

# Convenience Yields and Financial Repression\*

Preliminary Draft

Latest version: [https://jepayne.github.io/files/PS\\_CYRepress.pdf](https://jepayne.github.io/files/PS_CYRepress.pdf)

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## Abstract

US federal debt plays a special role in the US economy and so gives the US government a funding advantage, often summarized by the “convenience yield” on US debt. Why? One reason is that government design (and/or repression) of the financial sector influences asset pricing and helps make long term US federal debt a “safe-asset”. We study the macroeconomic consequences on government borrowing capacity, financial stability, and investment. We then test our theory using new historical data on US convenience yields going back to 1860.

JEL CLASSIFICATION: E31, E43, G12, N21, N41

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# 1 Introduction

Many researchers have documented that US federal debt plays a special role in the US economy and so gives the US government a funding advantage, often summarized by the “convenience yield”. Macro-finance models have frequently treated this as an immutable feature of the economic environment and encoded the “benefits” of holding US debt into agent preferences or the market structure. This means the government can easily “exploit” the convenience yield to increase spending. By contrast, historical studies suggest that the convenience yield emerged as part of a complicated, long-term government program to increase its borrowing capacity. Financial regulation and/or repression have been key tools in this process, particularly during crises when the government has needed to raise funding quickly. When viewed in this way, generating and exploiting a convenience yield imposes far reaching impacts on the economy. It links the stability of the financial sector to the stability of the government budget constraint. It distorts the portfolio of the financial sector, potentially increasing default and crowding out private liquidity creation and productive investment. In this paper, we study the mechanics and trade-offs involved with creating financial sector demand for government debt and relate our analysis to historical eras.

We start with an illustrative three period model, in which the banking sector is risky and, absent regulation, there is no special role for government debt. The economy is populated by households who need bank deposits to be able to consume in the middle period. Banks issue on-demand deposits and equity to households and invest in short assets, capital, and government bonds. In this sense, banks provide both liquidity and intermediation services to households. In the middle period, banks get heterogeneous deposit withdrawal shocks, which potentially cause them to default because their resource-drawing capacity (from the household sector) is constrained and the inter-bank asset markets are characterized by “fire-sale pricing”. The combination of households’ need for deposits and the possibility of costly default are the “frictions” in the economy that break [Modigliani and Miller \(1958\)](#) by driving a wedge between the stochastic discount factors of the household and banks. The government in our model cares about spending and household welfare but faces a constraint that taxation is determined by an exogenous political process. Instead, the government can place restrictions on the portfolios of the banks that potentially increase the price of government debt and expand their spending. We focus on restrictions that require the banks to maintain a particular ratio of weighted average assets to deposits. We interpret equal weighting on government debt and capital to be neutral regulation since, absent regulation, government debt does not play a special role in the economy. We interpret a higher weighting on government debt to be financial “repression” because it makes government debt a better asset for satisfying regulatory requirements.

We characterize how repression can generate a convenience yield on government debt both directly through forced portfolio choice and also indirectly by making government debt endogenously a “safer-asset” for economy. The key feature of our model is that the constraint on holding government debt binds more in the bad state of the world and so the relative price of government debt appreciates. This makes government debt a good “hedge” against bad shocks. So, forcing

banks to hold government debt in the interbank market also makes banks more willing to purchase government debt in the primary market to hedge risk, which opens up a convenience yield on government debt. In the terminology of the recent empirical finance literature on institutional asset pricing, the government is using regulation to make bond demand inelastic for banks and so generate price “under-reaction” in the market for that asset. The tractability of our model allows us to characterize how the shape parameters in the bond demand functions and convenience yield expressions depend explicitly on regulation and fiscal policy. In this sense, our model offers a rich understanding of how the convenience yield emerges from government policy.

We first use our model to show that generating a higher convenience yield comes at the cost of higher bank default, less bank liquidity creation, and lower investment into capital. The higher rate of bank default appears because financial repression inflates the debt price in the interbank market but also decreases the portfolio return for solvent banks and so makes the marginal bank more likely to default. The lower investment rate appears because government borrowing crowds out bank capital creation, as is standard in many macroeconomic models. In this sense, the government faces a trade-off between optimizing their fiscal capacity and having a well functioning financial sector. We characterize this trade-off and show that the optimal government policy requires some degree of repression. This result is different to some recent papers (e.g. [Chari et al. \(2020\)](#)) because we have placed restrictions on the tax process and because the banks in our model play roles as both liquidity providers and intermediaries.

We then show that government fiscal irresponsibility erodes the convenience yield. There are a number of reasons for this. First, government default in bad states of the world restricts the banking sector’s ability to use government debt to hedge aggregate risk, which makes it harder for repression to ensure government debt plays the “safe-asset” role in the economy. Second, repression ties the solvency of the banking sector to the solvency of the government. So, increasing government default makes government debt a worse hedge at the same time that it makes banks more concerned about finding a good hedge. Ultimately, this leads to a decrease in the convenience yield. This is very different to models with bond-in-the-utility or bond-in-advance. In these cases, the role of government debt is exogenous and its marginal usefulness increases as the market value of government debt declines. This means that as the government starts to default, the convenience yield increases. Or put another way, in these models the agents get utility from giving resources to the government so when the government starts to default, then they want more government debt. This highlights the importance of starting from a model where government is not exogenously important.

We “test” our model using a new data set containing prices and cash flow information for a large collection of corporate bonds from 1850-1940. To infer term structures of *yields* on US high grade corporate bonds, we deploy the techniques from [Payne et al. \(2022\)](#), which use a non-linear state space model with drifting parameters and stochastic volatility. We combine these estimates with existing bond indices for the modern period and estimates of the government yield curve from [Payne et al. \(2022\)](#) to calculate a term structure of spreads between government and corporate bonds form 1850-2022. We follow [Krishnamurthy and Vissing-Jorgensen \(2012\)](#) and

refer to this spread as the “convenience yield” of government debt and interpret it as reflecting the special role of government debt in the economy.

We infer a collection of stylized facts about relative government debt prices and how they are related to changes in financial regulation. First, we find there are low frequency movement in average convenience yields. During the late nineteenth century there was tight financial repression, high convenience yields, and frequent bank defaults, as predicted by our model. The relationship is very different after FDR introduces deposit insurance in the 1930s and the banking sector is stabilized. Second, we find that the elasticity of the convenience yield to government debt supply varies with regulation. In the late nineteenth century and the decades following World War II (times with high restrictions on the financial sector and bank balance sheets skewed towards government debt), the elasticity is close to zero while in the 1920s, 70s, and 80s (times with less restriction on the financial sector), the elasticity is strongly negative. This is consistent with our model, which suggests that the elasticity of the convenience yield is not a stable exploitable demand function but instead a reflection of particular regulations and government policies. Similar to the Phillips curve, the relationship breaks down as governments try to exploit it.

## 1.1 Related Literature (Incomplete)

Our equilibrium model of safe asset creation is part of a long literature attempting to understand how the financial sector and government can create safe assets (e.g. [Holmstrom and Tirole \(1997\)](#), [Holmström and Tirole \(1998\)](#), [Gorton and Ordonez \(2013\)](#), [Gorton \(2017\)](#), [He et al. \(2016\)](#), [He et al. \(2019\)](#)) and the macroeconomic implications of safe asset creation (e.g. [Caballero et al. \(2008\)](#), [Caballero et al. \(2017\)](#), [Caballero and Farhi \(2018\)](#)). Our focus is on trying to characterize how safe asset creation impacts fiscal capacity, financial stability, and investment.

Our government design problem is part of a literature studying optimal fiscal policy in economies with financial frictions and tax distortions ([Calvo \(1978\)](#), [Bhandari et al. \(2017b\)](#), [Bhandari et al. \(2017a\)](#), [Chari et al. \(2020\)](#), [Bassetto and Cui \(2021\)](#), [Sims \(2019\)](#), [Brunnermeier et al. \(2022\)](#)). In this paper we take the stand that the government follows a fiscal policy rule governed by political constraints. We do not address the question of whether a Ramsey planner without any political constraints would want to use financial repression. We believe that the historical evidence is clear that the government does use financial repression. We leave the questions of working out financial and political constraints could make this optimal for further work. Instead, our focus is on studying how the government chooses price processes in a economy with a financial sector.

Our historical comparisons extend existing studies on the convenience yield (e.g. [Krishnamurthy and Vissing-Jorgensen \(2012\)](#), [Choi et al. \(2022\)](#)) back to the mid nineteenth century. This makes us part of a literature attempting to connect historical time series for asset prices to government financing costs (e.g. [Payne et al. \(2022\)](#), [Jiang et al. \(2022a\)](#), [Chen et al. \(2022\)](#), [Jiang et al. \(2022b\)](#), [Jiang et al. \(2021b\)](#), [Jiang et al. \(2021a\)](#), [Jiang et al. \(2020\)](#)).

## 2 Model of Convenience Yields

In this section, we outline a three-period version of our model to illustrate how financial sector regulation can create a convenience yield on government debt.

### 2.1 Environment

*Setting:* The economy lasts for three periods:  $t \in \{0, 1, 2\}$ . We interpret  $t = 0$  as a primary asset market,  $t = 1$  as a morning inter-bank market, and  $t = 2$  as the afternoon competitive market. There is one consumption good. There is a continuum of islands,  $j \in [0, 1]$ , each with a unit measure of household members, indexed by  $h \in [0, 1]$ , and a unit measure of competitive banks, indexed by  $i \in [0, 1]$ . Each household can only participate in the financial market on their island. There are two production technologies in the economy: one that transforms  $m_0$  goods at time  $t = 0$  to  $z_1(s)m_0$  goods at time  $t = 1$  (short-term asset) and another one that transforms  $k_0$  goods at time  $t = 0$  to  $z_2(s)k_0$  goods at time  $t = 2$  (capital), where  $s$  is the aggregate state that has distribution  $\Pi(s)$  and is realized at the beginning of  $t = 1$ .

*Assets and Markets:* We use goods as the numeraire. At  $t = 0$ , the government issues bonds in the primary market at price  $q_0^b$  that pay  $\delta_2^b$  at time  $t = 2$ . At time  $t = 1$ , banks trade government bonds, at price  $q_1^b$ , and claims on capital, at price  $q_1^k$ , in the inter-bank market. We show production and bond payoffs and the timing of shocks graphically in Figure 1.

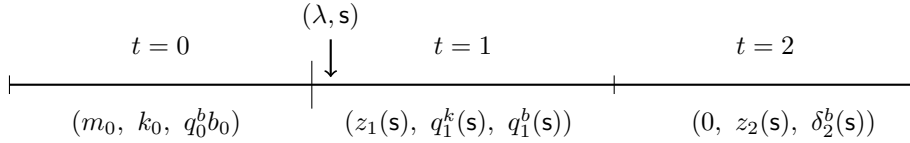


Figure 1: Timing of Payoffs

At  $t = 0$ , each bank issues demand deposits,  $d_0$ , and equity,  $e_0$ , to the households on their island at prices  $q_0^d$  and  $q_0^e$ , respectively.<sup>1</sup> Bank equity pays  $\delta_1^e$  at time 1 and  $\delta_2^e$  at time 2 and is not tradable after  $t = 0$ . Households can withdraw deposits at time  $t \in \{1, 2\}$  for resources  $\delta_t^d$ , where  $\delta_t^d = 1$  if the bank is solvent and  $1 > \delta_2^d \geq \delta_1^d$  if the bank is insolvent, where inequality is set so there is no run.

*Government:* The government ranks allocations according to:

$$\theta G + \mathcal{U}$$

where  $G$  is the provision of public goods by the government and  $\mathcal{U}$  is the aggregate lifetime household utility under equal Pareto weights. Parameter  $\theta$  is interpreted as the relative value of

<sup>1</sup>The deposit and equity prices are the same on each island because islands are ex-ante identical.

public goods. At  $t = 0$ , the government finances public good provision by issuing  $B_0$  bonds at price  $q_0^b$  leading to the  $t = 0$  budget constraint:

$$G \leq q_0^b B_0 \quad (2.1)$$

At time 2, the government raises taxes  $T_2(\mathbf{s})$  from households at  $t = 2$ , which it uses to repay  $\delta_2^b(\mathbf{s})$  per unit of bonds according to:

$$\delta_2^b(\mathbf{s}) B_0 \leq T_2(\mathbf{s}) \quad (2.2)$$

where  $\delta_2^b(\mathbf{s}) < 1$  is interpreted as “partial default” or “dilution” when the government decreases the real value of the bond principle. We refer to  $T_2(\mathbf{s})$  as the government “fiscal rule” and treat it as an exogenous outcome of an unmodelled political process. The exogenous  $T_2(\mathbf{s})$  pins down an upper bound on  $B_0$ , which means that the only way the government can increase  $G$  at time  $t = 0$  is by inflating the value of its debt  $q_0^b B_0$ . Motivated by this feature of our model, we refer to  $G$  as the government’s “fiscal capacity”. The government can try to increase its fiscal capacity by imposing portfolio restrictions on each bank at end of period 0 and period 1:

$$\varrho(q_0^d d_0^i) \leq q_0^b b_0^i + (1 - \kappa) k_0^i \quad (2.3)$$

$$\varrho d_1^i(\lambda) \leq q_1^b(\mathbf{s}) b_1^i(\lambda, \mathbf{s}) + (1 - \kappa) \left( q_1^k(\mathbf{s}) k_1^i(\lambda, \mathbf{s}) \right) \quad (2.4)$$

where  $(d_0^i, d_1^i)$  denote bank  $i$ ’s initial deposit issuance at  $t = 0$  and remaining deposit at the end of period 1, respectively, and similarly for the holdings of government debt  $(b_0^i, b_1^i)$ , and capital  $(k_0^i, k_1^i)$ . The pair  $(\varrho, \kappa)$  is a set of regulatory parameters:  $\varrho$  restricts the bank’s deposit-to-asset ratio, while  $\kappa$  is the relative “weight” on capital in the calculation of regulatory asset value, that we interpret as an extent of repression.  $\kappa = 0$  refers to a regulatory regime that treats government debt and capital symmetrically and just restricts bank risk taking.  $\kappa > 0$  incentivizes the holding of government debt over capital as regulatory collateral, while  $\kappa < 0$  corresponds to the opposite case. One historically relevant set of regulatory parameters is  $(\varrho, \kappa) = (\varrho, 1)$ , which corresponds to forcing to back each deposit by  $1/\varrho$  fraction of government debt, similar to what was done during the National Banking Era (1862-1913). For contemporary regulation, we can always find a combination of  $(\varrho, \kappa)$  such that  $1/\varrho$  and  $(1 - \kappa)/\varrho$  are the Basel III “risk weights” for calculating weighted bank assets.

*Household problem:* Households are uncertain about their own preferences. There are two “layers” of uncertainty: individual- and island-specific, both of which are resolved at the start of  $t = 1$ . On each island  $j$ , with probability  $\lambda^j$  agents are early consumers, who only value the good at period 1, and with probability  $1 - \lambda^j$  they are late consumers, who only value the good at period 2. The probability  $\lambda^j$  is island-specific and it follows the distribution  $\lambda \sim F(\lambda)$ . For convenience we drop the  $j$  superscript and index islands by  $\lambda$ . We denote the state of being an early consumer

by  $\zeta^h \in \{0, 1\}$ . At time 0, households rank allocations according to:

$$\mathcal{U} := \mathbb{E} [\zeta^h u(c_1^h(\lambda, \mathbf{s})) + (1 - \zeta^h) u(c_2^h(\lambda, \mathbf{s}))], \quad (2.5)$$

where  $c_t^h(\lambda, \mathbf{s})$  denotes consumption of household  $h$  on island  $\lambda$  in period  $t \in \{1, 2\}$  when aggregate state is  $\mathbf{s}$ . Each household is endowed with one unit of goods at  $t = 0$  and zero goods in the other periods. All agents have the time 0 budget constraint:

$$q_0^d d_0^h + q_0^e e_0^h \leq 1 \quad (2.6)$$

where  $d_0^h$  and  $e_0^h$  are household  $h$ 's deposit and equity holdings. Early consumers ( $\zeta_h = 1$ ) only consume at  $t = 1$  and face the deposit-in-advance constraint:<sup>2</sup>

$$c_1^h \leq \delta_1^d(\lambda, \mathbf{s}) d_0^h.$$

Late consumers ( $\zeta_h = 0$ ) do not consume at  $t = 0$  (leave all their deposits in their bank)<sup>3</sup> and face the following budget constraint in periods 1 and 2:

$$\begin{aligned} \delta_1^d(\lambda, \mathbf{s}) d_1^h &\leq \delta_1^d(\lambda, \mathbf{s}) d_0^h + \delta_1^e(\lambda, \mathbf{s}) e_0^h \\ c_2^h &\leq \delta_2^e(\lambda, \mathbf{s}) e_0^h + \delta_2^d(\lambda, \mathbf{s}) d_1^h - \tau(\mathbf{s}) \end{aligned} \quad (2.7)$$

where  $\tau(\mathbf{s})$  denotes (per capita) lump-sum taxes.

*Bank problem:* Each island has a representative bank owned by the households on that island. The bank's object is to maximize its market value at  $t = 0$ :

$$\underbrace{\mathbb{E} [\xi(\lambda, \mathbf{s}) \max \{0, \delta_1^e + \delta_2^e\}]}_{=q_0^e \text{ (price of equity at } t=0)} + q_0^d d_0^i - m_0^i - k_0^i - q_0^b b_0^i \quad (2.8)$$

where  $\xi(\lambda, \mathbf{s})$  denotes the representative household's stochastic discount factor on island  $\lambda$  when the aggregate state is  $\mathbf{s}$ . At  $t = 0$ , the bank chooses deposit issuance,  $d_0^i \geq 0$ , short asset holdings,  $m_0^i \geq 0$ , initial capital,  $k_0^i \geq 0$ , and initial government debt holding,  $b_0^i \geq 0$ , subject to the regulatory constraint (2.3) at  $t = 0$ . At  $t = 1$ , it chooses whether to default on its deposit

<sup>2</sup>For convenience, we assume that the equity of the early consumers is lost. This assumption is without loss of generality for the qualitative direction of our results.

<sup>3</sup>Late consumers have no incentive to run because the deposit contract payouts are restricted to give the late consumer at least as much as the early consumer.

(by paying  $\delta_1^d, \delta_2^d < 1$ ), and chooses new asset holdings  $b_1^i \geq 0$  and  $k_1^i \geq 0$ , subject to:

$$\delta_1^e + \delta_1^d \lambda d_0^i + q_1^k(s)k_1^i + q_1^b(s)b_1^i \leq z_1(s)m_0^i + q_1^k(s)k_0^i + q_1^b(s)b_1^i - \varsigma d_0^i \mathbb{1}\{\delta_1^d < 1\}, \quad (2.9)$$

$$0 \leq b_1^i, \quad 0 \leq k_1^i, \quad 0 \leq \delta_1^e \quad (2.10)$$

$$\begin{aligned} \delta_2^e + \delta_2^d(1 - \lambda)d_0^i &\leq z_2(s)k_1^i + \delta_2^b(s)b_1^i, & 0 \leq \delta_2^e, \\ \delta_1^d &\leq \delta_2^d, \end{aligned} \quad (2.11)$$

where  $\lambda d_0^i$  and  $(1 - \lambda)d_0^i$  represent early withdrawal and rolled over deposit, respectively,  $\delta_1^e$  and  $\delta_2^e$  are bank dividends paid at  $t = 1$  and at  $t = 2$ , while  $k_1^i$  and  $b_1^i$  denote the bank holdings of capital and government debt at the end of period  $t = 1$ —both of them are subject to short selling constraints. In addition, banks face the regulatory constraint (2.4) at  $t = 1$ .

The bank problem involves three key frictions. First, the deposit payout at  $t = 1$ ,  $\delta^d$ , cannot be freely conditioned on the state  $(\lambda, s)$ . Second, banks cannot issue equity at  $t = 1$  in the sense that

$$0 \leq \delta_1^e, \quad (2.12)$$

which—combined with (2.9)-(2.10)—implies that *at  $t = 1$  banks cannot get extra resources from the household*: they cannot raise equity and must cover their early withdrawals either by using their short asset holdings or by selling their long assets. This means that banks may potentially end up defaulting on deposits. The ability to do so is guaranteed by the third friction, namely, that banks have limited liability, in the sense that they cannot force negative dividends on their shareholders at  $t = 2$ :

$$0 \leq \delta_2^e.$$

If the limited liability constraint binds and the bank defaults, then it incurs a real dead-weight cost  $\varsigma$  at  $t = 1$  (proportional to its total outstanding deposit  $d_0^i$ ) and is contractually obligated to pay the maximum amount  $\delta_1^d$  to its early withdrawers subject to that it is able to pay at least as much to its late withdrawers at  $t = 2$  (so that late consumers have no incentive to run). The dead-weight cost  $\varsigma$  may include the loss of firm specific information, the destruction of consumer networks, etc. Banks take  $\varsigma$  as given but in equilibrium it is determined as an increasing function of the fraction of defaulting banks, that is, the cost of default is higher when a lot of banks default at the same time.

## 2.2 Equilibrium

**Definition 1** (Budget-feasible government policy). Given a fiscal rule  $T_2(s)$  and bond price  $q_0^b$ , a budget-feasible government policy is a tuple  $(G, B_0, \delta_2^b)$  s.t. (2.1) and (2.2) are satisfied with

$$T_2(s) = (1 - \Lambda)\tau(s)$$



where  $\Lambda := \int \lambda dF$  is the expected aggregate withdrawal rate.

**Definition 2** (Competitive Equilibrium). Given a fiscal rule  $T_2(\mathbf{s})$ , regulation  $(\varrho, \kappa)$ , and a budget-feasible government policy  $(G, B_0, \delta_2^b)$ , a competitive equilibrium is a set of prices  $(q_0^d, q_0^e, q_0^b)$  and  $(q_1^k(\mathbf{s}), q_1^b(\mathbf{s}))$ , payoffs  $(\delta_1^d(\lambda, \mathbf{s}), \delta_2^d(\lambda, \mathbf{s}), \delta_2^e(\lambda, \mathbf{s}))$ , household policies  $(d_0^h, e_0^h, c_1^h(\lambda, \mathbf{s}), c_2^h(\lambda, \mathbf{s}))$ , and bank policies  $(d_0^i, m_0^i, k_0^i, b_0^i)$  and  $(k_1^i(\lambda, \mathbf{s}), b_1^i(\lambda, \mathbf{s}))$ , such that

- Households maximize (2.5) subject to (2.6)-(2.7),
- Banks maximize (2.8) subject to (2.3)-(2.4) and (2.9)-(2.11),
- Markets clear:

$$\begin{aligned} G + m_0^i + k_0^i &= 1, & d_0^h &= d_0^i, & e_0^h &= 1, & b_0^i &= B_0, \\ \int b_1^i(\lambda, \mathbf{s}) dF &= B_0 & \int k_1^i(\lambda, \mathbf{s}) &= k_0^i & \int \lambda c_1^h(\lambda, \mathbf{s}) dF &= z_1(\mathbf{s}) m_0 - \varsigma(\lambda^*) d_0 \\ & & \int (1 - \lambda) c_2^h(\lambda, \mathbf{s}) dF &= z_2(\mathbf{s}) k_0 - \int \lambda \delta_2^e(\lambda, \mathbf{s}) dF(\lambda) \end{aligned}$$

We characterize equilibrium in the following way. First, we solve the optimization problem of the household in subsection 2.2.1. Second, we combine household and bank optimization with inter-bank asset market clearing to characterize equilibrium in the  $t = 1$  market for given  $t = 0$  choices in subsection 2.2.2. Finally, we characterize equilibrium in the  $t = 0$  market and discuss convenience yields in subsection 2.2.3.

### 2.2.1 Household Problem

We characterize the solution to the household problem in Proposition 1. The households choose their asset portfolio once and for all at  $t = 0$ , so that the choices satisfy the Euler equations (2.13) and (2.14). Given the household portfolio,  $(d_0^h, e_0^h)$ , early consumption  $c_1$  and late consumption  $c_2$  are determined as functions of asset payoffs  $(\delta_1^d, \delta_2^d, \delta_2^e)$  and idiosyncratic and aggregate shocks.

