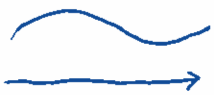
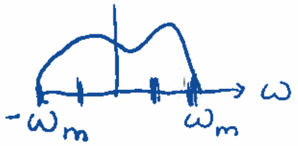
SIGSYS PS08 - JENNIFER WEI 03262015

1. Consider signal $x(t)$ band limited to ω_M

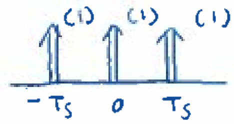
$x(t)$ $X(\omega)$





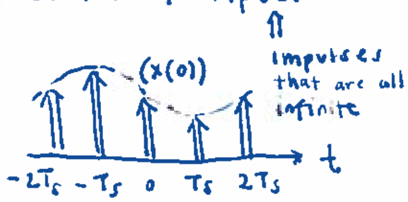
$p(t)$ = impulse train

$$p(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT_s)$$

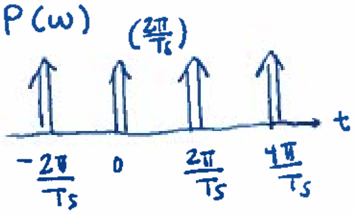


$$x_p(t) = x(t)\,p(t)$$

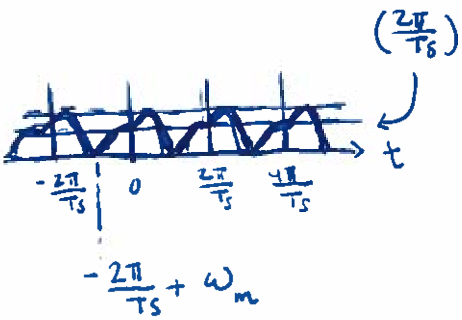
a Sketch of $x_p(t)$



b $P(\omega)$



c $X_p(\omega)$



d Relationship between T_s & ω_m

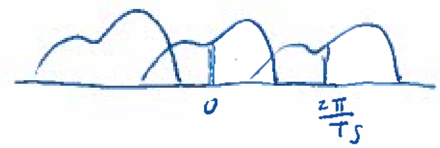
$$2\omega_m < \frac{2\pi}{T_s}$$

since

$$\frac{2\pi}{T_s} - \omega_m > \omega_m$$

$\therefore$ $2\omega_m < \omega_s$ guarantees that $X_p(\omega)$ contains all the information present in $X(\omega)$.

If $2\omega_m > \omega_s$:



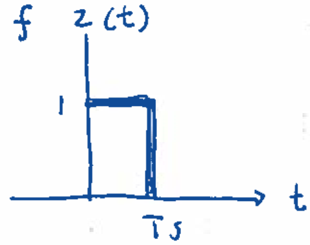
There would be overlaps of freq components

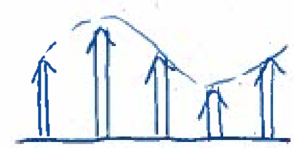
e HOW TO RECOVER $x(t)$ from $x_p(t)$

To recover $x(t)$ from $x_p(t)$, multiply the frequency by a box (a ~~[illegible]~~ filter with frequency $[-\omega_m, \omega_m]$) to isolate the middle frequency.

In the time domain, this would mean convolving with a sinc function.

f $z(t)$





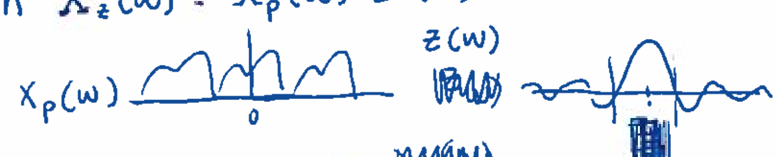
X_p [illegible]
(see Q1a)

g $x_z(t) = X_p * z(t)$

↓

zero-order hold reconstruction of $x_p(t)$

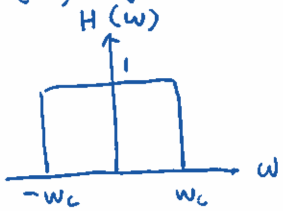
h $X_z(\omega) = X_p(\omega)\, Z(\omega)$



