# User's Guide for the Bayes GMM estimator

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This is a guide for the C++ program that implements the Bayesian GMM estimator of Gallant et al. (2017). The program is **heavily** built on Gallant's MLE package.

## **Directory Structure**

data: this folder contains the data (in file data.dat - variables in columns separated by white spaces) and the initial\_particle.dat file containing an intial draw of particles for the conditional particle filter. (TODO:)There is also a .cpp file that by using components of the usermodel class simulates these two files.

model: in addition to the subfolders detailed below, this folder contains the makefile that generates the main executable (bayes\_gmm) and the InputParamFile (named as \*\*\*.param.00\*) detailing the specifics of the estimation

initialize: reads the InputParamFile and defines the specification class

- source files: main.cpp, initialize.cpp
- header files: initialize.h

estimator: elements of the mcmc sampler/optimizer, generates an mcmc class

- source files: asymptotics.cpp, mcmc\_class.cpp, proposal.cpp
- header files: estimator\_base.h, estimator.h

libscl: slightly altered version of Gallant's statistical library (including the gmm class)

- usermodel: defines the usermodel class
  - source files: usermodel.cpp, moments.cpp, model.cpp, default\_params.cpp
  - header files: usermodel.h, moments.h, model.h, default\_params.h

result\_files: a plethora of .dat files generated by the estimator for diagnoses and further analyses plots: this folder contains the jupyter notebooks that read and visualize the result\_files.

## 1 Usage

Download bayes\_gmm\_test.tar from XXX. On a Unix machine use tar -xf bayes\_gmm\_test.tar to expand the tar archive into a directory that will be named bayes\_gmm\_test.

## 1.1 Sequential version

Change directory to bayes\_gmm\_test/model/ and type make and the bayes\_gmm executable will be built and ready to run.

This folder also contains a file called control.dat. This file contains the names of the InputParam Files (see later) and the prefix for the generated output files (denoted by \*\*\* later). Here is an example of a one line control.dat file:

```
test.param.000 test
```

The input paramfile is named test.param.000 (must be in the same folder) and all output files such as detail.dat, pi.000.dat, etc. are named test.detail.dat, test.pi.000.dat, etc.

As a result a lot of output files will be generated in the result\_files folder. The key ones are:

- detail.dat: Voluminous detailed output from the run.
- summary.dat: This file summarizes the output giving mean, mode, and standard errors
- theta.000.dat: contains the MCMC chain for  $\theta$
- stats.000.dat: contains the corresponding values of stats;
- pi.000.dat: Let  $\ell(\theta) = \exp(-ns_n(\theta))$ , and let  $p(\theta)$  denote the prior. This file contains three items corresponding to the MCMC chain for  $\theta$ : (1)  $\log \ell(\theta) + \log p(\theta)$ , (2)  $\log \ell(\theta)$ , (3)  $\log p(\theta)$ .
- reject.000.dat: this contains a matrix whose first column contains the rejection rate for each parameter followed by the overall rejection rate. ADD: other columns
- parmfile.fit: A copy of the InputParamFile with the parameter start values replaced by the mode and scaling variables recomputed so that proposal\_scale\_factor is 1.0. All else is the same as the InputParamFile.
- theta\_mode, theta\_mode, V\_hat\_hess, etc.: Statistics from the run in the form expected for reading with member vecread of class scl::realmat in library libscl.

## 1.2 Parallel version with OpenMPI

## 2 The InputParam File

The InputParam File (typically named as \*\*\*.param.000 contains several blocks of control information. The structure is the same as the paramfiles for Gallant's EMM and MLE packages. The followings are mostly extracts from his MLE guide.

#### 2.1 PARAMFILE HISTORY

This part is optional. It is written automatically by the program to the output paramfile called \*\*\*.paramfile.fit (located in the result\_files folder) at the end of every run. It consists of seven lines that begin with # that should be left alone. After these seven lines, the user can add additional lines<sup>1</sup> (e.g. describing the model) that begin with a # and these will get copied from the input parmfile to the output parmfile.

