EDA

회귀분석을 할 때 다중공선성이 발생하면, 데이터 분석의 신뢰성이나 예측 정확도를 떨어뜨린다. 이러한 문제를 하기 위한 방법 중 하나로 데이터 선정/전처리 과정에서 "변수선택"이 매우 중요하다. ADP 에서 종종 출제되는 가변수를 만드는 전처리 및 다중공선성 확인 및 정규성을 확인해보자

- 참고 데이터 : 도요타코롤라

범주형 파생변수 만들기

In [176…

```
import pandas as pd
import numpy as np
import statsmodels.api as sm
from sklearn.model_selection import train_test_split

# 데이터 불러오기

corolla = pd.read_csv("./ToyotaCorolla.csv")
corolla.info()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 1436 entries, 0 to 1435
Data columns (total 37 columns):
 #   Column            Non-Null Count  Dtype 
---  ------            --------------  ----- 
 0   Id                1436 non-null   int64 
 1   Model             1436 non-null   object
 2   Price             1436 non-null   int64 
 3   Age_08_04         1436 non-null   int64 
 4   Mfg_Month         1436 non-null   int64 
 5   Mfg_Year          1436 non-null   int64 
 6   KM                1436 non-null   int64 
 7   Fuel_Type         1436 non-null   object
 8   HP                1436 non-null   int64 
 9   Met_Color         1436 non-null   int64 
 10  Automatic         1436 non-null   int64 
 11  cc                1436 non-null   int64 
 12  Doors             1436 non-null   int64 
 13  Cylinders         1436 non-null   int64 
 14  Gears             1436 non-null   int64 
 15  Quarterly_Tax     1436 non-null   int64 
 16  Weight            1436 non-null   int64 
 17  Mfr_Guarantee     1436 non-null   int64 
 18  BOVAG_Guarantee   1436 non-null   int64 
 19  Guarantee_Period  1436 non-null   int64 
 20  ABS               1436 non-null   int64 
 21  Airbag_1          1436 non-null   int64 
 22  Airbag_2          1436 non-null   int64 
 23  Airco             1436 non-null   int64 
 24  Automatic_airco   1436 non-null   int64 
 25  Boardcomputer     1436 non-null   int64 
 26  CD_Player         1436 non-null   int64 
 27  Central_Lock      1436 non-null   int64 
 28  Powered_Windows   1436 non-null   int64 
 29  Power_Steering    1436 non-null   int64 
 30  Radio             1436 non-null   int64 
 31  Mistlamps         1436 non-null   int64 
 32  Sport_Model       1436 non-null   int64 
 33  Backseat_Divider  1436 non-null   int64 
 34  Metallic_Rim      1436 non-null   int64 
 35  Radio_cassette    1436 non-null   int64 
 36  Tow_Bar           1436 non-null   int64 
dtypes: int64(35), object(2)
memory usage: 415.2+ KB
```

- 적절한 범주형 변수를 찾아야 함

In [201…

```python
# Fuel_Type 변수 확인

corolla.Fuel_Type.unique()
```

Out[201]:

```
array(['Diesel', 'Petrol', 'CNG'], dtype=object)
```

- 연료타입이 3가지 종류가 있는 것으로 확인되어 이를 데이터 분석을 위한 수치화를 진행할 필요가 있어보인다.

In [177…

```python
# 가변수 생성

nCar = corolla.shape[0]
nVar = corolla.shape[1]
```

```
dummy_p = np.repeat(0,nCar)
dummy_c = np.repeat(0,nCar)
dummy_d = np.repeat(0,nCar)

# boolean 인덱싱을 통해 boolean index 행렬 생성

p_idx = np.array(corolla.Fuel_Type == "Petrol")
d_idx = np.array(corolla.Fuel_Type == "Diesel")
c_idx = np.array(corolla.Fuel_Type == "CNG")


# boolean index 행렬을 사용하여 가변수에 대입 ( True = 1, False = 0 )
dummy_p[p_idx] = 1
dummy_d[d_idx] = 1
dummy_c[c_idx] = 1


# 행렬로 존재하는 가변수 -> 데이터프레임으로 전환
Fuel = pd.DataFrame({"Petrol" : dummy_p, "Diesel" : dummy_d, "CNG" : dummy_c


# 불필요한 변수 삭제 및 가변수 붙이기
corolla_ = corolla.drop(["Id","Model","Fuel_Type"], axis = 1, inplace = Fals
mlr_data = pd.concat([corolla_,Fuel],1)


# bias를 위한 상수항 추가
mlr_data = sm.add_constant(mlr_data, has_constant = "add")
mlr_data
```

```
C:\Users\David\AppData\Roaming\Python\Python37\site-packages\ipykernel_launc
her.py:29: FutureWarning: In a future version of pandas all arguments of con
cat except for the argument 'objs' will be keyword-only
```

