Regression Analysis

```
import warnings
warnings.filterwarnings('ignore')
```

I. 단일회귀분석

1) Load Data

```
import pandas as pd

url = 'https://raw.githubusercontent.com/rusita-ai/pyData/master/Galton.txt'
DF = pd.read_table(url)

DF.info()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 898 entries, 0 to 897
Data columns (total 6 columns):
 #   Column  Non-Null Count  Dtype  
---  ------  --------------  -----  
 0   Family  898 non-null    object 
 1   Father  898 non-null    float64
 2   Mother  898 non-null    float64
 3   Gender  898 non-null    object 
 4   Height  898 non-null    float64
 5   Kids    898 non-null    int64  
dtypes: float64(3), int64(1), object(2)
memory usage: 42.2+ KB
```

```
DF.head()
```

	Family	Father	Mother	Gender	Height	Kids
0	1	78.5	67.0	M	73.2	4
1	1	78.5	67.0	F	69.2	4
2	1	78.5	67.0	F	69.0	4
3	1	78.5	67.0	F	69.0	4
4	2	75.5	66.5	M	73.5	4

2) 남자 데이터만 분리

```
DFS = DF.loc[DF.Gender == 'M', :]

DFS.head()
```

	Family	Father	Mother	Gender	Height	Kids
0	1	78.5	67.0	M	73.2	4
4	2	75.5	66.5	M	73.5	4
5	2	75.5	66.5	M	72.5	4
8	3	75.0	64.0	M	71.0	2
10	4	75.0	64.0	M	70.5	5

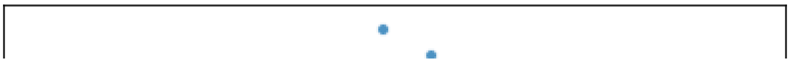
3) pearson 상관계수

```
from scipy import stats

stats.pearsonr(DFS.Father, DFS.Height)[0]
```

```
0.391317358141 79
```

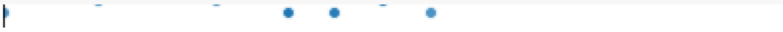
4) 회귀선 시각화

```
import matplotlib.pyplot as plt
import seaborn as sns

plt.figure(figsize = (7, 5))
sns.regplot(x = DFS.Father, y = DFS.Height,
            line_kws = {'color':'red'},
            scatter_kws = {'edgecolor' : 'white'})
plt.show()
```



5) Modeling

```
import statsmodels.formula.api as smf

Model_lm = smf.ols(formula = 'Height ~ Father',
                   data = DFS).fit()
```



6) Model Summary

- 잔차(residual) 검증
  - Prob(Omnibus) & Prob(JB) : 0.05보다 크면 정규분포
  - 왜도(Skew) : 정규분포는 '0', '0'보다 크면 오른쪽 자락이 길어짐
  - 첨도(Kurtosis) : 정규분포는 '3'
  - Durbin-Watson : 잔차의 자기상관 체크 지표 '2' 전후

```
Model_lm.summary(alpha = 0.05)
```

OLS Regression Results

Dep. Variable:	Height	**R-squared:**	0.153
Model:	OLS	**Adj. R-squared:**	0.151
Method:	Least Squares	**F-statistic:**	83.72
Date:	Thu, 11 Mar 2021	**Prob (F-statistic):**	1.82e-18
Time:	00:42:40	**Log-Likelihood:**	-1070.6
No. Observations:	465	**AIC:**	2145.
Df Residuals:	463	**BIC:**	2153.
Df Model:	1		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
Intercept	38.2589	3.387	11.297	0.000	31.604	44.914
Father	0.4477	0.049	9.150	0.000	0.352	0.544

Omnibus:	8.699	**Durbin-Watson:**	1.481
Prob(Omnibus):	0.013	**Jarque-Bera (JB):**	13.007
Skew:	-0.112	**Prob(JB):**	0.00150
Kurtosis:	3.788	**Cond. No.**	2.09e+03

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 2.09e+03. This might indicate that there are
strong multicollinearity or other numerical problems.