#### Example:

#### 2.2 ESTIMATION DESCRIPTION

The 13 lines specify values for parameters of the estimator (order matters!). Each line starts with 12 characters including the parameter value, then the following pieces of information separated by comma: (1) description, (2) variable name, (3) variable type.

proposaltype: Standard is the group move proposal which defaults to a single move proposal when the optional InputParamFile block PROPOSAL GROUPING is missing. When the PROPOSAL GROUPING block is missing, the proposaltype=0 proposal randomly selects an element of  $\theta$  to move and the draws from a normal; i.e. a move-one-at-a-time random walk. When PROPOSAL GROUPING block is present, the proposal randomly selects one of the groups defined therein to move and draws from a user specified multivariate normal.

<sup>&</sup>lt;sup>1</sup>Specifically, the three paramfiles (\*\*\*.paramfile.fit, \*\*\*.paramfile.alt, \*\*\*.paramfile.end) are being written by the function initialize::specification\_class::write\_params() which is defined in initialize.cpp.

ask\_print: If the value is 1, then voluminous debugging information is written to file \*\*\*.detail.dat in the result\_files.dat subdirectory. Setting it of 0 suppresses printing.

iseed: Seed for the MCMC chain.

num\_mcmc\_draws: The MCMC chain is broken up into pieces and written to files theta.000.dat, theta.001.dat, etc. This variable determines the number of draws per file.

num\_mcmc\_files:: Determines how many files in addition to theta.000.dat are generated. The total length of the MCMC chain is R= num\_mcmc\_draws\*(num\_mcmc\_files + 1). Many other files are produced to describe the chain such as reject.000.dat, pi.000.dat, stats.000.dat as well as summary files, files containing variance matrices, etc.

proposal\_scale\_factor: Rescales the proposal standard deviations that are set in the PROPOSAL SCALING block without changing relative values.

temperature: For Bayesian inference it is essential that temperature = 1! Otherwise, this variable controls the peakedness of the objective function (e.g. likelihood). Putting temperature = 2 is like doubling the number of observations from which the likelihood was computed, which makes the objective function more peaked. Putting temperature = 0.5 would be like halving them.

no\_sandwich: Computing sandwich standard errors is costly and often unnecessary, setting this variable to 1 will stop them from being computed. Even for an estimator that does require the computation of sandwich standard errors, one should set kilse = 1 during the early hill climbing phase of the chain. When the objective function has reached its plateau and the stationary portion of the chain has been reached, no\_sandwich can be set to 0.

lag\_hac: The number of lags to be used to compute the HAC information matrix in the middle of the sandwich variance estimator. Set lhac=0 if the scores are uncorrelated, in which case the estimator is heteroskedastic consistent.

thin: The program writes the MCMC chains to files of length num\_mcmc\_draws as explained above. If thin=1, every element of the chain is written. If thin=2, every other element is written and the length of an output files becomes num\_mcmc\_draws/2. Similarly for higher values of thin. Thin greater than one reduces memory requirements because values not written are not stored anywhere. One consequence of this is that statistics such as the Hessian are computed only from the elements of the MCMC chain that are written, not from all that are generated. The exceptions are that the mode and the rejection count are computed from all elements that were generated.

draw\_from\_prior: When the prior is proper, it is useful to be able to draw from the prior for at least two purposes. The first is to be able to compare the prior and posterior distribution of estimates of parameters and functionals. The other is as an intermediate step in computing posterior probabilities for model selection as discussed in, e.g., Gamerman and Lopes (2006, Section 7.2.1). The essential information for model selection is in the output files named pi.000.dat, pi.001.dat, etc. (to which a user

defined prefix is prepended). Their structure is discussed in more detail later but, briefly, the information one needs are the likelihood draws, in the second row, and the prior draws in the third row. When draw\_from\_prior=0 these will be draws made by comparing the posterior at the accept/reject step of the MCMC chain, as will be true of all other output files such as theta.000.dat, theta.001.dat, etc. When draw\_from\_prior=1 these will be draws made by comparing the prior at the accept/reject step of the MCMC chain, as will be true of all other output files. Setting draw\_from\_prior=1 when the prior is not proper is a ghastly error.