**Proposition 1** (Characterization of Household Problem). *The household portfolio choices at  $t = 0$  satisfy:*

$$q_0^d = \mathbb{E} \left[ \xi(\lambda, \mathbf{s}) \left( 1 + \nu(\lambda, \mathbf{s}) \right) \delta_1^d(\lambda, \mathbf{s}) \right] \quad (2.13)$$

$$q_0^e = \mathbb{E} \left[ \xi(\lambda, \mathbf{s}) \delta_2^e(\lambda, \mathbf{s}) \right] \quad (2.14)$$

We use the following notation for the stochastic discount factor (SDF) and the liquidity premium:

$$\xi(\lambda, \mathbf{s}) := \frac{(1 - \lambda) u'(c_2^h(\lambda, \mathbf{s}))}{\mu_0^e}, \quad \nu(\lambda, \mathbf{s}) := \frac{\lambda u'(c_1^h(\lambda, \mathbf{s}))}{(1 - \lambda) u'(c_2^h(\lambda, \mathbf{s}))}$$

where  $\mu_0^c > 0$  is the households' Lagrange multiplier on their period  $t = 0$  budget constraint and their consumption choices are

$$c_1(\lambda, \mathbf{s}) = \delta_1^d(\lambda, \mathbf{s})d_0^h, \quad \text{and} \quad c_2(\lambda, \mathbf{s}) = \delta_2^e(\lambda, \mathbf{s})e_0^h + \delta_2^d(\lambda, \mathbf{s})d_0^h - \tau(\mathbf{s}). \quad (2.15)$$

*Proof.* See Appendix A.1. □

Demand deposits provide liquidity services at  $t = 1$  to the early consumers, which introduces a wedge  $(1 + \nu)$  into the household's deposit Euler equation. The presence of this asset-specific wedge implies that households are willing to hold demand deposits at a discount, which leads to a "funding advantage" to the providers of such assets.

### 2.2.2 Equilibrium in the inter-bank markets ( $t = 1$ )

Proposition 2 characterizes equilibrium in the inter-bank markets at time  $t = 1$  for given initial asset holdings  $(m_0, k_0, b_0, d_0)$ . This involves combining household optimization with bank optimization and inter-bank market clearing, the later two of which are complicated by the possibility that banks can default. For the characterization of this default decision, it will be useful to define the banking sector's net worth at  $t = 1$  (after withdrawal) as:

$$a(\lambda, \mathbf{s}) := z_1(\mathbf{s})m_0 + q_1^k(\mathbf{s})k_0 + q_1^b(\mathbf{s})b_0 - \delta^d(\lambda, \mathbf{s})\lambda d_0,$$

and the share of government debt in the banking sector's period  $t = 1$  portfolio as:

$$\varphi(\lambda, \mathbf{s}) := \frac{q_1^b(\mathbf{s})b_1(\lambda, \mathbf{s})}{a(\lambda, \mathbf{s})}.$$

At the beginning of time  $t = 1$ , the island-specific withdrawal shock,  $\lambda d_0$ , leads to ex post heterogeneity among banks: those with low  $\lambda$  will have excess resources,  $z_1(\mathbf{s})m_0^i - \lambda d_0^i > 0$ , that they can use to purchase assets in the inter-bank markets, while those with  $\lambda$  such that  $z_1(\mathbf{s})m_0^i - \lambda d_0^i < 0$  will be forced to sell assets to cover early withdrawals at  $t = 1$ .

**Proposition 2** (Equilibrium at  $t = 1$ ). *Let  $\psi_1^e(\mathbf{s}) \geq 0$  and  $\mu_1^r(\mathbf{s}) \geq 0$  denote the Lagrange multipliers on the  $t = 1$  equity raising constraint (2.12) and the  $t = 1$  regulatory constraint (2.4), respectively. The following hold:*

- (i) *Portfolio choice: If  $\kappa = 0$  (symmetric regulatory penalty), then government debt and capital are perfect substitutes, with relative prices satisfying:*

$$\frac{q_1^b(\mathbf{s})}{q_2^k(\mathbf{s})} = \frac{\delta_1^b(\mathbf{s})}{z_2^k(\mathbf{s})} \quad (2.16)$$

*If  $\kappa \neq 0$  (asymmetric regulatory penalty), then the time  $t = 1$  regulatory constraint binds so for  $\kappa > 0$  ( $\kappa < 0$ ) the banks hold the minimum bonds (capital) required to satisfy the*

constraint. In both cases, this implies the portfolio shares:

$$\begin{aligned}\varphi(\lambda, \mathbf{s}) &= \frac{\varrho \delta^d(\lambda, \mathbf{s})(1 - \lambda)d_0 - (1 - \kappa)(a(\lambda, \mathbf{s}) - \varsigma \mathbb{1}\{\delta^d < 1\}d_0)}{\kappa a(\lambda, \mathbf{s})} \\ 1 - \varphi(\lambda, \mathbf{s}) &= \frac{-\varrho \delta^d(\lambda, \mathbf{s})(1 - \lambda)d_0 + (a(\lambda, \mathbf{s}) - \varsigma \mathbb{1}\{\delta^d < 1\}d_0)}{\kappa a(\lambda, \mathbf{s})}\end{aligned}$$

and the relative prices satisfy:

$$\frac{q_1^b(\mathbf{s})}{q_1^k(\mathbf{s})} = \frac{\delta_2^b(\mathbf{s})}{z_2^k(\mathbf{s}) - \kappa \mu_1^r(\mathbf{s}) q_1^k(\mathbf{s})}. \quad (2.17)$$

(ii) Default decision: The equity raising constraint at  $t = 1$  always binds,  $\psi_1^e > 0$ , and bank dividends at  $t = 2$  have the form:

$$\delta_2^e(\lambda, \mathbf{s}) = \max \left\{ 0, \left( 1 + \psi_1^e(\mathbf{s}) - \mu_1^r(\mathbf{s}) + \kappa \mu_1^r(\mathbf{s})(1 - \varphi(\lambda, \mathbf{s})) \right) a(\lambda, \mathbf{s}) - (1 - \lambda)d_0 \right\} \quad (2.18)$$

Banks default at  $t = 1$  iff they get a withdrawal shock with  $\lambda > \lambda^*$ , where the default cutoff is determined by  $\delta_2^e(\lambda^*, \mathbf{s}) = 0$ .

If  $\kappa \neq 0$ , we can use (2.17) and the definitions  $R^b := \delta_2^b/q_1^b$  and  $R^k := z_2/q_1^k$  for the returns on bonds and capital from  $t = 1$  to  $t = 2$ , to show that in equilibrium the term in the parentheses in (2.18) must be equal to the return on the bank's asset portfolio:

$$1 + \psi_1^e(\mathbf{s}) - \mu_1^r(\mathbf{s}) + \kappa \mu_1^r(\mathbf{s})(1 - \varphi(\lambda, \mathbf{s})) = R^b(\mathbf{s}) + (R^k(\mathbf{s}) - R^b(\mathbf{s}))(1 - \varphi(\lambda, \mathbf{s})) \quad (2.19)$$

(iii) Equilibrium prices: Inter-bank market prices  $(q_1^b(\mathbf{s}), q_1^k(\mathbf{s}), \mu_1^r)$  satisfy the aggregate resource constraint at  $t = 1$ :

$$\int \left( q_1^b(\mathbf{s}) b_1(\lambda, \mathbf{s}) + q_1^k(\mathbf{s}) k_1(\lambda, \mathbf{s}) \right) dF = \int \left( a(\lambda, \mathbf{s}) - \varsigma d_0 \mathbb{1}\{\delta^d < 1\} \right) dF$$

with one of the asset market clearing conditions at  $t = 1$  and either (2.16) (if  $\kappa = 0$ ) or (2.17) (if  $\kappa \neq 0$ ).

*Proof.* See Appendix A.2. □

For a given parameterization of the model, Figure 2 illustrates numerically how the idiosyncratic and aggregate shocks affect the banks' default decision and implied asset payoffs.<sup>4</sup> The vertical dashed lines depict default cutoffs  $\lambda^*$ , assuming that aggregate available resources  $z_1(\mathbf{s})m_0$  are high ("good state", blue color) or low ("bad state", orange color). A negative shock to the available aggregate resources at  $t = 1$  leads to falling prices in the inter-bank asset markets, a decline in bank net worth, and therefore, a fall in  $\lambda^*$  and an increase in the number of defaulting

<sup>4</sup>We can use (2.15) to transform these asset payoffs into household consumption.

banks. The left panel in Figure 2 shows how dead-weight default losses  $\varsigma$  introduce a discontinuity in the deposit payoff at  $\lambda^*$ . The right panel, depicting bank dividends at  $t = 2$ , defined in (2.18), illustrates how banks with low withdrawal shocks  $\lambda$  are benefited from falling inter-bank asset prices in the bad aggregate state.

The bottom two plots of Figure 2 depict bank deposit and equity payouts when financial repression ( $\kappa$ ) is increased. Evidently, an increase in financial repression leads to more default in the bad state and less default in the good state. To understand this, recall from part (ii) of Proposition 2 that banks default when the combination of withdrawals and asset sales force the dividend at  $t = 2$  to be negative. From equations (2.18)-(2.19), we can see that higher financial repression forces the bank portfolio share in bonds to increase. This increases their return in the good state of the world because  $R^k(s_H) > R^b(s_H)$ , which increases the default cut-off  $\lambda^*$ . However, it decreases their return in the bad state because  $R^k(s_L) < R^b(s_L)$ , which decreases the default cut-off  $\lambda^*$ . Or put another way, financial repression leads to redistribution from solvent to insolvent banks in the good state but redistribution from insolvent to solvent banks in the bad state.

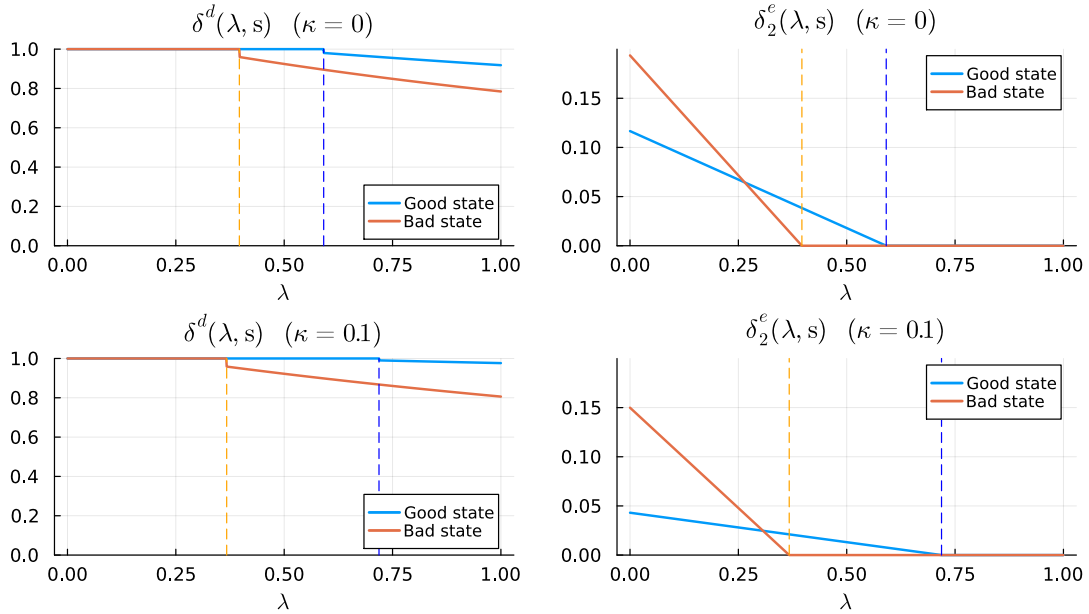


Figure 2: Equilibrium at  $t = 1$ : asset payoffs as functions of  $\lambda$  and  $s$ . The yellow vertical dashed line depicts  $\lambda^*$  in the bad state and the blue vertical dashed line depicts  $\lambda^*$  in the good state.

The bank portfolio decisions and the market clearing conditions in Proposition 2 characterize asset prices  $(q_1^k, q_1^b)$  in the inter-bank market at time  $t = 1$ . Evidently, two key features influence asset prices: the banking sector's inability to draw resources from the households (as characterized by the multiplier  $\psi_1^e$ ) and the regulatory constraint (as characterized by the multiplier  $\mu_1^r$ ). If neither of these features were present, then  $\psi_1^e = \mu_1^r = 0$  and so the prices of bonds and capital would be  $q_1^b = \delta_2^b$  and  $q_1^k = z_2$ . We refer to this as the assets being priced at their “fundamental

value”. Adding the equity raising friction introduces a link between the aggregate proceeds from bank short asset holdings,  $z_1 m_0$ , and aggregate asset demand, putting downward pressure on asset prices in the inter-bank market. This shows up as a wedge,  $\psi_1^e > 0$ , between the marginal value of income inside versus outside of a particular bank. In equilibrium this wedge manifests itself as “fire sale pricing” in the inter-bank asset markets in the sense that  $q_1^b < \delta_2^b$  and  $q_1^k < z_2$ , i.e., assets are traded below their “fundamental value” at  $t = 1$ .<sup>5</sup> Finally, we can consider the case with both equity raising frictions and regulation. If regulation is symmetric in its treatment of bonds and capital, then  $\kappa = 0$  and the relative price ratio is simply the ratio of  $t = 2$  payoffs (2.16). If regulation advantages government bonds, then  $\kappa > 0$  and relative price of government debt is higher and satisfies:

$$\frac{q_1^b(s)}{q_1^k(s)} = \frac{\delta_2^b(s)}{z_2^k(s) - \kappa \mu_1^r(s) q_1^k(s)}$$

In the bad state,  $s_L$ , there are fewer resources and so  $\mu_1^r(s_L)$  increases, which in turn increases  $q_1^b(s)/q_1^k(s)$ . In this sense, regulation makes banks more “captive buyers” for government debt in bad times. Both cases are depicted graphically in Figure 3.

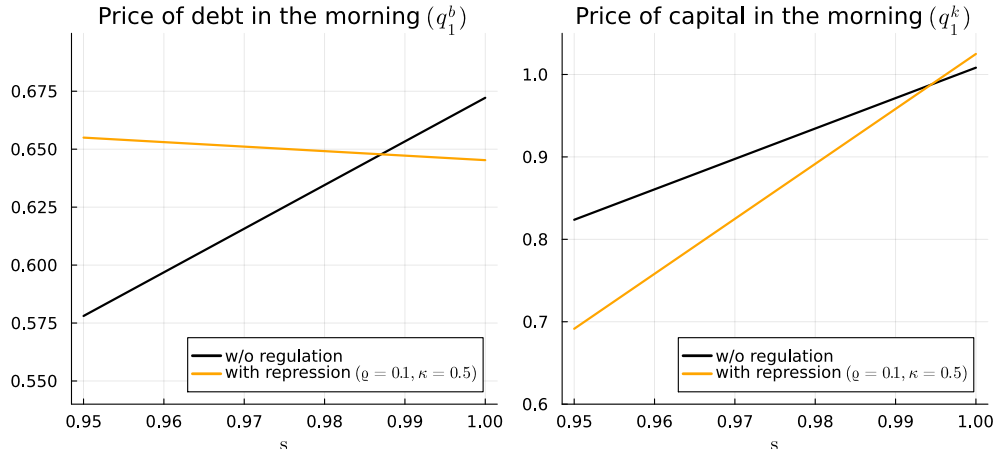


Figure 3

### 2.2.3 Equilibrium at time $t = 0$

We finish the characterization of equilibrium by studying agent decisions and market clearing at  $t = 0$  in Proposition 3.

**Proposition 3** (Equilibrium at  $t = 0$ ). *The household demand for banks deposit and the bank*

<sup>5</sup>The finance literature refers to this as “fire sale” Shleifer and Vishny (1992, 2011) or cash-in-the-market pricing Allen and Gale (1994, 1998). The monetary literature, starting with Lucas (1990), refers to this as “liquidity effect” which was taken up by the limited participation literature (Christiano and Eichenbaum, 1992, 1995).

supply of deposit are determined by the Euler equations:

$$\begin{aligned} q_0^d &= \mathbb{E} \left[ \xi(\lambda, \mathbf{s}) \left( 1 + \nu(\lambda, \mathbf{s}) \right) \delta^d(\lambda, \mathbf{s}) \right] \\ q_0^d(1 - \varrho \mu_0^r) &= \mathbb{E} \left[ \xi(\lambda, \mathbf{s}) \Omega(\lambda, \mathbf{s}) \Gamma(\lambda, \mathbf{s}) \delta^d(\lambda, \mathbf{s}) \right] \end{aligned}$$

where  $\mu_0^r \geq 0$  is the Lagrange multiplier on the  $t = 0$  regulatory constraint (2.3),  $\nu$  is the liquidity premium defined in Proposition 1 and  $\Omega$  and  $\Gamma$  satisfy:

$$\begin{aligned} \Omega(\lambda, \mathbf{s}) &:= \left( 1 + \psi_1^e \right) \times \begin{cases} \frac{(1+\nu(\lambda))}{(1+\psi_1^e)\lambda + (1+\varrho\mu_1^r)(1-\lambda)} + \varsigma \frac{\xi(\lambda^*)(1+\nu(\lambda^*))}{\xi(\lambda)\mathcal{R}\ell} \frac{f(\lambda^*)}{(1-F(\lambda^*))} & \lambda > \lambda^* \\ 1 & \lambda \leq \lambda^* \end{cases} \\ \Gamma(\lambda, \mathbf{s}) &:= \begin{cases} \ell(\mathbf{s})/\delta^d(\lambda, \mathbf{s}) & \lambda > \lambda^* \\ \frac{(1+\psi_1^e(\mathbf{s}))\lambda + (1+\varrho\mu_1^r(\mathbf{s}))(1-\lambda)}{(1+\psi_1^e(\mathbf{s}))(1-\varrho\mu_0^r)} & \lambda \leq \lambda^* \end{cases} \end{aligned}$$

The bank asset choices satisfy:

$$\begin{aligned} 1 &= \mathbb{E} \left[ \xi(\lambda, \mathbf{s}) \Omega(\lambda, \mathbf{s}) z_1(\mathbf{s}) \right] \\ 1 - (1 - \kappa) \mu_0^r &= \mathbb{E} \left[ \xi(\lambda, \mathbf{s}) \Omega(\lambda, \mathbf{s}) q_1^k(\mathbf{s}) \right] = \mathbb{E} \left[ \xi(\lambda, \mathbf{s}) \Omega(\lambda, \mathbf{s}) \left( \frac{1}{1 + \psi_1^e(\mathbf{s}) - (1 - \kappa) \mu_1^r(\mathbf{s})} \right) z_2(\mathbf{s}) \right] \\ q_0^b(1 - \mu_0^r) &= \mathbb{E} \left[ \xi(\lambda, \mathbf{s}) \underbrace{\Omega(\lambda, \mathbf{s}; \psi_1^e, \mu_1^r)}_{\text{default}} q_1^b(\mathbf{s}) \right] = \mathbb{E} \left[ \xi(\lambda, \mathbf{s}) \underbrace{\Omega(\lambda, \mathbf{s}; \psi_1^e, \mu_1^r)}_{\text{default}} \underbrace{\left( \frac{1}{1 + \psi_1^e(\mathbf{s}) - \mu_1^r(\mathbf{s})} \right)}_{\text{regulation}} \delta_2^b(\mathbf{s}) \right] \end{aligned}$$

From Proposition 3, we can see the two key features of the bank problem. First, the costly default wedge  $\Omega$  effectively makes the banking sector act as more “risk-averse” than the household sector even though they use the household’s SDF. Second, the optimal bank leverage choice at  $t = 0$  trades off earning the liquidity premium on deposits, as measured by  $\nu$ , against the cost of having a higher default probability, as captured by  $\Omega$ . In this sense, the combination of deposit liquidity services and costly default break Modigliani-Miller style results. We plot  $\Omega$  in Figure 4, where the left hand plot has symmetric treatment of assets and the right hand plot has financial repression.

**Corollary 1.** *If the government fiscal rule fully repays the debt,  $\delta_2^b(\mathbf{s}) = 1$ , then the regulatory Lagrange multiplier binds at  $t = 1$  ( $\mu_1^r > 0$ ) but not at  $t = 0$  ( $\mu_0^r = 0$ ).*

The multiplier  $\mu_0^r$  reflects the impact of forcing the banks to buy government debt in the primary market. This can be thought of as the “direct” impact of financial repression. The multiplier  $\mu_1^r$  reflects the impact of creating a captive secondary market for government debt in the interbank market. Ultimately, this changes the price process for government debt and makes government debt a “safe-asset” that banks want to hold at  $t = 0$ , which means that the constraint in the primary market no longer binds. Corollary 1 shows that the safe asset benefit is sufficiently

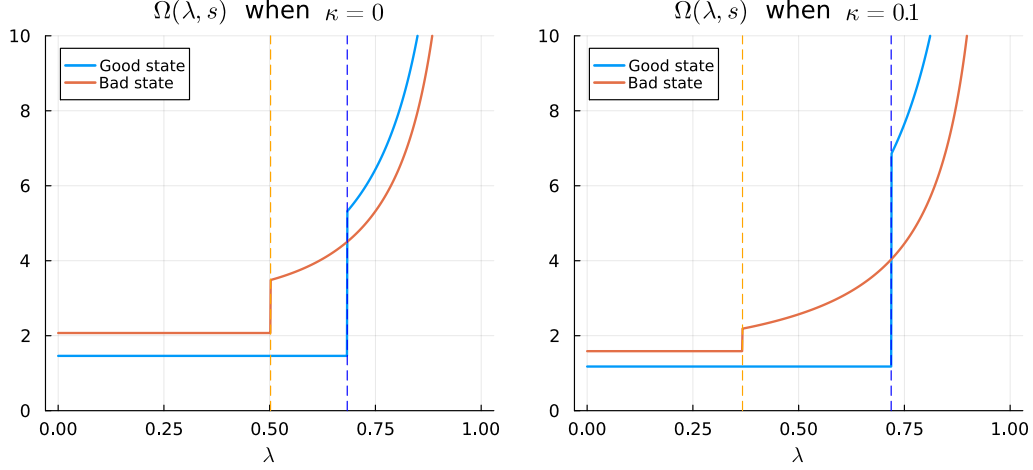


Figure 4

strong that the banks want to purchase more government debt in primary market than is required by regulation. That is, the bank have additional precautionary motive for holding government debt that further increases the convenience yield.

A key feature of our model is that the default cut-off  $\lambda^*$  and default wedge  $\Omega$  depends upon the policy parameters of the government:  $(\varrho, \kappa, \delta^b)$ . This means that regulation and fiscal irresponsibility not only directly change demand but also change the precautionary role for holding government debt. Taken together, our expression for  $\Omega(\lambda, s; \psi_1^e, \mu_1^r, \delta^b)/(1 + \psi_1^e - \mu^r)$  characterizes how government policy can create endogenous demand for government debt by distorting the SDF of the banks.

#### 2.2.4 Convenience Yields

We close this section by characterizing the convenience yield. The convenience is often defined to be the percentage deviation between the asset price and the expectation of stochastic discount factor:

$$\chi := \log(q_0^b) - \log(\mathbb{E}[\xi(\lambda, s)]).$$

However, for risky assets, this definition includes both the role of the asset and risk premium on that asset. In this sense, it helpful to break up the convenience yield into:

$$\chi = \log(q_0^b) - \log(\mathbb{E}[\xi(\lambda, s)\delta^b(s)]) + \log(\mathbb{E}[\xi(\lambda, s)\delta^b(s)]) - \log(\mathbb{E}[\xi(\lambda, s)])$$

where we refer to the first term as the “risk-adjusted” convenience yield:

$$\tilde{\chi} := \log(q_0^b) - \log(\mathbb{E}[\xi(\lambda, s)\delta^b(s)])$$

which is the difference between the price of government debt and an asset with the same cash flows as government debt that is not government debt. We interpret the “risk-adjusted” convenience yield as the value that households place on the special role that government debt plays. If we expand out covariances, then we get the following approximate expression for the risk adjusted convenience yield:

$$\tilde{\chi} \approx \mu_0^r + \log \left( \mathbb{E} \left[ \Omega(\lambda, \mathbf{s}) \left( \frac{1}{1 + \psi_1^e - \mu_1^r} \right) \right] \right) + \text{Cov} \left( \frac{\xi(\lambda) \delta_2^b}{\mathbb{E}[\xi(\lambda) \delta_2^b]}, \frac{\Omega(\lambda, \mathbf{s}) \left( \frac{1}{\psi_1^e - \mu_1^r} \right)}{\mathbb{E} \left[ \Omega(\lambda, \mathbf{s}) \left( \frac{1}{\psi_1^e - \mu_1^r} \right) \right]} \right)$$

From this expression we can see that the convenience yield is partly the direct impact coming from forced purchases of government debt in the primary market and the indirect impact coming from the that repression helps the government to hedge risk.

