Strategic Money and Credit Ledgers

Markus K. Brunnermeier and Jonathan Payne*

February 5, 2026

Abstract

A dominant BigTech trading platform can set up a ledger that allows agents to issue unsecured tradable IOUs ("digital tokens"). Unlike a stand-alone ledger, the platform can incentivize the use of the ledger and enforce repayment so long as it can compromise the universal liquidity of public money. This lowers the equilibrium interest rate, but also increases rent extraction by the platform. A CBDC provides competition but can undermine ledger credit enforcement by reintroducing a universal public money alternative. A higher inflation rate weakens the public money alternative, enabling platform-operated ledger credit enforcement.

Keywords: Ledgers, platform money, CBDC, currency competition, private currencies, industrial organization of payments, platforms, Bigtech.

*Brunnermeier: Princeton, Department of Economics, Bendheim Center for Finance, markus@princeton.edu. Payne: Princeton, Department of Economics, Bendheim Center for Finance, jepayne@princeton.edu. We are grateful to our discussants Charlie Kahn and Yao Zeng. We thank Weiqi Chen and Mark Walker for helpful research assistance. We are also grateful for comments from Joseph Abadi, Viral Acharya, Mark Aguiar, Jonathan Chui, Mikhail Golosov, Hanna Halaburda, Charlie Kahn, Nobu Kiyotaki, Thorsten V. Koeppl, Ye Li, Igor Makarov, Abdoulaye Ndiaye, Guillermo Ordoñez, Laura Veldkamp, and Chaojun Wang.

1 Introduction

Historically, payment and credit services have been provided differently from the current bank-centric arrangement. For example, in early modern London a credit system emerged that involved tradable, uncollateralized “I Owe You” promises (IOUs). Merchants bought grain from farmers in exchange for IOUs, sold the grain in London and subsequently repaid the IOU holders. This meant that the merchants became active participants in both the grain and secondary IOU markets. That is, a “bills-of-exchange” system emerged. However, scaling this system proved challenging because of limitations on record keeping and IOU enforcement. Many proposals were put forward to address these challenges (e.g. Smith (1776)) but ultimately a collateral-dependent bank lending system emerged. Recent advancements in ledger and trading platform technologies have led to renewed interest outside of the banking sector in scaling up an uncollateralized credit system. BigTech platforms have taken steps to overcome the record keeping and enforcement challenges. In China, Alibaba’s MyBank offers members of their ecosystem uncollateralized loans, while supply chains platforms organize tradable account receivables (e.g. Liu et al. (2022)). In India, FinTech startups have attempted to offer similar services without the “backing” of large trading platforms and with less success (e.g. Rishabh and Schäublin (2021)).

This paper investigates when and how a large private BigTech trading platform will provide a settlement ledger where suppliers issue and repay uncollateralized IOUs with each other. To study this, we build a general equilibrium model where producers need to issue uncollateralized IOU contracts to purchase input goods but cannot commit to repayment. The economy has two technologies for making payments. The first option is unmonitored “spot” exchange that requires a payment asset (e.g. cash) to be held in advance. The second option is that a profit maximizing intermediary can provide a centralized ledger for settling payments without cash and executing contracts. Our ledger technology can be thought of as analogous to Kocherlakota (1998). However, unlike in Kocherlakota (1998) where a benevolent planner organizes a monopolistic ledger, we study how different private profit-motivated operators could (or could not) offer a centralized ledger that competes with standard cash payments. We start with a two-period real model in Section 2 where the endowment good acts as "commodity money" in spot trades and then extend to an infinite-horizon macroeconomic model with fiat money in Section 3.

In the real model, we first consider a standalone operator that provides a centralized ledger to the economy. Producers issue IOUs to purchase input goods that are recorded on the ledger. Contracts are automatically enforced when payments are received through the ledger because the ledger automatically redirects payments to settle IOUs. Conceptually, this is similar to a worker who takes a loan from their employer and accepts a portion of his wages being deducted to repay the loan. The issue in our economy is that agents do not have to make payments through the ledger because there is also an outside option to conduct spot trade with "universally liquid" commodity money. When agents receive a spot payment, the producer may default on their IOUs and so producers can charge higher prices to buyers who can pay with commodity money. As a result, in equilibrium, all savers prefer to store commodity money and all producers plan to default on IOUs. In anticipation of this, no IOUs are accepted and funding dries up: the universal liquidity of commodity money prevents the issuance of uncollateralized IOUs.

We next consider a BigTech platform that operates both a trading platform and a centralized ledger. The trading platform insists that on-platform trades must be settled on the ledger. In doing so, it breaks the universal liquidity of commodity money. Now, agents who saved with commodity money, rather than with IOUs, may be stuck with an unusable payment instrument if they need to trade through the platform. If a sufficiently large fraction of trades occur on the platform, and the platform's markup charges are sufficiently low, then agents stop storing commodity money and it becomes impossible for producers to conduct spot trades in which they can default. Consequently, IOUs are perfectly enforced and defaults do not occur. The key insight is that producers can only default in a ledger economy if buyers choose to store the commodity money required for hidden trades. So, by changing incentives in the payment system, the platform can change the feasibility of default. The difficulty for policy makers is that the platform only finds it attractive to set up a ledger that incentivizes contract repayment if it controls a large fraction of the goods trading. That is, we have a "natural" monopoly on contract enforcement.

Our finding that the platform's ability to break the universality liquidity of money is essential for the emergence of an unsecured credit market has important implications for the introduction of a central bank digital currency (CBDC). CBDC can be thought of as a public government ledger. The design of the CBDC ledger determines whether an unsecured private credit market is viable. If the CBDC ledger is universal, in the sense that the platform is forced to accept CBDC payments in addition to platform ledger payments, and the CBDC ledger is set up to respect privacy (e.g. with zero-knowledge proofs), then it essentially reintroduces the option to conduct hidden spot trades and default. In this case, the unsecured private credit market is not viable. By contrast, if a CBDC is set up without privacy to record and enforce contracts, then it can implement the Kocherlakota (1998) environment. So the government faces a tradeoff between introducing a payment technology that respects privacy and one that ensures efficient contracting.

In Section 3, we extend our analysis to an infinite-horizon monetary model to study the relationship between the payment and trading systems. Settlement is made with two possible financial assets: "fiat" government money and claims to assets on the ledger. The ledger asset system could be interpreted as a "bills-of-exchange" model where tradable IOUs are used as the medium of exchange on the platform or as a model of "tokenization" where claims to future income become tradable tokens on the ledger. In our monetary model, we derive conditions under which the ledger-operating platform can and will ensure IOU repayment, generalizing our results from Section 2.

Our dynamic, monetary model yields two main additional insights. First, we show that the platform markup has large general equilibrium effects in the credit market. For a fixed interest rate, an increase in the markup leads to fewer agents trading on the platform and consequently lower IOU issuance.

To ensure the credit market clears, the interest rate must increase. This can offset the markup disincentive to trade through the platform. Ultimately, this allows the platform to increase markups without losing customers but the resulting higher interest rate leads to a large output decline. Our optimizing platform has to balance these general equilibrium trade-offs. Second, whether the platform ledger credit and payment system is viable depends on the inflation rate on public money. The platform has to offer a payment technology that is more attractive than holding public money. So, a higher the inflation rate makes public money less competitive and allows the platform to charge a higher markup without losing additional customers. In this sense, a platform ledger system is most likely to appear in economies where the government inflating their currency.

We conclude our analysis by studying competition between two *private* platforms, each providing its own trading and settlement technologies. We show that competing private platforms that bundle ledger and trading technologies will cooperate on contract enforcement as long as the gap between their respective trading technologies is not too large and financial frictions do not prevent the less efficient platform from committing to pay the more efficient platform. Otherwise, a dominant platform will emerge that will attract more trades and extract higher rents.

Literature Review. Our paper is related to several branches of research. First, it connects to the literature concerning the role of ledgers and settlement assets in organizing trading systems. Aiyagari and Wallace (1991) and Kocherlakota (1998) study how a planner can increase the contracting space by updating a common ledger with trading histories. Freeman (1996b,a) studies how the choice of settlement asset creates or mitigates trading frictions in the currency market. Our model shares many features with these papers. However, we consider an environment where a private profit-maximizing agent controls the ledger. This brings an industrial organization perspective to the literature on ledgers and settlement assets.

Second, we relate to the literature studying how FinTech can expand uncollateralized debt limits. In our model, the BigTech ledger operator can incentivize the repayment of debt contracts by forcing agents to use their ledger. In this sense, forcing the use of the ledger creates a form of "digital collateral" that can be used in automated contracting (e.g. Garber et al. (2021), Kahn and van Oordt (2022)). This would resolve the supply chain contracting issues presented in Bigio (2023). In Brunnermeier and Payne (2023), we extend our model to study strategic information portability decisions. Empirically, Liu et al. (2022) documents BigTech uncollateralized lending successes in China. Rishabh and Schäublin (2021) document for India that, without a coordinating BigTech platform, borrowers' non-cash revenue drops after fintech companies disbursed "digitally collateralized" loans. Copestake et al. (2025) documents the positive effects of interoperability from introducing India's Unified Payment Interface.

Third, we relate to the growing field of digital currencies. Like in our paper in Ozdenoren et al. (2025) the platform takes seigniorage income into when setting markup charges, but they do not include credit market implications. We are most closely related to the papers that study private Tech platforms providing centralized currencies, (e.g. Chiu and Wong (2020), Chiu and Koeppel (2025), Cong et al. (2020), Ahnert et al. (2022)). More generally, we touch upon the growing literature on central bank digital currency (e.g. Fernández-Villaverde et al. (2020), Keister and Sanches (2019), Kahn et al. (2019)) and decentralized, programmable cryptocurrencies (e.g., Fernández-Villaverde and Sanches (2018), Benigno et al. (2019), Abadi and Brunnermeier (2024), Schilling and Uhlig (2019), Cong et al. (2021)).

Fourth, we relate to the literature on currency competition (e.g., Hayek (1976), Kareken and Wallace (1981), Brunnermeier and Sannikov (2019)). Formally, our dynamic model in section 3 expands on the two currency cash-in-advance model from Svensson (1985) and endogenizes currency demand using search and trading frictions in the tradition of the new monetarist literature (e.g. Lagos and Wright (2005), Lagos et al. (2017)).

We structure the paper in the following way. Section 2 solves the two-period version of the model with a monopoly ledger controller and exogenous platform demand. Section 3 extends the analysis to an infinite-horizon macroeconomic model in which agents endogenously choose whether to trade on or off the platform. Section 4 concludes.

2 An Illustrative Model of Ledgers, Platforms, and Contract Enforcement

In this section, we outline a two-period version of our model. We use this model to highlight why a large trading platform is important for expanding uncollateralized lending in the economy. To do this, we first show that introducing an independent stand-alone common record keeping ledger is not sufficient to achieve first best allocations. We then show that a large trading platform can provide a ledger technology that leads to contract enforcement and the first-best allocation. Finally, we consider when the introduction of a public ledger alternative helps or undermines the provision of uncollateralized lending.

To keep the focus on enforcement issues, we start with a real model where goods are traded without fiat currency and where the ledger manages physical trades as well as record keeping. We consider a monetary economy in Section 3. Contracting in our economy is difficult because debtors can potentially sell output for commodity money off platform in private side-trades and default. The platform can eliminate this behavior by setting payment rules that encourage payment through a centralized ledger, and so discourage agents from storing the commodity money that facilitates private side trades. In doing so, they can create an equilibrium in which agents coordinate on contract enforcement through their choice of payment technology.

2.1 Environment and First Best Allocation

Setting, production, and preferences: Time lasts for two periods: $t \in \{0, 1\}$. The economy contains a collection of storable "endowment goods" (which can be interpreted as "commodity money") and a collection of perishable "output goods". The economy is populated by a unit continuum of savers and a unit continuum of producers. Each saver is born with one unit of an endowment good at $t = 0$. Each producer can transform one endowment good at $t = 0$ into $z \in (1, 2)$ output goods at $t = 1$. Each saver can store one endowment good at $t = 0$ to get one endowment good at $t = 1$. Savers get linear utility from consuming goods at the start of $t = 1$ while producers get linear utility from consuming goods at the end of $t = 1$. No agent gets utility from consuming the goods that they are endowed with or have produced. Instead, they only derive utility from goods endowed to or produced by other agents. Ultimately, this means that agents need to trade endowment goods in order to produce at $t = 0$ and output or endowment goods in order to consume at $t = 1$.

Information, commitment, and matching frictions: Agents have publicly verifiable identities and can therefore be identified at $t = 1$. However, agents' actions are not publicly observable, agents cannot commit, and there is no public legal system for contract enforcement.

A benevolent planner who cares equally about all agents, reallocates one endowment good to each producer at $t = 0$ to maximize production and reallocates output goods across agents. We refer to this as the **first-best allocation**. The goal of this section is to explore which private payment, record-keeping, and trading technologies are able to implement the best allocation despite information and commitment frictions in the economy.

2.2 Payment Technologies and a Common Ledger

In order to consider whether the private sector can implement the first-best allocation, we need to specify the trading and payment arrangements in the economy. In this section, we introduce a ledger technology that (i) collects and records trades and contracts, and (ii) settles all trades and contracts at the end of each period. In the next section we introduce a platform that controls the trading system.

Trading arrangements and payment technologies: At $t = 0$, a competitive market opens for IOU contracts between savers and producers that promise R goods in period $t = 1$ for each endowment good given in period $t = 0$.

At $t = 1$, producers start with output goods and savers start with either stored endowment goods or IOUs. At the beginning of the period, a competitive market opens for trading goods and IOUs but is subject to payment frictions. There are two payment technologies in the economy for settling goods trades at $t = 1$: spot payments and ledger payments. **Spot payments** (indexed by s) are not recorded and are settled immediately. We impose that spots trades can only be undertaken in exchanges that involve endowment goods. We interpret this as saying that endowment goods are universally liquid, whereas output goods are not. **Ledger payments** (indexed by l) are recorded and can be made with any goods. At the time of the transaction, agents give their output goods to the ledger. At the end of the period, agents can return to the ledger, and the ledger settles all transactions and IOUs with seniority given to earlier claims. This means that the ledger automatically uses revenue from ledger trades to settle contracts but cannot settle contracts using revenue from spot trades. Since all trades occur before IOU settlement, contracts are only enforced when the output producer trades through the ledger. We let ϕ denote the endogenous probability that an IOU is repaid. We use output goods as the numeraire and let (p, q) denote the number of endowment goods and IOUs required to purchase one output good (i.e. the prices at $t = 1$).

Producer's problem: At $t = 1$, a producer chooses whether to trade their output good for other output goods, IOUs, or endowment goods to solve:

$$V_1^p = \max\{z - R,\ \phi Rqz - R,\ pz\}, \tag{2.1}$$

where the first, second, and third terms are the payoff when the producer trades for output goods, IOUs, and endowment goods respectively. In the first and second cases, the producer must pay through the ledger and so cannot default. By contrast, in the third case they can undertake spot trade and default. At $t = 0$, producers issue IOUs if $V_1^p \geq 0$.

Saver's problem: At $t = 1$, a saver trades for output goods or other endowment goods. If they hold an endowment good, then they solve:

$$V_1^{s,e} = \max\{1,\ \phi Rq/p,\ 1/p\}, \tag{2.2}$$

where the first, second, and third terms are the payoff when they trade for other endowment goods, IOUs, or output goods. If they hold an IOU, then they solve:

$$V_1^{s,i} = \max\{p/q,\ \phi R,\ 1/q\}, \tag{2.3}$$

where the first, second, and third terms are the payoffs when they trade their IOU for endowment goods, other IOUs, or output goods. At $t = 0$, the saver's problem is to decide whether to store endowment goods or exchange endowment goods for IOUs. They make this choice by solving:

$$\max\{V_1^{s,e}, V_1^{s,i}\}. \tag{2.4}$$

Equilibrium: An equilibrium is a collection of prices (R, p, q) such that savers and borrowers make optimal choices satisfying (2.1), (2.2), (2.3), and (2.4) and markets clear at $t = 0$ and $t = 1$.

Proposition 1. *The only equilibrium is that savers store endowment goods, no IOUs are issued, and no production takes place.*

Proof. See Appendix A. □

The intuition for Proposition 1 is that savers prefer to store endowment goods to facilitate side trades rather than save by holding IOUs. If all the other savers are buying IOUs and so production takes place, then an individual saver would prefer to store endowment goods to conduct side trades rather than purchase IOUs. This is because producers offer a relatively high price for endowment goods (a high $1/p$) since they enable the producer to default. It follows that IOU issuance is not an equilibrium. Conversely, if all other savers are storing endowment goods and so there is no production, then an individual saver would also prefer to store endowment goods rather than holding IOUs because all future trades will be spot trades, and no IOUs will be repaid. It follows that endowment good storage is an equilibrium.

Proposition 1 highlights that introducing a stand-alone record keeping ledger does not resolve the contracting problems in the economy. This is because agents have an alternative payment option: a universally "liquid" endowment good that facilitates side default trades and offers a better saving vehicle for agents. Put another way, relative to Kocherlakota (1998), the ledger in our environment has competition from another payment technology and so agents need to be incentivized to use the ledger.

2.3 Trading Platform

We now introduce a large trading platform into the economy. This means that the economy has a large institution that can force trades onto the ledger and break the universal liquidity of endowment goods.

Trading technologies: Suppose there are now two trading technologies for connecting agents in the economy, indexed by $n \in \{o, p\}$. Trading technology $n = o$ is not controlled by anyone and is referred to as the **"open" public marketplace**. Trading technology $n = p$ is controlled by a profit maximizing organization, which we refer to as the **private platform**. Agents are randomly allocated to the technologies each period. With probability $1 - \eta$ producers are allocated to posting on the public marketplace and with probability η they post on the private platform. We endogenize η in Section 3. The platform charges a markup μ on the profit received by savers using their trading technology. We impose that all profits from markups collected by the platform at t are redistributed lump sum to the agents as dividends.

We assume that the platform provides both the trading technology and the common settlement ledger for the economy. We further assume that the platform can control which payment technology is used for its trades, forces agents using the platform to make payments using their ledger, and blocks agents from using endowment goods for trades on the ledger. We also allow the platform to guarantee IOUs.

Producer's problem: At $t = 1$, if the producer goes to the public marketplace, then their decision is the same as in Subsection (2.2). By contrast, if they go to the platform, then they are only allowed to post trades using the ledger and so they can only trade for output goods and IOUs. This means their value at time $t = 1$ becomes:

$$V_1^p = \eta \max\{z - R,\ \phi Rqz - R\} + (1 - \eta) \max\{z - R,\ \phi Rqz - R,\ pz\}$$

At $t = 0$, producers once again issue IOUs if $V_1^p \geq 0$.

Saver's problem: At $t = 1$, if the saver is assigned to the public marketplace, then their problem is the same as in (2.2). By contrast, if they go to the platform, then they can only trade if they saved with IOUs. Thus, the value of saving with endowment goods, $V_1^{s,e}$, and IOUs, $V_1^{s,i}$, become:

$$\begin{aligned} V_1^{s,e} &= (1 - \eta) \max\{1,\ \phi Rq/p,\ 1/p\} \\ V_1^{s,i} &= \eta(1 - \mu) \max\{\phi R,\ 1/q\} + (1 - \eta) \max\{p/q,\ \phi R,\ 1/q\} \end{aligned}$$

At $t = 0$, their optimization problem is once again $\max\{V_1^{s,e}, V_1^{s,i}\}$, as in equation (2.4).

Proposition 2. *For a sufficiently large η, the platform operates the ledger, offers to discount IOUs, and sets the maximum markup $\bar{\mu}$ that is incentive compatible with contract enforcement:*

$$\mu \leq 1 - \left(\frac{4}{z} - 1\right) \frac{1 - \eta}{\eta} =: \bar{\mu} \tag{2.5}$$

In this case, the first-best allocation is implemented. For a sufficiently low η, the platform does not set up a ledger and there is no production in the economy.

Proof. See Appendix A.2. □

The intuition for Proposition 2 is that the platform can incentivize agents to use the ledger. If $\eta = 1$, then it is clear that the platform can ensure contract enforcement because it can force all trades in the economy through their ledger. The reason why they can ensure full contract enforcement when $\eta < 1$ and they don't control all trade is more subtle. The platform makes two changes to the environment from Section 2.2: (i) they prevent savers from using endowment goods when trading on their platform, and (ii) they guarantee IOUs.

The first change, preventing endowment good payment, breaks the universal liquidity of endowment goods and creates an equilibrium with contract enforcement. This occurs because savers are discouraged from saving with endowment goods, which can be used for private side trades, and encouraged to save with IOUs, which can only be traded through the ledger technology and so are automatically used to settle contracts. So, blocking payment with endowment goods on *platform trades* leads to all savers choosing the payment technology that ensures contracts are enforced in *all* trades.

The second change, guaranteeing IOUs, eliminates the possibility of a second equilibrium in which all savers store endowment goods. This occurs because guaranteeing IOUs ensures that having an IOU repaid offers a higher return than storing IOUs when all other agents are storing endowment goods and the relative price of endowment goods at $t = 1$ is low. It is important to note that guaranteeing IOUs without blocking endowment good payment on the platform is not sufficient to create an equilibrium with full contract enforcement. This is because when all savers save with IOUs, endowment goods are relatively scarce at $t = 1$ and so have a relatively high price. This leads to savers switching to endowment storage unless the platform makes endowment good unattractive by blocking their use as payment on the platform.