Out[177]:

	const	Price	Age_08_04	Mfg_Month	Mfg_Year	KM	HP	Met_Color	Automatic
0	1.0	13500	23	10	2002	46986	90	1	0
1	1.0	13750	23	10	2002	72937	90	1	0
2	1.0	13950	24	9	2002	41711	90	1	0
3	1.0	14950	26	7	2002	48000	90	0	0
4	1.0	13750	30	3	2002	38500	90	0	0
...	...	...	...	...	...	...	...	...	...
1431	1.0	7500	69	12	1998	20544	86	1	0
1432	1.0	10845	72	9	1998	19000	86	0	0
1433	1.0	8500	71	10	1998	17016	86	0	0
1434	1.0	7250	70	11	1998	16916	86	1	0
1435	1.0	6950	76	5	1998	1	110	0	0

1436 rows × 38 columns

회귀분석

In [6]:

```
# 종속변수, 설명변수 준비
```

```
feature_columns = mlr_data.columns.difference(["Price"]) # Target column 빼기

X = mlr_data[feature_columns]
y = mlr_data.Price

train_x, test_x, train_y, test_y = train_test_split(X,y, train_size = 0.7, t

print(train_x.shape, test_x.shape, train_y.shape, test_y.shape)
```

```
(1005, 37) (431, 37) (1005,) (431,)
```

In [7]:

```
# train & 회귀모델 적합

full_model = sm.OLS(train_y,train_x)
fitted_full_model = full_model.fit()

fitted_full_model.summary()
```

Out[7]:

OLS Regression Results

Dep. Variable:	Price	**R-squared:**	0.907
Model:	OLS	**Adj. R-squared:**	0.904
Method:	Least Squares	**F-statistic:**	288.0
Date:	Tue, 29 Aug 2023	**Prob (F-statistic):**	0.00
Time:	07:14:12	**Log-Likelihood:**	-8453.4
No. Observations:	1005	**AIC:**	1.697e+04
Df Residuals:	971	**BIC:**	1.714e+04
Df Model:	33		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
ABS	-209.1348	135.532	-1.543	0.123	-475.104	56.834
Age_08_04	-121.4503	4.206	-28.872	0.000	-129.705	-113.196
Airbag_1	184.6335	270.467	0.683	0.495	-346.134	715.401
Airbag_2	-76.3266	139.305	-0.548	0.584	-349.701	197.048
Airco	166.1030	95.305	1.743	0.082	-20.924	353.131
Automatic	431.2130	159.784	2.699	0.007	117.652	744.774
Automatic_airco	2399.4228	198.598	12.082	0.000	2009.692	2789.153
BOVAG_Guarantee	353.2689	131.141	2.694	0.007	95.917	610.621
Backseat_Divider	-312.7795	134.836	-2.320	0.021	-577.383	-48.176
Boardcomputer	-337.2981	125.746	-2.682	0.007	-584.063	-90.533
CD_Player	294.0563	105.520	2.787	0.005	86.982	501.130
CNG	-1105.7947	236.346	-4.679	0.000	-1569.602	-641.987
Central_Lock	97.4224	165.192	0.590	0.555	-226.751	421.596
Cylinders	-0.0334	0.002	-14.757	0.000	-0.038	-0.029
Diesel	161.8499	194.900	0.830	0.407	-220.624	544.324
Doors	26.0380	42.259	0.616	0.538	-56.891	108.967
Gears	69.1401	196.987	0.351	0.726	-317.430	455.710
Guarantee_Period	56.8111	15.211	3.735	0.000	26.961	86.661
HP	22.9470	3.658	6.273	0.000	15.768	30.126
KM	-0.0156	0.001	-11.109	0.000	-0.018	-0.013
Met_Color	-15.2726	79.940	-0.191	0.849	-172.148	141.603
Metallic_Rim	246.2389	100.363	2.453	0.014	49.285	443.193
Mfg_Month	-98.9174	10.993	-8.998	0.000	-120.490	-77.344
Mfg_Year	1.6271	0.826	1.970	0.049	0.006	3.248
Mfr_Guarantee	249.2004	77.981	3.196	0.001	96.169	402.232
Mistlamps	37.4949	117.062	0.320	0.749	-192.229	267.218
Petrol	943.9365	199.032	4.743	0.000	553.354	1334.519

