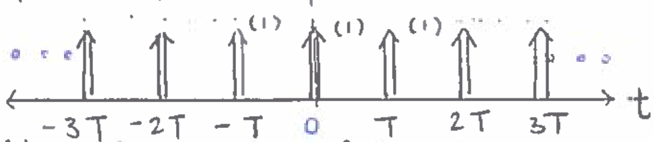
SIGSYS PS07 - Jennifer Wei 12 MARCH 2015

1. $p(t) = \sum_{k=-\infty}^{\infty} \delta(t-kT)$ T = time units

a. p(t) representation



Above is a series of impulses to represent p(t) that span from $-\infty$ to ∞ (in time).

b. Fourier Series Representation of p(t) with an ∞ # of terms

$$C_k = \frac{1}{T}\int_{-T/2}^{T/2} p(t)\, e^{-j\frac{2\pi}{T}kt}\, dt$$

$$p(t) = \sum_{k=-\infty}^{\infty} \delta(t-kT)$$

$$C_k = \frac{1}{T}\int_{-T/2}^{T/2} \sum_{k=-\infty}^{\infty} \delta(t-kT)\, e^{-j\frac{2\pi}{T}kt}\, dt$$

$$C_k = \frac{1}{T}\int_{-T/2}^{T/2} \delta(t)\, e^{-j\frac{2\pi}{T}kt}\, dt = \frac{1}{T}$$

$$\therefore\ p(t) = \sum_{k=-\infty}^{\infty} C_k \cdot e^{j\frac{2\pi}{T}kt}\, dt$$

$$\Downarrow$$

$$\underline{\mathbf{p(t+T)}} = p(t) = \frac{1}{T}\sum_{k=-\infty}^{\infty} e^{j\frac{2\pi}{T}kt}$$

c. $$x(t) = \sum_{k=-\infty}^{\infty} C_k\, e^{j\frac{2\pi}{T}kt}$$

$$x(t) = \sum_{k=-\infty}^{\infty} C_k\, e^{jw_0kt},$$

where w_0 is the fundamental frequency $= \frac{2\pi}{T}$

$$X(w) = \int_{-\infty}^{\infty} x(t)\, e^{-jwt}\, dt$$

unnecessary info

$$C_k = \frac{w_0}{2\pi}\int_{-\infty}^{\infty} x(t)\, e^{-jw_0kt}\, dt$$

$$C_k = \frac{1}{T}\int_{-\infty}^{\infty} x(t)\, e^{-jw_0kt}\, dt$$

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c cont'd

$$X(w) = \int_{-\infty}^{\infty} \sum_{k=-\infty}^{\infty} C_k\, e^{jw_0kt}\, e^{-jwt}\, dt$$

$$X(w) = \int_{-\infty}^{\infty} \sum_{k=-\infty}^{\infty} C_k\, e^{jt(w_0k-w)}\, dt$$

$$X(w) = \sum_{k=-\infty}^{\infty} C_k \cdot \int_{-\infty}^{\infty} e^{jt(w_0k-w)}\, dt$$

$$X(w) = \sum_{k=-\infty}^{\infty} C_k\, 2\pi\, \delta(w-kw_0)$$

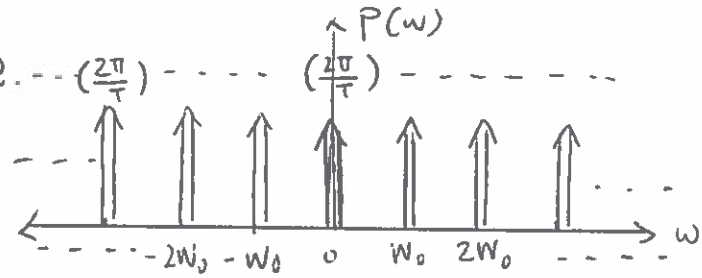
d. Using the answers to the previous two parts, we can find P(w)

$$P(w) = \sum_{-\infty}^{\infty} 2\pi\, C_k\, \delta(w-w_0k),$$

where $C_k = \frac{1}{T}$, $W_0 = \frac{2\pi}{T}$

$$\therefore\ P(w) = \sum_{k=-\infty}^{\infty} \frac{2\pi}{T}\, \delta\!\left(w - \frac{2\pi}{T}k\right)$$

e.



