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The effects of uncertainty on the WTA–WTP gap

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Abstract We analyze the effects of uncertainty on *WTA, WTP* and the *WTA–WTP* gap. Extending the approach of Weber (Econom Lett 80:311–315, 2003) to the case of lotteries, we develop an exact expression for the *WTA–WTP* gap that allows identification of its magnitude under different utility specifications. Reinterpreting and extending results by Gabillon(Econom Lett 116:157–160, 2012), we also identify generally the relationship between an agent's utility of income and the gap's algebraic sign, as well as the effects of risk increases on *WTA* and *WTP*. Finally, we derive the limit behavior of the gap as income or risk increase.

Keywords Compensating variation · Equivalent variation · Expected utility theory · Willingness to accept · Willingness to pay

JEL Classification D01 · D81

1 Introduction

The frequently observed disparity between willingness to pay (*WTP*) and willingness to accept (*WTA*) has been a subject of continuing attention among economists. The disparity is of interest for both policy and theoretical reasons. As a policy matter, analysts frequently find themselves in the position of needing to know the value stakeholders place on various alternatives, so determining the appropriate method for eliciting such information is of critical importance. Theoretically, the disparity is a curiosity because willingness to pay (measured as a compensating variation) and willingness to accept,

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(measured as equivalent variation), should be identical except for a small income effect (Willig 1976).[1]

An important branch of the literature studying *WTA* and *WTP* involves elicitations for goods of uncertain value. In a variety of contexts such as medical insurance, highway safety, and environmental quality enhancements, policymakers are interested in the value consumers place on goods whose usefulness contains a stochastic component. Laboratory implementations of value elicitations for such goods typically involve lotteries. In such experiments, the mean elicited *WTA* persistently exceeds its *WTP* counterpart, even under rigorous test conditions involving within-subject comparisons, using financially salient elicitations under an incentive compatible elicitation mechanism, and after practice decisions to ensure familiarity (see, e.g., Harless 1989; Eisenberger and Weber 1995; Schmidt and Traub 2009; Isoni et al. 2011).

Most theoretical work on the effects of uncertainty on *WTA* and *WTP* has focused on variations of reference dependent utility theory (*RDT*s), such as loss aversion (Kahneman and Tversky 1979), endowment effects (Thaler 1980), regret (Loomes and Sugden 1982), and status quo bias (Gal 2006). A conclusion of these analyses is that uncertainty creates a "reluctance to trade" by which *WTA* rises above and *WTP* falls below expected value.[2]

Although the magnitude of the gap documented in several empirical studies likely exceeds what we can reasonably expect standard expected utility theory to explain, we observe that it is also possible that a "reluctance to trade" may be a consequence of the way uncertainty affects *WTA* and *WTP* under standard expected utility theory (*EUT*). This possibility has received some limited attention. In separate studies, Isik (2004) and Okada (2010) both conclude that *EUT* can explain the observed reluctance to trade. Specifically, these authors find that the combination of risk aversion and uncertainty causes symmetric adjustments to *WTA* and *WTP*, with *WTA* rising above and *WTP* falling below a lottery's expected value. They both further conclude that increases in uncertainty prompt symmetric adjustments in *WTA* and *WTP* that increase the gap's magnitude. Unfortunately, however, both analyses contain important errors and/or assumptions that economists would find unacceptable.[3]

Still lacking in the literature, then, is a direct and correct analysis of the effects of uncertainty on the *WTA*–*WTP* gap under standard *EUT*. This paper removes this deficiency**.** Our development consists of three parts. First, we extend an approach

[1] Moreover, Horowitz and McConnell (2003) show that the typical *WTA*–*WTP* disparity is too large to reasonably be explained by income effects.

[2] Neilson et al. (2012) provide a clean statement of the effects of changes in uncertainty on the *WTA*–*WTP* gap for both a binary and uniformly distributed lottery under loss aversion, as well as for a binary lottery under rank dependent utility theory. In all cases Neilson, McKee and Berrens show that increases in uncertainty raise the *WTA*–*WTP* gap. We observe, however, that their treatment involves the special case of utility that is linear both above zero and below zero. Thus, all positive gambles are simply evaluated at their expected value, a consequence that abstracts from any possible effects of risk aversion.

