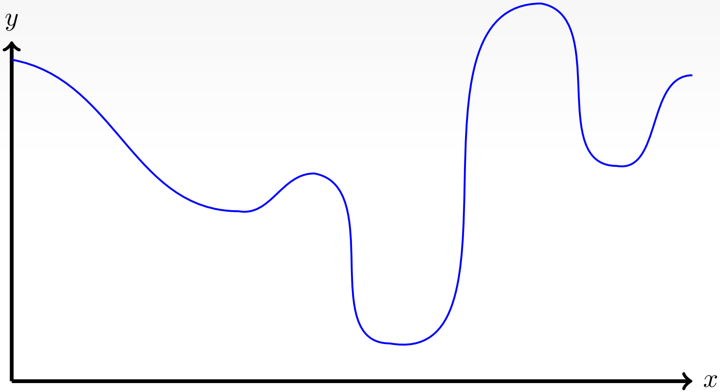
Artificial Intelligence: *Modeling Human Intelligence with Networks*

Jeová Farias Sales Rocha Neto
jeova_farias@brown.edu

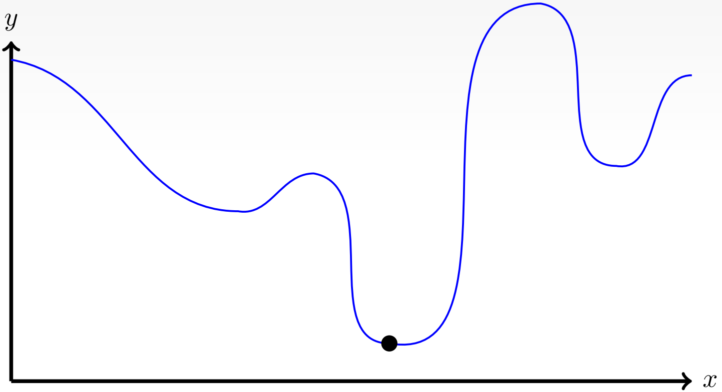
From the class yesterday

- Which points in the following curve have derivative equals to zero?



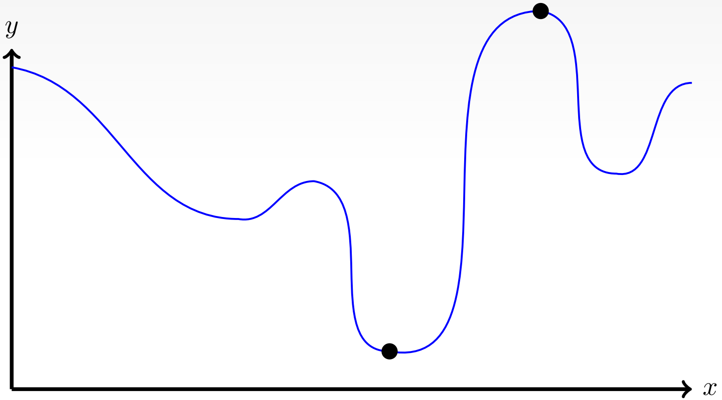
From the class yesterday

- Which points in the following curve have derivative equals to zero?



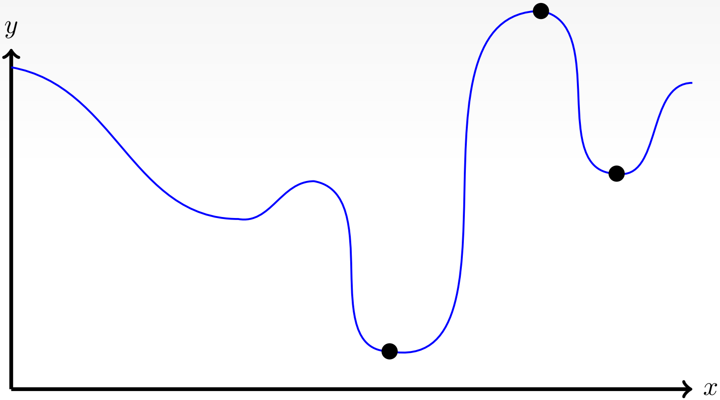
From the class yesterday

- Which points in the following curve have derivative equals to zero?



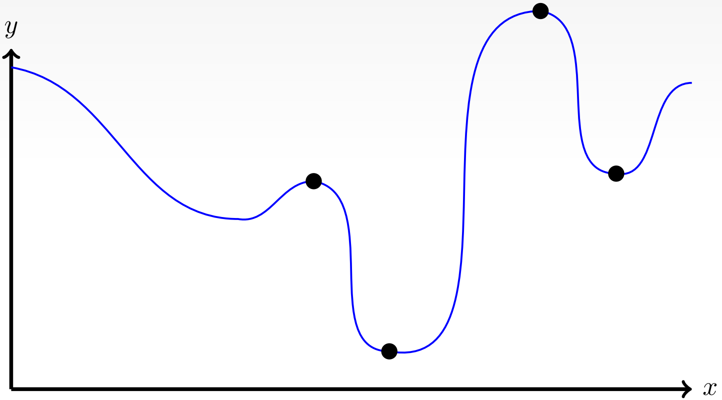
From the class yesterday

- Which points in the following curve have derivative equals to zero?



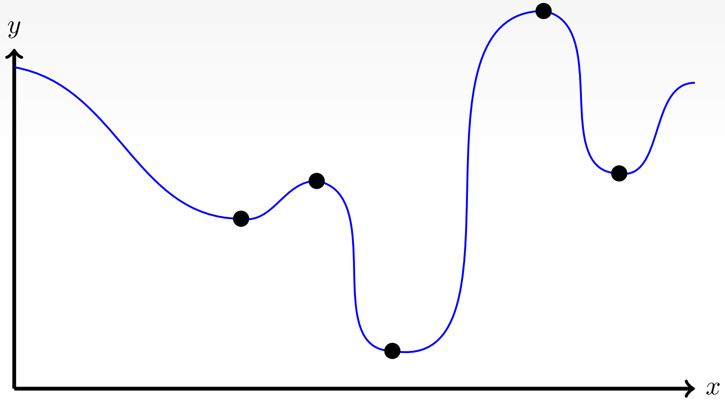
From the class yesterday

- Which points in the following curve have derivative equals to zero?



