CS500 Quiz on Background

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Use mathematical notations in your answer.

Problem 1 (0 pts)**.** Define the Cartesian product $A \times B$.

Answer. $A \times B = \{(a, b) \ : \ a \in A, b \in B\}$. □

Problem 2 (10 pts)**.** Define "set partition".

Answer. Given a set A, a set P is a *partition* of A if:

- For all $X \in P$, $X \subseteq A$;
- For all $X, Y \in P$, $X \cap Y = \emptyset$ or $X = Y$; and
- For all $X \in P$, $X \neq \emptyset$

□

Problem 3 (10 pts)**.** Let $f : \{0, 1, 2\} \to \{0, 1, 2\}$ be a function for which $f(0) = 0, f(1) = 1, f(2) = 1$. What are the *domain*, *co-domain*, and *range* of f?

Answer. The domain is $\{0, 1, 2\}$, the co-domain is $\{0, 1, 2\}$, and the range is $\{0, 1\}$

□

Problem 4 (50 pts)**.** Let $G = (V, E)$ be an undirected graph.

1. Define the set $Paths(G)$ of (possibly non-simple) paths of G.
2. Define the set $SimplPaths(G)$ of simple paths of G. You can use $Paths(G)$ in your answer. (From now on, you can use the notion defined in an earlier sub-problem in a later one.)
3. Define the set $SimplCycles(G)$ of simple cycles of G.
4. Define the predicate $Connected(G)$.
5. Define the predicate $IsPartition(P, G)$, where P is a set of undirected graphs.

Answer. as follows.

1. $Paths(G) = \{p \in V^+ | \forall i \in [0, |p| - 1), \{p_i, p_{i+1}\} \in E\}$.
2. $SimplPaths(G) = \{p \in Paths(G) \mid \forall i, j \in [0, |p|), p_i \neq p_j \text{ or } i = j \text{ or } \{i, j\} = \{0, |p| - 1\}\}$
3. $SimplCycles(G) = \{p \in SimplPaths(G) \mid p_0 = p_{|p|-1}\}$.
4. $Connected(G) = \forall u, v \in V.\ \exists p \in Paths(G).\ p_0 = u \wedge p_{|p|-1} = v$.
5. $IsPartition(P, G) = \exists P_v$.
   (P_v is a set partition of V) $\wedge\ P = \{(V', E') \mid V' \in P_v \wedge E' = \{\{a, b\} \in E \mid a, b \in V'\}\}$

□

Problem 5 (30 pts)**.** Let G be a tree rooted at v_0. Suppose G is *represented as an undirected graph* (on contrary to the textbook).

1. Define $Depth(G, v_0, v)$, where v_0's depth is 0.
2. Define $Height(G, v_0)$.
3. Define $IsLeaf(G, v_0, v)$.

Answer. as follows.

1. $Depth(G, v_0, v) = \mathrm{Min}_{p \in Paths(G)}((p_0 = v_0 \wedge p_{|p|-1} = v) \ ? \ |p| : \infty)$.
2. $Height(G, v_0) = \mathrm{Max}_{v \in V} Depth(G, V_0, v)$.
3. $IsLeaf(G, v_0, v) = \forall u \in N_G(v),\ Depth(G, v_0, u) < Depth(G, v_0, v)$.

$\square$