Artificial Intelligence: *Modeling Human Intelligence with Networks*

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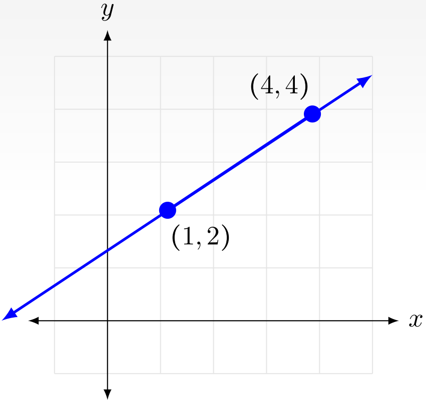
- But the world is much MESSIER!
- In order to solve real problems, we'll need first to unleash the power of DERIVATIVES!!

The slope of a line

What are derivatives?

- How do I compute the slope m of the following line?



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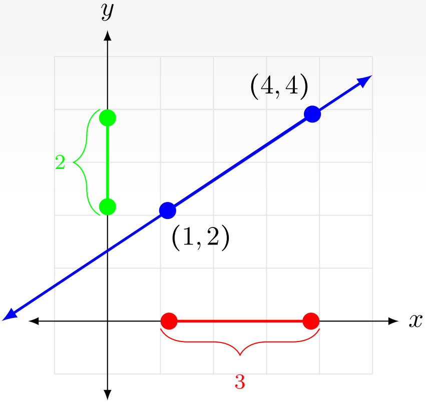


$$m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{2}{3} \approx 0.66$$

Let’s try it on the jupyter notebook! Do Section 6.1

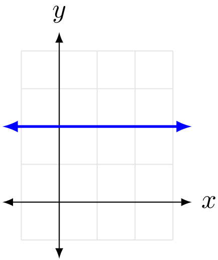
Derivatives

What are derivatives?

- The derivative of a function tells us the "slope" of that function a each point.
- Imagine you have a function $f(x)$ (ex.: $f(x) = x^2$).
- The derivative of $f(x)$ is also a function and is as:

$$\frac{\mathrm{d}f(x)}{\mathrm{d}x} \quad \text{or} \quad f'(x)$$

- Some examples:



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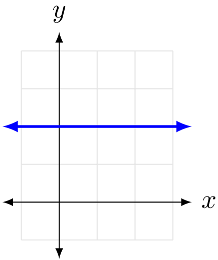

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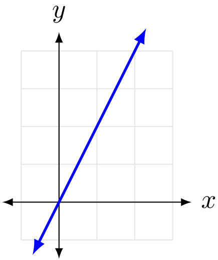

(b) $f(x) = 2x,\ f'(x) = 2$

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  - $h(x) = (x^3)^2 = \left(2 \times (x^3)\right) 3(x^2) = 6x^5$ (expected, since $(x^3)^2 = x^6$).

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- We then have the following rules:
  - **Addition:**

$$h(x) = f(x) + g(x) \Rightarrow h'(x) = f'(x) + g'(x)$$

Ex.: $h(x) = 2x + 4 \Rightarrow (f(x) = 2x, g(x) = 4) \Rightarrow h'(x) = f'(x) + g'(x) = 2 + 0 = 2.$

  - **Multiplication:**

$$h(x) = f(x)g(x) \Rightarrow h'(x) = f'(x)g(x) + f(x)g'(x)$$

Ex.: $h(x) = xe^x \Rightarrow (f(x) = x, g(x) = e^x) \Rightarrow h'(x) = f'(x)g(x) + f(x)g'(x) = e^x + xe^x = (x+1)e^x.$

  - **Division:**

$$h(x) = \frac{f(x)}{g(x)} \Rightarrow h'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$

Now, check Homework 3 on GitHub! Then do Sections 6.2 and 6.3 on the jupyter notebook.