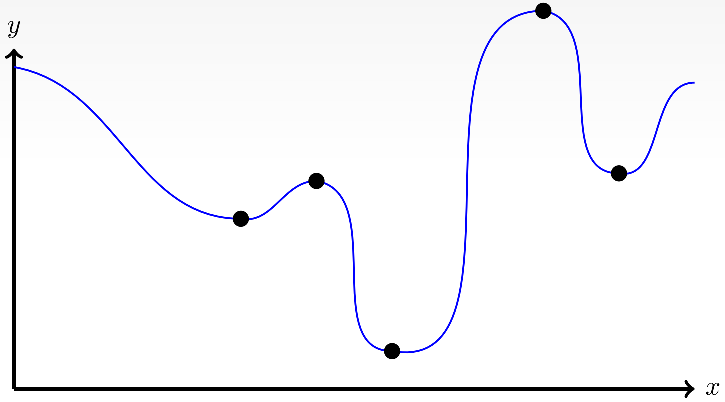
From the class yesterday

- Which points in the following curve have derivative equals to zero?



From the class yesterday

- Which points in the following curve have derivative equals to zero?



- Does anyone spot something important about (some of) these points?

Gradient Descent

The minimum value of a function

- Have we heard of any quantity we want to minimize in this class?

The minimum value of a function

- Have we heard of any quantity we want to minimize in this class?
- How about the **loss** of a classifier?

The minimum value of a function

- Have we heard of any quantity we want to minimize in this class?
- How about the **loss** of a classifier?
- For the simple linear classifier we saw, we know that for a dataset $X = \{x^1, x^2, \ldots, x^m\}$:

$$\mathcal{L}(X, w) = \sum_{i=1}^{m} [f_{act}(w \cdot x^i) \neq class_i]$$

The minimum value of a function

- Have we heard of any quantity we want to minimize in this class?
- How about the **loss** of a classifier?
- For the simple linear classifier we saw, we know that for a dataset $X = \{x^1, x^2, \ldots, x^m\}$:

$$\mathcal{L}(X, w) = \sum_{i=1}^{m} [f_{act}(w \cdot x^i) \neq class_i]$$

- In order to minimize it, we can't change the data X (it was given to us), but we can try to find the best w!

The minimum value of a function

- Have we heard of any quantity we want to minimize in this class?
- How about the **loss** of a classifier?
- For the simple linear classifier we saw, we know that for a dataset $X = \{x^1, x^2, \ldots, x^m\}$:

$$\mathcal{L}(X, w) = \sum_{i=1}^{m} [f_{act}(w \cdot x^i) \neq class_i]$$

- In order to minimize it, we can't change the data X (it was given to us), but we can try to find the best w!
- The **Gradient descent** algorithm!

Gradient descent

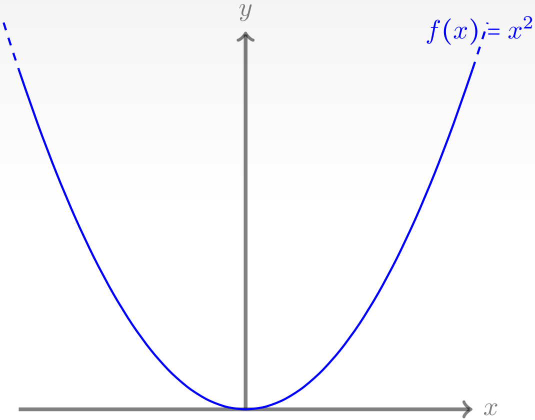
Algorithm 1 Gradient descent - v1

Input: A function $f(x)$
Output: A number x.

```
Initialize x
step_size = 1
for some repetitions do
    if Slope at f(x) is negative then
        x = x + step_size
    end if
    if Slope at f(x) is positive then
        x = x − step_size
    end if
end for
```

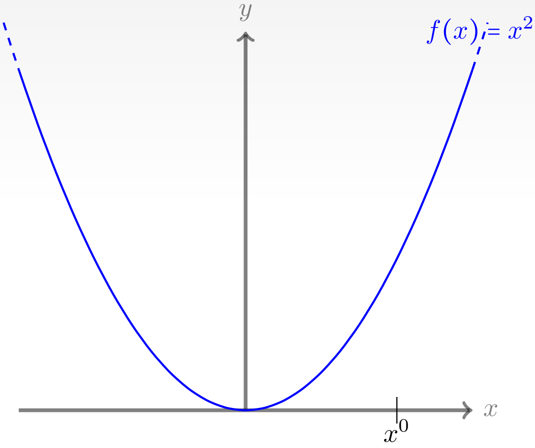
The minimum value of a function

- How do I compute x such that $f(x)$ is minimum?



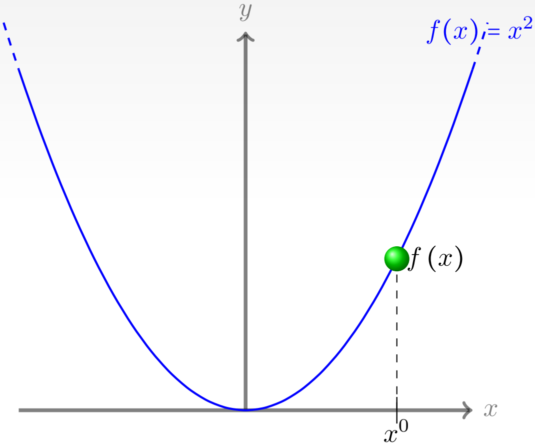
The minimum value of a function

- How do I compute x such that $f(x)$ is minimum?



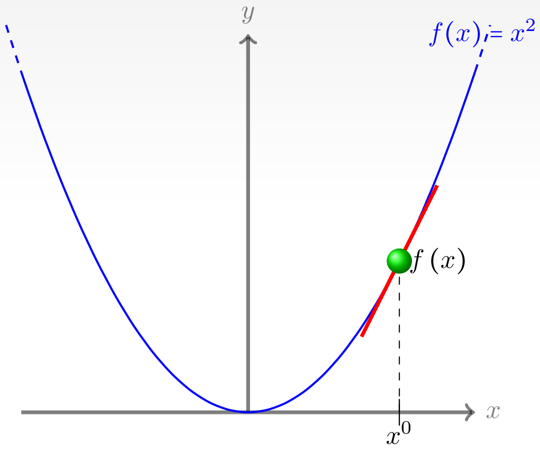
The minimum value of a function

- How do I compute x such that $f(x)$ is minimum?



