

Convenience Yields and Financial Repression*

Preliminary Draft

Latest version: https://jepayne.github.io/files/PS_CYRepress.pdf

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May 31, 2024

Abstract

US federal debt plays a special role in the US economy and so gives the US government a funding advantage, often summarized by the “convenience yield” on US debt. Why? One reason is that government regulation (and/or repression) of the financial sector influences asset pricing and helps make long term US federal debt a “safe-asset”. We study the mechanics and macroeconomic trade-offs involved with generating a convenience yield through restrictions on the financial sector. We then test our theory using new historical data on US convenience yields going back to 1860.

JEL CLASSIFICATION: E31, E43, G12, N21, N41

KEY WORDS: Convenience Yields, Government Debt Capacity, Ramsey Problems, Optimal Fiscal Policy.

*We thank Clemens Lehner for outstanding research assistance. We are also very grateful to the comments and discussion from Fernando Alvarez, Mark Aguiar, Jaroslav Borovička, Anmol Bhandari, Markus Brunnermeier, Lars Hansen, Greg Kaplan, Arvind Krishnamurthy, Moritz Lenel, Carolin Pflueger, Thomas J. Sargent, and Annette Vissing-Jorgensen. We are also thankful for the comments from participants in the Stanford SITE Conference of Government Financing, the finance seminar at the University of Maryland, the Blue Collar Working Group at the University of Chicago, and the Money and Banking Seminar at the University of Chicago.

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1 Introduction

Many researchers have documented that US federal debt plays a special role in the US economy and so gives the US government a funding advantage, often summarized by the “convenience yield”.¹ Macro-finance models have frequently treated this as an immutable feature of the economic environment and encoded the “benefits” of holding US debt into agent preferences or the market structure. This means the government can easily “exploit” the convenience yield to increase spending. By contrast, historical studies suggest that the convenience yield emerged as part of a complicated, long-term government program to increase its borrowing capacity. Financial regulation and/or repression have been key tools in this process, particularly during crises when the government has needed to raise funding quickly. When viewed in this way, generating and exploiting a convenience yield imposes far reaching impacts on the economy. It links the stability of the financial sector to the stability of the government budget constraint. It distorts the portfolio of the financial sector, potentially increasing default and crowding out private liquidity creation and productive investment. In this paper, we study the mechanics and trade-offs involved with creating financial sector demand for government debt and relate our analysis to historical eras.

We start with an illustrative three period model, in which the banking sector is risky and, absent regulation, there is no special role for government debt. The economy is populated by households who need bank deposits to be able to consume in the middle period. Banks issue on-demand deposits and equity to households and invest in short assets, capital, and government bonds. In this sense, banks provide both liquidity and intermediation services to households. In the middle period, banks get idiosyncratic deposit withdrawal shocks, which potentially cause them to default because their resource-drawing capacity is constrained and the inter-bank asset markets are characterized by “fire-sale pricing”. The combination of households’ need for deposits and the possibility of costly default are the “frictions” in the economy that break [Modigliani and Miller \(1958\)](#) by driving a wedge between the stochastic discount factors of the household and banks. The government in our model cares about spending and household welfare but faces a constraint that taxation is determined by an exogenous political process. Instead, the government can place restrictions on the portfolios of the banks that potentially increase the price of government debt and expand their spending. We focus on restrictions that require the banks to maintain a particular ratio of weighted average assets to deposits. We interpret equal weighting on government debt and capital to be neutral regulation and a higher weighting on government debt to be financial “repression”.

We characterize how repression can generate a convenience yield on government debt both directly through forced portfolio choice and also indirectly by making government debt an endogenously “safe-asset” in the economy. We show that the government can choose constraints on holdings of government debt that bind more for banks in the bad state of the world and so create “captive demand”. This can lead to an appreciation of the price of government debt in the secondary market in bad states. Ultimately, this makes government debt a good “hedge” against both aggregate shocks and idiosyncratic withdrawal risk, which also inflates the price of government debt in the primary

¹The convenience yield is often defined conceptually as the difference between the yield on US treasuries and the inverse of the expectation of the private sector stochastic discount factor. It is often measured as the difference between the yield on US treasuries and low risk corporate bonds.

market because banks voluntarily hold it to hedge their default risk. We interpret the inflated debt price as an embedded “convenience yield”, defined to be the difference between the yield on government debt and the inverse of the expectation of the undistorted stochastic discount factor for the economy. In our model, the convenience yield decomposes into two components: a term reflecting the direct effect of regulatory demand and a second term reflecting the hedging role of government debt. The tractability of our model allows us to characterize how these terms depend explicitly on both regulatory parameters and fiscal policy.

We use our model to show that government fiscal irresponsibility erodes the convenience yield, where we interpret fiscal irresponsibility to mean the (explicit or implicit) default on government debt in bad times. Once government debt carries default risk, it is important to consider a “risk-adjusted” convenience yield that subtracts the risk premium to remove the direct compensation for default risk (following [Jiang et al. \(2020b\)](#)). However, we show that doing so does not make the risk adjusted convenience yield independent of government default. In fact, both components of the convenience yield change. On the one hand, fiscal irresponsibility increases the convenience yield from direct regulatory demand because it makes the real value of government debt scarce and so the regulatory constraint harder to satisfy. On the other hand, it decreases the hedging role of government debt because it erodes the price of government debt in the secondary market in bad times. The second force is amplified by the fact that repression ties the solvency of the banking sector to the solvency of the government and so government default makes government debt a worse hedge at the same time that it makes banks less solvent and more concerned about finding a good hedge. Ultimately, the result of this feedback is that fiscal irresponsibility decreases the risk-adjusted convenience yield (and by extension the convenience yield). This is in sharp contrast to models with bond-in-the-utility or bond-in-advance where the role of government debt is exogenous and its marginal usefulness increases as the market value of government debt declines. This means that as the government starts to default, the risk-adjusted convenience yield increases. Or put another way, in these models the agents get utility from giving resources to the government so when the government starts to default, then they want more government debt. In this sense, the bond-in-utility and bond-in-advance models only capture the first component of the convenience yield in our model and miss the impact of the diminishing hedging role of government debt as the government starts to default. This highlights the importance of starting from a model where government is not exogenously important when we study fiscal policy.

We then use our model to show that generating a higher convenience yield comes at the cost of higher bank default, less bank liquidity creation, and lower investment into capital. The higher rate of bank default appears because financial repression inflates the debt price in the inter-bank market but also decreases the portfolio return for solvent banks and so makes the marginal bank more likely to default. The lower investment rate appears because government borrowing crowds out bank capital creation, as is standard in many macroeconomic models. In this sense, the government faces a trade-off between optimizing their fiscal capacity and having a well functioning financial sector. We characterize this trade-off and show that the optimal government policy requires some degree of repression. This result is different to some recent papers (e.g. [Chari et al. \(2020\)](#)) because we have placed restrictions on the tax process and because the banks in our model play roles as both

liquidity providers and intermediaries.

Our theoretical model provides sharp predictions about how government regulation and fiscal policy influence the convenience yield. In Section 3 we consider whether these predictions are consistent with empirical evidence. We focus on two datasets: a new collection of historical convenience yields in the US covering the period 1860-2022 (from our companion paper [Payne and Szőke \(2024\)](#)) and cross-sectional convenience yields across the Eurozone during the sovereign debt crisis covering the period 2003-2022 (following [Jiang et al. \(2020b\)](#)).

We use the historical US convenience yields to show that financial repression correlates with changes in the relationship between debt issuance and the convenience yield. We construct our historical US convenience yield estimates using a new data set containing prices and cash flow information for a large collection of corporate bonds from 1850-1940. To infer term structures of *yields* on US high grade corporate bonds, we deploy the techniques from [Payne et al. \(2023a\)](#), which use a non-linear state space model with drifting parameters and stochastic volatility. We combine these estimates with existing bond indices for the modern period and estimates of the government yield curve from [Payne et al. \(2023a\)](#) to calculate a term structure of spreads between government and corporate bonds from 1850-2022. We follow [Krishnamurthy and Vissing-Jorgensen \(2012\)](#) and interpret this spread as approximating the “convenience yield” on government debt. We infer a collection of stylized facts that are consistent with our model. First, we find there are low frequency movements in average convenience yields. During the late nineteenth century there was tight financial repression, high convenience yields, and frequent bank defaults, as predicted by our model. The relationship is very different after FDR introduces deposit insurance in the 1930s and the banking sector is stabilized. Second, we find that the relationship between the convenience yield to government debt supply varies with regulation. In the late nineteenth century and the decades following World War II (times with high restrictions on the financial sector and bank balance sheets skewed towards government debt), the convenience yield and debt-to-GDP are almost uncorrelated while in the 1920s, 70s, and 80s (times with less restriction on the financial sector), the correlation is strongly negative, following the pattern pointed out by [Krishnamurthy and Vissing-Jorgensen \(2012\)](#). This is consistent with our model, which suggests that the convenience yield does not reflect a stable exploitable demand function but instead is a reflection of particular regulations and government policies. Similar to the Phillips curve, the relationship breaks down as governments tries to exploit it.

We also study the Eurozone during the sovereign debt crisis because it offers a unique opportunity to “test” a second key prediction of our model: that increases in the likelihood of government default (implicit or explicit) erode the risk-adjusted convenience yield, even when government debt is privileged by regulation. For this period, unlike for the historical US period, we can use data from credit default swaps (CDS) to approximate the default risk premium and calculate risk-adjusted convenience yields (following the approach in [Jiang et al. \(2020b\)](#)). We find that countries facing fiscal crises and high CDS spreads (e.g. Ireland, Portugal, Spain, and Italy) experienced much larger decreases in risk-adjusted convenience yields than countries in relative strong positions (e.g. Germany, Netherlands, Finland, and France). This pattern holds true even after the ECB issued waivers for Greek debt (April 2010), Irish debt (March 2011), and Portuguese debt (July 2011) that

allowed banks to continue to use debt from those countries as collateral at the ECB despite their low credit ratings. So, the decrease in the funding advantage for Ireland, Portugal, Spain, and Italy cannot entirely be explained by risk premia and changes in their collateral value. This is consistent with our model, which suggests that the hedging role of these assets also changed leading to an erosion of the risk-adjusted convenience yield.

1.1 Related Literature

Our paper is part of a large literature studying financial and fiscal policies in non-Ricardian macroeconomic models. A recent branch of this literature studies the “fiscal-sustainability” of government debt taking fiscal policy and private sector pricing kernels as given (e.g. [Jiang et al. \(2022a,b\)](#); [Chen et al. \(2022\)](#)) or deriving private sector pricing kernels from a model with incomplete markets that generate a premium on government debt (e.g. [Reis \(2021b\)](#), [Reis \(2021a\)](#), [Brunnermeier et al. \(2022\)](#)). Our paper studies the feasibility and costs of using financial regulation as a means to “choose” private sector pricing kernels that increase government fiscal capacity. Another branch of this literature studies fiscal-monetary connections (e.g. [Sargent and Wallace \(1981\)](#)) and the “fiscal theory of the price level” papers such as [Leeper \(1991\)](#), [Sims \(1994\)](#), [Woodford \(1994\)](#), [Cochrane \(2023\)](#), [Bianchi et al. \(2023\)](#)). Unlike in these papers, government debt in our model is in part backed by financial regulation by creating captive demand within the financial sector and endogenously making government debt a safe asset. Ultimately, this means that fiscal policy not only backs government debt through the surplus process but also through effectiveness of its endogenous safe asset role.

Our government design problem is also related to a literature studying optimal policy in economies with financial frictions and tax distortions (e.g. [Calvo \(1978\)](#), [Bhandari et al. \(2017b\)](#), [Bhandari et al. \(2017a\)](#), [Chari et al. \(2020\)](#), [Bassetto and Cui \(2021\)](#), [Sims \(2019\)](#), [Brunnermeier et al. \(2022\)](#)). In this paper we take the stand that the government follows a fiscal policy rule governed by unmodeled political constraints but has flexibility in how it wants to restrict the financial sector. We believe this reflects the historical experience of many governments. We use this model to focus on microfounding the “costs” of using financial regulation to increase government fiscal capacity.

We are also part of a long literature attempting to understand how the financial sector and government can create safe assets (e.g. [Holmstrom and Tirole \(1997\)](#), [Holmström and Tirole \(1998\)](#), [Gorton and Ordonez \(2013\)](#), [Gorton \(2017\)](#), [He et al. \(2016\)](#), [He et al. \(2019\)](#)) and the macroeconomic implications of safe asset creation (e.g. [Caballero et al. \(2008\)](#), [Caballero et al. \(2017\)](#), [Caballero and Farhi \(2018\)](#)). Our contribution to this literature is to connect an endogenous safe asset model to a macroeconomy with a government that faces fiscal constraints.

Our historical comparisons extend existing studies on the convenience yield (e.g. [Krishnamurthy and Vissing-Jorgensen \(2012\)](#), [Choi et al. \(2022\)](#)) back to the mid nineteenth century. This makes us part of a literature attempting to connect historical time series for asset prices to government financing costs (e.g. [Payne et al. \(2023b\)](#), [Payne et al. \(2023a\)](#), [Jiang et al. \(2021b\)](#), [Jiang et al. \(2021a\)](#), [Jiang et al. \(2020a\)](#)).

2 A Model of Endogenous Convenience Yields

In this section, we outline a three-period model to illustrate how convenience yields on government debt are connected to the government budget constraint and influence government borrowing capacity. Many macroeconomic models generate convenience yields through an exogenous demand function. This gives the government an immutable financing advantage, which leads to counterfactual asset pricing and misleading implications for fiscal policy makers who can easily exploit the exogenous demand function. By contrast, we study an environment where government debt has no ex-ante advantages and convenience yields emerge endogenously from financial regulations and fiscal policies. We show how financial repression in the secondary asset market creates captive demand for government debt in bad times. This makes government debt a good hedge against aggregate risk, which generates a convenience yield in the primary market. However, this convenience yield can only emerge if the government runs a responsible fiscal policy. Unlike in typical macroeconomic models, implicit or explicit default erodes the convenience yield.

To make these points formally, we need a model with the following key features: (i) there are financial intermediaries that provide a service to households that exposes the intermediaries to risk, (ii) the financial intermediaries faces frictions that distort the way they price risky assets, and (iii) the government forces intermediaries to maintain a minimum level of government debt holdings, which induces crowding into the secondary government debt market in bad times. Our modeling in this section focuses on generating these features in a standard banking environment with default and a meaningful inter-bank market. However, the forces we discuss are more general.

2.1 Environment

Setting: The economy lasts for three periods: $t \in \{0, 1, 2\}$. We interpret $t = 0$ as a primary asset market, $t = 1$ as a morning secondary (inter-bank) asset market, and $t = 2$ as the afternoon settlement of long term assets. There is one consumption good. There is a continuum of islands, $j \in [0, 1]$, each with a unit measure of household members, indexed by $h \in [0, 1]$, and a unit measure of competitive banks, indexed by $i \in [0, 1]$. Each household can only participate in the financial market on their island. There are two production technologies in the economy: one that transforms m_0 goods at time $t = 0$ to $z_1(s)m_0$ goods at time $t = 1$ (short-term asset) and another one that transforms k_0 goods at time $t = 0$ to $z_2(s)k_0$ goods at time $t = 2$ (capital), where s is the aggregate state that has distribution $\Pi(s)$ and is realized at the beginning of $t = 1$.

Assets and Markets: We use goods as the numeraire. All markets are competitive. At $t = 0$, the government issues bonds in the primary market at price q_0^b that pay $\delta_2^b(s)$ at time $t = 2$. At time $t = 1$, banks trade government bonds, at price $q_1^b(s)$, and claims on capital, at price $q_1^k(s)$, in the inter-bank market. We show production and bond payoffs and the timing of shocks graphically in Figure 1.

At $t = 0$, each bank issues demand deposits, d_0 , and equity, e_0 , to the households on their island at prices q_0^d and q_0^e , respectively.² Bank equity pays δ_1^e at time 1 and δ_2^e at time 2 and is not tradable

²The deposit and equity prices are the same on each island because islands are ex-ante identical.

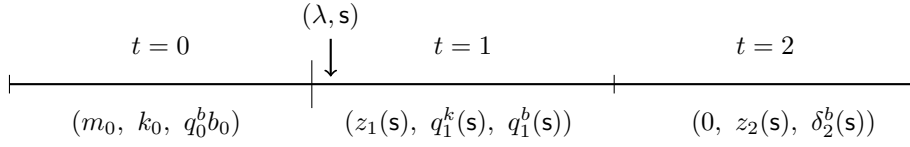


Figure 1: Timing of Payoffs

after $t = 0$. Households can withdraw deposits at time $t \in \{1, 2\}$ for resources δ_t^d , where $\delta_t^d = 1$ if the bank is solvent and $1 > \delta_2^d \geq \delta_1^d$ if the bank is insolvent, where inequality is set so that there is no run.

Government: The government ranks allocations according to:

$$\mathcal{U} + \theta G$$

where \mathcal{U} is the aggregate lifetime household utility under equal Pareto weights and G is the provision of public goods by the government. Parameter θ is interpreted as the relative value of public goods. At $t = 0$, the government finances public good provision by issuing B_0 bonds leading to the $t = 0$ budget constraint:

$$G \leq q_0^b B_0. \quad (2.1)$$

We refer to $q_0^b B_0$ as the government's "fiscal capacity". At time 2, the government raises lump-sum taxes $T_2(s)$ from households at $t = 2$, which it uses to repay $\delta_2^b(s)$ per unit of bonds according to:

$$\delta_2^b(s) B_0 \leq T_2(s), \quad (2.2)$$

where $\delta_2^b(s) < 1$ is interpreted as "partial default" or "dilution" when the government decreases the real value of the bond principal. We refer to $T_2(s)$ as the government "fiscal rule" and treat it as an exogenous outcome of an unmodelled political process. The exogenous fiscal rule $T_2(s)$ has two roles in our model: (i) it pins down an upper bound on B_0 and (ii) it determines the riskiness of the bond's cash-flow.

Because the government cannot choose B_0 , the only way it can increase G at time $t = 0$ is by increasing the value of its debt $q_0^b B_0$ through financial regulation—by imposing portfolio restrictions on each bank at end of period 0 and period 1:

$$\frac{\varrho}{2}(q_0^d d_0^i - m_0^i) \leq \kappa q_0^b b_0^i + (1 - \kappa)k_0^i \quad (2.3)$$

$$\frac{\varrho}{2}\delta_1^d(\lambda, s)d_1^i(\lambda, s) \leq \kappa q_1^b(s)b_1^i(\lambda, s) + (1 - \kappa)\left(q_1^k(s)k_1^i(\lambda, s)\right) \quad (2.4)$$

where (d_0^i, d_1^i) denote bank i 's initial deposit issuance at $t = 0$ and remaining deposit at the end of period 1, respectively, and similarly for the holdings of government debt (b_0^i, b_1^i) , and capital (k_0^i, k_1^i) , and m_0^i is the bank i 's holdings of the short asset at $t = 0$. The pair (ϱ, κ) is a set of regulatory parameters: ϱ is a leverage constraint that restricts the bank's deposit-to-asset ratio, while κ is the

relative “weight” on government debt in the calculation of regulatory asset value, which we interpret as the extent of repression. $\kappa = 0.5$ refers to a regulatory regime that treats government debt and capital symmetrically and just restricts bank risk taking. $\kappa > 0.5$ is a regime that incentivizes the holding of government debt over capital as regulatory collateral, while $\kappa < 0.5$ corresponds to the opposite case. We refer to $\kappa = 0.5$ as a “neutral” regulatory regime and $\kappa > 0.5$ as a “repression” regime.

Two of our main objectives are to understand: (i) how financial repression ($\kappa > 1/2$), by decreasing the riskiness of government debt, can generate a convenience yield and (ii) how irresponsible fiscal rules (implying $\delta_2^b(\mathbf{s}) < 1$ for “bad” \mathbf{s}), by increasing the riskiness of government debt, can restrict the government’s ability to exploit the convenience yield.

Household problem: Households are uncertain about their own preferences, in the manner of [Diamond and Dybvig \(1983\)](#) and [Allen and Gale \(1994\)](#). There are two “layers” of uncertainty: individual- and island-specific, both of which are resolved at the start of $t = 1$. On each island j , with probability λ^j agents are early consumers, who only value the good at period 1, and with probability $1 - \lambda^j$ they are late consumers, who only value the good at period 2. The probability λ^j is island-specific and it follows the distribution $\lambda \sim F(\lambda)$. For convenience we drop the j superscript and index islands by λ . We denote the state of being an early consumer by $\zeta^h \in \{0, 1\}$. At time 0, households rank allocations according to:

$$\mathcal{U} := \mathbb{E} [\zeta^h u(c_1^h(\lambda, \mathbf{s})) + (1 - \zeta^h) u(c_2^h(\lambda, \mathbf{s}))], \quad (2.5)$$

where $c_t^h(\lambda, \mathbf{s})$ denotes consumption of household h on island λ in period $t \in \{1, 2\}$ when the aggregate state is \mathbf{s} . Each household is endowed with one unit of goods at $t = 0$ and zero goods in the other periods. All agents have the time 0 budget constraint:

$$q_0^d d_0^h + q_0^e e_0^h \leq 1 \quad (2.6)$$

where d_0^h and e_0^h are household h ’s deposit and equity holdings. Early consumers ($\zeta^h = 1$) only consume at $t = 1$ and face the deposit-in-advance constraint:³

$$c_1^h(\lambda, \mathbf{s}) \leq \delta_1^d(\lambda, \mathbf{s}) d_0^h, \quad \forall \lambda, \mathbf{s}.$$

Late consumers ($\zeta^h = 0$) do not consume at $t = 0$ (leave all their deposits in their bank)⁴ and face the following budget constraint in periods 1 and 2:

$$\begin{aligned} \delta_1^d(\lambda, \mathbf{s}) d_1^h(\lambda, \mathbf{s}) &\leq \delta_1^d(\lambda, \mathbf{s}) d_0^h + \delta_1^e(\lambda, \mathbf{s}) e_0^h, \quad \forall \lambda, \mathbf{s} \\ c_2^h(\lambda, \mathbf{s}) &\leq \delta_2^e(\lambda, \mathbf{s}) e_0^h + \delta_2^d(\lambda, \mathbf{s}) d_1^h(\lambda, \mathbf{s}) - \tau(\mathbf{s}), \quad \forall \lambda, \mathbf{s} \end{aligned} \quad (2.7)$$

³For convenience, we assume that the equity of the early consumers is lost. This assumption is without loss of generality for the qualitative direction of our results.

⁴Late consumers have no incentive to run because the deposit contract payouts are restricted to give the late consumer at least as much as the early consumer.

where $\tau(\mathbf{s})$ denotes (per capita) lump-sum taxes.