Theorem 2 also shows that the dominance of the trading platform, as measured by η, characterizes whether the economy has a problem with monopoly "rent" extraction or a problem with credit "fragility". The intuition for this is the following. The platform derives profit from charging markups on trades. However, it also incentivizes loan repayment by discouraging savers from storing the endowment goods required to facilitate default. This means that increasing the markup to increase profits will also diminish the disincentive to store endowment goods. Together, these forces mean that the maximum markup the platform can charge while maintaining contract enforcement is given by (2.5). When η is high, the platform can maintain a positive markup while still disincentivizing default, and so is willing to set up the ledger. However, when η is low, the platform would have to offer a negative markup (i.e. a subsidy) to make exclusion from trade sufficiently costly to disincentivize default. In this case, it prefers to not set up the common ledger.

Corollary 1. *If the common settlement ledger and trading technology are operated independently by separate institutions and the institutions do not coordinate on blocking payment with commodity money, then all output good producers default and there is no production in the economy.*

Why is cooperation between the ledger operator and the platform required to ensure contract enforcement? If the ledger automatically enforces contracts, then it need cooperation from the platform. If the platform allows agents to undertake spot payments regardless of whether they have defaulted, then all agents use spot trades and default. If the platform attempts to ensure IOU repayment, then it needs cooperation from the ledger. Agents trade before contract settlement, which means the platform needs to force agents to use the ledger so that revenue can be seized to fulfill the contract. If the ledger operator does not allocate payments to settle contracts and instead gives the resources directly to the sellers, then the sellers can walk away and default.

Taking stock, we have the following key lessons from our stylized two-period model that we explore further in Section 3.

(i) **Ledgers are only useful if they are "backed":** The ledger record keeping technology is potentially very powerful in the economy but only if agents use it. This means that the platform controlling the trading technologies in the economy needs to "back" the ledger by forcing agents to use it. Otherwise, introducing the ledger technology will not change the equilibrium. In this sense, BigTech platforms are natural providers of currency ledgers. This suggests the dominance of Alibaba and WeChat in the Chinese payment system might reflect an underlying advantage or synergy in providing payments.

(ii) **The payment technology can collateralize sales revenue:** In this economy, the type of payment technology matters for the collateralizability of future sales revenue. Trades settled using the common ledger can always be used for the repayment of contracts and so essentially act as digital "collateral" for borrowing. Trades not settled using the ledger are not automatically used for the repayment of contracts and so can only be used as collateral if the agents coordinate on reporting and excluding defaulting agents. In this sense, the model is set up so the platform can choose to what extent future sales revenue can be effectively used as "collateral" across the economy. We extend this in the monetary model in Section 3, where "bills-of-exchange" compete with government fiat currency as a medium of exchange.

(iii) **A natural monopoly dilemma:** We can also see that there is a type of natural monopoly force in this economy. The more trades that use the ledger (the higher is η), the easier it is for the ledger controller to enforce contracts. For example, suppose that η must be greater than 1/2 in order for the no-default incentive compatibility constraint (2.5) to be satisfied for a positive markup. In this case, there is no way for multiple ledgers to operate in the economy and enforce contracts unless they cooperate on enforcement. In other words, one large ledger provider can better enforce contracts than a collection of non-cooperative smaller ledger providers. So, a regulator in this environment needs to find a way to get a monopoly ledger provider to behave more competitively or have multiple large platforms compete on markups while coordinating on contract enforcement. We take up these questions in Section 3.6.

2.4 "Central Bank Digital Currency" Ledger

So far we have only considered the private provision of a ledger technology. We now introduce a public ledger provided by the government, which can be thought of as a form of central bank digital currency (CBDC) or as broad access to the central bank payment system (FedNow). We show that the way the government designs the ledger has significant implications for contracting in the economy.

Public ledger: Suppose that now the government offers a public ledger technology that can be used to settle trades. The government can choose whether the ledger just provides payment settlement that respects agent privacy (similar to a private "payment" CBDC or anonymous FedNow) or whether the ledger also records and settles contracts (a "smart" CBDC). The former option can be thought of as providing a public technology for undertaking spot trades without needing to store endowment goods. The latter option can be thought of as a publicly provided version of the platform ledger from Section 2.5.

Corollary 2.

(i) If the government provides a privacy-respecting "payment" ledger and forces the platform to accept payments through the public ledger (i.e. makes it universal), then there is no equilibrium with full contract enforcement unless $\eta = 1$.

(ii) If the government provides a "smart" ledger and eliminates endowment good payments (i.e. blocks commodity money), then all contracts are enforced and first best production is achieved.

Proof. See Appendix A.2. □

Corollary 2 say that the government needs to be careful not to provide a public payment option that ends up reintroducing spot trade into the economy. A privacy-respecting government payment ledger essentially introduces a universally liquid payment technology that does not require agents to store endowment goods in advance. So, agents can always make side trades and default. By contrast, if the government gives up privacy protections and uses the CBDC ledger to enforce contracts, then it can implement the Kocherlakota (1998) ledger system. This poses a tradeoff for the government: introducing a public payment ledger that respects privacy makes the payment system work more effectively but prevents efficient contracting.

2.5 Extensions and Discussion

We close this section by discussing how effectively an uncollateralized IOU system could be created in extended environments with aggregate risk and/or other private institutional arrangements.

- *Production risk:* Our production process in Section 2.1 was risk free. If, instead, productivity z is a random variable, then savers and producers would need to write contracts to share output risk at $t = 1$. As long as the platform dis-incentivizes endowment good storage and all revenue goes through the ledger, the savers and producers can contract on any division of output. That is, as long as the IC constraint is satisfied, the platform-ledger arrangement brings the additional advantage that agents have access to complete contracting. In practical terms, this can be thought of as a modern variation on the "share-cropping" arrangements historically employed in the US South, where tenants were allowed to use land in exchange for a share of crop output, Stiglitz (1974).

- *Banks as ledger and uncollateralized credit providers:* There is a large literature showing that the threat of exclusion from future access to the loan market can incentivize the repayment of uncollateralized loans (e.g. Kehoe and Levine (1993)). This might suggest that traditional banks should be able to provide the ledger for uncollateralized loans instead of platforms. The reason why this is not possible in our model is because agents only need to access the loan market once, and so exclusion from future credit is not costly. If we expanded the model to have additional periods and repeated borrowing, then there could be equilibria with uncollateralized credit supported by exclusion from future lending, as in many finance models. We do not consider such equilibria for two reasons. First, this credit could be provided by *either* the bank or platform (or any other large agent in the economy). In this sense, the bank is not better at excluding agents from future credit than the platform. Second, exclusion from future credit has already been comprehensively studied in the literature, and we want to focus on new mechanisms related to the emergence of platforms and ledgers.

- *Industry supply chains as ledger and uncollateralized credit providers:* We could also ask whether a platform on which all goods are traded is a superior ledger provider compared to a supply chain platform that is restricted to a subset of goods, e.g., everything related to cars for an automobile supply chain platform. Answering this question requires us to extend our model so that agents get different utility from different types of goods. If the platform covers all types of goods, then IOUs on the platform ledger are denominated in the overall consumption basket. In this case, neither the platform nor the agents face any "exchange rate risk". By contrast, in a setting with a platform that covers only a subset of goods, IOUs on the platform ledger are denominated in a fraction of the consumption basket. In this case, agents face exchange rate risk, which makes the threat of exclusion less powerful.

3 Dynamic Model of Platform & Ledger Provision

The two-period model in Section 2 illustrates the value of having a large trading platform that provides a settlement ledger. In this section, we set up an infinite-horizon monetary model where agents choose whether to trade on-or-off-platform. There are now two key endogenous prices: the real exchange rate between money and ledger IOUs/tokens and the spread between the return on IOUs and money. This allows us to derive the following two results. First, platform markup choices impact both the real exchange rate and the interest spread in the economy. Second, the inflation rate on public money influences whether the platform ledger credit and payment system is viable.

3.1 Environment

Time is discrete with an infinite horizon. There is a collection of goods that can be used for production and consumption. The economy contains a continuum of agents, financial intermediaries, and a private platform that operates a ledger for the economy. There is a "fiat" money asset provided by the government, interperiod IOUs that agents can create through the ledger, and equity shares in the platform. There are two marketplaces for exchanging goods: a private platform and an open public marketplace. Each period is divided into a morning subperiod when a goods market opens and an evening subperiod when the asset market opens. Agents need money to make payments in the morning subperiod on the public marketplace, as in Svensson (1985) and Lagos and Wright (2005), but the platform prevents money being used for payment on its marketplace.

Production, preferences, and life-cycle: Each agent follows a "life-cycle" where they start as producers and then become savers looking to consume. An agent born in the morning of time t (at age 0) arrives without resources but with a technology to convert x_t goods at time t into $y_{t+1} = zx_t^{\alpha}$ goods at time $t+1$, where $z > 0$ is productivity and $\alpha \in (0, 1)$. At $t+1$ (at age 1), they sell their produced goods, consume, and save. At time $t+2$ (at age 2), they consume again and exit. Agents in generation t rank consumption allocations according to $(1-\beta)u(c_{1,t+1}) + \beta u(c_{2,t+2})$, where $u(c) = \log(c)$, $c_{\tau,t}$ is consumption of goods produced by other agents at time t when the agent's age is τ, and

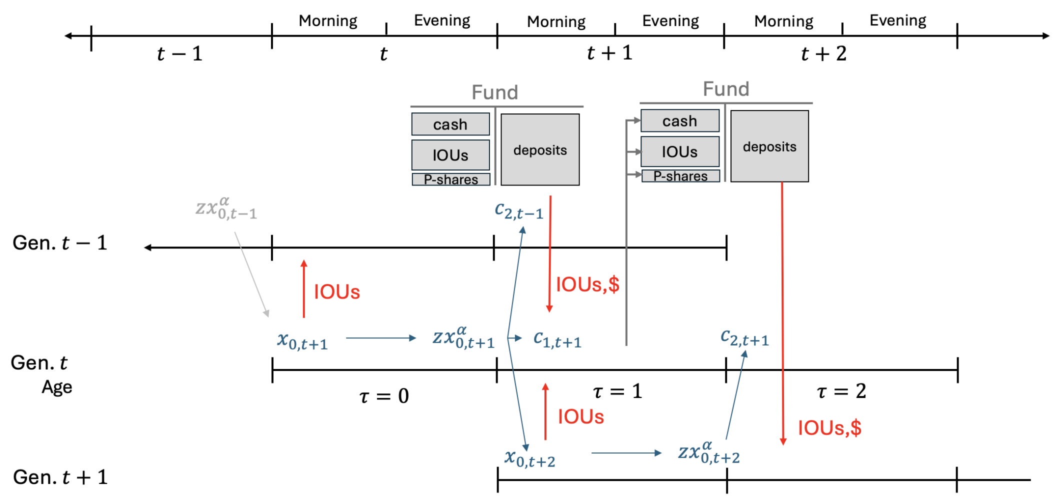

Figure 1: Timeline of the OLG Infinite Horizon Model.

$\beta \in [0, 1]$.

Timing: We show the timeline visually in Figure 1, sketch the key points below, and then subsequently describe the environment in detail. The timing in the *morning subperiod* is the following:

(i) Age-0 agents are born and begin the period without any assets or goods. Age-1 agents begin with produced goods and a payment type that they have chosen to accept. Age-2 agents begin with holdings of deposits in financial intermediaries, which hold a portfolio of money, old IOUs, and equity shares in the platform.

(ii) The goods markets open on each trading technology. Age-0 agents issue new IOUs to buy input goods. More specifically, if they buy on the platform, then they exchange IOUs directly for goods. If they buy off the platform, then they sell their IOUs to a financial intermediary in exchange for the money to make payments. Age-1 agents sell the goods they have produced. Age-2 agents agents withdraw their wealth from the financial intermediary in the required currency. All agents trading on a particular trading technology participate in a competitive goods market.[1]

(iii) At the close of the goods market, the ledger settles payments made through the ledger and IOUs issued in the previous period that are now due. If the ledger does not have sufficient payment revenue to settle an IOU, then it declared in default. Age-2 agents exit after consuming the goods they purchased.

In the *afternoon subperiod*, age-1 agents deposit revenue with the financial intermediary. The currency and asset markets open. In order to manage household currency needs, financial intermediaries choose their asset portfolio for the next period, including their currency reserves.

We now explain the operation of the different markets in more detail.

Goods trading frictions: As before, there is both a public marketplace and a private platform to connect buyers and sellers, indexed by $n \in \{o, p\}$ respectively. However, now, agents can choose where to trade at each age. We model this as a discrete choice problem. In addition to the pecuniary benefits from trading, we incorporate amenity benefits. Formally, at each time period t, for each technology $n \in \{o, p\}$, each agent i gets an idiosyncratic, independent amenity draw to trade on that platform.[2] The draw for agents of age τ is distributed according to $\tilde{\zeta}^{ni}_{\tau,t} \sim \log(\zeta^n_\tau) + \mathrm{Gu}(1/\gamma_\tau, -(1/\gamma_\tau)\mathcal{E})$, where Gu denotes the Gumbel distribution, $\mathcal{E}$ is the Euler–Mascheroni constant, ζ^n_τ is a technology-specific component that characterizes the average quality of service provided by the platform to sellers, and γ_τ is the elasticity of substitution. For convenience, we normalize $\zeta^o_\tau = 1$ and denote $\zeta^p_\tau = \zeta_\tau$.[3] At age 0,

[1]The search literature often studies models where pricing is determined through one-to-one matching and bargaining over prices. Throughout this paper, we instead consider segmented competitive markets. We believe this is a closer approximation to the markets we are studying, especially in later sections when we model trade taking place on platforms such as Amazon or Alibaba.

[2]We introduce idiosyncratic risk in order to avoid “bang-bang” solutions to the platform choice problem. Our model uses tools from the discrete choice literature. This is analogous to assuming a CES preference function across the trading technologies.

[3]We do not impose a physical interpretation on the amenity values but they could be modeled as idiosyncratic search costs or good quality. For the cost interpretation, note

in the morning subperiod, the agents observe their amenity shock for trading at age 0 and 1.[4] At age 2, in the morning subperiod, the agents observe their amenity shock for trading at age 2. As before, the platform charges a markup μ on buyers.

Assets and financial intermediaries: At each time t, newly born agents issue IOUs that promise to pay one good in the morning market at $t+1$. At time 0, the platform issues equity shares that pay firm profits π_t each period t. The government issues "fiat" public money, in supply M_t each period t, and rebates seigniorage to agents proportional to their wealth. We let g_t^M denote the growth rate of money supply.

There is a continuum of competitive financial intermediaries that take deposits and pool resources across agents to purchase assets. From an economic point of view, the intermediaries provide liquidity services and, from a technical point of view, they simplify the exposition of the asset pricing.[5] On the liability side, the financial intermediary issues one-period deposits that allow agents to withdraw IOUs or money in the morning subperiod. We allow the platform to block intermediaries from participating on the ledger if they have accepted defaulting agents. So financial intermediaries only accept depositors who have repaid IOUs.[6] Consequently, defaulting agents lose access to the platform and financial intermediary investment services, which leaves them only able to hold money.

that the Gumbel distribution takes values across the real line and so ζ_τ^{ni} would represent a normalized cost. For the good quality interpretation, observe that we can write the total utility a buyer receives as: $\log(e^{\zeta_\tau^{ni}} \zeta_\tau^n c)$, and so $e^{\zeta_\tau^{ni}} \zeta_\tau^n$ is essentially scaling the utility that the buyer gets from the good they consume.

[4]We make the assumption that agents observe their amenity shock for age 1 at age 0 for mathematical convenience so that we can get a closed-form solution to the agent problem.

[5]An equivalent structure would be to have a "Lucas family" that pools resources together but penalizes agents based on where they choose to trade or what type of currency they bring back to the family.

[6]We could equivalently consider a model where some intermediaries accept defaulting depositors and some do not. This is equivalent to our model because intermediaries accepting defaulting depositors can only hold money and so they are not able to provide any liquidity services.

Payment technologies in the morning markets: Each morning subperiod, a competitive goods market opens on each trading technology among the sellers and buyers who chose to go to that trading technology. Once again, there are two payment technologies in the economy: spot trade and ledger trade. However, now currencies and financial assets are used for payment. Spot transactions are not recorded and are subject to a resource-in-advance constraint: the payment must be made using goods and/or public money ("dollars") issued by the government. Let P_t^m denote the units of money required to purchase a good through the public marketplace in the morning market in period t.

The other payment technology is the digital ledger provided by the platform, which is not subject to a resource-in-advance constraint. During the morning market, the digital ledger allows agents to make payments on the platform up to the value of the assets they hold on the ledger. This means that agents can use risk-free future ledger income to purchase goods so the ledger is essentially creating "bills of exchange" or "tokenized" claims on the revenue within the ledger ecosystem. Let P_t^b and P_t^s respectively denote the number of $(t+1)$-maturity IOUs and shares required to purchase a good on the private platform in the morning market in period t. Since t-maturity IOUs are settled in goods at the end the period t morning market, they can be traded for goods at a 1-1 rate so long as there is no default. The financial intermediary buys IOUs issued by newly born agents (i.e. "discounts" them to money) wanting to trade on the public marketplace at rate $\breve{E}_t$ IOUs per unit of money.

As in Section 2, we impose that the platform restricts the liquidity of public money by preventing sellers from accepting money when trading on their platform. Rather than considering all the different choices of payment technology on the public marketplace, like we did in Section 2, we start by imposing that all trade on the public marketplace is spot-trade. In this sense, money is the currency for spot trade, while IOUs (and other financial assets on the ledger) are the currency for ledger trade.

Asset market in the afternoon: In the afternoon subperiod, a competitive asset market opens for trading all currencies and financial assets. Let E_t denote the number of $(t + 1)$-maturity IOUs required to purchase one dollar in the afternoon market in period t. i.e. the "nominal exchange rate" between IOUs and money. Let Q^s_t denote the number of $(t + 1)$-maturity IOUs required to purchase one platform share. Following the monetary literature, it is helpful to distinguish between "money-goods" traded in dollar transactions and "token-goods" traded in ledger transactions. We typically use platform-goods as the numeraire and use "real prices" to refer to prices in terms of platform-goods. We define the real prices of money, $(t+1)$-maturity IOUs, and platform shares as $q^m_t := E_t/P^b_t$, $q^b_t := 1/P^b_t$ and $q^s_t := Q^s_t/P^b_t$. We define the real exchange rate between marketplace-goods and platform-goods as $\epsilon_t := E_t P^m_t/P^b_t$ and the real price of shares as q^s_t. Where appropriate, we use $R^m_{t,t+1}$, $R^b_{t,t+1}$, and $R^s_{t,t+1}$ to denote the real return on the holdings of money, bonds, and equity shares between t and $t + 1$. We let $\mathcal{R}^{bn}_{t,t+1}$ denote the effective real borrowing rate when the agent issues an IOU to trade on market n (after any financial intermediary discounting to convert it into the required currency) and let $\mathcal{R}^{dn}_{t,t+1}$ denote the effective real deposit return set by the financial intermediary when agents withdraw deposits in the medium of exchange on market n. We summarize the prices and returns on the underlying assets in Table 1.

	Goods price (morning)	Real asset price (afternoon)	Real return
Money	P^m_t	q^m_t	$R^m_{t,t+1}$
IOUs	P^b_t	q^b_t	$R^b_{t,t+1}$
Platform shares	P^s_t	q^s_t	$R^s_{t,t+1}$

Table 1: Summary of prices and returns. The first column is the number of assets to purchase one good in the morning market. The second column is the real asset price in the afternoon market. The third column is the real one-period return from holding the asset.

Information frictions: The environment has analogous information frictions to Section 2.1 except for two differences. First, if an agent defaults on an IOU, then the IOU holder can recover a fraction $\chi \in [0, 1)$ of the input good and the producer keeps the rest of their production. Second, as outlined above, the platform now excludes the financial intermediaries accepting defaulting agents from the ledger technology rather than the individual agents.

3.2 Comparison to Other Models

Our environment attempts to nest or reflect canonical models with currency and settlement frictions. To illustrate this, we discuss how our model relates to key models in the literature and why we have made deviations.

(i) *Cash-in-advance models:* The timing of trade and construction of the "synthetic" real exchange rate between the two segmented markets is taken from the two-currency cash-in-advance model proposed by Svensson (1985). The difference in our setup is that the cash-in-advance constraint is only relevant off-platform because the platform allows trade using claims to future revenue on the ledger. That is, trade on the platform is settled using digital bills of exchange that are automatically enforced at the end of the morning market. This is similar to the existence of both cash-good trades and credit-good trades in Lucas Jr and Stokey (1985). However, in our model, the creation of bills-of-exchange is not exogenous. Instead, the platform endogenously chooses whether to set up trading rules to facilitate the creation of bills-of-exchange or credit-good trades in order to maximize their markup profit.

(ii) *Money search models:* Like in Lagos and Wright (2005), we adopt a morning-evening subperiod structure where search frictions in the morning market require that agents hold money or have access to a currency ledger. Unlike many papers in this literature, for simplicity, we abstract from bargaining between agents and instead consider segmented competitive markets. We also take the view that digital money trades occur with access to a ledger and so are necessarily monitored trades rather than anonymous trades in a decentralized markets. For this reason, we focus on how large institutions monitoring trades might supply digital currency ledgers rather than on how demand for digital money is different from demand for other monies.

(iii) *Social planner ledger provision:* Like in Kocherlakota (1998), we focus on how the introduction of a common record keeping technology can change equilibrium allocations. We have two main points of departure from Kocherlakota (1998): (a) we consider a ledger provided by a profit-maximizing platform and (b) agents choose whether to use the ledger or an outside payment technology. If we eliminated spot trade and treated the platform as a benevolent planner, then we would get the results in Kocherlakota (1998).

(iv) *OLG models with financial assets:* Our model nests a classic OLG environment in which agents need an asset that they can buy when they are young and sell when they are old (e.g. Samuelson (1958), Diamond (1965)). In our environment, there are multiple assets available for storage that are differentiated by their usefulness in trade. In this sense, the financial assets in our model have both a role of storage and of a medium of exchange. Like in these models, the risk-free rate in our model will end up being distorted. However, unlike in these models, it is the strategic behavior of the platform controlling trade that leads to the interest rate distortion.