II. 모델의 선형성

1) 예측값(fitted) 계산

```
fitted = Model_lm.predict(DFS.Father)
```

2) 잔차(residual) 계산

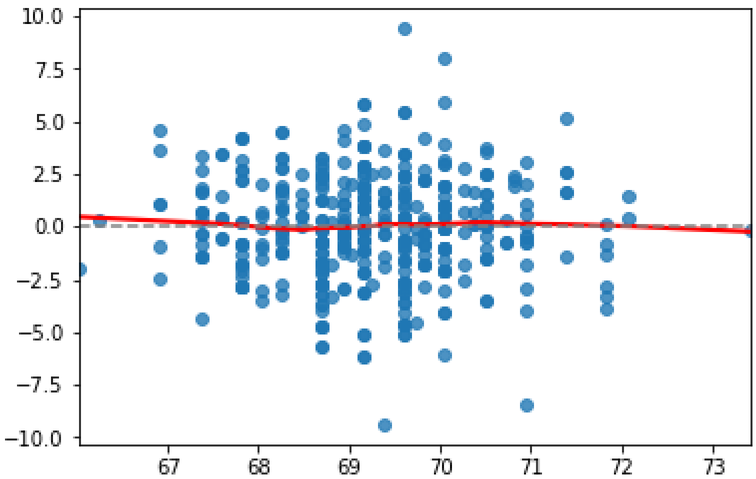
- 실제값과 예측값의 차이

```
residual = DFS.Height - fitted
```

3) 예측값과 잔차 비교

- 모든 예측값에서 잔차가 비슷하게 있어야 함
- 잔차의 추세 : 빨간실선
- 빨간실선이 회색점선을 크게 벗어난다면 예측값에 따라 잔차가 크게 달라지는 것을 의미

```
sns.regplot(fitted, residual,
            lowess = True,
            line_kws = {'color':'red'})
plt.plot([fitted.min(), fitted.max()],
         [0, 0], '--', color = 'gray')
plt.show()
```



III. 잔차분석

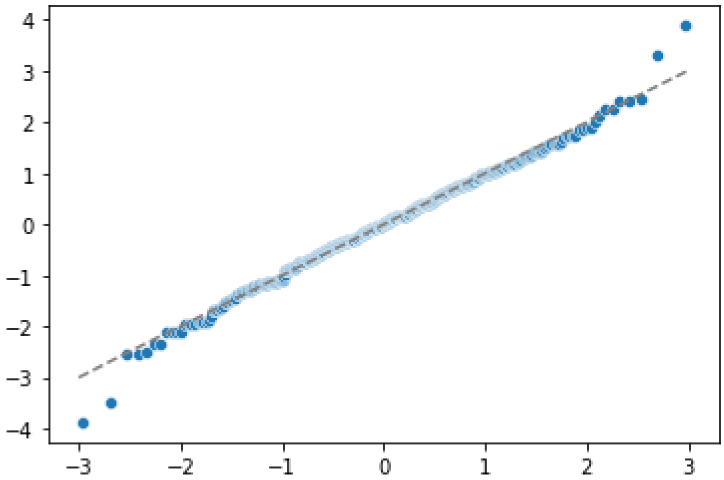
1) 잔차의 정규성

- 잔차가 정규분포를 따른다는 가정 검증

```
import scipy.stats

sr = scipy.stats.zscore(residual)
(x, y), _ = scipy.stats.probplot(sr)
```

- Q-Q 플롯
    - 잔차가 정규분포를 띄면 Q-Q 플롯에서 점들이 점선을 따라 배치

```
sns.scatterplot(x, y)
plt.plot([-3, 3], [-3, 3], '--', color = 'grey')
plt.show()
```



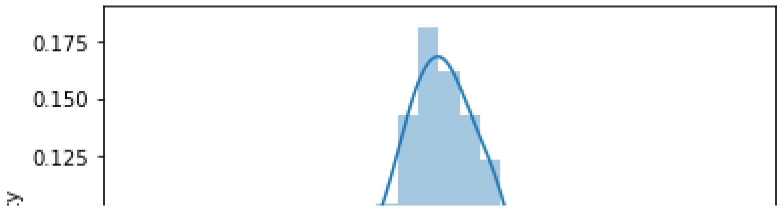
- shapiro Test
    - p값이 0.05보다 작아 잔차의 정규성을 따른다는 귀무가설을 기각
    - 유의수준 5%에서 잔차의 정규성 위반

```
scipy.stats.shapiro(residual)[1]
```

