ISE 5113 Advanced Analytics and Metaheuristics
Homework #1

Instructor: Charles Nicholson

Due: see website for due date/time

Requirement details

1. Homeworks should be submitted in a clean, clear, concise electronic format through the course website. You must show your logic, work, and/or code where appropriate. Any code (e.g. AMPL) is part of your solution – make sure to provide comments on what your code is doing.
2. Homeworks are to be completed in teams of two. If team members disagree on an answer, you can record solutions corresponding to each member (please clearly mark which solution belongs to which team member)
3. For any mathematical programming problem, in addition to solving the problem and responding to the questions, please ensure you *clearly define* the following elements in the your answer: (i) **any necessary assumptions**, (ii) **decision variables**, (iii) **objective and objective function**, and (iv) **constraints**.

   For example, don't simply use the variable x_1 and assume I can figure out what it means! Also, please label your constraints in meaningful ways, both in the the problem formulation text and in any relevant code. Points may be deducted if proper definitions/documentation is missing.

Question 1: Smullyan's Island Revisited (6 points)

Inhabitants on Smullyan's Island are either always telling the truth or always lying. You meet three people, Gregor, Tywin, and Catelyn, who make the following statements, respectively:

Gregor: "Exactly one of us is telling the truth."
Tywin: "We are all lying."
Catelyn: "The other two are lying."

From this, what can you tell about Gregor, Tywin, and Catelyn? (Use a truth-table!)

Question 2: Wildlings and Humans (10 points)

Jon Snow of Winterfell (and a brother of Night's Watch) is called to Castle Black to investigate some individuals. Arriving there, he found the country inhabited both by "wildlings" and humans. Wildlings (who look exactly like humans) always lie, whereas humans always tell the truth. However, half the inhabitants, both human and wildling, are insane and totally deluded in their beliefs: all true propositions they believe false, and all false propositions they believe true. The other half of the inhabitants are completely sane: all true statements they know to be true, and all false statements they know to be false. Thus sane humans and insane wildlings make only true statements; insane humans and sane wildlings make only false statements. Jon Snow met two sisters, Arya and Sansa. He knew that one was a wildling and one was a human, but knew nothing about the sanity of either.

Here is the investigation:

1. Jon (to Arya): Tell me about yourselves.
2. Arya: We are both insane.
3. Jon (to Sansa): Is that true?
4. Sansa: Of course not!

From this, Jon was able to prove which of the sisters was the wildling. Which one was it? (Make sure your logic is undeniably solid!, i.e., use a truth-table!)

Question 3: Working Capital Management (20 points)

You are the financial manager of company charged with maximizing the company's cash flow. You must manage short-term investments to maximize interest income, while making sure that funds are available to pay company expenditures. Consider these three investment options over the next 6 months:

- a 1-month investment that pays 0.5%
- a 3-month investment that pays 2.1%
- a 6-month investment that pays 3.5%

The net expenditures for the next 6 months are as follows: \$50k, (\$12k), \$23k, (\$20k), \$41k, (\$13k). Note that the dollar amounts in parentheses indicate a net inflow of cash (for example at the start of month 2, \$12,000 will be added to company accounts). The company currently has \$300k in cash.

(a) Clearly state and define the decision variables. How many decision variables exist?

(b) Clearly state the objective and objective function.

(c) Clearly define and determine a functional form for each constraint.

(d) Solve the problem using AMPL.

Question 4: Seasonal Demand (14 points)

You are the proud owner of "Sunny Sandals", a company which produces sandals. This is quite a seasonal business, but year to year, the demand patterns are relatively stable. Due to your semi-obsolete equipment, the total number sandals you can produce in a season is limited to 1,200 pairs. You can, however, keep an inventory of sandals at a holding cost of \$0.15 per pair per season. The demand in each season is as follows: 2800, 500, 100, 850. What is the best production plan that you can devise? (You must code and solve this in AMPL)

Question 5: Golden Canning Co (50 points)

On Monday, January 19, 2015, Mr. Ernesto Ochandio, Vice-President of Operations, asked the Controller, the Sales Manager, and the Production VP to meet with him to discuss the amount of tomato products to pack that season. The tomato crop, which had been purchased at planting, was beginning to arrive at the cannery, and packing operations would have to be started by the following Monday. Golden Canning Co. cans and distributes a variety of fruit and vegetable products under private brands in the US.

Mr. Dale Thomas, the Controller, and Ms. Michelle Bollman, the Sales Manager, were the first to arrive in Mr. Ochandio's office. Brett Jaggers, the Production VP, came in a few minutes later and said that he had picked up Produce Inspection's latest estimate of the quality of the incoming tomatoes. According to their report, about 20% of the crop was Grade "A" quality and the remaining portion of the 3,000,000-pound crop was Grade "B."

Ochandio asked Bollman about the demand for tomato products for the coming year. Bollman replied that they could sell all of the whole canned tomatoes they could produce. The expected demand for tomato juice and tomato paste, on the other hand, was limited. The Sales Manager then passed around the latest demand forecast, which is shown in Table 1. She reminded the group that the selling prices had been set in light of the long-term marketing strategy of the company, and potential sales had been forecast at these prices.

Dale Thomas, after looking at Bollman's estimates of demand, said that it looked like the company "should do quite well (on the tomato crop) this year." With the new accounting system that had been set up, he had been able to compute the contribution for each product, and according to his analysis the incremental profit on the whole tomatoes was greater than for any other tomato product. In May, after Golden Canning had signed contracts agreeing to purchase the grower's production at an average delivered price of 6 cents per pound, Thomas had computed the tomato products' contributions (see Table 2).