Changing T affects both p(t) & P(w).

P(w): When T increases, the impulses are then closer together since the fundamental frequency is smaller.

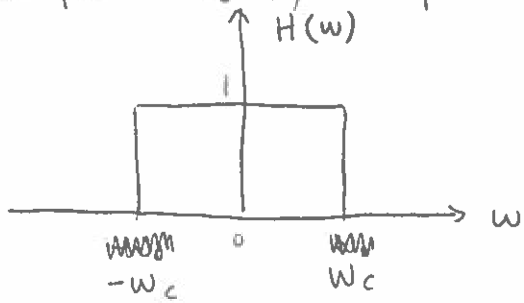
T↑ = Spacing ↓ (freq ↑)
= Magnitude (P(w)) ~~decrease~~ ~~constant~~ decreases & vice versa

p(t): when T increases, the spacing between inpulses also increases

T↑ = Spacing ↑ ... & vice versa

This makes sense that P(w) decreases when T increases & vice versa since P(w) is dependent on the

2 LTI System with impulse response $h(t)$, input $x(t)$, output $y(t)$



a. Find $h(t)$

$$h(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} h(w)\, e^{jwt}\, dw$$

$$h(w) = 1$$

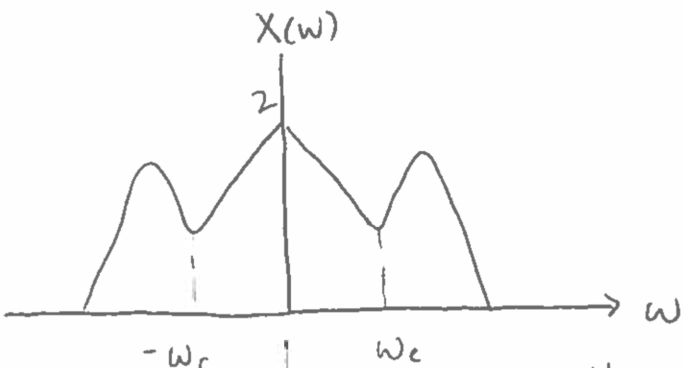
$$h(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{jwt}\, dw = \frac{1}{2\pi}\int_{-w_c}^{w_c} e^{jwt}\, dw$$

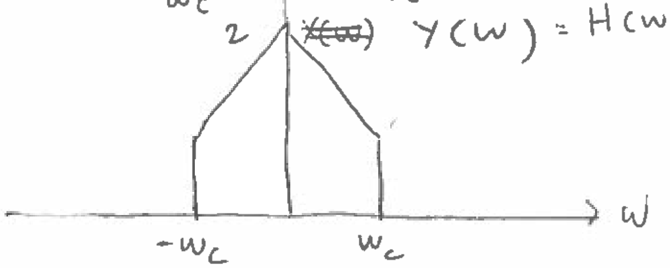
since $w > w_c$ & $w < -w_c = 0$

$$h(t) = \frac{1}{2\pi}\,\frac{1}{jt}\, e^{jwt}\Big|_{-w_c}^{w_c} = \frac{1}{2\pi jt}\left(e^{jw_ct} - e^{-jw_ct}\right)$$

$$h(t) = \frac{1}{\pi t}\left(\frac{1}{2j}e^{jw_ct} - \frac{1}{2j}e^{-jw_ct}\right) = \frac{1}{\pi t}\sin(w_ct)$$