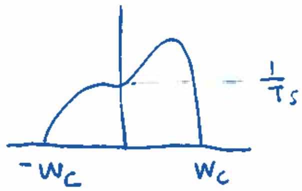
$\bar{X}(\omega) = X_t(\omega)\, H(\omega)$

$\hat{X}(\omega) = X_p(\omega)\, H(\omega)$

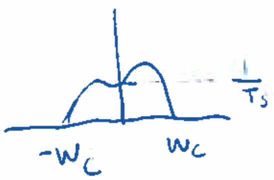
$H(\omega)$ (with $\omega_c = \frac{\pi}{T_s}$)



$\hat{X}(\omega)$



$\bar{X}(\omega)$



Since W_c happens to encapsulate one frequency component, this is ~~cse~~ essentially demonstrating ~~1e~~ ~~&~~ the whole 'isolate the middle frequency' idea

j. $\bar{X}(\omega)$ & $\hat{X}(\omega)$ are different since $\bar{X}(\omega)$ has a sinc function applied & decays more quickly (and is attenuated) as a result.

k. when $\omega_m = \frac{\pi}{T_s}$, the ratio of $\bar{X}(\omega_m)$ & $\hat{X}(\omega_m)$ is $\frac{0}{0}$ since both equal 0 at $\frac{\pi}{T_s}$

SIGSYS PS 08 - JENNIFER WEI

2 $y(t) = x_1(t)\cos(w_1 t) + x_2(t)\cos(w_2 t)$

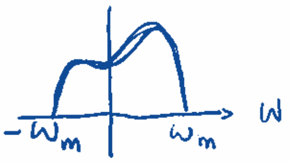
Suppose $X_1(W) = 0$, $X_2(W) = 0$ if $|w| > W_m$

ASSUME $W_1 >> W_m$

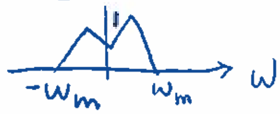
$W_2 >> W_m$

$W_1 + 2W_m < W_2$

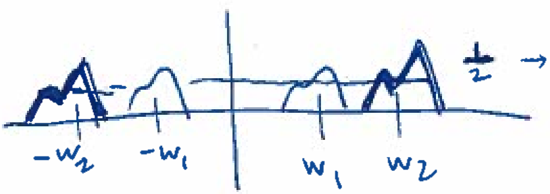
$X_1(W)$



$X_2(W)$

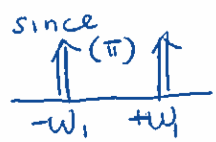


a $Y(\omega)$



$\frac{1}{2}$ →

FT{$\cos w_1 t$}

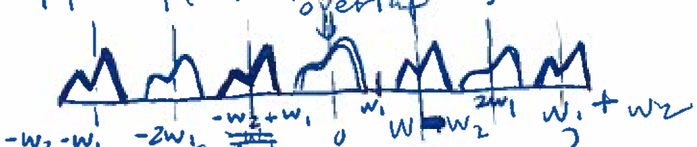


NOTE: The gap between W_1 & W_2 should be larger since $W_1 + 2W_m <$ W_2. (sorry I have poor drawing skills)

b FT of $\{y(t)\cos(w_1 t)\}$

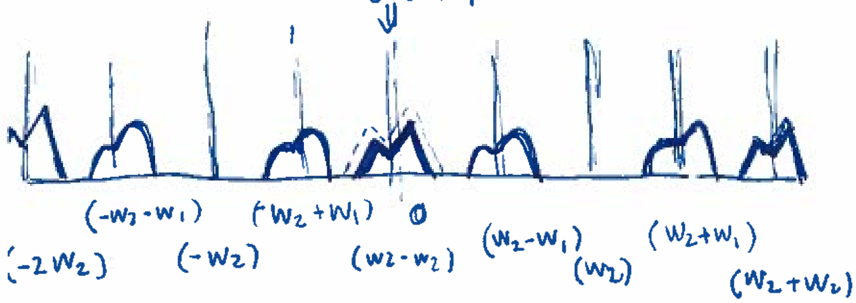
FT = $Y(w) \cdot$ FT{cos wt}

overlap



FT of $\{y(t)\cos(w_2 t)\}$

overlap



c TO RECOVER $x_1(t)$ & $x_2(t)$ from $y(t)$, where $y(t)$ is the received AM signal from two different AM transmitters using frequency W_1 & W_2

To recover $x_1(t)$ & $x_2(t)$, multiply $y(t)$ by $\cos(w_1 t)$ or $\cos(w_2 t)$ (respectively).