[3] Isik (2004) uses Taylor's series expansions to assess the effects of uncertainty regarding an environmental quality enhancement on the *WTA*–*WTP* gap. He errs with inconsistencies in his selection of reference income levels. Davis and Reilly (2012) detail the errors in this development. Okada (2010) studies the way uncertainty impacts buyer–seller interactions. Among other issues, to generate her principal results, Okada posits a negative marginal utility of income, which is of course fundamentally inconsistent with standard analysis.

taken by Weber (2003) to the general case of lotteries, in order to develop an exact expression for the size of the *WTA–WTP* gap under uncertainty. Second, we offer a pair of corollaries that reinterpret and extend an analysis of uncertainty on risk premia by Gabillon (2012) to the case of *WTA*, *WTP* and the sign of the *WTA–WTP* gap.[4] Third, we derive a pair of theorems that assess the limit behavior of the *WTA–WTP* gap's size as income or risk increases.

By way of pre-summary, we find the following. First, given risk aversion, uncertainty causes *both WTA* and *WTP* to fall below a lottery's expected value. Second, the algebraic sign of the *WTA–WTP* gap is determined by the behavior of the agent's coefficient of absolute risk aversion. Given decreasing absolute risk aversion *(DARA)* $WTA > WTP$. Given increasing absolute risk aversion *(IARA)* the reverse is true, and $WTA < WTP$. Third, we show that in the limit the magnitude of the *WTA–WTP* gap for favorable lotteries necessarily approaches zero as either income or riskiness increases. This convergence need not be monotone, however, and in the case of increasing risk, a common pattern is a gap that at first widens and then falls to zero. We illustrate this pattern with a representative simulation.

2 The size of the WTA–WTP GAP under uncertainty and the impact of income and risk on WTA and WTP

2.1 An exact expression of the WTA–WTP gap under uncertainty

We begin with some definitions. Let $u(y)$ denote an individual's von Neumann–Morgenstern utility of income function. Let y_0 denote an initial certain income and let z denote a bounded lottery over potential additions to and subtractions from that income such that $y_0 + \inf(z) > 0$. Define the expected value of lottery z, which we assume to be favorable, as $\mu > 0$. The expected value of the lottery with initial income is $y_0 + \mu$. Denote the utility of the lottery in combination with the initial income by the expected utility $\nu(y_0, z)$. A favorable z implies that $\nu(y_0, z) > v(y_0) = u(y_0)$.

For initial income y_0 let $C(y_0, z) \in [0, y_0]$ be the unique solution to

$$\nu[y_0 - C(y_0, z), z] = \nu(y_0), \tag{1}$$

and let $E(y_0, z) \in \left[0, \sup(z)\right]$ be the unique solution to

$$\nu(y_0, z) = \nu[y_0 + E(y_0, z)]. \tag{2}$$

Thus, $C(y_0, z)$ is the *WTP* (compensating variation) for the acquisition of lottery z from initial position y_0, i.e $E(y_0, z)$ is the *WTA* (equivalent variation) for the loss of

[4] Gabillon (2012) establishes several important properties of risk-avoiding and risk-taking premia in lotteries under expected utility. The former is what is commonly referred to simply as the risk premium (the expected value minus the certainty equivalent) and the latter is the difference between the expected value and the *WTP*.

lottery z given initial income y_0.[5] The favorability of z implies that both $C(y_0, z) > 0$ and $E(y_0, z) > 0$. Note that $E(y_0, z)$ is by definition also the certainty equivalent of lottery z at starting income y_0.

To assess the effects of uncertainty on the difference between *WTA* and *WTP*, we extend the approach taken by Weber (2003) for the analysis of non-market goods to the case of a favorable lottery. We begin by expressing *WTP* in terms of *WTA* at a different level of income. This allows us to express the difference between the two measures in terms of the effects of a change in income on the *WTP*. Manipulating this difference, and establishing that income has a monotonic effect on the *WTA*, we use the fundamental theorem of calculus to express *WTA*–*WTP* in terms of the change in *WTA* over the range of incomes that distinguish the two measures.

Define $y_1 = y_0 + E(y_0, z)$. Thus $y_1 > y_0$, and so $\nu(y_1) > \nu(y_0)$. First we establish that at any level of income, an added income term will leave expected indirect utility constant if and only if the expected value of the added term is zero.

Lemma 1 *For any* $y > 0$, $\nu(y, z) = \nu[y + \delta, z]$ *iff* $\delta = 0$.