#### Example:

### ESTIMATION DESCRIPTION (required)

- test Project name, project\_name, char\*
- 1.0 bayes\_gmm version, version, float
  - O Proposal type, O group\_move, 1 cond\_move, 2 usr, proposaltype, int
  - 1 Write detailed output if ask\_print=1, ask\_print, int
- 1741133992 Seed for MCMC draws, iseed, int
  - 10000 Number of MCMC draws per output file, num\_mcmc\_draws, int
    - 9 Number of MCMC output files beyond the first, num\_mcmc\_files, int
  - 10.0 Rescale prop scale block by this, proposal\_scale\_factor, float
  - 1.0 Rescale posterior by this val, temperature, float
    - 1 Sandwich variance not computed if no\_sandwich=1, no\_sandwich, int
    - O Number of lags in HAC middle of sandwich variance, lag\_hac, int
    - 5 The thinning parameter used to write MCMC draws, thin, int
    - O Draw from prior if draw\_from\_prior=1, draw\_from\_prior, int

### 2.3 DATA DESCRIPTION

In the block labeled DATA DESCRIPTION are parameters that specify the dimension of the data, the number of observations, and govern reading of the data. The data are presumed to be stored in a file containing rows that have values separated by blanks containing the data for each observation  $y_t$  and perhaps additional values such as dates or the index t. There should be one line for each t = 1, ..., n. The presence of the line terminating character is important because the C + + function getline does the reading.

M: The dimension of the vector  $y_t$ .

sample\_size: The number of observations to be read. The value can be smaller than the number of observations in the file in which case those at the end will not be read.

datafilename: The name of the file from which the data are to be read. The file must be located in the data directory.

var\_cols: Lastly, one has fields. One must use care here because errors can cause the program to crash with misleading diagnostic messages, if any at all. As just mentioned, the presumptions is that the data are arranged in a table with time t as the row index and the elements of  $y_t$  in the columns. The blank separated numbers here specify the variables (columns) of the data in the order in which they are to be assigned to the elements  $y_{1t}, y_{2t}, \ldots, y_{Mt}$  of  $y_t$ . It does not hurt to have too many fields listed because only the first M are read. The disaster is when there are too few (less than M) or one of them is larger than the actual number of columns in the data set. A few of the first and last values of  $y_t$  read in are printed in the file \*\*\*.detail.dat which should be checked to make sure the data were read correctly. var\_cols can be specified as a single digit or as a range. Thus, one can enter either 1 2 3 5 or 1:3 5.

#### Example:

```
DATA DESCRIPTION (required) (model constructor sees realmat data(M,sample_size))

12 Dimension of the data, M, int
200 Number of observations, sample_size, int
data.dat File name, any length, no embedded blanks, datafilename, string
1:12 Read these white space separated var_cols, var_cols, intvec
```

#### 2.4 MODEL DESCRIPTION

The MODEL DESCRIPTION block is straightforward, it gives the dimensions of the parameters of the model.

len\_model\_param: The dimension of  $\theta$ , which is the parameter vector of the model.

len\_model\_func: The dimension of stats, which is the vector of statistics (functionals) of the model that are computed from a simulation of the model. (NOT USED)

Example:

```
MODEL DESCRIPTION (required)

26 Number of model parameters, len_model_param, int

1 Number of model functionals, len_model_func, int
```

### 2.5 MODEL PARAMFILE

The vectors model\_paramfile\_lines and model\_addlines of type vector<string> that are passed to the usermodel constructor (in main.cpp) are defined in the MODEL PARAMFILE block.

model\_paramfile: This is the name of a file containing lines of the user's choosing. This file is read and passed to the usermodel constructor as the std::vector of std::string model\_paramfile\_lines. If there is no such file then code \_\_none\_\_ as the filename.

#begin additional\_lines, #end additional\_lines: Lines between these two markers are read and passed to the usermodel constructor as model\_addlines of type vector<string>. The two marker lines are passed as well so that the first user line is model\_addlines[1] and not model\_addlines[0].