Bank problem: Each island has a representative bank owned by the households on that island. The bank's objective is to maximize its market value at $t = 0$:

$$q_0^e(d_0^i, m_0^i, k_0^i, b_0^i) + q_0^d(d_0^i, m_0^i, k_0^i, b_0^i)d_0^i - m_0^i - k_0^i - q_0^b b_0^i \quad (2.8)$$

At $t = 0$, the bank chooses deposit issuance, $d_0^i \geq 0$, short asset holdings, $m_0^i \geq 0$, initial capital, $k_0^i \geq 0$, and initial government debt holding, $b_0^i \geq 0$, subject to the regulatory constraint (2.3) at $t = 0$. At $t = 1$, it chooses whether to default on its deposit (by paying $\delta_1^d, \delta_2^d < 1$), and chooses new asset holdings $b_1^i \geq 0$ and $k_1^i \geq 0$, subject to:

$$\delta_1^e(\lambda, \mathbf{s}) + \delta_1^d(\lambda, \mathbf{s})\lambda d_0^i + q_1^k(\mathbf{s})k_1^i(\lambda, \mathbf{s}) + q_1^b(\mathbf{s})b_1^i(\lambda, \mathbf{s}) \leq z_1(\mathbf{s})m_0^i + q_1^k(\mathbf{s})k_0^i + q_1^b(\mathbf{s})b_0^i - \varsigma \mathbb{1}^d d_0^i, \quad (2.9)$$

$$\delta_2^e(\lambda, \mathbf{s}) + \delta_2^d(\lambda, \mathbf{s})(1 - \lambda)d_0^i \leq z_2(\mathbf{s})k_1^i(\lambda, \mathbf{s}) + \delta_2^b(\mathbf{s})b_1^i(\lambda, \mathbf{s}), \quad (2.10)$$

$$\delta_1^d(\lambda, \mathbf{s}) \leq \delta_2^d(\lambda, \mathbf{s}), \quad (2.11)$$

$$0 \leq b_1^i(\lambda, \mathbf{s}), \quad 0 \leq k_1^i(\lambda, \mathbf{s}), \quad 0 \leq \delta_1^e(\lambda, \mathbf{s}), \quad 0 \leq \delta_2^e(\lambda, \mathbf{s}), \quad (2.12)$$

for all (λ, \mathbf{s}) where λd_0^i and $(1 - \lambda)d_0^i$ represent early withdrawal and rolled over deposits, respectively, $\delta_1^e(\lambda, \mathbf{s})$ and $\delta_2^e(\lambda, \mathbf{s})$ are bank dividends paid at $t = 1$ and at $t = 2$, while $k_1^i(\lambda, \mathbf{s})$ and $b_1^i(\lambda, \mathbf{s})$ denote the bank holdings of capital and government debt at the end of period $t = 1$ —both of them are subject to short selling constraints. In addition, banks face the regulatory constraint (2.4) at $t = 1$.

The bank problem involves three key frictions. First, the deposit payout at $t = 1$, δ_1^d , cannot be freely conditioned on the state (λ, \mathbf{s}) . Second, banks cannot issue equity at $t = 1$ in the sense that:

$$0 \leq \delta_1^e(\lambda, \mathbf{s}), \quad \forall \lambda, \mathbf{s} \quad (2.13)$$

which—combined with (2.9)-(2.12)—implies that *banks cannot get extra resources from the household at $t = 1$* : they cannot raise equity and must cover their early withdrawals either by using their short asset holdings or by selling their long assets. This means that banks may potentially end up defaulting on deposits by paying $\delta_1^d < 1$. The ability to do so is guaranteed by the third friction, namely, that banks have limited liability, in the sense that they cannot force negative dividends on their shareholders at $t = 2$:

$$0 \leq \delta_2^e(\lambda, \mathbf{s}), \quad \forall \lambda, \mathbf{s}.$$

If the bank defaults, denoted by the indicator $\mathbb{1}^d$, then it cannot pay dividends, incurs a real dead-weight cost ς at $t = 1$ (proportional to its total outstanding deposit d_0^i), and pays the maximum amount δ_1^d to its early withdrawers subject to the constraint that it is able to pay at least as much to its late withdrawers at $t = 2$ (so that late consumers have no incentive to run). The dead-weight cost ς can be interpreted as the loss of firm specific information, the destruction of consumer networks, and/or other costs associated with default. Banks take ς as given but in equilibrium it is determined as an increasing function of the fraction of defaulting banks, that is, the cost of default is higher

when many banks default at the same time.

Finally, we assume that after the shocks are realized, banks can avoid the $t = 1$ regulatory constraint (2.4) by paying a penalty αd_0 (measured in goods). The ability of banks to avoid the regulatory constraint at $t = 1$ gives rise to a participation constraint and prevents the government from forcing too many losses on the financial sector. Parameter α captures the degree of regulation enforcement: low values of α means that financial repression cannot impose significant losses on the financial sector because banks can opt out relatively easily; on the other hand, $\alpha = \infty$ means that the government can use financial repression “without limits”. In subsection 2.2, we consider $\alpha = \infty$ as our base case to focus on how, in general equilibrium, financial repression can influence the variability of asset prices, thereby generating a convenience yield on government debt. In Section 2.4, we relax this assumption and set $\alpha < \infty$ when we consider government debt devaluation.

Numerical Example: Throughout this section, we will use the following numerical example to illustrate the main forces in our model. The aggregate shock has a two-point support $\mathbf{s} \in \{\mathbf{s}_H, \mathbf{s}_L\}$ with probabilities $\mathbb{P}\{\mathbf{s} = \mathbf{s}_L\} = \pi$ and $\mathbb{P}\{\mathbf{s} = \mathbf{s}_H\} = 1 - \pi$. We will refer to \mathbf{s}_L as the “bad state” or a “liquidity crisis”, because it makes the return of the short-term asset fall at $t = 1$, such that $z_1(\mathbf{s}_H) = 1$ and $z_1(\mathbf{s}_L) = \bar{z}_1$ with $1 > \bar{z}_1$, but leaves the return of capital at $t = 2$ unchanged, $z_2(\mathbf{s}) = \bar{z}_2$ (i.e., the capital is risk-free). We parameterize the fiscal rule as $T_2(\mathbf{s}_H) = \bar{T}_H$ and $T_2(\mathbf{s}_L) = \bar{T}_L$ and assume that $\bar{T}_H \geq \bar{T}_L$. The island-specific idiosyncratic shock λ follows a Beta distribution, $F(\lambda) = \text{Beta}(\bar{\alpha}, \bar{\beta})$, with support $(0, 1)$. The household period utility function $u(\cdot)$ has a CRRA form with risk aversion parameter γ . In equilibrium, the dead-weight default cost is assumed to be linear in the fraction of defaulting banks with slope parameter $\bar{\varsigma}$.

2.2 Equilibrium

Definition 1 (Budget-feasible government policy). Given a fiscal rule T_2 and bond price q_0^b , a budget-feasible government policy is a tuple (G, B_0, δ_2^b) s.t. (2.1) and (2.2) are satisfied with

$$T_2(\mathbf{s}) = (1 - \Lambda)\tau(\mathbf{s})$$

where $\Lambda := \int \lambda dF$ is the expected aggregate withdrawal rate.

Definition 2 (Competitive Equilibrium). Given a fiscal rule T_2 , regulation (ϱ, κ) , and a budget-feasible government policy (G, B_0, δ_2^b) , a competitive equilibrium is a set of prices (q_0^d, q_0^e, q_0^b) and (q_1^k, q_1^b) , payoffs $(\delta_1^d, \delta_2^d, \delta_2^e)$, household policies $(d_0^h, e_0^h, c_1^h, c_2^h)$, and bank policies $(d_0^i, m_0^i, k_0^i, b_0^i)$ and (k_1^i, b_1^i) , such that

- Households maximize (2.5) subject to (2.6)-(2.7),
- Banks maximize (2.8) subject to (2.3)-(2.4) and (2.9)-(2.11),

- Markets clear:

$$\begin{aligned} G + m_0^i + k_0^i &= 1, & d_0^h &= d_0^i, & e_0^h &= 1, & b_0^i &= B_0, \\ \int b_1^i dF &= B_0 & \int k_1^i &= k_0^i & \int \lambda c_1^h dF &= z_1 m_0 - \varsigma(\cdot) d_0 \end{aligned} \quad (2.14)$$

$$\int (1 - \lambda) c_2^h dF = z_2 k_0 - \int \lambda \delta_2^e dF(\lambda) \quad (2.15)$$

We characterize equilibrium in the following way. First, we solve the optimization problem of the household in subsection 2.2.1. Second, we combine household and bank optimization with inter-bank asset market clearing to characterize equilibrium in the $t = 1$ market for given $t = 0$ choices in subsection 2.2.2. Finally, we characterize equilibrium in the $t = 0$ market and discuss the emergence of convenience yields in subsection 2.2.3.

The equilibrium looks complicated but ultimately ends up being very analytically tractable. The heart of the mechanism comes through the equilibrium in the secondary asset market at $t = 1$, which, for the case of $\alpha = \infty$, is characterized by bank's default policy, which follows a cutoff rule that banks default if and only if $\lambda > \lambda^*$. Ultimately, the cutoff decision is a function of bank leverage choices at $t = 0$, asset prices at $t = 1$, and regulatory restrictions in the secondary market. The subtlety is that equilibrium prices depend upon feedback between the default decision and regulatory constraints. The bank time $t = 0$ portfolio decisions are made to hedge their default risk, understanding how regulatory constraints are going to distort asset pricing in the secondary market.

2.2.1 Household Problem

We characterize the solution to the household problem in Proposition 1. The households choose their asset portfolio once and for all at $t = 0$, so that the choices satisfy the Euler equations (2.16) and (2.17). Given the household portfolio, (d_0^h, e_0^h) , early consumption c_1 and late consumption c_2 are determined as functions of asset payoffs $(\delta_1^d, \delta_2^d, \delta_2^e)$ and idiosyncratic and aggregate shocks.

Proposition 1 (Characterization of Household Problem). *The household portfolio choices at $t = 0$ satisfy (where the dependence on \mathbf{s} is implicit):*

$$q_0^d = \mathbb{E} \left[\xi(\lambda) \nu(\lambda) \delta_1^d(\lambda) \right] \quad (2.16)$$

$$q_0^e = \mathbb{E} \left[\xi(\lambda) \delta_2^e(\lambda) \right] \quad (2.17)$$

We use the following notation for the stochastic discount factor (SDF) and the liquidity premium:

$$\xi(\lambda) := \frac{(1 - \lambda) u'(c_2^h(\lambda))}{\mu_0^c}, \quad \nu(\lambda) := 1 + \frac{\lambda u'(c_1^h(\lambda))}{(1 - \lambda) u'(c_2^h(\lambda))}$$

where $\mu_0^c > 0$ is the households' Lagrange multiplier on their period $t = 0$ budget constraint and their consumption choices are

$$c_1(\lambda) = \delta_1^d(\lambda) d_0^h, \quad \text{and} \quad c_2(\lambda) = \delta_2^e(\lambda) e_0^h + \delta_2^d(\lambda) d_0^h - \tau.$$

Proof. See Appendix A.1. □

Demand deposits provide liquidity services at $t = 1$ to the early consumers, which introduces a wedge ν into the household's deposit Euler equation. The presence of this asset-specific wedge implies that households are willing to hold demand deposits at a discount, which leads to a “funding advantage” to the providers of such assets. This is the role of the first layer of idiosyncratic risk in the model (the individual specific risk, ζ^h)—it creates the need for a banking sector as insurers of individual risk and providers of liquidity services to households. As we will see in the bank problem, the second layer of idiosyncratic risk (the island specific risk, λ) introduces a need for an inter-bank market. Because this inter-bank market is frictional, it is costly for the bank to provide insurance and liquidity services to the households.

2.2.2 Equilibrium in the inter-bank markets ($t = 1$)

Proposition 2 characterizes equilibrium in the inter-bank markets at time $t = 1$ for given initial asset holdings (m_0, k_0, b_0, d_0) . This involves combining household optimization with bank optimization and inter-bank market clearing, the latter two of which are complicated by the possibility that banks can default. For the characterization of this default decision, it will be useful to define the deposit to asset ratio (which we refer to as “leverage”) at the end of time 0, the beginning of time 1, and the end of time 1 by:

$$\ell_0 = \frac{d_0}{m_0 + k_0 + q_0^b b_0}, \quad \tilde{\ell}_1 = \frac{d_0}{z_1 m_0 + q_1^k k_0 + q_1^b b_0}, \quad \ell_1 = \frac{d_1}{q_1^k k_1 + q_1^b b_1}.$$

At the beginning of time $t = 1$, the island-specific withdrawal shock, λd_0 , leads to ex post heterogeneity among banks: those with low λ will have excess resources, $z_1 m_0 - \lambda d_0 > 0$, that they can use to purchase assets in the inter-bank markets, while those with λ such that $z_1 m_0 - \lambda d_0 < 0$ will be forced to sell assets to cover early withdrawals at $t = 1$.

Proposition 2 (Equilibrium at $t = 1$). *Let $\mu_1^e \geq 0$ and $\mu_1^r \geq 0$ denote the Lagrange multipliers on the $t = 1$ equity raising constraint (2.13) and the $t = 1$ regulatory constraint (2.4), respectively. The following hold:*

(i) Secondary market asset prices: Given (μ_1^e, μ_1^r) , asset prices satisfy:

$$q_1^b = \frac{\delta_2^b}{1 + \mu_1^e - \kappa \mu_1^r}, \quad q_1^k = \frac{z_2}{1 + \mu_1^e - (1 - \kappa) \mu_1^r} \quad (2.18)$$

(ii) Default decision: Given $(\tilde{\ell}_1, \mu_1^e, \mu_1^r)$, banks default iff $\lambda > \lambda^*$, where λ^* is given by:

$$\lambda^* = \min \left\{ \frac{\left(\frac{1 + \mu_1^e}{1 + \varrho \mu_1^r} \right) \tilde{\ell}_1^{-1} - 1}{\left(\frac{1 + \mu_1^e}{1 + \varrho \mu_1^r} \right) - 1}, \quad \frac{\left(\frac{\kappa}{\varrho} \right) \tilde{\ell}_1^{-1} - 1}{\left(\frac{\kappa}{\varrho} \right) - 1} \right\} \quad (2.19)$$

The first term is the cutoff when the limited liability constraint triggers default and the second term is when the regulatory constraint triggers default.

(iii) *Other variables:* Given $(\tilde{\ell}_1, \mu_1^e, \mu_1^r)$, prices (q_1^b, q_1^k) satisfy (2.18), λ^* satisfies (2.19), the deposit payoffs $(\delta_1^d(\lambda), \delta_2^d(\lambda))$ are such that $\delta_1^d(\lambda) = \delta_2^d(\lambda)$ and their value is 1 if the bank does not default ($\lambda < \lambda^*$) and it comes either from the condition that $\delta_2^e(\lambda) = 0$ or from the binding $t = 1$ regulatory constraint if the bank defaults ($\lambda \geq \lambda^*$), and finally, the period $t = 2$ dividend $\delta_2^e(\lambda)$ can be derived from budget constraints (2.9)–(2.10). The equilibrium $(\tilde{\ell}_1, \mu_1^e, \mu_1^r)$ is determined so that the market clearing conditions (2.14)–(2.15) are satisfied.

Proof. See Appendix A.2. □

Corollary 1. Let $\mathcal{V}_1(\tilde{\ell}_1; \mathbf{s}) := \int \xi(\lambda, \mathbf{s}) (\max\{0, \delta_1^e(\lambda, \mathbf{s}) + \delta_2^e(\lambda, \mathbf{s})\} + \nu(\lambda, \mathbf{s})\delta_1^d(\lambda, \mathbf{s})) dF(\lambda)$ denote the expected bank continuation value conditional on the aggregate state \mathbf{s} . Then, given (ξ, μ_1^e, μ_1^r) , the bank's value is:

$$\begin{aligned} \mathcal{V}_1(\tilde{\ell}_1; \mathbf{s}) := & \int_{\lambda^*(\tilde{\ell}_1)}^{\lambda^*(\tilde{\ell}_1)} \xi(\lambda, \mathbf{s}) (\mu_1^e(\mathbf{s}) - \varrho \mu_1^r(\mathbf{s})) (\lambda^*(\tilde{\ell}_1) - \lambda) dF \\ & + \int_{\lambda^*(\tilde{\ell}_1)}^{\lambda^*(\tilde{\ell}_1)} \xi(\lambda, \mathbf{s}) \nu(\lambda, \mathbf{s}) dF \\ & + \int_{\lambda^*(\tilde{\ell}_1)} \xi(\lambda, \mathbf{s}) \nu(\lambda, \mathbf{s}) \left(\frac{(1 + \mu_1^e(\mathbf{s})) (\tilde{\ell}_1^{-1} - \varsigma)}{(1 + \mu_1^e(\mathbf{s})) \lambda + (1 + \varrho \mu_1^r(\mathbf{s})) (1 - \lambda)} \right) dF(\lambda), \end{aligned}$$

where $\lambda^*(\tilde{\ell}_1)$ is given by (2.19).

Proposition 2 characterizes asset prices (q_1^k, q_1^b) in the inter-bank market at time $t = 1$. Evidently, two key features influence equilibrium: the banking sector's inability to draw resources from the households (as characterized by the multiplier μ_1^e) and the regulatory constraint (as characterized by the multiplier μ_1^r). If neither of these features were present, then $\mu_1^e = \mu_1^r = 0$ and $\lambda^* = 1$ (no default). In this case, the bank's marginal value of an additional unit of wealth is equal to the household's marginal value: $\partial_{\tilde{\ell}_1} \mathcal{V}_1 = \int \xi dF(\lambda)$ and prices of bonds and capital would be $q_1^b = \delta_2^b$ and $q_1^k = z_2$. We refer to this as the assets being priced at their “fundamental value”.

The equity raising friction (2.13) introduces a link between the aggregate proceeds from bank short asset holdings, $z_1 m_0$, and aggregate asset demand in the sense that resource scarcity puts downward pressure on asset prices in the inter-bank market. This shows up as a wedge, $\mu_1^e > 0$, between the marginal value of income inside versus outside of a particular bank. In equilibrium this wedge manifests itself as “fire sale pricing” in the inter-bank asset markets in the sense that, on average, $q_1^b < \delta_2^b$ and $q_1^k < z_2$, i.e., assets are traded below their “fundamental value” at all states of the world in which $\mu_1^e > 0$.⁵ Moreover, because the multiplier μ_1^e increases when aggregate resources get relatively more scarce, the bank's marginal valuation of wealth $\partial_{\tilde{\ell}_1} \mathcal{V}_1$ goes up relatively more in bad times which makes the bank more “risk averse” than the household. As a result, banks will pay a premium (relative to households) for an asset that is a good hedge against aggregate risk. We show this visually in the left panel of Figure 2. Taking the household's and bank's SDFs as given we ask how much more the bank will pay, compared to the household, for an asset that has

⁵The finance literature refers to this as “fire sale” Shleifer and Vishny (1992, 2011) or cash-in-the-market pricing Allen and Gale (1994, 1998). The monetary literature, starting with Lucas (1990), refers to this as “liquidity effect” which was taken up by the limited participation literature (Christiano and Eichenbaum, 1992, 1995).

a mean price of \bar{q}_1^b but which pays $q_1^b(s_L)$ in the bad state. The solid lines depict this premium, $\mathbb{E}[\partial \mathcal{V}_1(s) q_1^b(s)] - \mathbb{E}[\xi(s) q_1^b(s)]$, as a function of $q_1^b(s_L)$ while keeping the mean price \bar{q}_1^b constant (dashed vertical line). As we move from left to right the asset becomes a better hedge against aggregate risk and in fact the premium the bank will pay for this asset increases.

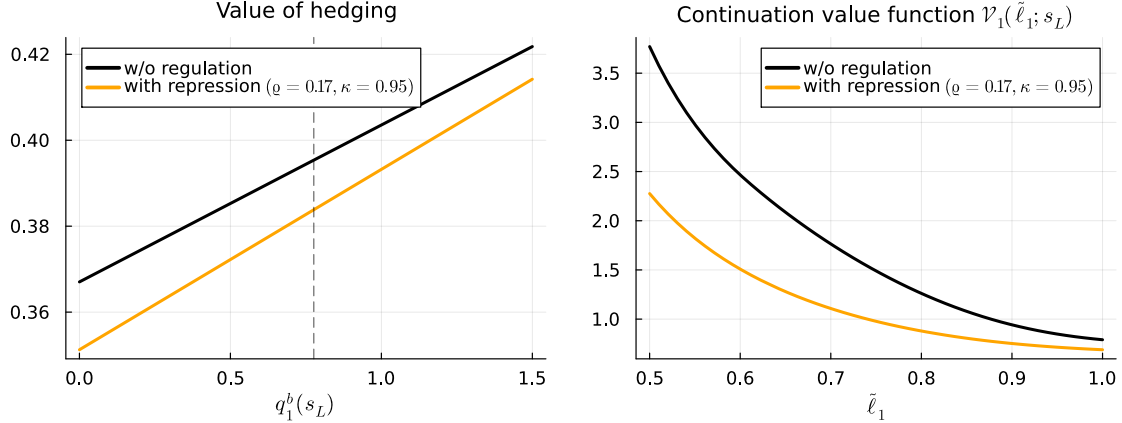


Figure 2: Financial regulation affects the banking sector's "risk aversion"

Left plot: solid lines depict how much more banks would pay for an asset that has a mean price of \bar{q}_1^b (dashed line) but which pays $q_1^b(s_L)$ in the bad state (x-axis). Formally, we measure this via $\mathbb{E}[\partial \mathcal{V}_1(s) q_1^b(s)] - \mathbb{E}[\xi(s) q_1^b(s)]$ (y-axis). *Right plot:* Continuation value function \mathcal{V}_1 (as a function of leverage $\tilde{\ell}_1$) conditional on the bad aggregate state s_L .

Finally, when both equity raising frictions and regulation are present, both Lagrange multipliers are positive $\mu_1^e > 0, \mu_1^r > 0$ at least for some banks. The multipliers appear in \mathcal{V}_1 as $\mu_1^e - \varrho \mu_1^r$, indicating that the regulatory constraint dampens the higher effective risk aversion of the bank. Indeed, comparing the orange and black lines in the left panel of Figure 2, we can see that the bank's hedging premium under repression (orange line) is consistently lower than the premium without government restrictions on bank's balance sheets (black line). In the right panel, we can see that this is reflected in a decrease in the concavity of the bank's continuation value because they now have an effective hedge. If regulation is symmetric in its treatment of bonds and capital, then $\kappa = 1/2$ and the relative price ratio is simply the ratio of $t = 2$ payoffs:

$$\frac{q_1^b(s)}{q_1^k(s)} = \frac{\delta_2^b(s)}{q_1^k(s)}$$

If regulation advantages government bonds, then $\kappa > 1/2$ and relative price of government debt is higher and satisfies:

$$\frac{q_1^b(s)}{q_1^k(s)} = \frac{\delta_2^b(s)}{z_2^k(s) - \kappa \mu_1^r(s) q_1^k(s)}$$

In the bad state, s_L , there are fewer resources and so $\mu_1^r(s_L)$ increases, which in turn increases $q_1^b(s)/q_1^k(s)$. In this sense, regulation makes banks more "captive buyers" for government debt in bad times. Both cases are depicted graphically in Figure 3. Without repression, government debt

prices fall in bad times. However, with repression, government debt prices increase in bad times because the banks crowd into the government debt market, making government debt a good hedge against aggregate risk.