Proof If $\delta = 0$, $\nu(y, z) = \nu[y + \delta, z]$ trivially. If $\delta > 0$, $\nu(y + \delta, z) = \nu(y, z + \delta) > \nu(y, z)$ by stochastic dominance. Similarly, if $\delta < 0$, $\nu(y + \delta, z) < \nu(y, z)$ by stochastic dominance. □

Lemma 1 allows us to develop an expression for the difference between equivalent and compensating variation in terms of the compensating variation over a change in income levels. Let $C(y_1, z)$ denote the unique solution to $\nu[y_1 - C(y_1, z), z] = \nu(y_1)$. Then, $\nu(y_0, z) = \nu[y_0 + E(y_0, z)] = \nu[y_1] = \nu[y_1 - C(y_1, z), z] = \nu[y_0 + E(y_0, z) - C(y_1, z), z]$.

Thus

$$\nu(y_0, z) = \nu[y_0 + E(y_0, z) - C(y_1, z), z]. \tag{3}$$

Lemma 1 and Eq. (3) imply that $E(y_0, z) = C(y_1, z)$. Thus we have

$$E(y_0, z) = C[y_0 + E(y_0, z), z]. \tag{4}$$

Noting again that $y_1 = y_0 + E(y_0, z)$, we may differentiate (4) with respect to y_0 to obtain

$$\begin{aligned}
\left.\frac{\partial E(y_0, z)}{\partial y}\right|_{y=y_0} &= \left(\left.\frac{\partial C(y_1, z)}{\partial y}\right|_{y=y_1}\right)\left(\left.\frac{\partial y_1}{\partial y_0}\right|_{y=y_0}\right) \\
&= \left(\left.\frac{\partial C(y_1, z)}{\partial y}\right|_{y=y_1}\right)\left(1 + \left.\frac{\partial E(y_0, z)}{\partial y}\right|_{y=y_0}\right).
\end{aligned}$$

[5] In what follows we use the terms *WTA* and $E(y_0, z)$, and the terms and *WTP* and $C(y_0, z)$ interchangeably. Also, *DARA*, *CARA*, and *IARA* refer to decreasing, constant, and increasing absolute risk aversion.

Solving the left and right equalities for $\left.\frac{\partial C(y_1)}{\partial y}\right|_{y=y_1}$ yields,

$$\left.\frac{\partial C\left(y_1, z\right)}{\partial y}\right|_{y=y_1} = \frac{\left.\frac{\partial E(y_0,z)}{\partial y}\right|_{y=y_0}}{1+\left.\frac{\partial E(y_0,z)}{\partial y}\right|_{y=y_0}}. \tag{5}$$

Now, from the fundamental theorem of calculus, we may write

$$C\left(y_1, z\right) = C\left(y_0, z\right) + \int\limits_{\delta=y_0}^{\delta=y_1} \left.\frac{\partial C\left(y, z\right)}{\partial y}\right|_{y=\delta} \mathrm{d}\delta.$$

Using $C\left(y_1, z\right) = E\left(y_0, z\right)$ we have

$$E\left(y_0, z\right) - C\left(y_0, z\right) = \int\limits_{\delta=y_0}^{\delta=y_1} \left.\frac{\partial C\left(y, z\right)}{\partial y}\right|_{y=\delta} \mathrm{d}\delta. \tag{6}$$

For reasons that will be made clear below, the right hand side of Eq. (6) is more usefully represented in terms of the equivalent variation (e.g., *WTA*) rather than the compensating variation (*WTP*). The following lemma enables such a representation.

Lemma 2 $y_1 = y_0 + E\left(y_0, z\right)$*is a monotonically increasing function of* y_0.

Proof Differentiating Eq. (2) with respect to y_0,

$$\frac{\partial v\left(y_0, z\right)}{\partial y_0} = \left.\frac{\partial v\left(y_1, z\right)}{\partial y}\right|_{y=y_1} \left(1 + \frac{\partial E\left(y_0, z\right)}{\partial y_0}\right).$$

Note that a positive marginal utility of income (or alternatively, stochastic dominance) implies that both $\frac{\partial v(y_0,z)}{\partial y_0} > 0$ and $\left.\frac{\partial v(y_1,z)}{\partial y}\right|_{y=y_1} > 0$. Thus from the above equality,

$$\frac{\partial E\left(y_0, z\right)}{\partial y_0} > -1.$$

Since $\frac{\partial y_1}{\partial y_0} = 1 + \frac{\partial E(y_0,z)}{\partial y_0}$, we have $\frac{\partial y_1}{\partial y_0} > 0$. □

Lemma 2 implies the existence of the inverse function $y_0 = g\left(y_1\right)$ for any initial y_0. Lemmas 1 and 2 are used in the proof of the following principal theorem.