## Example:

- 2 Number of observable risk factors, numb\_obs\_factor, int
- 1 Lags for observable risk factors, lag\_obs\_factor, int
- 8 Number of log returns, numb\_returns, int
- 1 Lags for HAC variance estimator (GMM objfun), lag\_hac\_gmm, int
- 500 Number of particles, N, int
- 100 Simulation size, len\_simul, int
- 50 Draws between particle filter updates, particle\_update, int

#### 2.6 PARAMETER START VALUES

#end additional lines

The block labeled PARAMETER START VALUES specifies the first value for the chain.<sup>2</sup> It must satisfy the support conditions; i.e. usermodel\_class::support must return true, and usermodel\_class::prior must return scl::dev\_val.positive = true for this initial value of  $\theta$ . The numbers to the right, 0 or 1, determine whether that element is held fixed or is active. If 0, then the proposal never moves that element of  $\theta$ . To the right of this 0 or 1 the user may add text such as the name of the parameter.

#### Example:

## PARAMETER START VALUES (required)

1.90970554334998099e-01	1	1	A11
3.24793056338963071e+00	1	2	A21
3.60281749990822575e-02	1	3	A12
8.48608238478818777e-02	1	4	A22

<sup>&</sup>lt;sup>2</sup>New files paramfile.fit, paramfile.end and paramfile.alt are written as the MCMC chain progress with the current putative mode of the objective function replacing the values in PARAMETER START VALUES for .fit and .alt and the last value of  $\theta$  in the chain in the case of .end. The paramfile.end is used to recommence where one left off; paramfile.fit is used to recommence starting at the mode, which is what one usually wants to do; and paramfile.alt is used when switching to the conditional move proposal (proposal\_type=1). If the number of parameters exceeds 20, then paramfile.alt will not be written. Once the mode has been found, it will not change.

#### 2.7 PROPOSAL SCALING

These are the standard deviations of the proposal. Ideally, they should be roughly proportional to the standard errors of the estimate of  $\theta$  if such is known. Altering their values is a way to affect the rejection probability.

#### Example:

#### PROPOSAL SCALING (required)

3.1250000000000017e-03	1	A11
3.1250000000000017e-03	2	A21
3.1250000000000017e-03	3	A12
3.1250000000000017e-03	4	A22

#### 2.8 PROPOSAL GROUPING

How to specify group moves in the InputParam File is discussed in Subsection 6.3 of Gallant's EMM User's Guide. Briefly, in each matrix, the first element of the first row gives the relative probability with which this group is selected. In the remaining columns of the first row are the indexes of the parameters in that group. The first column is the same as the first row. The submatrix bounded by the first row and column is a correlation matrix. The multivariate normal to move the group is determined by this correlation matrix and the values in the PROPOSAL SCALING block. Within the PROPOSAL GROUPING block, the index of every parameter must be accounted for. Those parameters that are not moved (i.e. have a 0 to their right in the PARAMETER START VALUES block) are collected into a group that is assigned zero probability of being selected. If the PROPOSAL GROUPING block is not present, then one is synthesized. One can view an example (the synthesized version) in the file \*\*\*.detail.dat, presuming ask\_print=1 in the ESTIMATION DESCRIPTION block.

## The model/initialize/main.cpp file

 The main is called with command arguments with possibly multiple rows, e.g. the file control.dat might contain

```
svsim.param.000 svsim_0
svsim.param.001 svsim_1
```

and the program reads rows iteratively, at each round argp[1] becomes paramfile, prefix

- 2. Create specification class (called specification) and take the rows of paramfile to pass them into specification.set\_params(). This initializes the private members of specification\_class
  - the (estblock, datablock, modelblock) triple using the first three blocks of the Input-ParamFile (see above)
  - set the value of theta (and theta\_fixed) using PARAMETER START VALUES block
  - set proposal\_scale as a product of estblock.proposal\_scale\_factor and the PRO-POSAL SCALING block
  - set proposal\_groups from the PROPOSAL GROUP block if provided
- 3. Read data from the data/ folder using datablock.read\_data()
  - data file must contain columns for observables
  - columns must be separated by whitespace
- 4. Create usermodel class (of usermodel\_type) named as usermodel with the arguments
  - just imported data
  - model\_blk.len\_model\_param from the InputParamFile
  - model\_blk.len\_model\_func from the InputParamFile
  - specification.get\_model\_addlines() from the InputParamFile