### 2.3 Costs of Generating Convenience Yields

In this subsection, we use our microfounded model to understand the costs of generating a convenience yield. Figure 5 plots the equilibrium outcomes at  $t = 0$  for different values of financial repression. Evidently, an increase in financial repression leads to a higher convenience yield and more fiscal capacity, as measured by the amount of government spending. However, it also leads to more bank default in the bad state of the world and lower bank investment into capital. The higher rate of bank default appears because financial repression inflates the debt price in the interbank market and so also decreases the portfolio return for solvent banks, which makes the marginal bank more likely to default. The lower investment rate appears because government borrowing crowds out bank capital creation, as is standard in many macroeconomic models. Together these effects lead to lower household consumption. In this sense, the government faces a trade-off between optimizing their fiscal capacity and having a well functioning financial sector. For our numerical example, we find that some degree of repression is optimal. This result is different to some recent papers (e.g. [Chari et al. \(2020\)](#)) because we have placed restrictions on the tax process and because the banks in our model play roles as both liquidity providers and intermediaries.

### 2.4 Comparison to Exogenous Bond Demand Functions

In this section, we consider two alternative model of bond demand that are frequently used in the macroeconomics literature: bond-in-the-utility and bond-in-advance.



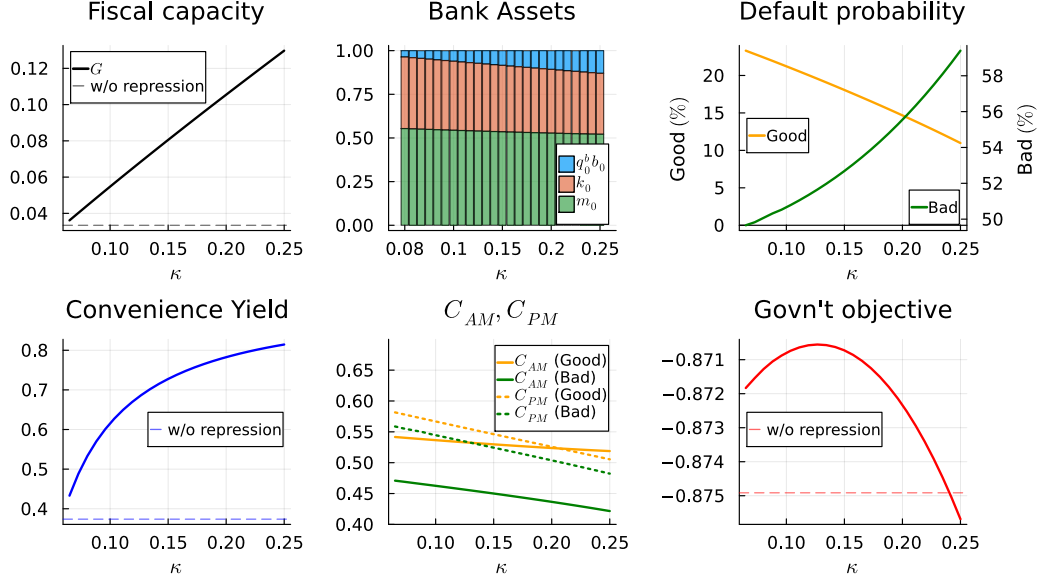


Figure 5: Equilibrium at  $t = 0$  for Different Levels of Repression.

*Bond-in-the-utility:* In this model, the household solves:

$$\begin{aligned} \max_{b_0, k_0, c_1} \quad & \{ \nu(q_0^b b_0) + \beta \mathbb{E}[u(c_2)] \} \quad s.t. \\ & q_0^b b_0 + k_0 \leq 1 \\ & c_2 \leq z_2(s)k_0 + \delta_2^b(s)b_0 - \tau(s) \end{aligned}$$

where  $\nu(q_0^b b_0)$  denotes the utility benefit from holding the real value of government debt.

*Bond-in-advance:* In this model, the household solves:

$$\begin{aligned} \max_{b_0^h, m_0^h, k_0^h, b_1^h, c} \quad & \mathbb{E}[\lambda u(c_1^h) + (1 - \lambda)u(c_2^h)] \quad s.t. \\ & q_0^b b_0^h + m_0^h + k_0^h \leq 1 \\ & c_1^h \leq q_1^b b_0^h \\ & c_{PM}^{h,i} \leq \frac{\delta_2^b}{q_1^b} (q_1^b b_0^h + z m_0 - c_1^h) + z_2 k_0^h - \tau(s) \end{aligned}$$

where the constraint  $c_1^h \leq q_1^b b_0^h$  says that the household need to use bonds to purchase consumption goods at  $t = 1$ .

The corresponding Euler equations for government debt the bond-in-the-utility (BIU) and

bond-in-advance (BIA) are given by:

$$q_0^b = \left(1 - \frac{\nu'(q_0^b b_0)}{\mu_0}\right)^{-1} \mathbb{E}[\xi \delta_2^b(s)] \quad \dots (\text{BIU})$$

$$q_0^b = \mathbb{E} \left[ \xi \left(1 - \frac{\mu_1^b(s)}{\lambda u'(c_1(s))}\right)^{-1} \delta_2^b(s) \right] \quad \dots (\text{BIA})$$

By contrast, the repression model has the Euler equations:

$$q_0^b = (1 - \mu_0^r)^{-1} \mathbb{E} \left[ \xi(\lambda, s) \Omega(\lambda, s; \psi_1^e, \mu_1^r) \left(1 + \psi_1^e(s) - \mu_1^r(s)\right)^{-1} \delta_2^b(s) \right]$$

Ultimately, we can see that the repression model “nests” the bond-in-the-utility and bond-in-advance models in the sense that we get terms that look similar to the bond-in-the-utility and bond-in-advance wedges. To illustrate this, in Figure 6 we plot the government bond demand functions from each model and show that they can have similar slopes. In this sense, the regulatory parameters map to the shape parameters in the reduced form bond demand functions. However, there are two important differences in our formulation: (i) we have an additional wedge coming from government default, and (ii) all our wedges are explicit functions of government policy. We use our microfoundation in the next section to show that convenience yield in our repression model reacts very differently to government default.

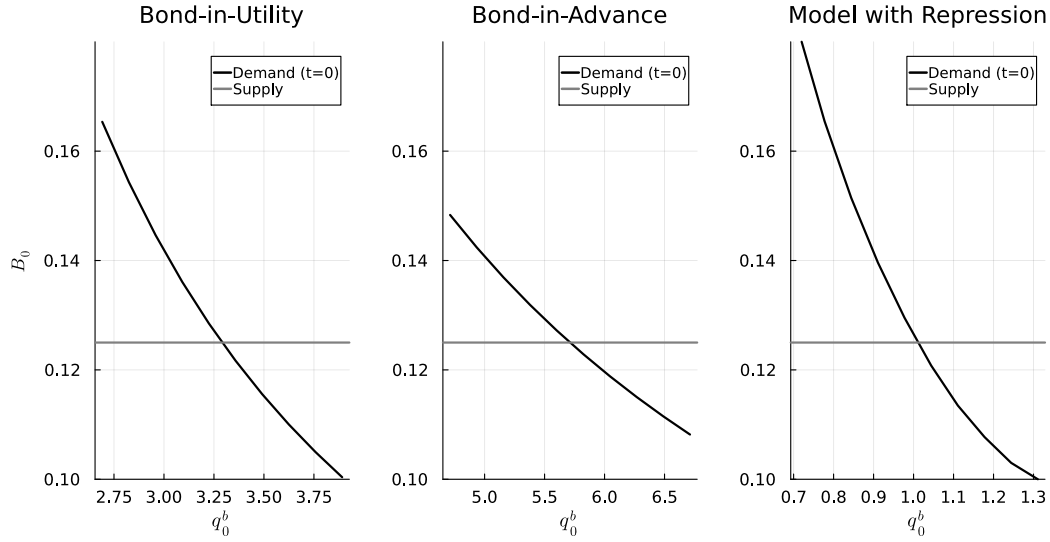


Figure 6

## 2.5 Government Default

We now use our model to explore how government fiscal irresponsibility impacts the convenience yield. Figure 7 shows the convenience yield as the government defaults more in the bad state. Evidently, in the repression model, an increase in government default leads to a decrease in the convenience yield. There are a number of reasons for this. First, government default in bad states of the world restricts the banking sector's ability to use government debt to hedge aggregate risk, which makes it harder for repression to ensure government debt plays the "safe-asset" role in the economy. Second, repression ties the solvency of the banking sector to the solvency of the government. So, increasing government default makes government debt a worse hedge at the same time that it makes banks more concerned about finding a good hedge. Ultimately, this leads to a decrease in the convenience yield.

This is very different to the models with bond-in-the-utility or bond-in-advance, where an increase in government default leads to a higher convenience yield. Why? In these models, the role of government debt is exogenous and its marginal usefulness increases as the market value of government debt declines. This means that as the government starts to default, the convenience yield increases. Or put another way, in these models the agents get utility from giving resources to the government so when the government starts to default, then they want to give more resources by buying government debt. This highlights the importance of making government debt endogenously important to the economy.

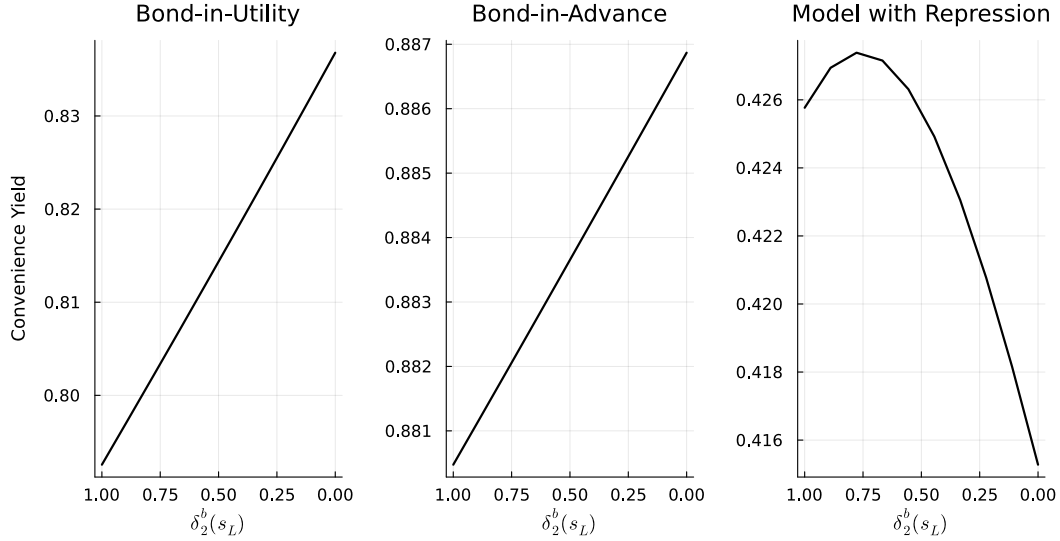


Figure 7

Figure has the corresponding plot for the fiscal capacity of the government. In this case government defaults leads to fall in government fiscal capacity whereas in the bond-in-utility model the decrease is much more muted and in bond-in-advance model government default actually leads to higher capacity, which is a counter intuitive result.

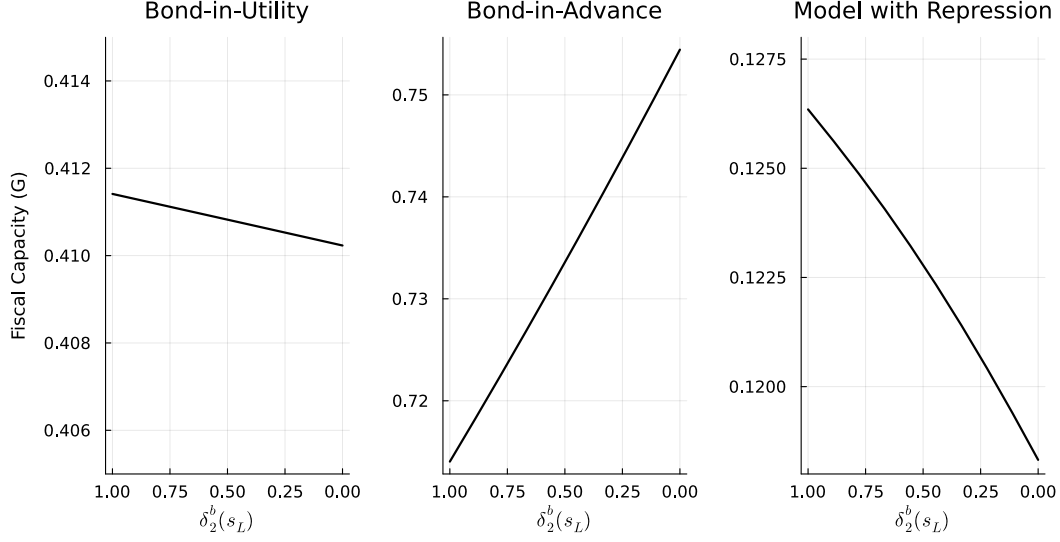


Figure 8

### 3 Empirical Connection

In Section 4.4 we saw that our model made sharp predictions for what how the design of the financial sector impacted the convenience yield. We close the paper by studying whether we see evidence for these relationships in the historical data.

#### 3.1 Data and Methodology

In a previous paper, [Payne et al. \(2022\)](#), we assembled prices and cash flows for the universe of government bonds and estimated the zero-coupon yield curve on US federal debt. For this paper, we assemble a companion data-set with a large collection of corporate bonds between 1860 and 1940. We describe the original sources and the details of the data collection in Appendix D. We use the classification system from [Macaulay et al. \(1938\)](#) to identify a collection of low risk corporate bonds (primarily railroad bonds) for the period before 1900 when there is no Moody’s rating system. We estimate the historical yield curve on low risk corporate debt using the empirical approach developed in [Payne et al. \(2022\)](#). We then combine our estimates for historical US Treasury yields and our estimates for historical corporate bonds, with existing modern series.

#### 3.2 Stylized Facts

We use our estimates of historical yields to construct a collection of stylized facts about the historical pricing of government debt.

*Fact 1: Low frequency movements in average convenience yields:* Figure 9 shows the time series

for the 10-year corporate yield, the 10-year treasury yield, and the “convenience yield”, as measured by the corporate yield minus the treasury yield. We can see that throughout the National Banking Era (1860-1917), the convenience yield was typically relatively high, around 1.5%. The convenience yield then drops down significantly to close to zero around WWII before spiking again during the 1970s.

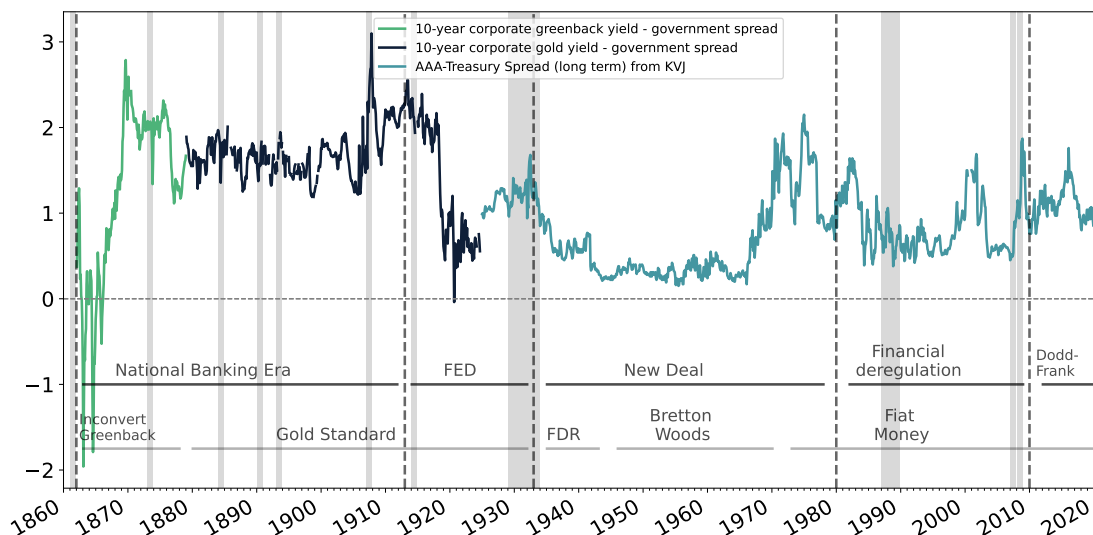


Figure 9: Government Yields, (high-grade) Corporate Yields, and the Convenience Yield: 1860-2020

*Fact 2: Low frequency movements in the elasticity of the convenience yield with respect to government debt supply.* Figure 10 shows a scatter plot with the ratio of the market value of government debt/GDP on the x-axis and the convenience yield on the y-axis. We can see that within the National Banking Era (1868-1914) and around WWII (1940-1965), the elasticity of the convenience yield is very low, even though the level of the convenience yield is very different. These are both periods, where the government intervened very directly in financial markets to try and create a market for government debt. The Period with the much studied downward sloping “demand curve is really the period in the interwar period and the period of financial deregulation. These are both periods, where the government relaxes demand for government. In the interwar period, Fed takes over money creation from the national banks and so banking sector does not need to hold as much government debt. In the last third of the 20th century, the government deregulates the financial sector. Ultimately, we interpret this plot as suggestive evidence that the elasticity has very different properties under different financial regulation regimes.

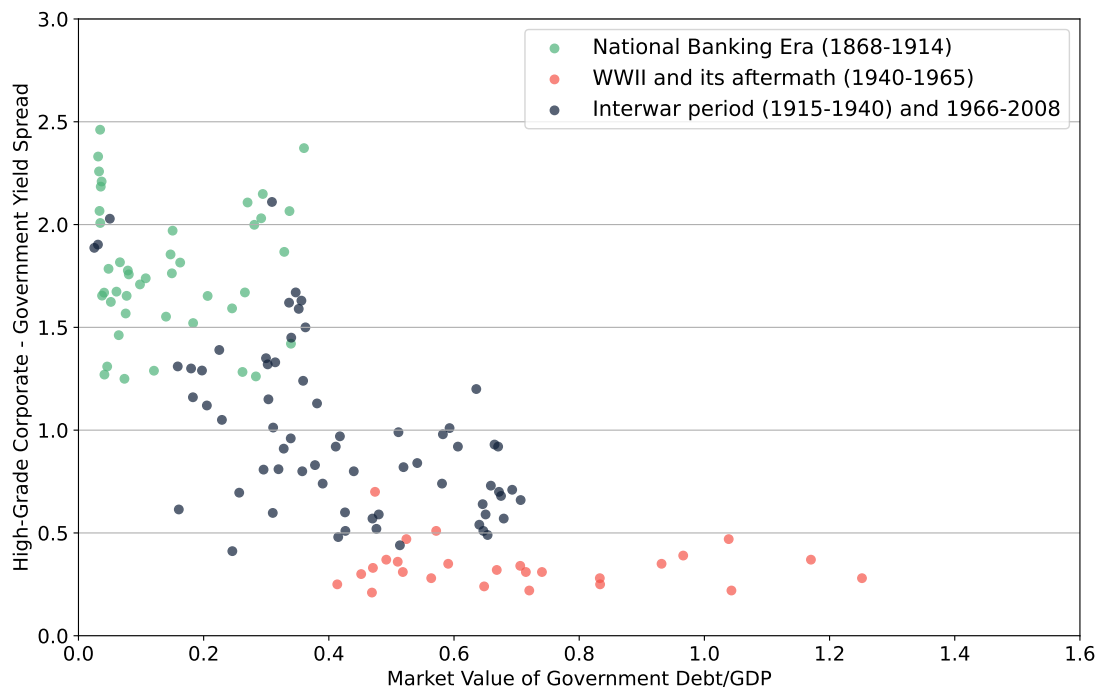


Figure 10: Convenience Yield vs Debt/GDP: 1868-2008 [JP: NEEDS TO BE UDPATED.]

*Fact 3: Short rate “disconnect” for most of the sample.* In Figure 11, we use our statistical model to examine whether a so called “short-rate disconnect” existed during the 19th century. The pale blue dots depict the difference between model-implied and observed yield-to-maturities for bonds with *less than one year* to maturity. Because we estimate our yield curve models using bonds with maturity greater than 1 year, these dots represent an “out-of-sample” fit at the short end of the yield curve. The solid blue line depicts the 15-year centered moving average of these blue dots. The orange solid line depicts the 15-year centered moving average of the difference between model-implied and observed yield-to-maturities for bonds with *more than one year* to maturity. Evidently, pricing errors average out for bonds with long maturities but are systematically positive for extended periods for bonds close to maturity. In particular, until the 1880s, bonds close to maturity traded with a premium in a range of 0.5 to 1.0 percentage points. The premium effectively disappeared from the 1880s until the First World War before reappearing in the 1920s. We interpret this as strong evidence that there has been a short rate disconnect through most US history, with a period towards the end of the 19th century when the short rate disconnect disappeared.

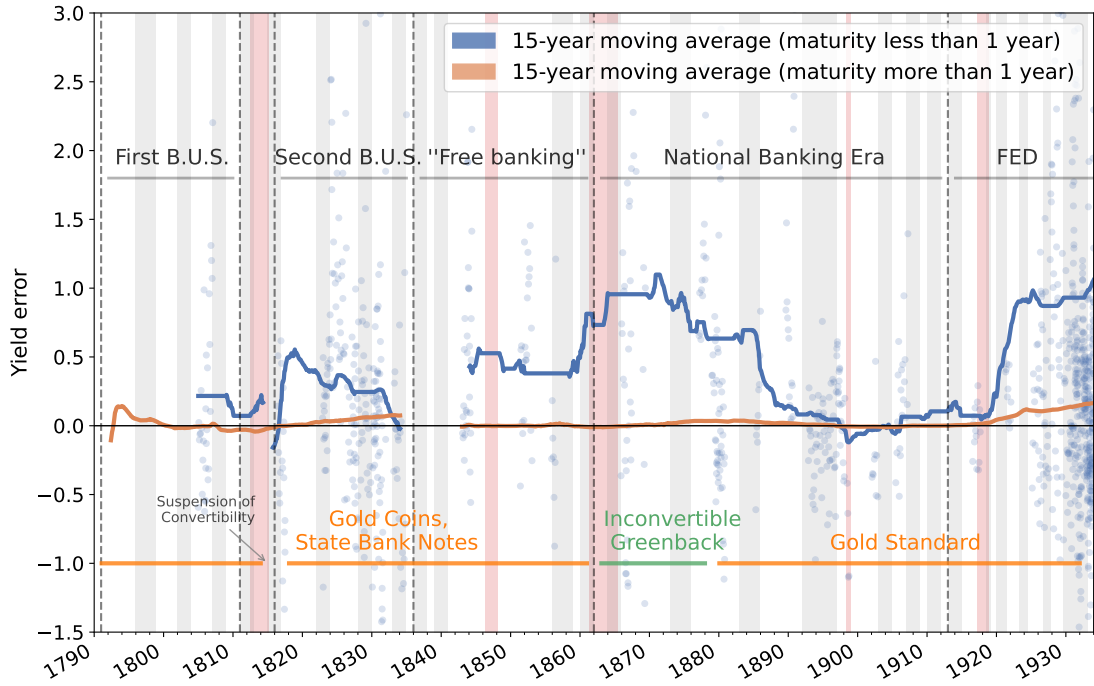


Figure 11: Short Rate Disconnect.

Pale blue dots depict the difference between model-implied and observed yield-to-maturities for bonds with *less than one year* to maturity. The solid blue line depicts the 15-year centered moving average of these dots excluding yield errors with magnitude greater than 4 (to handle potential outliers from data issues). The orange solid line depicts the 15-year centered moving average of the difference between model-implied and observed yield-to-maturities for bonds with *more than one year* to maturity. The light gray intervals depict recessions, and the light red intervals depict wars.