From there, like in the process described in 1e, filter with a cutoff @ [$-W_m$] and [W_m].

Since cos attenuates the signal, multiply by 2 to revert back.

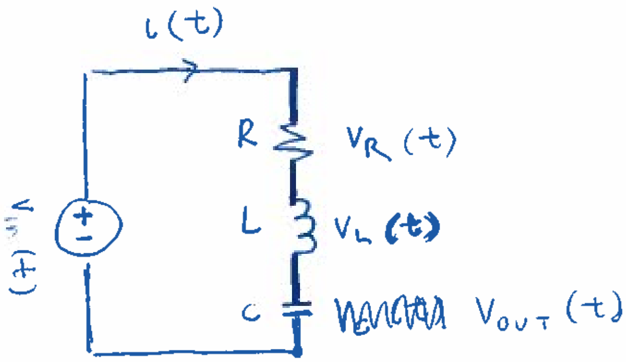
ANOTHER WAY TO SOLVE WOULD BE TO WORK OFF OF THE FTs of 2b.

TAKING THE FTs, APPLY A FILTER WITH FREQS [W_M, W_M].

~~UNDO THE ATTENUATION~~ & ~~use the inverse FT to get $X_1(t)$ & $X_2(t)$~~

UNDO THE ATTENUATION & use the inverse fourier to find $X_1(t)$ & $X_2(t)$

3.



$i(t) = C \frac{d}{dt} V_{OUT}(t)$

$V_L(t) = L \frac{d}{dt} i(t)$

$V_{OUT} = V_{IN} - V_R - V_L$

$V_{OUT} = V_{IN} - R\, i(t) - L \frac{d}{dt} i(t)$

$V_{OUT} = V_{IN} - R\left(C \frac{d}{dt} V_{OUT}(t)\right) - L \frac{d}{dt}\left(C \frac{d}{dt} V_{OUT}(t)\right)$

$V_{OUT} + RC \frac{d}{dt} V_{OUT}(t) + L \frac{d}{dt} C \frac{d}{dt} V_{OUT}(t) = V_{IN}$

$V_{IN} = V_{OUT}(t) + RC \frac{d}{dt} V_{OUT}(t) + CL \frac{d^2}{dt^2} V_{OUT}(t)$

$H(w) = \frac{V_{OUT}(t)}{V_{IN}(t)}$

$V_{IN}(t) = e^{jw_1 t}$

$V_{OUT}(t) = H(w) V_{in}(t) = H(w) e^{jw_1 t}$

$e^{jw_1 t} = H(w)\left(e^{jw_1 t} + RC\, jw_1 e^{jw_1 t} + j^2 w^2 CL e^{jw_1 t}\right)$

$\cancel{e^{jw_1 t}} = H(w)\left[\cancel{e^{jw_1 t}} + RCjw_1 \cancel{e^{jw_1 t}} - w^2 CL \cancel{e^{jw_1 t}}\right]$

$\frac{1}{1 + RCjw_1 - w^2 CL} = H(w)$ ☆

$\|H(w)\| = \sqrt{H(w)^2 + H(w)^2}$

$= \sqrt{\left(\frac{1}{1+RCjw - w^2CL}\right)^2 * 2}$

$= \sqrt{\frac{2}{C^2L^2w^4 + j(2CRw - 2C^2LRw^3) - C^2R^2w^2 - 2CLw^2 + 1}}$

$= \frac{\sqrt{2}}{-1 + Cw(Lw - jR))^2}$ (used Wolfram alpha)

d used Wolfram Alpha

$$\frac{-\sqrt{2}\, C(R + 2iLw)\sqrt{\frac{1}{(-1 + Cw(Lw - iR))^2}}}{-1 + Cw(R + iLw)}$$

