Graph Theory & Surface Reconstruction

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Abstract

The purpose of this paper is to utilize a technique for recreating models of 3-D objects. This process is referred to as surface reconstruction, which replaces a set of sample points using a faceted geometric model. Concepts of graph theory are used, specifically duals of graphs, n-regular graphs, bridgeless graphs, and matchings.

Introduction

The paper will refer to the triangulation of a 3-D surface given in the figure below.

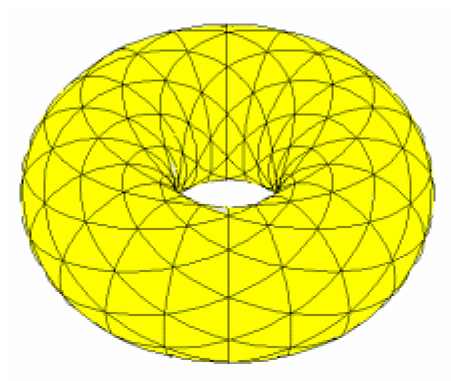

Figure 1: A torus.

The figure resembles a torus, which is created by revolving a circle along a line made by another circle. Divisions of polygonal faces that form the torus produces triangular faces throughout the torus.

Dual Graph

By definition, the planar dual of any planar graph, G, can be constructed by placing a vertex in each region of G and then adding edges between regions that share a border. The planar dual is denoted as D(G).

If a dual of the torus is to be constructed, the same procedure to create the dual of a planar graph will be used. A point will be placed in each of the triangular faces, then lines between pairs of vertices would be placed, if and only if, the corresponding faces share a common side. The dual of the torus, G, is constructed below.

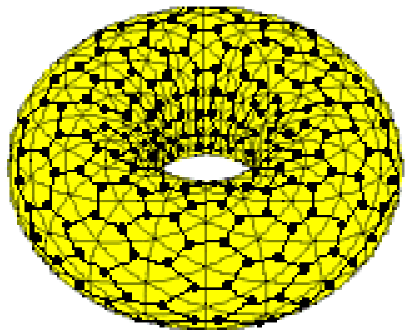

Figure 2: Dual of a torus, G.

As shown in the figure, the triangular faces of the original torus are hexagonal faces in the dual of the torus.

n-regular Graph

A graph is considered n-regular if every vertex in the graph is incident to exactly n edges.

The dual of the torus is a 3-regular graph. According to the definition, that would mean that every vertex in the dual of the torus, G, is incident to exactly 3 edges. The graph of the dual is a 3-regular because each face in the original graph is a complete graph, K_3, in which each vertex is connected to all the other vertices. A K_3 graph consists of 3 vertices that all have degree 2. The dual of a K_3 is just one vertex inside the cycle and one vertex on the outside of the cycle, which represents the infinite region. Each edge in K_3 has an edge in the dual graph crossing the edge to connect those two vertices, as shown below.

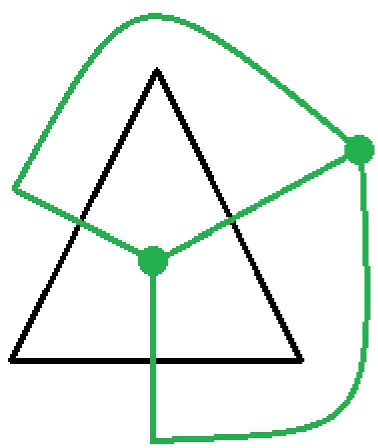

Figure 3: Dual of a complete graph, K_3.

By taking the dual of groups of K_3, a similar concept is used to connect the vertices for each region, excluding the infinite region. As a result of the three edges in each K_3, each vertex representing one region has three incident edges in the planar dual graph of each K_3. Thus, the dual of the torus is a 3-regular graph.

Bridgeless Graph

A graph is considered bridgeless if the removal of any single edge does not disconnect the graph.

The dual of the torus is bridgeless because the dual does not contain a cut-edge, whose removal would increase the number of components. Since the dual consists of a group of cycles, C_6, the edge connectivity of the dual would have to be, at minimum, the edge connectivity of C_6. The edge connectivity of C_6, $\kappa'(C_6)$, is greater than or equal to two because C_6 does not contain a cut-edge. As a result, the edge connectivity of the dual is at least two, as shown below.

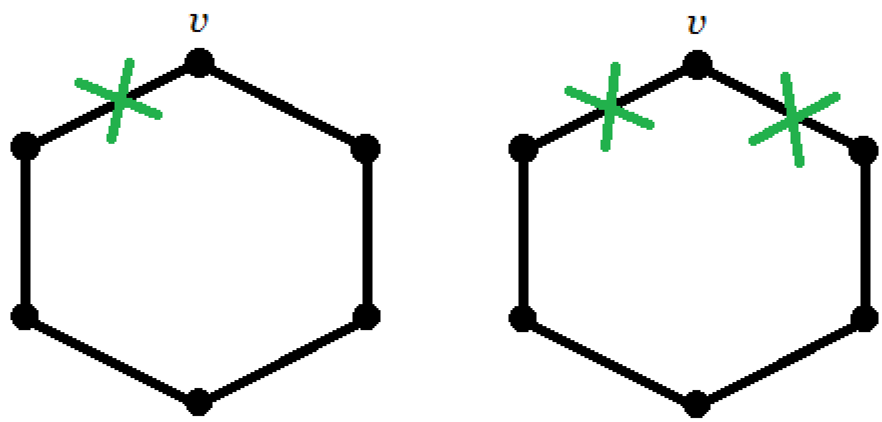

Figure 4: Connectivity of C_6.
Removal of one edge in the left graph of C_6 does not disconnect C_6.
Removal of two edges in the right graph of C_6 disconnects v from C_6.

Since a grouping of C_6 in the dual adds one edge to each vertex, the edge connectivity of the dual is increased to three. That edge is one extra edge needed to increase the number of components. As a result, the removal of at least three edges is required to disconnect the dual of the torus. Thus, the dual of the torus is bridgeless because the removal of any single edge does not disconnect the graph.