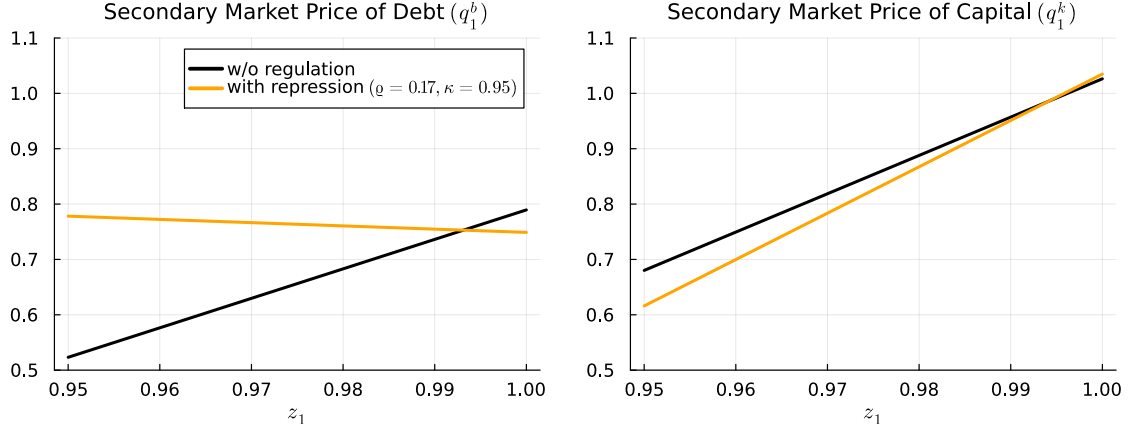


Figure 3: Financial regulation makes government debt a good hedge against aggregate risk

2.2.3 Equilibrium at time $t = 0$

We finish the characterization of equilibrium by studying agent decisions and market clearing at $t = 0$ in Proposition 3.

Proposition 3 (Equilibrium at $t = 0$). *The bank zero profit condition is satisfied:*

$$q_0^d d_0 + q_0^e = m_0 + k_0 + q_0^b b_0$$

and the bank's Euler equations are given by:

$$\begin{aligned} 1 - \varrho \mu_0^r &= \mathbb{E}[\xi \Omega z_1] \\ 1 - (1 - \kappa) \mu_0^r &= \mathbb{E}[\xi \Omega q_1^k] = \mathbb{E}\left[\xi \Omega \left(\frac{1}{1 + \mu_1^e - (1 - \kappa) \mu_1^r} \right) z_2\right] \\ q_0^b (1 - \kappa \mu_0^r) &= \mathbb{E}[\xi \Omega q_1^b] = \mathbb{E}\left[\xi \Omega \left(\frac{1}{1 + \mu_1^e - \kappa \mu_1^r} \right) \delta_2^b\right] \end{aligned}$$

where $\int \xi(\lambda, s) \Omega(\lambda, s) dF(\lambda) = \partial_{\bar{t}} \mathcal{V}_1(s)$ is the bank's stochastic discount factor for pricing the aggregate risk.

From Proposition 3, we can see the two key features of the bank problem. First, the costly default wedge Ω effectively makes the banking sector act as more “risk-averse” than the household sector even though they use the household's SDF. Second, the optimal bank leverage choice at $t = 0$ trades off earning the liquidity premium on deposits, as measured by ν , against the cost of having a higher default probability, as captured by Ω . In this sense, the combination of deposit liquidity services and costly default break Modigliani-Miller style results.

Corollary 2. *If the government fiscal rule fully repays the debt, $\delta_2^b(s) = 1$, then the regulatory Lagrange multiplier binds at $t = 1$ ($\mu_1^r > 0$) but not at $t = 0$ ($\mu_0^r = 0$).*

The multiplier μ_0^r reflects the impact of forcing the banks to buy government debt in the primary market. This can be thought of as the “direct” impact of financial repression. The multiplier μ_1^r reflects the impact of creating a captive secondary market for government debt in the interbank market. Ultimately, this changes the price process for government debt and makes government debt a “safe-asset” that banks want to hold at $t = 0$, which means that the constraint in the primary market no longer binds. Corollary 2 shows that the safe asset benefit is sufficiently strong that the banks want to purchase more government debt in primary market than is required by regulation. That is, the bank have additional precautionary motive for holding government debt that further increases the convenience yield.

A key feature of our model is that the default cut-off λ^* and default wedge Ω depends upon the policy parameters of the government: $(\varrho, \kappa, \delta^b)$. This means that regulation and fiscal irresponsibility not only directly change demand but also change the precautionary role for holding government debt. Taken together, our expression for $\Omega/(1 + \mu_1^e - \kappa\mu^r)$ characterizes how government policy can create endogenous demand for government debt by distorting the SDF of the banks.

2.2.4 Convenience Yields

We close this section by characterizing the convenience yield. This is often defined to be the log difference between the asset price and the expectation of stochastic discount factor:

$$\chi := \log(q_0^b) - \log(\mathbb{E}[\xi]).$$

However, for risky assets, it has been observed that this definition includes both the special role of the asset and risk premium on that asset (e.g. [Jiang et al. \(2020b\)](#)). In this sense, it helpful to break up the convenience yield in the following way:

$$\chi = \underbrace{\log(q_0^b) - \log(\mathbb{E}[\xi\delta_2^b])}_{\text{risk-adjusted convenience yield}} + \underbrace{\log(\mathbb{E}[\xi\delta_2^b]) - \log(\mathbb{E}[\xi])}_{\text{risk penalty}}$$

where we refer to the first term as the “risk-adjusted” convenience yield:

$$\tilde{\chi} := \log(q_0^b) - \log(\mathbb{E}[\xi\delta_2^b])$$

and the second term the risk premium. The risk adjusted convenience yield is the log difference between the price of government debt and the price of an asset with the same cash flows as government debt but without the special role of government debt. We interpret the “risk-adjusted” convenience yield as the value that households place on the special role that government debt plays.

We can decompose the risk-adjusted convenience yield further into:

$$\tilde{\chi} = \underbrace{\log \left(\mathbb{E} \left[\xi \Omega \left(\frac{(1 - \kappa \mu_0^r)^{-1}}{1 + \mu_1^e - \kappa \mu_1^r} \right) \delta_2^b \right] \right) - \log (\mathbb{E} [\xi \Omega \delta_2^b])}_{\text{direct effect of regulation} =: \tilde{\chi}_r} + \underbrace{\log (\mathbb{E} [\xi \Omega \delta_2^b]) - \log (\mathbb{E} [\xi \delta_2^b])}_{\text{indirect hedging premium} =: \tilde{\chi}_h}$$

From this expression we can see that the convenience yield comes from both the direct impact of forcing the banks to purchase government debt and indirect impact that creating a safe asset helps the banks to hedge risk. Expanding the first term gives:

$$\tilde{\chi}_r \approx \kappa \mu_0^r + \log \left(\mathbb{E} \left[(1 + \mu_1^e - \kappa \mu_1^r)^{-1} \right] \right) + \text{Cov} \left(\frac{\xi \Omega \delta_2^b}{\mathbb{E} [\xi \Omega \delta_2^b]}, \frac{(1 + \mu_1^e - \kappa \mu_1^r)^{-1}}{\mathbb{E} [(1 + \mu_1^e - \kappa \mu_1^r)^{-1}]} \right)$$

Expanding the second term gives:

$$\tilde{\chi}_h \approx \log (\mathbb{E} [\Omega]) + \text{Cov} \left(\frac{\xi \delta_2^b}{\mathbb{E} [\xi \delta_2^b]}, \frac{\Omega}{\mathbb{E} [\Omega]} \right)$$

In both cases, we can see that the covariance between the wedges in the bank Euler equation and the devaluation of government debt ($\delta^b < 1$ in some states of the world) are key components of the convenience yield. A goal of our paper is to show explicitly how these covariance terms are related to government regulation and fiscal policies. In Subsections 2.3 and 2.4, we show that models of exogenous government bond demand are missing these covariance terms and so give misleading implications for how government default impacts the convenience yield.

2.3 Comparison to Exogenous Bond Demand Functions

In this section, we consider two alternative models of bond demand that are frequently used in the macroeconomics literature: bond-in-utility and bond-in-advance (with uninsurable idiosyncratic risk).

Bond-in-utility (BIU): In this model, the household solves:

$$\begin{aligned} \max_{b_0, k_0, c_1} \quad & \{v(q_0^b b_0) + \beta \mathbb{E}[u(c_2)]\} \quad s.t. \\ q_0^b b_0 + k_0 & \leq 1 \\ c_2 & \leq z_2 k_0 + \delta_2^b b_0 - \tau \end{aligned} \tag{2.20}$$

where $v(q_0^b b_0)$ denotes the utility benefit from holding a real value of government debt $q_0^b b_0$ at time 0.

Bond-in-advance with uninsurable idiosyncratic risk (BIA): In this model, the household solves:

$$\begin{aligned}
& \max_{b_0, m_0, k_0, b_1, c} \mathbb{E}[\lambda u(c_1) + (1 - \lambda)u(c_2)] \quad s.t. \\
& q_0^b b_0 + m_0 + k_0 \leq 1 \\
& c_1 \leq q_1^b b_0 \\
& c_2 \leq z_2 k_0 + \delta_2^b \left(\frac{z_1 m_0 + q_1^b b_0 - c_1}{q_1^b} \right) - \tau
\end{aligned} \tag{2.21}$$

where the constraint $c_1 \leq q_1^b b_0$ says that the household needs to use bonds to purchase consumption goods at $t = 1$. Observe that our “bond-in-advance” model actually contains two frictions: the need to hold bonds for trading in the morning market and the need to hedge the idiosyncratic island risk. The former feature is analogous to a “cash-in-advance” constraint (e.g. Svensson (1985)) while the later feature is similar to a self-insurance motive (e.g. Bewley (1980, 1983)). We formulate the model this way because both of these forces are also present in our environment and we want to give the bond-in-advance model the best possible chance of delivering the forces in our endogenous bond demand model. More specifically, we can view the bond in advance model as our environment without the restrictions on conditioning deposit payouts on random variable (λ, s) and so without any bank default.

The corresponding Euler equations for government debt the bond-in-the-utility (BIU) and bond-in-advance (BIA) are given by:

$$\begin{aligned}
q_0^b &= \mathbb{E}[\xi \delta_2^b] \left(1 - \frac{v'(q_0^b b_0)}{\mu_0^c} \right)^{-1} & \dots \text{(BIU)} \\
q_0^b &= \mathbb{E} \left[\xi \delta_2^b \left(1 - \frac{\mu_1^b}{\lambda u'(c_1)} \right)^{-1} \right] & \dots \text{(BIA)}
\end{aligned}$$

where $\mu_0^c \geq 0$ denotes the Lagrange multiplier on the household’s period $t = 0$ budget constraint (2.20) and $\mu_1^b \geq 0$ is the multiplier on the bond-in-advance constraint (2.21). We specify the bond-in-utility benefit to be on time-0 asset purchases and the bond-in-advance constraint on the time-1 trading to mimic the way that bond-in-utility is often formulated but this is not a constraint. We could have also imposed bond-in-utility on time-1 asset purchases, in which case the Euler equations would look similar. By contrast, the repression model has the Euler equations:

$$q_0^b = (1 - \kappa \mu_0^r)^{-1} \mathbb{E} \left[\xi \delta_2^b \Omega \left(1 + \mu_1^e - \kappa \mu_1^r \right)^{-1} \right]$$

Ultimately, we can see that the repression model “nests” the bond-in-the-utility and bond-in-advance models in the sense that we get terms that look similar to the bond-in-the-utility and bond-in-advance wedges. To illustrate this, in Figure 4 we plot the government bond demand functions from each model and show that they can have similar slopes. In this sense, the regulatory parameters map to the shape parameters in the reduced form bond demand functions. However, there are two

important differences in our formulation: (i) we have an additional wedge coming from bank default, Ω , and (ii) all our wedges are explicit functions of government regulatory and fiscal policies. We use our microfoundation in the next section to show that the convenience yield in our repression model reacts very differently to government default.

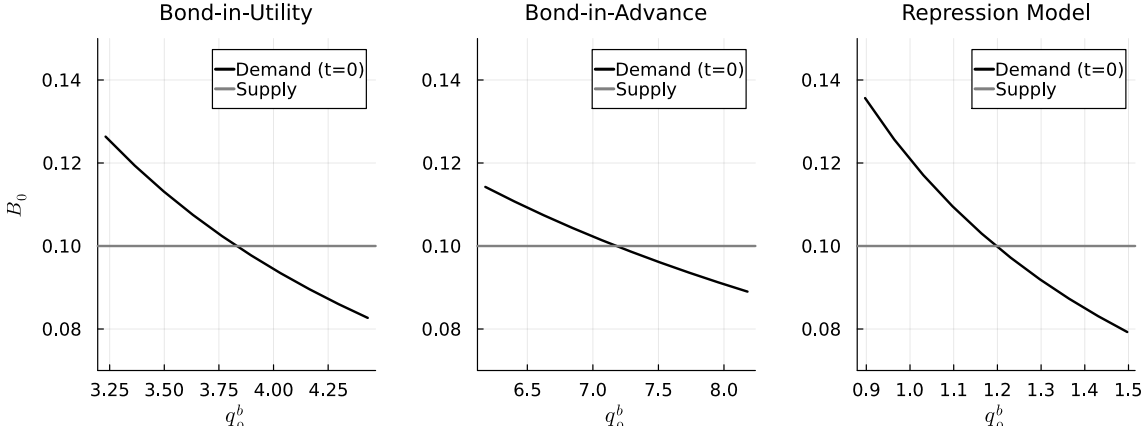


Figure 4: Private sector demand for government debt in alternative convenience yield models

2.4 Government Default

We now use our model to explore how government fiscal irresponsibility impacts the convenience yield. Figure 5 shows the convenience yield, the risk-adjusted convenience yield, and the risk-penalty in the BIU, BIA, and repression models as the government defaults more in the bad state, i.e., $\delta_2^b(s_L)$ decreases. The plots are constructed so that moving from left to right increases the risk on government debt. Evidently, in all models, the risk penalty becomes more negative as government debt becomes riskier. However, the behavior of the risk-adjusted convenience yield, which captures the special role of government debt, varies substantially.

In the BIU and BIA models the risk-adjusted convenience yield actually increases as the government debt becomes riskier. Why? In these models, the role of government debt is exogenous and its marginal usefulness increases as the market value of government debt decreases thereby making the special asset scarcer. This means that as the government starts to default, the risk-adjusted convenience yield increases. Or put another way, in these models the agents derive value from giving resources to the government so when the government starts to default, then they want to give more resources by buying government debt.

By contrast, in our repression model, the risk-adjusted convenience yield decreases as government debt becomes more risky. To understand this, in Figure 6, we plot the decomposition of the risk-adjusted convenience yield (and other prices) in our repression model. We can see that the direct effect of regulation on the risk-adjusted convenience yield, $\tilde{\chi}_r$, is increasing as government debt becomes riskier and so plays a very similar role to the terms in the BIU and BIA models. That is, it reflects captive demand within the banking sector for holding government debt, which increases as the market value of government drops. In our model, this force is dampened by allowing the banks

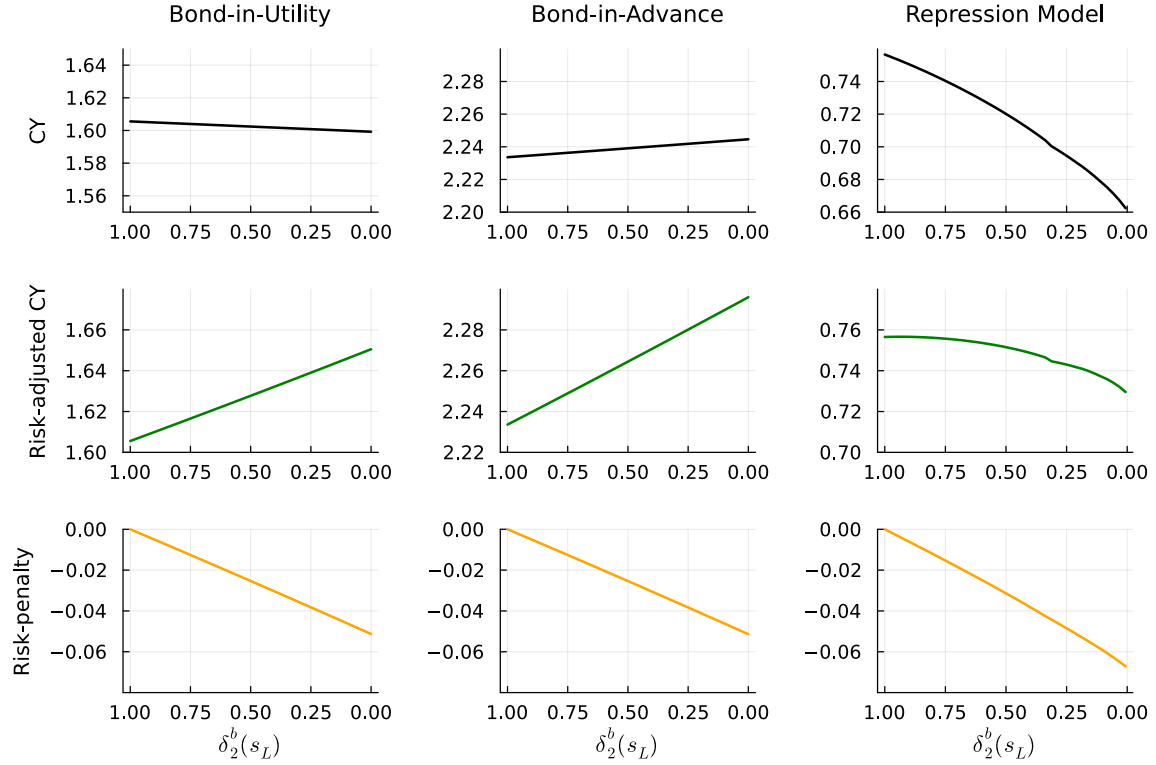


Figure 5: Convenience yields, risk-adjusted convenience yields, and risk-penalty for the Bond-in-Utility, Bond-in-Advance, and Repression models.

to pay a cost, α , and escape the financial regulation. The higher the cost of doing so, the stronger the direct impact of repression and more this term behaves like the BIU and BIA models. The large difference in our model is that our risk adjusted convenience yield also contains the hedging term, $\tilde{\chi}_h$, and this term decreases as government debt becomes riskier. Why? Agents in our model want an asset to hedge the idiosyncratic and aggregate risk but the effectiveness of government debt as a hedge against either shock depends on whether the government repression can actually make government debt a safe asset. As the government defaults more in the bad state, the hedging property of government debt erodes, as depicted in the top row of Figure 6. Ultimately, this decreases the covariance between $\xi\delta_2^b$ and Ω , which decreases the risk-adjusted convenience yield. That is, by modeling how financial regulation makes government debt a safe asset, we can show how fiscal irresponsibility erodes that role. This highlights the importance of microfounding government debt demand when it comes to questions about the government budget constraint and fiscal capacity.

2.5 Costs of Generating Convenience Yields

We use our microfounded model to understand the costs of generating a convenience yield. Figure 7 plots the equilibrium outcomes at $t = 0$ for different values of financial repression. Evidently, an increase in financial repression leads to a higher convenience yield and more fiscal capacity, as

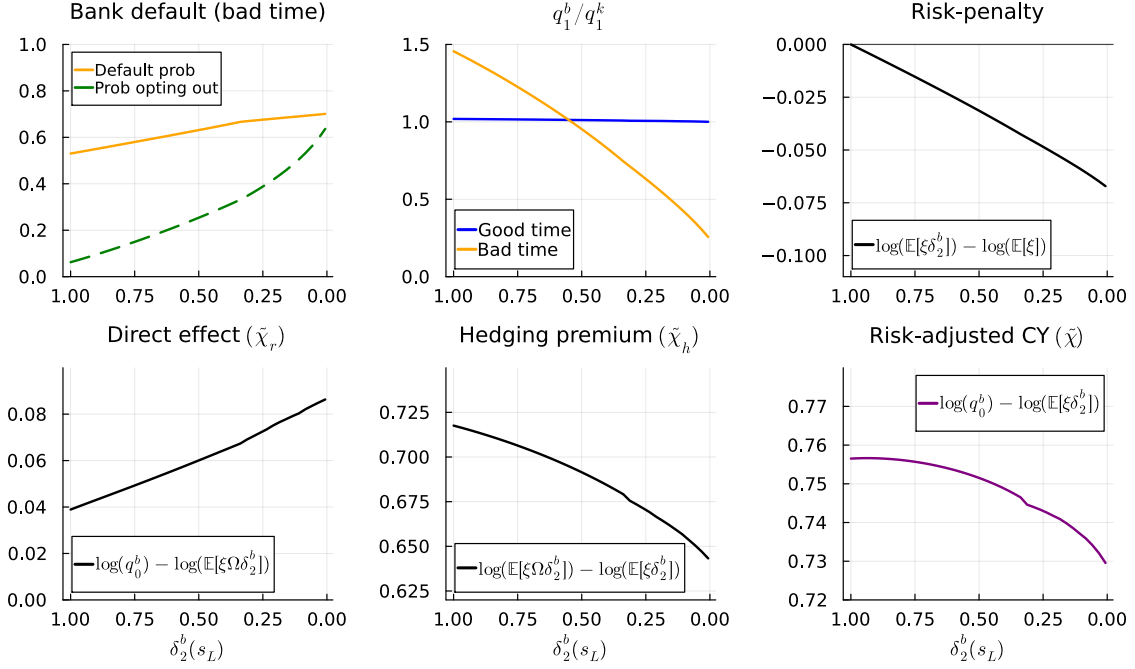


Figure 6: Decomposition of Equilibrium Pricing in the Repression Model

measured by the amount of government spending. However, it also leads to more bank default in the bad state of the world and lower bank investment into capital. The higher rate of bank default appears because financial repression inflates the debt price in the interbank market and so also decreases the portfolio return for solvent banks, which makes the marginal bank more likely to default. The lower investment rate appears because government borrowing crowds out bank capital creation, as is standard in many macroeconomic models. Together these effects lead to lower household consumption. In this sense, the government faces a trade-off between optimizing their fiscal capacity and having a well functioning financial sector. For our numerical example, we find that some degree of repression is optimal. This result is different to some recent papers (e.g. [Chari et al. \(2020\)](#)) because we have placed restrictions on the tax process and because the banks in our model play roles as both liquidity providers and intermediaries.

2.6 A Broader Interpretation of the Model

We have written the model to focus on how portfolio restrictions in the banking sector change the price process for government debt. However, the forces in the model generalize to other environments. Here we discuss two generalizations: alternative types of regulation and alternative types of financial intermediaries.