3.3 Market Equilibrium Without Default

In this subsection, we characterize the equilibrium with full IOU repayment. We first outline the buyer and seller problems under no-default. We then solve for market prices and show how platform markups affect general equilibrium. We use our characterization to study how platform decisions interact with general equilibrium. In the next subsection, we return to the difficulties of loan enforcement and derive the incentive compatibility constraint for no-default.

3.3.1 Agent Problem

We consider the problem of an agent in the generation born at time $t \geq 2$ and use the following terminology. Let n_τ denote the agent's choice of goods trading technology at age τ.

We now set up the budget constraints. Consider the agent at age 0. The agent chooses where to buy input goods, n_0, and where to sell output goods, n_1. If the agent purchases $x_{0,t}$ input goods, then they issue IOUs discounted through the financial intermediary at real borrowing cost $\mathcal{R}^{bn_0}_{t,t+1}(1+\mu^{n_0}_t)x_{0,t}$. Here, $\mu^{n_0}_t$ is the markup when buying on marketplace n_0 and $\mathcal{R}^{bn_0}_{t,t+1}$ is the effective borrowing rate when trading on platform n_0 (i.e. the borrowing rate when selling IOUs to producers on the platform $n = p$ or the borrowing rate when selling IOUs to financial intermediaries for money to use on the public marketplace $n = o$).

At age 1, the agent then produces and sells $y_{1,t+1} = z(x_{0,t})^\alpha$ goods, repays the IOU, consumes $c_{1,t+1}$ goods, and deposits $d_{1,t+1}$ in a financial intermediary. If the agent sells on the private platform, i.e. $n_1 = p$, then their revenue in platform goods is $y_{1,t+1}$. If the agent sells off-platform, i.e. $n_0 = o$, then their revenue in units of platform goods is $\epsilon_t y_{t+1}$. Thus, their budget constraint at $t = 1$ in real terms when they buy inputs on n_0 and sell on n_1 is:

$$d_{1,t+1} \leq \epsilon^{n_1}_{t+1}\Big(z(x_{0,t})^\alpha - (1+\mu^{n_1}_{t+1})c_{1,t+1}\Big) - \mathcal{R}^{bn_0}_{t,t+1}(1+\mu^{n_0}_{t+1})x_{0,t} \tag{3.1}$$

where ϵ^n_t denotes the real exchange rate between goods on the trading technology $n \in \{o, p\}$ and goods on the private platform, i.e. is ϵ_t if $n = o$ and 1 if $n = p$.

At age 2, the agent chooses a marketplace on which to consume, n_2. If they choose $n_2 \in \{o, p\}$, then the real value of their withdrawals is $\mathcal{R}^{dn_2}_{t+1,t+2}d_{1,t+1}$ and so the agent faces the budget constraint:

$$(1+\mu^{n_2}_{t+1})\epsilon^{n_2}_{t+2}c_{2,t+2} \leq \mathcal{R}^{dn_2}_{t+1,t+2}d_{1,t+1} \tag{3.2}$$

Finally, taking the price and return processes as given, at age 0, an agent in generation t solves problem (3.3) below:

$$\mathbb{E}_t\Big[\max_{x_0,c_1,c_2,d_1,\underline{\mathbf{n}}} \Big\{\tilde{\zeta}_{0,t}^{n_0} + \tilde{\zeta}_{1,t+1}^{n_1} + (1-\beta)u(c_{1,t+1}) + \beta(\tilde{\zeta}_{2,t+2}^{n_2} + u(c_{2,t+2}))\Big\}\Big] \quad s.t. \quad (3.1),(3.2). \tag{3.3}$$

Theorem 1. *An agent choosing trading technologies $\underline{\mathbf{n}} = (n_0, n_1, n_2)$, undertakes production:*

$$x_{0,t} = \left(\frac{\epsilon_{t+1}^{n_1}\alpha z}{(1+\mu_t^{n_0})\mathcal{R}_{t,t+1}^{bn_0}}\right)^{\frac{1}{1-\alpha}}, \qquad y_{1,t+1} = z\left(\frac{\epsilon_{t+1}^{n_1}\alpha z}{(1+\mu_t^{n_0})\mathcal{R}_{t,t+1}^{bn_0}}\right)^{\frac{\alpha}{1-\alpha}}, \tag{3.4}$$

$$\pi_{1,t+1} = \left(\frac{\epsilon_{t+1}^{n_1}\alpha z}{((1+\mu_t^{n_0})\mathcal{R}_{t,t+1}^{bn_0})^{\alpha}}\right)^{\frac{1}{1-\alpha}}\left(\frac{1-\alpha}{\alpha}\right).$$

and chooses consumption and saving:

$$c_{1,t+1} = \frac{(1-\beta)\pi_{1,t+1}}{\epsilon_{t+1}^{n_1}(1+\mu_{t+1}^{n_1})}, \qquad d_{1,t+1} = \beta\pi_{1,t+1}, \qquad c_{2,t+2} = \frac{\mathcal{R}_{t+1,t+2}^{dn_2}\beta\pi_{1,t+1}}{\epsilon_{t+2}^{n_2}(1+\mu_{t+2}^{n_2})}. \tag{3.5}$$

The fraction of producers and consumers respectively choosing trading technologies n_0, n_1, and n_2 are:

$$\eta_{0,t}^{n_0} = \frac{\left(\zeta_0^{n_0}((1+\mu_t^{n_0})\mathcal{R}_{t,t+1}^{bn_0})^{-\frac{\alpha}{1-\alpha}}\right)^{\gamma_0}}{\sum_{n_0'}\left(\zeta_0^{n_0'}((1+\mu_t^{n_0'})\mathcal{R}_{t,t+1}^{bn_0'})^{-\frac{\alpha}{1-\alpha}}\right)^{\gamma_0}} \tag{3.6}$$

$$\eta_{1,t+1}^{n_1} = \frac{\left(\zeta_1^{n_1}\,(\epsilon_{t+1}^{n_1})^{\frac{1}{1-\alpha}+\beta-1}\,(1+\mu_{t+1}^{n_1})^{\beta-1}\right)^{\gamma_1}}{\sum_{n_1'}\left(\zeta_1^{n_1'}\left(\epsilon_{t+1}^{n_1'}\right)^{\frac{1}{1-\alpha}+\beta-1}(1+\mu_{t+1}^{n_1'})^{\beta-1}\right)^{\gamma_1}} \tag{3.7}$$

$$\eta_{2,t+2}^{n_2} = \frac{\left(\zeta_2^{n_2}\mathcal{R}_{t+1,t+2}^{dn_2}/((1+\mu_{t+2}^{n_2})\epsilon_{t+2}^{n_2})\right)^{\gamma_2}}{\sum_{n_2'}\left(\zeta_2^{n_2'}\mathcal{R}_{t+1,t+2}^{dn_2'}/((1+\mu_{t+2}^{n_2'})\epsilon_{t+2}^{n_2'})\right)^{\gamma_2}} \tag{3.8}$$

Proof. See Appendix B. □

Theorem 1 shows that we get an intuitive closed form solution to the agent's problem. Holding all else constant, an increase in the private platform's trading advantage, $\zeta_\tau^p/\zeta_\tau^o$, leads to more agents using the private platform at each age. An increase in the effective cost of borrowing to purchase on a trading technology, $\mathcal{R}_{t,t+1}^{bn_0}$, leads to fewer agents purchasing input goods there. Likewise, an increase in the effective price of goods on a trading technology, $\epsilon_{t+2}^{n_2}$, leads to fewer agents purchasing consumption goods there. Finally, an increase in the markup, $(1+\mu_t)$, leads to fewer agents buying on the platform at all ages.

3.3.2 Financial Intermediary Problem

Financial intermediaries manage agents' wealth. From an economic point of view, the intermediaries provide liquidity services to agents in the sense that they ensure they have the currency they need to trade. From a modeling point of view, the intermediaries provide a convenient way of pricing the assets in the economy without complicating the agent problem.[7] In the evening of each period t, the financial intermediaries accept agents' wealth and purchase money M_t (to back deposit withdrawals in the morning market and purchase newly issued IOUs in the morning market), bonds B_t, and shares in the platform S_t. Their budget constraint in the evening at time t is:

$$q_t^m M_t + q_t^b B_t + q_t^s S_t \leq A_t.$$

In the morning of $t+1$, depositors who trade on the public marketplace ($n=o$) withdraw money, and depositors who trade on the private platform ($n=p$) exchange the market value of their remaining assets in the financial intermediary through the ledger. Lemma 1 shows that, in equilibrium, the effective agent borrowing and saving rates offset the financial intermediary's opportunity cost of holding money or IOUs.

Lemma 1. *If the financial intermediary holds excess reserves, then the borrowing rate and deposit rate faced by agents using trading technology n are*

[7] An alternative would be to open up a currency market in the morning market.

given by:

$$\mathcal{R}^{bn}_{t,t+1} = \epsilon^n_t \frac{R^b_{t-1,t}}{R^n_{t-1,t}} R^b_{t,t+1}, \qquad\qquad \mathcal{R}^{dn}_{t,t+1} = R^n_{t,t+1} \tag{3.9}$$

Proof. See Appendix B.3. □

3.3.3 Market Equilibrium

We define market equilibrium for an arbitrary markup policy.[8]

Definition 1. *Given a sequence of ledger policies, $\underline{\boldsymbol{\mu}}$, a competitive equilibrium is a collection of price and return sequences, $(\underline{\mathbf{R}}^b, \underline{\mathbf{R}}^m, \underline{\mathbf{r}}^d, \underline{\mathbf{r}}^b, \underline{\mathbf{q}}^s)$, and agent choice sequences, $(\{\underline{\boldsymbol{\eta}}_\tau\}_{\tau\leq 2},\ \underline{\boldsymbol{x}},\ \underline{\boldsymbol{y}},\ \underline{\boldsymbol{c}}_1,\ \underline{\boldsymbol{c}}_2)$, such that: (i) given prices, the agent choices solve optimization problem* (3.3)*, (ii) given prices, the financial intermediary choices solve equation* (3.9)*, and (iii) market clearing is satisfied for the goods market on each trading technology, the IOU market, the money market, and the equity market:*

$$\begin{aligned} Y^n_t &= X^n_{0,t} + C^n_{1,t} + C^n_{2,t}, \quad \forall n \in \{o, p\} \\ B_t &= I_{0,t} \\ M_t &= \bar{M}_t \\ S_t &= 1 \end{aligned}$$

where at time t on trading technology n:

- $Y^n_t := \eta^n_{1,t} \sum_{n_0} \eta^{n_0}_{0,t-1} y^{(n_0,n)}_{1,t}$ *is aggregate output,*
- $X^n_{0,t} := \eta^n_{0,t} \sum_{n_1} \eta^{n_1}_{1,t+1} x^{(n,n_1)}_{0,t}$ *is aggregate input good purchases,*
- $I_{0,t} := \sum_{n_0,n_1} \eta^{n_0}_{0,t} \eta^{n_1}_{1,t+1} \mathcal{R}^{bn}_{t,t+1} (1 + \mu^{n_0}_t) x^{(n_0,n_1)}_{0,t}$ *is aggregate IOU issuance,*
- $C^n_{1,t} := \eta^n_{1,t} \sum_{n_0} \eta^{n_0}_{0,t-1} c^{(n_0,n)}_{1,t}$ *is aggregate consumption by age-1 agents,*

[8] In numerical examples, we will ultimately focus on the steady-state limit with a fixed markup.

- $C_{2,t}^n := \eta_{2,t}^n \sum_{n_0,n_1} \eta_{0,t-2}^{n_0} \eta_{1,t-1}^{n_1} c_{2,t}^{(n_0,n_1,n)}$ *is aggregate consumption by age-*2 *agents.*

To develop intuition for the model, we discuss the characterization of each market's equilibrium separately.

3.3.4 Goods Markets

A major difference compared to Section 2 is that agents now choose on which platform to trade goods, which makes goods market clearing more involved. At each time t, the total goods supplied to each market in the morning subperiod is predetermined by the input good and trading marketplace choices that age-1 agents made in the previous period when they arrived in the economy. The relative goods price, ϵ_t, adjusts to ensure that age-0 and age-2 buyers spread out on-and-off-platform and each market clears. Thus, the goods market clearing condition on each market $n \in (o, p)$ becomes:

$$\eta_{1,t}^n \sum_{n_0} \eta_{0,t-1}^{n_0} y_{1,t}^{(n_0,n)} = \eta_{0,t}^n \sum_{n_1} \eta_{1,t+1}^{n_1} x_{0,t}^{(n,n_1)} + \eta_{1,t}^n \sum_{n_0} \eta_{0,t-1}^{n_0} c_{1,t}^{(n_0,n)} + \eta_{2,t}^n \sum_{n_0,n_1} \eta_{0,t-2}^{n_0} \eta_{1,t-1}^{n_1} c_{2,t}^{(n_0,n_1,n)}$$

where the left-hand-side is total produced goods brought to the market and the right-hand-side is the total purchases of goods as inputs or for consumption. Imposing the household choices and rearranging gives the closed form expression for ϵ_t in Theorem 2 below.

Theorem 2. *Suppose that* $\frac{1+\alpha\gamma_0}{1-\alpha} = 1 + \gamma_2$ *and* $(\zeta_0^n)^{\gamma_0} = (\zeta_2^n)^{\gamma_2}$.[9] *Then the real exchange rate satisfies:*

$$\epsilon_t = \left[\frac{\zeta_1^{\gamma_1}}{\zeta_2^{\gamma_2}} \frac{(1+\mu_t)^{-\gamma_1(1-\beta)+1+\gamma_2}}{(R_{t-1,t}^b / R_{t-1,t}^m)^{1+\gamma_2}} \left(\frac{1 - \frac{(1-\beta)(1-\alpha)}{1+\mu_t}}{1-(1-\beta)(1-\alpha)} \right) \right]^{\frac{1}{\frac{\alpha}{1-\alpha}(\gamma_1+1)+\gamma_1\beta+1+\gamma_2}} \tag{3.10}$$

Proof. See Appendix B.4. ☐

[9]These assumptions ensure that age-0 and age-2 agents have sufficiently similar marketplace choice functions that we can get a closed form expression for ϵ_t.

Theorem 2 illuminates how the variables affect the equilibrium prices. First, observe that, holding all else constant, the real exchange rate is decreasing in the excess return on bonds relative to money, $R^b_{t-1,t}/R^m_{t-1,t}$, and increasing in the markup, μ_t. This is because a decrease in $R^b_{t-1,t}/R^m_{t-1,t}$ and an increase in μ both encourage buyers to choose the public marketplace instead of the private platform, which increases demand on the public marketplace and so its relative price.

3.3.5 Asset Markets

Theorem 3 implicitly characterizes the equilibrium in the asset markets for IOUs, money, and platform equity.

Theorem 3. *Suppose that* $\frac{1+\alpha\gamma_0}{1-\alpha} = 1+\gamma_2$ *and* $(\zeta_0^n)^{\gamma_0} = (\zeta_2^n)^{\gamma_2}$. *Then the bond return, money return, and equity price satisfy:*

$$\eta^p_{2,t+1}\beta\Pi_{1,t} = \sum_{n_1}\left(\sum_{n_0}\eta^{n_0}_{0,t}\eta^{n_1}_{1,t+1}\epsilon^{n_0}_t\frac{R^b_{t-1,t}}{R^{n_0}_{t-1,t}}(1+\mu^{n_0}_{t+1})x^{(n_0,n_1)}_{0,t} + \eta^o_{0,t+1}\eta^{n_1}_{1,t+2}\frac{\epsilon_{t+1}}{R^m_{t,t+1}}x_{0,t+1}\right) + q^s_t \tag{3.11}$$

$$R^m_{t,t+1} = \left(\frac{\bar{M}_t}{\bar{M}_{t+1}}\right)\left(\frac{\eta^o_{2,t+2}\beta\Pi_{1,t+1} + \frac{1}{R^m_{t,t+2}}X^o_{0,t+2}}{\eta^o_{2,t+1}\beta\Pi_{1,t} + \frac{1}{R^m_{t,t+1}}X^o_{0,t+1}}\right) \tag{3.12}$$

$$q^s_t = \frac{1}{R^b_{t,t+1}}\left(\pi^s_{t+1} + q^s_{t+1}\right) \tag{3.13}$$

where $\Pi_{1,t} = \sum_{n_0,n_1}\eta^{n_0}_{0,t-1}\eta^{n_1}_{1,t}\pi^{(n_0,n_1)}_{1,t}$ *is aggregate agent profit at time* t *and* $X^o_{0,t} := \sum_{n_1}\eta^o_{0,t}\eta^{n_1}_{1,t+1}\epsilon_t x^{(o,n_1)}_{0,t}$ *is aggregate input purchases on marketplace* o *at time* t.

Proof. See Appendix B.4. □

Although the equations in Theorem 3 appear involved, they have a clear interpretation. The return on money satisfies equation (3.12), which is the ratio of money demand growth to money supply growth. This implies that, in a steady state with constant money growth rate g^M, we have the familiar relationship that $R^m_{t,t+1} = 1/g^M$. As is standard in "currency-in-advance" models, the environment has money neutrality in the sense that the level of money supply does not affect real variables. However, it does not have super-neutrality; the growth rate of money does affect real variables by impacting agent trading and borrowing decisions.

The return on IOUs satisfies (3.11). The left-hand-side of this equation denotes the supply of deposits that can be used to purchase ledger assets. The right-hand-side denotes the demand for ledger assets. Evidently, a decrease in the fraction of agents wanting to trade on the private platform, $\eta^p_{2,t+1}$, leads to a decrease in the supply of deposits available to purchase ledger assets (a decrease in supply in the loan market). Likewise, a decrease in input good purchases, $x^{(n_0,n_1)}_{0,t}$, leads to a decrease in the issuance of IOUs (and a decrease in demand in the loan market).

Finally, the price of equity is given by future dividends and capital gains discounted by the bond rate.

3.3.6 Numerical Illustration

To help illustrate these forces, we plot the steady state equilibrium in Figure 2 as a function of the platform markup. The blue dashed line shows the partial equilibrium allocations as μ varies when the interest rate is fixed at 3% and there is an exogenous external source of credit. The black solid line shows general equilibrium with an adjusting interest rate such that the credit market clears (i.e. IOU-borrowing equals IOU-saving). There a sharp difference between the partial and general equilibrium outcomes. In partial equilibrium, a higher markup leads to fewer agents going to the platform and more agents going to the public marketplace, which ultimately increases the real exchange rate (i.e., makes the public marketplace relatively cheaper). In general equilibrium, a higher markup leads to a higher equilibrium interest rate, R^b, less substitution away from the platform, and a greater decline in output. This is because the interest rate needs to increase to make the platform ecosystem sufficiently attractive to ensure there are enough IOU-savings to clear the credit market. In this sense, the platform trying to extract rents in the goods market pushes up the interest rate in the credit market and so restricts production in the economy.

Although our numerical example model is an illustrative experiment rather than a calibrated quantitative exercise, we can none-the-less see some quantitative features of the equilibrium. One feature is that interest rates are high and very sensitive to η_0, η_1, and η_2. This is because age-0 agents are only able to borrow if other agents are willing to save and spend using their uncollaterlized IOUs. There is no alternative collateralized debt market for borrowers to access. Quantitatively, this means that, in order to target an uncollaterlized interest rate less 10% at a markup of 5%, the equilibrium is such that η_0 and η_2 are close to 1. Finally, note that the apparent monotonicity in the η-functions with respect to μ does not persist higher levels of μ.

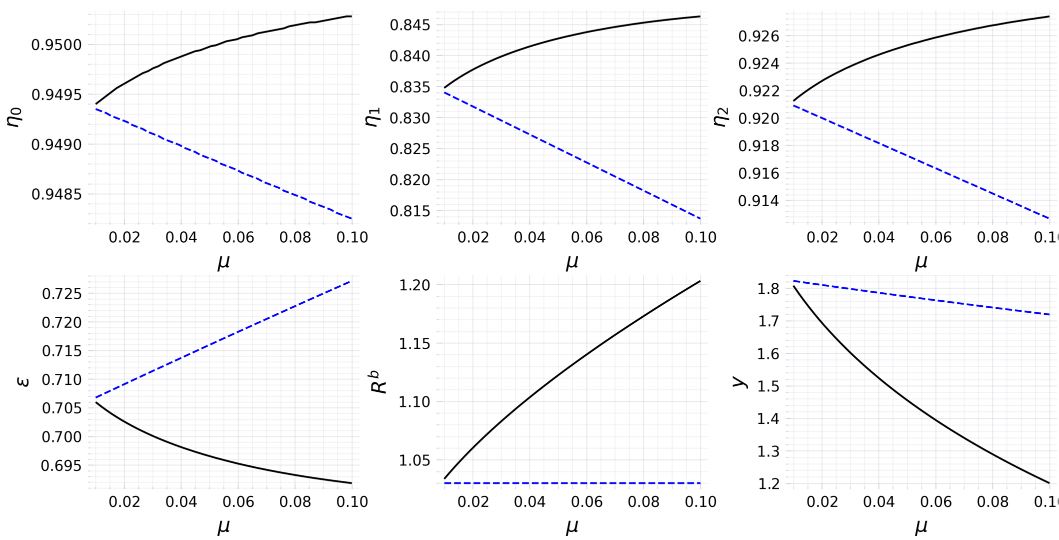

Figure 2: Equilibrium for $\mu \in [0, 0.1]$.

Other variables are $z = 1$, $\alpha = 0.6$, $\beta = 0.9$, $\gamma_1 = 1.9$, $\gamma_2 = 1.9$, and $\zeta = 1.0$. Black line denotes general equilibrium. The blue dashed line denotes partial equilibrium with fixed interest rate R^b.

