SIGSYS PS 10 - 04 · 05 · 2015 Jennifer Wei

1. $\dot{y} + y = x$ } transform

$sY + Y = X$

$H(s) = \frac{Y}{X} = \frac{1}{s+1}$

step response

$\mathcal{L}[u(t)] = \frac{1}{s}$

SOLVE FOR A + B of partial fraction to make the inverse Laplace EASIER

$\frac{1}{s} \cdot \frac{1}{s+1} = \frac{A}{s} + \frac{B}{s+1}$

$1 = A(s+1) + B(s)$

FOR $s = -1$, $B = -1$

FOR $s = 0$, $A = 1$

$\therefore \frac{1}{s(s+1)} = \frac{1}{s} - \frac{1}{s+1}$

USING THE GIVEN PROPERTIES

$e^{-at}u(t) \overset{\mathcal{L}}{\Longleftrightarrow} \frac{1}{s+a}$, $Re\{s\} > -a$

$u(t) \overset{\mathcal{L}}{\Longleftrightarrow} \frac{1}{s}$, $Re\{s\} > 0$

$\mathcal{L}^{-1}\left(\frac{1}{s} - \frac{1}{s+1}\right) = u(t) - u(t)e^{-t}$

$\therefore y(t) = (1 - e^{-t})(u(t))$ ✓

The easiest way to describe this process is with a diagram

$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$

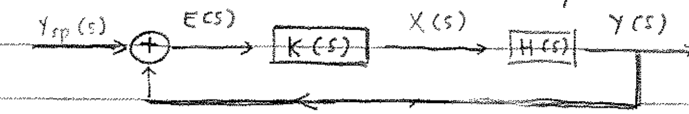
$\downarrow \mathcal{L}$ $\uparrow \mathcal{L}^{-1}$

$X(s) \longrightarrow \boxed{H(s)} \longrightarrow Y(s) = H(s)X(s)$

To get to $y(t)$, the easiest route would be to transform $x(t)$ to $X(s)$, multiply with $H(s)$, and use the inverse laplace transform.

We multiply by the Laplace of $u(t)$ since we want the step response of the system, so $x(t)$ in the diagram is our $u(t)$

2A Find the DC gain of $\frac{Y(s)}{Y_{sp}(s)}$ given integral controller $K(s) = \frac{K_I}{s}$ for any $H(s)$




Using Black's formula, we know

$\frac{Y}{Y_{sp}} = \frac{kH}{1+kH} = \frac{(K_I/s)H}{1+(K_I/s)H} = \frac{K_I H}{s} \div \left(\frac{s + K_I H}{s}\right)$

$\frac{Y}{Y_{sp}} = \frac{K_I H}{s} \cdot \frac{s}{s+K_I H} = \frac{K_I H}{s + K_I H}$

$DC_{gain} = \lim_{s \to 0}\left(\frac{Y}{Y_{sp}}\right) = \lim_{s \to 0}\left(\frac{K_I H}{s + K_I H}\right) = 1$

2B Assume $H(s) = \frac{1/\tau}{s + 1/\tau}$ & find $\frac{Y(s)}{Y_{sp}(s)}$

Find the pole(s) assuming $K \gg \frac{1}{\tau}$ *

$\frac{Y(s)}{Y_{sp}(s)} = \frac{K_I H}{s + K_I H} = \left(\frac{(K_I)/\tau}{s + 1/\tau}\right) \div \left(s + \frac{(K_I)/\tau}{s + 1/\tau}\right)$ ✓

$\left(\frac{K_I/\tau}{s + 1/\tau}\right) \div \left(\frac{s^2 + s/\tau + K_I/\tau}{s + 1/\tau}\right) = \frac{K_I/\tau}{s^2 + s/\tau + K_I/\tau} = \frac{Y(s)}{Y_{sp}(s)}$

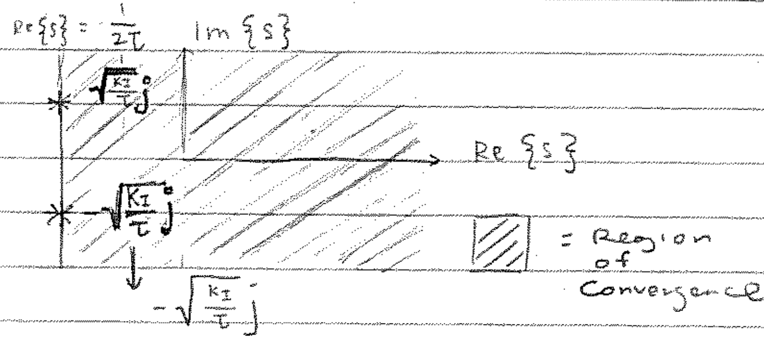
SOLVE FOR S USING QUADRATIC FOR THE POLES

$s = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$s = \frac{-(\frac{1}{\tau}) \pm \sqrt{(1/\tau)^2 - 4(1)(K_I/\tau)}}{2}$

$s = \frac{-\frac{1}{\tau} \pm \sqrt{\frac{1}{\tau^2} - 4K_I/\tau}}{2}$, where $K \gg \frac{1}{\tau}$

$s \approx -\frac{1}{2\tau} \pm \frac{\sqrt{-4K_I/\tau}}{2}$




2A Gain is not dependent on $K_I$ as it is always 1 when $S$ approaches 0.

Note: There are no zeros since the numerator is not dependent on $S$.

3E

Q3 [See code + Plots]

4 Stabilize $H(s) = \frac{1}{s^2 - 0.01s + 1}$

4A $u(t) * h(t) = $ step response

$p : \frac{KH}{1+KH}$

$Y(s) = X(s)\,H(s)$

$Y(s) = \frac{1}{s}\,\frac{1}{s^2 - 0.01s + 1} = \frac{1}{s(s^2 - 0.01s + 1)}$

SEE CODE + PLOTS

ATTACHED

4B PROPORTIONAL CTRL

$H(s) = \frac{1}{s^2 - 0.01s + 1} \quad K = K_p$

Transfer Function $= \frac{K_p}{s^2 - 0.01s + 1 + K_p}$

4c INTEGRAL CTRL

$K = \frac{K_I}{s}$

Transfer $= \frac{K_I}{s^3 - 0.01s^2 + s} \div 1 + \frac{K_I}{s^3 - 0.01s^2 + s}$

$\hookrightarrow = \frac{K_I}{s^3 - 0.01s^2 + s + K_I}$

4d DERIVATIVE CTRL

$K = sK_I$

Transfer $= \frac{sK_I}{s^2 - 0.01s + 1} \div 1 + \frac{sK_I}{s^2 - 0.01s + 1}$

$\hookrightarrow = \frac{K_I}{s^2 + K_I s - 0.01s + 1}$

[See plots]

http://nbviewer.ipython.org/github/jenwei/SigSys2015/blob/master/PS10.ipynb