Alternative regulations: We have interpreted κ as the weight in explicit macroprudential regulation. One alternative is that it could reflect implicit pressure on the banking sector to purchase government debt. Another alternative is that it could reflect collateral requirements at a government

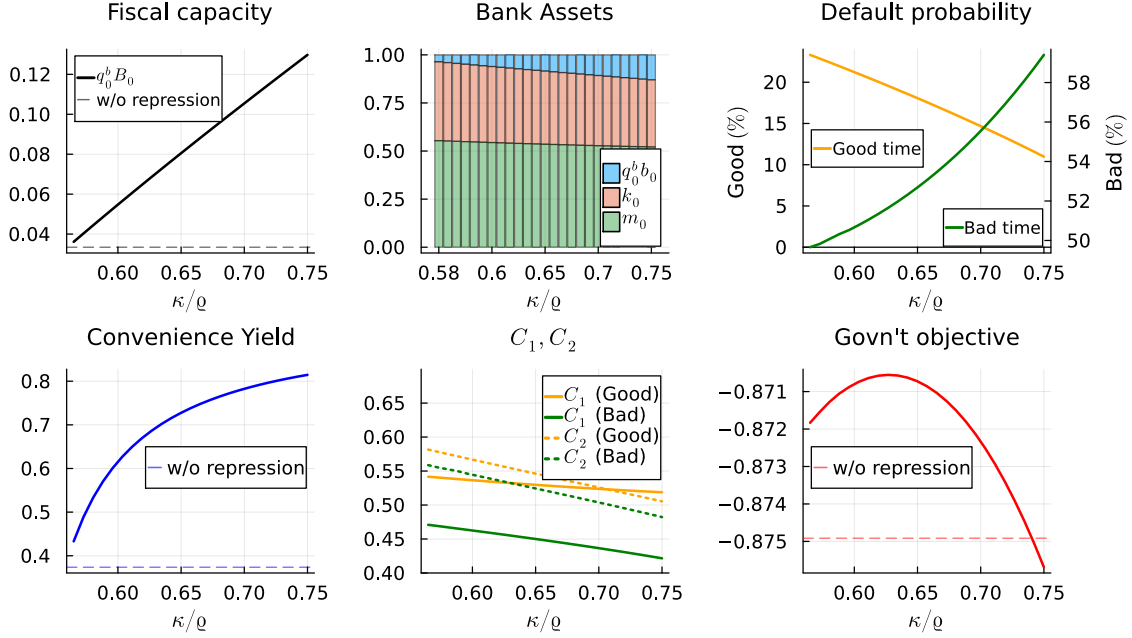


Figure 7: Equilibrium for Different Levels of Repression.

discount window. For the latter case, the regulatory requirement is only faced by banks that are in trouble rather than by all banks in the economy.

Alternative financial intermediaries: At a more abstract level, the key features of the model that we require are: (i) there is a financial intermediary that provides a service to households that exposes the intermediary to risk, (ii) the financial intermediary faces frictions that generate a wedge Ω in the intermediary Euler (e.g. equity raising constraints), (iii) the government restricts the portfolio that the financial intermediary must maintain when they trade in secondary asset markets. In this sense, the forces in our model could also apply to insurance companies or pension funds.

2.7 Connection to Different Fiscal Literatures:

Our paper is connected to a number of very large literatures studying fiscal and financial policy in general equilibrium models. Here, we provide some thoughts on how our analysis is distinct but complementary to these literatures:

- (i) *Ramsey and constrained planner models:*⁶ Our environment has two key frictions: an incomplete secondary interbank market at $t = 1$ and a default cost externality. From the constrained planner point of view, the first friction leads to fire-sale asset pricing and bank default at $t = 1$ and the second friction potentially means the banks take on too much risk when they choose portfolios at $t = 0$. Both frictions manifest as wedges in the private sector Euler equations. Consequently, the constrained planner would respond by reallocating resources across islands

⁶Chari et al. (2020), Bassetto and Cui (2021)

to liquidity constrained banks at $t = 1$ and across states by restricting the leverage of the banking sector at $t = 0$. In principle, a Ramsey planner could implement this without any “financial regulation” if it had a sufficiently large set of tax and transfer tools at $t = 0$ and $t = 1$. By contrast, our paper considers a government facing political restrictions that limit its policy choice set to financial regulation. This allows us to focus on the “costs” of using financial regulation to increase government fiscal capacity. We show that these costs involve subtle covariances between the different wedges on the private sector Euler equations and so the government faces a trade-off between expanding fiscal capacity and the stability of the financial sector. We view our work as microfounding the (implicit) cost of “taxing” the financial sector. Future work could consider how a Ramsey planner might balance this cost against the distortionary costs of other taxes.

- (ii) *Macroeconomic safe asset models:*⁷ In our model, the household need for deposits and the frictions on the banking sector create bank demand for a safe-asset that allows them to hedge default risk and the associated costs. In this sense, we have a similar argument to the “safe-asset” literature, which suggests government debt can earn a “convenience yield” by playing the role of the “liquid” or “safe” asset in the economy. However, this literature typically models the special role of government debt using an exogenous bond-in-utility or bond-in-advance formulation, which allows the government to easily increase fiscal capacity by exploiting the convenience yield. We believe this makes these models less suitable for studying fiscal policy. By contrast, we generate the convenience yield through government financial regulations that create a captive market for government debt in bad times, which endogenously makes government debt a good hedge against both aggregate and idiosyncratic risk. One benefit of endogenizing the convenience yield in this way is that we can show how irresponsible fiscal policy erodes the safe-asset role of government debt. Another benefit is that we can see that the full cost of making government debt a safe asset involves financial instability and the crowding out of real investment and private liquidity creation.
- (iii) *Non-Ricardian macro-fiscal models:*⁸ Similar to this literature, we are very interested in the trade-off about how the government backs its liabilities. In our model with aggregate risk, government debt is partially backed by an exogenous surplus process but also by restrictions that create captive demand within the financial sector in bad times and so change the price process of government debt. We believe this makes the following important contributions to this literature: (i) we provide a model of an endogenous convenience yield that, unlike other papers in the literature, is intimately related to government fiscal policy, and (ii) we relate the convenience yield to frictions within the financial sector that reflect some overlooked features of financial history. Ultimately, this means that, in our model, exploiting the convenience yield is hard work that depends very tightly on the fiscal rule, and doesn’t invalidate the key trade-offs involved in backing government debt with taxation. In this sense, we show that convenience

⁷Caballero et al. (2008), Caballero et al. (2017), Choi et al. (2022), Kekre and Lenel (2024).

⁸This includes (but is not limited to) Sargent and Wallace (1981) and the “fiscal theory of the price level” literature, e.g., Leeper (1991), Sims (1994), Woodford (1994), Cochrane (2023), Bianchi et al. (2023), and the recent literature on fiscal backing, e.g., Jiang et al. (2022a,b); Chen et al. (2022).

yields are not an alternative backing for all government debt. There is no free lunch. Overall, we believe we show how to introduce convenience yields while maintaining the importance of fiscal policy for determining the role of government debt.

3 Empirical Connections

Our model in Section 2 provides sharp predictions about how government regulation and fiscal policy influence the convenience yield. We now consider whether these predictions are consistent with empirical evidence. We focus on two datasets: a new collection of historical convenience yields in the US covering the period 1860-2022 (from our companion paper [Payne and Szöke \(2024\)](#)) and cross-sectional convenience yields across the Eurozone during the sovereign debt crisis covering the period 2003-2022. We use the first dataset to look for evidence that financial repression correlates with changes in the relationship between debt issuance and the convenience yield. We use the second dataset to study how fiscal difficulties correlate with the erosion of the risk-adjusted convenience yield, even when government debt is privileged by regulation.

3.1 Historical Convenience Yields in the US

An important prediction of our model is that restrictions on the financial sector change the convenience yield at which the government can issue debt. The US is a particularly interesting country for studying this prediction because different policy makers have organized very different financial systems. The challenge is that we need to work with historical data, which requires us to construct and analyse new estimates for historical convenience yields. In this section, we take up these challenges.

3.1.1 Context on Historical Financial Sector Regulation

To help interpret the historical data, we outline some key historical changes in monetary, financial, and fiscal policy. We provide a summary in Table 1 and a more comprehensive time-line in Appendix C.

1791-1862: Banks of The US and State Banks. Between April 1792 and February 1862, the federal government minted gold and silver coins but not paper notes. Instead, state legislatures chartered state banks, which could issue their own bank notes. Initially, the First (1791-1811) and Second (1816-1836) Banks of the United States operated at the national level and indirectly regulated state bank bank note creation but Andrew Jackson (1829-1837) allowed the Bank’s charter to expire (1836). In the subsequent decades (1837-1862), states expanded their banking sectors by allowing the automatic chartering of banks without requiring explicit approval from the state legislature. This period is often referred to as the “free banking era” and was perceived to be characterized by weak enforcement of bank portfolio restrictions, high bank risk taking, and discounted state bank notes. From the point of view of our model, we interpret this as a period with a low effective leverage requirement (low ϱ) and no particular weight on US federal debt ($\kappa = 0.5$).

Table 1: Summary of Financial Eras

	Regulation Parameters	Discussion
1791-1862	$\varrho \approx 0, \kappa = 0.5$	<i>Pre-Civil War:</i> bank regulation was typically at the state level, and regulation was not tightly enforced.
1862-1913	$\varrho = 0.9, \kappa = 1$ for $q^b \leq 1$	<i>National Banking Era:</i> has tight repression on the banking sector, which could only use government debt to back money creation.
1913-2007	$\varrho > 0, \kappa$ varying and more implicit	<i>FED and New Deal Regulation:</i> has implicit advantages for government debt through the acceptance of US debt at the FED discount window and the Bretton Woods reserve requirements (from 1944-1971).
2008-2024	$\varrho =$ leverage ratio, $\kappa =$ risk weight on US debt	<i>Basel III and Dodd-Frank Act:</i> led to increased regulation of the financial sector, with asset requirements based on their risk weights.

1862-1913: National Banking System. The outbreak of the Civil War in 1861 put significant strain on the monetary and financial systems, leading to major policy changes. On February 25, 1862, Congress passed a Legal Tender Act that authorized the Treasury to issue 150 million dollars of a paper currency known as greenbacks that the government did not promise immediately to exchange for gold dollars. In addition, between 1863-6, Congress passed a collection of National Banking Acts, which established a system of nationally chartered banks and the Office of the Comptroller of the Currency. National banks faced restrictions on what loans they could make⁹ and were allowed to issue bank notes up to 90% of the minimum of par and market value of qualifying US federal bonds.¹⁰ These national bank notes were intended to replace the state bank notes as a standardised currency that could be used across the country. In order to achieve this, Congress imposed a 10% annual tax on state bank notes, which was significantly greater than the 1% annual tax on national bank notes.¹¹ From the point of view of our model, the National Banking Era is a period of explicit financial repression with $\varrho = 0.9$ and $\kappa = 1$ when bonds trade below par (and $\kappa = q_t^b$ when bonds traded above par).

1915-1971: Establishment of the Federal Reserve Bank, Deposit Insurance, and Bretton Woods. Bank runs and stock market crashes were a common feature of all different monetary and banking policy arrangements during the 19th century. There were country wide bank panics in 1819, 1827, 1857, 1873, 1893, and 1907 as well as many other local bank panics in New York and other

⁹National banks could only operate one branch. They were restricted from making mortgages unless they were operating in rural areas, where they could make a limited range of loans collateralized by agricultural land.

¹⁰Technically, national banks could issue bank notes for circulation according to the following rules. Banks had to deposit certain classes of US Treasury bonds as collateral for note issuance. Permissible bonds were US federal registered bonds bearing coupons of 5% or more. Deposited bonds had to be at least one-third of the bank's capital (not less than \$30,000). Banks could issue bank notes up to an amount of 90% of the maximum of the market value of the bonds and the par value of the bonds. The 90% value was changed to 100% in 1900.

¹¹Before 1900, the banks had to pay 1.0% tax on the notes they had issued. After 1900, they had to pay a 0.5% tax.

financial hubs. In response, The Federal Reserve System Act was passed in 1913 to create a Federal Reserve Bank (FRB) to act as a reserve money creator of last resort to prevent bank runs. The Bank started operations in late 1914. The inability to prevent bank failures during the depression prompted Franklin D. Roosevelt to introduce a further reorganization of the financial sector. The 1933 Banking act introduced deposit insurance for retail banks, established the Federal Deposit Insurance Corporation (FDIC), and separated commercial and investment banking. The 1934 and 1938 household acts established the Federal National Mortgage Association (commonly known as Fannie Mae) to insure long term mortgages. These reforms ultimately relaxed the explicit financial repression from the National Banking Era. However, the FRB started to privilege government debt as collateral for discount window lending, which acted as an implicit advantage to government debt.

The difficulties of financing World War II led to the government “fixing” the yield curve from 1942-1951, with yields on long term bonds set at 2.5% (see [Garbade \(2020\)](#)). The policy was implemented through coordination between the Treasury and the Federal Reserve, with the Fed agreeing to absorb excess bond supply at the fixed price, and implicit coordination with the banking system, which ended up predominately holding government debt. This coordination ended in 1951 with the Treasury-Fed Accord that establishes official Fed independence from fiscal policy. At the international level, the 1944 Bretton Woods Agreement set up an international system of fixed exchange rate with US dollar convertible to gold.

1972-2007: Financial Deregulation. Internationally, the US effectively terminated the Bretton Woods systems in 1971 by ending convertibility to gold. Domestically, the government embarked on a program of financial deregulation. In 1994, the Riegle-Neal Interstate Banking and Branching Efficiency Act allowed banks to operate across states. In 1999, the Gramm-Leach-Bliley Act repealed the provisions of the Glass-Steagall Act that prohibited banks from holding other financial companies. In our model, we would interpret this as a decrease in the effective ϱ .

2008-2024: Financial Crisis, Basel-III, and Dodd-Frank Act. The 2007-9 financial crisis led to extensive new regulation on the banking sector and the Dodd-Frank Wall Street Reform and Consumer Protection Act. In addition, the Basel-III regulation introduces restrictions so that ϱ reflects the bank leverage requirement and κ is the “risk” weight on government debt for calculating risk weighted asset ratios.

3.1.2 Data and Methodology

In a previous paper, [Payne et al. \(2022\)](#), we assembled prices and cash flows for the universe of government bonds and estimated the zero-coupon yield curve on US federal debt. In our companion paper, [Payne and Szöke \(2024\)](#), we assemble a companion data-set with a large collection of corporate bonds between 1860 and 1940. We briefly describe the original sources and the details of the data collection in Appendix B. We use the classification system from [Macaulay et al. \(1938\)](#) to identify a collection of low risk corporate bonds (primarily railroad bonds) for the period before 1900 when there is no Moody’s rating system (and extend the classification using pricing errors to group bonds).

We estimate the historical yield curve on low risk corporate debt using the empirical approach developed in [Payne et al. \(2022\)](#). We then combine our estimates for historical US Treasury yields and our estimates for historical corporate bonds, with existing modern series.

The other series are taken from existing databases. Our series for the monthly market value of government debt is taken from [Hall et al. \(2018\)](#).¹² Our series for annual GDP (1790-2023) is taken from [Officer and Williamson \(2021\)](#). Our series for quarterly GDP (1947-2023) is taken from FRED (BEA Account Code: A191RC). Following [Krishnamurthy and Vissing-Jorgensen \(2012\)](#), our series for quarterly and annual volatility are calculated as the annualized standard deviation stock market returns. For 1815-1914 we use the NYSE Monthly Index from [Goetzmann et al. \(2001\)](#). For 1915-2023 we use the Dow Jones Monthly Index (NBER Indicator: m11009b). We calculate the slope of the government yield curve as the 10 year yield - 1 year yield using our yield curve estimates from [Payne et al. \(2022\)](#). Our series for the historical expected default rate on corporate debt comes from [Giesecke et al. \(2011\)](#).

3.1.3 Times Series For Convenience Yields

Figure 8 shows the time series for the 10-year corporate yield, the 10-year treasury yield, and the “convenience yield”, as measured by the corporate yield minus the treasury yield. The grey bands in the background represent banking crises. We can see that throughout the National Banking Era (1860-1917), the convenience yield was typically relatively high, around 1.5%, and banking crises were very frequent. Although we cannot make any causal claims and there are lot changes through this period, this observation is very consistent with our model. The convenience yield then drops down significantly to close to zero around WWII before spiking again during the 1970s.

¹²Available on George Hall’s website: <https://people.brandeis.edu/~ghall/>

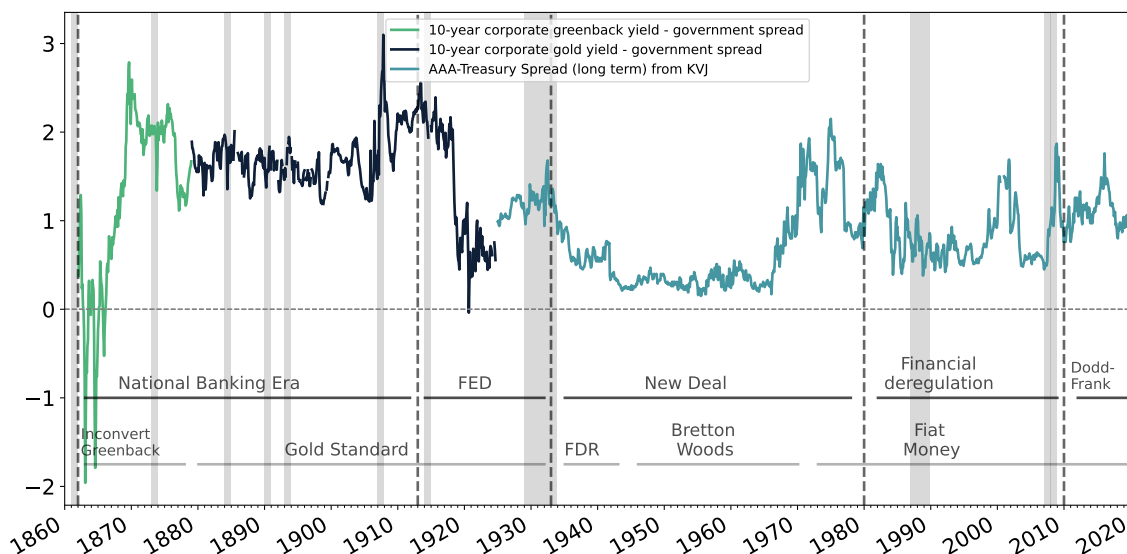


Figure 8: The Convenience Yield: 1860-2020

3.1.4 Relationship Between Convenience Yields and Government Debt Supply

An important prediction of our model is that the ability of the US government to issue government debt at low cost depends upon the restrictions on the financial sector. This implies that the relationship between the convenience yield and the debt-to-GDP ratio should vary across the different eras of financial sector regulation. We can now use our historical convenience yields to look for evidence that this took place.

Figure 9 shows a scatter plot with the ratio of the market value of government debt/GDP on the x-axis and the convenience yield on the y-axis. This extends the plot in [Krishnamurthy and Vissing-Jorgensen \(2012\)](#) from the period 1919-2007 to the period 1860-2022. We can see that within the National Banking Era (1868-1914) and around WWII (1940-1965), the elasticity of the convenience yield to government debt issuance is very low. These are the periods when the government intervened most aggressively in financial markets to try and create a market for government debt. The period with the much studied downward sloping “demand curve is essentially the interwar period and the last third of the 20th century. These are both periods, where the government relaxes demand for government. Ultimately, we interpret this plot as suggestive evidence that the equilibrium relationship between government debt supply and the convenience yield has very different properties under different financial regulation regimes.

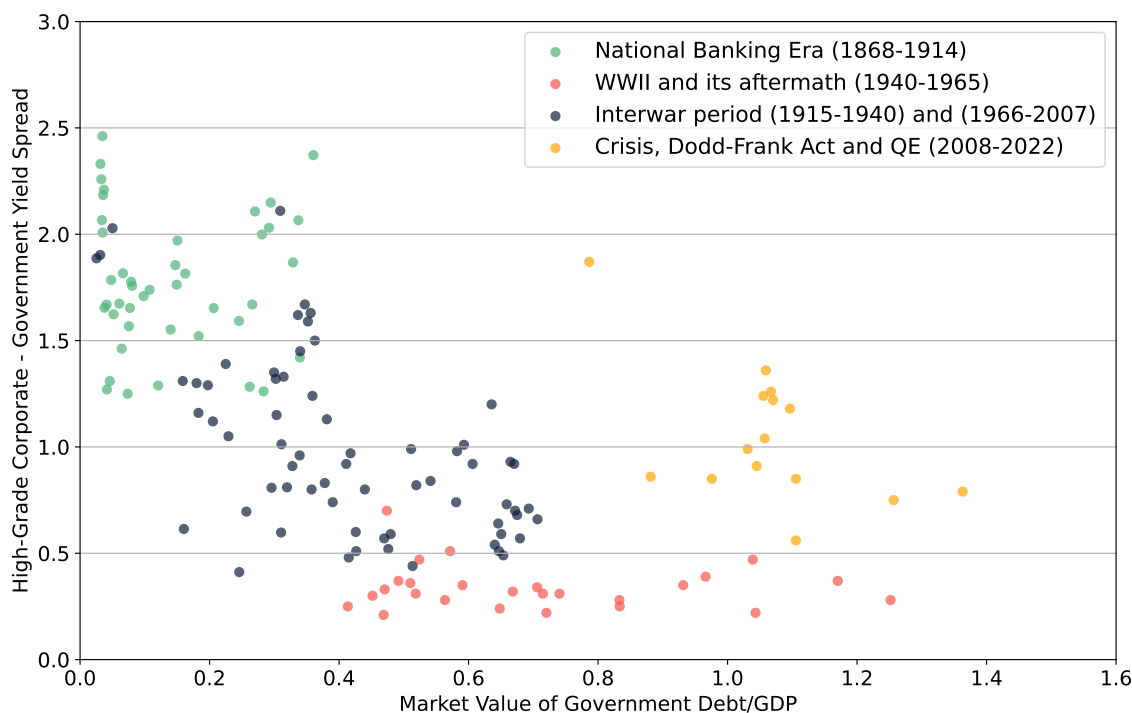


Figure 9: Convenience Yield vs Debt/GDP: 1868-2022, Annual

To study this more systematically, in Table 2 we rerun the regression from [Krishnamurthy and Vissing-Jorgensen \(2012\)](#) using different sub-samples from our extended dataset. The regressions confirm what can be seen visually in Figure 9:

1. For 1868-1914 the debt/GDP ratio is not significant and has a coefficient close to zero. We run the regression with and without the average corporate default rate during the period but find it makes little difference, which suggests it is not changes in the likelihood of corporate default that is driving the result.
2. For the overall period from 1915-2007, we find a significant downward sloping relationship between the convenience yield and government debt supply, with similar values to [Krishnamurthy and Vissing-Jorgensen \(2012\)](#).¹³ However, within this period there is a lot of variation in the relationship. The coefficient on $\log(Debt/GDP)$ is close to zero for the period of yield curve control and implicit financial repression. It also has different negative values for the periods 1915-33, 1952-1971, and 1972-2007.
3. For the Dodd-Frank period 2010-2022, it looks like the curve shifts and slope flattens.