3.4 Default and Incentive Compatibility

We now return to the question of incentive compatibility in the dynamic model. Agents can only access financial intermediaries if they repay loans. So, if agents default and deposit into a financial intermediary, then they are restricted to the monetary system. The agent problems are very similar to Subsection 3.3, so we defer the details to Appendix B. Theorem 4 states the main result: the incentive compatibility constraint to deter agents from defaulting.

Theorem 4. *No agents default if the following incentive compatibility constraint is satisfied* $\forall t \geq 0$*:*

$$\frac{\check{\pi}_{1,t}}{\pi_{1,t}} \leq \left(\frac{\bar{\nu}_{2,t+1}}{\check{\nu}_{2,t+1}}\right)^{\beta}, \tag{3.14}$$

where $\pi_{1,t}$ *is agents' profit at age* 1 *if they repay,* $\check{\pi}_{1,t}$ *is agent profit at age* 1 *if they default,* $\bar{\nu}_{2,t+1} := \left(\sum_{n_2}\left(\frac{\zeta_2^{n_2}\mathcal{R}^{dn_2}_{t,t+1}}{(1+\mu^{n_2}_{t+1})\epsilon^{n_2}_{t+1}}\right)^{\gamma_2}\right)^{1/\gamma_2}$ *is the average marginal utility of wealth at age* 2 *if the agent repays at age* 1*, and* $\check{\nu}_{2,t+1} := \frac{\zeta_2^{o}\mathcal{R}^{o}_{t,t+1}}{\epsilon^{o}_{t+1}}$ *is marginal utility of wealth at age* 2 *if the agent defaults at age* 1 *and can only trade with public money on the public marketplace. Imposing equilibrium, the incentive compatibility constraint becomes* $\forall t \geq 0$*:*

$$1+\mu_{t+1} \leq \frac{\zeta_2^{p}\epsilon_{t+1}R^{b}_{t,t+1}/R^{m}_{t,t+1}}{\left(\left(\chi^{-1}\min\{\epsilon^{n_0}_{t-1}R^{b}_{t-2,t-1}/R^{n_0}_{t-2,t-1},1\}R^{b}_{t-1,t}\right)^{\frac{\alpha\gamma_2}{1-\alpha}}-1\right)^{1/\gamma_2}}. \tag{3.15}$$

For $\chi > 0$*, the incentive compatibility constraint will be satisfied for sufficiently large* ζ_2*.*

Proof. See Appendix B.5. □

Contract enforcement is similar to that of the two-period model in Section 2. If agents trade using the platform, then they have to repay because the platform forces them to use the ledger. If they trade using the public marketplace, they can trade using money and subsequently default. As in Section 2, the difficulty for defaulting agents is that the platform has broken the universal liquidity of government money. Once an agent defaults, they are unable to bring money back to the financial intermediaries. This means they lose access to the assets on the ledger and the ability to trade on the platform. These conditions are stated formally in the IC constraint (3.14), which says that agents repay if the benefit of having access to ledger assets and the platform trading system at age 2 is greater than the additional profit from defaulting. This ultimately places an upper bound on how large the markup can be, as described by equation (3.15). In this sense, equation (3.15) is the dynamic analogue of equation (2.5) in Section 2.

To help illuminate the enforcement in the dynamic model, we consider a collection of special cases with different liquidity of the outside option.

Case: No government money. Suppose that $\beta = 1$ and there is no government money (so there is no universally liquid off-ledger payment option and no cash-in-advance constraint on the public marketplace). In this case, the presence of a ledger is sufficient to incentivise repayment, regardless of what the platform does. To see this, observe that when $\beta = 1$, agents do not value consumption at $t = 1$ and do not engage in barter trade with other agents of their generation for perishable goods. Instead, they only trade goods to agents of other generations in exchange for financial assets. Because there is no money, the age-1 agents selling goods can only receive newly issued IOUs from the age-0 agents and old IOUs from the age-2 agents purchasing goods. Since all IOUs are on the ledger, there is no way for age-1 agents to default, regardless of whether the platform excludes defaulting agents or not. In this sense, the agents are locked into the ledger payment system because they need a way to store wealth.

Case: Government money. Suppose that now we introduce government money into the model (so there is a universally liquid off-ledger payment option and a cash-in-advance constraint on the public marketplace). In this case, the agents are no longer locked into the ledger payment system because they can use government money to make side spot trades and subsequently default. The presence of a platform is now necessary to disincentivize government money storage by breaking its universal liquidity, as outlined in Theorem 4.

Case: Government money and ledger IOUs payments off-platform. So far, we have assumed that government money is used on the public marketplace and that defaulting agents cannot bring government money back to the platform. An alternative arrangement is that the platform allows IOUs to be used as payment on the public marketplace and prevents all agents from bringing public money back to the platform. This can be thought of as a closer analogue to the restrictions in Section 2. In this case, agents have no reason to use public money (so long as μ is set sufficiently low that IOUs are repaid) and so ledger payment dominates both markets, as in Section 2.

3.5 Platform Problem

We can now write down the problem of the private platform. Suppose the economy starts with an initial collection of age-1 agents with loans and goods $(b_{0,0}, y_{1,0})$ and a collection of age-2 agents with wealth $(a_{2,0})$ in a collection of financial intermediaries. We consider a platform that takes interest rates as given but internalizes equilibrium in the goods markets.[10] Specifically, taking the interest rate processes $\{\underline{\boldsymbol{R}}^b, \underline{\boldsymbol{R}}^m\}$ as given, the platform chooses a sequence

[10]We set up the platform problem in this way for a number of reasons. First, we believe it is realistic to study a trading platform that internalizes how their markups affect equilibrium on their goods market but does not internalize how their markups affect the discount factor that is used to price their equity. Second, for log utility, allowing the platform to internalize their impact on the household's discount factor leads to the result that the platform is indifferent about the amount of output produced in the problem because low output is exactly offset by a high household marginal value of output. These issues (and additional issues about the choice of numeraire) are discussed in Kelsey and Milne (2006) and Böhm (1994).

of markups $\underline{\boldsymbol{\mu}}$ to maximise their equity price by solving problem:

$$
\begin{aligned}
q_0^s = \max_{\underline{\boldsymbol{\mu}}} & \left\{ \sum_{t=0}^{\infty} \xi_{0,t} \pi_t^s \right\} \quad s.t. \\
& \pi_t^s = \mu_t \eta_{1,t}^p \sum_{n_0} \eta_{0,t-1} y_{1,t}^{(n_0,p)}, \quad t \geq 1 \\
& \pi_0^s = \mu_0 \eta_{1,0} y_{1,0} \\
& \text{Agent choices: } (3.4), (3.5), (3.6), (3.7), (3.8), \\
& \text{Equilibrium prices: } (3.10), (3.12), (3.13), (3.11)
\end{aligned}
\tag{3.16}
$$

where $\xi_{0,t} = \prod_{j=0}^{t} (R_{j,j+1}^b)^{-1}$ is the household SDF.

The market equilibrium in the economy is the collection of price and return sequences, $(\underline{\mathbf{R}}^b, \underline{\mathbf{R}}^m, \underline{\mathbf{r}}^d, \underline{\mathbf{r}}^b, \underline{\mathbf{q}}^s)$, and agent choice sequences, $(\{\underline{\boldsymbol{\eta}}_\tau\}_{\tau \leq 2}, \underline{\boldsymbol{x}}, \underline{\boldsymbol{y}}, \underline{\boldsymbol{c}}_1, \underline{\boldsymbol{c}}_2)$ such that Definition 1 is satisfied for the choice of markup sequence that solves the platform problem (3.16). That is, the market equilibrium that emerges from the platform's optimization problem.

Compared to Section 2, the platform problem is now more involved because the platform considers how its choice of markup affects agents' choices of where to trade (η_0, η_1, η_2). The resulting first order condition for the platform's choice of μ_t is:

$$
\begin{aligned}
0 = \xi_{0,t} & \frac{\partial \pi^s(\mu_t, \mu_{t-1}, \epsilon_t, \epsilon_{t-1}, \mathcal{R}_{\mathbf{t}})}{\partial \mu_t} + \xi_{0,t+1} \frac{\partial \pi^s(\mu_{t+1}, \mu_t, \epsilon_{t+1}, \epsilon_t, \mathcal{R}_{\mathbf{t+1}})}{\partial \mu_t} \\
& + \xi_{0,t} \frac{\partial \pi^s(\mu_t, \mu_{t-1}, \epsilon_t, \epsilon_{t-1}, \mathcal{R}_{\mathbf{t}})}{\partial \epsilon_t} \frac{\partial \epsilon(\mu_t, R_{t-1,t}^b, R_{t-1,t}^m)}{\partial \mu_t} \\
& + \xi_{0,t+1} \frac{\partial \pi^s(\mu_{t+1}, \mu_t, \epsilon_{t+1}, \epsilon_t, \mathcal{R}_{\mathbf{t+1}})}{\partial \epsilon_t} \frac{\partial \epsilon(\mu_t, R_{t-1,t}^b, R_{t-1,t}^m)}{\partial \mu_t}
\end{aligned}
$$

which we fully characterize in Appendix B.6. Evidently, the platform faces the standard "monopoly" tradeoff that increasing the markup increases their profit per trade ($\uparrow \mu_t$) but also discourages agents from coming to the platform ($\downarrow \eta_{1,t}$) and decreases production ($\downarrow y_{1,t}^{(n_0,p)}$). These tradeoffs appear in the first line of the first order condition. In addition, the platform has to consider how their decisions affect the real exchange rate, which is captured by the last two lines.

Credit Fragility: In Section 2, we explored how the platform's share of trade affects their willingness to set up the ledger. We have now endogenized trade shares, which illustrates how exchange rates, interest rates, and platform trading advantage impact the platform's willingness to set up a ledger and enforce contracts. From the IC constraint (3.14), we can see that a low ζ_2 makes it unprofitable for the platform to set up the ledger while age-2 elasticity of substitution, γ_2, determines how the exchange rates and interest rates influence the profitability of setting up the ledger.

Monetary Policy: Figure 3 depicts the platform's optimal markup charge, μ, for different rates of constant public money supply growth, g^M. The blue dashed line, μ^*, indicates the platform's choice of markup if the platform need not care about the incentive constraint. The incentive constraint requires that the markup is below the red dashed line, $\bar{\mu}^{IC}$. If either the IC constraint or competition with public money requires the markup to be negative, then the platform derives negative value from running a non-default ledger and so would not set up the ledger system. In our illustrative example, this occurs at around an inflation rate of 2% per annum. In this sense, the uncollateralized credit equilibrium is "fragile"; strong competition from the dollar – due to low dollar-inflation – makes it too costly for the platform to set up a no-default ledger. This implies that a platform ledger credit system is more likely to be set up in countries where the government runs high inflation.

3.6 Discussion of Regulatory Options

We have shown that a tech platform provides and "backs" a common settlement ledger in an unregulated economy if they have a sufficiently dominant trading technology. This incentivizes the financial sector to coordinate on enforcement, but also gives the platform market power to extract rents. In this sense, regulators have a "natural monopoly" dilemma. We discussed one po-

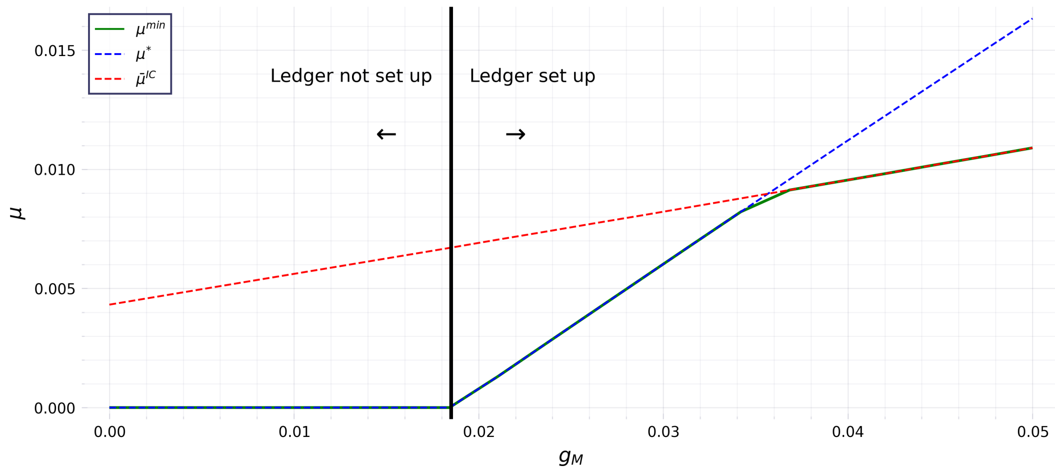

Figure 3: Markup as a function of public money supply growth rate

The blue dashed line depicts the platform's markup choice, μ^*, if they are not constrained by having to ensure no-default. The red dashed line depicts the maximum markup $\bar{\mu}$ at which financial intermediaries are deterred from defaulting. The green line depicts the markup for the equilibrium chosen by the platform, μ^{min}.

tential government response in Subsection 2.4 when we explored introducing a public ledger. We close the paper by discussing an alternative policy response: regulated competition between platforms and a public ledger option.

Environment changes: The environment is the same as in Subsection 3.1 but with the following changes. There are now two private platforms, labeled $n \in \{1, 2\}$. There is no public marketplace nor public currency. Both platforms manage their own ledger, charge a markup μ^n, and have an average trading advantage ζ^n. For simplicity, we assume that each platform chooses a fixed μ^n at time $t = 0$ for all periods. We let $\eta_{\tau,t}$ denote the fraction of agents at age τ choosing platform 1.

Since there is no public dollar, agents cannot undertake side payments; all transactions are observed by one of the two platforms. In other words, in this new environment the only way producers can default is by writing a contract on the ledger provided by platform n, then defaulting and trading on the other platform n'. We use the currency provided by ledger 1 as the numeraire for asset pricing. So, ϵ_t now refers to the real exchange rate from tokens provided by platform 1 to tokens provided by platform 2.

Regulation: The regulator allows the platforms to bargain at time $t = 0$ over committing to exclude financial intermediaries who allow their depositors to default on contracts on the other ledger. We assume that financial intermediaries face no borrowing constraints nor commitment problems during this bargaining and the Nash bargaining protocol is followed. The regulator does not allow the platforms to collude on setting markups at times $t \geq 0$.

Platform competition at $t = 0$: For $t > 0$, the equilibrium is the same as in the Subsection 3.3 but with $(1-\mu_t)$ replaced by $(1-\mu_t^1)/(1-\mu_t^2)$. Let q_0^{En} denote the price of equity in platform n at $t = 0$ under cooperation on enforcement for $t \geq 0$ and let $\tilde{q}_0^{En}$ denote price of equity in platform n at $t = 0$ if there is no cooperation on enforcement for $t \geq 0$. The surplus from cooperation is $S = q_0^{E2} - \tilde{q}_0^{E2} + q_0^{E1} - \tilde{q}_0^{E1}$. If the surplus is positive, then the platforms bargain over coordination on contract enforcement. We assume that platform 2 makes a (positive or negative) transfer T to platform 1 at time 0 and the payment is determined by a Nash Bargaining protocol. In particular, we have:

$$
\begin{aligned}
T = & \arg\max_{T} \left\{ \left(q_0^{E2} - T - \tilde{q}_0^{E2} \right) \left(q_0^{E1} + T - \tilde{q}_0^{E1} \right) \right\} \\
= & \frac{1}{2} \left(q_0^{E2} - \tilde{q}_0^{E2} - (q_0^{E1} - \tilde{q}_0^{E1}) \right)
\end{aligned}
$$

If the surplus is negative, then the platforms do not coordinate. Proposition 3 shows that, when platforms are symmetric, the outcome of the bargaining is contract enforcement on both ledgers whereas when platforms are asymmetric the dominant platform provides the ledger.

Proposition 3. *We have the following:*

(i) If the platforms are symmetric, then the outcome of the bargaining at time 0 *is that contracts are enforced on both ledgers and no transfer is made.*

(ii) If $\zeta := \zeta_\tau^1/\zeta_\tau^2 > 1$, then for χ and ζ sufficiently high, the outcome of the bargaining at $t = 0$ is platform 2 pays a transfer to platform 1 for providing the ledger in the economy.

Proof. See Appendix B. □

The first part of the proposition says that contract enforcement coordination is straightforward when the platforms are similar. The second part of theorem reinforces the market structure result in Section 2. Ultimately, it shows that the only viable ledger operators are those that also possess a platform trading technology. In other words, there is a natural bundling between offering ledger and trading services. This implies that a financial intermediary with no trading technology (which would be modeled as $\zeta_2 = 0$ in our environment) would never provide the ledger in equilibrium.

4 Conclusion

In this paper, we model the strategic decision making of a private controller of the ledger used for settling transactions and writing contracts. We find that in an unregulated economy "BigTech" platforms are likely to provide "FinTech" services. This brings both benefits and costs. Tech platforms can expand uncollateralized credit across a supply chain by exploiting their control of the payment system to break the universal liquidity of public money and better coordinate the financial system to enforce contracts. However, Tech platforms will also use their control of the ledger to increase their market power and charge high markups. We see these issues playing out in China where tech platforms Alibaba and WeChat have created a well-functioning payment system with very limited competition.

Ultimately, our model suggests that ledgers may need to be regulated like other natural monopolies. This could include restrictions on when ledgers can cooperate on contract enforcement and compete on markups. It could also include a competing public option in the form a programmable Central

Bank Digital Currency (CBDC) ledger. We consider further modeling of the government's regulatory options as important future work.

References

Abadi, J. and Brunnermeier, M. (2024). Blockchain economics. Technical report, National Bureau of Economic Research.

Ahnert, T., Hoffmann, P., and Monet, C. (2022). The digital economy, privacy, and CBDC.

Aiyagari, S. R. and Wallace, N. (1991). Existence of steady states with positive consumption in the Kiyotaki-Wright model. *The Review of Economic Studies*, 58(5):901–916.

Benigno, P., Schilling, L. M., and Uhlig, H. (2019). Cryptocurrencies, currency competition, and the impossible trinity. Technical report, National Bureau of Economic Research.

Bigio, S. (2023). A theory of payments-chain crises. Technical report, National Bureau of Economic Research.

Böhm, V. (1994). The foundation of the theory of monopolistic competition revisited. *Journal of Economic theory*, 63(2):208–218.

Brunnermeier, M. and Payne, J. (2023). Platform, ledgers, and interoperability. *Princeton Working Paper*.

Brunnermeier, M. K. and Sannikov, Y. (2019). International monetary theory: A risk perspective. Technical report.

Chiu, J. and Koeppel, T. (2025). Platform-based digital currencies.

Chiu, J. and Wong, R. (2020). Payments on digital platforms: Resilience, interoperability and welfare.

Cong, L. W., Li, Y., and Wang, N. (2020). Token-based platform finance. *Fisher College of Business Working Paper*, (2019-03):028.

Cong, L. W., Li, Y., and Wang, N. (2021). Tokenomics: Dynamic adoption and valuation. *The Review of Financial Studies*, 34(3):1105–1155.

Copestake, A., Kirti, D., Peria, M. S. M., and Zeng, Y. (2025). Integrating fragmented networks: The value of interoperability in money and payments.

Diamond, P. A. (1965). National debt in a neoclassical growth model. *The American Economic Review*, 55(5):1126–1150.

Fernández-Villaverde, J., Sanches, D., Schilling, L., and Uhlig, H. (2020). Central bank digital currency: Central banking for all? Technical report, National Bureau of Economic Research.

Fernández-Villaverde, J. and Sanches, D. R. (2018). On the economics of digital currencies.

Freeman, S. (1996a). Clearinghouse banks and banknote overissue. *Journal of Monetary Economics*, pages 101–115.

Freeman, S. (1996b). The payments system, liquidity, and rediscounting. *The American Economic Review*, pages 1126–1138.

Garber, P., Green, B., and Wolfram, C. (2021). Digital collateral. (28724).

Hayek, F. A. (1976). *Choice in Currency: A Way to Stop Inflation*, volume 48. Ludwig von Mises Institute.

Kahn, C. and van Oordt, M. R. (2022). The demand for programmable payments. Technical report, http://dx.doi.org/10.2139/ssrn.4218966.

Kahn, C. M., Rivadeneyra, F., and Wong, T.-N. (2019). Should the central bank issue e-money? *Money*, pages 01–18.

Kareken, J. and Wallace, N. (1981). On the indeterminacy of equilibrium exchange rates. *The Quarterly Journal of Economics*, 96(2):207–222.

Kehoe, T. J. and Levine, D. K. (1993). Debt-constrained asset markets. *The Review of Economic Studies*, 60(4):865–888.

Keister, T. and Sanches, D. R. (2019). Should central banks issue digital currency?

Kelsey, D. and Milne, F. (2006). Externalities, monopoly and the objective function of the firm. *Economic Theory*, 29:565–589.

Kocherlakota, N. R. (1998). Money is memory. *Journal of Economic Theory*, 81(2):232–251.

Lagos, R., Rocheteau, G., and Wright, R. (2017). Liquidity: A new monetarist perspective. *Journal of Economic Literature*, 55(2):371–440.

Lagos, R. and Wright, R. (2005). A unified framework for monetary theory and policy analysis. *Journal of political Economy*, 113(3):463–484.

Liu, L., Lu, G., and Xiong, W. (2022). The big tech lending model. (30160).

Lucas Jr, R. E. and Stokey, N. L. (1985). Money and interest in a cash-in-advance economy. Technical report, National Bureau of Economic Research.

Ozdenoren, E., Tian, Y., and Yuan, K. (2025). Platform money.

Rishabh, K. and Schäublin, J. (2021). Payment fintechs and debt enforcement.