What does this do? (see model/usermodel/usermodel.cpp)

- (a) read initial\_particle.dat to set the value of saved\_particle member of usermodel
- (b) initialize moment\_cond=moments()
  - default: data(0), particle(0), n(0), lag\_gmm(0), theta(4, 1)
  - theta gets initialized from default\_params.h default values
- (c) initialize gmm\_objfun=gmm(moment\_cond, lag\_gmm, &data.ncol(), lag\_hac\_gmm) with regularized W (ridge= 1.0e-3)

- default: mf(moment\_cond), mfl(lag\_gmm), data, sample\_size(data.ncol()),
   Lhac(lag\_hac\_gmm), correct\_for\_mean(true), regularize\_W(false), ridge(0.0),
   warning\_messages(true)
- reset lag\_gmm, data, n for moment\_cond from these (!!!)
- run gmm\_objfun->set\_regularize\_W(true, 1.0e-3)
- (d) initialize model=model(moment\_cond, gmm\_objfun)
  - default: parameters (default\_params.h), moment\_cond and gmm\_objfun
  - run gmm\_objfun->set\_moment\_function(moment\_cond), i.e.
    - set\_lag\_gmm(), set\_data() and set\_sample\_size() for moment\_cond from provided objects (!!!) (this is insurance, should be redundant)
- (e) lag\_gmm, lag\_hac\_gmm, N, len\_simul, particle\_update get read from specification.get\_model\_addlines()
- (f) instantiate draws, smooth and filter of usermodel with length N
- 5. Create proposal class (of proposal\_base type) named proposal using
  - estblock.proposaltype (group\_move or conditional\_move) and
  - specification.get\_proposal\_groups() defined above
- 6. Create the mcmc class using the proposal and usermodel classes
  - default: proposal(proposal), usermodel(usermodel), simulation\_size(1), thin(1), draw\_from\_posterior(true), temperature(1.0), posterior\_mode(), posterior\_maxval(-REAL\_MAX)
  - reset elements from the ESTIMATION DESCRIPTION block:
    - simulation\_size becomes len\_mcmc\_draws
    - reset thin, draw\_from\_posterior, temperature from est\_blk
- 7. Create asymptotics class (of asymptotics\_base type) with args data, usermodel, mcmc
  - default: data(data), T(data.get\_cols()), mcmc(mcmc),
    len\_theta(usermodel.get\_len\_theta()), theta\_sum(len\_theta, 1, 0.0),
    mean\_old(len\_theta, 1, 0.0), theta\_sse(len\_theta, len\_theta, 0.0),
    mean(len\_theta, 1, 0.0), posterior\_mode(len\_theta, 1, 0.0),
    cov(len\_theta, len\_theta, 0.0), foc(len\_theta, 1, 0.0), cum\_sample\_size(0),
    posterior\_maxval(-REAL\_MAX)
- 8. Define realmat theta = specification.get\_theta(); and INT\_32BIT seed = est\_blk.seed;
- 9. Loop num\_mcmc\_files many times over the sampler of size num\_mcmc\_draws starting at theta

```
[A] mcmc.draw(seed, theta_start, theta_sim, stats_sim, pi_sim);
      • this function communicates with PF in usermodel.likelihood()
         - upon new proposal mcmc class tells usermodel the new and old theta's
         - it draws particle_update many proposals before a new set of particles
         - it returns the rejection probabilities as a table: rejection/total
      • realmat theta_old = theta_start;
        usermodel.set_theta(theta_start);
        usermodel.set_theta_old(theta_old);
        usermodel.get_stats(stats_old);
        den_val likehood_old = usermodel.likelihood();
        den_val prior_old = usermodel.prior(theta_old, stats_old);
        den_val pi_old = prior_old;
        if (draw_from_posterior) pi_old += likehood_old;
        posterior_mode = theta_start;
        posterior_maxval = pi_old.log_den;
        if (pi_old.positive) pi_old.log_den *= temperature;
      • Define *_new variables from *_old ones
      • num_mcmc_draws many times
         - proposal.draw(jseed, theta_old, theta_new);
           usermodel.set_theta(theta_new);
           usermodel.set_theta_old(theta_old);
         - if (usermodel.support(theta_new) && usermodel.get_stats(stats_new))
            * call usermodel.likelihood()
            * update likelihood_new, prior_new, pi_new
            * if pi_new > high, reset posterior_mode and posterior_maxval
            * calculate alpha from pi_old and pi_new and rejection/acceptance step
      • last simulated value becomes theta_start, so we can use that again in the num_mcmc_files
        many iterations
[B] asymptotics.set_asymptotics(theta_sim);
      • taking theta_sum, this updates asymptotics' mean, posterior_mode, cov, cum_sample_size
        and posterior_maxval
[C] asymptotics.get_asymptotics(theta_hat, V_hat, T);
      - assigns mean, cov/T, T to arguments
[C] asymptotics.get_asymptotics(theta_mean, theta_mode, posterior_high, I, invJ,
```