## 4 Infinite Horizon Macroeconomic Model

The previous sections illustrated how the regulation of bank balance sheets can generate a convenience yield. In this section, we move to an infinite horizon general equilibrium model. We show that the impact of financial repression depends on “stickiness” of deposit demand. We use the model to explore how the government can use the financial sector to smooth borrowing costs across bad shocks.

### 4.1 Environment

*Setting:* Time is discrete in infinite horizon. There is one consumption good. The economy is populated by a representative household that directly or indirectly owns all claims to production. The economy also contains a representative firm, a representative financial intermediary, and a government, all of which issue securities. The firm issues equity claims and creates capital

to produce consumption goods. The intermediary issues deposits and equity. The government issues geometrically decaying long-term bonds that pay repay a fraction  $\zeta$  of the principal each period. The high level relationship is given in figure 12.

Lucas Tree		Government		Banks		Household	
A	L	A	L	A	L	A	L
Goods	Shares	Taxes	Bonds	Shares	Deposits	Deposits	Net worth
				Bonds	Equity	Equity	
						Shares	
						Bonds	

Figure 12: Agent balance sheets

*Representative household:* ranks allocations according to:

$$\mathbb{E}_0 \left[ \sum_{t=0}^{\infty} \beta^t u(c_t) + \nu(d_t^h + \zeta b_t^h) - \Psi_{t+1}(a_{t+1}^f) e_t^h - \Omega_{t+1}(\tau_{t+1}) \right]$$

where  $c_t$  is household consumption at time  $t$ ,  $d_t^h$  is the household holdings of financial intermediary deposits,  $b_t^h$  is household holdings of government debt,  $a_{t+1}^f$  is the net-worth of the financial intermediary,  $e_t^h$  is household equity holdings, and  $\tau_{t+1}$  is the tax rate. The function  $\nu(\cdot)$  is increasing and captures the non-pecuniary benefit of holding “safe-assets”. The function  $\Psi(\cdot)$  is decreasing and captures the cost of bank “insolvency”. The function  $\Omega(\cdot)$  is increasing and captures the distortion from raising taxes. The household also faces the short selling constraints  $d_t^h \geq 0$ ,  $s_t^h \geq 0$ , and  $b_t^h \geq 0$ , where  $s_t^h$  is household holdings of shares in the firm. At time 0, the household is endowed with capital,  $k_0$ , and sells it to the representative firm. Each period, the household is endowed with a unit of labor,  $l_t = 1$ .

*Representative firm:* has a Cobb-Douglas production technology subject to stochastic productivity  $z_t$ :

$$y = z_t k_{t-1}^\alpha l_t^{1-\alpha}$$

$$\log(z_t) = (1 - \eta) \log(\bar{z}) + \eta \log(z_{t-1}) + \epsilon_t$$

where  $l_t$  is labor hired by the firm and  $k_{t-1}$  is firm capital stock. The evolution of capital stock



is given by the constant-return-to-scale technology:

$$k_t = (1 - \delta)k_{t-1} + \Phi(\iota_{t-1})k_{t-1}$$

where  $\iota_{t-1} := \frac{i_{t-1}}{k_{t-1}}$  is the investment-capital ratio and  $\Phi(\cdot)$  is an “adjustment” function.

*Representative financial intermediary:* On the liability side of their balance sheet, the intermediary issues “safe-assets”,  $d_t^f$ , that each pay 1 good at  $t + 1$  and equity,  $e_t^f$ , that pays a dividend  $\delta_{t+1}^e$  at  $t + 1$ . On the asset side, they purchase shares in the firm,  $s_t^f$ , and government debt,  $b_t^f$ . The intermediary faces a regulatory collateral constraint that at any point in time, a proportion  $\kappa^b$  of the maturing safe asset must be backed by the market value of government debt:

$$(\zeta + (1 - \zeta)q_t^b)b_t^f \geq \kappa^b d_t^f$$

where  $q_t^b$  is the price of government debt.

*Government:* Each period, the government raises lump sum taxes,  $\tau_t$ , issue bonds,  $b_t$ , and undertakes spending  $g_t = g(z_t)y_t$  that is a function of the aggregate state, where  $g(\cdot)$  is a decreasing function. Bonds are issued at par and repay a fraction  $\zeta$  of the principal each period. They face the inter-temporal budget constraint that:

$$g_t + \zeta b_{t-1} \leq \tau_t + q_t^b(b_t - (1 - \zeta)b_{t-1}).$$

Following [Bohn \(1998\)](#) and [Bai and Leeper \(2017\)](#), we impose that the government sets a budget feasible tax policy to target a long run debt to GDP ratio:

$$\hat{\tau}_t - \hat{\tau}^* = \gamma (\hat{b}_{t-1} - \hat{b}^*)$$

where  $\hat{\tau}_t := \tau_t/y_t$  and  $\hat{b}_{t-1} := b_{t-1}/y_{t-1}$ . The government also chooses regulatory portfolio restriction  $\kappa^b \geq 0$ .

*Markets:* All markets are competitive. Let  $q_t^s$  denote the firm equity price. Let  $q_t^b$  denote the government bond price. Let  $(q_t^e, q_t^d)$  denote the time- $t$  price of equity and safe assets issued by the financial intermediary. We use upper case  $R$  for the gross return and  $r$  for the yield. Let  $w_t$  denote the wage rate. We are focusing on the case when  $\zeta$  is a parameter and to simplify notation we define  $\tilde{q}_t^b := \zeta + (1 - \zeta)q_t^b$ .

*Functional Forms:* We impose

$$u(c) = \log c, \quad \nu = \log(\exp(-r_t^d)d_{t+1}^h + \zeta b_{t+1}^h)$$

capital adjustment cost

$$\Phi(\iota) = \phi_0 + \frac{\bar{\phi}}{1 - \phi} \iota^{1 - \phi}$$

**Discussion of environment frictions:** This environment is characterized by two key distortions. The first distortion is that the households get additional utility from holding safe assets through the  $\nu$  function. The second is distortion is that the the financial intermediaries, who have the technology to create safe assets, face the cost function,  $\Psi_{t+1}$ , when they become insolvent. This effectively makes the safe asset issuers less willing to take on risk than the households. Although we are not modelling the microfoundations for these distortions, we believe the model captures the key friction in macro-finance models.

**Discussion of government policy rule:** We interpret our government tax and spending policies as arising from unmodelled political frictions that induce the government to run deficits during recessions and then surpluses in expansions to return to a target long-run debt-to-GDP ratio. This policy potentially imposes welfare costs if running surpluses induces the government to move the tax rate around. We are going to study how financial regulation and changes in the convenience yield on government debt influence the welfare cost of running such a fiscal policy.

**Discussion of regulatory constraints:** In addition to the environmental frictions, the environment also contains regulatory constraints that restrict the portfolio choices of agents and so change asset demand elasticities. The key constraint is the collateral requirement that the market value of government debt cannot fall below  $\kappa^b d_t$ . Effectively, this constraint means that the government only allows the financial sector to use their financial technology to issue safe-assets to households if they hold government bonds. In this sense, the government is repressing the financial sector to create demand for their debt and so drive up the price of their debt when the collateral constraint binds. The other regulatory constraint is that the household may not hold government bonds and firm equity. This segments the market for government debt so that the only agents trading government debt are the financial intermediaries facing the collateral constraint requiring them to hold government bonds. Ultimately, these regulatory constraints will allow the government to indirectly tax the value that the financial intermediaries generate through safe asset creation.

## 4.2 Competitive Equilibrium

In this subsection, we set up the agent problems and characterize the competitive equilibrium.

### 4.2.1 Household Problem

We set up the household problem recursively. The (individual) state variable for the household is  $a_t^h$ , which denotes the wealth of the household at the start of period  $t$ . The household solves

problem (4.1) below:

$$\begin{aligned}
V_t(d_t^h) &= \max_{c_t, d_t^h, b_t^h, e_t^h, s_t^h} \left\{ u(c_t) + \nu(d_t^h + \zeta b_t^h) - \mathbb{E}_t [\Psi_{t+1} e_t^h + \Omega_{t+1} + \beta V_{t+1}(a_{t+1}^h)] \right\} \\
\text{s.t. } c_t + q_t^e e_t^h + q_t^s s_t^h + q_t^b b_t^h + q_t^d d_t^h &\leq a_t^h \\
a_{t+1}^h &= (\delta_{t+1}^e + q_{t+1}^e) e_t^h + ((1 - \tau_{t+1}) \delta_{t+1}^s + q_{t+1}^s) s_t^h + \tilde{q}_{t+1}^b b_t^h + d_t^h \\
0 \leq d_t^h, \quad 0 \leq b_t^h, \quad 0 \leq s_t^h
\end{aligned} \tag{4.1}$$

Taking first order conditions and imposing the envelope condition gives the “asset-demand” equations:

$$\begin{aligned}
[d_t^h] : \quad q_t^d &= \mathbb{E}[\xi_{t,t+1}] + \frac{\nu'(d_t^h + \zeta b_t^h)}{u'(c_t)} + \frac{\lambda_t^d}{u'(c_t)} \\
[b_t^h] : \quad q_t^b &= \mathbb{E}[\xi_{t,t+1} \tilde{q}_{t+1}^b] + \zeta \frac{\nu'(d_t^h + \zeta b_t^h)}{u'(c_t)} + \frac{\lambda_t^b}{u'(c_t)} \\
[e_t^h] : \quad q_t^e &= \mathbb{E}[\xi_{t,t+1} (\delta_{t+1}^e + q_{t+1}^e)] - \frac{\mathbb{E}_t[\Psi_{t+1}]}{u'(c_t)} \\
[s_t^h] : \quad q_t^s &= \mathbb{E}[\xi_{t,t+1} ((1 - \tau_{t+1}) \delta_{t+1}^s + q_{t+1}^s)] + \frac{\lambda_t^s}{u'(c_t)}
\end{aligned}$$

where  $\xi_{t,t+1} := \beta u'(c_{t+1})/u'(c_t)$  is the household stochastic-discount-factor (SDF) and where  $\lambda_t^d \geq 0$ ,  $\lambda_t^b \geq 0$ , and  $\lambda_t^s \geq 0$  are the multipliers on the household portfolio constraints on  $d_t^h$ ,  $b_t^h$ , and  $s_t^h$ . Observe the Euler equations for  $d_t^h$  and  $b_t^h$  have been “distorted” by the household demand for safe assets,  $\nu$ . Observe that the Euler equation for bank equity can be rewritten as:

$$q_t^e = \mathbb{E}_t \left[ \xi_{t,t+1} \left( \delta_{t+1}^e + q_{t+1}^e - \frac{\Psi_{t+1}}{u'(c_{t+1})} \right) \right]$$

so we can see that the insolvency costs distort the price of price of bank equity.

#### 4.2.2 Financial Intermediary Problem

The financial intermediary chooses a collection of asset portfolio and dividend payouts to maximise its market value by solving problem:

$$\begin{aligned}
V_0 &= \max_{\delta^e, s^f, b^f, d^f} \{ q_0^e + q_0^h h_1 - q_0^s s_1 - q_0^b b_1 \} \quad \text{s.t.} \\
\delta_t^e + q_t^s s_t^f + q_t^b b_t^f - q_t^d d_t^f &= a_t^f \\
a_{t+1}^f &= ((1 - \tau_{t+1}) \delta_{t+1}^s + q_{t+1}^s) s_t^f + \tilde{q}_{t+1}^b b_t^f - d_t^f \\
q_t^e &= \mathbb{E}_t \left[ \xi_{t,t+1} \left( \delta_{t+1}^e + q_{t+1}^e - \frac{\Psi_{t+1}(a_{t+1}^f)}{u'(c_{t+1})} \right) \right] \\
\tilde{q}_{t+1}^b b_t^f &\geq \kappa_t^b d_t^f \\
s_t^f &\geq 0
\end{aligned}$$

The first order conditions gives the following financial intermediary asset demand and supply equations :

$$\begin{aligned} [s_{t+1}^f] \quad 0 &= -q_t^s \left( 1 - \frac{\partial_{e\delta} \Psi_t}{u'(c_t)} \right) + \mathbb{E}_t \left[ \xi_{t,t+1} \left( 1 - \frac{\partial_{e\delta} \Psi_{t+1}}{u'(c_{t+1})} - \frac{\partial_{ea} \Psi_{t+1}}{u'(c_{t+1})} \right) (\delta_{t+1}^s + q_{t+1}^s) \right] + \mu_t^s \\ [b_{t+1}^f] \quad 0 &= -q_t^b \left( 1 - \frac{\partial_{e\delta} \Psi_t}{u'(c_t)} \right) + \mathbb{E}_t \left[ \xi_{t,t+1} \left( 1 - \frac{\partial_{e\delta} \Psi_{t+1}}{u'(c_{t+1})} - \frac{\partial_{ea} \Psi_{t+1}}{u'(c_{t+1})} + \mu_{t+1}^b \right) \hat{q}_{t+1}^b \right] \\ [\hat{d}_{t+1}^f] \quad 0 &= \left( 1 - \frac{\partial_{e\delta} \Psi_t}{u'(c_t)} \right) - \mathbb{E}_t \left[ \xi_{t,t+1} \left( 1 - \frac{\partial_{e\delta} \Psi_{t+1}}{u'(c_{t+1})} - \frac{\partial_{ea} \Psi_{t+1}}{u'(c_{t+1})} + \kappa_t^b \mu_{t+1}^b \right) \exp(r_t^h) \right] \end{aligned}$$

In equilibrium, market clearing and the regulatory constraint on household portfolio  $s_t^h \leq \kappa_t^s$  with  $\kappa_t^s \geq 0$  implies that the short-selling constraint for the financial intermediary never binds ( $\mu_t^s = 0$ ).

### 4.2.3 Firm Problem

Taking prices and the shareholder's SDF as given, firms solve:

$$V_t(k_{t-1}) = \max_{\iota_t, l_t} \left\{ z_t k_{t-1}^\alpha l_t^{1-\alpha} - w_t l_t - \iota_t k_{t-1} + \mathbb{E}_t \left[ \hat{\xi}_{t,t+1} V_{t+1}(k_t) \right] \right\}$$

where  $\hat{\xi}_{t,t+1}$  is the weighted average of the household and firm stochastic discount factors and the firm is subject to the capital accumulation technology:

$$k_t = (1 - \delta + \Phi(\iota_t)) k_{t-1}$$

where  $\iota_t := \frac{i_t}{k_{t-1}}$  is the investment-capital ratio. The first order conditions are:

$$\begin{aligned} [w_t] : \quad & 0 = (1 - \alpha) z_t k_{t-1}^\alpha l_t^{-\alpha} - w_t \\ [\iota_t] : \quad & 0 = -k_{t-1} + \mathbb{E}_t [\hat{\xi}_{t+1} \partial_k V_{t+1}(k_t) \Phi'(\iota_t) k_{t-1}] \end{aligned}$$

Guess the form  $V_t = v_t k_{t-1}$ , then the first order condition for  $\iota_t$  becomes:

$$\Phi'(\iota_t) = \mathbb{E}_t [\hat{\xi}_{t+1} \partial_k V_{t+1}(k_t)]^{-1} = \mathbb{E}_t [\hat{\xi}_{t+1} v_{t+1}]^{-1}$$

The Bellman equation becomes:

$$\begin{aligned} v_t k_{t-1} &= \left( \left( \frac{z_t (1 - \alpha)}{w_t^{1-\alpha}} \right)^{1/\alpha} \left( \frac{\alpha}{1 - \alpha} \right) - \iota_t \right) k_{t-1} + \mathbb{E}_t [\hat{\xi}_{t,t+1} v_{t+1} k_t] \\ \Rightarrow v_t &= \left( \alpha \frac{y_t}{k_{t-1}} - \iota_t \right) + (1 - \delta + \Phi(\iota_t)) \mathbb{E}_t [\hat{\xi}_{t,t+1} v_{t+1}] \\ \Rightarrow v_t &= \left( \alpha \frac{y_t}{k_{t-1}} - \iota_t \right) + \frac{1 - \delta + \Phi(\iota_t)}{\Phi'(\iota_t)} \end{aligned}$$

Let  $\hat{r}_t^Y := \alpha \frac{y_t}{k_{t-1}}$  be the marginal return to capital (from production) and  $\hat{r}_t^K = \frac{\Phi(\iota_t)}{\Phi'(\iota_t)} - \iota_t$  be the marginal return to capital (from reducing future adjustment costs<sup>6</sup>). Then, the value function becomes:

$$\begin{aligned} V_t &= (\hat{r}_t^Y - \iota_t)k_{t-1} + \frac{k_t}{\Phi'(\iota_t)} \\ &= \underbrace{(\hat{r}_t^Y + \hat{r}_t^K)k_{t-1}}_{\text{return on capital}} + \underbrace{\frac{(1-\delta)k_{t-1}}{\Phi'(\iota_t)}}_{\text{capital stock after production}} \end{aligned}$$

and so the dividend and ex-dividend price are:

$$\begin{aligned} \delta_t^s &= (\hat{r}_t^Y - \iota_t)k_{t-1} \\ q_t^s &= \frac{k_t}{\Phi'(\iota_t)} \end{aligned}$$

### 4.3 Financial Repression and Asset Pricing

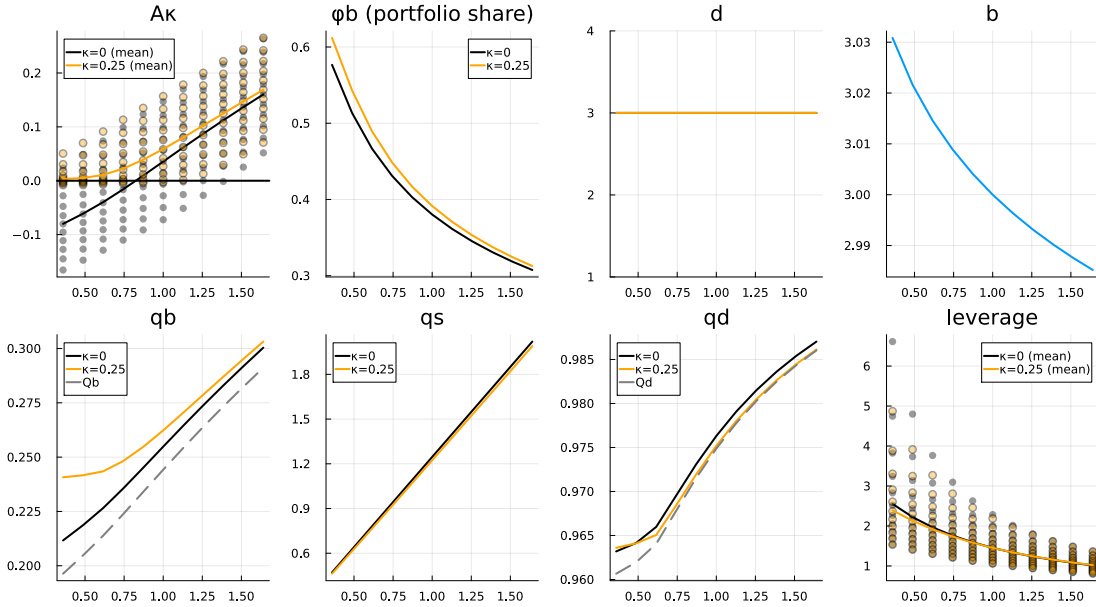


Figure 13: Inelastic Deposit Demand:  $\kappa$  makes  $z$ -shock gov. debt demand shock

<sup>6</sup>This is the capital goods producer's return.

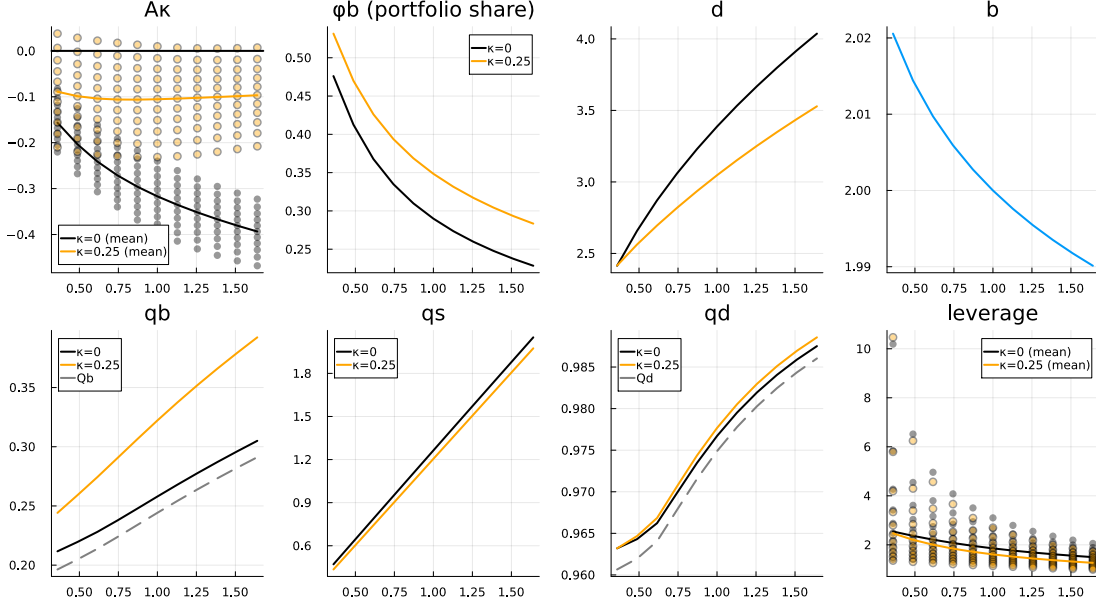


Figure 14: Elastic Deposit Demand:  $\kappa$  makes  $q^b$  more procyclical

#### 4.4 Fiscal Capacity Over the Business Cycle

Government funding advantage in our model:

$$\text{“Convenience yield”} = \mathbb{E}_t[\xi_{t,t+1}/\zeta]^{-1} - \zeta \log(1/q_t^b)$$

In our model, the “convenience yield” on long-term government debt can potentially come (i) regulation (the constraint  $\varrho_b$ ) leading banks to buy more debt in recessions and (ii) the government reducing bond supply in recessions. However, the empirical fiscal policies shut down this second channel. To understand how this plays out in equilibrium, we simulate economy under different regulatory policies and plot:

$$\underbrace{\mathbb{E}_t[\xi_{t,t+1}/\zeta]^{-1} - \zeta \log(1/q_t^b)}_{\text{Convenience yield / “funding advantage”}} \sim \underbrace{q_t^b b_{t+1}/y_t}_{\text{Market Value of Debt to GDP}}$$

We then show how convenience yield moves along the equilibrium path.

We plot the results in Figure (15). The top line shows the simulated relationship between the market value of debt-to-GDP and the convenience yield in an economy without regulation. The middle line shows the simulated relationship with loose regulation and elastic demand. The final line shows the simulated relationship with tight regulation and inelastic demand. Evidently, the shape of the relationship between the convenience yield and debt-to-GDP changes from downward

sloping to flat once tight regulation is introduced. To understand this, consider the impact of a recession in the model. A decrease in productivity,  $\downarrow z$ , lead the government to increase debt/GDP. But the decrease in productivity also causes the regulatory constraint to bind and so increases bank demand for government debt. Thus, under tight regulation, the government can increase the debt/GDP ratio without facing an increasing interest rate.

