Written Exam Economics winter 2021-22

Advanced Macroeconomics: Structural Vector Autoregressive (VAR) Analysis

January 15 to 17, 2022

This exam question consists of 7 pages in total

Answers only in English.

A take-home exam paper cannot exceed 10 pages – and one page is defined as 2400 keystrokes

The paper must be uploaded as <u>one PDF document</u>. The PDF document must be named with exam number only (e.g. '1234.pdf') and uploaded to Digital Exam.

Be careful not to cheat at exams!

Exam cheating is for example if you:

- Copy other people's texts without making use of quotation marks and source referencing, so that it may appear to be your own text
- Use the ideas or thoughts of others without making use of source referencing, so it may appear to be your own idea or your thoughts
- Reuse parts of a written paper that you have previously submitted and for which you have received a pass grade without making use of quotation marks or source references (self-plagiarism)
- Receive help from others in contrary to the rules laid down in part 4.12 of the Faculty of Social Science's common part of the curriculum on cooperation/sparring

You can read more about the rules on exam cheating on your Study Site and in part 4.12 of the Faculty of Social Science's common part of the curriculum.

Exam cheating is always sanctioned by a written warning and expulsion from the exam in question. In most cases, the student will also be expelled from the University for one semester.

Practical Information

Note the following formal requirements:

- This is an individual examination. You are not allowed to cooperate with other students or other people, see the focus on exam cheating above.
- The assignment consists of Sections 1-7 with 20 questions to be answered. Please answer all questions.
- The exam paper should not exceed 10 pages. A maximum of 10 pages of supporting material (graphs, estimation output, etc.) can accompany the paper as appendices. You may refer to the computer output in the appendices when answering the questions or you can include tables and/or graphs in the main text. Also, you may add clarifying comments in the output as part of your answer. It is not necessary to provide the MATLAB code you are using, unless otherwise stated in the question.
- All pages must be numbered consecutively and marked with your exam number. **You should not write your name on the exam paper.**
- Your paper must be uploaded on the course page in Absalon at the given time. The exam paper (including supporting material) must be in PDF-format and collected in one file only; the uploaded file must be named 1234.pdf, where 1234 is your exam number.

Regarding the data for the exam paper, please note the following:

- All assignments are based on different data sets. You should use the data set (quarterly data covering the period 1985:1-2021:4) located in the MATLAB file 1234.mat, where 1234 is your exam number. This MATLAB file contains the data (x), the dates (dates) and the name of the variables (names). You can load this file into MATLAB directly using 'load 1234.mat'. In case you cannot find your exam number, you can use the 1000.mat file.
- To avoid that some data sets are more difficult to handle than others, the data sets are artificial (simulated from a known data generating process), and they behave, as close as possible, like actual data.

Background

The topic for this project examination is monetary policy in a small open economy using inflation targeting. The purpose of the examination is to assess your understanding of structural vector autoregressive (VAR) models. Substantial emphasis will be placed on using your programming skills in MATLAB. Specifically, the examination assesses theoretical and practical knowledge of structural vector autoregressive models within stationary and non-stationary frameworks including assessing empirical results, using different approaches to identify VAR models and be able to use MATLAB to generate empirical results. You can use any MATLAB functions that you have programmed yourself or any function uploaded to Absalon during the course except when otherwise stated. You are not allowed to use other programs or built-in MATLAB functions except for those that are specified in the questions below. The assignment requires some additional coding.

Most questions in the examination are applied, concerning the empirical example outlined below. When you answer these empirical questions, please explain and motivate your answers as detailed as possible, preferably with reference to the underlying theory.

Consider the following data generating process (DGP). Let the time series vector $x_t = \begin{bmatrix} \ln Y_t & \ln P_t & i_t & \ln e_t \end{bmatrix}'$ where Y_t is GDP, P_t is the price level, i_t is the interest rate (the policy rate) and e_t is the exchange rate measured as home currency units per unit foreign currency. We assume that the time series vector is generated by the following VAR model

$$x_t = \nu + A_1 x_{t-1} + \ldots + A_p x_{t-p} + u_t$$

where ν is a constant and u_t is a vector of reduced form residuals with covariance matrix Σ_u. Assume that this time series vector is integrated of order one and that the cointegration rank is r.

The assignment will guide you through an empirical analysis of the time series vector stated above including estimation and analysis of the cointegrated VAR model, identification of the structural cointegrated VAR model and robustness analysis.

The data

1. The data is already in natural logarithms (except the interest rate) and ordered as in x_t. Plot the data and perform graphical analysis in order to assess the degree of integration of all four variables.

The Vector Error Correction Model

2. Show in detail that we can rewrite the VAR model as the following Vector Error Correction (VEC) model

$$\Delta x_t = \nu + \Pi x_{t-1} + \Gamma_1 \Delta x_{t-1} + \ldots + \Gamma_2 \Delta x_{t-2} + u_t \tag{1}$$

where

$$\Pi = -\left(I_4 - A_1 - \ldots - A_3\right)$$

and

$$\Gamma_i = -\left(A_{i+1} + \cdots + A_3\right).$$

3. Assume that the rank $r = 1$ and that the cointegration vector is given by

$$\beta' = \begin{bmatrix} 0 & -1 & 0 & 1 \end{bmatrix}.$$

Note that this vector can be interpreted as the PPP relation implying that $\beta' x_t$ is the real exchange rate. This interpretation follows if we interpret P_t as the relative price levels or if we assume that the foreign price level is constant. What conditions must the adjustment coefficients in $\alpha = \begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 \end{bmatrix}'$ satisfy for this system to have 1 cointegration vector?