foc\_hat, reps);

```
- assigns mean, posterior_mode, posterior_maxval, I = null; invJ = cov; foc_hat = null; reps = 0; to arguments
```

- [D] usermodel.set\_theta(theta\_mode): reset usermodel's theta with the new mode
- [E] specification.write\_params(paramfile, prefix, seed, theta, theta\_mode, invJ/T)
   usermodel.set\_theta(theta\_mode);
   filename = "../result\_files/" + prefix + ".usrvar";
   usermodel.write\_usrvar(filename.c\_str());

## [F] call output function

- theta\_mean -> prefix + ".theta\_mean.dat";
- theta\_mode -> prefix + ".theta\_mode.dat";
- V\_hat\_hess -> prefix + ".V\_hat\_hess.dat";
- theta\_mean, theta\_mode, V\_hat\_hess -> prefix + ".summary.dat";
- theta\_sim -> prefix + ".theta." + number + ".dat";
- stats\_sim -> prefix + ".stats." + number + ".dat";
- pi\_sim -> prefix + ".pi." + number + ".dat";
- reject -> prefix + ".reject." + number + ".dat";

### The usermodel\_class

The class is declared in usermodel.h and defined in usermodel.cpp. Most importantly, it has three key linked private members: these are pointers to (1) a moments class, (2) a scl::gmm class, and (3) a model class<sup>3</sup>

#### model class:

- its likelihood member calculates the value of the GMM representation of the measurement
  density evaluated at the data y<sub>t</sub> for given t ≤ sample\_size and a given set of particles. For
  this step, it first set\_particle and set\_sample\_size(t) of the moments class, then calls
  scl::gmm for the current theta value.
- it has two members that draw\_x0() and draw\_xt(xlag) (latent variable) and a wrapper prop\_yt(t, particle) for the likelihood which are called by the particle filter

moments class: evaluates the moment conditions  $g_t(\theta, y_t, x_t)$  (for a given period t) given theta, the data, and the particle. It inherits the scl::moment\_function\_base and provides a way to specify the moment conditions.

## scl::gmm class :

- it returns  $g'_n S_n^{-1} g_n$  where  $g_n = \frac{1}{n} \sum_{T_{\min}}^n g_t(\theta, y_t, x_t)$  and  $T_{\min} \leq n \leq \text{sample\_size}$  and  $S_n$  is
  - calculated from centered sample moments if correct\_for\_mean==true
  - if regularize\_W==true, it is regularized with parameterridge to make the inverse well conditioned
  - HAC corrected if lag\_hac\_gmm>0
- its set\_\*() methods are expected to update the moments class as well.