Theorem 1

$$E\left(y_0, z\right) - C\left(y_0, z\right) = \int\limits_{\delta=y_0}^{\delta=y_1} \frac{\left.\frac{\partial E(g(\delta),z)}{\partial y}\right|_{y=g(\delta)}}{1+\left.\frac{\partial E(g(\delta),z)}{\partial y}\right|_{y=g(\delta)}} d\delta \tag{7}$$

is an exact expression of the WTA–WTP gap for any favorable lottery z.

Proof Using the existence of the inverse function g, we may rewrite (5) as

$$\left.\frac{\partial C\left(\delta, z\right)}{\partial y}\right|_{y=\delta} = \frac{\left.\frac{\partial E(g(\delta),z)}{\partial y}\right|_{y=g(\delta)}}{1+\left.\frac{\partial E(g(\delta),z)}{\partial y}\right|_{y=g(\delta)}} \tag{8}$$

Substituting the right hand side of this equality for the integrand in (6) yields (8). □

2.2 Some general features of WTA, WTP, and the sign of the WTA–WTP gap under uncertainty

Theorem 1 allows us to precisely express the sign and magnitude of the *WTA–WTP* gap, given the utility of income function, a specific lottery, and an initial income level.[6] Reinterpreting and extending work by Gabillon (2012) further allows us to express some additional general features of *WTA, WTP* and the *WTA–WTP* gap, with the following two corollaries.[7]

Corollary 1 (*i*) *For a favorable lottery z,*
(*a*) $C(y_1, z) = E(y_0, z)$ *where* $y_1 = y_0 + E(y_0, z)$
(*b*) $C(y_0, z) = E(y_2, z)$ *where* $y_2 = y_0 - C(y_0, z)$,

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[6] For example, consider the *DARA* utility function $u\left(y\right) = ln\left(y\right)$ in combination with the lottery form $z = (x_1, x_2, 0.5)$. For any initial income y_0, $E\left(y_0, z\right) = \sqrt{y_0^2 + x_1 y_0 + x_2 y_0 + x_1 x_2} - y_0$, and $C\left(y_0, z\right) = \frac{(2y_0+x_1+x_2)-\sqrt{(2y_0+x_1+x_2)^2-4(x_1y_0+x_2y_0+x_1x_2)}}{2}$. Given the specific lottery $z = (5, 40, 0.5)$ and initial income $y_0 = 20$, $E\left(y_0, z\right) - C\left(y_0, z\right) = 2.805198$. Further, for this utility function, we can verify the equality in (7). For any income y, $\frac{\partial E(y,z)}{\partial y} = \frac{(2y+x_1+x_2)}{2\sqrt{y^2+x_1y+x_2y+x_1x_2}} - 1$, and the inverse function for $\delta = y_0 + E\left(y_0, z\right)$ is given by $g\left(\delta\right) = \frac{-(x_1+x_2)+\sqrt{(x_1+x_2)^2-4\left(x_1+x_2-\delta^2\right)}}{2}$. Using these expressions for $\frac{\partial E}{\partial y}$ and $g\left(\bullet\right)$ in the right hand side of (7) and employing numerical integration in the open-source *R* statistical programming language, the right hand side of (7) yields the same 2.805198.

[7] Corollary 1 reinterprets Gabillon's Proposition 3 to corresponding properties of *WTA* and *WTP*, and reinterprets and extends that portion of her Corollary 3 relating to risk premia in favorable lotteries to the properties of *WTA* and *WTP* under *DARA, CARA*, and *IARA* utility functions. Corollary 2 recasts the strict form of Gabillon's Corollary 1 to focus on *WTA* and *WTP* for a single individual over two different lotteries rather than on the risk premia for two different individuals on a single lottery. Since these corollaries are relatively straightforward extensions of Gabillon (2012), we relegate their proofs to appendices.