Power_Steering	17.1144	303.273	0.056	0.955	-578.031	612.260
Powered_Windows	244.0205	164.575	1.483	0.138	-78.944	566.985
Quarterly_Tax	14.8763	1.981	7.508	0.000	10.988	18.764
Radio	-681.9936	810.016	-0.842	0.400	-2271.578	907.591
Radio_cassette	712.4387	812.196	0.877	0.381	-881.423	2306.300
Sport_Model	267.3095	94.393	2.832	0.005	82.072	452.547
Tow_Bar	-161.4976	83.827	-1.927	0.054	-326.000	3.005
Weight	9.6194	1.229	7.828	0.000	7.208	12.031
cc	-0.1060	0.078	-1.351	0.177	-0.260	0.048
const	-0.0083	0.001	-14.757	0.000	-0.009	-0.007

Omnibus:	110.330	**Durbin-Watson:**	1.977
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	853.390
Skew:	0.096	**Prob(JB):**	4.88e-186
Kurtosis:	7.510	**Cond. No.**	1.32e+16

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The smallest eigenvalue is 3.37e-20. This might indicate that there are
strong multicollinearity problems or that the design matrix is singular.

다중공산성 확인

In [ ]:
```
# VIF를 통한 다중공선성 확인

from statsmodels.stats.outliers_influence import variance_inflation_factor

vif = pd.DataFrame()
vif["VIF Factor"] = [variance_inflation_factor(mlr_data.values, i) for i in
vif["features"] = mlr_data.columns
vif[['features','VIF Factor']].sort_values('VIF Factor',ascending=False)
```

```
C:\Dev\Miniconda\lib\site-packages\statsmodels\regression\linear_model.py:17
36: RuntimeWarning: divide by zero encountered in double_scalars
  return 1 - self.ssr/self.centered_tss
C:\Dev\Miniconda\lib\site-packages\statsmodels\stats\outliers_influence.py:1
95: RuntimeWarning: divide by zero encountered in double_scalars
  vif = 1. / (1. - r_squared_i)
```

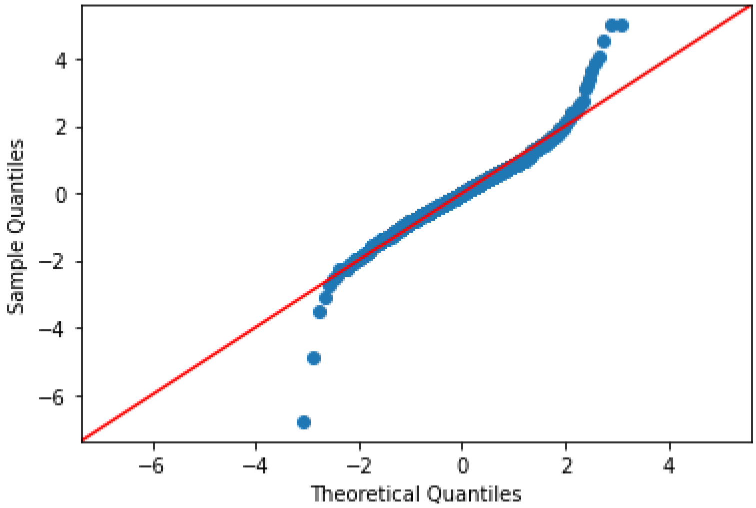
Out[ ]:

	features	VIF Factor
37	CNG	inf
36	Diesel	inf
2	Age_08_04	inf
3	Mfg_Month	inf
4	Mfg_Year	inf
35	Petrol	inf
28	Radio	62.344621
33	Radio_cassette	62.172860
1	Price	10.953474
13	Quarterly_Tax	5.496805
26	Powered_Windows	4.676311
25	Central_Lock	4.593157
14	Weight	4.487491
20	Airbag_2	3.106933
31	Backseat_Divider	2.702141
23	Boardcomputer	2.647036
6	HP	2.621514
5	KM	2.400334
18	ABS	2.276617
29	Mistlamps	2.076846
22	Automatic_airco	2.009866
21	Airco	1.846429
19	Airbag_1	1.612758
27	Power_Steering	1.582829
17	Guarantee_Period	1.573026
24	CD_Player	1.564446
30	Sport_Model	1.510131
16	BOVAG_Guarantee	1.392485
10	Doors	1.352288
32	Metallic_Rim	1.349642
12	Gears	1.271814
9	cc	1.258641
15	Mfr_Guarantee	1.210815
34	Tow_Bar	1.153760
7	Met_Color	1.143778
8	Automatic	1.121303
11	Cylinders	0.000000