    0.04990636184811592

- Residual Visualization

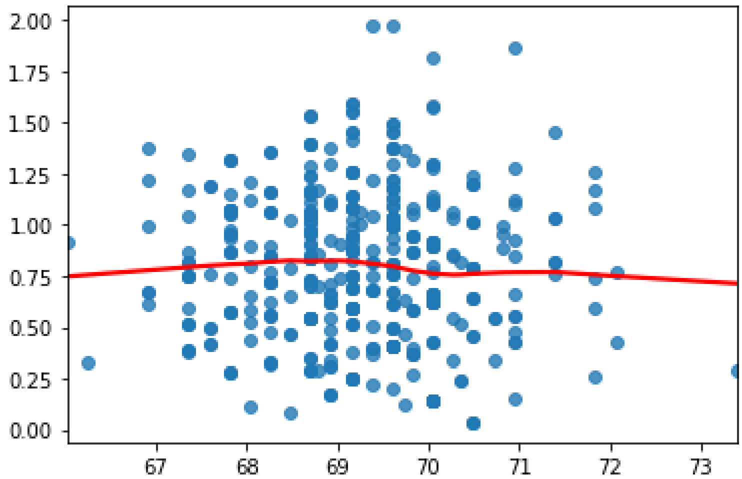
```
sns.distplot(residual)
plt.show()
```



2) 잔차의 등분산성

- 예측된 값이 크던 작던, 모든 값들에 대하여 잔차의 분산이 동일하다는 가정
    - 예측값(가로축)에 따라 잔차가 어떻게 달라지는지 시각화
    - 빨간실선이 수평선을 그리는 것이 이상적

```
import numpy as np

sns.regplot(fitted,
            np.sqrt(np.abs(sr)),
            lowess = True,
            line_kws = {'color': 'red'})
plt.show()
```



3) 잔차의 독립성

- 회귀분석에서 잔차는 정규성, 등분산성 그리고 독립성을 가지는 것으로 가정
- 자료 수집 시 Random Sampling을 하였다면, 잔차의 독립성은 만족하는 것으로 봄

4) 극단값

- Cook's distance
    - 극단값을 나타내는 지표

```
from statsmodels.stats.outliers_influence import OLSInfluence
```

```
cd, _ = OLSInfluence(Model_lm).cooks_distance
```

- 59번자료가 예측에서 많이 벗어남을 확인

```
cd.sort_values(ascending = False).head()
```

```
59     0.050149
22     0.026499
868    0.023930
17     0.020738
125    0.019052
dtype: float64
```

IV. 다중회귀분석

1) Load Data

```
import seaborn as sns
```

```
DF2 = sns.load_dataset('iris')

DF2.info()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 150 entries, 0 to 149
Data columns (total 5 columns):
 #   Column        Non-Null Count  Dtype  
---  ------        --------------  -----  
 0   sepal_length  150 non-null    float64
 1   sepal_width   150 non-null    float64
 2   petal_length  150 non-null    float64
 3   petal_width   150 non-null    float64
 4   species       150 non-null    object 
dtypes: float64(4), object(1)
memory usage: 6.0+ KB
```

```
DF2.head()
```

	sepal_length	sepal_width	petal_length	petal_width	species
0	5.1	3.5	1.4	0.2	setosa
1	4.9	3.0	1.4	0.2	setosa
2	4.7	3.2	1.3	0.2	setosa
3	4.6	3.1	1.5	0.2	setosa
4	5.0	3.6	1.4	0.2	setosa

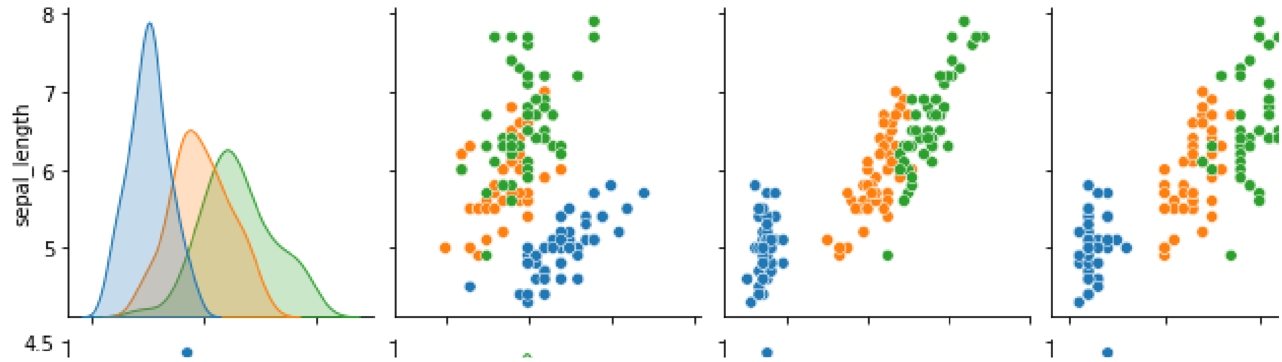
2) pearson 상관계수

```
DF2.corr()
```

	sepal_length	sepal_width	petal_length	petal_width
sepal_length	1.000000	-0.117570	0.871754	0.817941
sepal_width	-0.117570	1.000000	-0.428440	-0.366126
petal_length	0.871754	-0.428440	1.000000	0.962865
petal_width	0.817941	-0.366126	0.962865	1.000000