(*ii*) *Under DARA (CARA, IARA), $E(y, z)$ and $C(y, z)$ are strictly increasing (constant, decreasing) in income.*
(*iii*) *Under strict DARA (CARA, IARA), $E(y_0, z) - C(y_0, z) > (=, <)\ 0$.*

Proof See Appendix 1. □

Corollary 2 *Consider two favorable lotteries z_1 and z_2 where z_2 can be obtained from z_1 by a series of mean-preserving spreads (MPS's) and is thus riskier than z_1. For any strictly concave utility function, $E\,(y_0, z_1) > E\,(y_0, z_2)$ and $C\,(y_0, z_1) > C\,(y_0, z_2)$.*

Proof See Appendix 2. □

Part (iii) of Corollary 1 is especially important here as it identifies a relationship between the curvature of an agent's utility of income function and the algebraic sign of the *WTA–WTP* gap. Under *EUT*, the gap takes on a positive sign when the underlying utility of income function exhibits *DARA*. This is in contrast to predictions from *RDT*'s, for example, where a predicted "reluctance to trade" uniformly implies that the *WTA–WTP* gap is always positive, independent of the curvature of an agent's utility of income function.

Corollary 2 specifies the effect of increases in risk on *WTA* and *WTP*. Here, and again in distinction to many *RDT*'s and previous related analyses, increases in uncertainty cause both *WTA* and *WTP* to fall (rather than *WTA* increasing and *WTP* falling).[8] Moreover, starting from a lottery that is initially degenerate on its expected value μ, it follows that an increase in uncertainty causes *both WTA* and *WTP* to fall below the lottery's expected value.

3 The effects of income and risk on the limit behavior of the *WTA–WTP* gap

Finally, we consider the effects of income and risk on the *WTA–WTP* gap in the limit, as either income or risk increase. We consider first the limiting effects of income increases, with the following Theorem 2.

Theorem 2 *Under DARA or IARA,* $\lim_{y_0 \to +\infty} [E(y_0, z) - C(y_0, z)] = 0$.

Proof Since under *DARA*, by Corollary 1.ii above, $E\,(y, z)$ and $C\,(y, z)$ are both continuous strictly increasing functions of y which are bounded above by μ_z, they must each possess a least upper bound. Denote $\lim_{y_0 \to +\infty} E\,(y_0, z) = H$. Again define $y_2 = y_0 - C\,(y_0, z)$. Note that $\lim_{y_0 \to +\infty} y_2 = \lim_{y_0 \to +\infty} y_0 = +\infty$. Hence, $H = \lim_{y_0 \to +\infty} E\,(y_0, z) = \lim_{y_2 \to +\infty} E\,(y_2, z) = \lim_{y_0 \to +\infty} E\,(y_2, z)$. By Corollary 1.i.b, $C\,(y_0, z) = E\,(y_2, z)$, and so $\lim_{y_0 \to +\infty} C\,(y_0, z) = H$. Thus $\lim_{y_0 \to +\infty} [E\,(y_0, z) - C\,(y_0, z)] = \lim_{y_0 \to +\infty} E\,(y_0, z) - \lim_{y_0 \to +\infty} C\,(y_0, z) = H - H = 0$, establishing the result under *DARA*.

[8] For example, it can be shown in all three cases analyzed in Neilson et al. (2012) that uncertainty causes *WTA* to increase above and *WTP* to fall below a lottery's expected value.

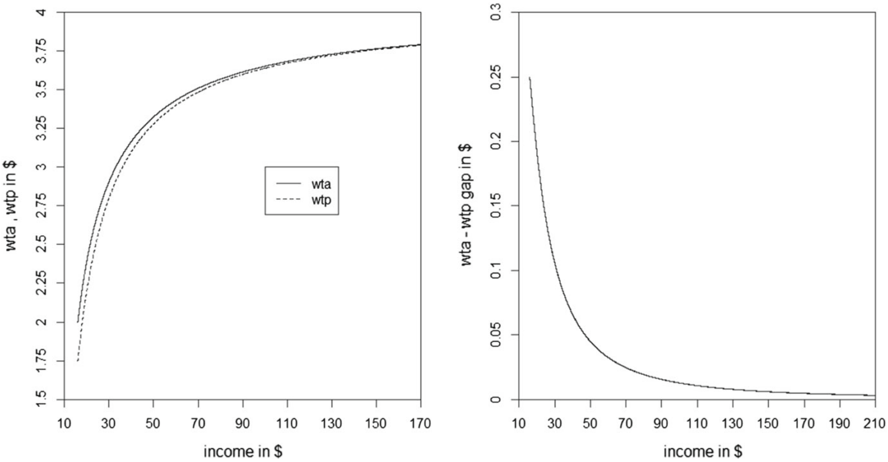

Fig. 1 Income effects for *WTA* and *WTP* (*left panel*) and *WTA–WTP* (*right panel*) for the *DARA* utility function $U(y) = y^{0.5}$, using a $\left(\$16, -\$8, 0.5\right)$ lottery