	features	VIF Factor
0	const	0.000000

잔차 및 정규성 확인

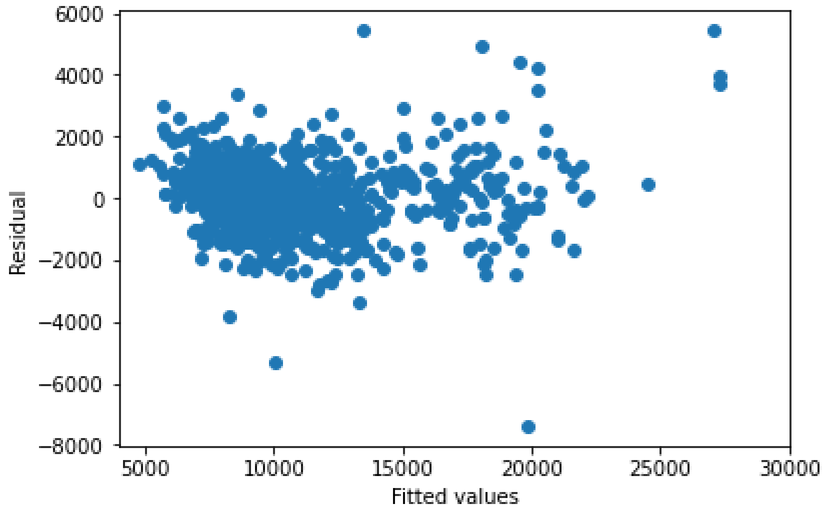
In [9]:
```
# 학습데이터의 잔차 확인
res = fitted_full_model.resid

# q-q plot을 통한 잔차의 정규분포 확인
fig = sm.qqplot(res,fit = True, line = '45')
```



q-q plot은 잔차의 정규성을 확인할 때 쓰는 그래프 중 하나로, statsmodels.api 라이브러리 내에 존재한다. 이 q-q plot은 y=x의 형태를 띄어야 정규성을 보이는 것이다. 위의 그래프를 보면 잔차가 완전한 정규성을 보이지 않는 것을 확인할 수 있다. 하지만, 실제데이터는 잔차가 완전한 정규성을 띄는 것을 확인하기 힘들다. 보통 위의 그래프처럼 실제에서도 꼬리부분의 값을이 정규성을 띄지 않는 경우가 많다. 그래도 위의 그래프는 꼬리부분 약간의 데이터를 제외하고 정규성을 띄고 있다고 판단되어 양호하다는 판단이 가능하다.

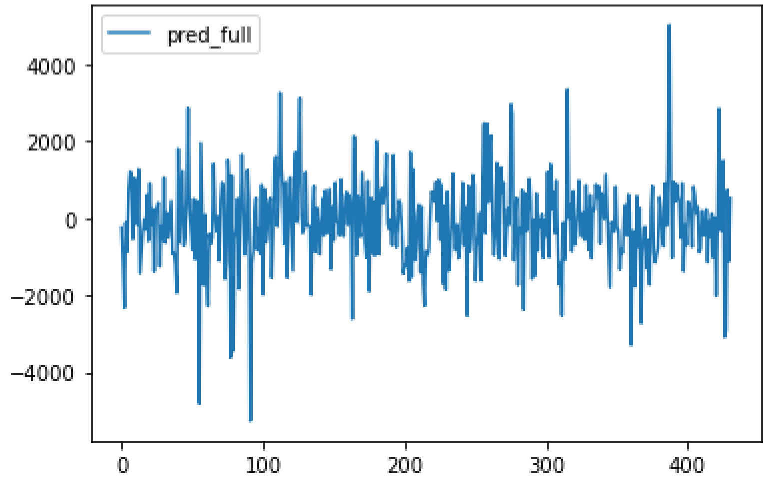
In [11]:
```
# 잔차패턴 확인

pred_y = fitted_full_model.predict(train_x)

import matplotlib.pyplot as plt

plt.scatter(pred_y, res)

plt.xlim(4000,30000)
plt.xlabel('Fitted values')
plt.ylabel('Residual')
```

Out[11]:
```
Text(0, 0.5, 'Residual')
```



위의 그래프는 모든 변수를 사용해 학습된 모델의 예측값과 잔차간의 상관관계를 확인하기 위해 나타낸 산점도이다. 이를 보면 특별한 경향이 발견되지 않으므로 잔차가 균등하게 나와있다고 판단가능하다.