3) Visualization

```
import matplotlib.pyplot as plt
import seaborn as sns

sns.pairplot(hue = 'species', data = DF2)
plt.show()
```



4) Modeling

```
import statsmodels.formula.api as smf

Model = smf.ols(formula = 'sepal_length ~ sepal_width + petal_length + petal_width',
                data = DF2)
```

```
Model_mr = Model.fit()
```

5) Model Summary

```
Model_mr.summary(alpha = 0.05)
```

OLS Regression Results

Dep. Variable:	sepal_length	**R-squared:**	0.859
Model:	OLS	**Adj. R-squared:**	0.856
Method:	Least Squares	**F-statistic:**	295.5
Date:	Thu, 11 Mar 2021	**Prob (F-statistic):**	8.59e-62
Time:	00:42:50	**Log-Likelihood:**	-37.321
No. Observations:	150	**AIC:**	82.64
Df Residuals:	146	**BIC:**	94.69
Df Model:	3		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
Intercept	1.8560	0.251	7.401	0.000	1.360	2.352
sepal_width	0.6508	0.067	9.765	0.000	0.519	0.783
petal_length	0.7091	0.057	12.502	0.000	0.597	0.821
petal_width	-0.5565	0.128	-4.363	0.000	-0.809	-0.304

Omnibus:	0.345	**Durbin-Watson:**	2.060
Prob(Omnibus):	0.842	**Jarque-Bera (JB):**	0.504
Skew:	0.007	**Prob(JB):**	0.777
Kurtosis:	2.716	**Cond. No.**	54.7

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

V. 다중공선성(Multicollinearity)

- 공선성(Collinearity) : 독립변수가 다른 독립변수로 잘 예측되는 경우
    - 또는 서로 상관이 높은 경우
- 다중공선성(Multicollinearity) : 독립변수가 다른 여러 개의 독립변수들로 잘 예측되는 경우

```
from statsmodels.stats.outliers_influence import variance_inflation_factor
```

1) 독립변수 확인

```
Model.exog_names
```

```
['Intercept', 'sepal_width', 'petal_length', 'petal_width']
```

2) 다중공선성 진단

- 분산팽창계수(VIF:Variance Inflation Factor)
    - 엄밀한 기준은 없으나 보통 10보다 크면 다중공선성이 있다고 판단
    - 5를 기준으로 하기도 함

- 'sepal_width'의 VIF

```
variance_inflation_factor(Model.exog, 1)
```

```
1.270814929344654
```

- 'petal_length'의 VIF

```
variance_inflation_factor(Model.exog, 2)
```

```
15.097572322915717
```

- 'petal_width'의 VIF

```
variance_inflation_factor(Model.exog, 3)
```

```
14.234334971742083
```

- pearson 상관계수

```
DF2.corr()
```

	sepal_length	sepal_width	petal_length	petal_width
sepal_length	1.000000	-0.117570	0.871754	0.817941
sepal_width	-0.117570	1.000000	-0.428440	-0.366126
petal_length	0.871754	-0.428440	1.000000	0.962865
petal_width	0.817941	-0.366126	0.962865	1.000000

3) 다중공선성 해결

- VIF가 큰 독립변수를 제거 후 모델링
  - 'petal_width' 제거

```
Model_VIF = smf.ols(formula = 'sepal_length ~ sepal_width + petal_length',
                    data = DF2).fit()
```

- 다중공선성 처리 후

```
Model_VIF.summary()
```

OLS Regression Results

Dep. Variable:	sepal_length	**R-squared:**	0.840
Model:	OLS	**Adj. R-squared:**	0.838
Method:	Least Squares	**F-statistic:**	386.4
Date:	Thu, 11 Mar 2021	**Prob (F-statistic):**	2.93e-59
Time:	00:42:50	**Log-Likelihood:**	-46.513
No. Observations:	150	**AIC:**	99.03
Df Residuals:	147	**BIC:**	108.1
Df Model:	2		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	2.2491	0.248	9.070	0.000	1.759	2.739
sepal_width	0.5955	0.069	8.590	0.000	0.459	0.733
petal_length	0.4719	0.017	27.569	0.000	0.438	0.506

Omnibus:	0.164	**Durbin-Watson:**	2.021
Prob(Omnibus):	0.921	**Jarque-Bera (JB):**	0.319
Skew:	-0.044	**Prob(JB):**	0.853
Kurtosis:	2.792	**Cond. No.**	48.3

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

#

#

#

The End

#

#

#