## 5 Conclusion

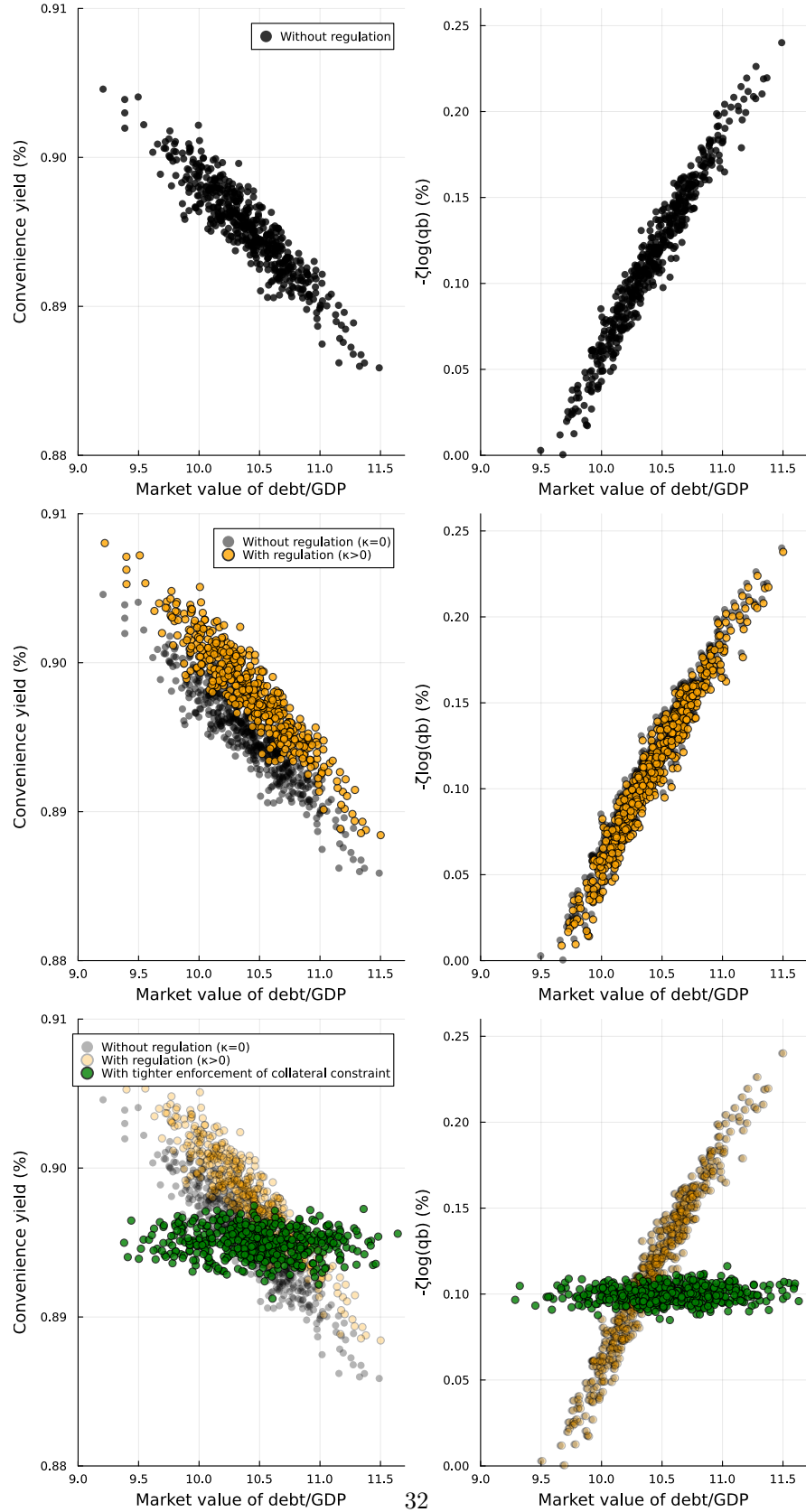


Figure 15: Top line: no regulation. Middle line: loose regulation and elastic demand for deposits. Bottom line: tight regulation and inelastic demand for deposits.



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## A Details on the Model in Section 2

**Notation:** There is a continuum of islands,  $i \in [0, 1]$ , each with a unit measure of household members, indexed by  $h \in [0, 1]$ , and a unit measure of competitive banks, indexed by  $f \in [0, 1]$ . Index  $h$  can be replaced by the binary idiosyncratic shock  $\zeta \in \{0, 1\}$  (the probability of which is island-specific), while  $i$  can be replaced by the idiosyncratic shock  $\lambda$ .

### A.1 Household problem

Taking prices  $(q_0^d, q_0^e)$  and payoffs  $\left\{ \left( \delta^d(\lambda), \delta_{PM}^e(\lambda) \right) \right\}_\lambda$  as given, the household solves (each of them being able to buy assets from only one bank):

$$\begin{aligned} \max_{d_0, e_0, c_0, c, d} \quad & \mathbb{E} \left[ \zeta_{h,i} u(c_{AM}^{h,i}) + (1 - \zeta_{h,i}) u(c_{PM}^{h,i}) \right] \quad s.t. \\ q_0^d d_0^h + q_0^e e_0^h \leq 1 = a_0^h = & \left( \delta_0^e + q_0^e \right) e_{-1}^h & \left( \mu_0^{c,h} \right) \\ c_{AM}^{h,i} \leq \delta^{d,i} d_0^h \quad \forall i & & \left( \psi_{AM}^{h,i} \right) \\ c_{AM}^{h,i} \leq \delta^{d,i} (d_0^h - d_{AM}^{h,i}) + \delta_{AM}^{e,i} e_0^h \quad \forall i & & \left( \mu_{AM}^{c,h,i} \right) \\ c_{PM}^h \leq \delta_{PM}^{e,i} e_0^h + \delta^{d,i} d_{AM}^{h,i} - \tau_{PM} & & \left( \mu_{PM}^{c,h} \right) \\ 0 \leq c_{AM}^{h,i}, c_{PM}^{h,i}, d_0^h, d_{AM}^{h,i} & & \left( \underline{\mu}_t^{j,h} \right) \end{aligned}$$

At time  $t = 0$ , the household sells its initial consumption goods  $a_0^h = z_{-1} k_{-1}$  to the two types of capital goods producers (for price of one). For a given island  $i$ , the FOCs of an individual household  $h$  are

$$\begin{aligned} [c_{AM}^h] \quad & 0 = \zeta_h u'(c_{AM}^h) - \mu_{AM}^{c,h} - \psi_{AM}^h + \underline{\mu}_{AM}^{c,h} \\ [c_{PM}^h] \quad & 0 = (1 - \zeta_h) u'(c_{PM}^h) - \mu_{PM}^{c,h} + \underline{\mu}_{PM}^{c,h} \\ [d_{AM}^h] \quad & 0 = -\delta^d \mu_{AM}^{c,h} + \mathbb{E}[\delta^d \mu_{PM}^{c,h}] + \underline{\mu}_{AM}^{d,h} \end{aligned}$$

For early consumers ( $\zeta = 1$ ), the marginal value of income in the PM is zero:  $\mu_{PM}^c(1) = \underline{\mu}_{PM}^c(1) = 0$ , while the marginal utility of consumption is equal to the marginal cost of consumption which in the AM is equal to the marginal value of income adjusted by the extra cost from the CIA constraint:  $u'(c_{AM}(1)) = \mu_{AM}^c(1) + \psi(1)$ . This implies that early households want to sell all of their assets in the AM,  $\underline{\mu}_{AM}^d(1) > 0$  and  $d_{AM}(1) = 0$ . Their supply is inelastic irrespective of which island they are on. The CIA constraint binds  $\psi_{AM} > 0$  and the income constraint is satisfied with  $d_{AM}(1) = 0$ , nevertheless  $\underline{\mu}_{AM}^d(1) = \mu_{AM}^c(1) = 0$ . In this sense, CIA constraint is equivalent with the households' inability to trade assets in the AM. In other words, we could “drop” the CIA constraint from the above problem. The key for this is that early households don't care about the potential continuation value in the portfolio-adjustment sub-period. If they do care about the PM period, we need to keep the explicit CIA constraint.

For late consumers ( $\zeta = 0$ ), the marginal value of income in the PM equals to the marginal utility of consumption,  $\mu_{PM}^c(0) = u'(c_{PM}(0))$ . It follows from their FOCs for deposit that their marginal utility of income in the AM must be strictly positive as well,  $\mu_{AM}^c(0) = \mathbb{E}[\mu_{PM}^c(0)] > 0$  and  $\underline{\mu}_{AM}^d(0) = \underline{\mu}_{AM}^e(0) = 0$ , due to the fact that they can use idle AM income to save for the PM. Their deposit roll-over decision depends on the relative returns on deposit vs alternative investment opportunities between the AM and PM (we assume that there is none). Strictly positive value of AM income and the lack of utility from AM consumption implies that late consumers set  $c_{AM}(0) = 0$  and so  $\psi(0) = 0$ . As a result, being “cash constrained” is equivalent with being an early consumer ( $\zeta = 1$ ).

The FOCs with respect to period  $t = 0$  choices are

$$\begin{aligned}
[d_0^h] \quad q_0^d \mu_0^c &= \mathbb{E} \left[ \int \left( \lambda \left( \mu_{AM}^c(1, \lambda) + \psi_{AM}(1, \lambda) \right) + (1 - \lambda) \mu_{AM}^c(0, \lambda) \right) \delta^d(\lambda) dF(\lambda) \right] \\
&= \mathbb{E} \left[ \int \left( \lambda u'(c_{AM}(1, \lambda)) + (1 - \lambda) \mathbb{E}[u'(c_{PM}(0, \lambda))] \right) \delta^d(\lambda) dF(\lambda) \right] \\
&= \mathbb{E} \left[ \int (1 - \lambda) \mathbb{E}[u'(c_{PM}(0, \lambda))] \underbrace{\left( 1 + \frac{\lambda u'(c_{AM}(1, \lambda))}{(1 - \lambda) \mu_{AM}^c(0, \lambda)} \right)}_{=: 1 + \nu(\lambda)} \delta^d(\lambda) dF(\lambda) \right] \\
q_0^d &= \mathbb{E} \left[ \int \xi_{AM}(\lambda) (1 + \nu(\lambda)) \delta^d(\lambda) dF(\lambda) \right]
\end{aligned}$$

$$\begin{aligned}
[e_0^h] \quad q_0^e \mu_0^c &= \mathbb{E} \left[ \int \left( \lambda \mu_{AM}^c(1, \lambda) + (1 - \lambda) \mu_{AM}^c(0, \lambda) \right) \delta_{AM}^e(\lambda) dF(\lambda) \right] \\
&\quad + \mathbb{E} \left[ \int \left( \lambda \mu_{PM}^c(1, \lambda) + (1 - \lambda) \mu_{PM}^c(0, \lambda) \right) \delta_{PM}^e dF(\lambda) \right] \\
&= \mathbb{E} \left[ \int (1 - \lambda) \mu_{AM}^c(0, \lambda) \delta_{AM}^e(\lambda) dF(\lambda) \right] \\
&\quad + \mathbb{E} \left[ \int (1 - \lambda) u'(c_{PM}(0, \lambda)) \delta_{PM}^e(\lambda) dF(\lambda) \right] \\
q_0^e &= \mathbb{E} \left[ \int \xi_{AM}(\lambda) \underbrace{\left( \delta_{AM}^e(\lambda) + \frac{\xi_{PM}(\lambda)}{\xi_{AM}(\lambda)} \delta_{PM}^e(\lambda) \right)}_{=: V_{AM}(\lambda)} dF(\lambda) \right]
\end{aligned}$$

where we used the notations for the stochastic discount factor

$$\xi_{AM}(\lambda) := \frac{(1 - \lambda) \mathbb{E}[u'(c_{PM}(0, \lambda))]}{\mu_0^c} \quad \xi_{PM}(\lambda) := \frac{(1 - \lambda) u'(c_{PM}(0, \lambda))}{\mu_0^c}$$

The individual consumption choices are

$$\begin{aligned} c_{AM}(0, \lambda) &= 0 & c_{AM}(1, \lambda) &= \delta^d(\lambda) d_0^h \\ c_{PM}(0, \lambda) &= \delta_{PM}^e(\lambda) e_0^h + \delta^d(\lambda) d_0^h - \tau_{PM} & c_{PM}(1, \lambda) &= 0 \end{aligned}$$

**Remark:** A (“shadow”) asset that promises to pay a risk-free unit of goods in the PM but otherwise plays no special role in the AM market would be priced as

$$q_0^s = \mathbb{E} \left[ \int \xi_{PM}(\lambda) dF(\lambda) \right] = \mathbb{E} \left[ \int \xi_{AM}(\lambda) dF(\lambda) \right]$$

Comparing this expression with the deposit Euler equation, we can define the liquidity premium on bank deposit as

$$\left( \frac{r_0^s - r_0^d}{1 + r_0^d} = \right) q_0^d (q_0^s)^{-1} - 1 = \mathbb{E} \left[ \frac{\xi_{AM}(\lambda)}{\mathbb{E}[\xi_{AM}(\lambda)]} (1 + \nu(\lambda)) \delta^d(\lambda) \right] > 0$$

In yield terms we can write this condition as

$$(\text{cost of deposit financing}) \quad r_0^d < r_0^s \quad (\text{cost of equity financing})$$

This is the source of “funding advantage of bank deposit”: in equilibrium the household’s required (risk-adjusted) return on bank equity is  $r_0^s$ , while the required (pecuniary) return on bank deposit is  $r_0^d$ . However, it is also clear that as  $\delta^d$  gets lower (due to bankruptcy) the liquidity premium is also decreasing.

## A.2 Bank problem

Default is costly for two reasons: (i) there are deadweight costs of default (proportional to outstanding deposit  $d_0$ ) and denoted by  $\varsigma$ . While banks take  $\varsigma$  as given, in equilibrium  $\varsigma$  is an increasing function of the fraction of defaulting banks that leads to “too much deposit issuance” (individual costs < social costs); (ii) forced selling results in the sale of assets at prices below their “fundamental value” because of market illiquidity. This is a transfer of value from the seller to the buyer, so it leads to “too little deposit issuance” (individual costs > social costs). Taking prices  $(q_{AM}^b, q_{PM}^b)$  as given, the bank solves

$$\begin{aligned} \max_{d_0, m_0, k_0, b_0} & \left\{ \delta_0^e + \mathbb{E} \left[ \int \xi_{AM}(\lambda) V_{AM} \left( d_0, m_0, k_0, b_0; \lambda, \mathbf{s} \right) dF(\lambda) \right] \right\} \quad s.t. \\ \delta_0^e + m_0 + k_0 + q_0^b b_0 & \leq q_0^d d_0 \\ \varrho(q_0^d d_0) & \leq \kappa^m m_0 + \kappa^b (q_0^b b_0) + \kappa^k k_0 \\ 0 & \leq d_0, m_0, k_0, b_0 \\ q_0^d & = \mathbb{E} \left[ \int \xi_{AM}(\lambda) (1 + \nu(\lambda)) \delta^d(\lambda) dF(\lambda) \right] \end{aligned}$$

where

$$\begin{aligned}
V_{AM}(d_0, m_0, k_0, b_0; \lambda, s) &= \max \left\{ 0, \delta_{AM}^e + \mathbb{E} \left[ \left( \frac{\xi_{PM}(\lambda)}{\mathbb{E}[\xi_{PM}(\lambda)]} \right) \delta_{PM}^e \right] \right\} \quad s.t. \\
\delta_{AM}^e + q_{AM}^k k_{AM} + q_{AM}^b b_{AM} &\leq z_{AM} m_0 + q_{AM}^k k_0 + q_{AM}^b b_0 - \delta^d \lambda d_0 - \varsigma d_0 \mathbb{1}\{\delta^d < 1\} \\
\delta_{PM}^e &\leq z_{PM} k_{AM} + \delta_{PM}^b b_{AM} - \delta^d (1 - \lambda) d_0 \\
0 &\leq \delta_{AM}^e, k_{AM} \\
\underbrace{\varrho \delta^d (1 - \lambda) d_0}_{\text{rolled-over deposit}} &\leq \underbrace{q_{AM}^b b_{AM}}_{\text{market value of debt}} + (1 - \kappa)(q_{AM}^k k_{AM}) \quad \varrho \geq 0, \kappa \leq 1
\end{aligned}$$

Function  $\varsigma(\cdot)$  denotes real dead-weight losses from default that may include the loss of firm specific information, the destruction of capital/consumer networks, etc. The  $\varsigma(\cdot)$  function is a feature of the environment that the government cannot overcome per se, but they can internalize the externality that it represents.

Parameters  $(\varrho, \kappa)$  are regulatory parameters:

- $\varrho$  restricts the banks' deposit-to-asset ratio ("leverage constraint").  $\varrho = 0$  corresponds to the case of no financial regulation. We call  $\varrho$  the regulation parameter.
- $\kappa$  measures the amount of repression.  $\kappa = 0$  corresponds to symmetric regulatory treatment of the two assets, while  $\kappa \neq 0$  introduces asymmetric treatment. When  $\kappa$  is positive, government debt is preferred to capital, when  $\kappa$  is negative capital is preferred relative to debt.  $\kappa = 1$  corresponds to the extreme case when capital is excluded completely.

### A.2.1 Portfolio choice in the AM and default

Given period  $t = 0$  choices  $(m_0, k_0, b_0, d_0)$  and the aggregate shock, there is a bank-specific withdrawal shock of size  $\lambda d_0$ . Because of financial frictions, liquidity is limited in the AM: (i) there is no equity injection  $\delta_{AM}^e \geq 0$ , and (ii) there is no un-collateralized debt issuance  $b_{AM}, k_{AM} \geq 0$  (i.e., banks are borrowing constrained). Market illiquidity introduces a wedge between the asset's market price and "fundamental value" which makes AM asset sales costly. Nevertheless, because of (i) and (ii), withdrawals must be financed either by cash-on-hand or by costly asset sales/borrowing, both of which affect the shareholders' dividend payment in the PM. Shareholders of the bank have limited liability, so when the bank value in the AM becomes negative, they choose to default.

To see how the wedges between asset prices and fundamental value appear in the AM, we

study the FOCs with respect to AM choices:

$$\begin{aligned} [\delta_{AM}^e] \quad & \mu_{AM} = 1 + \psi_{AM}^e \\ [b_{AM}] \quad & q_{AM}^b(\mu_{AM} - \mu_{AM}^r) = \mathbb{E} \left[ \left( \frac{u'(c_{PM}(0, \lambda))}{\mathbb{E}[u'(c_{PM}(0, \lambda))]} \right) \delta_{PM}^b \right] \end{aligned} \quad (\text{A.1})$$

$$[k_{AM}] \quad q_{AM}^k(\mu_{AM} - (1 - \kappa)\mu_{AM}^r) = \mathbb{E} \left[ \left( \frac{u'(c_{PM}(0, \lambda))}{\mathbb{E}[u'(c_{PM}(0, \lambda))]} \right) z_{PM} \right] \quad (\text{A.2})$$

where  $\psi_{AM}^e \geq 0$  is the Lagrange multiplier on the equity raising constraint,  $\mu_{AM} \geq 0$  is the multiplier on the period  $t = 1$  budget constraint and  $\mu_{AM}^r \geq 0$  is the multiplier on the  $t = 1$  regulatory constraint. The RHS of (A.1) and (A.2) represent the two long-term assets' fundamental value from the AM point of view. When there is no aggregate risk in the PM, the expressions become  $\delta_{PM}^b$  and  $z_{PM}$ , respectively.

**(I). No equity raising + No regulation:** Consider the case with  $\varrho = 0$ , when  $\mu_{AM}^r = 0$  and the two long-term assets are perfect substitutes. When the AM equity raising constraint binds,  $\psi_{AM}^e(\mathbf{s}) > 0$ , the “liquidity shortage” makes AM asset prices lower than what the household would be willing to pay for them (“fundamental value”), i.e.  $q_{AM}^b(\mathbf{s}) < \delta_{PM}^b(\mathbf{s})$  and  $q_{AM}^k(\mathbf{s}) < z_{PM}(\mathbf{s})$ . In fact, the presence of the idiosyncratic shock  $\lambda$  makes the equity raising constraint always bind in the AM: low- $\lambda$  banks would raise equity to buy assets cheaply, high- $\lambda$  banks would raise equity to avoid costly default.<sup>7</sup> We can use the  $t = 1$  Euler equations and combine the  $t = 1$  and  $t = 2$  budget constraints to get

$$\delta_{AM}^e + \frac{\delta_{PM}^e}{1 + \psi_{AM}^e} = z_{AM}m_0 + \frac{z_{PM}k_0 + \delta_{PM}^b b_0}{1 + \psi_{AM}^e} - \underbrace{\delta^d \left[ \lambda + \frac{1 - \lambda}{1 + \psi_{AM}^e} \right] d_0 - \varsigma d_0 \mathbb{1}\{\delta^d < 1\}}_{\text{exposure to } \lambda \text{ shock}}$$

which shows how the “missing morning markets” causes troubles for the banks to move resources between the AM and PM. Effectively, it changes the banks' inter-temporal marginal rate of substitution between the AM and PM (recall that shareholders want to maximize  $\delta_{AM}^e + \delta_{PM}^e$ , i.e. their IMRS is one). It also shows that without the equity raising friction, shareholder value would not depend on  $\lambda$ . We can rearrange this “consolidated” budget constraint to write dividends in the PM as:

$$\delta_{PM}^e = \left(1 + \psi_{AM}^e\right) \left(z_{AM}m_0 + q_{AM}^k k_0 + q_{AM}^b b_0 - \delta^d \lambda d_0 - \varsigma d_0 \mathbb{1}\{\delta^d < 1\} - \delta_{AM}^e\right) - \delta^d (1 - \lambda) d_0$$

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<sup>7</sup>The relationship works vice versa: without equity raising friction the idiosyncratic shock  $\lambda$  has no bite.



The bank defaults when this term becomes negative *assuming*  $\delta^d = 1$ . This leads to the cutoff

$$\begin{aligned}\lambda^* &:= \frac{(1 + \psi_{AM}^e)(z_{AM}m_0 + q_{AM}^k k_0 + q_{AM}^b b_0) - d_0}{\psi_{AM}^e d_0} \\ &= \underbrace{\frac{z_{AM}m_0}{d_0}}_{=\lambda^0} + \frac{z_{AM}m_0 + z_{PM}k_0 + \delta_{PM}^b b_0 - d_0}{\psi_{AM}^e d_0}\end{aligned}$$

This expression (that we got by substituting out prices using the  $t = 1$  Euler equations) clarifies that the reason why  $\lambda$  is an issue and certain banks default is the equity raising friction. In the event of default,  $\lambda > \lambda^*$ , deposit payout is

$$\delta^d(\lambda) = \frac{(1 + \psi_{AM}^e)(z_{AM}m_0 + q_{AM}^k k_0 + q_{AM}^b b_0 - \varsigma d_0)}{\left[(1 + \psi_{AM}^e)\lambda + (1 - \lambda)\right] d_0} = \frac{1 + \psi_{AM}^e(\lambda^* - \varsigma) - \varsigma}{1 + \psi_{AM}^e \lambda}$$

We can use the expression for the default cutoff to write dividends in the PM as

$$\delta_{PM}^e = \max \left\{ 0, \psi_{PM}^e (\lambda^* - \lambda) d_0 \right\}$$

which shows that the source of positive dividend (and a non-zero  $q_0^e$  is the equity raising friction in the morning).

**(II). Free equity raising + Regulation:** Consider the case without the  $\delta_{AM}^e \geq 0$  constraint, when  $\psi_{AM}^e = 0$  and there is no default. Looking at the Euler equations reveals that a binding  $t = 1$  regulatory constraint has an opposite effect on AM asset prices to the equity raising constraint, i.e.,  $\mu_{AM}^r > 0$  raises  $q_{AM}^k$  and  $q_{AM}^b$  above their fundamental values. However, it is not clear that the  $t = 1$  regulatory constraint binds. To see this, plug the  $t = 1$  budget constraint into the regulatory constraint:

$$\varrho(1 - \lambda)d_0 \leq z_{AM}m_0 + (1 - \mu_{AM}^r)^{-1}(z_{PM}k_0 + \delta_{PM}^b b_0) - \lambda d_0 - \delta_{AM}^e$$

which shows that banks can always avoid the  $t = 1$  regulatory constraint by raising equity. In this economy, the  $\lambda$  shock has no bite, so for all purposes, this looks like one with a  $t = 0$  regulatory constraint without a morning market.