In [12]:
```
# 검증데이터에 대한 예측

pred_y2 = fitted_full_model.predict(test_x)

# 예측데이터 잔차

plt.plot(np.array(test_y - pred_y2), label = "pred_full")
plt.legend()
plt.show()
```



테스트값에 대한 잔차를 확인하면, 하나의 수치를 제외하면 균등하게 분포되어있음

In [13]:
```
#MSE 구하기

from sklearn.metrics import mean_squared_error

mean_squared_error(y_true = test_y, y_pred = pred_y2)
```

Out[13]: 1315085.7258945287

이러한 과정으로 데이터를 확인하고 변수를 제거해야하는지 확인하는 작업이 사전에 필요하다. 그리고 다중공선성/과적합 등 문제가 발생하면 변수를 가공하고, 범주형 변수는 가변수생성을 통해 추가적인 변수로 변환할 필요가 있다. 이 과정 후에 변수선택법을 적용하여 변수를 선택하고 축소하는 과정이 이어진다.

변수선택법

In [15]:
```
import time
import itertools

#변수선택을 통해 형성한 모델의 AIC를 구하는 함수
# AIC가 낮을 수록 모델이 좋다고 평가된다.
def processSubset(X,y,feature_set):
    model = sm.OLS(y,X[list(feature_set)]) # Modeling
    regr = model.fit() # model fitting
    AIC = regr.aic # model's AIC
    return {"model" : regr, "AIC" : AIC}

# getBest : 가장 낮은 AIC를 가지는 모델을 선택하고 저장하는 함수
def getBest(X,y,k):
    tic = time.time() # 시작 시간
    results = [] # 결과 저장 공간

    # 각 변수 조합을 고려한 경우의수
    for combo in itertools.combinations(X.columns.difference(['const']),k):
        combo = (list(combo)+['const']) # 상수항을 추가하여 combo를 결성
        results.append(processSubset(X,y,feature_set = combo)) # 모델링된것을 저

    models = pd.DataFrame(results) # 데이터프레임으로 모델결과 변환
    best_model = models.loc[models['AIC'].argmin()] # argmin은 최소값의 인덱스를
    toc = time.time() # 종료 시간
    print("Processed", models.shape[0], "models on", k, "predictors in",(toc
    return best_model

getBest(X=train_x, y = train_y, k=2)
```

```
Processed 630 models on 2 predictors in 1.134995460510254 seconds.
```

Out[15]:
```
model    <statsmodels.regression.linear_model.Regressio...
AIC                                           17499.544708
Name: 211, dtype: object
```

전진선택법

In [65]:
```
def forward(X,y,predictors):
    # predictor - 현재 선택되어있는 변수
    # 데이터 변수들이 미리정의된 predictors에 있는지 없는지 확인 및 분류
    remaining_predictors = [p for p in X.columns.difference(['const']) if p
    tic = time.time()
    results = []
    for p in remaining_predictors :
        results.append(processSubset(X=X,y=y,feature_set=predictors+[p]+['co

    # 데이터프레임으로 변환
    models = pd.DataFrame(results)

    # AIC가 가장 낮은 것을 선택
    best_model = models.loc[models['AIC'].argmin()]

    toc = time.time()
    # print(best_model)
    print(f"Processed {models.shape[0]}, models on {len(predictors)+1},predi
    print(f"Selected predictors: {best_model['model'].model.exog_names} \nAI
    print()
    return best_model
```

```
def forward_model(X,y):
    Fmodels = pd.DataFrame(columns=["AIC","model"])
    tic = time.time()
    predictors = []

    # 변수 1~10개 : 0-9 -> 1-10
    for i in range(1, len(X.columns.difference(['const']))+1):
        Forward_result = forward(X=X , y=y, predictors=predictors)
        if i > 1 :
            if Forward_result["AIC"] > Fmodel_before:
                break
        Fmodels.loc[i] = Forward_result
        predictors = Fmodels.loc[i]["model"].model.exog_names
        Fmodel_before = Fmodels.loc[i]["AIC"]
        predictors = [k for k in predictors if k != 'const']
    toc = time.time()
    print("Total elapsed time:",(toc-tic), "seconds.")

    return (Fmodels['model'][len(Fmodels['model'])])
```