¹³[Krishnamurthy and Vissing-Jorgensen \(2012\)](#) uses a different range and finds the coefficient to be in the range of -0.55 to -1.66 depending upon the exact time period and bond yield chosen.

	1868-1914 (NBE)		1915-2007 (FED, Macroprudential Regulation)						2008-2022
	no EDF	with EDF	1915-33	1942-1951	1952-1971	1972-2007	1952-2007	All	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
<i>Log(debt/GDP)</i>	-0.023 (0.065)	-0.033 (0.073)	-0.500*** (0.086)	0.125*** (0.027)	-1.061*** (0.086)	-0.887*** (0.083)	-1.067*** (0.062)	-0.496*** (0.413)	-0.605 (1.262)
<i>Volatility</i>	-0.123 (0.462)	-0.181 (0.500)	-0.552 (0.799)	3.281*** (0.265)	5.496** (1.784)	2.794*** (0.835)	4.307*** (0.746)	0.464 (0.799)	2.847*** (0.318)
<i>Slope</i>	-0.069** (0.462)	-0.066** (0.034)	0.167** (0.071)	-0.057** (0.265)	0.214*** (0.050)	0.036* (0.021)	0.097*** (0.019)	0.031 (0.036)	0.060 (0.036)
<i>EDF</i>		0.006 (0.020)							
<i>constant</i>	1.808*** (0.235)	1.778*** (0.256)	0.252 (0.257)	-0.004 (0.937)	-1.037*** (0.133)	-0.378** (0.130)	-0.829*** (0.090)	0.264* (0.096)	0.484* (0.175)
<i>R</i> ²	0.115	0.117	0.727	0.970	0.702	0.593	0.662	0.492	0.326
F-test	0.196	0.316	0.0003	5.91e-05	3.76e-19	8.01e-21	7.03e-44	5.91e-13	0.0001
AIC	25.86	27.74	6.426	-51.86	-3.041	13.26	40.02	63.64	-3.085
N	42 (A)	42 (A)	18 (A)	10 (A)	77 (Q)	111 (Q)	191 (Q)	92 (A)	56 (Q)

Nota: * $p < 0.1$; ** $p < 0.05$; *** $p < 0.01$

Table 2: Relationship Between Treasury Supply and Bond Spreads

The scatter plot in Figure 9 and whole sample regression in Table 2 have been interpreted as reflecting an exploitable stable demand function. Many macro models capture this using bond-in-the-utility or some other exogenous bond demand function. However, our illustrative model and our sub-sample regressions suggest that researchers should be very cautious about this exercise because attempts to exploit the relationship in Figure 9 are very vulnerable to the Lucas critique. Financial regulation and government fiscal policy change the bond demand function, which ends up being reflected in the relationship between government debt supply and the convenience yield. In this sense, the convenience yield curve in macro-public finance is very similar to the Phillips curve relationship in macro-labor—neither are stable to changes in government policy. We return to the question of generating the different elasticities in Table 2 using a microfounded model when we construct the infinite horizon version of our model in Section 4.

3.2 Cross Section of Convenience Yields in the Eurozone

The historical US data provides a comparison across very different regulatory eras. However, it is difficult to isolate changes in the role of government debt from changes in the risk on government debt. For the modern period, we can use data from credit default swaps (CDS) to approximate risk-adjusted convenience yields. In this subsection, we follow [Jiang et al. \(2020b\)](#) and do this for European countries during the Eurozone crisis (2009-15). This allows us to study a second important prediction of our model: increases in the likelihood of government default (implicit or explicit) erode the risk-adjusted convenience yield.

3.2.1 Regulatory Context

In the Eurozone context, there are a number of components of regulation that are particularly important to our analysis and are well captured by our model. The first is the treatment of government debt from European countries as collateral by the European Central Bank (ECB). Before 2005, the ECB decided collateral terms using a private discretionary rating system that could deviate from those of private credit agencies. In 2005, the ECB moved to a market based criteria that linked the collateral value to a combination of the credit ratings from different agencies. In principle, this meant that the government debt of a number of European countries (particularly Greece and Cyprus) should have become ineligible as collateral during the Eurozone crisis (2009-2015). However, the ECB repeatedly relaxed the criteria. In 2008, they lowered the minimum market credit rating requirement and then announced waivers for Greek debt (April 2010), Irish debt (March 2011), and Portuguese debt (July 2011). From May 2010, the ECB started to purchase Greek, Portuguese, and Irish bonds as part of its “Security Markets Programme” (SMP), which was extended to Spanish and Italian bonds in 2011. We interpret the April 2010 announcement as resolving uncertainty that European government debt could lose its collateral status. Ultimately, the ECB treatment of Greek, Irish, Portuguese, Spanish, and Italian debt as collateral allowed the European banks to take low interest loans from the ECB and purchase high yielding government assets without increasing their risk-weighted assets or their TIER 1 capital ratio.

In addition, the deposit insurance system in Europe does not have the same backing as in the

US. All European Union member states are required to maintain a minimum government deposit guarantee. However, this guarantee is not backed by the ECB or the European Union but instead by the independent member state. So, for countries in the Eurozone, they cannot easily create money to recapitalize their banking sectors. In this sense, as in our model, the Eurozone deposits are not necessarily risk free, particularly when the government is unable to access debt markets. We saw this risk materialize in Iceland, Cyprus, and Greece during the Eurozone crisis.

3.2.2 Risk Adjusted Convenience Yields

We can express the yield on a government bond from Eurozone country i with maturity h and price $q_t^{i,h}$ as:

$$y_t^{i,h} = r_t^h - \chi_t^{i,h}$$

where $y_t^{i,h} = -\frac{1}{h} \log(q_t^{i,h})$ is the yield on the bond, $r_t^h = -\frac{1}{h} \log \mathbb{E}[\Xi_{t,t+h}]$ is the expectation of the h period (nominal) SDF pricing government debt, and $\chi_t^{i,h}$ is the convenience yield on the bond. In analogous manner to Section 2.2.4, we breakup the convenience yield into:

$$\chi_t^{i,h} = \tilde{\chi}_t^{i,h} - s_t^{i,h}$$

where $s_t^{i,h} = -\frac{1}{h} \log \mathbb{E}_t \left[\Xi_{t,t+h} \prod_{j=1}^h (1 - \mathfrak{d}_{t+j}^i) \right] + \frac{1}{h} \log \mathbb{E}[\Xi_{t,t+h}]$ is market rate for default risk insurance, \mathfrak{d}_{t+j}^i is the probability of government default, and $\tilde{\chi}_t^{i,h}$ is the risk-adjusted convenience yield on the bond. Following the approach in [Jiang et al. \(2020b\)](#), we proxy $s_t^{i,h}$ by the credit default spread and, instead of estimating r_t , we focus on the difference between the convenience yield in country i and Germany. Assuming that there is a common SDF across the Eurozone, we have that:

$$\tilde{\chi}_t^{i,h} - \tilde{\chi}_t^{DE} = s_t^{i,h} - s_t^{DE,h} - (y_t^{i,h} - y_t^{DE,h})$$

We plot the risk-adjusted convenience yield differentials in Figure 10 for key Eurozone countries over the period from 2004 to 2024 which includes the European Sovereign Debt Crisis. The top row are countries that maintained relatively strong fiscal positions during the Eurozone crisis while the bottom row are countries that faced ratings downgrades and speculation about their fiscal sustainability. For calculations, we use Euro denominated 5 year CDS spreads from Markit and 5 year sovereign yields from Global Financial Data. Evidently, risk-adjusted convenience yields decreased significantly more in the countries on the bottom row. In Figure 11 we plot the risk-adjusted convenience yield against the CDS spread and show that the negative relationship we saw in the cross-section is also true in the time series. These plots suggest that, even after controlling for the different risk characteristics of the sovereign bonds, there was a higher erosion of sovereign debt premia in the countries facing fiscal challenges during the crisis. As we saw in Subsection 2.4, this is a puzzle for workhorse macroeconomic models that use BIU or BIA formulations to generate convenience yields because those models predict the risk-adjusted convenience yield increases when the market value of government debt falls. By contrast, our model suggests a potential resolution: that an increase

in the probability of government default lead to a decrease in the risk adjusted convenience yield because the hedging role of Irish, Italian, Portuguese, and Spanish debt diminished ($\tilde{\chi}_h$ decreased) even though their collateral role at the ECB stayed the same ($\tilde{\chi}_r$ stayed the same). A complementary explanation is proposed by Jiang et al. (2020b), which suggests that the heterogeneous decreases in the risk-adjusted convenience yields reflect how different fiscal policies during the crisis lead to different expectations about post-crisis debt issuance. We nest both explanations in our infinite horizon macroeconomic model in Section 4.

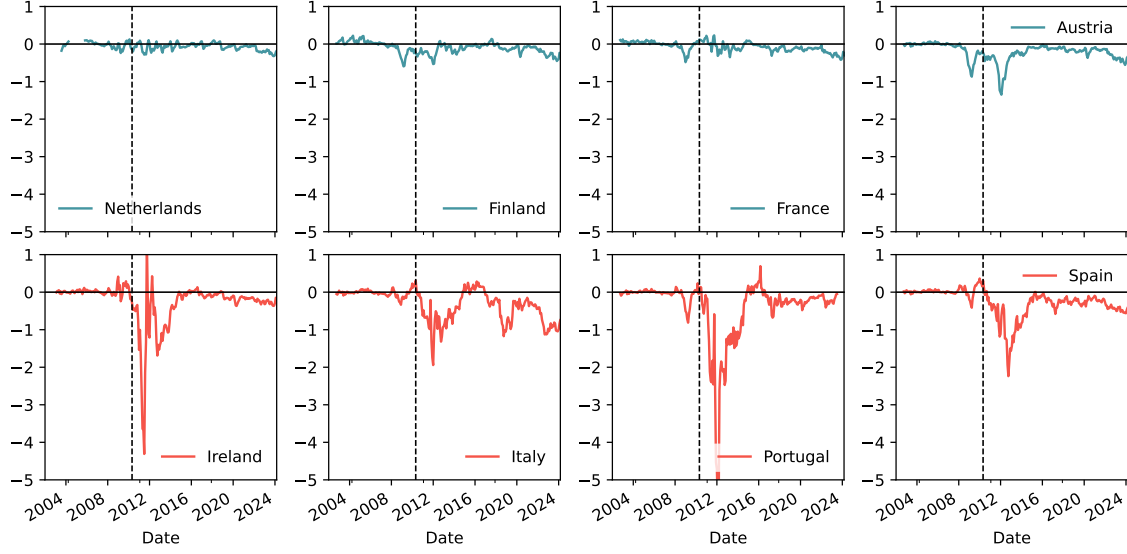


Figure 10: Difference in Risk Adjusted Convenience Yields to Germany.

The dashed line is at April 2010, the date at which the ECB announced the waiver for Greek debt.

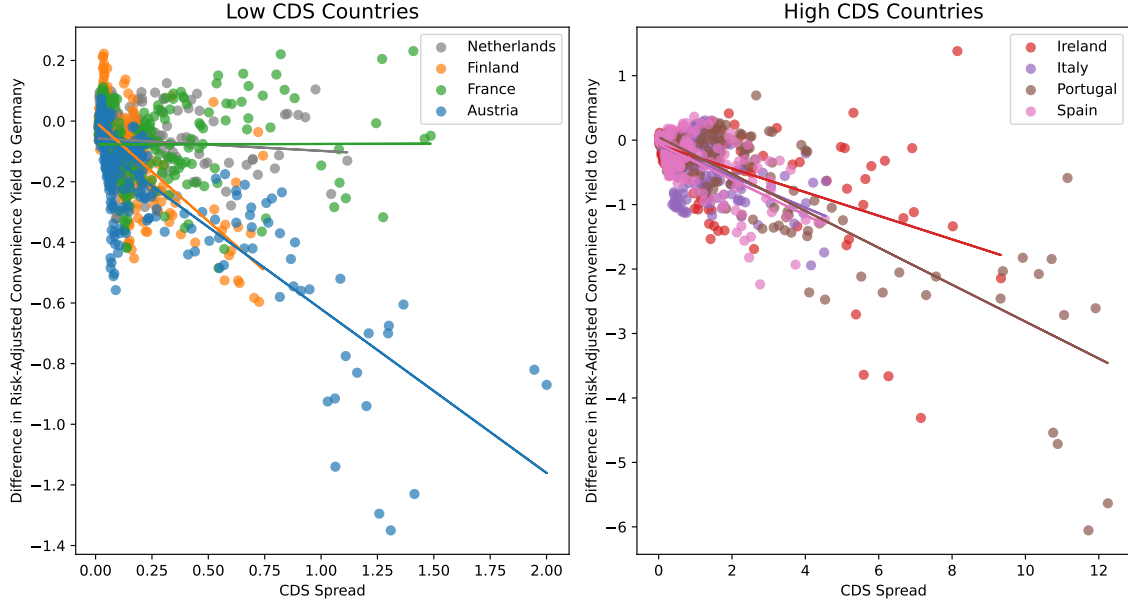


Figure 11: Difference in Risk Adjusted Convenience Yields to Germany vs CDS Spreads.

The left plot show countries that maintained low CDS spreads during the Eurozone crisis while the right plots shows countries that had high CDS spreads. The dots represent monthly observations and the lines represent linear regressions for each country.

4 Infinite Horizon Macroeconomic Model (Preliminary—To be Revised)

Our finite horizon model illustrates how the regulation of bank balance sheets can generate a convenience yield. However, it has limited ability to speak to the time series data. For this reason, we now move to an infinite horizon general equilibrium model with long-term government debt, a fiscal rule for debt issuance, and a mean reverting productivity process. We use this model to make the following additional points:

1. Varying the level and tightness of financial repression allows us to generate the different patterns (i.e. the shifts and slope changes) we observed in the historical Convenience Yield vs Debt/GDP plots.

4.1 Environment

Setting: Time is discrete in infinite horizon. There is one consumption good. The economy is populated by a representative household that directly or indirectly owns all claims to production. The economy also contains a representative firm, a representative financial intermediary, and a government, all of which issue securities. The firm issues equity claims and creates capital to produce

consumption goods. The intermediary issues deposits and equity. The government issues geometrically decaying long-term bonds that pay repay a fraction ζ of the principal each period. The high level relationship is given in figure 12.

Firm		Government		Bank		Household	
A	L	A	L	A	L	A	L
Capital	Shares	Taxes	Bonds	Shares	Deposits	Deposits	Net worth
				Bonds	Equity	Equity	
						Shares	
						Bonds	

Figure 12: Agent balance sheets

Representative household: ranks allocations according to:

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t) + \nu(d_t^h + \zeta b_t^h) - \Psi_{t+1}(a_{t+1}^f) e_t^h - \Omega_{t+1}(\tau_{t+1}) \right]$$

where c_t is household consumption at time t , d_t^h is the household holdings of financial intermediary deposits, b_t^h is household holdings of government debt, a_{t+1}^f is the net-worth of the financial intermediary, e_t^h is household equity holdings, and τ_{t+1} is the tax rate. The function $\nu(\cdot)$ is increasing and captures the non-pecuniary benefit of holding “safe-assets”. The function $\Psi(\cdot)$ is decreasing and captures the cost of bank “insolvency”. The function $\Omega(\cdot)$ is increasing and captures the distortion from raising taxes. The household also faces the short selling constraints $d_t^h \geq 0$, $s_t^h \geq 0$, and $b_t^h \geq 0$, where s_t^h is household holdings of shares in the firm. At time 0, the household is endowed with capital, k_0 , and sells it to the representative firm. Each period, the household is endowed with a unit of labor, $l_t = 1$.

Representative firm: has a Cobb-Douglas production technology subject to stochastic productivity z_t :

$$y = z_t k_{t-1}^\alpha l_t^{1-\alpha}$$

$$\log(z_t) = (1 - \eta) \log(\bar{z}) + \eta \log(z_{t-1}) + \epsilon_t$$

where l_t is labor hired by the firm and k_{t-1} is firm capital stock. The evolution of capital stock is

given by the constant-return-to-scale technology:

$$k_t = (1 - \delta)k_{t-1} + \Phi(\iota_{t-1})k_{t-1}$$

where $\iota_{t-1} := \frac{i_{t-1}}{k_{t-1}}$ is the investment-capital ratio and $\Phi(\cdot)$ is an “adjustment” function.

Representative financial intermediary: On the liability side of their balance sheet, the intermediary issues “safe-assets”, d_t^f , that each pay 1 good at $t + 1$ and equity, e_t^f , that pays a dividend δ_{t+1}^e at $t + 1$. On the asset side, they purchase shares in the firm, s_t^f , and government debt, b_t^f . The intermediary faces a regulatory collateral constraint that at any point in time, a proportion κ^b of the maturing safe asset must be backed by the market value of government debt:

$$(\zeta + (1 - \zeta)q_t^b)b_t^f \geq \kappa^b d_t^f$$

where q_t^b is the price of government debt.

Government: Each period, the government raises lump sum taxes, τ_t , issue bonds, b_t , and undertakes spending $g_t = g(z_t)y_t$ that is a function of the aggregate state, where $g(\cdot)$ is a decreasing function. Bonds are issued at par and repay a fraction ζ of the principal each period. They face the inter-temporal budget constraint that:

$$g_t + \zeta b_{t-1} \leq \tau_t + q_t^b(b_t - (1 - \zeta)b_{t-1}).$$

Following [Bohn \(1998\)](#) and [Bai and Leeper \(2017\)](#), we impose that the government sets a budget feasible tax policy to target a long run debt to GDP ratio:

$$\hat{\tau}_t - \hat{\tau}^* = \gamma (\hat{b}_{t-1} - \hat{b}^*)$$

where $\hat{\tau}_t := \tau_t/y_t$ and $\hat{b}_{t-1} := b_{t-1}/y_{t-1}$. The government also chooses regulatory portfolio restriction $\kappa^b \geq 0$.

Markets: All markets are competitive. Let q_t^s denote the firm equity price. Let q_t^b denote the government bond price. Let (q_t^e, q_t^d) denote the time- t price of equity and safe assets issued by the financial intermediary. We use upper case R for the gross return and r for the yield. Let w_t denote the wage rate. We are focusing on the case when ζ is a parameter and to simplify notation we define $\tilde{q}_t^b := \zeta + (1 - \zeta)q_t^b$.

Functional Forms: We impose the utility forms:

$$u(c) = \log c, \quad \nu = \log(\exp(-r_t^d)d_{t+1}^h + \zeta b_{t+1}^h)$$

and the capital adjustment cost:

$$\Phi(\iota) = \phi_0 + \frac{\bar{\phi}}{1 - \phi} \iota^{1 - \phi}$$

Discussion of environment frictions: This environment is characterized by two key distortions. The first distortion is that the households get additional utility from holding safe assets through the ν function. The second is distortion is that the the financial intermediaries, who have the technology to create safe assets, face the cost function, Ψ_{t+1} , when they become insolvent. This effectively makes the safe asset issuers less willing to take on risk than the households. Although we are not modelling the microfoundations for these distortions, we believe the model captures the key friction in macro-finance models.

Discussion of government policy rule: As in our three period model, we interpret our government tax and spending policies as arising from unmodelled political frictions. The difference is that now these frictions induce the government to run deficits during recessions and then surpluses in expansions to return to a target long-run debt-to-GDP ratio. This policy potentially imposes welfare costs if running surpluses induces the government to move the tax rate around. We are going to study how financial regulation and changes in the convenience yield on government debt influence the welfare cost of running such a fiscal policy.

Discussion of regulatory constraints: In addition to the environmental frictions, the environment also contains regulatory constraints that restrict the portfolio choices of agents and so change asset demand elasticities. The key constraint is the collateral requirement that the market value of government debt cannot fall below $\kappa^b d_t$. Effectively, this constraint means that the government only allows the financial sector to use their financial technology to issue deposits to households if they hold government bonds. In this sense, the government is repressing the financial sector to create demand for their debt and so drive up the price of their debt when the collateral constraint binds. The other regulatory constraint is that the household may not hold government bonds and firm equity. This segments the market for government debt so that the only agents trading government debt are the financial intermediaries facing the collateral constraint requiring them to hold government bonds. Ultimately, these regulatory constraints will allow the government to indirectly tax the value that the financial intermediaries generate through safe asset creation.

4.2 Competitive Equilibrium

In this subsection, we set up the agent problems and characterize the competitive equilibrium.

4.2.1 Household Problem

We set up the household problem recursively. The (individual) state variable for the household is a_t^h , which denotes the wealth of the household at the start of period t . The household solves problem

(4.1) below:

$$\begin{aligned}
V_t(a_t^h) &= \max_{c_t, d_t^h, b_t^h, e_t^h, s_t^h} \left\{ u(c_t) + \nu(d_t^h + \zeta b_t^h) - \mathbb{E}_t [\Psi_{t+1} e_t^h + \Omega_{t+1} + \beta V_{t+1}(a_{t+1}^h)] \right\} \\
\text{s.t. } & c_t + q_t^e e_t^h + q_t^s s_t^h + q_t^b b_t^h + q_t^d d_t^h \leq a_t^h \\
& a_{t+1}^h = (\delta_{t+1}^e + q_{t+1}^e) e_t^h + ((1 - \tau_{t+1}) \delta_{t+1}^s + q_{t+1}^s) s_t^h + \tilde{q}_{t+1}^b b_t^h + d_t^h \\
& 0 \leq d_t^h, \quad 0 \leq b_t^h, \quad 0 \leq s_t^h
\end{aligned} \tag{4.1}$$

Taking first order conditions and imposing the envelope condition gives the “asset-demand” equations:

$$\begin{aligned}
[d_t^h] : & \quad q_t^d = \mathbb{E}[\xi_{t,t+1}] + \frac{\nu'(d_t^h + \zeta b_t^h)}{u'(c_t)} + \frac{\lambda_t^d}{u'(c_t)} \\
[b_t^h] : & \quad q_t^b = \mathbb{E}[\xi_{t,t+1} \tilde{q}_{t+1}^b] + \zeta \frac{\nu'(d_t^h + \zeta b_t^h)}{u'(c_t)} + \frac{\lambda_t^b}{u'(c_t)} \\
[e_t^h] : & \quad q_t^e = \mathbb{E}[\xi_{t,t+1} (\delta_{t+1}^e + q_{t+1}^e)] - \frac{\mathbb{E}_t[\Psi_{t+1}]}{u'(c_t)} \\
[s_t^h] : & \quad q_t^s = \mathbb{E}[\xi_{t,t+1} ((1 - \tau_{t+1}) \delta_{t+1}^s + q_{t+1}^s)] + \frac{\lambda_t^s}{u'(c_t)}
\end{aligned}$$

where $\xi_{t,t+1} := \beta u'(c_{t+1})/u'(c_t)$ is the household stochastic-discount-factor (SDF) and where $\lambda_t^d \geq 0$, $\lambda_t^b \geq 0$, and $\lambda_t^s \geq 0$ are the multipliers on the household portfolio constraints on d_t^h , b_t^h , and s_t^h . Observe the Euler equations for d_t^h and b_t^h have been “distorted” by the household demand for safe assets, ν . Observe that the Euler equation for bank equity can be rewritten as:

$$q_t^e = \mathbb{E}_t \left[\xi_{t,t+1} \left(\delta_{t+1}^e + q_{t+1}^e - \frac{\Psi_{t+1}}{u'(c_{t+1})} \right) \right]$$

so we can see that the insolvency costs distort the price of price of bank equity.

