Information Theory

1. Bayes Theorem (베이즈 정리 확장)

$$P(B) = \sum_i^n P(B \cap A_i) = \sum_i^n P(B|A_i) \cdot P(A_i)$$

$$P(A_i|B) = \frac{P(B|A_i)P(A_i)}{\sum_i^n P(B|A_i)P(A_i)}$$

$P(A_i|B)$: 사후 확률

$P(B|A_i)$: 우도

$P(A_i)$: 사전 확률

2. ML & MAP

ML

우도($P(x|C_k)$)를 알고 있다면

Maximum Likelihood 최대우도 방법을 통해서 분류

$$K^* = \arg\max_{k=1...K} P(x_{new}|C_k)$$

MAP

사전확률($P(C_k)$)을 알고 있다면

Maximum A Posteriori 방법을 통해서 분류

$$K^* = \arg\max_{k=1...K} P(x_{new}|C_k)P(C_k)$$

3. Expectations

Conditional Expectations

$$E(Y|X) = \begin{cases} \sum_k y_k \cdot P(y_k|X) & Y = \text{discrete} \\ \int_{-\infty}^{\infty} y f(y|x)\, dy & Y = \text{Continuous} \end{cases}$$

Function of R.V

$$E(g(x)) = \begin{cases} \sum_k g(x_k) P(x_k) & Y = \text{discrete} \\ \int_{-\infty}^{\infty} g(x) f(x)\, dx & Y = \text{continuous} \end{cases}$$

4. Correlation, Covariance

Correlation

$E(XY)$

직교: $E(XY) = 0$

독립: $E(XY) = E(X)E(Y)$

Covariance

$$Cov(X,Y) = \sigma_{XY} = E[(X - E(X))(Y - E(Y))]$$
$$= E(XY) - E(X)E(Y)$$

독립: $\sigma_{XY} = 0$

Correlation Coefficient

$$\rho_{XY} = \frac{\sigma_{XY}}{\sigma_X \sigma_Y}$$

5. Source Coding Theorem

$L \geq H(X)$ → Encoder Length

$L = \sum_i^n P(x_i) n_i$ Length

$H(X) = -\sum_i^m P(x_i) \log P(x_i)$ Entropy