In [ ]:
```
Forward_best_model = forward_model(X=train_x, y=train_y)
```

- 변수를계속 추가하며 AIC가 증가하는 경우가 생기면, 이전 모델을 선택하는 학습과정을 진행한다

In [66]:
```
Forward_best_model.aic
```

Out[66]:
```
16959.00706023506
```

In [67]:
```
Forward_best_model.summary()
```

Out[67]:

OLS Regression Results

Dep. Variable:	Price	**R-squared:**	0.907
Model:	OLS	**Adj. R-squared:**	0.905
Method:	Least Squares	**F-statistic:**	415.5
Date:	Tue, 29 Aug 2023	**Prob (F-statistic):**	0.00
Time:	07:32:39	**Log-Likelihood:**	-8455.5
No. Observations:	1005	**AIC:**	1.696e+04
Df Residuals:	981	**BIC:**	1.708e+04
Df Model:	23		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
Mfg_Year	1457.5891	49.114	29.678	0.000	1361.208	1553.970
Automatic_airco	2402.2817	182.567	13.158	0.000	2044.016	2760.548
Weight	9.8614	1.142	8.637	0.000	7.621	12.102
KM	-0.0155	0.001	-11.141	0.000	-0.018	-0.013
HP	21.9823	3.542	6.206	0.000	15.032	28.933
Central_Lock	116.9404	160.998	0.726	0.468	-199.000	432.880
Quarterly_Tax	14.7153	1.920	7.666	0.000	10.948	18.482
Petrol	-7.287e+05	2.45e+04	-29.778	0.000	-7.77e+05	-6.81e+05
Guarantee_Period	59.0242	14.826	3.981	0.000	29.930	88.118
Metallic_Rim	263.4380	93.367	2.822	0.005	80.217	446.659
Mfr_Guarantee	244.4188	76.659	3.188	0.001	93.983	394.854
Diesel	-7.295e+05	2.45e+04	-29.804	0.000	-7.78e+05	-6.82e+05
Boardcomputer	-334.2517	123.482	-2.707	0.007	-576.572	-91.932
BOVAG_Guarantee	380.0324	129.019	2.946	0.003	126.848	633.217
CD_Player	265.6198	101.702	2.612	0.009	66.042	465.197
Mfg_Month	21.3081	10.714	1.989	0.047	0.284	42.332
Automatic	387.8602	156.387	2.480	0.013	80.969	694.752
ABS	-222.7873	104.094	-2.140	0.033	-427.061	-18.514
Sport_Model	254.1427	88.623	2.868	0.004	80.231	428.055
Backseat_Divider	-311.1664	120.477	-2.583	0.010	-547.589	-74.744
Airco	165.5736	91.564	1.808	0.071	-14.111	345.258
Tow_Bar	-154.1662	81.231	-1.898	0.058	-313.572	5.240
Powered_Windows	250.7068	162.230	1.545	0.123	-67.650	569.064
CNG	-7.308e+05	2.45e+04	-29.822	0.000	-7.79e+05	-6.83e+05
const	-2.189e+06	7.34e+04	-29.803	0.000	-2.33e+06	-2.04e+06

Omnibus:	112.461	**Durbin-Watson:**	1.981
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	881.914

Skew:	0.116	**Prob(JB):**	3.12e-192
Kurtosis:	7.583	**Cond. No.**	2.67e+20

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The smallest eigenvalue is 8.21e-29. This might indicate that there are
strong multicollinearity problems or that the design matrix is singular.