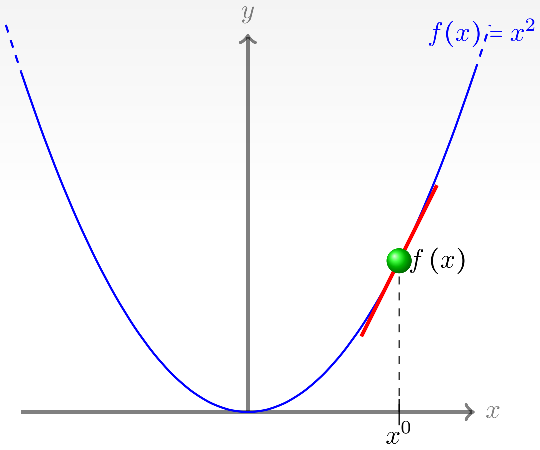
The minimum value of a function

- How do I compute x such that $f(x)$ is minimum?



The minimum value of a function

- How do I compute x such that $f(x)$ is minimum?

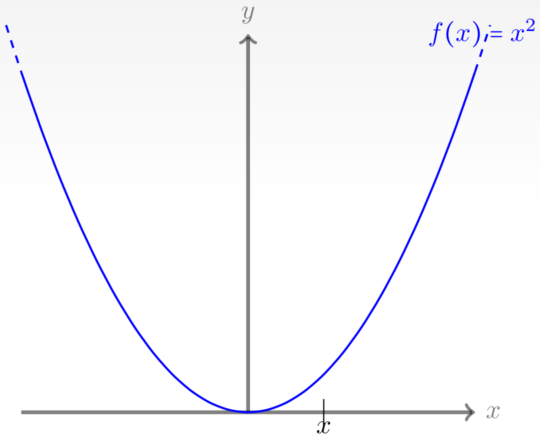


Positive slope

$\longleftarrow x$

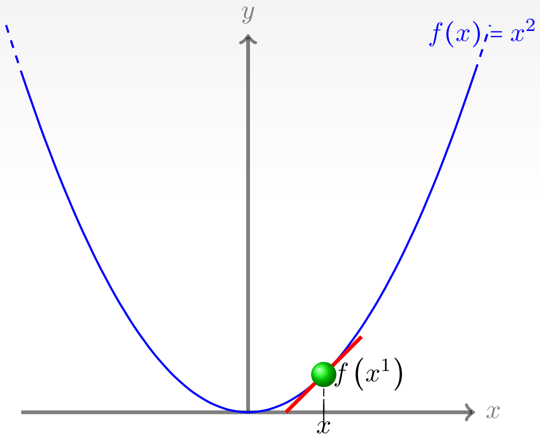
The minimum value of a function

- How do I compute x such that $f(x)$ is minimum?



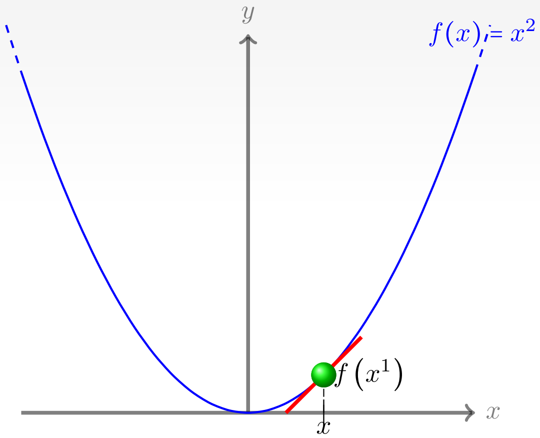
The minimum value of a function

- How do I compute x such that $f(x)$ is minimum?



The minimum value of a function

- How do I compute x such that $f(x)$ is minimum?

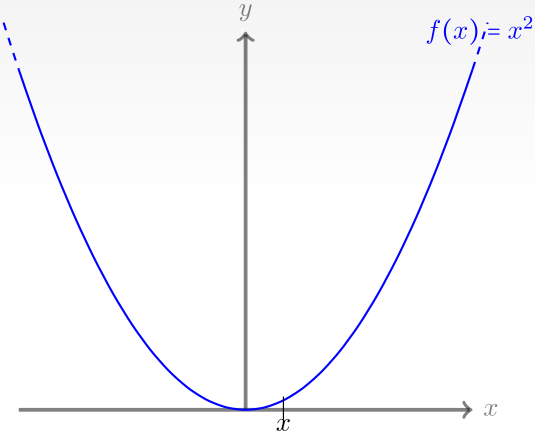


Positive slope

$\longleftarrow x$

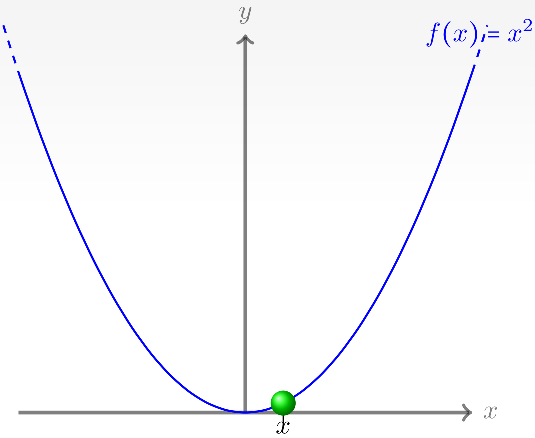
The minimum value of a function

- How do I compute x such that $f(x)$ is minimum?



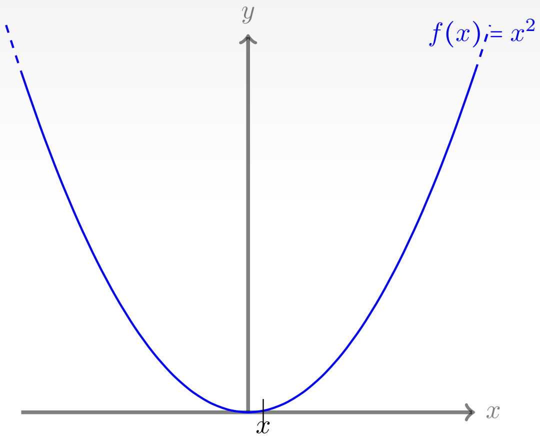
The minimum value of a function

- How do I compute x such that $f(x)$ is minimum?