**(III). No equity raising + Regulation:** This is the case when the two non-negative multipliers might appear together with opposite effects on AM asset prices. In other words, the fact that banks cannot easily avoid the regulatory constraint makes  $\mu_{AM}^r > 0$  more likely. Following the same logic as before, we can use the consolidated budget constraint to write PM dividends as

$$\delta_{PM}^e = (1 + \psi_{AM}^e - \mu_{AM}^r)(q_{AM}^k k_{AM} + q_{AM}^b b_{AM}) - \delta^d(1 - \lambda)d_0$$

and the default cutoff as

$$\begin{aligned}\lambda^* &:= \frac{\left(1 + \psi_{AM}^e - \mu_{AM}^r\right)\left(z_{AM}m_0 + q_{AM}^k k_0 + q_{AM}^b b_0\right) - d_0}{\left(\psi_{AM}^e - \mu_{AM}^r\right)d_0} \\ &= \underbrace{\frac{z_{AM}m_0}{d_0}}_{=\lambda^0} + \frac{z_{AM}m_0 + z_{PM}k_0 + \delta_{PM}^b b_0 - d_0}{\left(\psi_{AM}^e - \mu_{AM}^r\right)d_0}\end{aligned}$$

which shows that *for a given initial balance sheet,  $(d_0, m_0, k_0, b_0)$ , and a given  $\psi_{AM}^e$* , a binding  $t = 1$  regulatory constraint can help reduce the probability of default. This works through a valuation effect: by linking asset trading in the morning markets to the bank's (fixed) outstanding liabilities, regulation can keep morning asset prices relatively high. However, in equilibrium higher morning asset prices mean that the equity raising constraint is less of a problem and  $\psi_{AM}^e$  falls. Similarly, higher morning prices mean that the regulatory constraint binds less. Plugging the  $t = 1$  budget constraint into the regulatory constraint leads to

$$\delta^d \lambda d_0 + \varrho(1 - \lambda)d_0 \leq z_{AM}m_0 + \left(\frac{z_{PM}k_0 + \delta_{PM}^b b_0}{1 + \psi_{AM}^e - \mu_{AM}^r}\right)$$

so it is unclear if the  $t = 1$  regulatory constraint can be “stronger” than the  $t = 0$  constraint. In the event of default,  $\lambda > \lambda^*$ , deposit payout is

$$\delta^d(\lambda) = \frac{1 + \left(\psi_{AM}^e - \mu_{AM}^r\right)(\lambda^* - \varsigma) - \varsigma}{1 + \left(\psi_{AM}^e - \mu_{AM}^r\right)\lambda}$$

We can use the expression for the default cutoff to write dividends in the PM as

$$\delta_{PM}^e = \max\left\{0, \left(\psi_{AM}^e - \mu_{AM}^r\right)(\lambda^* - \lambda)d_0\right\}$$

which shows that the effect of regulation on PM dividends is ambiguous. The increasing  $\lambda^*$  is beneficial, but the “return” on morning resources is lower.

**(IV). No equity raising + Repression:** It is clear from the Euler equations, that  $\kappa \neq 0$  introduces a wedge between the AM asset returns. Asymmetric regulatory treatment makes the otherwise perfectly substitutable assets different. This implies that banks will have a clear preference which asset they want to hold and make morning trades so that the  $t = 1$  regulatory constraint always binds. In particular,

$$\frac{\delta_{PM}^b}{q_{AM}^b} + \kappa \mu_{AM}^r = \frac{z_{PM}}{q_{AM}^k} \quad \Leftrightarrow \quad \frac{q_{AM}^b}{q_{AM}^k} = \frac{\delta_{PM}^b}{z_{PM} - \kappa \mu_{AM}^r q_{AM}^k}$$

The binding regulatory constraint implies the following “asset demand functions”:

$$\kappa q_{AM}^b b_{AM} = \varrho \delta^d (1 - \lambda) d_0 - (1 - \kappa) (q_{AM}^b b_{AM} + q_{AM}^k k_{AM})$$

and

$$\begin{aligned} \kappa q_{AM}^k k_{AM} &= -\varrho \delta^d (1 - \lambda) d_0 + (q_{AM}^b b_{AM} + q_{AM}^k k_{AM}) \\ &= -\varrho \delta^d (1 - \lambda) d_0 + (\ell_{AM} - \delta^d \lambda) d_0 \end{aligned}$$

The consolidated budget constraint can be used to express dividend in the PM

$$\begin{aligned} \delta_{PM}^e(\lambda) &= (1 + \psi_{AM}^e - \mu_{AM}^r) (q_{AM}^k k_{AM} + q_{AM}^b b_{AM}) + \kappa \mu_{AM}^r q_{AM}^k k_{AM}(\lambda) - \delta^d (1 - \lambda) d_0 \\ &= (1 + \psi_{AM}^e) (\ell_{AM} - \delta^d \lambda) d_0 - (1 + \varrho \mu_{AM}^r) \delta^d (1 - \lambda) d_0 \end{aligned}$$

where the orange term highlights the effect of the asymmetric regulatory treatment of assets. The second equality uses the binding regulatory constraint to substitute out the orange term. The default cutoff is (where  $\mathbb{1}$  denotes the event  $\{\kappa \neq 0\}$ ):

$$\lambda^* := \frac{(1 + \psi_{AM}^e - \mu_{AM}^r + \mathbb{1} \mu_{AM}^r) \ell_{AM} - (1 + \mathbb{1} \varrho \mu_{AM}^r)}{(\psi_{AM}^e - \mu_{AM}^r + \mathbb{1} (1 - \varrho) \mu_{AM}^r)}$$

In the event of default,  $\lambda > \lambda^*$ , deposit payout is

$$\delta^d(\lambda) = \frac{(1 + \psi_{AM}^e - \mu_{AM}^r + \mathbb{1} \mu_{AM}^r) (\ell_{AM} - \varsigma)}{(1 + \psi_{AM}^e - \mu_{AM}^r + \mathbb{1} \mu_{AM}^r) \lambda + (1 + \mathbb{1} \varrho \mu_{AM}^r) (1 - \lambda)}$$

We can use the expression for the default cutoff to write dividends in the PM as

$$\delta_{PM}^e = \max \left\{ 0, \left( \psi_{AM}^e - \mu_{AM}^r + \mathbb{1} (1 - \varrho) \mu_{AM}^r \right) (\lambda^* - \lambda) d_0 \right\}$$

### A.2.2 Market clearing in AM asset markets

- (i)  $\kappa = 0$ : ( $\delta_{PM}^b / q_{AM}^b = z_{PM} / q_{AM}^k$ ): The portfolio shares are indeterminate, but the two asset markets must clear at the aggregate:

$$\begin{aligned} \int (q_{AM}^k \Delta k + q_{AM}^b \Delta b) dF &= 0 \\ \int_{\lambda^*}^{\lambda^*} (z_{AM} m_0 - \lambda d_0) dF &= - \int_{\lambda^*} (z_{AM} m_0 - \varsigma d_0 - \delta^d(\lambda) \lambda d_0) dF \end{aligned}$$

(ii)  $\kappa \neq 0$ : Market clearing on the debt market requires  $\int b_{AM} = b_0$  which becomes:

$$\int \frac{\varrho \delta^d(\lambda)(1-\lambda) - (1-\kappa) \left( \ell_{AM} - \varsigma \mathbb{1}\{\delta^d < 1\} - \delta^d(\lambda)\lambda \right)}{\kappa} dF = \frac{q_{AM}^b b_0}{d_0}$$

Market clearing on the capital market requires  $\int k_{AM} = k_0$  which becomes:

$$\int \frac{-\varrho \delta^d(\lambda)(1-\lambda) + \left( \ell_{AM} - \varsigma \mathbb{1}\{\delta^d < 1\} - \delta^d(\lambda)\lambda \right)}{\kappa} dF = \frac{q_{AM}^k k_0}{d_0}$$

### A.2.3 Aggregate resource constraints

The banks aggregated budget constraints in the AM can be written as (where we also use  $\int \Delta k dF = \int \Delta b dF = 0$ ):

$$\underbrace{\left[ \int^{\lambda^*} \lambda dF(\lambda) + \int_{\lambda^*} \delta^d(\lambda) \lambda dF(\lambda) \right]}_{\text{aggregate payout to early households}} d_0 = z_{AM} m_0 - \varsigma d_0 (1 - F(\lambda^*))$$

where the last term is equal to aggregate consumption (from household BC)

$$\int \lambda c_{AM}(1, \lambda) dF(\lambda) = \left[ \int^{\lambda^*} \lambda dF(\lambda) + \int_{\lambda^*} \delta^d(\lambda) \lambda dF(\lambda) \right] d_0$$

The aggregated bank budget constraint in the PM is

$$\int \left( \delta_{PM}^e(\lambda) + (1-\lambda) \delta^d(\lambda) d_0 \right) dF(\lambda) = z_{PM} k_0 + \delta_{PM}^b b_0$$

while aggregate consumption in the PM (from the household budget constraint) is

$$\begin{aligned} \int (1-\lambda) c_{PM}(0, \lambda) dF(\lambda) &= \int (1-\lambda) \left( \delta_{PM}^e(\lambda) + \delta^d(\lambda) d_0 - \tau \right) dF(\lambda) \\ &= z_{PM} k_0 + \delta_{PM}^b b_0 - \int \lambda \delta_{PM}^e(\lambda) dF(\lambda) - T_{PM} \end{aligned}$$

#### A.2.4 Choice of initial portfolio

The FOCs with respect to period  $t = 0$  choices are

$$\begin{aligned}
[m_0] \quad & 0 = -1 + \frac{\partial q_0^d}{\partial m_0} d_0 + \frac{\partial q_0^e}{\partial m_0} + \kappa^m \mu_0^r \\
[k_0] \quad & 0 = -1 + \frac{\partial q_0^d}{\partial k_0} d_0 + \frac{\partial q_0^e}{\partial k_0} + (1 - \kappa) \mu_0^r \\
[b_0] \quad & 0 = -q_0^b + \frac{\partial q_0^d}{\partial b_0} d_0 + \frac{\partial q_0^e}{\partial b_0} + \mu_0^r q_0^b \\
[d_0] \quad & 0 = q_0^d + \frac{\partial q_0^d}{\partial d_0} d_0 + \frac{\partial q_0^e}{\partial d_0} - \varrho \mu_0^r \left( q_0^d + \frac{\partial q_0^d}{\partial d_0} d_0 \right)
\end{aligned}$$

Before we get to the partial derivatives of prices, note that for  $x \in \{m_0, k_0, b_0, d_0\}$

$$\frac{\partial \delta^d}{\partial x} = \left( \frac{\partial \lambda^*}{\partial x} \right) \left( \frac{\psi_{AM}^e - \mu_{AM}^r + \mathbb{1}(1 - \varrho) \mu_{AM}^r}{(1 + \psi_{AM}^e - \mu_{AM}^r + \mathbb{1} \mu_{AM}^r)(\ell_{AM} - \varsigma)} \right) \delta^d$$

and for simplicity, let's define

$$\mathcal{R} := \psi_{AM}^e - \mu_{AM}^r + \mathbb{1}(1 - \varrho) \mu_{AM}^r$$

We use the Leibnitz integral rule several times to obtain for  $x \in \{m_0, k_0, b_0, d_0\}$

$$\frac{\partial q_0^e}{\partial x} = \frac{1}{\mu_0^e} \mathbb{E} \left[ \left( \int^{\lambda^*} \xi(\lambda) dF(\lambda) \right) \mathcal{R} \left( \frac{\partial \lambda^*}{\partial x} \right) d_0 \right] + \frac{q_0^e}{d_0} \mathbb{1}_{\{x=d_0\}}$$

The partial derivatives of the deposit price w.r.t.  $x \in \{m_0, k_0, b_0, d_0\}$  are

$$\begin{aligned}
\frac{\partial q_0^d}{\partial x} = & \mathbb{E} \left[ \int_{\lambda^*} \xi(\lambda) (1 + \nu(\lambda)) \left( \frac{\partial \lambda^*}{\partial x} \right) \left( \frac{\mathcal{R}}{(1 + \psi_{AM}^e - \mu_{AM}^r + \mathbb{1} \mu_{AM}^r)(\ell_{AM} - \varsigma)} \right) \delta^d(\lambda) dF(\lambda) \right] + \\
& + \mathbb{E} \left[ \left( \frac{\partial \lambda^*}{\partial x} \right) \xi(\lambda^*) (1 + \nu(\lambda^*)) (1 - \delta^d(\lambda^*)) f(\lambda^*) \right]
\end{aligned}$$

where the last term comes from the discontinuity in the deposit payoff at  $\lambda^*$ , and

$$1 - \delta^d(\lambda^*) = \frac{\varsigma}{\ell_{AM}}$$

Using these objects, we can write  $x \in \{m_0, k_0, b_0\}$

$$\begin{aligned} \frac{\partial q_0^d}{\partial x} d_0 + \frac{\partial q_0^e}{\partial x} = \mathbb{E} \left[ \left( \frac{\partial \lambda^*}{\partial x} d_0 \mathcal{R} \right) \left\{ \int_{\lambda^*} \xi(\lambda) \left( \frac{(1 + \nu(\lambda))}{(1 + \psi_{AM}^e - \mu_{AM}^r + \mathbb{1} \mu_{AM}^r) \lambda + (1 + \mathbb{1} \varrho \mu_{AM}^r)(1 - \lambda)} \right) dF(\lambda) + \right. \right. \\ \left. \left. + \int_{\lambda^*} \xi(\lambda) \left( \frac{\xi(\lambda^*) (1 + \nu(\lambda^*))}{\xi(\lambda) \mathcal{R}} \frac{(1 - \delta^d(\lambda^*)) f(\lambda^*)}{(1 - F(\lambda^*))} \right) dF + \int^{\lambda^*} \xi(\lambda) dF(\lambda) \right\} \right] \end{aligned}$$

and

$$\begin{aligned} (1 - \varrho \mu_0^r) \frac{\partial q_0^d}{\partial d_0} d_0 + \frac{\partial q_0^e}{\partial d_0} = \mathbb{E} \left[ (-\mu_{AM} \ell_{AM}) \left\{ \int_{\lambda^*} \xi(\lambda) \left( \frac{(1 + \nu(\lambda))}{\mu_{AM} (\ell_{AM} - \varsigma)} \right) \delta^d(\lambda) dF(\lambda) (1 - \varrho \mu_0^r) + \right. \right. \\ \left. \left. + \int_{\lambda^*} \xi(\lambda) \left( \frac{\xi(\lambda^*) (1 + \nu(\lambda^*))}{\xi(\lambda) \delta^d(\lambda) \mathcal{R}} \frac{(1 - \delta^d(\lambda^*)) f(\lambda^*)}{(1 - F(\lambda^*))} \right) \delta^d(\lambda) dF (1 - \varrho \mu_0^r) \right. \right. \\ \left. \left. + \int^{\lambda^*} \xi(\lambda) \left( \frac{\mu_{AM} \lambda + (1 + \varrho \mu_{AM}^r)(1 - \lambda)}{\mu_{AM} \ell_{AM}} \right) dF(\lambda) \right\} \right] \end{aligned}$$

Using

$$\frac{\partial \lambda^*}{\partial m_0} d_0 \mathcal{R} = \mu_{AM} z_{AM}, \quad \frac{\partial \lambda^*}{\partial k_0} k_0 \mathcal{R} = \mu_{AM} q_{AM}^k, \quad \frac{\partial \lambda^*}{\partial b_0} d_0 \mathcal{R} = \mu_{AM} q_{AM}^b, \quad \frac{\partial \lambda^*}{\partial m_0} d_0 \mathcal{R} = -\frac{\mu_{AM} \ell_{AM}}{d_0}$$

the FOCs become

$$\begin{aligned} [m_0] \quad & (1 - \kappa^m \mu_0^r) = \mathbb{E} [\xi(\lambda) \Omega(\lambda) z_{AM}] \\ [k_0] \quad & (1 - \kappa^k \mu_0^r) = \mathbb{E} [\xi(\lambda) \Omega(\lambda) q_{AM}^k] \\ [b_0] \quad & q_0^b (1 - \kappa^b \mu_0^r) = \mathbb{E} [\xi(\lambda) \Omega(\lambda) q_{AM}^b] \\ [d_0] \quad & q_0^d = \mathbb{E} [\xi(\lambda) \Omega(\lambda) \tilde{\Omega}(\lambda) \delta^d(\lambda)] \end{aligned}$$

where

$$\Omega(\lambda) := \begin{cases} \frac{(1 + \nu(\lambda)) \delta^d(\lambda)}{(\ell_{AM} - \varsigma)} + \mu_{AM} \frac{\xi(\lambda^*) (1 + \nu(\lambda^*))}{\xi(\lambda) \mathcal{R}} \frac{\varsigma}{\ell_{AM}} \frac{f(\lambda^*)}{(1 - F(\lambda^*))} & \lambda > \lambda^* \\ \mu_{AM} & \lambda \leq \lambda^* \end{cases}$$

and

$$\tilde{\Omega}(\lambda) := \begin{cases} \ell_{AM} / \delta^d(\lambda) & \lambda > \lambda^* \\ \frac{\mu_{AM} \lambda + (1 + \varrho \mu_{AM}^r)(1 - \lambda)}{\mu_{AM} (1 - \varrho \mu_0^r)} & \lambda \leq \lambda^* \end{cases}$$

Looking at the formula for  $\Omega$  it is pretty clear that it arises from the equity raising cost (see the presence of  $\mu_{AM}$ ) and regulation has only an indirect affect through  $\lambda^*$  (and  $\mathcal{R}$  and  $\delta^d$ ), which is why this decomposition makes sense (sort of).

### A.3 Convenience yield decomposition (start)

The multiplicative term in the Euler equation for deposit supply (when  $\kappa^b = \bar{\kappa}^b$ ) is

$$\Omega(\lambda)\tilde{\Omega}(\lambda) = \begin{cases} \frac{1+\nu(\lambda)}{1-\varsigma/\ell_{AM}} + \frac{g(\lambda^*)}{\xi(\lambda)\delta^d(\lambda)} & \lambda > \lambda^* \\ \frac{\mu_{AM}\lambda + (1+\varrho\mu_{AM}^r)(1-\lambda)}{(1-\varrho\mu_0^r)} & \lambda \leq \lambda^* \end{cases}$$

If there is no equity raising friction,  $\lambda^* = 1$  and  $\mu_{AM} = 1$ , so this becomes

$$\Omega(\lambda)\tilde{\Omega}(\lambda) = \frac{1 + \varrho\mu_{AM}^r(1-\lambda)}{(1-\varrho\mu_0^r)}$$

If we also drop financial regulation, we get

$$\Omega(\lambda)\tilde{\Omega}(\lambda) = 1$$

Turning to the government debt Euler equation, there are two ways we can express the price of debt (i) one that uses the bank SDF that prices morning payoffs:

$$q_0^b(1 - \kappa^b\mu_0^r) = \mathbb{E}[\xi(\lambda)\Omega(\lambda)q_{AM}^b]$$

(ii) one that expresses the time 0 price as a function of the afternoon payoffs (that we get by plugging in the morning Euler equations to replace  $q_{AM}^b$ ):

$$q_0^b(1 - \kappa^b\mu_0^r) = \mathbb{E}\left[\xi(\lambda)\Omega(\lambda)\left(\frac{1}{\mu_{AM} - \kappa^b\mu_{AM}^r}\right)\delta_{PM}^b\right]$$

## B Jonathan's Random Notes

### B.1 Bank Problem

*Bank problem at  $t = 1$ :* Given a choice of  $(d_0, m_0, k_0, b_0)$  and a realization of  $(\lambda, \mathbf{s})$ , the bank solves:

$$\begin{aligned}
& \max_{\delta_1^e, k_1, b_1} \{V_1 = \delta_1^e + \delta_2^e\} \quad s.t. \\
& \delta_1^e + q_1^k k_1 + q_1^b b_1 + \delta_1^d \lambda d_0 + \varsigma 1(\delta_2^d < 1) \leq z_1 m_0 + q_1^k k_0 + q_1^b b_0 \\
& (1 - \lambda) \delta_2^e + \delta_2^d (1 - \lambda) d_0 \leq z_2 k_1 + \delta_2^b b_1 \\
& \delta_2^d \geq \delta_1^d \\
& \kappa \delta_2^d (1 - \lambda) d_0 \leq q_1^b b_1 \\
& (1 - \delta_1^d)(\delta_1^e + \delta_2^e) = 0 \\
& (1 - \delta_2^d)(\delta_1^e + \delta_2^e) = 0 \\
& k_1, b_1, \delta_1^e, \delta_2^e \geq 0
\end{aligned}$$

The bank cannot default and pay positive dividends so it only defaults when honoring deposit contracts leads to negative dividends.

*Bank is solvent:* Suppose the bank is solvent  $\delta_1^e + \delta_2^e \geq 0$ . Then  $\delta_1^d = \delta_2^d = 1$ . The Lagrangian is:

$$\begin{aligned}
\mathcal{L} = & \delta_1^e + \delta_2^e + \mu_1(z_1 m_0 + q_1^k k_0 + q_1^b b_0 - \delta_1^e - q_1^k k_1 - q_1^b b_1 - \lambda d_0) \\
& + \mu_2(z_2 k_1 + \delta_2^b b_1 - (1 - \lambda) \delta_2^e - (1 - \lambda) d_0) \\
& + \mu_\delta(\delta_2^d - \delta_1^d) + \mu_r(q_1^b b_1 - \kappa \delta_2^b (1 - \lambda) d_0) \\
& + \underline{\mu}^k q_1^k k_1 + \underline{\mu}_1^e \delta_1^e + \underline{\mu}_2^e \delta_2^e
\end{aligned}$$

The first order conditions are:

$$\begin{aligned}
[\delta_1^e] : & \quad 0 = 1 - \mu_1 + \underline{\mu}_1^e \\
[\delta_2^e] : & \quad 0 = 1 - \mu_2(1 - \lambda) + \underline{\mu}_2^e \\
[k_1] : & \quad 0 = -\mu_1 q_1^k + \mu_2 z_2 + \underline{\mu}^k q_1^k \\
[b_1] : & \quad 0 = -\mu_1 q_1^b + \mu_2 \delta_2^b + \mu_r q_1^b
\end{aligned}$$

Define the returns  $R^b := \delta_2^b / q_1^b$  and  $R^k := z_2 / q_1^k$ . We have that the lower bound on dividends binds so  $\delta_1^e = 0$ . Case 1:  $R^k > R^b$  and so the regulatory constraint binds  $\mu_r > 0$ . This implies that:

$$q_1^b b_1 = \kappa \delta_2^b (1 - \lambda) d_0, \quad q_1^k k_1 = z_1 m_0 + q_1^k k_0 + q_1^b b_0 - \kappa \delta_2^b (1 - \lambda) d_0 - \lambda d_0$$



and so:

$$\begin{aligned}
(1 - \lambda)\delta_2^e &= z_2 k_1 + \delta_2^b b_1 - (1 - \lambda)d_0 \\
&= R^k q_1^k k_1 + R^b q_1^b b_1 - (1 - \lambda)d_0 \\
&= R^k (z_1 m_0 + q_1^k k_0 + q_1^b b_0 - \lambda d_0) - (R^k - R^b)\kappa\delta_2^b(1 - \lambda)d_0 - (1 - \lambda)d_0 \\
&= (R^k(\ell^{-1} - \lambda) - (R^k - R^b)\kappa\delta_2^b(1 - \lambda) - (1 - \lambda)) d_0
\end{aligned}$$

where

$$\ell = \frac{d_0}{z_1 m_0 + q_1^k k_0 + q_1^b b_0}$$

So, the cutoff at which  $\delta_2^2 = 0$  satisfies:

$$\begin{aligned}
0 &= R^k \left( \frac{\ell^{-1} - \lambda^*}{1 - \lambda^*} \right) - (R^k - R^b)\kappa\delta_2^b - 1 \\
\Rightarrow \lambda^* &= \frac{R^k \ell^{-1} - (R^k - R^b)\kappa\delta_2^b - 1}{R^k - (R^k - R^b)\kappa\delta_2^b - 1}
\end{aligned}$$

Case 2:  $R^k = R^b$ . In this case the bank portfolio is indeterminate, except that the regulatory constraint must be satisfied. Case 3:  $R^k > R^b$ . In this case the banks hold no capital stock and so the capital market does not clear.