후진제거법

In [92]:
```
def backward(X,y,predictors):
    tic = time.time()
    results = []
    for combo in itertools.combinations(predictors, len(predictors) - 1):
        results.append(processSubset(X=X,y=y,feature_set=list(combo)+['const
    models = pd.DataFrame(results)
    # 가장 낮은 AIC를 가진 모델을 선택
    best_model = models.loc[models['AIC'].argmin()]
    toc = time.time()

    print(f"Processed {models.shape[0]} models on {len(predictors) - 1} pred
    print(f'Selected predictors: {best_model["model"].model.exog_names} \n A
    print()

    return best_model

def backward_model(X,y) :
    Bmodels = pd.DataFrame(columns=["AIC","model"], index = range(1,len(X.co
    tic = time.time()
    predictors = X.columns.difference(['const'])
    Bmodel_before = processSubset(X, y, predictors)['AIC']
    while (len(predictors) > 1):
        Backward_result = backward(X=train_x, y= train_y,predictors=predicto
        if Backward_result['AIC'] > Bmodel_before : break
        Bmodels.loc[len(predictors) -1] = Backward_result
        predictors = Bmodels.loc[len(predictors) - 1]['model'].model.exog_na
        Bmodel_before = Backward_result["AIC"]
        predictors = [k for k in predictors if k != 'const']

    toc = time.time()
    print("Total elapsed time:",(toc-tic),"seconds.")
    return Bmodels["model"].dropna().iloc[0]
```

In [ ]:
```
Backward_best_model = backward_model(X=train_x, y= train_y)
```

In [94]:
```
Backward_best_model.aic
```

Out[94]: 16957.54740210653

In [95]:
```
Backward_best_model.summary()
```

Out[95]:

OLS Regression Results

Dep. Variable:	Price	**R-squared:**	0.907
Model:	OLS	**Adj. R-squared:**	0.905
Method:	Least Squares	**F-statistic:**	434.6
Date:	Tue, 29 Aug 2023	**Prob (F-statistic):**	0.00
Time:	07:49:22	**Log-Likelihood:**	-8455.8
No. Observations:	1005	**AIC:**	1.696e+04
Df Residuals:	982	**BIC:**	1.707e+04
Df Model:	22		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
ABS	-223.5094	104.065	-2.148	0.032	-427.724	-19.295
Airco	173.5813	90.876	1.910	0.056	-4.753	351.915
Automatic	387.3712	156.348	2.478	0.013	80.557	694.185
Automatic_airco	2402.0330	182.522	13.160	0.000	2043.854	2760.212
BOVAG_Guarantee	378.1191	128.961	2.932	0.003	125.049	631.189
Backseat_Divider	-310.4403	120.444	-2.577	0.010	-546.798	-74.083
Boardcomputer	-334.4778	123.452	-2.709	0.007	-576.738	-92.217
CD_Player	269.3438	101.548	2.652	0.008	70.068	468.619
Cylinders	-6.864e+05	2.3e+04	-29.807	0.000	-7.32e+05	-6.41e+05
Diesel	1226.1547	375.588	3.265	0.001	489.107	1963.202
Guarantee_Period	59.8978	14.773	4.054	0.000	30.907	88.889
HP	22.0835	3.538	6.241	0.000	15.140	29.027
KM	-0.0155	0.001	-11.138	0.000	-0.018	-0.013
Metallic_Rim	264.3685	93.335	2.832	0.005	81.209	447.528
Mfg_Month	20.9865	10.702	1.961	0.050	-0.015	41.988
Mfg_Year	1456.3956	49.075	29.677	0.000	1360.092	1552.699
Mfr_Guarantee	245.5440	76.625	3.204	0.001	95.176	395.912
Petrol	2082.0000	379.333	5.489	0.000	1337.603	2826.397
Powered_Windows	349.6680	88.046	3.971	0.000	176.889	522.447
Quarterly_Tax	14.7169	1.919	7.668	0.000	10.951	18.483
Sport_Model	256.4151	88.546	2.896	0.004	82.653	430.177
Tow_Bar	-153.6848	81.209	-1.892	0.059	-313.047	5.677
Weight	9.8772	1.141	8.654	0.000	7.638	12.117
const	-1.716e+05	5757.312	-29.807	0.000	-1.83e+05	-1.6e+05

Omnibus:	112.135	**Durbin-Watson:**	1.981
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	874.553
Skew:	0.117	**Prob(JB):**	1.24e-190

Kurtosis:	7.564	Cond. No.	7.28e+19

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The smallest eigenvalue is 1.1e-27. This might indicate that there are
strong multicollinearity problems or that the design matrix is singular.