Samuelson, P. A. (1958). An exact consumption-loan model of interest with or without the social contrivance of money. *Journal of political economy*, 66(6):467–482.

Schilling, L. and Uhlig, H. (2019). Some simple bitcoin economics. *Journal of Monetary Economics*, 106:16–26.

Smith, A. (1776). *The wealth of nations*.

Stiglitz, J. E. (1974). Incentives and risk sharing in sharecropping12. *The Review of Economic Studies*, 41(2):219–255.

Svensson, L. E. (1985). Currency prices, terms of trade, and interest rates: A general equilibrium asset-pricing cash-in-advance approach. *Journal of International Economics*, 18(1-2):17–41.

A Supplementary Proofs For Section 2 (Online Appendix)

A.1 Proof of Proposition 1

Proof. The key equilibrium objects in the $t = 1$ market are (p, q), the price of endowment goods and IOUs in terms of output goods, and the key equilibrium objects in the $t = 0$ market are (φ, R), the fraction of savers who store endowment goods (which ultimately pins down ϕ the fraction of IOUs that are repaid) and the interest rate on IOUs. We proceed by backwards induction. We first characterize the equilibrium (p, q) at $t = 1$ for different values of (φ, R). We then characterize the possible equilibrium values of (φ, R) at $t = 0$.

The $t = 1$ market. Suppose that (φ, R) is the outcome in the $t = 0$ market. All production decisions are taken at $t = 0$ so quantities cannot adjust at $t = 1$. Instead, the prices adjust to ensure competitive market clearing (sometimes referred to as "resource-in-the-market" or "cash-in-the-market" pricing).

If $\varphi = 1$, then only endowment goods are stored so savers enter with endowment goods and no production takes place. In this case, savers trade endowment goods with each other at a 1:1 rate.

If $\varphi = 0$, then no endowment goods are stored, so savers enter with IOUs and producers enter with output goods. In this case, producers potentially trade output goods with each at a 1:1 rate and savers trade IOUs for output goods at the price $q = 1/(\phi R)$ at which producers are indifferent between purchasing IOUs and output goods. (Observe that savers do not trade IOUs with each other because they need to consume in the morning of $t = 1$).

Finally, if $\varphi \in (0, 1)$, then both endowment goods and output goods are potentially traded in equilibrium, so the equilibrium is more involved. Since the market is competitive, the price is determined by relative scarcity. If the quantity of production goods is greater than the quantity of endowment goods, $(1 - \varphi)z > \varphi$ (equivalently $0 < \varphi < z/(1 + z)$), then endowment goods are relatively scarce. In this case, the prices are $p = (z - R)/z < 1$ and $q = 1/(\phi R)$ so all savers trade endowment goods for output goods and producers are indifferent between trading output goods for other output goods, IOUs, and endowment goods. If the quantity of endowment goods is greater than the quantity of production goods, $\varphi > (1 - \varphi)z$ (equivalently $z/(1 + z) < \varphi < 1$), then output goods are relatively scarce. In this case, the price is $p = 1$ and $q = 1/(\phi R)$ so all output producers trade output goods with savers and all savers are indifferent between trading with producers and with each other.

Observe that the price p is lower (i.e. better for endowment good holders) when endowment goods are scarce because then producers end up giving all the surplus from defaulting to the endowment good holders.

The $t = 0$ equilibrium. We now look for the values of (φ, R) that are possible equilibria at $t = 0$ given agent behavior at $t = 1$.

First observe that $(\varphi = 1, R < \infty)$ is an equilibrium. In this case, there is no production so the producer's problem is irrelevant. This also means that no loans are repaid so $\phi = 0$, which implies that savers strictly prefer endowment goods and so (2.4) is satisfied.

Now consider $\varphi \in [0, 1)$. If $(1 - \varphi)z > \varphi$, then, at $t = 1$, endowment goods are scarce and producer indifference prices the assets. So, the prices are $p = (z-R)/z$ and $q = 1/(\phi R)$. Producers default if they trade with endowment good savers and so the probability of default is $1 - \phi = \varphi/((1 - \varphi)z)$. Since some savers store endowment goods and some savers buy IOUs, in equilibrium savers must be indifferent between the two options. This implies that:

$$\begin{aligned} 1/q &= 1/p \\ \Rightarrow \phi R &= z/(z - R) \\ \Rightarrow R &= \frac{z \pm \sqrt{z^2 - 4z/\phi}}{2} \end{aligned}$$

which cannot have a real solution since $z \in (1, 2)$ implies that:

$$\begin{aligned} (1 - \phi)z < (1 - \phi)2 < 4 \\ \Rightarrow z^2 < 4z/(1 - \phi) \end{aligned}$$

Thus, by proof by contradiction, there is no equilibrium (φ, R) with $0 < \varphi \leq z/(1 + z)$.

Alternatively, if $(1 - \varphi)z \leq \varphi$, then, at $t = 1$, $p = 1$, $q = 1/(\phi R)$ and all producers trade for endowment goods, which implies that $\phi = 0$. In this case, indifference in the $t = 0$ IOU market requires:

$$\phi R = 1$$

but this cannot be satisfied because $\phi = 0$. Thus, by proof by contradiction, there is no equilibrium (φ, R) with $z/(1 + z) < \varphi \leq 1$. $\square$

A.2 Proof of Proposition 2

Proof. As in Proposition 1, the key equilibrium objects in the $t = 1$ market are (p, q), the price of endowment goods and IOUs in terms of output goods, and the key equilibrium objects in the $t = 0$ market are (φ, R), the fraction of savers who store endowment goods (which ultimately pins down ϕ the fraction of IOUs that are repaid) and the interest rate on IOUs. We need to find conditions on μ such that there is an equilibrium with $\varphi = 0$ (and full repayment $\phi = 1$). We also need to show there is no equilibrium with $\varphi > 0$ (and imperfect repayment $\phi < 1$). We proceed by backwards induction. We first characterize the equilibrium (p, q) at $t = 1$ for different values of (φ, R). We then characterize the possible equilibrium values of (φ, R) at $t = 0$.

The $t = 1$ market. The equilibrium on the public marketplace is the same as in Subsection 2.2 and the proof of Proposition 1. The equilibrium on the platform is that no endowment goods are traded and agents trade output goods and IOUs with price $q = 1/(\phi R)$.

The $t = 0$ market. We start by finding the condition on μ under which $\varphi = 0$ (no default $\phi = 1$) is an equilibrium. In this case, the equilibrium at $t = 1$ on the platform is the price $q = 1/R$ and the equilibrium on the public marketplace is $(p, q) = ((z - R)/z,\ 1/R)$ since endowment goods are scarce. So, the saver at time 0 chooses IOUs if:

$$\begin{aligned} \eta(1-\mu)R + (1-\eta)R &\geq \frac{(1-\eta)z}{z-R} \\ \Rightarrow R^2 - zR + \frac{(1-\eta)z}{\eta(1-\mu)+1-\eta} &\leq 0 \\ \Rightarrow \frac{z - \sqrt{z^2 - \frac{4(1-\eta)z}{\eta(1-\mu)+1-\eta}}}{2} &\leq R \leq \frac{z + \sqrt{z^2 - \frac{4(1-\eta)z}{\eta(1-\mu)+1-\eta}}}{2} \end{aligned}$$

So, the maximum markup for which there exists an IOU interest rate at which the sellers are willing to purchase IOUs must satisfy:

$$\begin{aligned} z^2 &\geq \frac{4(1-\eta)z}{\eta(1-\mu)+1-\eta} \\ \Rightarrow \mu &\leq 1 - \left(\frac{4}{z} - 1\right)\frac{1-\eta}{\eta} \end{aligned} \tag{A.1}$$

So, if (A.1) is satisfied, then the equilibrium at $t = 0$ is:

$$\varphi = 0, \;\; R = \frac{z + \sqrt{z^2 - \frac{4(1-\eta)z}{\eta(1-\mu)+1-\eta}}}{2} < z.$$

Finally, we show that there is no equilibrium with $\varphi > 0$. From the point of view of the saver, they price loans assuming repayment because the platform has guaranteed the IOUs. So, in this case, the equilibrium at $t = 1$ on the platform is the price $q = 1/R$ and the equilibrium at $t = 1$ on the public marketplace is $(p, q) = (1, 1/R)$ since endowment goods are plentiful. Thus, savers choose IOUs if μ satisfies (A.1) because:

$$\eta(1-\mu)R + (1-\eta)R \geq \frac{(1-\eta)z}{z-R} > 1-\eta$$

and so $\varphi = 0$. So, by proof by contradiction, there is no equilibrium with $\varphi > 0$.

□

Proof of Corollary 2. (i) If the government offers a payment CBDC ledger that respects privacy and forces the platform to accept payment using CBDC, then agents can always undertake spot trade regardless of whether endowment goods are stored. As a result, all agents on the public marketplace use CBDC and default.

(ii) This follows immediately from observing that every trade goes through the ledger. □

B Supplementary Proofs For Section 3 (Online Appendix)

B.1 Discrete Choice Problems

This section of the appendix contains the derivation of the discrete choice problems. Since these are standard results, we provided limited detail.

Lemma 2. *Let $\{\zeta^n\}_{n \leq N}$ be a collection of independent draws from $Gu(\mu, \gamma)$, where $\gamma = -\mu\mathcal{E}$ and $\mathcal{E}$ represents the Euler–Mascheroni constant. Let $u(c) =$*

log(c). *Then:*

$$\max_{n\leq N}\{\zeta^n+\varphi^n u(\pi^n)\}\sim Gu\left(\mu+\gamma\log\left(\sum_n(\pi^n)^{\varphi^n/\gamma}\right),\gamma\right) \tag{B.1}$$

and so we have:

$$\mathbb{E}[\max_n\{\zeta^n+\varphi^n\log(\pi^n)\}]=\gamma\log\left(\sum_n(\pi^n)^{\varphi^n/\gamma}\right),$$

$$\mathbb{P}\left(n=argmax_{n'}\left\{\zeta^{n'}+\varphi^{n'}\log(\pi^{n'})\right\}\right)=\frac{(\pi^n)^{\varphi^n/\gamma}}{\sum_{n'}\left(\pi^{n'}\right)^{\varphi^{n'}/\gamma}}$$

Proof. Using the definition of the Gumbel distribution and the independence of the N draws, we have that:

$$\begin{aligned}\mathbb{P}(\max_n\{\zeta^n+\varphi^n u(\pi^n)\}\leq k) &= \prod_n\mathbb{P}(\zeta^n+\varphi^n u(\pi^n)\leq k)\\ &= \exp\left(\sum_n -e^{-(k-\mu)/\gamma}e^{\varphi^n u(\pi^n)/\gamma}\right)\\ &= \exp\left(-e^{-\left(k-\mu-\gamma\log\left(\sum_n e^{\varphi^n u(\pi^n)/\gamma}\right)\right)/\gamma}\right)\end{aligned}$$

which implies result (B.1). From the properties of the Gumbel distribution, the expectation is:

$$\begin{aligned}\mathbb{E}[\max_n\{\zeta^n+\varphi^n\log(\pi^n)\}] &= \left[\mu+\gamma\log\left(\sum_n(\pi^n)^{\varphi^n/\gamma}\right)\right]+\gamma\mathcal{E}\\ &= \gamma\log\left(\sum_n(\pi^n)^{\varphi^n/\gamma}\right)\end{aligned}$$

and the probability of choosing n is:

$$\begin{aligned}\mathbb{P}\left(n=\operatorname{argmax}_{n'}\left\{\zeta^{n'}+\varphi^{n'}\log(\pi^{n'})\right\}\right) &= \frac{e^{\varphi^n u(\pi^n)/\gamma}}{\sum_{n'}e^{\varphi^{n'}u(\pi^{n'})/\gamma}}\\ &= \frac{(\pi^n)^{\varphi^n/\gamma}}{\sum_{n'}\left(\pi^{n'}\right)^{\varphi^{n'}/\gamma}}\end{aligned}$$

$\square$

B.2 Agent Problem and Proof of Theorem 1

Proof of Theorem 1. We solve the problem recursively.

At age 2:, taking price processes as given, an agent with deposits d chooses on which platform to search to solve problem (B.6) below (dropping the explicit i superscript and the time subscript on the choice n_2):

$$\begin{aligned} V_{2,t+2}(d) = \mathbb{E}\left[\max_{c,n}\left\{\tilde{\zeta}^n_{2,t+2} + u(c)\right\}\right] \\ s.t. \quad (1+\mu^n_{t+2})c \leq \mathcal{R}^{dn}_{t+1,t+2} d/\epsilon^n_{t+2}, \quad \forall n \in \{0,1\}, \end{aligned} \tag{B.2}$$

where $V_{2,t+2}$ is the value function at the start of the agent's final period. Using standard discrete choice results (summarized in Lemma 2 in the Appendix), the value function satisfies:

$$V_{2,t+2}(d) = \log\left(\bar{\nu}_{2,t+2} d\right) \tag{B.3}$$

where the average purchasing power at time τ is:

$$\bar{\nu}_{2,t+2} := \left(\sum_{n_2}\left(\frac{\zeta_2^{n_2}\mathcal{R}^{dn_2}_{t+1,t+2}}{(1+\mu^{n_2}_{t+2})\epsilon^{n_2}_{t+2}}\right)^{\gamma_2}\right)^{1/\gamma_2} \tag{B.4}$$

and the fraction of buyers who choose n_2 at time $t+2$ is given by:

$$\eta^{n_2}_{2,t+2} = \frac{\left(\zeta_2^{n_2}\mathcal{R}^{dn_2}_{t+1,t+2}/((1+\mu^{n_2}_{t+2})\epsilon^{n_2}_{t+2})\right)^{\gamma_2}}{\sum_{n_2'}\left(\zeta_2^{n_2'}\mathcal{R}^{dn_2'}_{t+1,t+2}/((1+\mu^{n_2'}_{t+2})\epsilon^{n_2'}_{t+2}\right)^{\gamma_2}}$$

At age 1:, after selling production goods, taking price processes as given, an agent who has made profit π in token goods selling on platform n solves the problem:

$$\begin{aligned} V^n_{1,t+1}(\pi) = \max_{c,d}\left\{(1-\beta)u(c) + \beta V_{2,t+2}(d)\right\} \\ s.t. \quad d \leq \pi - \epsilon^n_{t+1}(1+\mu^n_{t+1})c, \end{aligned}$$

Substituting in the constraint gives the standard consumption saving decision:

$$\max_d\left\{(1-\beta)u\left(\frac{\pi-d}{\epsilon^n_{t+1}(1+\mu^n_{t+1})}\right) + \beta V_{2,t+2}(d)\right\}$$

The first order condition gives:

$$0 = -\frac{(1-\beta)u'(c_{1,t+1})}{\epsilon^n_{t+1}(1+\mu^n_{t+1})} + \beta V'_{2,t+2}(d)$$

Imposing the functional forms and rearranging gives:

$$\frac{1-\beta}{\epsilon^n_{t+1}(1+\mu^n_{t+1})c} = \frac{\beta}{d} = \frac{\beta}{\pi - \epsilon^n_{t+1}(1+\mu^n_{t+1})c}$$

and so:

$$c_{1,t+1} = \frac{(1-\beta)\pi}{\epsilon^n_{t+1}(1+\mu^n_{t+1})}, \qquad d_{1,t+1} = \beta\pi$$

And so we have:

$$\begin{aligned} V^n_{1,t+1}(\pi) &= (1-\beta)\log\left(\frac{(1-\beta)\pi}{\epsilon^n_{t+1}(1+\mu^n_{t+1})}\right) + \beta\log\left(\bar{\nu}_{2,t+2}\beta\pi\right) \\ &= \log\left((1-\beta)^{1-\beta}\beta^{\beta}\right) + \beta\log\left(\bar{\nu}_{2,t+2}\right) - (1-\beta)\log\left(\epsilon^n_{t+1}(1+\mu^n_{t+1})\right) + \log(\pi) \end{aligned}$$

At age 0: Now consider the problem of an agent at age 0 during the morning market. They choose where to purchase input goods, where to sell output goods, and the quantity of input goods to solve (B.7) below:

$$\begin{aligned} V_{0,t} &= \mathbb{E}_t\left[\max_{n_0,n_1,x,\pi}\left\{\tilde{\zeta}^{n_0}_{0,t} + \tilde{\zeta}^{n_1}_{1,t+1} + V^{n_1}_{1,t+1}(\pi)\right\}\right] \quad s.t. \\ \pi &= \epsilon^{n_1}_{t+1}zx^{\alpha} - (1+\mu^{n_0}_t)\mathcal{R}^{bn_0}_{t,t+1}x \end{aligned} \tag{B.5}$$

For a given choice of $\underline{\mathbf{n}}_1$, the agent chooses x to maximize:

$$\max_x\left\{\epsilon^{n_1}_{t+1}zx^{\alpha} - (1+\mu^{n_0}_t)\mathcal{R}^{bn_0}_{t,t+1}x\right\}$$

Taking the FOC gives that producer labor demand, output, and profit are given by:

$$x_{0,t} = \left(\frac{\epsilon^{n_1}_{t+1}\alpha z}{(1+\mu^{n_0}_t)\mathcal{R}^{bn_0}_{t,t+1}}\right)^{\frac{1}{1-\alpha}}, \qquad y^{n_0}_{1,t+1} = z\left(\frac{\epsilon^{n_1}_{t+1}\alpha z}{(1+\mu^{n_0}_t)\mathcal{R}^{bn_0}_{t,t+1}}\right)^{\frac{\alpha}{1-\alpha}},$$

$$\pi_{1,t+1} = \left(\frac{\epsilon^{n_1}_{t+1}\alpha z}{((1+\mu^{n_0}_t)\mathcal{R}^{bn_0}_{t,t+1})^{\alpha}}\right)^{\frac{1}{1-\alpha}}\left(\frac{1-\alpha}{\alpha}\right).$$

And so we have:

$$\begin{aligned}
V_{0,t} &= \mathbb{E}_t\left[\max_{n_0,n_1,x,\pi}\left\{\tilde{\zeta}^{n_0}_{0,t} + \tilde{\zeta}^{n_1}_{1,t+1} + V^{n_1}_{1,t+1}(\pi^{(n_0,n_1)}_{1,t+1})\right\}\right] \\
&= \mathbb{E}_t\left[\max_{n_0,n_1}\left\{\tilde{\zeta}^{n_0}_{0,t} + \tilde{\zeta}^{n_1}_{1,t+1} - \log\left(\left(\epsilon^{n_1}_{t+1}(1+\mu^{n_1}_{t+1})\right)^{1-\beta}\right)\right.\right. \\
&\quad \left.\left. + \log\left(\left(\frac{\epsilon^{n_1}_{t+1}\alpha z}{((1+\mu^{n_0}_t)\mathcal{R}^{bn_0}_{t,t+1})^{\alpha}}\right)^{\frac{1}{1-\alpha}}\left(\frac{1-\alpha}{\alpha}\right)\right)\right\}\right] \\
&\quad + \log\left((1-\beta)^{1-\beta}\beta^{\beta}\right) + \beta\log\left(\bar{\nu}_{2,t+2}\right) \\
&= \mathbb{E}_t\left[\max_{n_0,n_1}\left\{\tilde{\zeta}^{n_0}_{0,t} + \tilde{\zeta}^{n_1}_{1,t+1} + \log\left(\frac{(\epsilon^{n_1}_{t+1})^{\frac{1}{1-\alpha}-(1-\beta)}(1+\mu^{n_1}_{t+1})^{-(1-\beta)}}{\left((1+\mu^{n_0}_t)\mathcal{R}^{bn_0}_{t,t+1}\right)^{\frac{\alpha}{1-\alpha}}}\right)\right\}\right] \\
&\quad + \log\left((\alpha z)^{\frac{1}{1-\alpha}}\left(\frac{1-\alpha}{\alpha}\right)\right) \\
&\quad + \log\left((1-\beta)^{1-\beta}\beta^{\beta}\right) + \beta\log\left(\bar{\nu}_{2,t+2}\right)
\end{aligned}$$

and Lemma 2 implies that the fraction of agents at age 0 choosing to purchase inputs on n_0 and at age 1 choosing to purchase on n_1 satisfies:

$$\begin{aligned}
\eta^{n_0}_{0,t} &= \frac{\left(\zeta^{n_0}_0((1+\mu^{n_0}_t)\mathcal{R}^{bn_0}_{t,t+1})^{-\frac{\alpha}{1-\alpha}}\right)^{\gamma_0}}{\sum_{n_0'}\left(\zeta^{n_0'}_0((1+\mu^{n_0'}_t)\mathcal{R}^{bn_0'}_{t,t+1})^{-\frac{\alpha}{1-\alpha}}\right)^{\gamma_0}} \\
\eta^{n_1}_{1,t+1} &= \frac{\left(\zeta^{n_1}_1\,(\epsilon^{n_1}_{t+1})^{\frac{1}{1-\alpha}+\beta-1}\,(1+\mu^{n_1}_{t+1})^{\beta-1}\right)^{\gamma_1}}{\sum_{n_1'}\left(\zeta^{n_1'}_1\left(\epsilon^{n_1'}_{t+1}\right)^{\frac{1}{1-\alpha}+\beta-1}(1+\mu^{n_1'}_{t+1})^{\beta-1}\right)^{\gamma_1}}
\end{aligned}$$

and:

$$\begin{aligned} V_{0,t} = \; & \log(\bar{\nu}_{0,t}) + \log(\bar{\nu}_{1,t+1}) + \beta \log\left(\bar{\nu}_{2,t+2}\right) \\ & + \log\left((\alpha z)^{\frac{1}{1-\alpha}} \left(\frac{1-\alpha}{\alpha}\right)\right) + \log\left((1-\beta)^{1-\beta}\beta^{\beta}\right) \end{aligned}$$

where:

$$\begin{aligned} \bar{\nu}_{0,t} &:= \left(\sum_{n_0} \left(\zeta_0^{n_0} \left((1+\mu_t^{n_0})\mathcal{R}^{bn_0}_{t,t+1}\right)^{-\frac{\alpha}{1-\alpha}}\right)^{\gamma_0}\right)^{1/\gamma_0} \\ \bar{\nu}_{1,t+1} &:= \left(\sum_{n_1} \left(\zeta_1^{n_1} \left(\epsilon^{n_1}_{t+1}\right)^{\frac{1}{1-\alpha}+\beta-1} (1+\mu^{n_1}_{t+1})^{\beta-1}\right)^{\gamma_1}\right)^{1/\gamma_1} \end{aligned}$$

and $\bar{\nu}_{2,t+2}$ is given by (B.4). □

B.3 Financial Intermediary Problem and Proof of Lemma 1

Before starting the proof, we need to set up the budget constraints for the financial intermediaries. In the evening of each period t, the financial intermediaries take real deposit wealth A_t from agents. They then purchase money M_t to back deposit withdrawals and discount the issuance of IOUs in dollars, purchase bonds B_t, and purchase shares in the platform, S_t. So their budget constraint at t is:

$$q_t^m M_t + q_t^b B_t + q_t^s S_t \leq A_t.$$

In the morning of $t+1$, depositors who trade on the public marketplace ($n = o$) withdraw money, and depositors who trade on the private platform ($n = p$) exchange the market value of the remaining assets in the financial intermediary. The financial intermediary offers depositors withdrawing in money the return on money and the other depositors the return on assets that can be used in exchange on the ledger. So the "money-in-advance" constraint on the financial intermediary is:

$$R^m_{t,t+1}\eta^m_{2,t+1}A_t \leq q^m_{t+1}M_t \quad \Leftrightarrow \quad \eta^m_{2,t+1}A_t \leq q^m_t M_t$$

Let $\breve{M}_t := M_t - \eta^m_{2,t+1}A_t/q^m_t$ denote the excess money holdings that can be used to discount new IOUs in the morning of $t+1$. In addition, the intermediaries offer to discount new IOUs into money at rate $\breve{E}_{t+1}$ IOUs per unit of money.