4.2.2 Financial Intermediary Problem

The financial intermediary chooses a collection of asset portfolio and dividend payouts to maximise its market value by solving problem (4.2) below:

$$\begin{aligned}
V_0 &= \max_{\delta^e, s^f, b^f, d^f} \{ q_0^e + q_0^h h_1 - q_0^s s_1 - q_0^b b_1 \} \quad \text{s.t.} \\
& \delta_t^e + q_t^s s_t^f + q_t^b b_t^f - q_t^d d_t^f = a_t^f \\
& a_{t+1}^f = ((1 - \tau_{t+1}) \delta_{t+1}^s + q_{t+1}^s) s_t^f + \tilde{q}_{t+1}^b b_t^f - d_t^f \\
& q_t^e = \mathbb{E}_t \left[\xi_{t,t+1} \left(\delta_{t+1}^e + q_{t+1}^e - \frac{\Psi_{t+1}(a_{t+1}^f)}{u'(c_{t+1})} \right) \right] \\
& \tilde{q}_{t+1}^b b_t^f \geq \kappa_t^b d_t^f \\
& s_t^f \geq 0
\end{aligned} \tag{4.2}$$

The first order conditions gives the following financial intermediary asset demand and supply equations :

$$\begin{aligned} [s_{t+1}^f] \quad 0 &= -q_t^s \left(1 - \frac{\partial_{e\delta}\Psi_t}{u'(c_t)} \right) + \mathbb{E}_t \left[\xi_{t,t+1} \left(1 - \frac{\partial_{e\delta}\Psi_{t+1}}{u'(c_{t+1})} - \frac{\partial_{ea}\Psi_{t+1}}{u'(c_{t+1})} \right) (\delta_{t+1}^s + q_{t+1}^s) \right] + \mu_t^s \\ [b_{t+1}^f] \quad 0 &= -q_t^b \left(1 - \frac{\partial_{e\delta}\Psi_t}{u'(c_t)} \right) + \mathbb{E}_t \left[\xi_{t,t+1} \left(1 - \frac{\partial_{e\delta}\Psi_{t+1}}{u'(c_{t+1})} - \frac{\partial_{ea}\Psi_{t+1}}{u'(c_{t+1})} + \mu_{t+1}^b \right) \tilde{q}_{t+1}^b \right] \\ [\hat{d}_{t+1}^f] \quad 0 &= \left(1 - \frac{\partial_{e\delta}\Psi_t}{u'(c_t)} \right) - \mathbb{E}_t \left[\xi_{t,t+1} \left(1 - \frac{\partial_{e\delta}\Psi_{t+1}}{u'(c_{t+1})} - \frac{\partial_{ea}\Psi_{t+1}}{u'(c_{t+1})} + \kappa_t^b \mu_{t+1}^b \right) \exp(r_t^h) \right] \end{aligned}$$

In equilibrium, market clearing and the regulatory constraint on household portfolio $s_t^h \leq \kappa_t^s$ with $\kappa_t^s \geq 0$ implies that the short-selling constraint for the financial intermediary never binds ($\mu_t^s = 0$).

4.2.3 Firm Problem

Taking prices and the shareholder's SDF as given, firms solve:

$$V_t(k_{t-1}) = \max_{\iota_t, l_t} \left\{ z_t k_{t-1}^\alpha l_t^{1-\alpha} - w_t l_t - \iota_t k_{t-1} + \mathbb{E}_t \left[\hat{\xi}_{t,t+1} V_{t+1}(k_t) \right] \right\} \quad (4.3)$$

where $\hat{\xi}_{t,t+1}$ is the weighted average of the household and firm stochastic discount factors and the firm is subject to the capital accumulation technology:

$$k_t = (1 - \delta + \Phi(\iota_t)) k_{t-1}$$

where $\iota_t := \frac{i_t}{k_{t-1}}$ is the investment-capital ratio. The first order conditions are:

$$\begin{aligned} [w_t] : \quad & 0 = (1 - \alpha) z_t k_{t-1}^\alpha l_t^{-\alpha} - w_t \\ [\iota_t] : \quad & 0 = -k_{t-1} + \mathbb{E}_t [\hat{\xi}_{t+1} \partial_k V_{t+1}(k_t) \Phi'(\iota_t) k_{t-1}] \end{aligned}$$

Guess the form $V_t = v_t k_{t-1}$, then the first order condition for ι_t becomes:

$$\Phi'(\iota_t) = \mathbb{E}_t [\hat{\xi}_{t+1} \partial_k V_{t+1}(k_t)]^{-1} = \mathbb{E}_t [\hat{\xi}_{t+1} v_{t+1}]^{-1}$$

The Bellman equation gives:

$$v_t = \left(\alpha \frac{y_t}{k_{t-1}} - \iota_t \right) + \frac{1 - \delta + \Phi(\iota_t)}{\Phi'(\iota_t)}$$

Let $\hat{r}_t^Y := \alpha \frac{y_t}{k_{t-1}}$ be the marginal return to capital (from production) and $\hat{r}_t^K = \frac{\Phi(\iota_t)}{\Phi'(\iota_t)} - \iota_t$ be the marginal return to capital (from reducing future adjustment costs¹⁴). Then, the value function

¹⁴This is the capital goods producer's return.

becomes:

$$V_t = \underbrace{(\hat{r}_t^Y + \hat{r}_t^K)k_{t-1}}_{\text{return on capital}} + \underbrace{\frac{(1-\delta)k_{t-1}}{\Phi'(\iota_t)}}_{\text{capital stock after production}}$$

and so the dividend and ex-dividend price are:

$$\delta_t^s = (\hat{r}_t^Y - \iota_t)k_{t-1}, \quad q_t^s = \frac{k_t}{\Phi'(\iota_t)}$$

4.2.4 Equilibrium Definition

Definition 3 (Competitive Equilibrium). Given a government fiscal rule and initial capital, k_0 , a competitive equilibrium is a sequence of prices $\{q_t^d, q_t^e, q_t^b, q_t^s, w_t\}_{t \geq 0}$, household choices, $\{c_t, d_t^h, b_t^h, e_t^h, s_t^h\}_{t \geq 0}$, financial intermediary choices, $\{\delta_t^e, d_t^f, b_t^f, s_t^f\}_{t \geq 0}$, and firm choices, $\{\iota_t, l_t\}_{t \geq 0}$ such that: (i) given prices, households solve equation (4.1), financial intermediaries solve equation (4.2), and firms solve equation (4.3), and (ii) markets clear:

$$d_t^h = d_t^f, \quad e_t^h = 1, \quad b_t^h + b_t^f = b_t, \quad s_t^h + s_t^f = 1, \quad l_t = 1,$$

$$y_t - \Psi_t = c_t + \iota_t k_{t-1} + g_t,$$

We solve the model globally using a collocation approach with parameters $\beta = 0.99$, $\alpha = 0.36$, $\delta = 0.025$, $\phi = 2.0$, $\zeta = 0.025$, and $(\eta, \sigma_z) = (0.97, 0.01)$.

4.3 Fiscal Capacity Over the Business Cycle

The convenience yield in our model is defined to be:

$$\chi_{t,\zeta} := \mathbb{E}_t[\xi_{t,t+1/\zeta}]^{-1} - \zeta \log(1/q_t^b)$$

which, as in the finite horizon model, is interpreted as the funding advantage of the government. In our dynamic model, the “convenience yield” on long-term government debt can potentially come from (i) regulation leading banks to buy more debt in recessions and (ii) the government reducing bond supply in recessions. However, our fiscal policy rule restricts the second channel because it increases bond issuance during recessions. To understand how this plays out in equilibrium, we simulate economy under different regulatory policies and plot:

$$\underbrace{\mathbb{E}_t[\xi_{t,t+1/\zeta}]^{-1} - \zeta \log(1/q_t^b)}_{\text{Convenience yield / “funding advantage”}} \sim \underbrace{q_t^b b_{t+1}/y_t}_{\text{Market Value of Debt to GDP}}$$

We then show how convenience yield moves along the equilibrium path.

We plot the results in Figure (13). The top line shows the simulated relationship between the market value of debt-to-GDP and the convenience yield in an economy without regulation. The

middle line shows the simulated relationship with loose regulation and elastic demand. The final line shows the simulated relationship with tight regulation and inelastic demand. Evidently, the shape of the relationship between the convenience yield and debt-to-GDP changes from downward sloping to flat once tight regulation is introduced. To understand this, consider the impact of a recession in the model. A decrease in productivity, $\downarrow z$, leads the government to increase debt/GDP. But the decrease in productivity also causes the regulatory constraint to bind and so increases bank demand for government debt. Thus, under tight regulation, the government can increase the debt/GDP ratio without losing their convenience yield and face a higher interest rate.

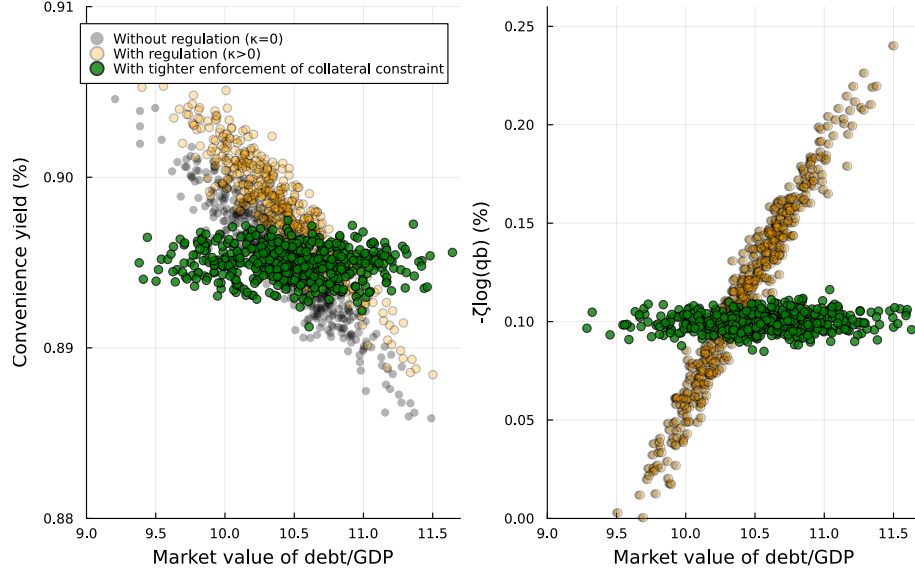


Figure 13: Convenience Yields and Debt/GDP

The gray dots denote the simulation without regulation. The orange dots denotes the simulation with regulation that is loosely enforced. The green dots denote the simulation with tightly enforced regulation.

5 Conclusion

In this paper, we show how the government can generate a convenience yield through restrictions on the financial sector that make government debt a “safe-asset” for the economy. Endogenizing the convenience yield in this way allows us to characterize how it is related to financial and fiscal policy. We show that government default erodes the risk-adjusted convenience yield because it changes the role that government debt plays in the financial sector and so changes the debt demand function. This is very different to bond-in-utility and bond-in-advance models where bond demand is exogenous and the risk-adjusted convenience yield increases when the government starts to default (because the real value of government debt becomes scarce). Our results suggest that macroeconomists should be very cautious about modeling convenience yields using exogenous, immutable demand functions that fit empirical “safe-asset” curves. Like for the Phillips Curve, these relationships break down

once the government attempts to exploit them.

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A Details of the Finite-Horizon Model

Notation: There is a continuum of islands, each with a unit measure of household members, indexed by $h \in [0, 1]$, and a unit measure of competitive banks, indexed by $j \in [0, 1]$. Index h can be replaced by the binary idiosyncratic shock $\zeta \in \{0, 1\}$ (the probability of which is island-specific), while islands can be indexed by the idiosyncratic shock λ .

A.1 Household problem

Taking prices (q_0^d, q_0^e) and payoffs $\left\{ \left(\delta_1^d(\lambda), \delta_2^d(\lambda), \delta_1^e(\lambda), \delta_2^e(\lambda) \right) \right\}_\lambda$ as given, the household solves (each of them being able to buy assets from only one bank):

$$\begin{aligned}
 & \max_{d_0, e_0, c_0, \mathbf{c}, \mathbf{d}} \quad \mathbb{E} \left[\zeta^{h, \lambda} u(c_1^h(\lambda)) + (1 - \zeta^{h, \lambda}) u(c_2^h(\lambda)) \right] \quad s.t. \\
 & q_0^d d_0^h + q_0^e e_0^h \leq 1 \quad \left(= a_0^h \right) \quad \left(\mu_0^{c, h} \right) \\
 & c_1^h(\lambda) \leq \delta_1^d(\lambda) d_0^h \quad \forall \lambda \quad \left(\mu_1^{d, h}(\lambda) \right) \\
 & c_1^h(\lambda) \leq \delta_1^d(\lambda) (d_0^h - d_1^h(\lambda)) + \delta_1^e(\lambda) e_0^h \quad \forall \lambda \quad \left(\mu_1^{c, h}(\lambda) \right) \\
 & c_2^h(\lambda) \leq \delta_2^e(\lambda) e_0^h + \delta_2^d(\lambda) d_1^h(\lambda) - \tau_2 \quad \forall \lambda \quad \left(\mu_2^{c, h}(\lambda) \right) \\
 & 0 \leq c_1^h(\lambda), c_2^h(\lambda), d_0^h, d_1^h(\lambda) \quad \left(\underline{\mu}_t^{x, h}(\lambda) \right)
 \end{aligned}$$

For a given island λ , the FOCs of an individual household h are

$$\begin{aligned}
 [c_1^h] \quad & 0 = \zeta^h u'(c_1^h) - \mu_1^{c, h} - \mu_1^{d, h} + \underline{\mu}_1^{c, h} \\
 [c_2^h] \quad & 0 = (1 - \zeta^h) u'(c_2^h) - \mu_2^{c, h} + \underline{\mu}_2^{c, h} \\
 [d_1^h] \quad & 0 = -\delta_1^d \mu_1^{c, h} + \delta_2^d \mu_2^{c, h} + \underline{\mu}_1^{d, h}
 \end{aligned}$$

For early consumers ($\zeta = 1$), the marginal value of income at $t = 2$ is zero: $\mu_2^c(1) = \underline{\mu}_2^c(1) = 0$, while the marginal utility of consumption is equal to the marginal cost of consumption which at $t = 1$ is equal to the marginal value of income adjusted by the extra cost from the deposit-in-advance (DIA) constraint: $u'(c_1(1)) = \mu_1^c(1) + \mu_1^d(1)$. This implies that early households want to sell all of their assets in the morning, $\underline{\mu}_1^d(1) > 0$ and $d_1(1) = 0$. Their supply is inelastic irrespective of which island they are on. The DIA constraint binds $\psi_1^d > 0$ and the income constraint is satisfied with $d_1(1) = 0$, nevertheless $\underline{\mu}_1^d(1) = \mu_1^c(1) = 0$.¹⁵

For late consumers ($\zeta = 0$), the marginal value of income at $t = 2$ equals to the marginal utility of consumption, $\mu_2^c(0) = u'(c_2(0))$. It follows from their FOCs for deposit that their marginal utility of income in the morning must be strictly positive as well, $\delta_1^d \mu_1^c(0) = \delta_2^d \mu_2^c(0) > 0$ and

¹⁵In this sense, DIA constraint is equivalent with the households' inability to trade assets in the morning. In other words, we could "drop" the DIA constraint from the above problem. The key for this is that early households don't care about the potential continuation value in the portfolio-adjustment sub-period. If they do care about consumption at $t = 2$ period, we need to keep the explicit DIA constraint.

$\underline{\mu}_1^d(0) = \underline{\mu}_1^e(0) = 0$, due to the fact that they can use idle morning income to save for $t = 2$. Their deposit roll-over decision depends on the relative returns on deposit vs alternative investment opportunities between the morning and afternoon (we assume that there is none). Strictly positive value of $t = 1$ income and the lack of utility from morning consumption implies that late consumers set $c_1(0) = 0$ and so $\mu^d(0) = 0$. As a result, being “cash constrained” is equivalent with being an early consumer ($\zeta = 1$).

The FOCs with respect to period $t = 0$ choices are

$$\begin{aligned}
[d_0^h] \quad q_0^d \mu_0^c &= \mathbb{E} \left[\int \left(\lambda \underbrace{(\mu_1^c(1, \lambda) + \mu_1^d(1, \lambda))}_{=0} + (1 - \lambda) \mu_1^c(0, \lambda) \right) \delta_1^d(\lambda) dF(\lambda) \right] \\
&= \mathbb{E} \left[\int \mu_1^c(0, \lambda) \left(\lambda \frac{u'(c_1(1, \lambda))}{\mu_1^c(0, \lambda)} + (1 - \lambda) \right) \delta_1^d(\lambda) dF(\lambda) \right] \\
&= \mathbb{E} \left[\int (1 - \lambda) \mu_1^c(0, \lambda) \underbrace{\left(1 + \frac{\lambda u'(c_1(1, \lambda))}{(1 - \lambda) \mu_1^c(0, \lambda)} \right)}_{=: \nu(\lambda)} \delta_1^d(\lambda) dF(\lambda) \right] \\
q_0^d &= \mathbb{E} \left[\int \xi_{0,1}(\lambda) \nu(\lambda) \delta_1^d(\lambda) dF(\lambda) \right] \\
[e_0^h] \quad q_0^e \mu_0^c &= \mathbb{E} \left[\int \left(\lambda \mu_1^c(1, \lambda) + (1 - \lambda) \mu_1^c(0, \lambda) \right) \delta_1^e(\lambda) dF(\lambda) \right] \\
&\quad + \mathbb{E} \left[\int \left(\lambda \mu_2^c(1, \lambda) + (1 - \lambda) \mu_2^c(0, \lambda) \right) \delta_2^e(\lambda) dF(\lambda) \right] \\
&= \mathbb{E} \left[\int (1 - \lambda) \mu_1^c(0, \lambda) \delta_1^e(\lambda) dF(\lambda) \right] + \mathbb{E} \left[\int (1 - \lambda) u'(c_2(0, \lambda)) \delta_2^e(\lambda) dF(\lambda) \right] \\
q_0^e &= \mathbb{E} \left[\int \xi_{0,1}(\lambda) \underbrace{(\delta_1^e(\lambda) + \xi_{1,2}(\lambda) \delta_2^e(\lambda))}_{=: V_1(\lambda)} dF(\lambda) \right]
\end{aligned}$$

where we used the notations for the stochastic discount factor

$$\xi_{0,1}(\lambda) := \frac{(1 - \lambda) \mu_1^c(0, \lambda)}{\mu_0^c} \quad \xi_{1,2}(\lambda) := \frac{u'(c_2(0, \lambda))}{\mu_1^c(0, \lambda)} = \frac{\delta_1^d(\lambda)}{\delta_2^d(\lambda)}$$

The individual consumption choices are

$$\begin{aligned}
c_1(0, \lambda) &= 0 & c_1(1, \lambda) &= \delta_1^d(\lambda) d_0^h \\
c_2(0, \lambda) &= \delta_2^e(\lambda) e_0^h + \delta_2^d(\lambda) d_0^h - \tau_2 & c_2(1, \lambda) &= 0
\end{aligned}$$

A.2 Bank problem

Default is costly for two reasons: (i) there are deadweight costs of default (proportional to outstanding deposit d_0) and denoted by ς . While banks take ς as given, in equilibrium ς is an increasing function of the fraction of defaulting banks that tends to lead to “too much deposit issuance” (in-

dividual costs < social costs); (ii) forced selling results in the sale of assets at prices below their “fundamental value” because of market illiquidity. This is a transfer of value from the seller to the buyer, so it tends to lead to “too little deposit issuance” (individual costs > social costs). Taking prices (q_1^b, q_1^k) as given, the bank maximizes shareholder value:

$$\begin{aligned} \max_{d_0, m_0, k_0, b_0} & \left\{ \delta_0^e + \mathbb{E} \left[\int \xi_{0,1}(\lambda) V_1(d_0, m_0, k_0, b_0; \lambda, \mathbf{s}) dF(\lambda) \right] \right\} \quad s.t. \\ & \delta_0^e + m_0 + k_0 + q_0^b b_0 \leq q_0^d d_0 \\ & \varrho(q_0^d d_0 - m_0) \leq \kappa(q_0^b b_0) + (1 - \kappa)k_0 \\ & 0 \leq d_0, m_0, k_0, b_0 \\ & q_0^d = \mathbb{E} \left[\int \xi_{0,1}(\lambda) \nu(\lambda) \delta_1^d(\lambda) dF(\lambda) \right] \end{aligned}$$

where the time $t = 1$ problem is given by

$$\begin{aligned} V_1(d_0, m_0, k_0, b_0; \lambda, \mathbf{s}) &= \max \left\{ \delta_1^e + \xi_{1,2} \delta_2^e \right\} \quad s.t. \\ & \delta_1^e + q_1^k k_1 + q_1^b b_1 \leq z_1 m_0 + q_1^k k_0 + q_1^b b_0 - \delta_1^d \lambda d_0 - \varsigma d_0 \mathbb{1}_1^d - \alpha d_0 \mathbb{1}_1^e \\ & \delta_2^e \leq z_2 k_1 + \delta_2^b b_1 - \delta_2^d (1 - \lambda) d_0 \\ & 0 \leq \delta_1^e, k_1, b_1 \\ & (1 - \mathbb{1}_1^e) \varrho \delta_1^d (1 - \lambda) d_0 \leq \kappa q_1^b b_1 + (1 - \kappa)(q_1^k k_1) \quad \varrho \in [0, 1/2), \quad \kappa \in [0, 1] \end{aligned}$$

The indicators $\mathbb{1}_1^d$ and $\mathbb{1}_1^e$ represent bank default and the bank’s choice of opting out of the regulatory framework (and paying the linear cost α), respectively. Function $\varsigma(\cdot)$ denotes real dead-weight losses from default that may include the loss of firm specific information, the destruction of capital/consumer networks, etc. The $\varsigma(\cdot)$ function is a feature of the environment that the government cannot overcome per se, but they can internalize the externality that it represents.

Parameters (ϱ, κ) are regulatory parameters:

- ϱ restricts the banks’ ability to back its deposit issuance with long-term assets (“leverage constraint”). $\varrho = 0$ corresponds to the case of no financial regulation. We call ϱ the regulation parameter.
- κ measures the amount of repression. $\kappa = 1/2$ corresponds to symmetric regulatory treatment of the two assets, while $\kappa \neq 1/2$ introduces asymmetric treatment. When $\kappa > 1/2$, government debt is preferred to capital, when $\kappa < 1/2$, capital is preferred relative to debt. $\kappa = 1$ corresponds to the extreme case when capital has no collateral value (“pure repression”).