## C Exogenous Bond Demand Functions

High level, I see a collection of points about the traditional models for generating bond demand functions:

- I think perhaps we should have three comparisons in the model: (1) bond-in-the-utility at  $t = 0$  and (2) Bewley style idiosyncratic risk insurance. I have worked through these three comparisons over the next few subsections.
- The common link between Bond-in-the-Utility, Bond-in-Advance, and Bewley models is that holding a real value of government debt is helpful to households regardless of the price process for government debt. For the Bond-in-the-Utility model, this is because the functional form does not depend upon price process of government debt. For Bond-in-Advance, this is because the need for debt to trade is not related to riskiness of government debt. For Bewley models, it is because the idiosyncratic risk that is self insured by holding government debt is orthogonal to the price process of government debt. In our model, that would be that the idiosyncratic island risk insured by the bonds is orthogonal to the aggregate risk hitting the economy.
- There are some differences in the repression model. One difference is that the government

debt is not exogenously useful for any activity in the economy. There should be a way to see that switching from a model where agents like to finance the government to a model where agents are forced to finance the government changes something in the problem. A related difference is that we allow the banking sector to substitute away from using government debt as the asset that backs its deposit creation.

## C.1 Exogenous Bond Demand: Idiosyncratic Consumption Risk and Bond-in-Advance

### C.1.1 Environment

*Setting:* The economy lasts for three periods:  $t \in \{0, 1, 2\}$ . We interpret  $t = 0$  as a primary asset market,  $t = 1$  as a morning market, and  $t = 2$  as the following period. There is one consumption good. There are two production technologies in the economy: one that transforms  $m_0$  goods at time  $t = 0$  to  $z_1(s_1)m_0$  goods at time  $t = 1$  (short-term asset) and another one that transforms  $k_0$  goods at time  $t = 0$  to  $z_2(s_1)k_0$  goods at time  $t = 2$  (capital), where  $s_1$  is the aggregate state that has distribution  $\Pi(s_1)$  and is realized at the beginning of  $t = 1$ .

*Assets and Markets:* We use goods as the numeraire. At  $t = 0$ , the government issues bonds in the primary market at price  $q_0^b$  that pay  $\delta_2^b$  at time  $t = 2$ . At  $t = 1$ , the agents are only able to trade bonds for goods at price  $q_1^b$ . They cannot trade capital.

*Government:* The government ranks allocations according to:

$$\theta G + \mathcal{U}$$

where  $G$  is the provision of public goods by the government and  $\mathcal{U}$  is the aggregate lifetime household utility under equal Pareto weights. Parameter  $\theta$  is interpreted as the relative value of public goods. At  $t = 0$ , the government finances public good provision by issuing  $B_0$  bonds at price  $q_0^b$  leading to the  $t = 0$  budget constraint:

$$G \leq q_0^b B_0$$

At time 2, the government raises taxes  $T_2(s_1)$  from households at  $t = 2$ , which it uses to repay  $\delta_2^b(s_1)$  per unit of bonds according to:

$$\delta_2^b(s_1) B_0 \leq T_2(s_1)$$

where  $\delta_2^b(s_1) < 1$  is interpreted as “partial default” or “dilution” when the government decreases the real value of the bond principle. We refer to  $T_2(s_1)$  as the government “fiscal rule” and treat it as an exogenous outcome of an unmodelled political process. The exogenous  $T_2(s_1)$  pins down an upper bound on  $B_0$ .

*Household problem:* Agents cannot consume their own goods. Instead, they can only consume goods produced by other agents. Agents are distributed across islands. All agents on island  $\lambda$  rank consumption allocations at  $t = 1$  and  $t = 2$  according to:

$$\lambda u(c_1) + (1 - \lambda)u(c_2)$$

where  $\lambda \sim F(\lambda)$  is revealed at  $t = 1$ . At time 0, households rank allocations according to:

$$\mathcal{U} := \mathbb{E} [\lambda u(c_1^h) + (1 - \lambda)u(c_2^h)],$$

where  $c_t^{h,i}$  denotes consumption of household  $h$  on island  $i$  in period  $t \in \{1, 2\}$ . Each household is endowed with one unit of good at  $t = 0$  and zero goods in the other periods. All agents have the time 0 budget constraint:

$$q_0^b b_0^h + m_0^h + k_0^h \leq 1$$

where  $b_0^h$ ,  $m_0^h$ , and  $k_0^h$  are household  $h$ 's bond, short asset, and capital holdings. At  $t = 1$ , the households face the budget constraint and the bond-in-advance constraint:

$$\begin{aligned} c_1^h + q_1^h b_1^h &\leq q_0^b b_0^h + z_1 m_0 \\ c_1^h &\leq q_1^h b_0^h \end{aligned}$$

At  $t = 2$ , the households face the budget constraint:

$$c_2^{h,i} \leq \delta_2^b b_1^h + z_2 k_0^h - \tau(\mathbf{s})$$

### C.1.2 Household Problem

Taking prices  $(q_0^b, q_1^b)$  as given, the household solves:

$$\begin{aligned} \max_{b_0^h, m_0^h, k_0^h, b_1^h, \mathbf{c}} \quad & \mathbb{E}[\lambda u(c_1^h) + (1 - \lambda)u(c_2^h)] \quad s.t. \\ q_0^b b_0^h + m_0^h + k_0^h &\leq 1 \\ c_1^h &\leq q_1^b b_0^h \\ c_2^{h,i} &\leq \frac{\delta_2^b}{q_1^h} (q_1^b b_0^h + z m_0 - c_1^h) + z_2 k_0^h - \tau(\mathbf{s}) \\ b_0^h, m_0^h, k_0^h, b_1^h &\geq 0 \end{aligned}$$

Lagrangian is (leaving the short selling constraints implicit):

$$\begin{aligned}\mathcal{L} = & \mathbb{E} \left[ \lambda u(c_1^h(\lambda, \mathbf{s})) + (1 - \lambda) u \left( \frac{\delta_2^b(\mathbf{s})}{q_1^b(\mathbf{s})} (q_1^b(\mathbf{s}) b_0^h + z_1(\mathbf{s}) m_0 - c_1^h(\lambda, \mathbf{s})) + z_2(\mathbf{s}) k_0^h - \tau(\mathbf{s}) \right) \right] \\ & + \mu_0 (1 - q_0^b b_0^h - m_0^h - k_0^h) \\ & + \mathbb{E} [\mu_1^b(\lambda, \mathbf{s}) (q_1^b(\mathbf{s}) b_0^h - c_1^h(\lambda, \mathbf{s}))]\end{aligned}$$

The first order conditions are following: (Note that this maps to the bank problem if they can choose  $\delta_1^d$  freely as a function of the state).

$$\begin{aligned}[c_1^h(\lambda, \mathbf{s})] : \quad & 0 = \lambda u'(c_1^h(\lambda, \mathbf{s})) - (1 - \lambda) u'(c_2^h(\lambda, \mathbf{s})) \frac{\delta_2^b(\mathbf{s})}{q_1^b(\mathbf{s})} - \mu_1^b(\lambda, \mathbf{s}) \\ [m_0^h] : \quad & 0 = -\mu_0 + \mathbb{E} \left[ (1 - \lambda) u'(c_2^h(\lambda, \mathbf{s})) \frac{\delta_2^b(\mathbf{s})}{q_1^b(\mathbf{s})} z_1(\mathbf{s}) \right] \\ [k_0^h] : \quad & 0 = -\mu_0 + \mathbb{E} [(1 - \lambda) u'(c_2^h(\lambda, \mathbf{s})) z_2(\mathbf{s})] \\ [b_0^h] : \quad & 0 = -\mu_0 q_0^b + \mathbb{E} \left[ (1 - \lambda) u'(c_2^h(\lambda, \mathbf{s})) \left( 1 + \frac{\mu_1^b(\lambda, \mathbf{s})}{(1 - \lambda) u'(c_2^h(\lambda, \mathbf{s}))} \frac{q_1^b(\mathbf{s})}{\delta_2^b(\mathbf{s})} \right) \delta_2^b(\mathbf{s}) \right] \\ & = -\mu_0 q_0^b + \mathbb{E} \left[ (1 - \lambda) u'(c_2^h(\lambda, \mathbf{s})) \left( \frac{\lambda u'(c_1^h(\lambda, \mathbf{s}))}{(1 - \lambda) u'(c_2^h(\lambda, \mathbf{s}))} \right) \frac{q_1^b(\mathbf{s})}{\delta_2^b(\mathbf{s})} \delta_2^b(\mathbf{s}) \right] \\ & = -\mu_0 q_0^b + \mathbb{E} \left[ (1 - \lambda) u'(c_2^h(\lambda, \mathbf{s})) \left( 1 - \frac{\mu_1^b(\lambda, \mathbf{s})}{\lambda u'(c_1)} \right)^{-1} \delta_2^b(\mathbf{s}) \right]\end{aligned}$$

where

$$\frac{\delta_2^b(\mathbf{s})}{q_1^b(\mathbf{s})} = \frac{\lambda u'(c_1) - \mu_1^b}{(1 - \lambda) u'(c_2)}$$

Let  $\lambda^*(\mathbf{s})$  denote the  $\lambda$  at which the bond-in-advance constraint binds (if such a  $\lambda^*(\mathbf{s})$  exists). Then, for  $\lambda \geq \lambda^*(\mathbf{s})$ , we have that  $(c_1(\lambda, \mathbf{s}), b_1(\lambda, \mathbf{s}))$  satisfies:

$$\begin{aligned}c_1(\lambda, \mathbf{s}) &= q_1^b(\mathbf{s}) b_0 \\ q_1^b(\mathbf{s}) b_1(\lambda, \mathbf{s}) &= z_1(\mathbf{s}) m_0\end{aligned}$$

And for  $\lambda < \lambda^*(\mathbf{s})$ , we have that  $(c_1(\lambda, \mathbf{s}), b_1(\lambda, \mathbf{s}))$  satisfies:

$$\begin{aligned}\lambda u'(c_1^h(\lambda, \mathbf{s})) &= (1 - \lambda) u' \left( \frac{\delta_2^b(\mathbf{s})}{q_1^b(\mathbf{s})} q_1^b(\mathbf{s}) b_1(\lambda, \mathbf{s}) + z_2(\mathbf{s}) k_0^h - \tau(\mathbf{s}) \right) \frac{\delta_2^b(\mathbf{s})}{q_1^b(\mathbf{s})} \\ q_1^b(\mathbf{s}) b_1(\lambda, \mathbf{s}) &= z_1(\mathbf{s}) m_0 + q_1^b(\mathbf{s}) b_0^h - c_1^h(\lambda, \mathbf{s})\end{aligned}$$

To obtain an explicit expression for  $c_1$  as a function of  $\lambda$  (which makes the solution an order of magnitude easier), we use the CRRA form of  $u$  to find an expression for the  $\lambda$ -specific growth

rate of consumption, then plug that into the life-time budget constraint (which is not  $\lambda$ -specific):

$$\begin{aligned}\frac{c_2(\lambda, \mathbf{s})}{c_1(\lambda, \mathbf{s})} &= \left[ \frac{1-\lambda}{\lambda} R^b(\mathbf{s}) \right]^{1/\gamma} \\ c_1(\lambda, \mathbf{s}) \left( 1 + \frac{c_2/c_1}{R^b(\mathbf{s})} \right) &= z_1(\mathbf{s})m_0 + q_1^b(\mathbf{s})b_0 + \frac{z_2(\mathbf{s})k_0 - \tau(\mathbf{s})}{R^b(\mathbf{s})} \\ \Rightarrow c_1(\lambda, \mathbf{s}) &= \frac{R^b(\mathbf{s}) \left( z_1(\mathbf{s})m_0 + q_1^b(\mathbf{s})b_0 \right) + z_2(\mathbf{s})k_0 - \tau(\mathbf{s})}{R^b(\mathbf{s}) + \left[ \frac{1-\lambda}{\lambda} R^b(\mathbf{s}) \right]^{1/\gamma}}\end{aligned}$$

which can be solved for  $(c_1(\lambda, \mathbf{s}), c_2(\lambda, \mathbf{s}), b_1(\lambda, \mathbf{s}))$ .

The cutoff  $\lambda^*(\mathbf{s})$  is pinned down by the condition:

$$\begin{aligned}\lambda u'(q_1^b(\mathbf{s})b_0) &= (1-\lambda)u' \left( \frac{\delta_2^b(\mathbf{s})}{q_1^h(\mathbf{s})} (z_1(\mathbf{s})m_0) + z_2(\mathbf{s})k_0^h - \tau(\mathbf{s}) \right) \frac{\delta_2^b(\mathbf{s})}{q_1^b(\mathbf{s})} \\ \Rightarrow \lambda^*(\mathbf{s}) &= \frac{u' \left( \frac{\delta_2^b(\mathbf{s})}{q_1^h(\mathbf{s})} (z_1(\mathbf{s})m_0) + z_2(\mathbf{s})k_0^h - \tau(\mathbf{s}) \right) \frac{\delta_2^b(\mathbf{s})}{q_1^b(\mathbf{s})}}{u'(q_1^b(\mathbf{s})b_0) + u' \left( \frac{\delta_2^b(\mathbf{s})}{q_1^h(\mathbf{s})} (z_1(\mathbf{s})m_0) + z_2(\mathbf{s})k_0^h - \tau(\mathbf{s}) \right) \frac{\delta_2^b(\mathbf{s})}{q_1^b(\mathbf{s})}} \\ &= \frac{1}{1 + \frac{u'(q_1^h(\mathbf{s})b_0)}{u' \left( \frac{\delta_2^b(\mathbf{s})}{q_1^h(\mathbf{s})} (z_1(\mathbf{s})m_0) + z_2(\mathbf{s})k_0^h - \tau(\mathbf{s}) \right) \frac{\delta_2^b(\mathbf{s})}{q_1^b(\mathbf{s})}}}\end{aligned}$$

### C.1.3 Market Clearing

The market clearing conditions at  $t = 0$  are:

$$b_0 = B_0, \quad q_0^b b_0 + m_0 + k_0 = 1$$

The market clearing conditions at  $t = 1$  are:

$$\int b_1(\lambda, \mathbf{s}) dF(\lambda) = B_0, \quad \int c_1(\lambda, \mathbf{s}) dF(\lambda) = z_1(\mathbf{s})m_0$$

The market clearing condition at  $t = 2$  is:

$$\int c_2(\lambda, \mathbf{s}) dF(\lambda) = z_2(\mathbf{s})k_0$$

#### C.1.4 Equilibrium Characterization

We start by considering market clearing at  $t = 1$ . We have that the bond and goods market clearing conditions must satisfy:

$$\begin{aligned}\int^{\lambda^*(s)} b_1(\lambda, s) dF(\lambda) + \frac{z_1(s)m_0}{q_1^b(s)}(1 - F(\lambda^*)) &= B_0 \\ \int^{\lambda^*(s)} c_1(\lambda, s) dF(\lambda) + q_1^b(s)b_0(1 - F(\lambda^*)) &= z_1(s)m_0\end{aligned}$$

In summary, the 11 equilibrium variables  $(b_0, m_0, k_0, c_1(\lambda, s), c_2(\lambda, s), b_1(\lambda, s), \lambda^*(s), \mu_0, \mu_1^b(\lambda, s), q_0^b, q_1^b(s))$  satisfy: [\[JP: I feel like I doubled counted here.\]](#)

$$\begin{aligned}
0 &= \lambda u'(c_1^h(\lambda, s)) - (1 - \lambda) u'(c_2^h(\lambda, s)) \frac{\delta_2^b(s)}{q_1^b(s)} - \mu_1^b(\lambda, s) \\
\mu_0 &= \mathbb{E} \left[ (1 - \lambda) u'(c_2^h(\lambda, s)) \frac{\delta_2^b(s)}{q_1^b(s)} z_1(s) \right] \\
\mu_0 &= \mathbb{E} \left[ (1 - \lambda) u'(c_2^h(\lambda, s)) z_2(s) \right] \\
\mu_0 q_0^b &= \mathbb{E} \left[ (1 - \lambda) u'(c_2^h(\lambda, s)) \left( 1 + \frac{\mu_1^b(\lambda, s)}{(1 - \lambda) u'(c_2^h(\lambda, s))} \frac{q_1^b(s)}{\delta_2^b(s)} \right) \delta_2^b(s) \right] \\
\lambda^*(s) &= \frac{u' \left( \frac{\delta_2^b(s)}{q_1^b(s)} (z_1(s) m_0) + z_2(s) k_0^h - \tau(s) \right) \frac{\delta_2^b(s)}{q_1^b(s)}}{u'(q_1^b(s) b_0) + u' \left( \frac{\delta_2^b(s)}{q_1^b(s)} (z_1(s) m_0) + z_2(s) k_0^h - \tau(s) \right) \frac{\delta_2^b(s)}{q_1^b(s)}} \\
b_0 &= B_0 \\
1 &= q_0^b b_0 + m_0 + k_0 \\
B_0 &= \int^{\lambda^*(s)} b_1(\lambda, s) dF(\lambda) + \frac{z_1(s) m_0}{q_1^b(s)} (1 - F(\lambda^*)) \\
z_1(s) m_0 &= \int^{\lambda^*(s)} c_1(\lambda, s) dF(\lambda) + q_1^b(s) b_0 (1 - F(\lambda^*)) \\
z_2(s) k_0 &= \int c_2(\lambda, s) dF(\lambda) \\
c_2(\lambda, s) &= \frac{\delta_2^b(s)}{q_1^b(s)} (q_1^b(s) b_0^h + z_1(s) m_0 - c_1^h(\lambda, s)) + z_2(s) k_0^h - \delta_2^b(s) B_0
\end{aligned}$$

## C.2 Exogenous Bond Demand: Bond-in-Advance

### C.2.1 Environment

*Setting:* The economy lasts for three periods:  $t \in \{0, 1, 2\}$ . We interpret  $t = 0$  as a primary asset market,  $t = 1$  as a morning market, and  $t = 2$  as the following period. There is one consumption good. There are two production technologies in the economy: one that transforms  $m_0$  goods at time  $t = 0$  to  $z_1(s_1) m_0$  goods at time  $t = 1$  (short-term asset) and another one that transforms  $k_0$  goods at time  $t = 0$  to  $z_2(s_1) k_0$  goods at time  $t = 2$  (capital), where  $s_1$  is the aggregate state that has distribution  $\Pi(s_1)$  and is realized at the beginning of  $t = 1$ .

*Assets and Markets:* We use goods as the numeraire. At  $t = 0$ , the government issues bonds in the primary market at price  $q_0^b$  that pay  $\delta_2^b$  at time  $t = 2$ . At  $t = 1$ , the agents are only able to trade bonds for goods at price  $q_1^b$ . They cannot trade capital.

*Government:* The government ranks allocations according to:

$$\theta G + \mathcal{U}$$

where  $G$  is the provision of public goods by the government and  $\mathcal{U}$  is the aggregate lifetime household utility under equal Pareto weights. Parameter  $\theta$  is interpreted as the relative value of public goods. At  $t = 0$ , the government finances public good provision by issuing  $B_0$  bonds at price  $q_0^b$  leading to the  $t = 0$  budget constraint:

$$G \leq q_0^b B_0$$

At time 2, the government raises taxes  $T_2(\mathbf{s}_1)$  from households at  $t = 2$ , which it uses to repay  $\delta_2^b(\mathbf{s}_1)$  per unit of bonds according to:

$$\delta_2^b(\mathbf{s}_1) B_0 \leq T_2(\mathbf{s}_1)$$

where  $\delta_2^b(\mathbf{s}_1) < 1$  is interpreted as “partial default” or “dilution” when the government decreases the real value of the bond principle. We refer to  $T_2(\mathbf{s}_1)$  as the government “fiscal rule” and treat it as an exogenous outcome of an unmodelled political process. The exogenous  $T_2(\mathbf{s}_1)$  pins down an upper bound on  $B_0$ .

*Household problem:* Agents cannot consume their own goods. Instead, they can only consume goods produced by other agents. All agents rank consumption allocations at  $t = 1$  and  $t = 2$  according to:

$$\lambda u(c_1) + (1 - \lambda)u(c_2).$$

At time 0, households rank allocations according to:

$$\mathcal{U} := \mathbb{E} [\lambda u(c_1^h) + (1 - \lambda)u(c_2^h)],$$

where  $c_t^{h,i}$  denotes consumption of household  $h$  on island  $i$  in period  $t \in \{1, 2\}$  and the expectation is only taken over  $\mathbf{s}$ . Each household is endowed with one unit of good at  $t = 0$  and zero goods in the other periods. All agents have the time 0 budget constraint:

$$q_0^b b_0^h + m_0^h + k_0^h \leq 1$$

where  $b_0^h$ ,  $m_0^h$ , and  $k_0^h$  are household  $h$ 's bond, short asset, and capital holdings. At  $t = 1$ , the



households face the budget constraint and the bond-in-advance constraint:

$$\begin{aligned} c_1^h + q_1^h b_1^h &\leq q_0^h b_0^h + z_1 m_0 \\ c_1^h &\leq q_1^h b_0^h \end{aligned}$$

At  $t = 2$ , the households face the budget constraint:

$$c_2^{h,i} \leq \delta_2^b b_1^h + z_2 k_0^h - \tau(s)$$

### C.2.2 Household Problem

Taking prices  $(q_0^b, q_1^b)$  as given, the household solves:

$$\begin{aligned} \max_{b_0^h, m_0^h, k_0^h, b_1^h, c} \quad & \mathbb{E}[\lambda u(c_1^h) + (1 - \lambda)u(c_2^h)] \quad s.t. \\ q_0^b b_0^h + m_0^h + k_0^h &\leq 1 \\ c_1^h &\leq q_1^b b_0^h \\ c_2^{h,i} &\leq \frac{\delta_2^b}{q_1^h} (q_1^b b_0^h + z m_0 - c_1^h) + z_2 k_0^h - \tau(s) \\ b_0^h, m_0^h, k_0^h, b_1^h &\geq 0 \end{aligned}$$

Lagrangian is (leaving the short selling constraints implicit):

$$\begin{aligned} \mathcal{L} = \mathbb{E} \bigg[ & \lambda u(c_1^h(s)) + (1 - \lambda)u \left( \frac{\delta_2^b(s)}{q_1^h(s)} (q_1^b(s) b_0^h + z_1(s) m_0 - c_1^h(s)) + z_2(s) k_0^h - \tau(s) \right) \bigg] \\ & + \mu_0 (1 - q_0^b b_0^h - m_0^h - k_0^h) \\ & + \mathbb{E} [\mu_1^b(s) (q_1^b(s) b_0^h - c_1^h(s))] \end{aligned}$$

The first order conditions are following: (Note that this maps to the bank problem if they can choose  $\delta_1^d$  freely as a function of the state).

$$\begin{aligned} [c_1^h(s)] : \quad & 0 = \lambda u'(c_1^h(s)) - (1 - \lambda)u'(c_2^h(s)) \frac{\delta_2^b(s)}{q_1^h(s)} - \mu_1^b(s) \\ [m_0^h] : \quad & 0 = -\mu_0 + \mathbb{E} \left[ (1 - \lambda)u'(c_2^h(s)) \frac{\delta_2^b(s)}{q_1^h(s)} z_1(s) \right] \\ [k_0^h] : \quad & 0 = -\mu_0 + \mathbb{E} [(1 - \lambda)u'(c_2^h(s)) z_2(s)] \\ [b_0^h] : \quad & 0 = -\mu_0 q_0^b + \mathbb{E} \left[ (1 - \lambda)u'(c_2^h(s)) \left( 1 + \frac{\mu_1^b(s)}{(1 - \lambda)u'(c_2^h(s))} \frac{q_1^b(s)}{\delta_2^b(s)} \right) \delta_2^b(s) \right] \end{aligned}$$

If the bond-in-advance constraint binds, then we have that  $(c_1(s), b_1)$  satisfies:

$$\begin{aligned} c_1(s) &= q_1^b(s)b_0 \\ q_1^b(s)b_1 &= z_1(s)m_0 \end{aligned}$$

If the bond-in-advance constraint doesn't bind, then we have that  $(c_1(s), b_1)$  satisfies:

$$\begin{aligned} \lambda u'(c_1^h(s)) &= (1 - \lambda)u' \left( \frac{\delta_2^b(s)}{q_1^h(s)} (q_1^b(s)b_0^h + z_1(s)m_0 - c_1^h(s)) + z_2(s)k_0^h - \tau(s) \right) \frac{\delta_2^b(s)}{q_1^b(s)} \\ q_1^b(s)b_1 &= q_1^b(s)b_0^h + z_1(s)m_0 - c_1^h(s) \end{aligned}$$

### C.2.3 Market Clearing

The market clearing conditions at  $t = 0$  are:

$$b_0 = B_0, \quad q_0^b b_0 + m_0 + k_0 = 1$$

The market clearing conditions at  $t = 1$  are:

$$b_1(s) = B_0, \quad c_1(s) = z_1(s)m_0$$

The market clearing condition at  $t = 2$  is:

$$c_2(s) = z_2(s)k_0$$

### C.2.4 Equilibrium Characterization

We start by considering market clearing at  $t = 1$ . Assuming that the bond-in-advance constraint binds, then the bond market and goods market must satisfy:

$$q_1^b(s) = \frac{z_1(s)m_0}{B_0}, \quad c_1(s) = z_1(s)m_0$$

In summary, the 9 equilibrium variables  $(b_0, m_0, k_0, c_1(s), c_2(s), \mu_0, \mu_1^b(s), q_0^b, q_1^b(s))$  satisfy (if the bond-in-advance constraint binds):

$$\begin{aligned}
0 &= \lambda u'(c_1(s)) - (1 - \lambda) u'(c_2(s)) \frac{\delta_2^b(s)}{q_1^b(s)} - \mu_1^b(s) \\
\mu_0 &= \mathbb{E} \left[ (1 - \lambda) u'(c_2(s)) \frac{\delta_2^b(s)}{q_1^b(s)} z_1(s) \right] \\
\mu_0 &= \mathbb{E} [(1 - \lambda) u'(c_2(s)) z_2(s)] \\
\mu_0 q_0^b &= \mathbb{E} \left[ (1 - \lambda) u'(c_2(s)) \left( 1 + \frac{\mu_1^b(s)}{(1 - \lambda) u'(c_2(s))} \frac{q_1^b(s)}{\delta_2^b(s)} \right) \delta_2^b(s) \right] \\
b_0 &= B_0 \\
1 &= q_0^b b_0 + m_0 + k_0 \\
q_1^b(s) &= \frac{z_1(s) m_0}{B_0} \\
c_1(s) &= z_1(s) m_0 \\
c_2(s) &= z_2(s) k_0
\end{aligned}$$

Combining the equations we get that:

$$\begin{aligned}
1 + \frac{\mu_1^b(s)}{(1 - \lambda) u'(c_2(s))} &= 1 + \frac{\lambda u'(c_1(s)) - (1 - \lambda) u'(c_2(s)) \frac{\delta_2^b(s)}{q_1^b(s)}}{(1 - \lambda) u'(c_2(s)) \frac{\delta_2^b(s)}{q_1^b(s)}} \\
&= \frac{\lambda u'(c_1(s))}{(1 - \lambda) u'(c_2(s)) \frac{\delta_2^b(s)}{q_1^b(s)}} \\
&= \frac{\lambda u'(c_1(s))}{(1 - \lambda) u'(c_2(s)) \frac{\delta_2^b(s) B_0}{z_1(s) m_0}}
\end{aligned}$$

Thus, the bond market Euler equation becomes:

$$\begin{aligned}
\mu_0 q_0^b &= \mathbb{E} \left[ (1 - \lambda) u'(c_2(s)) \left( 1 + \frac{\mu_1^b(s)}{(1 - \lambda) u'(c_2(s))} \frac{q_1^b(s)}{\delta_2^b(s)} \right) \delta_2^b(s) \right] \\
&= \mathbb{E} \left[ (1 - \lambda) u'(c_2(s)) \frac{\lambda u'(c_1(s))}{(1 - \lambda) u'(c_2(s)) \frac{B_0}{z_1(s) m_0}} \right] \\
&= \mathbb{E} \left[ \lambda u'(c_1(s)) \frac{z_1(s) m_0}{B_0} \right] \\
&= \mathbb{E} \left[ \lambda u'(z_1(s) m_0) \frac{z_1(s) m_0}{B_0} \right]
\end{aligned}$$

which seems to be independent of  $\delta_2^b(s)$  [JP: Did I make a mistake here?]