Thus, the real wealth available to depositors trading on the platform is:

$$\breve{A}_t = q^b_{t+1}\breve{E}_{t+1}\breve{M}_t + B_t + q^s_{t+1}S_t$$

Proof of Lemma 1. Afternoon at time t: Let A_t denote the real value of deposits in the afternoon sub-period at time t. The financial intermediary invests the portfolio by purchasing B_t $(t+1)$-IOUs, M_t money, and S_t shares subject to the budget constraint:

$$q^b_t B_t + q^m_t M_t + q^s_t S_t \leq A_t$$

The financial intermediary offers the return on money for money withdrawals so it must hold that:

$$\begin{aligned} q^m_{t+1}M_t &\geq R^m_{t,t+1}\eta^o_{2,t+1}A_t \\ \Rightarrow q^m_t M_t &\geq \eta^o_{2,t+1}A_t \end{aligned}$$

Morning at time t: Let $\breve{M}_t := M_t - \eta^o_{2,t+1}A_t/q^m_t$ denote excess holdings of money beyond the money-in-advance constraint. The financial intermediary uses its excess money to discount new IOUs. This leaves it with $\breve{E}_t\breve{M}_t$ IOUs. Incorporating discounting, the wealth available to depositors trading on the platform is $\breve{M}_t\breve{E}_t$ $(t+2)$-IOUs, B_t $(t+1)$-IOUs, and S_t shares. In real terms, this gives wealth:

$$\begin{aligned} \breve{A}_{t+1} &= \frac{1}{P^b_{t+1}}\breve{M}_t\breve{E}_{t+1} + B_t + \frac{1}{P^s_{t+1}}S_t \\ &= q^b_{t+1}\breve{M}_t\breve{E}_{t+1} + B_t + (q^s_{t+1} + \pi^s_{t+1})S_t \\ &= \left(\frac{q^b_{t+1}\breve{E}_{t+1}}{q^m_t}\breve{\theta}^m_t + \frac{1}{q^b_t}\theta^b_t + \frac{q^s_{t+1} + \pi^s_{t+1}}{q^s_t}\theta^s_t\right)A_t \end{aligned}$$

where $\breve{\theta}^m_t := q^m_t\breve{M}_t/A_t$, $\theta^b_t := q^b_t B_t/A_t$, and $\theta^s_t := q^s_t S_t/A_t$.

Optimization and equilibrium: In an equilibrium with discounting of IOUs, optimizing financial intermediaries must be indifferent across asset classes. That is, the risk free returns must equate:

$$\frac{q^b_{t+1}\breve{E}_{t+1}}{q^m_t} = \frac{1}{q^b_t} = \frac{q^s_{t+1} + \pi^s_{t+1}}{q^s_t}$$

so we have that:

$$\breve{E}_{t+1} = \frac{1}{q_t^b}\frac{q_t^m}{q_{t+1}^b} = \frac{1}{q_t^b}\frac{q_t^m}{q_{t+1}^m/E_{t+1}} = E_{t+1}\frac{R_{t,t+1}^b}{R_{t,t+1}^m}$$

and the depositor returns are:

$$\mathcal{R}_{t,t+1}^{dp} = R_{t,t+1}^b$$
$$\mathcal{R}_{t,t+1}^{do} = R_{t,t+1}^m$$

Entrepreneur borrowing costs: Now we can return to the problem of the entrepreneur. The entrepreneur needs to purchase $x_{0,t}$ goods. If they trade on the platform, then they issue $P_t^b(1+\mu_t^{n_0})x_{0,t}$ IOUs. If they trade on the public marketplace, then they issue $\breve{E}_t P_t^m(1+\mu_t^{n_0})x_{0,t}$ IOUs. Each IOU promises 1 good so their total borrowing costs are:

$$(1+\mu_t^{n_0})\breve{E}_t^{n_0}P_t^{n_0}x_{0,t}.$$

That is, $\mathcal{R}_{t,t+1}^{bn} := \breve{E}_t^{n_0}P_t^{n_0}$ is given by (imposing the indifference condition):

$$\mathcal{R}_{t,t+1}^{bp} := P_t^b = \frac{1}{q_t^b} = R_{t,t+1}^b$$
$$\mathcal{R}_{t,t+1}^{bo} := \breve{E}_t P_t^m = E_t\frac{R_{t-1,t}^b}{R_{t-1,t}^m}P_t^m = \frac{R_{t-1,t}^b}{R_{t-1,t}^m}\epsilon_t P_t^b = \frac{R_{t-1,t}^b}{R_{t-1,t}^m}\epsilon_t R_{t,t+1}^b$$

Equilibrium balance sheet: In equilibrium, the financial intermediary must hold enough money at t to discount all IOUs and fulfill withdrawal requests at $t+1$:

$$\begin{aligned} M_t &= \frac{R_{t,t+1}^m}{q_{t+1}^m}\eta_{2,t+1}^o A_t + \sum_{n_1}\eta_{0,t+1}^o\eta_{1,t+2}^{n_1}P_{t+1}^m(1+\mu_{t+1}^o)x_{0,t+1}^{(o,n_1)} \\ &= \frac{\eta_{2,t+1}^o}{q_t^m}A_t + \frac{1}{q_{t+1}^m}\sum_{n_1}\eta_{0,t+1}^o\eta_{1,t+2}^{n_1}\epsilon_{t+1}(1+\mu_{t+1}^o)x_{0,t+1}^{(o,n_1)} \\ &= \frac{\eta_{2,t+1}^o}{q_t^m}A_t + \frac{1}{q_t^m R_{t,t+1}^m}\sum_{n_1}\eta_{0,t+1}^o\eta_{1,t+2}^{n_1}\epsilon_{t+1}(1+\mu_{t+1}^o)x_{0,t+1}^{(o,n_1)} \end{aligned}$$

where $\mu_{t+1}^o = 0$. So, in real terms the portfolio is:

$$q_t^m M_t = \eta_{2,t+1}^o A_t + \frac{1}{R_{t,t+1}^m}\sum_{n_1}\eta_{0,t+1}^o\eta_{1,t+2}^{n_1}\epsilon_{t+1}x_{0,t+1}^{(o,n_1)}$$

$\square$

B.4 Equilibrium Characterization

Summarising the equilibrium and optimization equations gives the following characterization of equilibrium. Given states $\{M_t, \mu_{t-2}, \mu_{t-1}, R_{t-1}\}$ and current policy μ_t, we can solve for the equilibrium variables at time t:

$$\left(\underline{\mathbf{c}}_{1,t}^{\mathbf{n}_t}, \underline{\mathbf{c}}_{2,t}^{\mathbf{n}_t}, \underline{\mathbf{x}}_t^{\mathbf{n}_{t+1}}, \underline{\mathbf{y}}_t^{\mathbf{n}_t}, \underline{\boldsymbol{\pi}}_t^{\mathbf{n}_t}, \underline{\mathbf{d}}_t^{\mathbf{n}_t}, \epsilon_t, R_t^{bn}, R_t^{dn}, \underline{\boldsymbol{\eta}}_t^{\mathbf{n}_t}\right)$$

using the equations for agent choices:

$$x_{0,t}^{\mathbf{n}_1} = \left(\frac{\epsilon_{t+1}^{n_1}\alpha z}{(1+\mu_t^{n_0})\mathcal{R}_{t,t+1}^{bn_0}}\right)^{\frac{1}{1-\alpha}} \qquad y_{1,t}^{\mathbf{n}_1} = z\left(\frac{\epsilon_t^{n_1}\alpha z}{(1+\mu_{t-1}^{n_0})\mathcal{R}_{t-1,t}^{bn_0}}\right)^{\frac{\alpha}{1-\alpha}}$$

$$\pi_{1,t}^{\mathbf{n}_1} = \left(\frac{\epsilon_t^{n_1}\alpha z}{((1+\mu_{t-1}^{n_0})\mathcal{R}_{t-1,t}^{bn_0})^{\alpha}}\right)^{\frac{1}{1-\alpha}}\left(\frac{1-\alpha}{\alpha}\right) \qquad c_{1,t}^{\mathbf{n}_1} = \frac{(1-\beta)\pi_{1,t}^{\mathbf{n}_1}}{(1+\mu_t^{n_1})\epsilon_t^{n_1}}$$

$$d_{1,t}^{\mathbf{n}_1} = \beta\pi_{1,t}^{\mathbf{n}_1} \qquad c_{2,t}^{\mathbf{n}_2} = \frac{\mathcal{R}_{t-1,t}^{dn_2}\beta\pi_{1,t-1}^{\mathbf{n}_1}}{(1+\mu_t^{n_2})\epsilon_t^{n_2}}$$

$$\eta_{0,t}^{n_0} = \frac{\left(\zeta_0^{n_0}((1+\mu_t^{n_0})\mathcal{R}_{t,t+1}^{bn_0})^{-\frac{\alpha}{1-\alpha}}\right)^{\gamma_0}}{\sum_{n_0'}\left(\zeta_0^{n_0'}((1+\mu_t^{n_0'})\mathcal{R}_{t,t+1}^{bn_0'})^{-\frac{\alpha}{1-\alpha}}\right)^{\gamma_0}} \qquad \eta_{2,t}^{n_2} = \frac{\left(\zeta_2^{n_2}\mathcal{R}_{t-1,t}^{dn_2}/((1+\mu_t^{n_2})\epsilon_t^{n_2})\right)^{\gamma_2}}{\sum_{n_2'}\left(\zeta_2^{n_2'}\mathcal{R}_{t-1,t}^{dn_2'}/((1+\mu_t^{n_2'})\epsilon_t^{n_2'}\right)^{\gamma_2}}$$

$$\eta_{1,t}^{n_1} = \frac{\left(\zeta_1^{n_1}\left(\epsilon_t^{n_1}\right)^{\frac{1}{1-\alpha}+\beta-1}(1+\mu_t^{n_1})^{\beta-1}\right)^{\gamma_1}}{\sum_{n_1'}\left(\zeta_1^{n_1'}\left(\epsilon_t^{n_1'}\right)^{\frac{1}{1-\alpha}+\beta-1}(1+\mu_t^{n_1'})^{\beta-1}\right)^{\gamma_1}}$$

financial intermediary equations:

$$\mathcal{R}_{t,t+1}^{bn} = \epsilon_t^n\frac{R_{t-1,t}^b}{R_{t-1,t}^n}R_{t,t+1}^b \qquad \mathcal{R}_{t,t+1}^{dn} = R_{t,t+1}^n$$

$$A_t = q_t^m M_t + q_t^b B_t + q_t^s S_t \qquad \eta_{2,t+1}^m A_t \leq q_t^m M_t$$

$$A_t = \sum_{n_0,n_1}\eta_{0,t-1}^{n_0}\eta_{1,t}^{n_1}d_{1,t}^{(n_0,n_1)}$$

and the market clearing conditions:

$$
\begin{aligned}
\eta^n_{1,t}\sum_{n_0}\eta^{n_0}_{0,t-1}y^{(n_0,n)}_{1,t} &= \eta^n_{0,t}\sum_{n_1}\eta^{n_1}_{1,t+1}x^{(n,n_1)}_{0,t} + \eta^n_{1,t}\sum_{n_0}\eta^{n_0}_{0,t-1}c^{(n_0,n)}_{1,t} \\
&\quad + \eta^n_{2,t}\sum_{n_0,n_1}\eta^{n_0}_{0,t-2}\eta^{n_1}_{1,t-1}c^{(n_0,n_1,n)}_{2,t} \\
q^b_t\breve{B}_t &= \sum_{n_0,n_1}\eta^{n_0}_{0,t}\eta^{n_1}_{1,t+1}\mathcal{R}^{bn_0}_{t,t+1}(1+\mu^{n_0}_{t+1})x^{(n_0,n_1)}_{0,t} \\
\breve{B}_t &= B_t \\
q^m_t\bar{M}_t &= \eta^o_{2,t+1}A_t + \frac{1}{R^m_{t,t+1}}\sum_{n_1}\eta^o_{0,t+1}\eta^{n_1}_{1,t+2}\epsilon_{t+1}x^{(o,n_1)}_{0,t+1} \\
S_t &= 1
\end{aligned}
$$

Proof of Theorem 2. We start by solving for ϵ_t. Substituting the financial intermediary returns into the agent choices gives:

$$
\eta^{n_0}_{0,t} = \frac{\left(\zeta^{n_0}_0((1+\mu^{n_0}_t)R^{n_0}_{t,t+1})^{-\frac{\alpha}{1-\alpha}}\right)^{\gamma_0}}{\sum_{n_0'}\left(\zeta^{n_0'}_0((1+\mu^{n_0'}_t)R^{n_0'}_{t,t+1})^{-\frac{\alpha}{1-\alpha}}\right)^{\gamma_0}} \quad \eta^{n_2}_{2,t} = \frac{\left(\zeta^{n_2}_2R^{n_2}_{t-1,t}/((1+\mu^{n_2}_t)\epsilon^{n_2}_t)\right)^{\gamma_2}}{\sum_{n_2'}\left(\zeta^{n_2'}_2R^{n_2'}_{t-1,t}/((1+\mu^{n_2'}_t)\epsilon^{n_2'}_t\right)^{\gamma_2}}
$$

Now, return to the goods market clearing condition. After rearranging, we have:

$$
\eta^n_{1,t}\sum_{n_0}\eta^{n_0}_{0,t-1}\left(y^{(n_0,n)}_{1,t} - c^{(n_0,n)}_{1,t}\right) = \eta^n_{0,t}\sum_{n_1}\eta^{n_1}_{1,t+1}x^{(n,n_1)}_{0,t} + \eta^n_{2,t}\sum_{n_0,n_1}\eta^{n_0}_{0,t-2}\eta^{n_1}_{1,t-1}c^{(n_0,n_1,n)}_{2,t}
$$

where the LHS can be computed using:

$$
\begin{aligned}
&y^{(n_0,n)}_{1,t} - c^{(n_0,n)}_{1,t} \\
&\quad = z\left(\frac{\alpha z\epsilon^n_t}{(1+\mu^{n_0}_{t-1})\mathcal{R}^{bn_0}_{t-1,t}}\right)^{\frac{\alpha}{1-\alpha}} - \frac{(1-\beta)}{(1+\mu^n_t)\epsilon^n_t}\left(\frac{\alpha z\epsilon^n_t}{((1+\mu^{n_0}_{t-1})\mathcal{R}^{bn_0}_{t-1,t})^{\alpha}}\right)^{\frac{1}{1-\alpha}}\left(\frac{1-\alpha}{\alpha}\right) \\
&\quad = \left(1 - \frac{(1-\beta)(1-\alpha)}{1+\mu^n_t}\right)z\left(\frac{\alpha z\epsilon^n_t}{(1+\mu^{n_0}_{t-1})\mathcal{R}^{bn_0}_{t-1,t}}\right)^{\frac{\alpha}{1-\alpha}} \\
&\quad = \left(1 - \frac{(1-\beta)(1-\alpha)}{1+\mu^n_t}\right)y^{(n_0,n)}_{1,t}
\end{aligned}
$$

and so:

$$
\begin{aligned}
&\eta^n_{1,t} \sum_{n_0} \eta^{n_0}_{0,t-1} \left(y^{(n_0,n)}_{1,t} - c^{(n_0,n)}_{1,t} \right) \\
&\qquad = \eta^n_{1,t} \sum_{n_0} \eta^{n_0}_{0,t-1} \left(1 - \frac{(1-\beta)(1-\alpha)}{1+\mu^n_t} \right) z \left(\frac{\alpha z \epsilon^n_t}{(1+\mu^{n_0}_{t-1}) \mathcal{R}^{bn_0}_{t-1,t}} \right)^{\frac{\alpha}{1-\alpha}} \\
&\qquad = \eta^n_{1,t} \left(1 - \frac{(1-\beta)(1-\alpha)}{1+\mu^n_t} \right) z (\alpha z \epsilon^n_t)^{\frac{\alpha}{1-\alpha}} \sum_{n_0} \frac{\eta^{n_0}_{0,t-1}}{((1+\mu^{n_0}_{t-1}) \mathcal{R}^{bn_0}_{t-1,t})^{\frac{\alpha}{1-\alpha}}}
\end{aligned}
$$

and where the RHS can be computed using:

$$
\begin{aligned}
&\eta^n_{0,t}\sum_{n_1}\eta^{n_1}_{1,t+1}x^{(n,n_1)}_{0,t}+\eta^n_{2,t}\sum_{n_0,n_1}\eta^{n_0}_{0,t-2}\eta^{n_1}_{1,t-1}c^{(n_0,n_1,n)}_{2,t}\\
&\quad=\eta^n_{0,t}\sum_{n_1}\eta^{n_1}_{1,t+1}\left(\frac{\epsilon^{n_1}_{t+1}\alpha z}{(1+\mu^n_t)\mathcal{R}^{bn}_{t,t+1}}\right)^{\frac{1}{1-\alpha}}\\
&\qquad+\eta^n_{2,t}\sum_{n_0,n_1}\eta^{n_0}_{0,t-2}\eta^{n_1}_{1,t-1}\frac{\mathcal{R}^{dn}_{t-1,t}\beta}{(1+\mu^n_t)\epsilon^n_t}\left(\frac{\epsilon^{n_1}_{t-1}\alpha z}{((1+\mu^{n_0}_{t-2})\mathcal{R}^{bn_0}_{t-2,t-1})^{\alpha}}\right)^{\frac{1}{1-\alpha}}\left(\frac{1-\alpha}{\alpha}\right)\\
&\quad=\frac{\left(\zeta^n_0((1+\mu^n_t)\mathcal{R}^{bn}_{t,t+1})^{-\frac{\alpha}{1-\alpha}}\right)^{\gamma_0}}{\sum_{n'}\left(\zeta^{n'}_0((1+\mu^{n'}_t)\mathcal{R}^{bn'}_{t,t+1})^{-\frac{\alpha}{1-\alpha}}\right)^{\gamma_0}}\sum_{n_1}\eta^{n_1}_{1,t+1}\left(\frac{\epsilon^{n_1}_{t+1}\alpha z}{(1+\mu^n_t)\mathcal{R}^{bn}_{t,t+1}}\right)^{\frac{1}{1-\alpha}}\\
&\qquad+\frac{\left(\zeta^{n_2}_2\mathcal{R}^{dn_2}_{t-1,t}/((1+\mu^{n_2}_t)\epsilon^{n_2}_t)\right)^{\gamma_2}}{\sum_{n'_2}\left(\zeta^{n'_2}_2\mathcal{R}^{dn'_2}_{t-1,t}/((1+\mu^{n'_2}_t)\epsilon^{n'_2}_t\right)^{\gamma_2}}\\
&\qquad\sum_{n_0,n_1}\eta^{n_0}_{0,t-2}\eta^{n_1}_{1,t-1}\frac{\mathcal{R}^{dn}_{t-1,t}\beta}{(1+\mu^n_t)\epsilon^n_t}\left(\frac{\epsilon^{n_1}_{t-1}\alpha z}{((1+\mu^{n_0}_{t-2})\mathcal{R}^{bn_0}_{t-2,t-1})^{\alpha}}\right)^{\frac{1}{1-\alpha}}\left(\frac{1-\alpha}{\alpha}\right)\\
&\quad=(\zeta^n_0)^{\gamma_0}\left((1+\mu^n_t)\mathcal{R}^{bn}_{t,t+1}\right)^{-\frac{1+\alpha\gamma_0}{1-\alpha}}\frac{1}{\bar{\nu}^{\gamma_0}_{0,t}}\sum_{n_1}\eta^{n_1}_{1,t+1}\left(\epsilon^{n_1}_{t+1}\alpha z\right)^{\frac{1}{1-\alpha}}\\
&\qquad+(\zeta^n_2)^{\gamma_2}\left(\frac{\mathcal{R}^{dn}_{t-1,t}}{(1+\mu^n_t)\epsilon^n_t}\right)^{1+\gamma_2}\frac{1}{\bar{\nu}^{\gamma_2}_{2,t}}\sum_{n_0,n_1}\eta^{n_0}_{0,t-2}\eta^{n_1}_{1,t-1}\beta\left(\frac{\epsilon^{n_1}_{t-1}\alpha z}{((1+\mu^{n_0}_{t-2})\mathcal{R}^{bn_0}_{t-2,t-1})^{\alpha}}\right)^{\frac{1}{1-\alpha}}\left(\frac{1-\alpha}{\alpha}\right)\\
&\quad=(\zeta^n_0)^{\gamma_0}\left((1+\mu^n_t)\epsilon^n_t\frac{R^b_{t-1,t}R^b_{t,t+1}}{R^n_{t-1,t}}\right)^{-\frac{1+\alpha\gamma_0}{1-\alpha}}\frac{1}{\bar{\nu}^{\gamma_0}_{0,t}}\sum_{n_1}\eta^{n_1}_{1,t+1}\left(\epsilon^{n_1}_{t+1}\alpha z\right)^{\frac{1}{1-\alpha}}\\
&\qquad+(\zeta^n_2)^{\gamma_2}\left(\frac{R^n_{t-1,t}}{(1+\mu^n_t)\epsilon^n_t}\right)^{1+\gamma_2}\frac{1}{\bar{\nu}^{\gamma_2}_{2,t}}\sum_{n_0,n_1}\eta^{n_0}_{0,t-2}\eta^{n_1}_{1,t-1}\beta\left(\frac{\epsilon^{n_1}_{t-1}\alpha z}{((1+\mu^{n_0}_{t-2})\mathcal{R}^{bn_0}_{t-2,t-1})^{\alpha}}\right)^{\frac{1}{1-\alpha}}\left(\frac{1-\alpha}{\alpha}\right)
\end{aligned}
$$