Moreover, corresponding to the particle filter, it has scl::realmat members

- saved\_particle: used for the conditioning on the last trajectory in the conditioned PF
- draws (unweighted whole set), filter (reweighted using draws until t), smooth (reweighted using the whole set):  $T \times N$  matrices to store the particles from the last updates. They are needed to calculate the mean path and standard deviations
- gibbs\_draws separately collects the theta mcmc draws for periods when the saved\_particle gets updated

<sup>&</sup>lt;sup>3</sup>Defined in this order(!) using the previously defined objects, i.e. scl::gmm is defined using moments and model is defined using moments and scl::gmm.

As for the methods, there are three important ones

- 1. usermodel.support(theta): boolean variable to determine whether the given theta is inside the support of the prior
- 2. usermodel.prior(theta): calculates the log prior density evaluated at a specified  $\theta$ . The default is a flat prior with log p = 0.
- 3. usermodel.likelihood(): this is the particle filter
  - Before it is called, the mcmc.draw() function sets theta\_old and theta (the proposed  $\theta$ )
  - when it is called, the first thing to do is resetting theta for the \*moments and \*model class members and extract sample\_size and T0 from the data and \*moments class (get\_Tmin) members respectively
  - set the data and sample size for the scl::gmm class

```
bool dset = gmm_objfun->set_data(&data);
bool nset = gmm_objfun->set_sample_size(n+1);
```

- calculates the likelihood of theta
  - (a) until counter < particle\_update
    - use saved\_particle and t=sample\_size + 1 to call \*model.likelihood()
    - increment counter and return likelihood(theta\_new)
  - (b) when counter >= particle\_update, run the particle filter:
    - reset theta\_old for \*moments and \*models
    - add theta\_old to gibbs\_draws and initiate draws, smooth, filter with the saved\_particle (0th entry of the vector of scl::realmats)
    - fill the first  $T_0 = T_{\min}$  elements with \*model.draw\_x0 and \*model.draw\_xt
    - Importance sampling step uses \*model.prob\_yt (again \*model.likelihood) for the weights (in the background, scl::gmm call with theta\_old and data)
    - reset saved\_particle with the last trajectory and return likelihood(theta\_new)
       (i.e. with the old particles and new theta)

## Default usermodel

Necessary parameters to be given

K or numb\_obs\_factor: Number of observable macro risk variables

L or lag\_obs\_factor: Number of lags for the observable macro risk variables

I or numb\_returns: Number of returns used in the estimation

The data.dat file has the following format

• Let  $Y_t$  be the vector of **observable** risk factors in a demeaned format, i.e.

$$Y_t := \left[ y_t^1 - \mu_y^1, y_t^2 - \mu_y^2, \dots, y_t^K - \mu_y^K \right]' \qquad \qquad \mu_y^k := \frac{1}{T} \sum_{t=1}^T y_t^k, \quad \forall k \in \{1, \dots, K\}$$

and these constitute the first KL columns.

- The remaining columns contain the log returns  $\log R_t^i$  for various assets  $i \in \mathcal{I}$ .
- (?) Maybe after that could come the conditioning variables

The statistical model is the following

$$\begin{bmatrix} Y_{t+1} \\ X_{t+1} \end{bmatrix} = \begin{bmatrix} A_y & 0 \\ 0 & A_x \end{bmatrix} \begin{bmatrix} Y_t \\ X_t \end{bmatrix} + \begin{bmatrix} C_y & 0 \\ 0 & C_x \end{bmatrix} \varepsilon_{t+1}$$

$$\lambda_t = \lambda_0 + \begin{bmatrix} \lambda_y & \lambda_x \end{bmatrix} \begin{bmatrix} Y_t \\ X_t \end{bmatrix} \quad \text{with} \quad \lambda_t \in \mathbb{R}^{\dim(\varepsilon)}$$

$$\log \left( \frac{S_{t+1}}{S_t} \right) = -\delta_0 - \begin{bmatrix} \delta_y & \delta_x \end{bmatrix} \begin{bmatrix} Y_t \\ X_t \end{bmatrix} - \frac{|\lambda_t|^2}{2} - \lambda_t \cdot \varepsilon_{t+1}$$