### C.2.5 Convenience yields

Define the price of the “synthetic asset” by:

$$\mu_0 \tilde{q}_0^b := \mathbb{E} [(1 - \lambda) u'(c_2(\mathbf{s})) \delta_2^b(\mathbf{s})]$$

So, I think the convenience yield is:

$$\log(q_0^b) - \log(\tilde{q}_0^b) = \log \left( \mathbb{E} \left[ \lambda u'(z_1(\mathbf{s}) m_0) \frac{z_1(\mathbf{s}) m_0}{B_0} \right] \right) - \log (\mathbb{E} [(1 - \lambda) u'(c_2(\mathbf{s})) \delta_2^b(\mathbf{s})])$$

which is decreasing in  $\delta_2^b$  because the bond-in-advance constraint makes ends up making  $q_0^b$  independent of  $\delta_2^b$ .

### C.3 Bond in Utility

Consider the bond-in-utility model where the household solves:

$$\begin{aligned} \max_{b_0, k_0, c_1} \quad & \left\{ \nu(q_0^b b_0) + \beta \mathbb{E}[u(c_1)] \right\} \quad s.t. \\ & q_0^b b_0 + k_0 \leq 1 \\ & c_1 \leq z_1(\mathbf{s}) k_0 + \delta_1^b(\mathbf{s}) b_0 \end{aligned}$$

Let  $\mu_0$  be the Lagrange multiplier on time  $t = 0$  budget constraint. The first order conditions are:

$$\begin{aligned} q_0^b (\mu_0 - \nu'(q_0^b b_0)) &= \mathbb{E}[u'(z_1(\mathbf{s}) k_0 + \delta_1^b(\mathbf{s}) b_0) \delta_1^b(\mathbf{s})] \\ \mu_0 &= \mathbb{E}[u'(z_1(\mathbf{s}) k_0 + \delta_1^b(\mathbf{s}) b_0) z_1(\mathbf{s})] \end{aligned}$$

Define the price of the synthetic asset by:

$$\mu_0 \tilde{q}_0^b = \mathbb{E}[u'(z_1(\mathbf{s}) k_0 + \delta_1^b(\mathbf{s}) b_0) \delta_1^b(\mathbf{s})]$$

So the convenience yield is:

$$\log(q_0^b) - \log(\tilde{q}_0^b) \approx \frac{\nu'(q_0^b b_0)}{\mu_0}$$

Again, since  $q_0^b$  is increasing in  $\delta_1^b(\mathbf{s})$  and  $\nu'(\cdot)$  is a decreasing function, a decrease in  $\delta_1^b(\mathbf{s})$  leads to an increase in  $\nu'(q_0^b b_0)$ .

## C.4 Comparing Different Convenience Yield Definitions (APPENDIX)

In this subsection we consider the “convenience-yield” that the government earns in the government debt market at  $t = 0$ . [BS: Define convenience yield.] We do this by considering a collection of special cases.

PLAN:

- Standard material on safe asset and covariance terms [JP: (i) How does the regulation create a safe asset and (ii) how does this generate a convenience yield. For the first point, the makes the asset special in the interbank market. There are missing morning markets. The government could fix but doesn't because it can exploit this to generate a safe asset. But this makes the problem worse.]
- Comparison with a canonical BIU formulation, map marginal utility to multiplier on regulatory constraint for the case of frictionless banking ( $\Omega = 1$ )
- Potentially: KVJ plots in the two setups

**A special case:** Equity injection in the morning is possible. In this case, we have  $\mu_1^f(\mathbf{s}) = 1$ . There is no default,  $\lambda^* = 1$ , so  $\Omega = 1$ . In addition,  $\left(1 - \underline{\mu}_1^b(\mathbf{s})\right)^{-1} \delta_2^b(\mathbf{s}) = q_1^b(\mathbf{s})$ . If the shadow price is  $q_0^{sfs} = \mathbb{E}[\xi]$ , then the the government debt Euler equation becomes

$$q_0^b = q_0^s q_1^b$$

so the convenience yield is:

$$q_0^b (q_0^s)^{-1} - 1 = q_1^b - 1$$

while without equity raising in the morning, we would have

$$q_0^b (q_0^s)^{-1} - 1 = q_1^b \left( \mathbb{E}[\Omega R^k] + \mathbb{C}[(q_0^s)^{-1} \xi, \Omega R^k] \right) - 1$$

Interestingly,  $\delta_2^b(\mathbf{s})$  does not directly appear. However,  $\lambda^*$  and therefore  $\Omega$  will be affected. See the government default subsection.

(I). **BIU:** There are various cases that we can consider.

- BIU (non-separable case): KVJ (2012), KL (2022), Lenel and Kekre (2024) [JP: I think that  $q_1^b = \delta_1^b$ ?] [BS: Yes, of course. Fixed it] [JP: What is  $B_1$  referring to? I think it should be in the  $t = 1$  budget constraint, right?] [JP: I think the better comparison to our model is probably to just have the bond in the utility at  $t = 0$ ?]

$$\begin{aligned} \max \quad & u(C_0, q_0^b B_0/P_0) + \beta \mathbb{E}[u(C_1, \delta_1^b B_1/P_1)] \\ & P_0 C_0 + q_0^b B_0 + K_0 \leq Y_0 + \delta_0^b B_{-1} + (1 + r_0)K_{-1} \\ & P_1 C_1 \leq Y_1 + \delta_1^b B_0 + (1 + r_1)K_0 \end{aligned}$$

which leads to

$$\begin{aligned} 0 &= u_c(0) - \mu_0 P_0 \\ 0 &= u_b(0) q_0^b / P_0 - q_0^b \mu_0 + \mathbb{E}[\beta \mu_1 \delta_1^b] \end{aligned}$$

where  $u_c(0) = u_c(C_0, q_0^b B_0/P_0)$  so the Euler equation is

$$q_0^b = \mathbb{E} \left[ \left( \beta \frac{u_c(C_1, B_1/P_1)}{u_c(C_0, B_0/P_0)} \right) \left( \frac{P_0}{P_1} \right) \left( \frac{\delta_1^b}{1 - \frac{u_b(0)}{u_c(0)}} \right) \right]$$

Lenel and Kekre (2024) use  $u(C_0, q_0^b B_0/P_0) = (C_0 \Omega(q_0^b B_0/P_0))^{1 - \frac{1}{\psi}}$ , which leads to

$$\frac{u_b(0)}{u_c(0)} = \frac{C_0 \Omega'_0}{\Omega_0} =: \omega_0$$

which is their convenience yield. To see this, we note that  $\omega_0$  is known at  $t = 0$ , so we can pull it out from the expectation operator:

$$q_0^b(1 - \omega_0) = \mathbb{E}[\xi \delta_1^b] =: \tilde{q}_0^b$$

where  $\tilde{q}_0^b$  is the price of a synthetic asset that pays the same cash-flow as government debt (but provides no extra utility service). We can write

$$\omega_0 = \frac{q_0^b - \tilde{q}_0^b}{q_0^b} > 0$$

Lenel and Kekre consider government debt as completely safe, so  $\delta_1^b = 1$ . In this case it makes sense to define  $1 + i_0 := (q_0^b)^{-1}$  and  $1 + i_0^s := \mathbb{E}[\xi]$  and given that the asset has no risk-premium, we can write the above expression as

$$\omega_0 = \frac{i_0^s - i_0}{1 + i_0^s}$$

The old school monetary literature and KVJ motivate “demand regressions” by using the logged version

$$\log \left( \frac{i_0^s - i_1}{1 + i_0^s} \right) = \log C_0 + \log \left( \frac{\Omega'(B_0/P_0)}{\Omega(B_0/P_0)} \right) = \log C_0 + \log \left( \omega_t^d - \frac{1}{\epsilon^d} \frac{B_0}{P_0} \right)$$

This is the odd  $\Omega$  function by Lenel and Kekre. The more standard form would be

$$\log \left( \frac{i_0^s - i_1}{1 + i_0^s} \right) = \log C_0 - \frac{1}{\eta} \log \left( \frac{B_0}{P_0} \right)$$

- BIA constraint with Svensson timing as in my JMP: let  $v_0 := \frac{P_0 C_0}{\delta_0^b B_{-1}}$ , then

$$\begin{aligned} \max \quad & u(C_0) + \beta \mathbb{E}[u(C_1)] \\ P_0 C_0 + \nu(v_0) \delta_0^b B_{-1} + q_0^b B_0 + K_0 & \leq Y_0 + \delta_0^b B_{-1} + (1 + r_0) K_{-1} \\ P_1 C_1 + \nu(v_1) \delta_1^b B_0 & \leq Y_1 + \delta_1^b B_0 + (1 + r_1) K_0 \end{aligned}$$

which leads to

$$\begin{aligned} 0 &= u_c(0) - \mu_0 P_0 (1 + \nu'(v_0)) \\ 0 &= -q_0^b \mu_0 + \mathbb{E}[\delta_1^b \beta \mu_1] - \mathbb{E}[\beta \mu_1 (\nu(v_1) \delta_0^b - \nu'(v_1) \delta_0^b v_1)] \end{aligned}$$

so the Euler equation is

$$\begin{aligned} q_0^b &= \mathbb{E} \left[ \left( \beta \frac{\mu_1}{\mu_0} \right) (1 + \nu'(v_1) v_1 - \nu(v_1)) \delta_1^b \right] \\ &= \mathbb{E} \left[ \left( \beta \frac{u'(C_1)}{u'(C_0)} \frac{(1 + \nu'(v_0))}{(1 + \nu'(v_1))} \right) \left( \frac{P_0}{P_1} \right) (1 + \nu'(v_1) v_1 - \nu(v_1)) \delta_1^b \right] \end{aligned}$$

This can be written as

$$\begin{aligned} q_0^b &= \mathbb{E}[\xi_1 \delta_1^b] \mathbb{E}[(1 + \nu'(v_1) v_1 - \nu(v_1))] + \mathbb{C}[\xi_1 \delta_1^b, (1 + \nu'(v_1) v_1 - \nu(v_1))] \\ \frac{q_0^b - \tilde{q}_0^b}{\tilde{q}_0^b} &= \mathbb{E}[(\nu'(v_1) v_1 - \nu(v_1))] + \mathbb{C} \left[ \frac{\xi_1 \delta_1^b}{\mathbb{E}[\xi_1 \delta_1^b]}, (\nu'(v_1) v_1 - \nu(v_1)) \right] \end{aligned}$$

so in this case the convenience yield has a component that arises from the covariance between the non-pecuniary return and the SDF. Using the functional form  $\nu(v) = \frac{\bar{\nu}}{\eta} v^\eta$ , we can write the RHS as

$$\left( \frac{(\eta - 1)\bar{\nu}}{\eta} \right) \left\{ \mathbb{E} \left[ \left( \frac{C_0}{C_1} \frac{P_0}{P_1} \delta_1^b \right)^{-\eta} \right] + \mathbb{C} \left[ \frac{\xi_1 \delta_1^b}{\mathbb{E}[\xi_1 \delta_1^b]}, \left( \frac{C_0}{C_1} \frac{P_0}{P_1} \delta_1^b \right)^{-\eta} \right] \right\} C_0^{-\eta} \left( \frac{B_0}{P_0} \right)^{-\eta}$$

which suggests that the covariance term is negative. As  $\delta_1^b$  starts falling in bad times (when

$\xi_1$  is high) the product  $\xi_1 \delta_1^b$  is less volatile (and has a lower mean) than  $\xi_1$  alone, suggesting that the first term in the curly braces increases?! In other words, defaulting on the asset that appears on the RHS of the CIA constraint makes the constraint bind more increasing the value of holding the asset. [JP: Default makes the asset “scarce” in the bad states of the world and so acts as a backdoor way of reducing the real supply in the bad times to prop up prices without having to raise taxes. This has a connection to your job market paper.]

**Aside:** In the monetary context, as inflation erodes the value of money, the nominal interest rate (the convenience yield on fiat money) increases through the Fisher effect. Higher nominal interest rate is consistent with lower demanded real balances (we are moving along the money demand function). However, there is a seignorage-Laffer-curve, i.e. for a given money demand function, seignorage first rises than falls with higher inflation? Let me remind myself of the argument. . . Seignorage is

$$\underbrace{\frac{M_t - M_{t-1}}{P_t}}_{\text{seignorage}} = \frac{M_t}{P_t} - \frac{M_{t-1}}{P_{t-1}} \frac{P_{t-1} - P_t + P_t}{P_t} = \frac{M_t}{P_t} - \frac{M_{t-1}}{P_{t-1}} + \underbrace{\frac{P_t - P_{t-1}}{P_t} \frac{M_{t-1}}{P_{t-1}}}_{\text{inflation tax}}$$

or

$$\frac{M_t - M_{t-1}}{P_t} = \underbrace{\frac{M_t - M_{t-1}}{M_t}}_{\text{money growth}} \frac{M_t}{P_t} (i_t)$$

As the money growth increases, inflation is increasing the nominal interest rate (convenience yield on money) thereby reducing the demanded value of real balances. We are increasing the inflation tax, but the tax base is shrinking, hence total revenue has a hump.

With long-term government debt, the real revenue from debt issuance is

$$q_t^b (b_t - (1 - \zeta)b_{t-1}) = \frac{(b_t - (1 - \zeta)b_{t-1})}{b_t} (q_t^b b_t) (i_t^s - i_t)$$

increasing debt  $b_t$  can both decrease and increase the convenience yield depending on the elasticity of  $q_t^b$ .

- Debt is on-demand asset in the morning (closest to our setup):

$$q_0^b = \mathbb{E} \left[ \xi(\lambda) (1 + \nu(\lambda)) \delta_2^b \right]$$

Making it on-demand in the morning is costly in our setup (because banks/government must come up with goods to give them to the household, when  $\nu$  is from utility, it isn't).



## D Data Sources

We combined existing historical databases with transcription from the digital archives of newspapers and government reports. Before 1884, we take bond data from Global Financial Data (GFD). From 1884 to 1940, we collect digitize and organize data from The New York Times, the Commercial & Financial Chronicle, Merchant’s Magazine, and [Macauley et al. \(1938\)](#). We use the risk classifications from [Macauley et al. \(1938\)](#) to create a collection of high-grade corporate bonds.

## E Historical Time Line

The text references many changes to monetary and financial regulation. In this section, we collect those events into a historical timeline, which is shown in table 1. The time line is broken up into a collection a collection of banking “eras”. The first era is from 1791-1836, during which the First and Second Banks of the US operated alongside state banks. The second era is from 1837-1962, during which state banks could automatically gain bank charters without a congressional review process, often referred to as the “free banking” era. The third era is from 1863-1913, during which the federal government chartered national banks that issued bank notes backed by US federal government debt. The fourth era is from 1913-1933, during which the Federal Reserve Bank was introduced to act as lender-of-last resort to the banking sector. The fifth era is from 1934-1980, during which the New Deal financial regulations were in place. The sixth era is from 1980s-2009, during which the New Deal financial regulations were gradually unwound. Finally, there is the era from 2010 to the present day, during which the Dodd-Frank Act another financial crisis legislation are in place.

Table 1 Time Line of Monetary and Financial Events

1791	• Congress charters the First Bank of the US. The bank is privately owned. It operates as a commercial bank but also has the special privileges of acting as banker for the federal government (storing tax revenue and making loans) and being able to operate across states. It shares responsibility with state banks for bank note issuance. It influences state bank money and credit issuance by setting the rate at which it redeems state notes collected as tax revenue into gold.
1792	• Coinage Act of 1792. Authorizes the US to issue a new currency, the US gold dollar.
1811	• Charter of the First Bank of the US expires and is not renewed.
1812-5	• War of 1812. Convertibility to bank notes to gold is suspended. Government issues Treasury Notes to finance the war.

1816	• Congress charters the Second Bank of the U.S.
1819	• Panic of 1819. Cotton prices fall, farms go bankrupt, and banks fail.
1832	• Jackson vetoes bill to recharter Second Bank.
1833	• Jackson removes federal deposits from Second Bank of the US
1834	• Coinage Act of 1834. Changes the ratio of silver to gold from 15:1 to 16:1.
1836	• Charter of the Second Bank of the US expires and is not renewed. The Second Bank becomes a private corporation.
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1837	• “Free Banking” Era begins. Michigan Act allows the automatic chartering of banks (without requiring explicit approval from state legislature) that issue bank notes backed by specie (gold and silver coins). Over the next few years, other states pass similar laws.
1837	• Panic of 1837. Sharp decrease in real estate prices leads to large bank losses. In New York, every bank suspends payment in gold and silver coinage. Many banks fail.
1857	• Coinage Act of 1857. Foreign coins can longer be legal tender.
1857	• Panic of 1857. Railroad company stocks drop sharply. Ohio Life Insurance and Trust company fails, which prompts a collapse in stock prices and widespread failures across mercantile firms.
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1861-5	• Civil War.
1862	• Legal Tender Act. Authorizes the federal government to use nonconvertible greenback paper dollars to pay its bills.
1863-4	• The National Bank Acts. The National Currency Act (1863) and The National Bank Act (1864) establish a system of nationally chartered banks and the Office of the Comptroller of the Currency. National banks can issue national bank notes up to 90% of the minimum of par and market value of qualifying US federal bonds. Limit on aggregate national bank note issuance is \$300 million. Banks must pay a 1% annual tax per on outstanding national bank notes backed by US federal bonds. State banks must start paying a 2% annual tax on state bank notes.
1865-6	• Additional National Bank Acts. State banks must start paying a 10% annual tax on state bank notes.
1870	• Limit on aggregate national bank note issuance increases to \$354 million.

1873	Bank panic of 1873. Widespread failure of railroad firms leads to stock market crash and bank failures. Jay Cooke and Company goes bankrupt.
1875	Congress repeals limit on aggregate national bank note issuance.
1879	US Treasury starts to promise to convert greenbacks to dollars one-for-one.
1893	Bank panic. A combination of falling commodity prices, oversupply of silver, and a fall in US Treasury gold reserves prompted a run on bank deposits.
1896	Cross of Gold Speech. Democratic presidential candidate William Jennings Bryan gives a speech in favor of allowing unlimited coinage of silver into money demand (“free silver”).
1900	Tax on national bank notes backed by US federal bonds paying coupons less than or equal to 2% is reduced to 0.5% per annum.
1900	Gold Standard Act. The gold dollar becomes the standard unit of account (further restricting the possibility of “free silver”).
1907	Panic of 1907. The Knickerbocker Trust Company collapses prompting a bank run. J.P. Morgan organizes New York bankers to provide liquidity to shore up the banking system.
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1913	Federal Reserve Act. Establishment of the Federal Reserve Bank to act as a reserve money creator of last resort during financial panics.
1914-8	World War I.
1917	2nd Liberty Loan Act establishes a \$15 billion aggregate limit on the amount of government bonds issued.
1929	Stock market crash and start of the Great Depression.
1929	US issues first Treasury Bill.
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1933	Banking Act (“Glass-Steagall Act”). Establishes the Federal Deposit Insurance Corporation (FDIC). Separates commercial and investment banking. Introduces cap on deposit interest rate (“Regulation Q”).
1933	President Roosevelt issues an Executive Order requiring people and businesses to sell their gold to the government at \$20.67 per ounce.
1934	Gold Reserve Act.
1934	National Housing Act. Establishes the Federal Savings and Loan Insurance Corporation (FSLIC).
1935	The last national bank notes are replaced by Federal Reserve notes.

1938	Amendment to the National Housing Act established the Federal National Mortgage Association (FNMA), commonly known as Fannie Mae.
1939-45	World War II.
1942	The Treasury and Federal Reserve agree to fix the yield curve on Treasury securities.
1944	Bretton Woods Agreement.
1951	Treasury-Fed Accord ends the fixed yield curve on Treasury securities and establishes the Fed's policy independence from fiscal concerns.
1968	Housing and Urban Development Act of 1968. Creates the Government National Mortgage Association (GNMA), commonly known as Ginnie Mae.
1966	Fed applies Regulation Q to impose deposit rate ceiling for the first time.
1971	US effectively terminates the Bretton Woods system by ending the convertibility of the US dollar to gold.
1977	Congress issues the Fed with the dual mandate to "promote effectively the goals of maximum employment, stable prices, and moderate long term interest rates".
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1980	Depository Institutions Deregulation and Monetary Control Act of 1980 starts to phase out Regulation Q.
1986-1989	Savings and loan crisis.
1994	Riegle-Neal Interstate Banking and Branching Efficiency Act. Allows banks to operate across states.
1999	Gramm-Leach-Bliley Act. Repeals provisions of the Glass-Steagall Act that prohibited a bank holding company from owning other financial companies.
2007-9	Great Financial Crisis.
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2010	Dodd-Frank Wall Street Reform and Consumer Protection Act.