4. Assume now that the rank still is equal to 1 but that the cointegration vector is

$$\beta' = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}.$$

In this case we have that the interest rate is stationary. Assume that the real interest rate as well as inflation expectations are stationary. The Fisher relation then implies that the interest rate is stationary. What conditions must the adjustment coefficients in $\alpha = \begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 \end{bmatrix}'$ satisfy for this system to have rank equal to 1?

Lag order determination and diagnostic testing

5. Formulate a well-specified VAR model for x_t. Explain your workflow and how you argue for your choice of the number of autoregressive lags in the VAR model.

6. Test for multivariate autocorrelation, heteroscedasticity and normality. Does your model satisfy the underlying assumptions? If the multivariate tests of autocorrelation and ARCH reject the null hypotheses, apply univariate tests for autocorrelation and ARCH in the residuals in each equation. You are allowed to use the built-in MATLAB functions lbqtest and archtest as alternatives to own functions.

Testing for Cointegration

7. For your preferred model, proceed by testing for cointegration using the MATLAB function jcitest. Explain your approach and how you find the number of cointegration

vectors, that is, the rank r, in the system allowing for (i) a constant term in the cointegration vector and (ii) both a constant and a linear trend in the cointegration vector. Do you use different sources of information when determining the rank? If so, explain how you arrive at your decision.

8. Impose your preferred rank and test the null hypothesis that there is no linear trend in the cointegration vectors.

9. Impose your preferred rank and the preferred specification of the deterministic component in the cointegration vector found in the previous question and test hypotheses on the cointegration space using the MATLAB function jcontest. Test for exclusion, stationarity and weak exogeneity. Explain the meanings of these tests.

10. Interpret the unrestricted estimated cointegration vectors in light of the theoretical cointegration vectors in questions 3 and 4. Do you find plausible values of the parameters in the estimated cointegration vectors?

11. Test formally whether the theoretical cointegration vectors are in line with the information in the data using the MATLAB function jcontest. Explain how these tests relate to the exclusion and stationarity tests.

12. Plot both the unrestricted and theoretical cointegration vectors. Interpret your results.

Identification of Structural Model

13. Impose $r = 2$ and the theoretical cointegration vectors and re-estimate the VEC model using the full sample and using your preferred lag length found above. Interpret the adjustment coefficients in light of the PPP relation and the interest rate differential. Do you find the expected signs on the adjustment coefficients?

14. Suggest an identification scheme including names of the four structural shocks in the VAR/VEC system using a combination of long-run and short-run restrictions. If you cannot provide names for these shocks, try to explain how they affect the data under the maintained assumptions. State the long-run effects of the shocks on the data.

Impulse Responses and Forecast Error Variance Decomposition

15. Implement the identification scheme using the MATLAB solver. Check that the solver provides a valid identification and compute the variance-covariance matrix of the identified structural shocks. Please, provide the MATLAB code you are using to identify the shocks in the appendix. It must include a description of the restrictions you impose to identify the structural model.

    If you fail computing the B_0^{-1} matrix using the MATLAB solver, please use the ident.p file. This file works as a standard m-file but the coding is concealed and there is no way to convert the p-file into an m-file. Note that the ident.p file is set up to use a closed form solution to compute the B_0^{-1} matrix using a generic identification based on estimates from the VEC model. To use this function, you need to add the following code to your MATLAB m-file and you need to set the rank equal to 2. You can use any number of lags. Note: Make sure that you don't have any ident.m files in the same folder. The same function can be used in a bootstrap. Note also that it will be impossible to interpret the impulse response function and the variance decomposition using economic theory as the identification is incorrect.

```
% Use generic identification
% Input:
% alpha is the K x r adjustment coefficient matrix
% beta is the r x K cointegration vector
% Gamma = [ Gamma(1) Gamma(2) ....  Gamma(p-1)] coefficent matrix
% sigmahat = Sigma_u (the residual covariance matrix)
% K is the number of variables in the VAR
% p is the number of lags in underlying VAR
% r is the rank
%
% Output:
% invB0 is the inverse of the B0 matrix
% Xi is the C(1) matrix
[invB0,Xi]=ident(alpha,beta,Gamma,sigmahat,K,p,r)
```

16. Compute impulse response functions and forecast error variance decomposition of the data. Report only the effects of monetary policy shocks on the data. Use bootstrap (the standard residual based recursive design bootstrap or wild bootstrap) to compute confidence intervals based on bootstrap standard error estimates (for the impulse responses) and Efron's percentile intervals (for the variance decomposition). It is important that you clearly state which bootstrap method you use and how you

check the order condition in each bootstrap replication. You can show forecast error variance decomposition either in a table or in a graph. Interpret your results.

17. Show the impulse responses of the data to the remaining three shocks. Discuss whether the impulse responses are consistent with economic intuition.

18. Instead of implementing the theoretical cointegration vector you can use the estimated cointegration vector still imposing $r = 2$. Identify the structural shocks using the same identification scheme as above and plot the implied impulse responses of the data to monetary policy shocks together with confidence bands. Compare your results to what you previously found using the theoretical cointegration vector. Comment!

    If you fail computing the B_0^{-1} matrix using the MATLAB solver, please use the ident.p file again. The code is generic and works for any cointegration vector. However, you may not be able to interpret your results in light of economic theory.

Extensions

19. An alternative to using the MATLAB solver to compute the B_0^{-1} matrix is to use the approach suggested by Warne (1993). Outline this approach and show how the Warne approach can be used to identify the structural shocks in your preferred VEC model. Discuss both the identification of permanent and transitory shocks.

20. Code this identification and compute the implied B_0^{-1} matrix and show that it is identical to the one found by the solver. Please, provide the code you are using in the appendix.