Brett Jaggers brought to Thomas' attention that, although there was ample production capacity, it was impossible to produce all whole tomatoes as too small a portion of the tomato crop was "A" quality. Golden Canning used a numerical scale to record the quality of both raw produce and prepared products. This scale ran from zero to ten, the higher number representing better quality. Rating tomatoes according to this scale, "A" tomatoes averaged nine points per pound and "B" tomatoes averaged five points per pound. Jaggers noted that the minimum average input quality for canned whole tomatoes was eight and for juice it was six points per pound. Paste could be made entirely from "B" grade tomatoes. This meant that whole tomato production was limited to 800,000 pounds.

Ochandio stated that this was not a real limitation. He has been recently solicited to purchase 80,000 pounds of Grade "A" tomatoes at 8.5 cents per pound and at that time had turned down the offer. He felt, however, that the tomatoes were still available.

Bollman, who had been doing some calculations, said that although she agreed that the company "should do quite well this year," it would not be by canning whole tomatoes. It seemed to her that the tomato cost should be allocated on the basis of quality and quantity rather than by quantity only as Thomas had done. Therefore, she had recomputed the marginal profit on this basis (see Table 3), and from her results, Golden Canning should use 2,000,000 pounds of the "B" tomatoes for paste, and the remaining 400,000 pounds of "B" tomatoes and all of the "A" tomatoes for juice – no tomatoes should be sold has whole canned. If the demand expectations were realized, a contribution of $48,000 would be made on this year's tomato crop.

Assignment

(a) Why does Jaggers state that the whole tomato production is limited to 800,000 pounds (i.e., where does the number 800,000 come from)?

(b) How does Bollman compute the tomato costs in Table 3? How does she reach the conclusion that the company should use 2,000,000 pounds "B" tomatoes for paste, the remaining 400,000 pounds of "B" tomatoes, and all of the "A" in juice? What is wrong with her reasoning?

(c) Ignoring for the moment the chance to buy additional A-grade tomatoes. Formulate the production question as a LP problem, solve with AMPL, or an equivalent software package, and answer the following questions.

i. How much whole, juice, and paste should be made?

ii. What is the contribution to profit? 

iii. Are there any tomatoes left over? If so, of what grade?

iv. What is the average quality point count of whole, juice, and paste?

v. What would be the worth of one additional pound of A-grade tomatoes?

vi. Should Golden Canning Co buy the extra 80,000 pounds of A-grade tomatoes at the offered price?

(d) For the following questions, assume that the 80,000 pounds of A-grade is not in the formulation.

i. What product mix would result if Thomas's contribution figures were used? Is Thomas's contribution more or less than the contribution to your (earlier) Part (c) solution? Why the difference?

ii. Alternatively, what product mix would result if Bollman's profit figures were used? What would the profit be? Is this more or less than you found in Part (c)?

iii. Suppose an unlimited supply of A-grade tomatoes were available at \$0.085 per lb. How much should Golden Canning Co buy? What would be the product mix?

Table 1: Demand Forecasts

Product	Selling Price per Case	Demand Forecast (Cases)
24 - $2\frac{1}{2}$ whole tomatoes	\$4.00	800,000
24 - $2\frac{1}{2}$ choice peach halves	\$5.40	10,000
24 - $2\frac{1}{2}$ peach necture	\$4.60	5,000
24 - $2\frac{1}{2}$ tomato juice	\$4.50	50,000
24 - $2\frac{1}{2}$ cooking apples	\$4.90	15,000
24 - $2\frac{1}{2}$ tomato paste	\$3.80	80,000

Table 2: Product Item Profitability

Product	24 - $2\frac{1}{2}$ Canned Whole Tomatoes	24 - $2\frac{1}{2}$ Peach Halves	24 - $2\frac{1}{2}$ Peach Nectar	24 - $2\frac{1}{2}$ Tomato Juice	24 - $2\frac{1}{2}$ Cooking Apples	24 - $2\frac{1}{2}$ Tomato Paste
Selling Price (\$)	4.00	5.40	4.60	4.50	4.90	3.80
Variable Costs (\$)						
Direct Labor	1.18	1.40	1.27	1.32	1.70	0.54
Variable OHD.	0.24	0.32	0.23	0.36	0.22	0.26
Variable Selling	0.40	0.30	0.40	0.85	0.28	0.38
Packaging Material	0.70	0.56	0.60	0.65	0.70	0.77
Fruit*	1.08	1.80	1.70	1.20	0.90	1.50
Total Variable Costs	3.60	4.38	4.20	4.38	3.80	3.45
Contribution to profit	0.40	1.02	0.40	0.12	1.10	0.35
Less Allocated OHD	0.28	0.70	0.52	0.21	0.75	0.23
Net Profit	0.12	0.32	-0.12	-0.09	0.35	0.12

*Product usage is given as follows:

Product	Pounds per case
whole tomatoes	18
peach halves	18
peach nectar	17
tomato juice	20
cooking apples	27
tomato paste	25

Table 3: Bollman's marginal analysis of tomato products

1. Let z = cost per pound of A tomatoes in cents
2. Let y = cost per pound of B tomatoes in cents
3. (600,000 lb. $\times$ z) + (2,400,000 lb. $\times$ y) = (3,000,000 lb. $\times$ 6)
4. $\frac{z}{9} = \frac{y}{5}$
5. Therefore, $z = 9.32$ cents per pound; $y = 5.18$ cents per pound.

Product	Canned Whole Tomatoes	Tomato Juice	Tomato Paste
Selling Price	$4.00	$4.50	$3.80
Variable Costs (excluding tomato costs)	2.52	3.18	1.95
	$1.48	$1.32	$1.85
Tomato cost	1.49	1.24	1.30
Marginal profit	-$0.01	$0.08	$0.55