Quantitative Problems

1. Compute the dot product of the vectors $x = [5, 2, 6, 2, 4, 7]$ and $y = [2, 5, 4, 7, 3, 9]$.

2. Compute the dot product of the vectors $x = [8, 3, 6, 6, 0, 3]$ and $y = [3, 8, 6, 7, 3, 0]$.

3. Given the vector $x = [3, 1]$, find another nonzero vector y such that $x \cdot y = 0$. Then plot the vectors x and y in the xy-plane.

4. Let $x = [0, 1]$ be a vector. Compute the following dot products, the norm of each y, and plot the vectors in the xy-plane:

   (a) $x \cdot y$ with $y = [0, 1]$

   (b) $x \cdot y$ with $y = \left[\frac{1}{2}, \frac{\sqrt{3}}{2}\right]$

   (c) $x \cdot y$ with $y = \left[\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right]$

   (d) $x \cdot y$ with $y = \left[\frac{\sqrt{3}}{2}, \frac{1}{2}\right]$

   (e) $x \cdot y$ with $y = [1, 0]$

   Do you notice a pattern with the magnitude of the dot products and geometry of the vectors plotted in the xy-plane.

5. Suppose a neuron has five inputs that are weighted by $w = [1, 9, 0, 2, 4]$. Draw a model of the neuron showing the weights, inputs, and outputs. Then compute the output of the neuron for the input $x = [3, 4, 1, 0, 8]$.

6. Now suppose you have a neural network with three neurons. The first and second neuron both have three inputs that are weighted by $w_1 = [5, 3, 7]$ and $w_2 = [3, 7, 5]$. The output of these neurons is the input to the third neuron and you can choose the weights on the dendrites of the third neuron.

   (a) Draw a diagram of this neuron model.

   (b) Compute the output of this network for the inputs $x_1 = [3, 8, 2]$ and $x_2 = [2, 6, 5]$ which are fed into the first and second neuron, respectively.

7. (Challenge) Suppose you have network such that the output of the i-th neuron is the input of the $i + 1$-th neuron and the weight on each connection is $w_i = 3/2$ for all i. Suppose that there is a single dendrite on the first neuron and the input to this system of neurons is $x = 1$

   (a) What is the output of this network when there are n neurons?

   (b) What is the output of this network as the number of neurons goes to infinity?

   (c) How could you assign the weights on each connection so that the output is finite?

8. (Challenge) Now suppose you have a network with one neuron with n inputs such that weight on each connection is $3/2$. What is the output of this network

(a) What is the output of this network?

(b) What would the output be if the number of inputs goes to infinity?

(c) How could you assign the weights on each connection so that the output is finite?

Conceptual Problems

1. In what ways is the connectome model of the human brain an oversimplification of how our brain works?

2. Can you think of another technology that you use on a regular basis that may rely on a machine learning classification algorithm, which was not mentioned in class. For example, an email browser classifies emails as spam or non-spam.

3. What kinds of applications of machine learning do you think sound interesting and why? (i.e. image understanding, speech understanding, building intelligent robots, medical tools, game playing machines)