Under the assumption that $\frac{1+\alpha\gamma_0}{1-\alpha}=1+\gamma_2$ and $(\zeta^n_0)^{\gamma_0}=(\zeta^n_2)^{\gamma_2}$, we can take out the n specific component to get:

$$\eta^n_{0,t}\sum_{n_1}\eta^{n_1}_{1,t+1}x^{(n,n_1)}_{0,t}+\eta^n_{2,t}\sum_{n_0,n_1}\eta^{n_0}_{0,t-2}\eta^{n_1}_{1,t-1}c^{(n_0,n_1,n)}_{2,t}$$
$$=(\zeta^n_2)^{\gamma_2}\left(\frac{R^n_{t-1,t}}{(1+\mu^n_t)\epsilon^n_t}\right)^{1+\gamma_2}\left[\frac{1}{\bar{\nu}^{\gamma_0}_{0,t}}\sum_{n_1}\eta^{n_1}_{1,t+1}\left(\frac{\alpha z\epsilon^{n_1}_{t+1}}{(R^b_{t-1,t}R^b_{t,t+1})^{1+\alpha\gamma_0}}\right)^{\frac{1}{1-\alpha}}\right.$$
$$\left.+\frac{1}{\bar{\nu}^{\gamma_2}_{2,t}}\sum_{n_0,n_1}\eta^{n_0}_{0,t-2}\eta^{n_1}_{1,t-1}\beta\left(\frac{\alpha z\epsilon^{n_1}_{t-1}}{((1+\mu^{n_0}_{t-2})\mathcal{R}^{bn_0}_{t-2,t-1})^{\alpha}}\right)^{\frac{1}{1-\alpha}}\left(\frac{1-\alpha}{\alpha}\right)\right]$$

So, the market clearing condition in goods market n becomes:

$$\eta^n_{1,t}\left(1-\frac{(1-\beta)(1-\alpha)}{1+\mu^n_t}\right)z(\alpha z\epsilon^n_t)^{\frac{\alpha}{1-\alpha}}\sum_{n_0}\frac{\eta^{n_0}_{0,t-1}}{((1+\mu^{n_0}_{t-1})\mathcal{R}^{bn_0}_{t-1,t})^{\frac{\alpha}{1-\alpha}}}$$
$$=(\zeta^n_2)^{\gamma_2}\left(\frac{R^n_{t-1,t}}{(1+\mu^n_t)\epsilon^n_t}\right)^{1+\gamma_2}\left[\frac{1}{\bar{\nu}^{\gamma_0}_{0,t}}\sum_{n_1}\eta^{n_1}_{1,t+1}\left(\frac{\alpha z\epsilon^{n_1}_{t+1}}{(R^b_{t-1,t}R^b_{t,t+1})^{1+\alpha\gamma_0}}\right)^{\frac{1}{1-\alpha}}\right.$$
$$\left.+\frac{1}{\bar{\nu}^{\gamma_2}_{2,t}}\sum_{n_0,n_1}\eta^{n_0}_{0,t-2}\eta^{n_1}_{1,t-1}\left(\frac{\alpha z\epsilon^{n_1}_{t-1}}{((1+\mu^{n_0}_{t-2})\mathcal{R}^{bn_0}_{t-2,t-1})^{\alpha}}\right)^{\frac{1}{1-\alpha}}\left(\frac{1-\alpha}{\alpha}\right)\right]$$

Dividing the market clearing condition in market n by the market clearing condition in market n' gives:

$$\left(\frac{\zeta^n_1\,(\epsilon^n_t)^{\frac{1}{1-\alpha}+\beta-1}\,(1+\mu^n_t)^{\beta-1}}{\zeta^{n'}_1\left(\epsilon^{n'}_t\right)^{\frac{1}{1-\alpha}+\beta-1}(1+\mu^{n'}_t)^{\beta-1}}\right)^{\gamma_1}\frac{\left(1-\frac{(1-\beta)(1-\alpha)}{1+\mu^n_t}\right)}{\left(1-\frac{(1-\beta)(1-\alpha)}{1+\mu^{n'}_t}\right)}\left(\frac{\epsilon^n_t}{\epsilon^{n'}_t}\right)^{\frac{\alpha}{1-\alpha}}=\left(\frac{\zeta^n_2}{\zeta^{n'}_2}\right)^{\gamma_2}\left(\frac{\frac{R^n_{t-1,t}}{(1+\mu^n_t)\epsilon^n_t}}{\frac{R^{n'}_{t-1,t}}{(1+\mu^{n'}_t)\epsilon^{n'}_t}}\right)^{1+\gamma_2}$$

which implies that:

$$\left(\frac{\epsilon^n_t}{\epsilon^{n'}_t}\right)^{\frac{\alpha}{1-\alpha}(\gamma_1+1)+\gamma_1\beta+1+\gamma_2}\left(\frac{1+\mu^n_t}{1+\mu^{n'}_t}\right)^{-\gamma_1(1-\beta)+1+\gamma_2}\frac{\left(1-\frac{(1-\beta)(1-\alpha)}{1+\mu^n_t}\right)}{\left(1-\frac{(1-\beta)(1-\alpha)}{1+\mu^{n'}_t}\right)}$$
$$=\left(\frac{\zeta^n_2}{\zeta^{n'}_2}\right)^{\gamma_2}\left(\frac{\zeta^n_1}{\zeta^{n'}_1}\right)^{-\gamma_1}\left(\frac{R^n_{t-1,t}}{R^{n'}_{t-1,t}}\right)^{1+\gamma_2}$$

which gives:

$$\frac{\epsilon_t^n}{\epsilon_t^{n'}} = \left[\left(\frac{\zeta_2^n}{\zeta_2^{n'}}\right)^{\gamma_2}\left(\frac{\zeta_1^n}{\zeta_1^{n'}}\right)^{-\gamma_1}\left(\frac{R_{t-1,t}^n}{R_{t-1,t}^{n'}}\right)^{1+\gamma_2}\left(\frac{1+\mu_t^n}{1+\mu_t^{n'}}\right)^{\gamma_1(1-\beta)-(1+\gamma_2)}\left(\frac{1-\frac{(1-\beta)(1-\alpha)}{1+\mu_t^{n'}}}{1-\frac{(1-\beta)(1-\alpha)}{1+\mu_t^n}}\right)\right]^{\frac{1}{\frac{\alpha}{1-\alpha}(\gamma_1+1)+\gamma_1\beta+1+\gamma_2}}$$

and so for $n = o$ and $n' = p$ we have:

$$\epsilon_t = \left[\zeta_1^{\gamma_1}\zeta_2^{-\gamma_2}\left(\frac{R_{t-1,t}^m}{R_{t-1,t}^b}\right)^{1+\gamma_2}(1+\mu_t)^{-\gamma_1(1-\beta)+1+\gamma_2}\left(\frac{1-\frac{(1-\beta)(1-\alpha)}{1+\mu_t}}{1-(1-\beta)(1-\alpha)}\right)\right]^{\left(\frac{\alpha}{1-\alpha}(\gamma_1+1)+\gamma_1\beta+1+\gamma_2\right)^{-1}}$$

□

Proof of Theorem 3. (i) Solve for $R_{t,t+1}^b$: We have the following equations for the morning "primary" IOU market, the morning deposit market, the afternoon financial intermediary budget constraint, the afternoon bond market, the afternoon money market, and the afternoon equity market.

$$\begin{aligned}
q_t^b \breve{B}_t &= \sum_{n_0,n_1} \eta_{0,t}^{n_0}\eta_{1,t+1}^{n_1}\mathcal{R}_{t,t+1}^{bn_0}(1+\mu_{t+1}^{n_0})x_{0,t}^{(n_0,n_1)} \\
A_t &= \sum_{n_0,n_1} \eta_{0,t-1}^{n_0}\eta_{1,t}^{n_1}d_{1,t}^{(n_0,n_1)} \\
A_t &= q_t^b B_t + q_t^m M_t + q_t^s S_t \\
B_t &= \breve{B}_t \\
q_t^m M_t &= \eta_{2,t+1}^o A_t + \frac{1}{R_{t,t+1}^m}\sum_{n_1}\eta_{0,t+1}^o\eta_{1,t+2}^{n_1}\epsilon_{t+1}x_{0,t+1}^{(o,n_1)} \\
S_t &= 1
\end{aligned}$$

Substituting the other equations into the financial intermediary budget constraint:

$$\begin{aligned}
\sum_{n_0,n_1}\eta_{0,t-1}^{n_0}\eta_{1,t}^{n_1}d_{1,t}^{(n_0,n_1)} = &\sum_{n_0,n_1}\eta_{0,t}^{n_0}\eta_{1,t+1}^{n_1}\epsilon_t^{n_0}\frac{R_{t-1,t}^b}{R_{t-1,t}^{n_0}}R_{t,t+1}^b(1+\mu_{t+1}^{n_0})x_{0,t}^{(n_0,n_1)} \\
&+\eta_{2,t+1}^o\sum_{n_0,n_1}\eta_{0,t-1}^{n_0}\eta_{1,t}^{n_1}d_{1,t}^{(n_0,n_1)} \\
&+\sum_{n_1}\eta_{0,t+1}^o\eta_{1,t+2}^{n_1}\frac{\epsilon_{t+1}}{R_{t,t+1}^m}x_{0,t+1} + q_t^s
\end{aligned}$$

and so after rearranging and imposed $d_{t,1} = \beta\pi_{1,t}$ we have:

$$
\begin{aligned}
&(1-\eta^o_{2,t+1}) \sum_{n_0,n_1} \eta^{n_0}_{0,t-1}\eta^{n_1}_{1,t}\beta\pi^{(n_0,n_1)}_{1,t} \\
&\quad = \sum_{n_1}\Bigg(\sum_{n_0} \eta^{n_0}_{0,t}\eta^{n_1}_{1,t+1}\epsilon^{n_0}_t \frac{R^b_{t-1,t}}{R^{n_0}_{t-1,t}} R^b_{t,t+1}(1+\mu^{n_0}_{t+1})x^{(n_0,n_1)}_{0,t} \\
&\qquad + \eta^o_{0,t+1}\eta^{n_1}_{1,t+2}\frac{\epsilon_{t+1}}{R^m_{t,t+1}} x^{(o,n_1)}_{0,t+1}\Bigg) + q^s_t
\end{aligned}
$$

which gives the expression in the main text.

(ii) Return on Money: Rearranging the money market clearing condition:

$$
\begin{aligned}
q^m_t \bar{M}_t &= \eta^o_{2,t+1}A_t + \frac{1}{R^m_{t,t+1}} \sum_{n_1} \eta^o_{0,t+1}\eta^{n_1}_{1,t+2}\epsilon_{t+1}x^{(o,n_1)}_{0,t+1} \\
&= \eta^o_{2,t+1}\beta\Pi_{1,t} + \frac{1}{R^m_{t,t+1}} X^o_{0,t+1}
\end{aligned}
$$

gives that:

$$
R^m_{t,t+1} = \frac{q^m_{t+1}}{q^m_t} = \left(\frac{\bar{M}_t}{\bar{M}_{t+1}}\right)\left(\frac{\eta^o_{2,t+2}\beta\Pi_{1,t+1} + \frac{1}{R^m_{t,t+2}}X^o_{0,t+2}}{\eta^o_{2,t+1}\beta\Pi_{1,t} + \frac{1}{R^m_{t,t+1}}X^o_{0,t+1}}\right)
$$

so in the steady state, this is:

$$
\overline{R}^m = 1/g^M
$$

(iii) Equity Price: The equity price satisfies:

$$
q^s_t = \frac{1}{R^b_{t,t+1}}\left(\pi^s_{t+1} + q^s_{t+1}\right)
$$

In the steady state, this implies that:

$$
\bar{q}^s = \frac{\bar{\pi}^s}{\bar{R}^b - 1}
$$

$\square$

B.5 Incentive Compatibility

Proof of Theorem 4. We consider the value for an agent who chooses to default when other agents are choosing to not default. We solve the problem recursively.

At age 2: an agent who has defaulted and joined a financial intermediary taking defaulting agents can only hold cash and so only trade on the public marketplace. Taking price processes as given, an agent with deposits d chooses on which platform to search to solve problem (B.6) below (dropping the explicit i superscript and the time subscript on the choice n_2):

$$\begin{aligned} \check{V}_{2,t+2}(d) &= \mathbb{E}\left[\max_{c}\left\{\tilde{\zeta}^o_{2,t+2} + u(c)\right\}\right] \\ s.t. \quad & c \leq \mathcal{R}^m_{t+1,t+2} d / \epsilon^o_{t+2}, \end{aligned} \tag{B.6}$$

where $\check{V}_{2,t+2}$ is the value function at the start of the agent's final period. Evaluating this expression gives:

$$\check{V}_{2,t+2}(d) = \log\left(\check{\nu}_{2,t+2} d\right)$$

where $\check{\nu}_{2,t+2} := \zeta^o_2 \mathcal{R}^m_{t+1,t+2}/\epsilon^o_{t+2} = \mathcal{R}^m_{t+1,t+2}/\epsilon^o_{t+2}$.

At age 1: the agent cannot default if they end up trading on the private platform. So, their value is:

$$\begin{aligned} V^p_{1,t+1}(\pi) = & \max_{c,d}\left\{(1-\beta)u(c) + \beta V_{2,t+2}\left(d\right)\right\} \\ s.t. \quad & d \leq \pi - \epsilon^p_{t+1}(1+\mu^p_{t+1})c, \end{aligned}$$

and from the proof of Theorem 1 we have:

$$\begin{aligned} V^p_{1,t+1}(\pi) &= (1-\beta)\log\left(\frac{(1-\beta)\pi}{\epsilon^p_{t+1}(1+\mu^p_{t+1})}\right) + \beta\log\left(\bar{\nu}_{2,t+2}\beta\pi\right) \\ &= \log\left((1-\beta)^{1-\beta}\beta^{\beta}\right) + \beta\log\left(\bar{\nu}_{2,t+2}\right) - (1-\beta)\log\left(\epsilon^p_{t+2}(1+\mu^p_{t+1})\right) + \log(\pi) \end{aligned}$$

However, if they trade on the public marketplace, then they can trade with cash, default, and go to a bank accepting cash trades without reporting them to the ledger. In this case, their value at $t = 1$ is given by:

$$\begin{aligned}\check{V}^{o}_{1,t+1}(\pi) = & \max_{c,d} \left\{(1-\beta)u(c) + \beta \check{V}_{2,t+2}\left(d\right)\right\} \\ & s.t. \quad d \leq \check{\pi} - \epsilon^{o}_{t+1} c,\end{aligned}$$

where $\check{\pi}$ is the profit if the agent defaults. As before, we have that:

$$c_{1,t+1} = \frac{(1-\beta)\check{\pi}}{\epsilon_{t+1}}, \qquad\qquad d_{1,t+1} = \beta\check{\pi}$$

And so we have:

$$\begin{aligned}\check{V}^{o}_{1,t+1}(\pi) &= (1-\beta)\log\left(\frac{(1-\beta)\check{\pi}}{\epsilon^{o}_{t+1}}\right) + \beta\log\left(\check{\nu}_{2,t+2}\beta\check{\pi}\right) \\ &= \log\left((1-\beta)^{1-\beta}\beta^{\beta}\right) + \beta\log\left(\check{\nu}_{2,t+2}\right) - (1-\beta)\log\left(\epsilon^{o}_{t+2}\right) + \log(\check{\pi})\end{aligned}$$

At Age 0: Now consider the problem of an agent at age 0 conditional on choosing to sell on the public marketplace $(n_0, n_1) = (n_0, o)$.

If they intend to repay at age 1, then they solve the same problem as before:

$$\begin{aligned}V_{0,t|n_0,n_1=o} = \tilde{\zeta}^{n_0}_{0,t} + \tilde{\zeta}^{o}_{1,t+1} + \max_{x,\pi}\left\{V^{o}_{1,t+1}(\pi)\right\} \quad s.t. \\ \pi = \epsilon^{o}_{t+1} z x^{\alpha} - (1+\mu^{n_0}_{t})\mathcal{R}^{bn_0}_{t,t+1} x\end{aligned} \tag{B.7}$$

Taking the FOCs gives profit:

$$\pi^{(n_0,o)}_{1,t+1} = \left(\frac{\epsilon^{o}_{t+1}\alpha z}{((1+\mu^{n_0}_{t})\mathcal{R}^{bn_0}_{t,t+1})^{\alpha}}\right)^{\frac{1}{1-\alpha}} \left(\frac{1-\alpha}{\alpha}\right).$$

From the proof of Theorem 1, for the optimal choice of (x, π), we have that:

$$\begin{aligned}V^{o}_{1,t+1} = & \log\left((1-\beta)^{1-\beta}\beta^{\beta}\right) + \beta\log\left(\bar{\nu}_{2,t+2}\right) - (1-\beta)\log\left(\epsilon^{o}_{t+1}\right) \\ & + \log(\pi^{(n_0,o)}_{1,t+1}) \\ = & \log\left((1-\beta)^{1-\beta}\beta^{\beta}\right) + \beta\log\left(\bar{\nu}_{2,t+2}\right) - (1-\beta)\log\left(\epsilon^{o}_{t+1}\right) \\ & + \log\left(\left(\frac{\epsilon^{o}_{t+1}\alpha z}{((1+\mu^{n_0}_{t})\mathcal{R}^{bn_0}_{t,t+1})^{\alpha}}\right)^{\frac{1}{1-\alpha}}\left(\frac{1-\alpha}{\alpha}\right)\right)\end{aligned}$$

If they intend to default at age 1, then they solve (B.8) below:

$$\check{V}_{0,t|n_0,n_1=o} = \tilde{\zeta}_{0,t}^{n_0} + \tilde{\zeta}_{1,t+1}^{o} + \max_{x,\pi}\left\{\check{V}_{1,t+1}^{o}(\check{\pi})\right\} \quad s.t. \tag{B.8}$$
$$\check{\pi} = \epsilon_{t+1}^{o} z x^{\alpha} - \chi(1+\mu_t^{n_0})x$$

For a given choice of n_0, they choose x to maximize:

$$\max_x \left\{\epsilon_{t+1}^{o} z x^{\alpha} - \chi(1+\mu_t^{n_0})x\right\}$$

Taking the FOC gives profit:

$$\check{\pi}_{1,t+1}^{(n_0,o)} = \left(\frac{\epsilon_{t+1}^{o}\alpha z}{(\chi(1+\mu_t^{n_0}))^{\alpha}}\right)^{\frac{1}{1-\alpha}}\left(\frac{1-\alpha}{\alpha}\right).$$

So, for the optimal choice of $(\check{x},\check{\pi})$, their value is:

$$\begin{aligned}
\check{V}_{1,t+1}^{o} =\ & \log\left((1-\beta)^{1-\beta}\beta^{\beta}\right) + \beta\log\left(\check{\nu}_{2,t+2}\right) - (1-\beta)\log\left(\epsilon_{t+1}^{o}\right) \\
& + \log(\check{\pi}_{1,t+1}^{(n_0,o)}) \\
=\ & \log\left((1-\beta)^{1-\beta}\beta^{\beta}\right) + \beta\log\left(\check{\nu}_{2,t+2}\right) - (1-\beta)\log\left(\epsilon_{t+1}^{o}\right) \\
& + \log\left(\left(\frac{\epsilon_{t+1}^{o}\alpha z}{(\chi(1+\mu_t^{n_0}))^{\alpha}}\right)^{\frac{1}{1-\alpha}}\left(\frac{1-\alpha}{\alpha}\right)\right)
\end{aligned}$$