$\Downarrow$

$$\frac{-\sqrt{2}\, C(R + 2iLw)}{[-1 + Cw(R + iLw)][-1 + Cw(Lw - iR)]}$$

Something went wrong
Trying part (c) equation AGAIN

c. $\|H(w)\| = \sqrt{H(w)\, H^*(w)}$

$H^*(w) = \frac{1}{-w^2LC + 1 - jwRC}$

$\|H(w)\| = \frac{1}{\sqrt{((-w^2LC+1) + jwRC)((-w^2LC+1) - jwRC)}}$

$\|H(w)\| = \frac{1}{\sqrt{(-w^2LC+1)^2 - j^2w^2(RC)^2}}$

$\|H(w)\| = \frac{1}{\sqrt{(-w^2LC+1)^2 + w^2(RC)^2}}$

d. $\frac{d}{dw}\left(\sqrt{(-w^2LC+1)^2 + (wRC)^2}\right) = 0$ find max $H(w)$

$\frac{d}{dw}\left[(-w^2LC+1)^2 + (wRC)^2\right] = 0$

$\frac{d}{dw}\left[((-w^2LC)^2 - 2w^2LC + 1) + (wRC)^2\right] = 0$

$\frac{d}{dw}\left[w^4L^2C^2 - 2w^2LC + 1 + w^2R^2C^2\right] = 0$

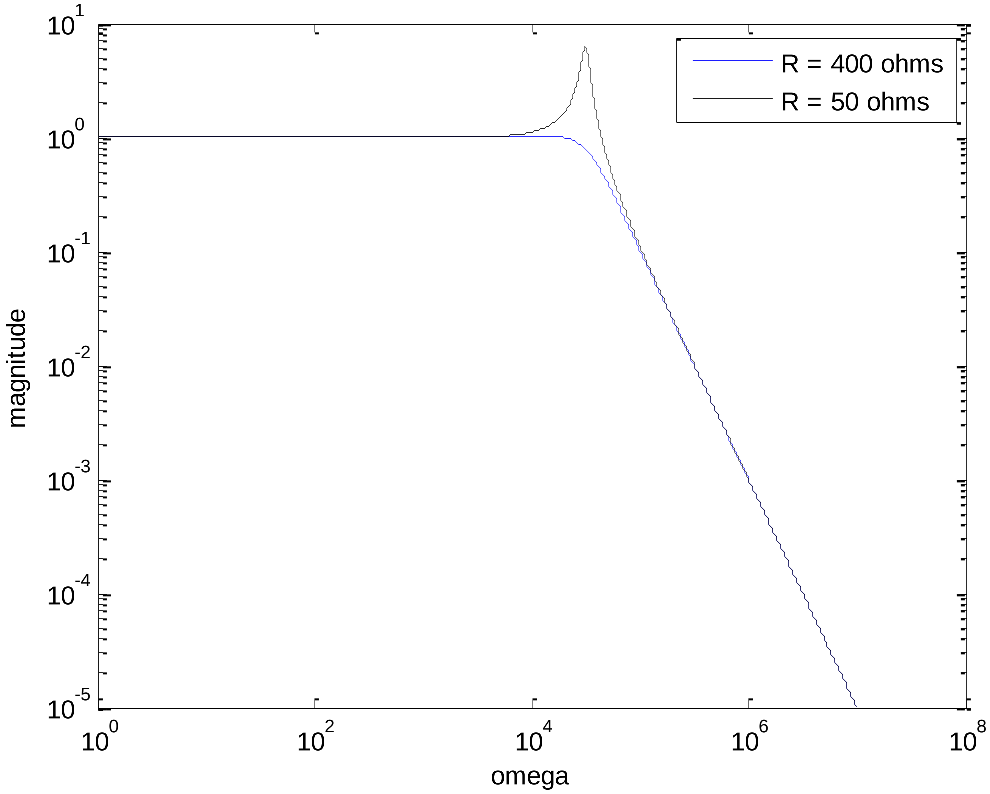
$4L^2C^2w^3 - 4LCw + 2R^2C^2w = 0$

$2L^2Cw^2 - 2L + R^2C = 0$

$2L^2Cw^2 = 2L - R^2C$

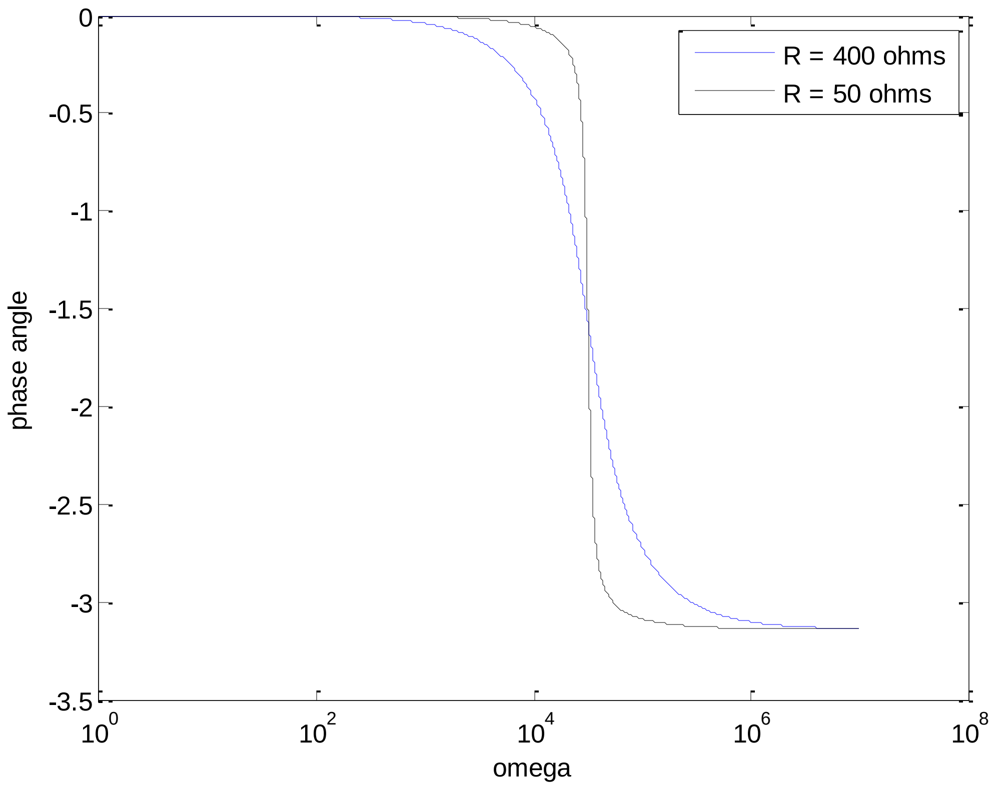