Under *IARA*, again by Corollary 1.ii above, $E(y, z)$ and $C(y, z)$ are both continuous strictly decreasing functions of y which, for any favorable lottery, are bounded below by 0. Thus they each possess a greatest lower bound. Denote $\lim_{y_0 \to +\infty} E(y_0, z) = L$. Again, define $y_2 = y_0 - C(y_0, z)$. $\lim_{y_0 \to +\infty} y_2 = \lim_{y_0 \to +\infty} y_0 = +\infty$. Thus $L = \lim_{y_0 \to +\infty} E(y_0, z) = \lim_{y_2 \to +\infty} E(y_2, z) = \lim_{y_0 \to +\infty} E(y_2, z)$. Again by Corollary 1.i.b, $C(y_0, z) = E(y_2, z)$, and so $\lim_{y_0 \to +\infty} C(y_0, z) = L$. Hence $\lim_{y_0 \to +\infty} [E(y_0, z) - C(y_0, z)] = \lim_{y_0 \to +\infty} [E(y_0, z) - E(y_2, z)] = \lim_{y_0 \to +\infty} E(y_0, z) - \lim_{y_0 \to +\infty} E(y_2, z) = L - L = 0$, establishing the result under *IARA*. □

Figure 1 illustrates convergence under Theorem 2 for the *DARA* utility function $U(y) = y^{1-r}$ with $r = 0.5$, and a $\left(\$16, -\$8, 0.5\right)$ lottery, where 0.5 is the probability of the higher outcome. Consistent with Theorem 2, observe that the gap falls to zero as income increases.

The following Theorem 3 addresses the effects of increasing risk on the *WTA–WTP* gap.

Theorem 3 *For any strictly concave utility function, define a sequence of favorable lotteries* $\{z_i\}, i = 1, 2, 3, \ldots$ *where* z_i *can be obtained from* z_{i-1} *by a series of mean-preserving spreads (MPS's) such that the greatest lower bound of the corresponding strictly decreasing sequence* $\{v(y_0, z_i)\}$ *is* $v(y_0)$. *Then* $\lim_{i \to \infty} \left[E(y_0, z_i) - C(y_0, z_i)\right] = 0$.

Proof Since the bounded strictly decreasing sequence $\{v(y_0, z_i)\}$ necessarily converges to its greatest lower bound $v(y_0)$ as $i \to \infty$, the corresponding strictly decreasing sequences $\{E(y_0, z_i)\}$ and $\{C(y_0, z_i)\}$ necessarily converge to 0. Thus the limit of the sequence $\{E(y_0, z_i) - C(y_0, z_i)\}$ necessarily converges to 0. □

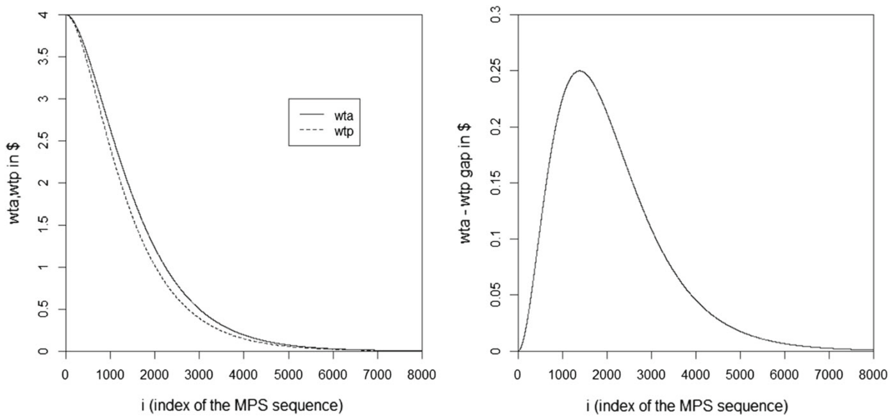

Fig. 2 The effects of increasing mean preserving spreads on *WTA* and *WTP* (*left panel*) and *WTA–WTP* (*right panel*) for the *DARA* utility function $U(y) = y^{0.5}$ with fixed initial income $y_o = \$16$. The values depicted are for the sequence of mean preserving spread lotteries $\boldsymbol{i}\{z_i\} = \left\{4 + 16\left(1 - \frac{1}{e^i}\right), 4 - 16\left(1 - \frac{1}{e^i}\right), 0.5\right\}$, $i = 0, 1, 2, 3, \ldots$, each of which has a mean of 4, and for which the corresponding sequence of expected utility values $\{v(y_0, z_i)\}$ converges to $\nu(y_0)$