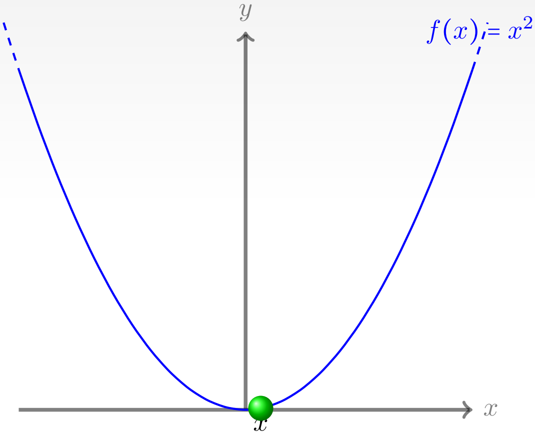
The minimum value of a function

- How do I compute x such that $f(x)$ is minimum?



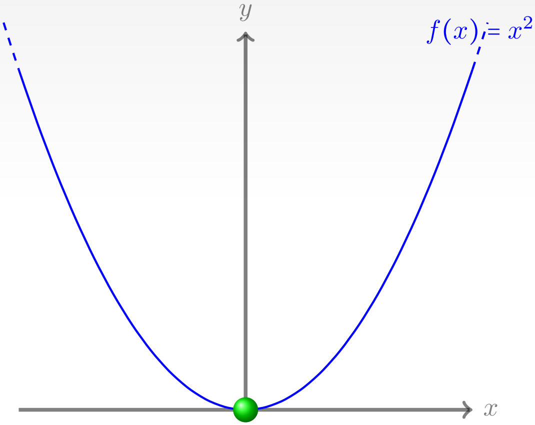
The minimum value of a function

- How do I compute x such that $f(x)$ is minimum?



The minimum value of a function

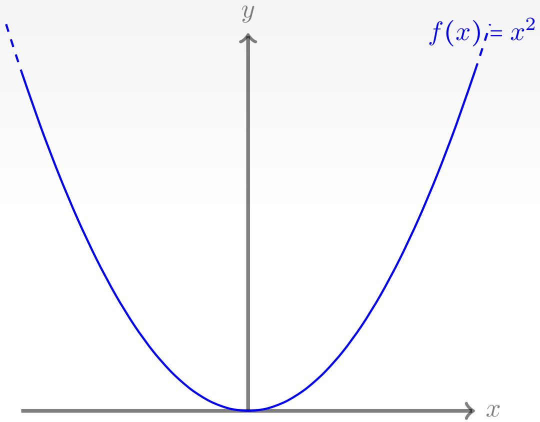
- How do I compute x such that $f(x)$ is minimum?



That's the minimum!!

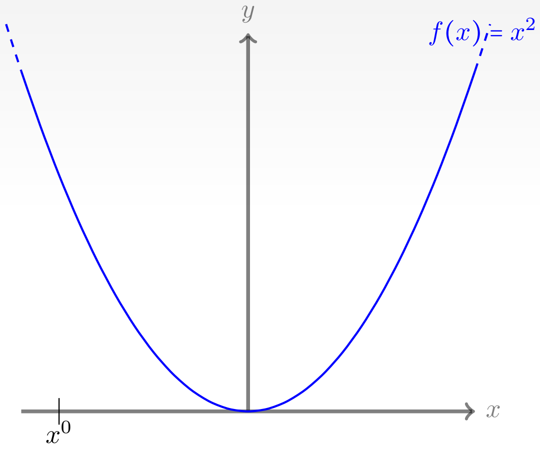
The minimum value of a function

- What if we had started here:



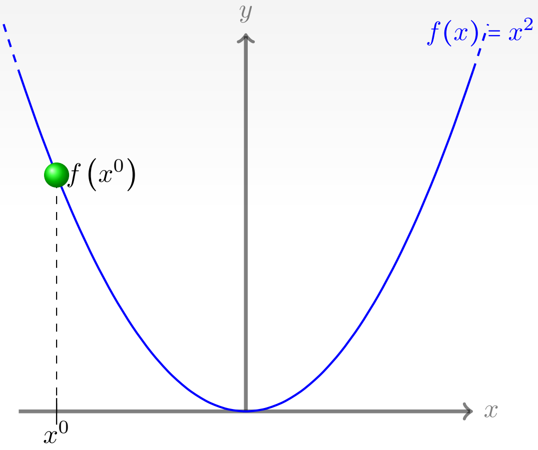
The minimum value of a function

- What if we had started here:



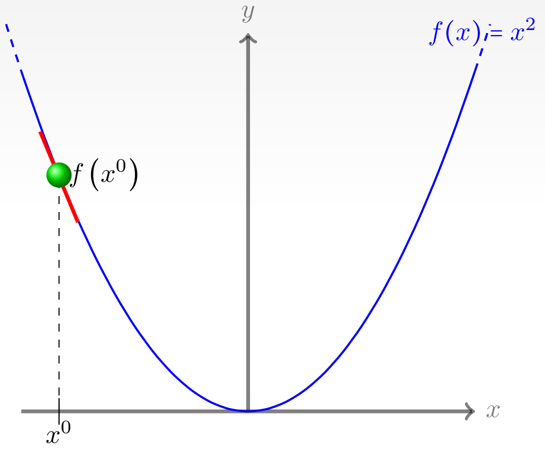
The minimum value of a function

- What if we had started here:



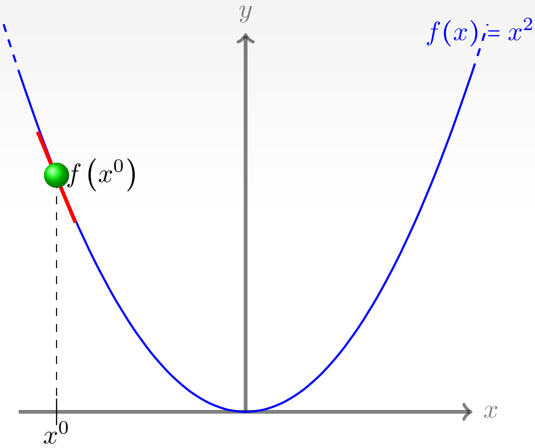
The minimum value of a function

- What if we had started here:



The minimum value of a function

- What if we had started here:

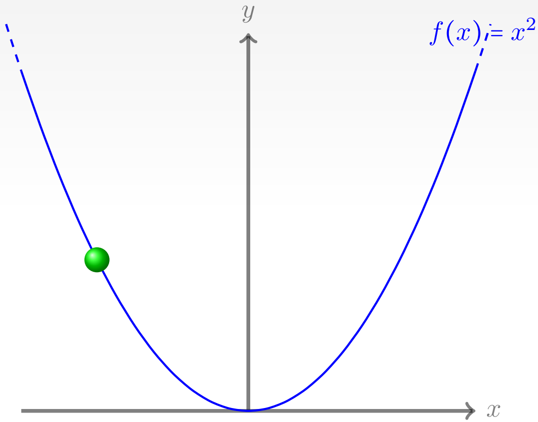


Negative slope

$x^1 \longrightarrow$

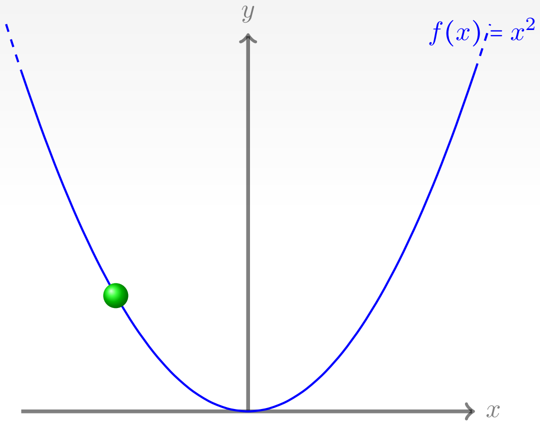
The minimum value of a function

- What if we had started here:



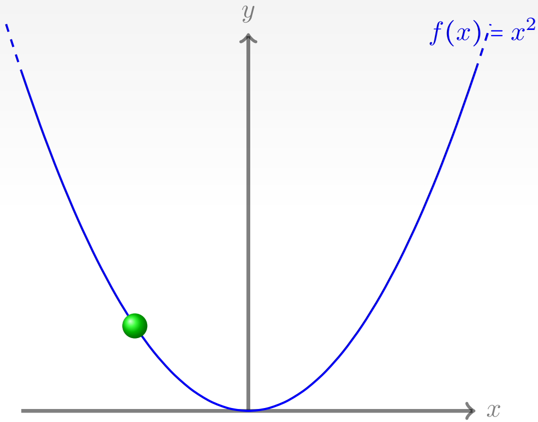
The minimum value of a function

- What if we had started here:



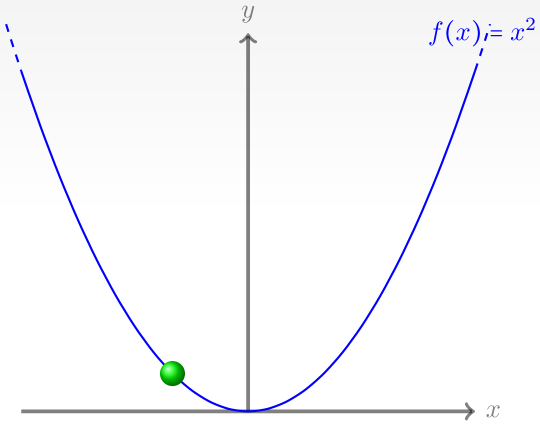
The minimum value of a function

- What if we had started here:



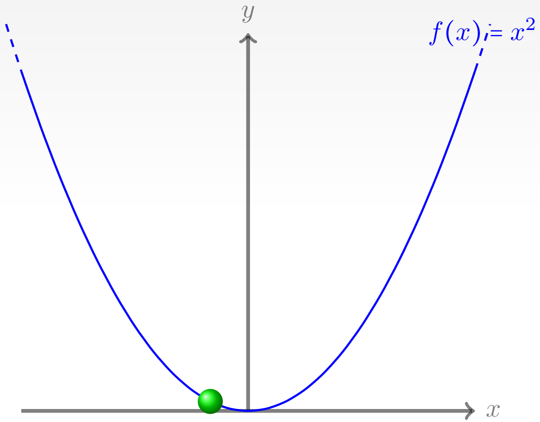
The minimum value of a function

- What if we had started here:



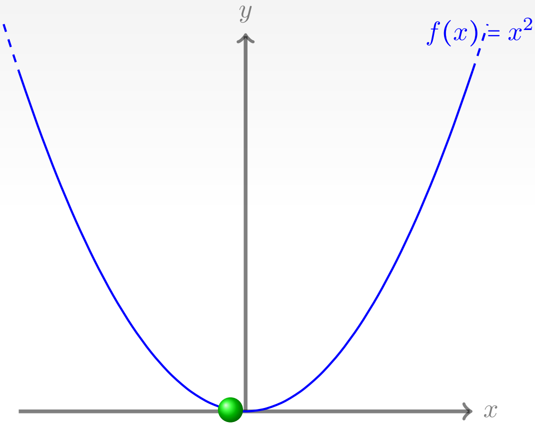
The minimum value of a function

- What if we had started here:



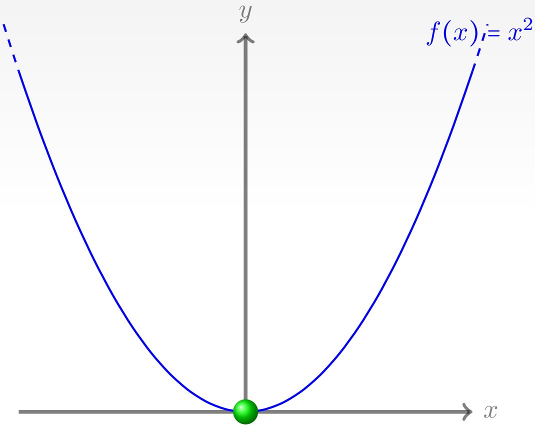
The minimum value of a function

- What if we had started here:



The minimum value of a function

- What if we had started here:



That's the minimum!!

Gradient descent

Algorithm 2 $GradientDescent - v1$

Input: A function $f(x)$
Output: A number x.

Initialize x
$step\_size = 1$
for some repetitions **do**
    if Slope at $f(x)$ is negative **then**
        $x = x + step\_size$
    end if
    if Slope at $f(x)$ is positive **then**
        $x = x - step\_size$
    end if
end for

Gradient descent

Algorithm 3 $GradientDescent - v2$

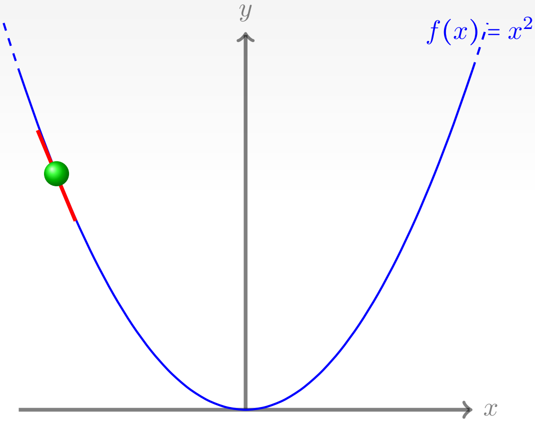
Input: A function $f(x)$
Output: A number x.

```
Initialize x
step_size = 1
for some repetitions do
    if f'(x) < 0 then
        x = x + step_size
    end if
    if f'(x) > 0 then
        x = x - step_size
    end if
end for
```

Python Notebooks!