$w^2 = \frac{2L - R^2C}{2L^2C} \rightarrow w = \sqrt{\frac{2L - R^2C}{2L^2C}}$

NOTE: This was done as a team by Ryan and Jennifer




```
C = 10^-7; %F
L = 10^-2; %H
R1 = 400; %ohms
R2 = 50; %ohms


w = 1:1:10000000;
Mag1 = (1.0./( (w*R1*C).^2 + (1-(w.^2)*L*C).^2 ).^(0.5));
Mag2 = (1.0./( (w*R2*C).^2 + (1-(w.^2)*L*C).^2 ).^(0.5));
loglog(w,Mag1,'b',w,Mag2,'k');
legend('R = 400 ohms', 'R = 50 ohms');
xlabel('omega');
ylabel('magnitude');
```




```
figure;
Freq1 = 1.0./(1+(R1.*C.*j.*w)-((w.^2).*C.*L));
Freq2 = 1.0./(1+(R2.*C.*j.*w)-((w.^2).*C.*L));
phase1= angle(Freq1);
phase2 = angle(Freq2);
%phase1 = atan((j.*w.*R1.*C)./(-(w.^2).*L.*C).^2).*(360/2*pi);
%phase2 = atan((j.*w.*R2.*C)./(-(w.^2).*L.*C).^2).*(360/2*pi);
semilogx(w,phase1,'b',w,phase2,'k');
legend('R = 400 ohms', 'R = 50 ohms');
xlabel('omega');
ylabel('phase angle');
```