b. Suppose that $X(w)$ is ↓ Sketch $Y(w)$

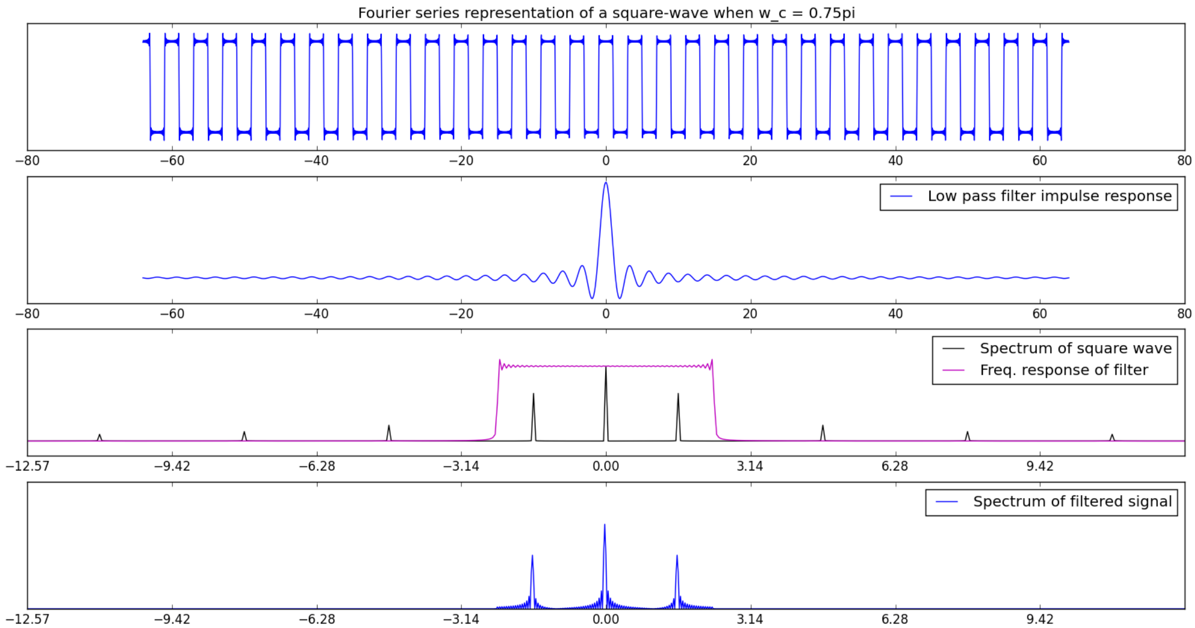


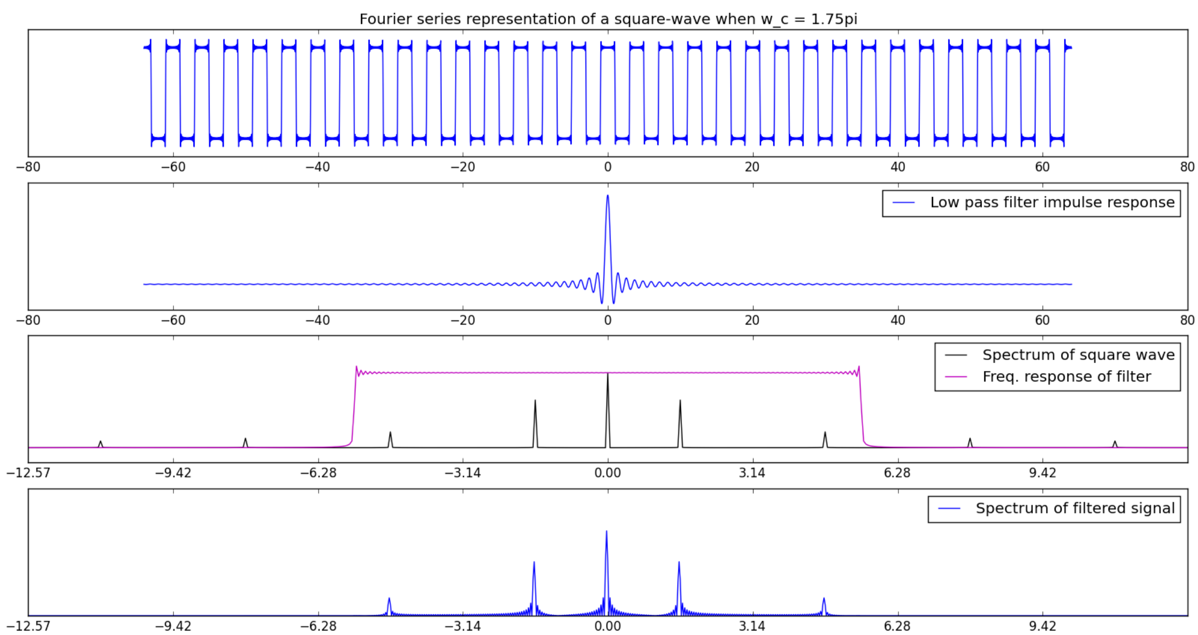


$Y(w) = H(w)\,X(w)$

c. This system acts as a low-pass filter as the frequencies (and their magnitudes) between $-w_c$ & w_c are preserved, while frequencies above w_c are cut off.

d. [See attached plots]

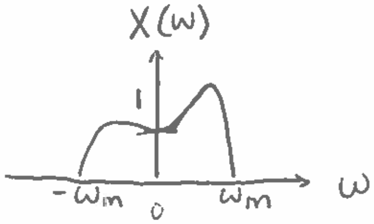




The graphs above were generated using code from SquareWaveFilterExercise.ipynb. The only alterations to the code were with w_c (when changed from the default of 0.75*pi to 1.75*pi) and with h (plugging in the solved h(t) from question 2a of the problem set). In both sets of plots, the “low-pass filter” quality of the system is validated. The last plot of each set is most telling, as the filtered signals are visibly cut off between 0 and pi for w_c = 0.75*pi and between pi and 2pi for w_c = 1.75*pi.