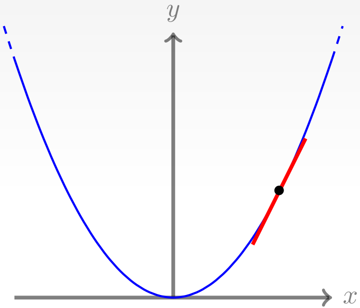
The minimum value of a function

- What if we had started here:



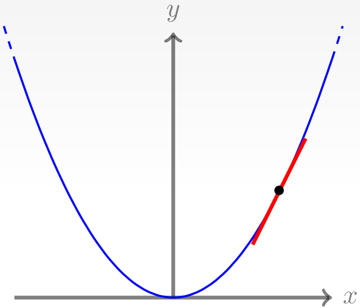
The minimum value of a function

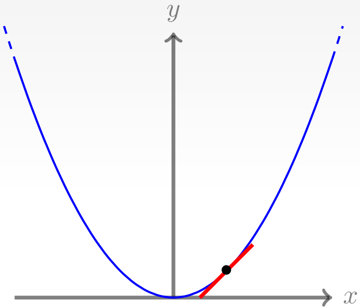
- Take this two points:



The minimum value of a function

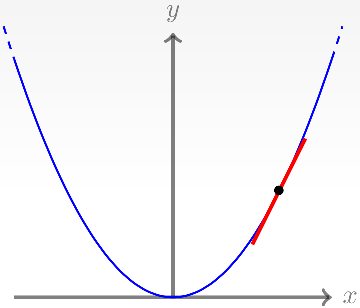
- Take this two points:





The minimum value of a function

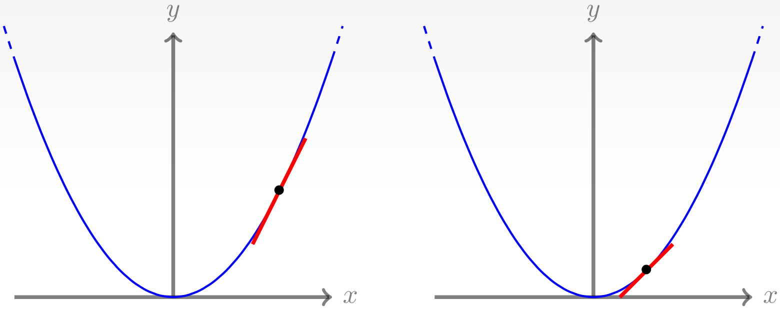
- Take this two points:



- Which one of the following is the closest to the minumum?

The minimum value of a function

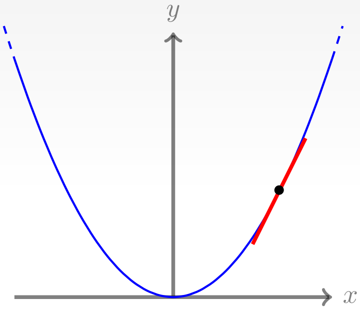
- Take this two points:

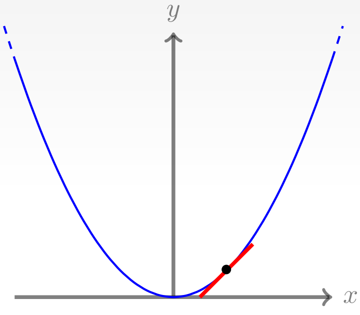


- Which one of the following is the closest to the minumum?
- Which one has the steeper slope?

The minimum value of a function

- Take this two points:





- Which one of the following is the closest to the minumum?
- Which one has the steeper slope?
- What if we could walk diferent step sizes in diferent points?

Gradient descent

Algorithm 4 $GradientDescent - v3$

Input: A function $f(x)$
Output: A number x.

Initialize x
$step\_size = 1$
for some repetitions **do**
    if $f'(x) < 0$ **then**
        $x = x + step\_size \times f'(x)$
    end if
    if $f'(x) > 0$ **then**
        $x = x - step\_size \times f'(x)$
    end if
end for

Gradient descent

Algorithm 5 $GradientDescent - Final$

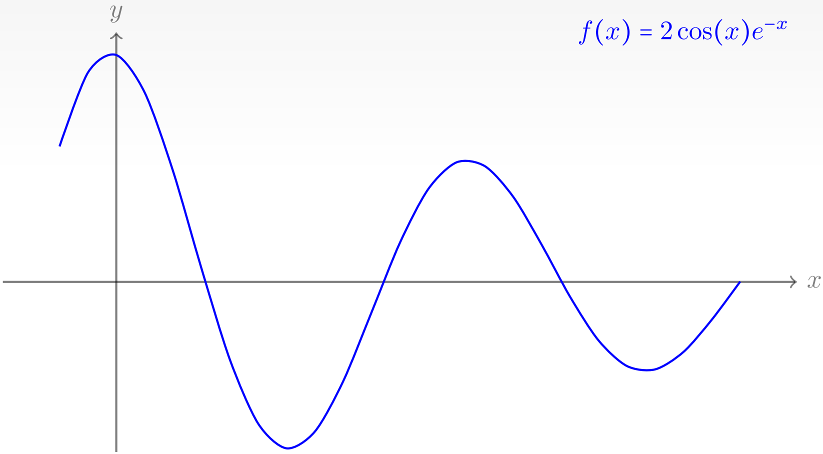
Input: A function $f(x)$

Output: A number x.

