Quantitative Problems

1. Find the extrema of the following functions:
   (a) $f(x) = \frac{x^4}{4} + \frac{x^3}{3} - \frac{x^2}{2} - x + 6$

   (b) $f(x) = 2(x^2 + x + 1)^2$

   (c) $f(x) = 1 - (x+1)^{3/2}$

   (d) $f(x) = x^2 \cdot e^{-x}$

2. Every polynomial function of degree 2 can be written as $f(x) = ax^2 + bx + c$ with $a, b, c \in \mathbb{R}$. Derive a formula that computes the extrema of second degree polynomials.

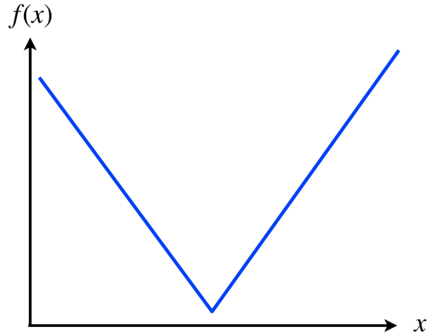
3. (Challenge) Derive a formula that computes the extrema of any polynomial of degree 3.

4. Suppose that the population of Rhode Island can be modeled by the equation $P(x) = \frac{1}{2}e^{kt}$ for some $k > 0$.
   (a) What is the rate of growth of the population

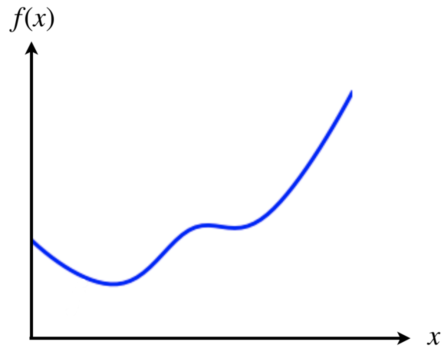
   (b) How does the population change as $t \to \infty$?

   (c) Can you think of any reasons why this model may not be very realistic?

5. Would the gradient descent algorithm converge when the loss function is the function shown in the graph below and why?



6. Would the gradient descent algorithm converge when the loss function is the function shown in the graph below and why?



7. Suppose that we would like to find the minimum of the function $f(x) = 2x^2 - 3x + 8$ by using gradient descent

   (a) In this algorithm, we start some initial point x_0 and make updates by $x_{n+1} = F(x_n)$. Let $step_size$ be the step size, then what is F given the loss function f?

   (b) How should we choose the initial point x_0?

   (c) Compute the first three updates of this algorithm with your choice of step size and initial points. Will this algorithm converge, why or why not?