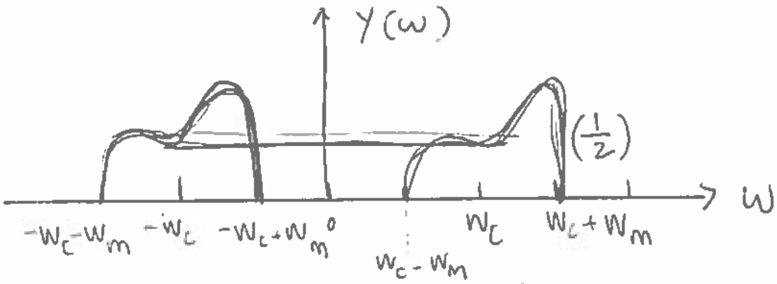
3. Signal $x(t)$ band-limited to $[-\omega_M, \omega_M]$

$$X(\omega) = \begin{cases} 0 & \text{for } \omega < -\omega_m \\ & \omega > \omega_m \end{cases}$$



Let $y(t) = x(t)\cos(W_c t)$

where $\omega_c \gg \omega_m$



$$Y(W) = H(W)\,X(W)$$

We know that:

$$y(t) = x(t)\,h(t)$$

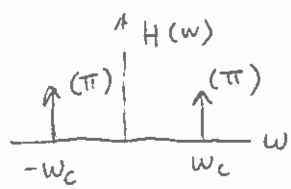
&

$$Y(\omega) = \frac{1}{2\pi} X * H(\omega)$$

$\therefore$ $h(t) = \cos(W_c t)$

$\hookrightarrow$ ~~H(W)~~

$$H(W) = \pi\,\delta(W - W_c) + \pi\,\delta(W + W_c)$$



$$Y(W) = \frac{1}{2\pi} X \cdot \left(\pi\,\delta(W - W_c) + \pi\,\delta(W + W_c)\right)$$

$$Y(W) = \frac{1}{2} X(\delta(W - W_c)) + \frac{1}{2} X(\delta(W + W_c))$$

$Y(W)$, as drawn above, is $X(W)$ shifted to be centered at the impulses of $H(W)$ and scaled to half