That implies the following moment conditions

$$\begin{aligned} \mathbf{0}_{K \times KL} &= \mathbf{m}_{1}(y_{t+1}, y_{t}, \theta) = E\left[\left(Y_{t+1} - \sum_{l=1}^{L} A_{y,l} Y_{t+1-l}\right) \left[Y_{t}' \quad Y_{t-1}' \quad \dots \quad Y_{t-L}'\right]\right] \\ \mathbf{0}_{K \times K} &= \mathbf{m}_{2}(y_{t+1}, y_{t}, \theta) = E\left[\left(Y_{t+1} - \sum_{l=1}^{L} A_{y,l} Y_{t+1-l}\right) \left(Y_{t+1} - \sum_{l=1}^{L} A_{y,l} Y_{t+1-l}\right)'\right] - C_{y} C_{y}' \\ 0 &= \mathbf{m}_{3}(x_{t+1}, x_{t}, \theta) = E\left[\left(X_{t+1} - A_{x} X_{t}\right) X_{t}\right] \\ 0 &= \mathbf{m}_{4}(x_{t+1}, x_{t}, \theta) = E\left[\left(X_{t+1} - A_{x} X_{t}\right)^{2}\right] - C_{x}^{2} \\ 0 &= \mathbf{m}_{5}^{i}(z_{t+1}, z_{t}, x_{t+1}, x_{t}, \theta) = E\left[\exp\left(-\left[\delta_{0} \quad \delta_{y} \quad \delta_{x}\right] \begin{bmatrix} 1 \\ Y_{t} \\ X_{t} \end{bmatrix} - \frac{1}{2} \begin{bmatrix} 1 \\ Y_{t} \\ X_{t} \end{bmatrix} \Lambda' \Lambda \begin{bmatrix} 1 \\ Y_{t} \\ X_{t} \end{bmatrix} - \left(\Lambda \begin{bmatrix} 1 \\ Y_{t} \\ X_{t} \end{bmatrix}\right)' \begin{bmatrix} C_{y} \quad 0 \\ 0 \quad C_{x} \end{bmatrix}^{-1} \left[\left(Y_{t+1} - \sum_{l=1}^{L} A_{y,l} Y_{t+1-l}\right) + \log R_{t+1}^{i} - 1\right] \end{aligned}$$

where  $\Lambda := \begin{bmatrix} \lambda_0 & \lambda_y & \lambda_x \end{bmatrix}$ . If  $Y_t$  includes the risk-free interest rate, then  $\delta_0 = \delta_y = \delta_{y, \neq r} = 0$ .

The  $\theta$  vector is of length  $K^2L + \frac{K(K+1)}{2} + 2 + K(K+2) + (K+2)$  with

$$\theta := \begin{bmatrix} \operatorname{vec}(A_{y,1})' & \dots & \operatorname{vec}(A_{y,L})' & \operatorname{vec}(C_y) & \rho & \sigma & \operatorname{vec}(\Lambda)' & \delta_0 & \delta_y' & \delta_x \end{bmatrix}$$

TODO

## Notes

- Both specification class and usermodel class have theta member
  - During the mcmc sampling: only the usermodel's theta private member gets updated (both theta and theta\_old) and while it calls usermodel.likelihood(), usermodel automatically updates \*\_model and \*\_moments classes as well
  - The specification class's theta on the other hand does not get updated, so it is always stays at the start values of theta (could rename actually)
- Both mcmc class and asymptotics class have posterior\_mode and posterior\_maxval members
  - mcmc versions get updated just before the acceptance/rejection step
  - temperature for posterior affects the acceptance/rejection step
  - after num\_mcmc\_draws many draws the mcmc versions are passed to asymptotics => theta\_mode
     and posterior\_high get updated in asymptotics along with the cumulative mean theta and
     cov => everything gets written into summary.dat files

# References

A. Ronald Gallant, Raffaella Giacomini, and Giuseppe Ragusa. Bayesian estimation of state space models using moment conditions. *Journal of Econometrics*, forthcoming, 2017.