단계적선택법

In [153…

```
def Stepwise_model(X,y):
    Stepmodels = pd.DataFrame(columns = ["AIC","model"])
    tic = time.time()
    predictors = []
    # 변수 1~10개 0-9 -> 1-10
    for i in range(len(X.columns.difference(['const']))) :
        Forward_result = forward(X=X, y=y, predictors = predictors) # consta
        print('forward')
        Stepmodels.loc[i] = Forward_result
        predictors = Stepmodels.loc[i]['model'].model.exog_names

        predictors = [k for k in predictors if k != 'const']
        Backward_result = backward(X=X, y=y, predictors = predictors)
        Smodel_before = Stepmodels.loc[i]["AIC"]

        if Backward_result["AIC"] < Smodel_before:
            Stepmodels.loc[i] = Backward_result
            predictors = Stepmodels.loc[i]["model"].model.exog_names
            Smodel_before = Stepmodels.loc[i]["AIC"]
            predictors = [k for k in predictors if k != "const"]
            print('backward')
        if Stepmodels.loc[i]["AIC"] > Smodel_before:
            break
        else :              Smodel_before = Stepmodels.loc[i]["AIC"]

    toc = time.time()
    print("Total elapsed time:",(toc-tic),"seconds.")
    return (Stepmodels["model"][len(Stepmodels["model"])-1])
```

In [ ]:

```
Stepwise_best_model = Stepwise_model(X=train_x, y= train_y)
```

In [163…

```
Stepwise_best_model.aic
```

Out[163]:

```
16957.547402106567
```

In [164…

```
Stepwise_best_model.summary()
```

Out[164]:

OLS Regression Results

Dep. Variable:	Price	**R-squared:**	0.907
Model:	OLS	**Adj. R-squared:**	0.905
Method:	Least Squares	**F-statistic:**	434.6
Date:	Tue, 29 Aug 2023	**Prob (F-statistic):**	0.00
Time:	08:30:36	**Log-Likelihood:**	-8455.8
No. Observations:	1005	**AIC:**	1.696e+04
Df Residuals:	982	**BIC:**	1.707e+04
Df Model:	22		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
Mfg_Year	1456.3956	49.075	29.677	0.000	1360.092	1552.699
Automatic_airco	2402.0330	182.522	13.160	0.000	2043.854	2760.212
Weight	9.8772	1.141	8.654	0.000	7.638	12.117
KM	-0.0155	0.001	-11.138	0.000	-0.018	-0.013
HP	22.0835	3.538	6.241	0.000	15.140	29.027
Quarterly_Tax	14.7169	1.919	7.668	0.000	10.951	18.483
Petrol	-5.51e+04	1878.033	-29.341	0.000	-5.88e+04	-5.14e+04
Guarantee_Period	59.8978	14.773	4.054	0.000	30.907	88.889
Metallic_Rim	264.3685	93.335	2.832	0.005	81.209	447.528
Mfr_Guarantee	245.5440	76.625	3.204	0.001	95.176	395.912
Diesel	-5.596e+04	1884.956	-29.687	0.000	-5.97e+04	-5.23e+04
Boardcomputer	-334.4778	123.452	-2.709	0.007	-576.738	-92.217
BOVAG_Guarantee	378.1191	128.961	2.932	0.003	125.049	631.189
CD_Player	269.3438	101.548	2.652	0.008	70.068	468.619
Mfg_Month	20.9865	10.702	1.961	0.050	-0.015	41.988
Automatic	387.3712	156.348	2.478	0.013	80.557	694.185
ABS	-223.5094	104.065	-2.148	0.032	-427.724	-19.295
Sport_Model	256.4151	88.546	2.896	0.004	82.653	430.177
Backseat_Divider	-310.4403	120.444	-2.577	0.010	-546.798	-74.083
Airco	173.5813	90.876	1.910	0.056	-4.753	351.915
Tow_Bar	-153.6848	81.209	-1.892	0.059	-313.047	5.677
Powered_Windows	349.6680	88.046	3.971	0.000	176.889	522.447
CNG	-5.718e+04	1915.373	-29.856	0.000	-6.09e+04	-5.34e+04
Cylinders	-6.73e+05	2.26e+04	-29.802	0.000	-7.17e+05	-6.29e+05
const	-1.682e+05	5645.443	-29.802	0.000	-1.79e+05	-1.57e+05

Omnibus:	112.135	**Durbin-Watson:**	1.981
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	874.553

Skew:	0.117	**Prob(JB):**	1.24e-190
Kurtosis:	7.564	**Cond. No.**	1.80e+21

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The smallest eigenvalue is 1.81e-30. This might indicate that there are strong multicollinearity problems or that the design matrix is singular.