<u>Incentive compatibility:</u> An agent choosing (n_0, o) chooses to repay if:

$$\begin{aligned}
V_{0,t|n_0,n_1=o} &\geq \check{V}_{0,t|n_0,n_1=o} \\
\Rightarrow V_{1,t+1}^{o} &\geq \check{V}_{1,t+1}^{o} \\
\Rightarrow \beta\log\left(\bar{\nu}_{2,t+2}\right) + \log(\pi_{1,t+1}^{(n_0,o)}) &\geq \beta\log\left(\check{\nu}_{2,t+2}\right) + \log(\check{\pi}_{1,t+1}^{(n_0,o)}) \\
\Rightarrow \beta\log\left(\frac{\bar{\nu}_{2,t+2}}{\check{\nu}_{2,t+2}}\right) &\geq \log\left(\frac{\check{\pi}_{1,t+1}^{(n_0,o)}}{\pi_{1,t+1}^{(n_0,o)}}\right) \\
\Rightarrow \beta\log\left(\frac{\bar{\nu}_{2,t+2}}{\check{\nu}_{2,t+2}}\right) &= \left(\frac{\alpha}{1-\alpha}\right)\log\left(\frac{\mathcal{R}_{t,t+1}^{bn_0}}{\chi}\right)
\end{aligned}$$

where:

$$\frac{\bar{\nu}_{2,t+2}}{\check{\nu}_{2,t+2}} = \frac{\left(\sum_{n_2}\left(\frac{\zeta_2^{n_2}\mathcal{R}^{dn_2}_{t+1,t+2}}{(1+\mu^{n_2}_{t+2})\epsilon^{n_2}_{t+2}}\right)^{\gamma^2}\right)^{1/\gamma_2}}{\frac{\mathcal{R}^m_{t+1,t+2}}{\epsilon^o_{t+2}}} = \left(1+\left(\zeta_2^p\left(\frac{R^b_{t+1,t+2}}{R^m_{t+1,t+2}}\right)\left(\frac{\epsilon^o_{t+2}}{1+\mu_{t+2}}\right)\right)^{\gamma_2}\right)^{1/\gamma_2}$$

and

$$\frac{\breve{\pi}^{(n_0,o)}_{1,t+1}}{\pi^{(n_0,o)}_{1,t+1}} = \frac{\left(\frac{\epsilon^o_{t+1}\alpha z}{(\chi(1+\mu^{n_0}_t))^\alpha}\right)^{\frac{1}{1-\alpha}}\left(\frac{1-\alpha}{\alpha}\right)}{\left(\frac{\epsilon^o_{t+1}\alpha z}{((1+\mu^{n_0}_t)\mathcal{R}^{bn_0}_{t,t+1})^\alpha}\right)^{\frac{1}{1-\alpha}}\left(\frac{1-\alpha}{\alpha}\right)} = \left(\frac{\mathcal{R}^{bn_0}_{t,t+1}}{\chi}\right)^{\frac{\alpha}{1-\alpha}}$$

So, the incentive compatibility constraint becomes:

$$\left(1+\left(\left(\frac{R^b_{t+1,t+2}}{R^m_{t+1,t+2}}\right)\left(\frac{\zeta_2^b\epsilon^m_{t+2}}{1+\mu_{t+2}}\right)\right)^{\gamma_2}\right)^{1/\gamma_2} \geq \left(\frac{\mathcal{R}^{bn_0}_{t,t+1}}{\chi}\right)^{\frac{\alpha}{1-\alpha}}$$

Rearranging, this becomes:

$$\frac{\zeta_2^p\epsilon^o_{t+2}}{1+\mu_{t+2}} \geq \left(\frac{R^m_{t+1,t+2}}{R^b_{t+1,t+2}}\right)\left(\left(\frac{\mathcal{R}^{bn_0}_{t,t+1}}{\chi}\right)^{\frac{\alpha\gamma_2}{1-\alpha}}-1\right)^{1/\gamma_2}$$

which implies:

$$\begin{aligned}1+\mu_{t+2} &\leq \zeta_2^p\epsilon^o_{t+2}\left(\frac{R^b_{t+1,t+2}}{R^m_{t+1,t+2}}\right)\left(\left(\frac{\mathcal{R}^{bn_0}_{t,t+1}}{\chi}\right)^{\frac{\alpha\gamma_2}{1-\alpha}}-1\right)^{-1/\gamma_2}\\ &= \zeta_2^p\epsilon^o_{t+2}\left(\frac{R^b_{t+1,t+2}}{R^m_{t+1,t+2}}\right)\left(\left(\frac{\epsilon^{n_0}_t\frac{R^b_{t-1,t}}{R^{n_0}_{t-1,t}}R^b_{t,t+1}}{\chi}\right)^{\frac{\alpha\gamma_2}{1-\alpha}}-1\right)^{-1/\gamma_2}\end{aligned}$$

So, the condition is:

$$1+\mu_{t+2} \leq \zeta_2^p \epsilon_{t+2}^o \frac{R_{t+1,t+2}^b}{R_{t+1,t+2}^m} \left(\left(\frac{\min\{\epsilon_t^{n_0} \frac{R_{t-1,t}^b}{R_{t-1,t}^{n_0}}, 1\} R_{t,t+1}^b}{\chi} \right)^{\frac{\alpha\gamma_2}{1-\alpha}} - 1 \right)^{-1/\gamma_2}$$

$\square$

B.6 Platform Problem

The platform earns profit:

$$\pi_t^s = \mu_t \Big(\eta_{0,t}^n \sum_{n_1} \eta_{1,t+1}^{n_1} x_{0,t}^{(n,n_1)} + \eta_{1,t}^n \sum_{n_0} \eta_{0,t-1}^{n_0} c_{1,t}^{(n_0,n)} + \eta_{2,t}^n \sum_{n_0,n_1} \eta_{0,t-2}^{n_0} \eta_{1,t-1}^{n_1} c_{2,t}^{(n_0,n_1,n)} \Big).$$

Imposing the equilibrium objects that the platform takes as given (the goods market and the agent decisions about where to trade), the platform profit can be re-expressed as:

$$\begin{aligned}
\pi_t^s &= \mu_t \eta_{1,t}^p \sum_{n_0} \eta_{0,t-1}^{n_0} y_{1,t}^{(n_0,p)} \\
&= \mu_t \left(\frac{1}{1 + \left(\frac{1}{\zeta}\right)^{\gamma_1} \epsilon_t^{(1/(1-\alpha)+\beta-1)\gamma_1} (1+\mu_t)^{(\beta-1)\gamma_1}} \right) \\
&\quad \times \sum_{n_0} \left(\frac{1}{1 + \left(\frac{\zeta^{n_0'}}{\zeta^{n_0}}\right)^{\gamma_0} \left(\frac{(1+\mu_{t-1}^{n_0'})\mathcal{R}_{t-1,t}^{bn_0'}}{(1+\mu_{t-1}^{n_0})\mathcal{R}_{t-1,t}^{bn_0}} \right)^{-\frac{\alpha\gamma_0}{1-\alpha}}} \right) z \left(\frac{\alpha z}{(1+\mu_{t-1}^{n_0})\mathcal{R}_{t-1,t}^{bn_0}} \right)^{\frac{\alpha}{1-\alpha}} \\
&= \mu_t \left(\frac{1}{1 + \left(\frac{1}{\zeta}\right)^{\gamma_1} \epsilon_t^{(1/(1-\alpha)+\beta-1)\gamma_1} (1+\mu_t)^{(\beta-1)\gamma_1}} \right) z \left(\frac{\alpha z}{R_{t-1,t}^b} \right)^{\frac{\alpha}{1-\alpha}} \\
&\quad \times \left(\frac{\left(\epsilon_{t-1} \frac{R_{t-2,t-1}^b}{R_{t-2,t-1}^m}\right)^{-\frac{\alpha}{1-\alpha}}}{1 + (\zeta)^{\gamma_0} \left(\frac{1+\mu_{t-1}}{(\epsilon_{t-1} R_{t-2,t-1}^b)/R_{t-2,t-1}^m} \right)^{-\frac{\alpha\gamma_0}{1-\alpha}}} + \frac{(1+\mu_{t-1})^{-\frac{\alpha}{1-\alpha}}}{1 + \left(\frac{1}{\zeta}\right)^{\gamma_0} \left(\frac{(\epsilon_{t-1} R_{t-2,t-1}^b)/R_{t-2,t-1}^m}{1+\mu_{t-1}} \right)^{-\frac{\alpha\gamma_0}{1-\alpha}}} \right) \\
&=: \Pi(\mu_t, \epsilon_t, \mathcal{R}_\mathbf{t}) \boldsymbol{\Gamma}(\boldsymbol{\mu_{t-1}}, \boldsymbol{\epsilon_{t-1}}, \mathcal{R}_\mathbf{t}) \\
&=: \pi^s(\mu_t, \mu_{t-1}, \epsilon_t, \epsilon_{t-1}, \mathcal{R}_\mathbf{t})
\end{aligned}$$

and ϵ_t is given by:

$$\begin{aligned}
\epsilon_t &= \left[\frac{\zeta_1^{\gamma_1}}{\zeta_2^{\gamma_2}} \frac{(1+\mu_t)^{-\gamma_1(1-\beta)+1+\gamma_2}}{(R_{t-1,t}^b/R_{t-1,t}^m)^{1+\gamma_2}} \left(\frac{1 - \frac{(1-\beta)(1-\alpha)}{1+\mu_t}}{1-(1-\beta)(1-\alpha)} \right) \right]^{\frac{1}{\frac{\alpha}{1-\alpha}(\gamma_1+1)+\gamma_1\beta+1+\gamma_2}} \\
&=: \epsilon(\mu_t, \mathcal{R}_\mathbf{t})
\end{aligned}$$

where $\mathcal{R}_\mathbf{t} = (R_{t-1,t}^b, R_{t-2,t-1}^b, R_{t-1,t}^m, R_{t-2,t-1}^m)$ is the set of returns that the platform does not internalize.

The Lagrangian for the platform is:

$$\mathcal{L} = \sum_{t=0}^{\infty} \xi_{0,t} \pi^s(\mu_t, \mu_{t-1}, \epsilon_t, \epsilon_{t-1}, \mathcal{R}_\mathbf{t})$$

So, for all $t \geq 0$, the FOC for μ_t is given by:

$$\begin{aligned}
0 = {} & \xi_{0,t} \frac{\partial \pi^s(\mu_t, \mu_{t-1}, \epsilon_t, \epsilon_{t-1}, \mathcal{R}_\mathbf{t})}{\partial \mu_t} + \xi_{0,t+1} \frac{\partial \pi^s(\mu_{t+1}, \mu_t, \epsilon_{t+1}, \epsilon_t, \mathcal{R}_\mathbf{t+1})}{\partial \mu_t} \\
& + \xi_{0,t} \frac{\partial \pi^s(\mu_t, \mu_{t-1}, \epsilon_t, \epsilon_{t-1}, \mathcal{R}_\mathbf{t})}{\partial \epsilon_t} \frac{\partial \epsilon(\mu_t, \mathcal{R}_\mathbf{t})}{\partial \mu_t} \\
& + \xi_{0,t+1} \frac{\partial \pi^s(\mu_{t+1}, \mu_t, \epsilon_{t+1}, \epsilon_t, \mathcal{R}_\mathbf{t+1})}{\partial \epsilon_t} \frac{\partial \epsilon(\mu_t, \mathcal{R}_\mathbf{t})}{\partial \mu_t}
\end{aligned}$$

where

$$\begin{aligned}
& \frac{\partial \pi^s(\mu_t, \mu_{t-1}, \epsilon_t, \epsilon_{t-1}, \mathcal{R}_\mathbf{t})}{\partial \mu_t} \\
& = \Gamma(\mu_{t-1}, \epsilon_{t-1}, \mathcal{R}_t) z \left(\frac{\alpha z}{R^b_{t-1,t}} \right)^{\frac{\alpha}{1-\alpha}} \left(\frac{1}{1 + \left(\frac{1}{\zeta}\right)^{\gamma_1} \epsilon_t^{(1/(1-\alpha)+\beta-1)\gamma_1} (1+\mu_t)^{(\beta-1)\gamma_1}} \right. \\
& \quad \left. - \frac{\mu_t \left(\frac{1}{\zeta}\right)^{\gamma_1} \epsilon_t^{(1/(1-\alpha)+\beta-1)\gamma_1} (\beta-1)\gamma_1 (1+\mu_t)^{(\beta-1)\gamma_1 - 1}}{\left(1 + \left(\frac{1}{\zeta}\right)^{\gamma_1} \epsilon_t^{(1/(1-\alpha)+\beta-1)\gamma_1} (1+\mu_t)^{(\beta-1)\gamma_1}\right)^2} \right) \\
& = \Gamma(\mu_{t-1}, \epsilon_{t-1}, \mathcal{R}_t) z \left(\frac{\alpha z}{R^b_{t-1,t}} \right)^{\frac{\alpha}{1-\alpha}} \frac{1}{1 + \left(\frac{1}{\zeta}\right)^{\gamma_1} \epsilon_t^{(1/(1-\alpha)+\beta-1)\gamma_1} (1+\mu_t)^{(\beta-1)\gamma_1}} \\
& \quad \times \left(1 - \frac{\mu_t \left(\frac{1}{\zeta}\right)^{\gamma_1} \epsilon_t^{(1/(1-\alpha)+\beta-1)\gamma_1} (\beta-1)\gamma_1 (1+\mu_t)^{(\beta-1)\gamma_1 - 1}}{1 + \left(\frac{1}{\zeta}\right)^{\gamma_1} \epsilon_t^{(1/(1-\alpha)+\beta-1)\gamma_1} (1+\mu_t)^{(\beta-1)\gamma_1}} \right)
\end{aligned}$$

and

$$\frac{\partial \pi^s(\mu_{t+1}, \mu_t, \epsilon_{t+1}, \epsilon_t, \mathcal{R}_{\mathbf{t+1}})}{\partial \mu_t}$$
$$= \Pi(\mu_{t+1}, \epsilon_{t+1}, \mathcal{R}_{\mathbf{t+1}})$$
$$\times \left(\frac{\left(\epsilon_t \frac{R^b_{t-1,t}}{R^m_{t-1,t}}\right)^{-\frac{\alpha(1-\gamma_0)}{1-\alpha}} (\zeta)^{\gamma_0} \left(\frac{\alpha\gamma_0}{1-\alpha}\right)(1+\mu_t)^{-\frac{\alpha\gamma_0}{1-\alpha}-1}}{\left(1 + (\zeta)^{\gamma_0}\left(\frac{1+\mu_t}{\epsilon_t R^b_{t-1,t}/R^m_{t-1,t}}\right)^{-\frac{\alpha\gamma_0}{1-\alpha}}\right)^2} - \right.$$
$$\left. \frac{\left(\frac{\alpha}{1-\alpha}\right)(1+\mu_t)^{-\frac{\alpha}{1-\alpha}-1}\left(1 + \left(\frac{1}{\zeta}\right)^{\gamma_0}\left(\frac{\epsilon_t R^b_{t-1,t}/R^m_{t-1,t}}{1+\mu_t}\right)^{-\frac{\alpha\gamma_0}{1-\alpha}} + (1+\mu_t)^{\frac{\alpha\gamma_0}{1-\alpha}} \frac{\gamma_0\left(\epsilon_t R^b_{t-1,t}/R^m_{t-1,t}\right)^{-\frac{\alpha\gamma_0}{1-\alpha}}}{\zeta^{\gamma_0}}\right)}{\left(1 + \left(\frac{1}{\zeta}\right)^{\gamma_0}\left(\frac{\epsilon_t R^b_{t-1,t}/R^m_{t-1,t}}{1+\mu_t}\right)^{-\frac{\alpha\gamma_0}{1-\alpha}}\right)^2} \right)$$

and

$$\frac{\partial \pi^s(\mu_t, \mu_{t-1}, \epsilon_t, \epsilon_{t-1}, \mathcal{R}_{\mathbf{t}})}{\partial \epsilon_t}$$
$$= -\frac{\Gamma(\mu_{t-1}, \epsilon_{t-1}, \mathcal{R}_t) z \mu_t \left(\frac{\alpha z}{R^b_{t-1,t}}\right)^{\frac{\alpha}{1-\alpha}} \epsilon_t^{-1}}{\left(1 + \zeta^{-\gamma_1}\epsilon_t^{(1/(1-\alpha)+\beta-1)\gamma_1}(1+\mu_t)^{(\beta-1)\gamma_1}\right)}$$
$$\times \left(\frac{\left(\frac{1}{\zeta}\right)^{\gamma_1}\left(\frac{1}{1-\alpha} + \beta - 1\right)\gamma_1 \epsilon_t^{(1/(1-\alpha)+\beta-1)\gamma_1}(1+\mu_t)^{(\beta-1)\gamma_1}}{1 + \left(\frac{1}{\zeta}\right)^{\gamma_1}\epsilon_t^{(1/(1-\alpha)+\beta-1)\gamma_1}(1+\mu_t)^{(\beta-1)\gamma_1}} \right)$$

and

$$\frac{\partial \pi^s(\mu_{t+1}, \mu_t, \epsilon_{t+1}, \epsilon_t, \mathcal{R}_{\mathbf{t+1}})}{\partial \epsilon_t}$$
$$= \Pi(\mu_{t+1}, \epsilon_{t+1}, \mathcal{R}_{\mathbf{t+1}}) \times$$
$$\left(\frac{-\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{1}{\epsilon_t R^b_{t-1,t}/R^m_{t-1,t}}\right)^{\frac{\alpha}{1-\alpha}} \frac{1}{\epsilon_t}\left(1+(1+\gamma_0)\zeta^{\gamma_0}\left(\frac{1+\mu_t}{\epsilon_t R^b_{t-1,t}/R^m_{t-1,t}}\right)^{-\frac{\alpha\gamma_0}{1-\alpha}}\right)}{\left(1+\zeta^{\gamma_0}\left(\frac{1+\mu_t}{\epsilon_t R^b_{t-1,t}/R^m_{t-1,t}}\right)^{-\frac{\alpha\gamma_0}{1-\alpha}}\right)^2} \right.$$
$$\left. + \frac{\left(\frac{1}{1+\mu_t}\right)^{\frac{\alpha}{1-\alpha}}\left(\frac{1}{\zeta}\right)^{\gamma_0}\left(\frac{\alpha\gamma_0}{1-\alpha}\right)\left(\frac{R^b_{t-1,t}/R^m_{t-1,t}}{1+\mu_t}\right)^{-\frac{\alpha\gamma_0}{1-\alpha}}(\epsilon_t)^{-\frac{\alpha\gamma_0}{1-\alpha}-1}}{\left(1+\left(\frac{1}{\zeta}\right)^{\gamma_0}\left(\frac{\epsilon_t R^b_{t-1,t}/R^m_{t-1,t}}{1+\mu_t}\right)^{-\frac{\alpha\gamma_0}{1-\alpha}}\right)^2} \right)$$

and

$$\frac{\partial \epsilon(\mu_t, \mathcal{R}_{\mathbf{t}})}{\partial \mu_t}$$
$$= \left[\frac{\zeta_1^{\gamma_1}}{\zeta_2^{\gamma_2}} \frac{(1+\mu_t)^{-\gamma_1(1-\beta)+1+\gamma_2}}{(R^b_{t-1,t}/R^m_{t-1,t})^{1+\gamma_2}} \left(\frac{1-\frac{(1-\beta)(1-\alpha)}{1+\mu_t}}{1-(1-\beta)(1-\alpha)}\right)\right]^{\left(\frac{\gamma_1+\alpha}{1-\alpha}+1+\gamma_2-\gamma_1(1-\beta)\right)^{-1}}$$
$$\times \left(\frac{\gamma_1+\alpha}{1-\alpha}+1+\gamma_2-\gamma_1(1-\beta)\right)^{-1}$$
$$\left[\frac{-\gamma_1(1-\beta)+1+\gamma_2}{1+\mu_t} + \frac{(1-\beta)(1-\alpha)}{(1+\mu_t)^2}\left(1-\frac{(1-\beta)(1-\alpha)}{1+\mu_t}\right)^{-1}\right]$$

We can now characterize steady-state equilibrium with an optimizing platform.

B.7 Proofs for Platform Competition

Proof of Proposition 3. (i) The market equilibrium is the same as in subsection 3.3 except that now $(1-\mu_t)$ is replaced by $(1-\mu_t^1)/(1-\mu_t^2)$. If χ is sufficiently large that the threat of exclusion from either platform is sufficient to incentive financial intermediaries to repay loans on that ledger, then $q^{En} = \tilde{q}^{En}$ and there is no need to bargain over enforcement because it doesn't require cooperation. If χ is sufficiently low that only exclusion from both platforms is sufficient to incentivize repayment, then for both platforms n, we have $q^{En} = q^E$ and

$\tilde{q}^{En} = 0$ so outcome of the Nash Bargaining is cooperation on enforcement without a transfer $T = 0$.
(ii) For ζ close to 1, when the trading advantage of platform 1 is not too large, the possible outcomes look like those in subsection 3.3. That is, if χ is large, then both platforms are able to enforce contract without cooperation and if χ is small, then cooperation is required for any contract enforcement. However, when χ and ζ are large, it is possible that, under non-cooperation, platform 1 can enforce contracts while platform 2 cannot.

Platform bargaining at $t = 0$ is now more complicated because the outside option for platform 1 is more complicated. If χ and ζ are sufficiently large that ledger 1 can incentivize contract enforcement on their ledger without cooperation and $\varsigma\epsilon < 1$, then $\tilde{q}^{E1} > q^{E1}$ and so ledger 1 prefers the non-cooperative outcome. This means that the transfer platform 2 would have to pay to get enforcement leads to negatives surplus:

$$q^{E2} - \tilde{q}^{E2} - T = \frac{1}{2}\left(q_0^{E2} - \tilde{q}_0^{E2} + q_0^{E1} - \tilde{q}_0^{E1}\right) < 0$$

if $\tilde{q}_0^{E1} - q_0^{E1} > q_0^{E2} - \tilde{q}_0^{E2}$and so the bargaining breaks down.

□