A common scenario is for the *WTA–WTP* gap to widen initially as risk increases, but eventually fall as the attractiveness of the increasingly risky lotteries deteriorates. Figure 2 illustrates using the same utility function as in Fig. 1 (i.e., $U(y) = y^{1-r}$, where $r = 0.5$ and $y_0 = \$16$). The values depicted in the two panels are for the sequence of mean preserving spread lotteries $\{z_i\} = \left\{4 + 16\left(1 - \frac{1}{e^i}\right), 4 - 16\left(1 - \frac{1}{e^i}\right), 0.5\right\}$, $i = 0, 1, 2, 3, \ldots$, each of which has a mean of \$4, and for which the corresponding sequence of expected utility values $\{v(y_0, z_i)\}$ converges to $\nu(y_0)$.

Under *DARA*, the non-monotonic pattern in which increases in risk at first expand and then contract the *WTA–WTP* gap will frequently be the case. Intuitively, an increase in risk may affect the *WTA–WTP* gap in two, offsetting ways. First, the risk increase makes the lottery less favorable, reducing both the *WTA* and *WTP,* as stated in Corollary 2 above. Second, this reduction in favorability may be more severe at lower incomes, leading to a more pronounced drop in the *WTP* than in the *WTA* (recall that $WTP_{z,y_0} = WTA_{z,y_0-WTP_{z,y_0}}$). If the latter effect is initially dominant, risk increases will expand the gap. Beyond some point, however, the inevitable decline in both *WTA* and *WTP* toward their lower bound of zero becomes the dominant effect, causing the gap to contract and eventually fall to zero.

4 Concluding comments

This paper evaluates the theoretical effects of uncertainty on *WTA* (equivalent variation), *WTP* (compensating variation), and the *WTA–WTP* gap under expected utility theory. Adapting an analysis by Weber (2003) to the case of lotteries, we develop an exact representation of the *WTA–WTP* difference. We further show that for any concave utility function, the introduction of uncertainty causes both *WTA* and *WTP* to fall, thus making the algebraic sign of *WTA–WTP* a function of the factor that falls by the largest margin. Additionally, we establish that the relative concavity of an individual's utility of income function determines the algebraic sign of the gap under uncertainty: *WTA–WTP* will be positive if the utility of income is *DARA*, and negative if the utility of income is *IARA*. Finally, we develop a pair of theorems establishing that increases in either income or risk drive the *WTA–WTP* gap for favorable lotteries to zero in the limit, but also show that the convergence need not be monotonic.

We do not assert from our analysis that standard expected utility theory explains the magnitude of the differences between equivalent and compensating variations observed in laboratory studies of lotteries. Rather, we show that expected utility theory both predicts the existence of a gap and implies specific comparative static behavior of the sign and size of the gap in response to changes in income and risk. These results under standard *EUT* should be a considered as factors potentially influencing observed behavior, regardless of whether they in themselves constitute a complete explanation of that behavior. The capacity of standard *EUT* to organize responses to changes in income and risk is an interesting behavioral question that we intend to explore with laboratory methods.

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Appendix 1

Proof of corollary 1

Proof of part (i) (a) was established as Eq. (4) above. For (b), define $y_2 = y_0 - C(y_0, z)$. By definition of $C(y_0, z)$, $v(y_0) = \nu(y_2, z)$. Further, by definition of $E(y_2, z)$, $v(y_2, z) = v[y_2 + E(y_2, z)]$, Hence $v(y_0) = v[y_0 - C(y_0, z) + E(y_2, z)]$, and so $C(y_0, z) = E(y_2, z)$. □

Proof of part (ii) Theorem 2 in Pratt (1964) establishes the equivalence between strict *DARA* and a strictly decreasing risk premium, between *CARA* and a constant risk premium, and between strict *IARA* and a strictly increasing risk premium. Denoting the expected value of lottery z by μ_z and recalling that the risk premium is defined as $\mu_z - E(y, z)$, Pratt's theorem establishes *(ii)* for $E(y, z)$. □

