Artificial Intelligence: *Modeling Human Intelligence with Networks*

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Some news

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- Next Friday presentations.

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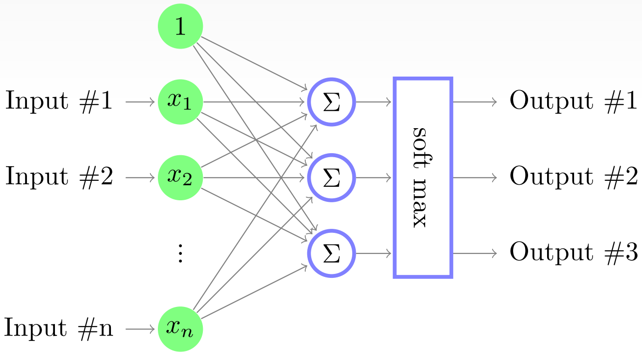

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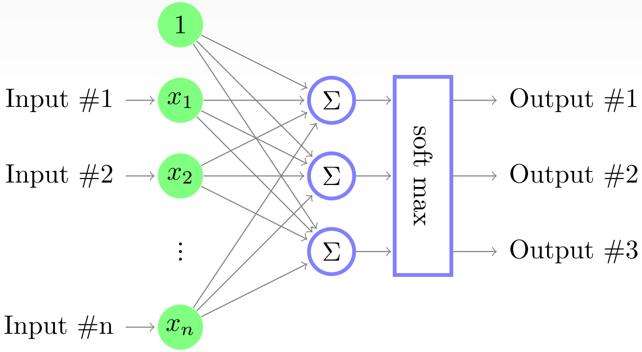

$$\text{Output} = o = [o_1, o_2, o_3] = \text{softmax}(Wx)$$

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- What to do? Cry?

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- Jupyter Notebooks!

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- And finally:

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\frac{\mathrm{d}\mathcal{L}(X,W)}{\mathrm{d}W} &= \frac{\mathrm{d}\left(\sum_{i=1}^{m} h_i(W)\right)}{\mathrm{d}W} \\
&= \sum_{i=1}^{m} \frac{\mathrm{d}h_i(W)}{\mathrm{d}W} \\
&= \sum_{i=1}^{m} (y^i - \mathrm{softmax}(Wx^i)) \cdot x^i, \\
&= \sum_{i=1}^{m} (o^i - y^i) \cdot x^i, \\
&= (O - Y)X,
\end{aligned}$$

  with y_i being a one-hot vector and Y the concatenation of y^i.
- So the Gradient Descent update rule is

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Algorithm 1 $GradientDescent - MultilayerPerceptron$

