutions to these problems. Thinking about a problem, even if you do not solve it will greatly increase your understanding of the underlying concepts.

Problems

Problem 1. Use the space-efficient version of Bellman-Ford to determine the shortest paths from vertex s to all other vertices in this graph. Afterwards, indicate to which vertices s has a well defined shortest path, and which do not by indicating the distance as $-\infty$. Draw the resulting shortest path tree containing the vertices with well defined shortest paths. For consistency, you should relax the edges in the following order: $s \to a$, $s \to b$, $a \to c$, $b \to a$, $b \to d$, $c \to b$, $c \to d$, $c \to e$, $d \to f$, $e \to d$ and $f \to e$.



Solution

The distances at each iteration are shown below. If you followed the order specified, you should have the same distances. Relaxing the edges in a different order may lead to a different table, but the end distances should be the same (except for the vertices reachable via negative cycles).

Vertex	Iteration						
	0	1	2	3	4	5	6
s	0	0	0	0	0	0	0
a	∞	5	5	5	5	5	5
b	∞	2	2	2	2	2	2
c	∞	4	4	4	4	4	4
d	∞	-4	-8	-9	-9	-10	-10
e	∞	0	-4	-4	-5	-5	-6
f	∞	-10	-14	-14	-15	-15	-16

Since another round of relaxation would decrease the distance of vertex d, there must exist a negative cycle in the graph. All vertices reachable from negative cycles have undefined distance estimates as it is possible to keep decreasing the distances further by going through a negative cycle more times. We normally mark such nodes as having distance $-\infty$. The final distances are therefore

s	a	b	c	d	e	f
0	5	2	4	$-\infty$	$-\infty$	$-\infty$

The shortest path tree is shown below