1: Initialize x
2: $step\_size = 1$
3: **for** some repetitions **do**
4: $\quad x = x - step\_size \times f'(x)$
5: **end for**

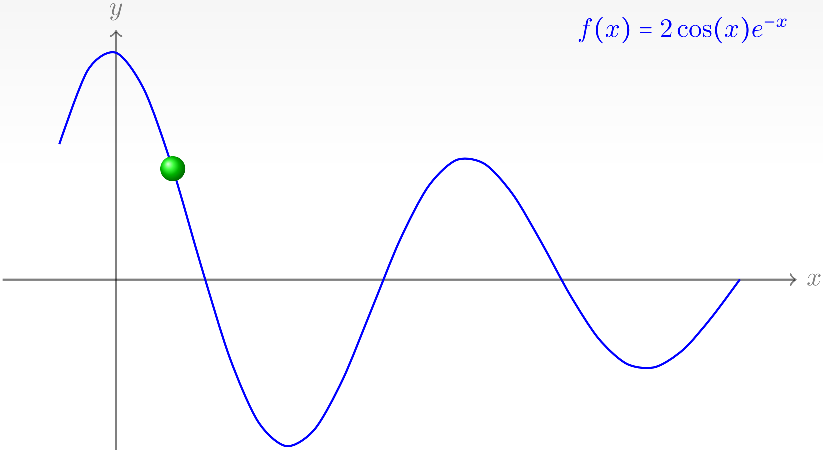
Convexity

- How about this function? Where would the gradient descent algorithm lead you to?



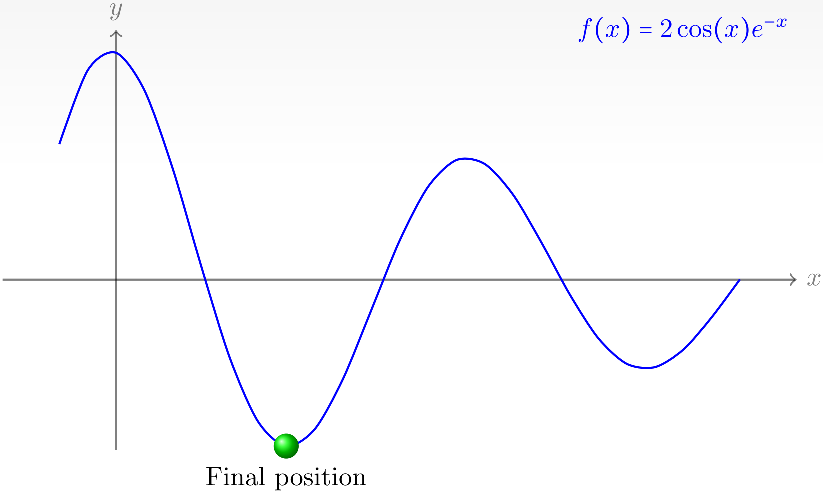
Convexity

- How about this function? Where would the gradient descent algorithm lead you to?



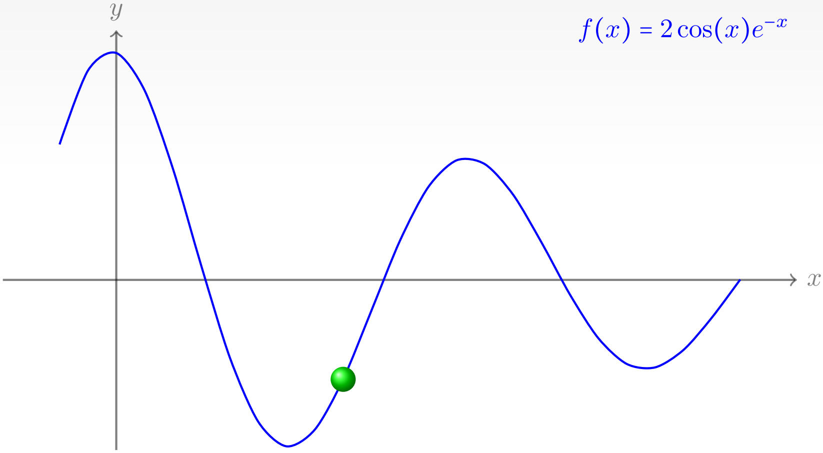
Convexity

- How about this function? Where would the gradient descent algorithm lead you to?



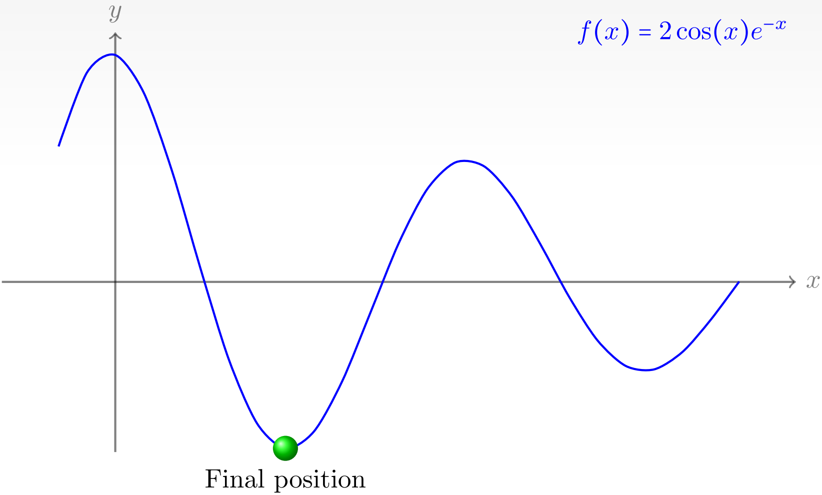
Convexity

- How about this function? Where would the gradient descent algorithm lead you to?



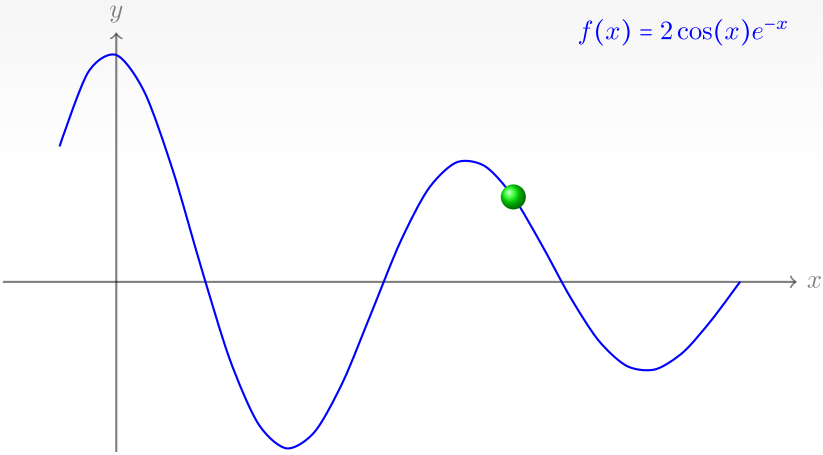
Convexity

- How about this function? Where would the gradient descent algorithm lead you to?