Input: A matrix of points X, a vector of classes y

Output: A matrix of weights W.

```
Add bias to X
Find Y as the one hot version of y.
Initialize W
η = 1
for a given number of epochs (repetitions) do
    O = softmax(WX) ⟼ Feed Forward step
    dL(X,W)/dW = (O − Y)X ⟼ Back Propagation step
    W = W − η dL(X,W)/dW
end for
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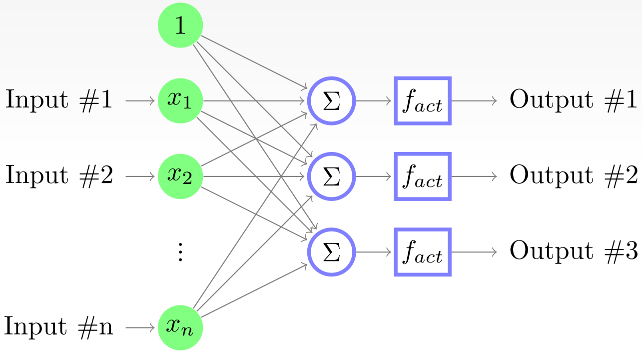
Neural Evolution (*Part II*)

Neural Evolution



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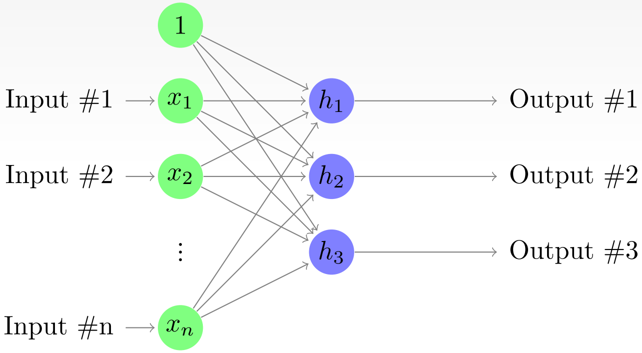

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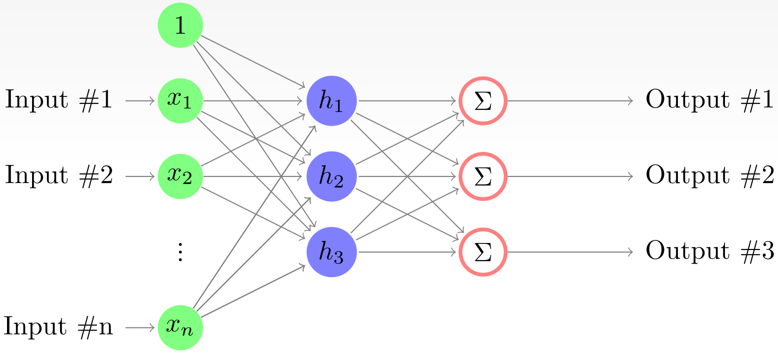

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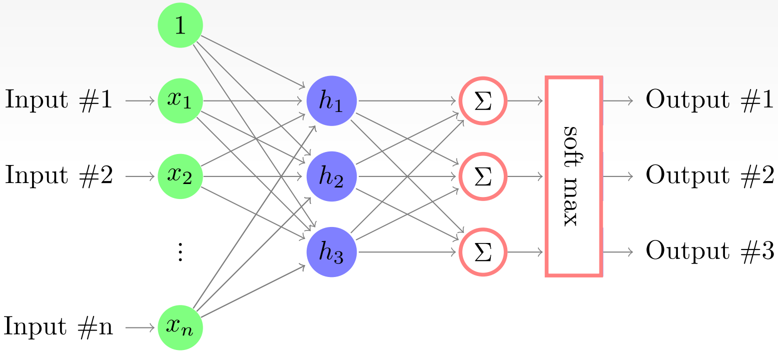