Notation: We define the “intra-period” returns as:

$$R_{1,2}^k := \frac{z_2}{q_1^k} \quad R_{1,2}^b := \frac{\delta_2^b}{q_1^b} \quad \Delta R_{1,2} := R_{1,2}^k - R_{1,2}^b$$

Let φ_1 be the portfolio share of government debt at the end of period $t = 1$ and $\mathcal{R}_{1,2}^\varphi$ be the return

on the bank portfolio between $t = 1$ and $t = 2$:

$$\varphi_1 := \frac{q_1^b b_1}{q_1^b b_1 + q_1^k k_1} \quad \mathcal{R}_{1,2}^\varphi := R_{1,2}^b \varphi_1 + R_{1,2}^k (1 - \varphi_1) = R_{1,2}^k - \varphi_1 \Delta R_{1,2}$$

In addition, let the “aggregated regulatory value” of the bank portfolio be:

$$\mathcal{K}_{1,2}^\varphi := \left(\frac{\kappa}{\varrho} \right) \varphi_1 + \left(\frac{1 - \kappa}{\varrho} \right) (1 - \varphi_1) = \left(\frac{1 - \kappa}{\varrho} \right) + \varphi_1 \left(\frac{2\kappa - 1}{\varrho} \right)$$

The bank net worth is defined as

$$a(\lambda) := z_1 m_0 + q_1^k k_0 + q_1^b b_0 - \varsigma d_0 \mathbb{1}_1^d - \alpha d_0 \mathbb{1}_1^e - \delta^d(\lambda) \lambda d_0$$

so $\delta_1^e = 0$ and the $t = 1$ budget constraint imply that $a(\lambda) = q_1^b b_1 + q_1^k k_1$. Similarly, let the bank’s period $t = 0$ (ex ante) leverage be

$$\ell_0 := \frac{d_0}{m_0 + k_0 + q_0^b b_0}$$

and define the morning returns

$$R_{0,1}^k := q_1^k \quad R_{0,1}^b := \frac{q_1^b}{q_0^b}$$

and let $\mathcal{R}_{0,1}^\varphi$ be the return on bank portfolio between $t = 0$ and $t = 1$:

$$\varphi_0 := \frac{q_0^b b_0}{m_0 + k_0 + q_0^b b_0} \quad \mathcal{R}_{0,1}^\varphi := z_1 \varphi_0^m + R_{0,1}^k (1 - \varphi_0^m - \varphi_0) + R_{0,1}^b \varphi_0$$

Finally, define ex post leverage at $t = 1$ as follows:

$$\mathcal{L}_0 := \frac{z_1 m_0 + q_1^b b_0 + q_1^k k_0}{d_0} = \mathcal{R}_{0,1}^\varphi \ell_0^{-1}$$

Reformulation of the problem: The budget constraints at $t = 1$ and $t = 2$ will bind. Using this fact, we can combine them to get the bank’s consolidated budget constraint at $t = 1$:

$$\delta_1^e + \frac{\delta_2^e}{\mathcal{R}_{1,2}^\varphi} = z_1 m_0 + q_1^k k_0 + q_1^b b_0 - \mathbb{1}_1^d \varsigma d_0 - \mathbb{1}_1^e \alpha d_0 - \delta^d(\lambda) \left[\lambda + \frac{(1 - \lambda)}{\mathcal{R}_{1,2}^\varphi} \right] d_0$$

As long as $\mathcal{R}_{1,2}^\varphi > 1$, the marginal cost of $t = 2$ dividend is strictly lower than the marginal cost of $t = 1$ dividend (which is one), which makes the bank’s IMRS different from the household’s IMRS (which is one). In other words, due to the “missing morning markets”, the bank has trouble moving resources between the morning and afternoon, and as a result, bank will value morning income relatively more than the household.¹⁶ This implies that the equity raising constraint $\delta_1^e \geq 0$ always

¹⁶The consolidated budget constraint also shows that if $\mathcal{R}^\varphi = 1$, shareholder value does not depend on λ .

bind. Using this fact, we can rearrange the consolidated budget constraint to express shareholder value as:

$$\delta_2^e(\lambda) = \mathcal{R}_{1,2}^\varphi \left(z_1 m_0 + q_1^k k_0 + q_1^b b_0 - \varsigma \mathbb{1}_1^d d_0 - \alpha \mathbb{1}_1^e d_0 \right) - \delta^d(\lambda) [\mathcal{R}_{1,2}^\varphi \lambda + (1 - \lambda)] d_0 \quad (\text{A.1})$$

On any given island λ , banks will choose portfolios, $\varphi_1 \in [0, 1]$, default $\mathbb{1}_1^d \in \{0, 1\}$ and regulatory escape decision $\mathbb{1}_1^e \in \{0, 1\}$, so that (A.1) is maximized, subject to the regulatory constraint that can be written as

$$\mathcal{K}_{1,2}^\varphi \left(z_1 m_0 + q_1^k k_0 + q_1^b b_0 - \varsigma d_0 \mathbb{1}_1^d \right) - \delta^d(\lambda) [\mathcal{K}_{1,2}^\varphi \lambda + (1 - \lambda)] d_0 \geq 0 \quad (\text{A.2})$$

unless $\mathbb{1}_1^e = 1$ in which case the regulatory constraint (A.2) does not need to be satisfied.

A.2.1 Characterization of the $t = 1$ problem

Given period $t = 0$ choices (m_0, k_0, b_0, d_0) and the aggregate shock, there is a bank-specific withdrawal shock of size λd_0 . Because of financial frictions, liquidity is limited at $t = 1$: (i) there is no equity injection $\delta_1^e \geq 0$, and (ii) there is no un-collateralized debt issuance $b_1, k_1 \geq 0$ (i.e., banks are borrowing constrained). Market illiquidity introduces a wedge between the asset's market price and “fundamental value” which makes morning asset sales costly. Nevertheless, because of (i) and (ii), withdrawals must be financed either by cash-on-hand or by costly asset sales/borrowing, both of which affect the shareholders' dividend payment at $t = 2$. Limited liability implies that whenever $\delta_2^e < 0$, the bank will default (partially) on its deposits.

Intra-period returns and portfolios: To see how the equity raising and regulatory frictions put wedges between asset prices and the assets' fundamental value, we first study the FOCs with respect to morning choices:

$$\begin{aligned} [\delta_1^e] \quad & \mu_1^\delta(\lambda) = 1 + \mu_1^e(\lambda) \\ [b_1] \quad & q_1^b \left(\mu_1^\delta(\lambda) - \kappa \mu_1^r(\lambda) - \underline{\mu}_1^b(\lambda) \right) = \xi_{1,2} \delta_2^b \\ [k_1] \quad & q_1^k \left(\mu_1^\delta(\lambda) - (1 - \kappa) \mu_1^r(\lambda) - \underline{\mu}_1^k(\lambda) \right) = \xi_{1,2} z_2 \end{aligned}$$

where $\psi_1^e \geq 0$ is the Lagrange multiplier on the equity raising constraint, $\mu_1^\delta \geq 0$ is the multiplier on the period $t = 1$ budget constraint and $\mu_1^r \geq 0$ is the multiplier on the $t = 1$ regulatory constraint.

Portfolio choice (conditional on $(\mathbb{1}_1^d, \delta^d)$ and $\mathbb{1}_1^e$): The bank portfolio decision will be driven by the spread between the two long-term assets $\Delta R_{1,2}$. Whenever $\Delta R_{1,2} > 0$, banks will try to keep as little government debt as possible in their portfolio. Suppose first that $\mathbb{1}_1^e(\lambda) = 1$, so the regulatory constraint does not need to hold for bank λ . In this case, $\varphi_1(\lambda) = 0$, i.e., the bank will invest all of their funds into high-yielding capital and make their debt short-selling constraint bind. On the other hand, if $\mathbb{1}_1^e(\lambda) = 0$, so the regulatory constraint is in effect, the bank will hold as little debt as

possible.

- If $\kappa = 1$ (pure repression), what constitutes “as little as possible” is determined by the $t = 1$ regulatory constraint: all banks, irrespective of λ will choose to hold φ_1 so that the regulatory constraint binds.¹⁷

$$\varphi_1(\lambda; \mathbb{1}^d, \delta^d) = \varrho \left(\frac{1 - \lambda}{\frac{\mathcal{L}_0 - \varsigma \mathbb{1}_1^d}{\delta^d(\lambda)} - \lambda} \right)$$

Intuitively, in the pure repression case, the banks can choose their portfolio’s regulatory value arbitrarily in the range $\mathcal{K}_{1,2}^\varphi \in [0, 1/\varrho]$ by varying $\varphi_1 \in [0, 1]$. This implies that for each λ , the regulatory constraint will always bind (at least as long as $d_0 > 0$) before the short selling constraint $\varphi \geq 0$ could kick in. On the other hand, if $\Delta R_{1,2} < 0$, banks will set $\varphi_1 = 1$ irrespective of λ . The regulatory constraint is satisfied, but at least for some banks it doesn’t bind. Of course, this policy cannot clear the secondary capital market, so pure repression requires $\Delta R_{1,2} > 0$ in equilibrium.

- If $\kappa < 1$ (capital has some collateral value) it is possible that the bank can satisfy the regulatory constraint by holding only capital and setting $\varphi_1 = 0$. The portfolio share that makes the regulatory constraint bind is now

$$\varphi_1(\lambda; \mathbb{1}^d, \delta^d) = \frac{\varrho}{2\kappa - 1} \left(\frac{1 - \lambda}{\frac{\mathcal{L}_0 - \varsigma \mathbb{1}_1^d}{\delta^d(\lambda)} - \lambda} \right) - \frac{1 - \kappa}{2\kappa - 1} \quad (\text{A.3})$$

If $\mathbb{1}_1^d = 0$ and $\delta^d = 1$, $\varphi_1(\lambda)$ is an increasing function of λ (as long as $\mathcal{L}_0 < 1$) and it converges to $\frac{\varrho(\mathcal{L}_0)^{-1} - (1 - \kappa)}{2\kappa - 1}$ as $\lambda \searrow 0$. If $\mathbb{1}_1^d = 1$, $\varphi_1(\lambda)$ is a decreasing function of λ as long as $\mathcal{L}_0 - \varsigma - \delta^d > 1$, i.e. the bank’s net worth is positive for all $\lambda \leq 1$, it converges to $\frac{1 - \kappa}{2\kappa - 1}$ as $\lambda \nearrow 1$. This means that if capital has some collateral value, there is always going to be at least some defaulting banks (those with the largest λ , hence the lowest amount of outstanding debt) who will set $\varphi_1 = 0$ and satisfy the regulatory constraint with capital only. Similarly, it is possible that some non-defaulting banks (those with the lowest λ , hence largest net worth) will be able to satisfy the regulatory constraint with only capital. Intuitively, the higher the bank’s ex post leverage, the more banks will choose this option. As the number of banks who needs to hold positive amount of debt falls, the price of debt q_1^b must fall which pushes up $R_{1,2}^b$ and decreases $\Delta R_{1,2} = 0$.

The key insight is that the regulatory constraint (A.2) implied feasible set (of period $t = 2$ income as a function of λ) can be visualized as an area enclosed within two affine functions of λ , corresponding to the policies $\varphi_1(\lambda) = 0$ and $\varphi_1(\lambda) = 1$ or $\mathcal{K}_{1,2}^\varphi = \frac{1 - \kappa}{\varrho}$ and $\mathcal{K}_{1,2}^\varphi = \frac{\kappa}{\varrho}$, respectively. The points at which these affine functions crosses the zero line (RHS of (A.2)) give us the λ cutoffs at which the

¹⁷It can be shown that φ_1 is increasing in λ over the no-default region ($\mathbb{1}^d = 0$, $\delta^d = 1$). Moreover, as long as $\mathcal{L}_0 - \varsigma - \delta^d > 1$, i.e. the bank’s net worth is positive for all $\lambda \leq 1$, $\varphi_1(\lambda)$ is non-negative and decreasing in λ over the default region ($\mathbb{1}^d = 1$) and becomes 0 at $\lambda = 1$.

regulatory constraint goes from being slack to bind and then from binding to being so tight that it triggers default. We can characterize these cutoffs by using the function:

$$\lambda^r(x) := \frac{x\mathcal{L}_0 - 1}{x - 1}$$

so that the two regulatory cutoffs can be written as¹⁸

$$\lambda_L^r := \lambda^r\left(\frac{1 - \kappa}{\varrho}\right) \quad \lambda_U^r := \lambda^r\left(\frac{\kappa}{\varrho}\right)$$

and φ_1 is interior as long as $\lambda \in (\lambda_L^r, \lambda_U^r)$.

Dividend function under no default and regulatory escape: The period 2 dividend as a function of (φ_1, δ^d) and λ can be written as

$$\begin{aligned} \delta_2^e(\lambda) &= \mathcal{R}_{1,2}^\varphi \left(z_1 m_0 + q_1^k k_0 + q_1^b b_0 - \delta_1^d \lambda d_0 - \varsigma d_0 \mathbb{1}_1^d - \alpha \mathbb{1}_1^e d_0 \right) - \delta_1^d (1 - \lambda) d_0 \\ &= \left(R_{1,2}^k - \varphi_1 \Delta R_{1,2} \right) \left(z_1 m_0 + q_1^k k_0 + q_1^b b_0 - \mathbb{1}_1^d \varsigma d_0 - \mathbb{1}_1^e \alpha d_0 \right) \\ &\quad - \delta^d(\lambda) \left[\left(R_{1,2}^k - \varphi_1 \Delta R_{1,2} \right) \lambda + (1 - \lambda) \right] d_0 \end{aligned}$$

Given the form of optimal portfolios discussed above, we know that $\delta_2^e(\lambda)$ is a piece-wise linear function under the optimal (φ_1, δ^d) . In particular, for $\lambda \in [\lambda_L^r, \lambda_U^r]$, we have

$$\begin{aligned} \delta_2^e(\lambda) &= \left(R_{1,2}^k + (1 - \mathbb{1}_1^e)(1 - \kappa) \frac{\Delta R_{1,2}}{2\kappa - 1} \right) \left(z_1 m_0 + q_1^k k_0 + q_1^b b_0 - \mathbb{1}_1^e(\lambda) \alpha d_0 \right) \\ &\quad - \delta_1^d(\lambda) \left[\left(R_{1,2}^k + (1 - \mathbb{1}_1^e)(1 - \kappa) \frac{\Delta R_{1,2}}{2\kappa - 1} \right) \lambda + \left(1 + (1 - \mathbb{1}_1^e) \varrho \frac{\Delta R_{1,2}}{2\kappa - 1} \right) (1 - \lambda) \right] d_0 \end{aligned}$$

and, from the period $t = 1$ Euler equations, we know that

$$\bar{\mu}_1^\delta = R_{1,2}^k + (1 - \kappa) \frac{\Delta R_{1,2}}{2\kappa - 1} \quad \bar{\mu}_1^r = \frac{\Delta R_{1,2}}{2\kappa - 1}$$

More generally, we can write dividends under no-default as a continuous function of λ :

$$\frac{\delta_2^e(\mathbb{1}_1^d = 0)}{d_0} = \begin{cases} (R_{1,2}^k - 1) \left(\lambda^r(R_{1,2}^k) - \lambda \right) & \lambda < \lambda_L^r \\ (R_{1,2}^k - 1) \left(\lambda^r(R_{1,2}^k) - \lambda \right) - \left(\frac{1 - \kappa}{\varrho} - 1 \right) \left(\varrho \frac{\Delta R_{1,2}}{2\kappa - 1} \right) (\lambda - \lambda_L^r) & \lambda \in [\lambda_L^r, \lambda_U^r] \\ (R_{1,2}^b - 1) \left(\lambda^r(R_{1,2}^b) - \lambda \right) & \lambda_U^r < \lambda \end{cases}$$

Given that under no default we have $\delta_1^d = 1$, $\delta_2^e(\lambda; \mathbb{1}_1^e)$ is an affine function of λ (over the no-default

¹⁸It is straightforward to show that $\varphi_1(\lambda_L^r) = 0$ and $\varphi_1(\lambda_U^r) = 1$.

region) with

$$\begin{array}{ll}
[\text{intercept } (\mathbb{1}_1^e = 0)] & R_{1,2}^k (z_1 m_0 + q_1^k k_0 + q_1^b b_0) - d_0 + \left((1 - \kappa) \mathcal{R}_{0,1}^\varphi \ell_0 - \varrho \right) \bar{\mu}_1^r d_0 \\
[\text{slope } (\mathbb{1}_1^e = 0)] & - \left(R_{1,2}^k - 1 + ((1 - \kappa) - \varrho) \bar{\mu}_1^r \right) d_0 \\
[\text{intercept } (\mathbb{1}_1^e = 1)] & R_{1,2}^k (z_1 m_0 + q_1^k k_0 + q_1^b b_0) - d_0 - R_{1,2}^k \bar{\psi} d_0 \\
[\text{slope } (\mathbb{1}_1^e = 1)] & - \left(R_{1,2}^k - 1 \right) d_0
\end{array}$$

The two affine functions (with different slopes) will cross each other at most once, and only when

$$\begin{aligned}
0 &= \left((1 - \kappa) \mathcal{L}_0 - \varrho \right) \bar{\mu}_1^r + R_{1,2}^k \alpha - \left((1 - \kappa) - \varrho \right) \bar{\mu}_1^r \lambda^\dagger \\
\lambda^\dagger &= \frac{\left((1 - \kappa) \mathcal{L}_0 - \varrho \right) \bar{\mu}_1^r + R_{1,2}^k \alpha}{\left((1 - \kappa) - \varrho \right) \bar{\mu}_1^r}
\end{aligned}$$

With pure repression ($\kappa = 1$) this becomes

$$\lambda^\dagger = 1 - \frac{R_{1,2}^k \alpha}{\varrho \bar{\mu}_1^r}$$

which is independent of $t = 0$ choices, so banks “take λ^\dagger as given”. Banks with $\lambda < \lambda^\dagger$ will choose to pay the cost to escape the regulatory constraint.

Default $\mathbb{1}_1^d$: There are two sources of default:

1. Default is triggered by the regulatory constraint: even if the bank sets $\varphi_1 = 1$, the regulatory value of assets are not sufficient to cover the amount of rolled over deposits.
2. Default is triggered by the limited liability constraint: shareholders choose to default because doing so gives them non-negative dividends even after paying the default cost ςd_0 .

Starting with the regulatory constraint (A.2), we need to consider two cases

- If $\kappa = 1/2$, the area allowed by the regulatory constraint is a line that crosses the x-axis at $\lambda_L^r = \lambda^r(1/(2\varrho)) = \lambda_U^r$. Banks with $\lambda > \lambda_U^r$ have only one choice to satisfy the regulatory constraint: pay the default cost and devalue their deposits. Doing so changes their regulatory constraint so that it is always satisfied and always binds. In other words, in the case the regulatory constraint binds only for the defaulting banks. A necessary condition for an equilibrium is $\Delta R_{1,2} = 0$.
- As κ gets larger than $1/2$ for a given $\varrho > 0$ (repression), the singel line becomes a “tunnel” with the edges crossing the x-axis at λ_L^r and λ_U^r . As $\kappa \nearrow 1$, the lower cutoff λ_L^r converges to 0, while the upper cutoff λ_U^r converges to 1.¹⁹ Suppose that $\Delta R_{1,2} > 0$, so each bank wants

¹⁹The function λ^r has a discontinuity at $x = 1$. As $(1 - \kappa)/\varrho$ becomes less than 1, the slope of the south edge of the tunnel switches signs (from negative to positive) and it remains negative over the $\lambda \in (0, 1)$ region. In other words, as long as $(1 - \kappa) < \varrho$, we have $\lambda_L^r = 0$ and an interior φ_1 is always available for banks with $\lambda < \lambda_U^r$.

to hold as little government debt as possible. Banks with $\lambda < \lambda_L^r$ can satisfy their regulatory constraint (without making it bind) by setting $\varphi_1 = 0$. What determines the portfolio of these banks is their short-selling constraint, not the regulatory constraint. As in the $\kappa = 1/2$ case, banks with $\lambda > \lambda^r(\kappa/\varrho)$ cannot satisfy their regulatory constraint (even with $\varphi_1 = 1$ which leads to the highest attainable regulatory value), so in equilibrium these banks must default. When they default, they can satisfy the regulatory constraint by setting φ_1 according to (A.3). As a result, the relevant default cutoff is $\lambda^r(\kappa/\varrho)$ and repression makes the regulatory induced default less likely. Finally, banks with $\lambda \in [\lambda_L^r, \lambda_U^r]$ are constrained by the regulatory constraint: they would want to set $\varphi_1 = 0$ as the other banks, but the regulatory constraint prevents them from doing so, they will choose an interior φ_1 according to (A.3) (unless they default for other reasons).