From part 1.i.b., $C(y_0, z) = E(y_2, z)$, so,

$$\frac{\partial C(y_0, z)}{\partial y_0} = \frac{\partial E[y_2, z]}{\partial y_0} = \frac{\partial E[y_2, z]}{\partial y_2}\frac{\partial y_2}{\partial y_0} = \frac{\partial E[y_2, z]}{\partial y_2}\left(1 - \frac{\partial C(y_0, z)}{\partial y_0}\right). \quad (9)$$

Since by definition $\nu(y_0) = \nu[y_0 - C(y_0, z), z]$ holds for all y_0, we may differentiate it with respect to y_0 to obtain

$$\frac{\partial \nu(y_0)}{\partial y_0} = \frac{\partial \nu(y_2, z)}{\partial y_2}\left(1 - \frac{\partial C(y_0, z)}{\partial y_0}\right).$$

Note that positive marginal utility of income (or alternatively, stochastic dominance) implies that both $\frac{\partial v(y_0)}{\partial y_0} > 0$ and $\frac{\partial v(y_2,z)}{\partial y_2} > 0$. Thus from the above equality, $\left(1 - \frac{\partial C(y_0,z)}{\partial y_0}\right) > 0$.

This result, together with $\frac{\partial E[y_2,z]}{\partial y_2} > (=, <)0$ under *DARA (CARA,IARA)* established for the $E(y, z)$ part of (ii), guarantees that the right hand side of Eq. (9) $> (=<)0$ under *DARA (CARA,IARA)*. Hence $\frac{\partial C(y_0,z)}{\partial y_0} > (=<)0$ under *DARA (CARA,IARA)*, establishing the $C(y_0, z)$ part of (ii).

Proof of part (iii) In the proof of Lemma 2 it was established that for any initial income y_0, $1 + \frac{\partial E(y_0,z)}{\partial y_0} > 0$. Thus the denominator of the integrand on the right hand side of (7) is positive for all δ in the interval of integration.

Theorem 2 in Pratt (1964) establishes the equivalence between strict *DARA* and a strictly decreasing risk premium, and between strict *IARA* and a strictly increasing risk premium. In the development here, the risk premium of the lottery z at initial income y_0 is the expected outcome of the lottery minus its certainty equivalent. For a risk averter, the certainty equivalent of lottery z is necessarily less than its expected outcome. As noted earlier, the certainty equivalent of the lottery at initial income y_0 is the willingness to accept, $E(y_0, z)$.

Consider first the case of *DARA*. A decreasing risk premium for z, together with a constant expected value, implies a rising certainty equivalent, i.e., that $\frac{\partial E(y_0,z)}{\partial y_0} > 0$. Thus, the numerator of the integrand on the right hand side of (7) is positive for any initial income y_0. Hence, the integrand is positive across the entire interval of integration. This implies that the integral itself is necessarily positive. Thus, under *DARA*, the right hand side of (7) is positive.

Matters are reversed in the case of *IARA*. An increasing risk premium combined with a positive expected value implies a falling certainty equivalent, that is, $\frac{\partial E(y_0,z)}{\partial y_0} < 0$ and thus the numerator of the integrand on the right hand side of (7) is negative. Using the same logic as in the case of *DARA* the integrand across the entire interval of integration is negative, meaning that the right hand side of (7) is negative.

Under *CARA*, the risk premium is constant and thus $\frac{\partial E(y_0,z)}{\partial y_0} = 0$. Hence, the numerator of the integrand on the right hand side of (7) is identically zero, and thus the integral is zero as well, establishing that the equivalent and compensating variations are equal in that case. □

Appendix 2

Proof of corollary 2

Proof Consider first the inequality on the left hand side. In their Theorem 2, Rothschild and Stiglitz (1970) establish, that given an income level y_0, $v(y_0, z_2) < v(y_0, z_1)$. Hence, $v(y_0 + E[y_0, z_2]) < v(y_0 + E[y_0, z_1])$, and so by $v_y > 0$, $E(y_0, z_1) > E(y_0, z_2)$.

Now consider the right hand inequality. Suppose that $C(y_0, z_1) < C(y_0, z_2)$. Then $v(y_0) = v(y_0 - C[y_0, z_1], z_1) \geq v(y_0 - C[y_0, z_2], z_1) > v(y_0 - C[y_0, z_2], z_2) =$

$\nu(y_0)$ where the strict inequality is again implied by Theorem 2 of Rothschild and Stiglitz. Thus $\nu(y_0) > \nu(y_0)$, a contradiction. Hence $C(y_0, z_1) > C(y_0, z_2)$. □

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