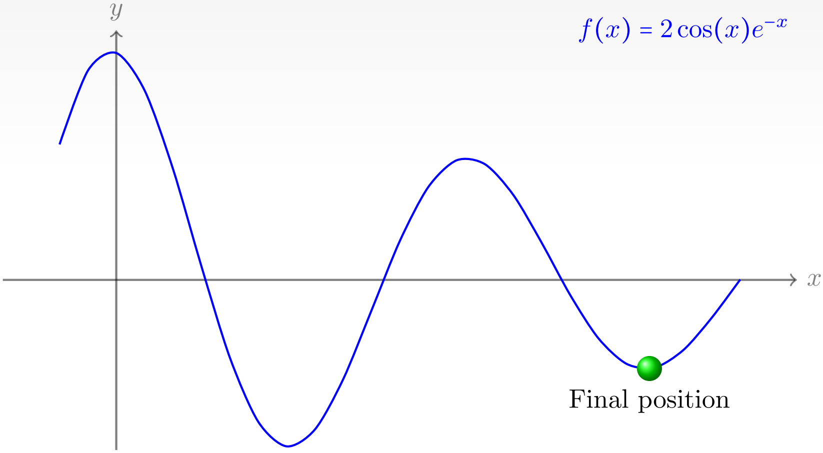
Convexity

- How about this function? Where would the gradient descent algorithm lead you to?



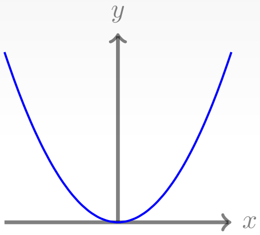
Convexity

- How about this function? Where would the gradient descent algorithm lead you to?



Convexity

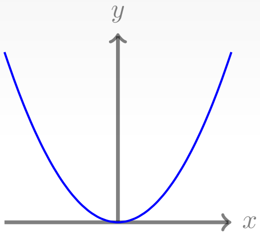
- What is the difference between these functions?



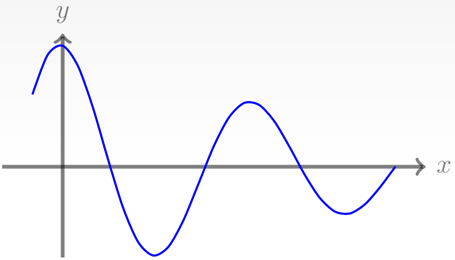
(a) $f(x) = x^2$

Convexity

- What is the difference between these functions?



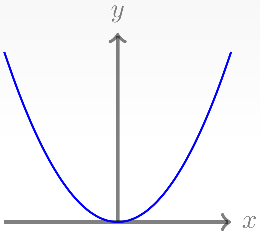
(a) $f(x) = x^2$



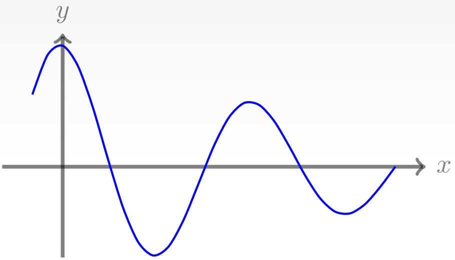
(b) $f(x) = 2\cos(x)e^{-x}$

Convexity

- What is the difference between these functions?



(a) $f(x) = x^2$

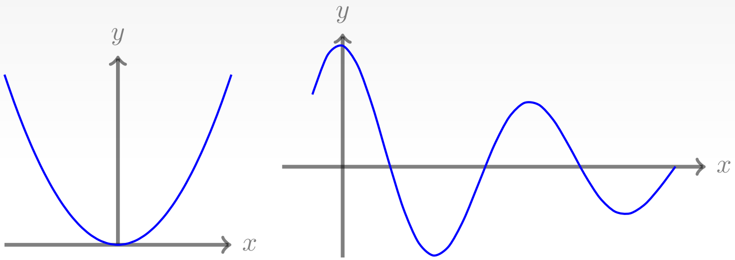


(b) $f(x) = 2\cos(x)e^{-x}$

- One is convex (a) and the other is non-convex (b).

Convexity

- What is the difference between these functions?

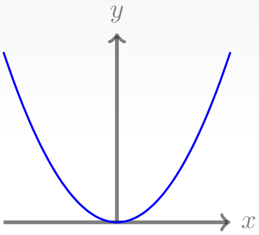


(a) $f(x) = x^2$ *(b)* $f(x) = 2\cos(x)e^{-x}$

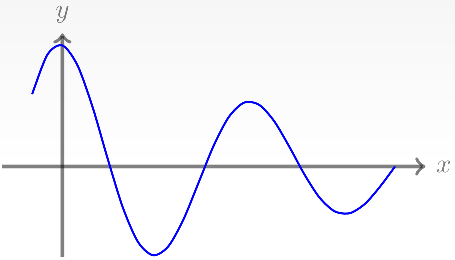
- One is convex (a) and the other is non-convex (b).
- Wait,

Convexity

- What is the difference between these functions?



(a) $f(x) = x^2$

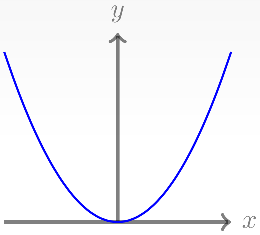


(b) $f(x) = 2\cos(x)e^{-x}$

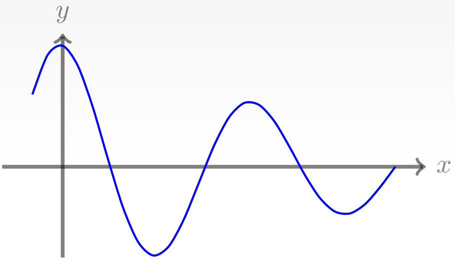
- One is convex (a) and the other is non-convex (b).
- Wait, WHAT THE HECK IS CONVEXITY???

Convexity

- What is the difference between these functions?



(a) $f(x) = x^2$



(b) $f(x) = 2\cos(x)e^{-x}$

- One is convex (a) and the other is non-convex (b).
- Wait, WHAT THE HECK IS CONVEXITY???
- How does this influence the gradient descent algorithm?

Convexity