The second reason why the bank might default is their limited liability. Banks default on their deposit at $t = 1$ when they cannot guarantee that $\delta_2^e > 0$. The point λ^* at which the relevant fraction of the dividend function crosses the zero line is given by

$$\lambda^* = \max \left\{ \lambda^*(0), \lambda^*(1) \right\}$$

where

$$\begin{aligned} \lambda^*(\mathbb{1}_1^e) &= \frac{\bar{\mu}_1^\delta \mathcal{L}_0 - \delta^d (1 + \varrho \bar{\mu}_1^r) + \mathbb{1}_1^e (\delta^d \varrho \bar{\mu}_1^r - \alpha R_{1,2}^k - (1 - \kappa) \bar{\mu}_1^r \mathcal{L}_0)}{\delta^d (\bar{\mu}_1^\delta - (1 + \varrho \bar{\mu}_1^r) + \mathbb{1}_1^e (\varrho \bar{\mu}_1^r - (1 - \kappa) \bar{\mu}_1^r))} \\ &= 1 - \frac{R_{1,2}^k (1 + \alpha \mathbb{1}_1^e - \left(\frac{z_1 m_0 + q_1^k k_0 + q_1^b b_0}{d_0} \right))}{R_{1,2}^k - (1 + \varrho \bar{\mu}_1^r) + \mathbb{1}_1^e \varrho \bar{\mu}_1^r} \end{aligned}$$

Using this definition of λ^* , we can rewrite $t = 2$ dividends as

$$\delta_2^e(\lambda) = \begin{cases} \max \left\{ 0, \left(R_{1,2}^k - 1 \right) (\lambda^*(1) - \lambda) d_0 \right\} & \lambda < \max\{\lambda^\dagger, 0\} \\ \max \left\{ 0, \left(\bar{\mu}_1^\delta - (1 + \varrho \bar{\mu}_1^r) \right) (\lambda^*(0) - \lambda) d_0 \right\} & \lambda \geq \max\{\lambda^\dagger, 0\} \end{cases}$$

In default, deposit payoff is determined by the condition $\delta_2^e(\lambda) = 0$ (using max dividend function):

$$\begin{aligned} \delta^d(\lambda) &= \begin{cases} \frac{R_{1,2}^k (z_1 m_0 + q_1^k k_0 + q_1^b b_0 - \varsigma d_0)}{[R_{1,2}^k \lambda + (1 + \varrho \bar{\mu}_1^r)(1 - \lambda)] d_0} = \frac{R_{1,2}^k (\mathcal{L}_0 - \varsigma)}{[R_{1,2}^k \lambda + (1 + \varrho \bar{\mu}_1^r)(1 - \lambda)]} & \lambda \geq \max\{\lambda^\dagger, \lambda^*(0)\} \\ \frac{R_{1,2}^k (z_1 m_0 + q_1^k k_0 + q_1^b b_0 - \varsigma d_0 - \alpha d_0)}{[R_{1,2}^k \lambda + (1 - \lambda)] d_0} = \frac{R_{1,2}^k (\mathcal{L}_0 - \varsigma - \alpha)}{[R_{1,2}^k \lambda + (1 - \lambda)]} & \lambda < \max\{\lambda^\dagger, \lambda^*(0)\} \end{cases} \\ &= \begin{cases} 1 - \frac{(R_{1,2}^k - (1 + \varrho \bar{\mu}_1^r))(\lambda - \lambda^*(0)) + R_{1,2}^k \varsigma}{(1 + \varrho \bar{\mu}_1^r) + (R_{1,2}^k - (1 + \varrho \bar{\mu}_1^r))\lambda} & \lambda^*(0) \geq \lambda^\dagger \\ 1 - \frac{(R_{1,2}^k - 1)(\lambda - \lambda^*(1)) + R_{1,2}^k \varsigma}{1 + (R_{1,2}^k - 1)\lambda} & \lambda^*(0) < \lambda^\dagger \end{cases} \end{aligned}$$

This form of deposit payoff guarantees that over the default region $\delta_2^e = 0$. For simplicity, let's define

functions of multipliers:

$$\begin{aligned} A(\mathbb{1}_1^e) &:= R_{1,2}^k - (1 + \varrho \bar{\mu}_1^r) + \mathbb{1}_1^e(\lambda) \varrho \bar{\mu}_1^r \\ B(\mathbb{1}_1^e) &:= 1 + \varrho \bar{\mu}_1^r - \mathbb{1}_1^e(\lambda) \varrho \bar{\mu}_1^r \end{aligned}$$

Continuation value: We can combine the banks optimal $t = 1$ choices to write the bank's value (per unit of deposit) at $t = 0$ as

$$\mathcal{V}_0(\lambda^*) := \frac{q_0^d d_0 + q_0^e}{d_0} = \mathbb{E} \left[\mathcal{V}_1 \left(\lambda^*(s) \right) \right]$$

where the continuation value function \mathcal{V}_1 is given by:

$$\begin{aligned} \mathcal{V}_1(\lambda^*) &:= \int^{\lambda^*} \xi(\lambda) \nu(\lambda) dF(\lambda) + \int^{\lambda^*} \xi(\lambda) A(\mathbb{1}_1^e(\lambda)) \left(\lambda^*(\mathbb{1}_1^e(\lambda)) - \lambda \right) dF(\lambda) \\ &\quad + \int_{\lambda^*} \xi(\lambda) \nu(\lambda) \left(1 - \frac{A(\mathbb{1}_1^e(\lambda)) (\lambda - \lambda^*(\mathbb{1}_1^e(\lambda))) + R_{1,2}^k \varsigma}{R_{1,2}^k \lambda + B(\mathbb{1}_1^e(\lambda))(1 - \lambda)} \right) dF(\lambda) \\ &= \int \xi(\lambda) \nu(\lambda) dF(\lambda) + \int^{\lambda^*} \xi(\lambda) A(\mathbb{1}_1^e(\lambda)) \left(\lambda^*(\mathbb{1}_1^e(\lambda)) - \lambda \right) dF(\lambda) \\ &\quad - \int_{\lambda^*} \xi(\lambda) \left(\frac{\nu(\lambda)}{R_{1,2}^k \lambda + B(\mathbb{1}_1^e(\lambda))(1 - \lambda)} \right) \left(A(\mathbb{1}_1^e) (\lambda - \lambda^*(\mathbb{1}_1^e(\lambda))) + R_{1,2}^k \varsigma \right) dF(\lambda) \end{aligned}$$

which, for given prices, is a non-linear function of λ^* . The partial derivative is

$$\begin{aligned} \frac{\partial \mathcal{V}_1}{\partial \lambda^*} &= \int^{\lambda^*} \xi(\lambda) A(\mathbb{1}_1^e(\lambda)) dF(\lambda) + \int_{\lambda^*} \xi(\lambda) \left(\frac{\nu(\lambda)}{R_{1,2}^k \lambda + B(\mathbb{1}_1^e(\lambda))(1 - \lambda)} \right) A(\mathbb{1}_1^e(\lambda)) dF(\lambda) + \\ &\quad + \varsigma \xi(\lambda^*) \left(\frac{\nu(\lambda^*)}{R_{1,2}^k \lambda^* + B(\mathbb{1}_1^e(\lambda^*))(1 - \lambda^*)} \right) R_{1,2}^k f(\lambda^*) \end{aligned}$$

For simplicity, let's define

$$\Omega \left(\lambda; \lambda^*, \mu_1^\delta, \mu_1^r \right) := \begin{cases} \frac{\nu(\lambda)}{R_{1,2}^k \lambda + B(\mathbb{1}_1^e(\lambda))(1 - \lambda)} + \varsigma \frac{\xi(\lambda^*)}{\xi(\lambda)} \left(\frac{\nu(\lambda^*)}{A(\mathbb{1}_1^e(\lambda))(\mathcal{L}_0 - \mathbb{1}_1^e(\lambda)\alpha)} \right) \frac{f(\lambda^*)}{1 - F(\lambda^*)} & \lambda > \lambda^* \\ 1 & \lambda \leq \lambda^* \end{cases}$$

A.2.2 Portfolio choice at $t = 0$

Let the bank's period $t = 0$ (ex ante) leverage be

$$\ell_0 := \frac{d_0}{m_0 + k_0 + q_0^b b_0}$$

The bank's objective function at $t = 0$ can be written as

$$\max_{\ell_0, \lambda^*} \left(-\ell_0^{-1} + \mathcal{V}_0(\lambda^*) \right) d_0$$

which shows that one of the necessary equilibrium conditions is that the bank's profit is zero (otherwise either $d_0 = \infty$ or $d_0 = -\infty$). The regulatory constraint at $t = 0$ is

$$\begin{aligned}\varrho q_0^d d_0 &\leq \varrho m_0 + (1 - \kappa)k_0 + \kappa q_0^b b_0 \\ \varrho q_0^d &\leq \left(\varrho \varphi_0^m + (1 - \kappa)(1 - \varphi_0^m - \varphi_0) + \kappa \varphi_0 \right) \ell_0^{-1}\end{aligned}$$

The sense in which banks “choose” their default probability at $t = 0$ is that the default cutoff can be written as a function of ℓ_0 and the portfolio returns between $t = 0$ and $t = 1$:

$$\mathcal{R}_{0,1}^\varphi := \frac{z_1 m_0 + q_1^k k_0 + q_1^b b_0}{m_0 + k_0 + q_0^b b_0} = z_1 \varphi_0^m + R_{0,1}^k (1 - \varphi_0^m - \varphi_0) + R_{0,1}^b \varphi_0$$

which is influenced by the bank through the portfolio shares (φ_0^m, φ_0) . The fact that $\mathcal{V}_0(\cdot)$ is a non-linear function of λ^* , and therefore also $(\ell_0, \varphi_0^m, \varphi_0)$, means that the bank's effective risk aversion is influenced by the curvature of \mathcal{V}_0 . The bank's FOCs are

$$\begin{aligned}[\ell_0^{-1}] \quad & 1 - \mu_0^r (1 - \kappa(1 - \varphi_0^m - \varphi_0)) = \mathbb{E} \left[\int \xi(\lambda) \Omega(\lambda) \bar{\mu}_1^\delta \mathcal{R}_{0,1}^\varphi dF(\lambda) \right] \\ [\varphi_0^m] \quad & \kappa \mu_0^r = \mathbb{E} \left[\int \xi(\lambda) \Omega(\lambda) \bar{\mu}_1^\delta (R_{0,1}^k - z_1) dF(\lambda) \right] \\ [\varphi_0] \quad & \kappa \mu_0^r = \mathbb{E} \left[\int \xi(\lambda) \Omega(\lambda) \bar{\mu}_1^\delta (R_{0,1}^k - R_{0,1}^b) dF(\lambda) \right]\end{aligned}$$

and the zero profit condition holds

$$m_0 + k_0 + q_0^b b_0 = q_0^d d_0 + q_0^e$$

Aside: The function $\mathcal{V}_0(\lambda^*)$ is the bank's expected continuation value, which depends on asset holdings $(m_0, k_0, q_0^b b_0)$ and deposit $q_0^d d_0$ in the spirit of bond-in-utility models. The key difference from those models is that here the relative weight of the assets in the bank's “utility function” is endogenous—it depends on how asset returns interact with the bank's default decision.

Individual Euler equations: The FOCs with respect to period $t = 0$ choices are

$$\begin{aligned}[m_0] \quad & 0 = -1 + \mathbb{E} \left[\frac{\partial \mathcal{V}_1}{\partial \lambda^*} \frac{\partial \lambda^*}{\partial m_0} \right] d_0 + \mu_0^r \\ [k_0] \quad & 0 = -1 + \mathbb{E} \left[\frac{\partial \mathcal{V}_1}{\partial \lambda^*} \frac{\partial \lambda^*}{\partial k_0} \right] d_0 + (1 - \kappa) \mu_0^r \\ [b_0] \quad & 0 = -q_0^b + \mathbb{E} \left[\frac{\partial \mathcal{V}_1}{\partial \lambda^*} \frac{\partial \lambda^*}{\partial b_0} \right] d_0 + \mu_0^r q_0^b \\ [d_0] \quad & 0 = \frac{\partial q_0^e}{\partial d_0} + (1 - \varrho \mu_0^r) \left(q_0^d + \frac{\partial q_0^d}{\partial d_0} d_0 \right)\end{aligned}$$

where

$$\begin{aligned}\frac{\partial q_0^e}{\partial d_0} &= \mathbb{E} \left[\left(\int^{\lambda^*} \xi(\lambda) A(\mathbb{1}_1^e) \left(\frac{\partial \lambda^*}{\partial d_0} \right) d_0 dF(\lambda) \right) \right] + \mathbb{E} \left[\left(\int^{\lambda^*} \xi(\lambda) A(\mathbb{1}_1^e) (\lambda^* - \lambda) dF(\lambda) \right) \right] \\ &= -\mathbb{E} \left[\left(\int^{\lambda^*} \xi(\lambda) \left(\bar{\mu}_1^\delta \lambda + B(\mathbb{1}_1^e)(1 - \lambda) + \alpha \mathbb{1}_1^e \right) dF(\lambda) \right) \right]\end{aligned}$$

and

$$\begin{aligned}\frac{\partial q_0^d}{\partial d_0} d_0 &= -\mathbb{E} \left[\int_{\lambda^*} \xi(\lambda) \frac{\nu(\lambda)}{\bar{\mu}_1^\delta \lambda + B(\mathbb{1}_1^e(\lambda))(1 - \lambda)} \bar{\mu}_1^\delta \mathcal{R}_{0,1}^\varphi \ell_0^{-1} dF(\lambda) \right] + \\ &\quad - \varsigma \mathbb{E} \left[\int_{\lambda^*} \xi(\lambda) \left(\frac{\xi(\lambda^*)}{\xi(\lambda)} \right) \left(\frac{\nu(\lambda^*)}{A(\mathbb{1}_1^e(\lambda^*))} \right) \bar{\mu}_1^\delta \frac{f(\lambda^*)}{1 - F(\lambda^*)} dF(\lambda) \right]\end{aligned}$$

Combining these expressions we get the bank pricing equation for deposit (deposit supply)

$$\begin{aligned}[d_0] \quad q_0^d &= \mathbb{E} \left[\int_{\lambda^*} \xi(\lambda) \Omega(\lambda) \bar{\mu}_1^\delta \mathcal{R}_{0,1}^\varphi \ell_0^{-1} dF(\lambda) \right] + \\ &\quad + \mathbb{E} \left[\int^{\lambda^*} \xi(\lambda) \Omega(\lambda) \bar{\mu}_1^\delta \left(\frac{\bar{\mu}_1^\delta \lambda + B(\mathbb{1}_1^e)(1 - \lambda) + \alpha \mathbb{1}_1^e}{\bar{\mu}_1^\delta (1 - \varrho \mu_0^r)} \right) dF(\lambda) \right]\end{aligned}$$

while the other pricing equations are

$$\begin{aligned}[m_0] \quad & (1 - \mu_0^r) = \mathbb{E} \left[\xi(\lambda) \Omega(\lambda) \bar{\mu}_1^\delta z_1 \right] \\ [k_0] \quad & (1 - (1 - \kappa) \mu_0^r) = \mathbb{E} \left[\xi(\lambda) \Omega(\lambda) \bar{\mu}_1^\delta q_1^k \right] \\ [b_0] \quad & q_0^b (1 - \mu_0^r) = \mathbb{E} \left[\xi(\lambda) \Omega(\lambda) \bar{\mu}_1^\delta q_1^b \right]\end{aligned}$$

A.2.3 Market clearing in the $t = 1$ asset markets

- (i) $\kappa = 1/2$: ($R_{1,2}^b = R_{1,2}^k$): The portfolio shares are indeterminate, but the two asset markets must clear at the aggregate:

$$\int a(\lambda) dF = q_1^k k_0 + q_1^b b_0$$

- (ii) $\kappa \neq 1/2$: Market clearing on the debt market requires $\int b_1 = b_0$ which becomes:

$$\int_{\lambda^\dagger} \frac{\varrho \delta^d(\lambda)(1 - \lambda) d_0}{\kappa} dF = q_1^b b_0$$

Market clearing on the capital market requires $\int k_1 = k_0$ which becomes:

$$\int^{\lambda^\dagger} \frac{-(1 - \omega) \varrho \delta^d(\lambda)(1 - \lambda) d_0 + a(\lambda)}{\kappa} dF + \int_{\lambda^\dagger} \frac{-\varrho \delta^d(\lambda)(1 - \lambda) d_0 + a(\lambda)}{\kappa} dF = q_1^k k_0$$

A.2.4 Aggregate resource constraints

The banks aggregated budget constraints at $t = 1$ can be written as ($\int \Delta k dF = \int \Delta b dF = 0$):

$$\underbrace{\left[\int^{\lambda^*} \lambda dF(\lambda) + \int_{\lambda^*} \delta^d(\lambda) \lambda dF(\lambda) \right]}_{\text{aggregate payout to early households}} d_0 = z_1 m_0 - \varsigma d_0 (1 - F(\lambda^*)) - \alpha d_0 F(\lambda^\dagger)$$

where the last term is equal to aggregate consumption (from household BC)

$$\int \lambda c_1(1, \lambda) dF(\lambda) = \left[\int^{\lambda^*} \lambda dF(\lambda) + \int_{\lambda^*} \delta^d(\lambda) \lambda dF(\lambda) \right] d_0$$

The aggregated bank budget constraint at $t = 2$ is

$$\int \left(\delta_2^e(\lambda) + (1 - \lambda) \delta^d(\lambda) d_0 \right) dF(\lambda) = z_2 k_0 + \delta_2^b b_0$$

while aggregate consumption in the PM (from the household budget constraint) is

$$\begin{aligned} \int (1 - \lambda) c_2(0, \lambda) dF(\lambda) &= \int (1 - \lambda) \left(\delta_2^e(\lambda) + \delta^d(\lambda) d_0 - \tau \right) dF(\lambda) \\ &= z_2 k_0 + \delta_2^b b_0 - \int \lambda \delta_2^e(\lambda) dF(\lambda) - T_2 \end{aligned}$$

B Data Sources

We combined existing historical databases with transcription from the digital archives of newspapers and government reports. Before 1884, we take bond data from Global Financial Data (GFD). From 1884 to 1940, we collect digitize and organize data from The New York Times, the Commercial & Financial Chronicle, Merchant’s Magazine, and [Macaulay et al. \(1938\)](#). We use the risk classifications from [Macaulay et al. \(1938\)](#) to create a collection of high-grade corporate bonds.

C US Historical Time Line

The text references many changes to monetary and financial regulation. In this section, we collect those events into a historical timeline, which is shown in table 3. The time line is broken up into a collection a collection of banking “eras”. The first era is from 1791-1836, during which the First and Second Banks of the US operated alongside state banks. The second era is from 1837-1962, during which state banks could automatically gain bank charters without a congressional review process, often referred to as the “free banking” era. The third era is from 1863-1913, during which the federal government chartered national banks that issued bank notes backed by US federal government debt. The fourth era is from 1913-1933, during which the Federal Reserve Bank was introduced to act as lender-of-last resort to the banking sector. The fifth era is from 1934-1980, during which the New Deal financial regulations were in place. The sixth era is from 1980s-2009, during which the New Deal financial regulations were gradually unwound. Finally, there is the era from 2010 to the present day, during which the Dodd-Frank Act another financial crisis legislation are in place.

Table 3 Time Line of Monetary and Financial Events

1791	Congress charters the First Bank of the US. The bank is privately owned. It operates as a commercial bank but also has the special privileges of acting as banker for the federal government (storing tax revenue and making loans) and being able to operate across states. It shares responsibility with state banks for bank note issuance. It influences state bank money and credit issuance by setting the rate at which it redeems state notes collected as tax revenue into gold.
1792	Coinage Act of 1792. Authorizes the US to issue a new currency, the US gold dollar.
1811	Charter of the First Bank of the US expires and is not renewed.
1812-5	War of 1812. Convertibility to bank notes to gold is suspended. Government issues Treasury Notes to finance the war.
1816	Congress charters the Second Bank of the U.S.
1819	Panic of 1819. Cotton prices fall, farms go bankrupt, and banks fail.

1832	Jackson vetoes bill to recharter Second Bank.
1833	Jackson removes federal deposits from Second Bank of the US
1834	Coinage Act of 1834. Changes the ratio of silver to gold from 15:1 to 16:1.
1836	Charter of the Second Bank of the US expires and is not renewed. The Second Bank becomes a private corporation.
<hr/>	
1837	“Free Banking” Era begins. Michigan Act allows the automatic chartering of banks (without requiring explicit approval from state legislature) that issue bank notes backed by specie (gold and silver coins). Over the next few years, other states pass similar laws.
1837	Panic of 1837. Sharp decrease in real estate prices leads to large bank losses. In New York, every bank suspends payment in gold and silver coinage. Many banks fail.
1857	Coinage Act of 1857. Foreign coins can longer be legal tender.
1857	Panic of 1857. Railroad company stocks drop sharply. Ohio Life Insurance and Trust company fails, which prompts a collapse in stock prices and widespread failures across mercantile firms.
<hr/>	
1861-5	Civil War.
1862	Legal Tender Act. Authorizes the federal government to use nonconvertible greenback paper dollars to pay its bills.
1863-4	The National Bank Acts. The National Currency Act (1863) and The National Bank Act (1864) establish a system of nationally chartered banks and the Office of the Comptroller of the Currency. National banks can issue national bank notes up to 90% of the minimum of par and market value of qualifying US federal bonds. Limit on aggregate national bank note issuance is \$300 million. Banks must pay a 1% annual tax per on outstanding national bank notes backed by US federal bonds. State banks must start paying a 2% annual tax on state bank notes.
1865-6	Additional National Bank Acts. State banks must start paying a 10% annual tax on state bank notes.
1870	Limit on aggregate national bank note issuance increases to \$354 million.
1873	Bank panic of 1873. Widespread failure of railroad firms leads to stock market crash and bank failures. Jay Cooke and Company goes bankrupt.
1875	Congress repeals limit on aggregate national bank note issuance.
1879	US Treasury starts to promise to convert greenbacks to dollars one-for-one.

1893	Bank panic. A combination of falling commodity prices, oversupply of silver, and a fall in US Treasury gold reserves prompted a run on bank deposits.
1896	Cross of Gold Speech. Democratic presidential candidate William Jennings Bryan gives a speech in favor of allowing unlimited coinage of silver into money demand (“free silver”).
1900	Tax on national bank notes backed by US federal bonds paying coupons less than or equal to 2% is reduced to 0.5% per annum.
1900	Gold Standard Act. The gold dollar becomes the standard unit of account (further restricting the possibility of “free silver”).
1907	Panic of 1907. The Knickerbocker Trust Company collapses prompting a bank run. J.P. Morgan organizes New York bankers to provide liquidity to shore up the banking system.
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1913	Federal Reserve Act. Establishment of the Federal Reserve Bank to act as a reserve money creator of last resort during financial panics.
1914-8	World War I.
1917	2nd Liberty Loan Act establishes a \$15 billion aggregate limit on the amount of government bonds issued.
1929	Stock market crash and start of the Great Depression.
1929	US issues first Treasury Bill.
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1933	Banking Act (“Glass-Steagall Act”). Establishes the Federal Deposit Insurance Corporation (FDIC). Separates commercial and investment banking. Introduces cap on deposit interest rate (“Regulation Q”).
1933	President Roosevelt issues an Executive Order requiring people and businesses to sell their gold to the government at \$20.67 per ounce.
1934	Gold Reserve Act.
1934	National Housing Act. Establishes the Federal Savings and Loan Insurance Corporation (FSLIC).
1935	The last national bank notes are replaced by Federal Reserve notes.
1938	Amendment to the National Housing Act established the Federal National Mortgage Association (FNMA), commonly known as Fannie Mae.
1939-45	World War II.
1942	The Treasury and Federal Reserve agree to fix the yield curve on Treasury securities.

1944	•	Bretton Woods Agreement.
1951	•	Treasury-Fed Accord ends the fixed yield curve on Treasury securities and establishes the Fed's policy independence from fiscal concerns.
1968	•	Housing and Urban Development Act of 1968. Creates the Government National Mortgage Association (GNMA), commonly known as Ginnie Mae.
1966	•	Fed applies Regulation Q to impose deposit rate ceiling for the first time.
1971	•	US effectively terminates the Bretton Woods system by ending the convertibility of the US dollar to gold.
1977	•	Congress issues the Fed with the dual mandate to "promote effectively the goals of maximum employment, stable prices, and moderate long term interest rates".
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1980	•	Depository Institutions Deregulation and Monetary Control Act of 1980 starts to phase out Regulation Q.
1986-1989	•	Savings and loan crisis.
1994	•	Riegle-Neal Interstate Banking and Branching Efficiency Act. Allows banks to operate across states.
1999	•	Gramm-Leach-Bliley Act. Repeals provisions of the Glass-Steagall Act that prohibited a bank holding company from owning other financial companies.
2007-9	•	Great Financial Crisis.
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2010	•	Dodd-Frank Wall Street Reform and Consumer Protection Act.