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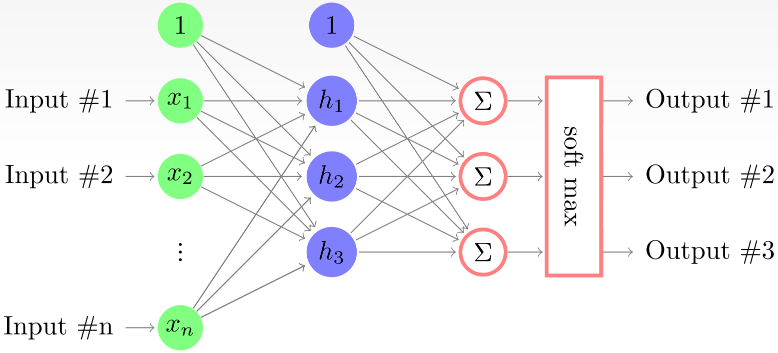

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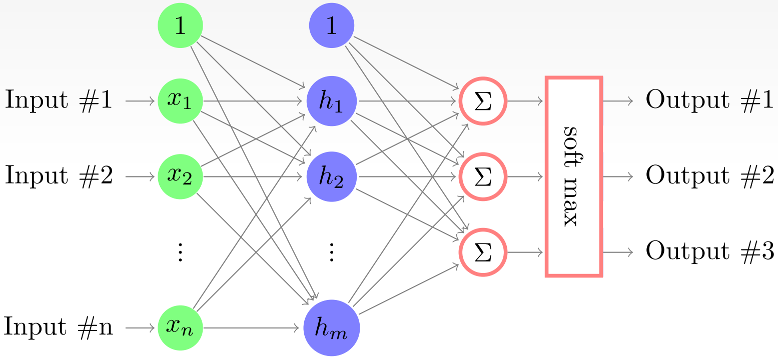

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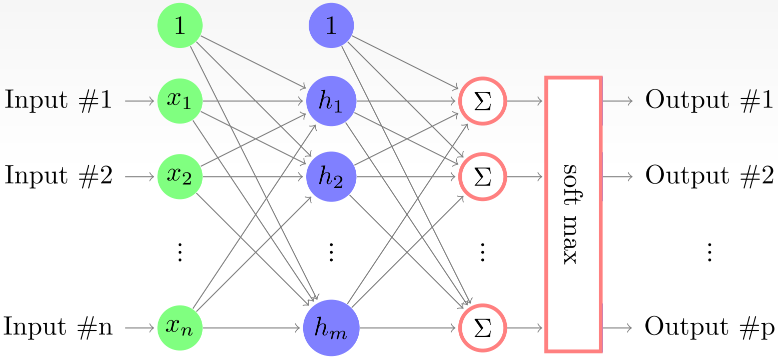
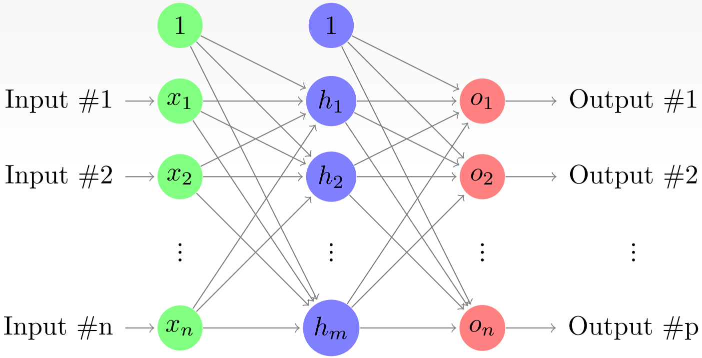
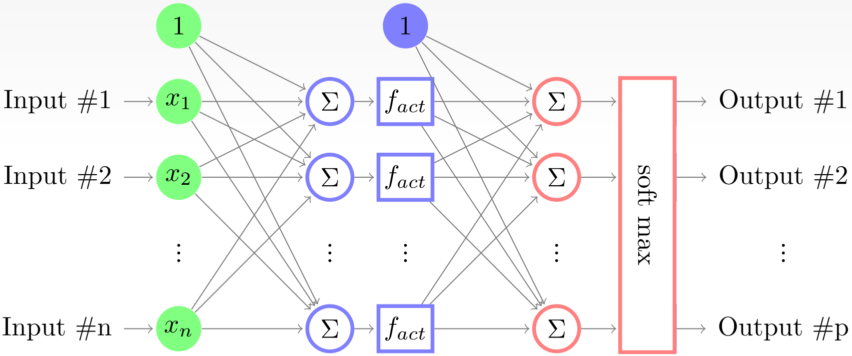


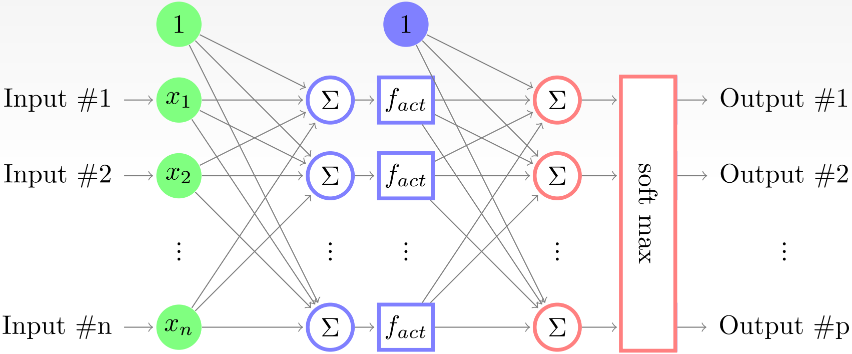
Neural Evolution



Neural Evolution

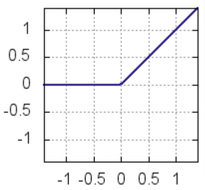


Neural Evolution

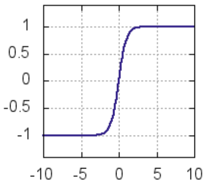


$$o = \text{softmax}(Wh) = \text{softmax}(W(f_{\text{act}}(W_h x)))$$

Activation Functions



(a) ReLU: $f(x) = \max(0, x)$



(b) tanh: $f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$