Quantitative Problems

1. Compute the dot product of the vectors $x = [5, 2, 6, 2, 4, 7]$ and $y = [2, 5, 4, 7, 3, 9]$.

2. Compute the dot product of the vectors $x = [8, 3, 6, 6, 0, 3]$ and $y = [3, 8, 6, 7, 3, 0]$.

3. Given the vector $x = [3, 1]$, find another nonzero vector y such that $x \cdot y = 0$. Then plot the vectors x and y in the xy-plane.

4. Let $x = [0, 1]$ be a vector. Compute the following dot products, the norm of each y, and plot the vectors in the xy-plane:

   (a) $x \cdot y$ with $y = [0, 1]$

   (b) $x \cdot y$ with $y = \left[\frac{1}{2}, \frac{\sqrt{3}}{2}\right]$

   (c) $x \cdot y$ with $y = \left[\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right]$

   (d) $x \cdot y$ with $y = \left[\frac{\sqrt{3}}{2}, \frac{1}{2}\right]$

   (e) $x \cdot y$ with $y = [1, 0]$

   Do you notice a pattern with the magnitude of the dot products and geometry of the vectors plotted in the xy-plane.

5. Suppose a neuron has five inputs that are weighted by $w = [1, 9, 0, 2, 4]$. Draw a model of the neuron showing the weights, inputs, and outputs. Then compute the output of the neuron for the input $x = [3, 4, 1, 0, 8]$. Remember that the activation function f_{act} is the sign(x).

6. Now suppose you have a neural network with three neurons. The first and second neuron both have three inputs that are weighted by $w_1 = [5, 3, 7]$ and $w_2 = [3, 7, 5]$. The output of these neurons is the input to the third neuron and you can choose the weights on the dendrites of the third neuron.

   (a) Draw a diagram of this neuron model.

   (b) Compute the output of this network for the inputs $x_1 = [3, 8, 2]$ and $x_2 = [2, 6, 5]$ which are fed into the first and second neuron, respectively.

Conceptual Problems

1. What does it mean when two vectors are orthogonal?

2. How can you explain with works what the norm of a vector is? What does it mean to normalize a vector?

3. What is matrix for you? What is a column vector? And a row vector?