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46750 - Optimization in modern power systems

Optimal scheduling and strategic offering problem in day ahead electricity markets

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Contents

Workload distribution

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Problem Statement and Motivation	25%	25%	25%	25%
Optimization Formulation and Solution Method	25%	25%	25%	25%
Implementation	25%	25%	25%	25%
Numerical Validation	25%	25%	25%	25%

Nomenclature

Decision variables

η^{ω} Auxiliary variable for CVaR in scenario ω

λ Market clearing price (dual variable for balancing constraint)

$\overline{\mu}_g$ Dual variable for generator production upper bounds

$\overline{\mu}_w$ Dual variable for wind production upper bounds

$\overline{\sigma}_d$ Dual variable for load upper bounds

$\underline{\mu}_g$ Dual variable for generator production lower bounds

$\underline{\mu}_w$ Dual variable for wind production lower bounds

$\underline{\sigma}_d$ Dual variable for load lower bounds

ξ Value at Risk (VaR)

l_d Dispatched load in the day-ahead (DA) market

p_{G1}^{DA} Day-ahead bid quantity for generator G1

p_g Dispatched generator production in the DA market

p_w Dispatched wind farm production in the DA market

Parameters

α Risk level for CVaR (eg, 95%)

β Weighting factor for CVaR optimization

$\overline{L}_d$ Maximum load demand in the DA market

$\overline{P}^{G1}$ Maximum generator capacity for G1 in the DA market

$\overline{P}_g$ Maximum generator capacity in the DA market

$\overline{P}_w$ Maximum wind farm capacity in the DA market

π_{ω} Probability of scenario ω

c_d^D Cost of load demand

c_g^G Cost of generator production

c_W^W Cost of wind farm production

c_{G1} Marginal cost of G1 as a price-maker

M Big-M constant for linearizing constraints

Sets

Ω Set of scenarios

D Set of loads

G Set of generators

H Set of hours

W Set of wind farms

1 Problem Statement and motivation

The liberalization of the European electricity market was partly driven by the aim of ensuring optimal market prices for both producers and consumers. One of the primary goals behind this initiative was to maximize social welfare by clearing the day-ahead (DA) electricity market at a uniform price level.

Achieving true maximum social welfare requires all market participants to act as price-takers. On the generator side, a price-taker submits bids in the DA market that accurately reflect their true marginal costs and production capacities. However, in certain scenarios, participants can act as price-makers by exercising market power to influence prices in their favor. By doing so, they aim to increase their profits beyond what would be possible if all participants behaved as price-takers. This strategy, however, is most fruitful when a participant's generation unit plays a role as a marginal price setter in the DA market.

The objective of this project is to evaluate the need for market regulations to address strategic bidding behavior, as such behavior is expected to reduce social welfare by driving up DA prices, excluding consumers from trading in the market. As a consequence of this, the DA price is increased and results in more expensive electricity for consumers.

This project focuses on analyzing the role of a thermal power plant operator and their impact on social welfare when acting as a price-maker in the DA market. The analysis also considers the generation mix of different units. The current electricity mix is increasingly characterized by a high share of renewable energy sources (RES), such as wind and solar power. The variable and uncertain nature of RES production introduces additional challenges for conventional market participants attempting to model and simulate outcomes in the DA market.

Finally, the project compares a market with relatively few participants holding significant market shares to one with many participants holding smaller shares, examining how a larger market size can reduce incentives for price-maker behavior.

To capture the influence of a participant acting as a price maker, different setups are investigated. This involves solving the market clearing problem for a chosen day, and compare it to the solution of solving the profit maximization of the price maker given the same wind power production. Given that the wind power production is associated with great uncertainty, it is investigated how maximizing the expected profit for the price maker given multiple scenarios can be achieved. The basis of comparison is accomplished by solving the market clearing problem given the different scenarios. This leads to a series of optimization problems with progressively greater complexity. These are listed below:

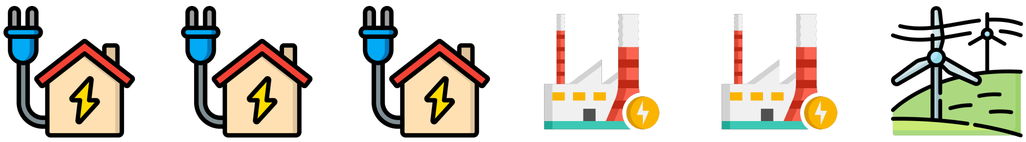
- Deterministic market clearing problem
- Deterministic pricer-maker profit maximization problem
- Market clearing problem solved across multiple scenarios
- Stochastic price-maker profit maximization problem with different add-ons

Problem statement:

"How does the price-maker behavior of a thermal power plant affect social welfare in the day-ahead market, considering the varying penetration of renewable energy sources and the role of market size in mitigating strategic bidding behavior?"

2 Study framework and methodology

The following sections present the applied framework and methodology which has been applied in this case study. As a baseline for investigation, we have chosen a small system consisting of two thermal generators, one wind turbine farm and three demands. Figure 1 shows the utility of demands and cost of generation in EUR/MWh and total capacities in MWh. The idea of operating with a small system is that we can more clearly display the consequences of a market player acting strategically as a price-maker. However, a case study is conducted in section 9 to analyze how increasing the number of market participants affects a price-maker's market power.



	D1	**D2**	**D3**	**G1**	**G2**	**W1**
Utility/Cost	$c_1^D = 140$	$c_2^D = 100$	$c_3^D = 60$	$c_1^G = 50$	$c_2^G = 90$	$c_1^W = 0$
Capacities	$\overline{P_1^D} = 450$	$\overline{P_2^D} = 650$	$\overline{P_3^D} = 800$	$\overline{P_1^G} = 800$	$\overline{P_2^G} = 1000$	$\overline{P_1^W} = 600$

Figure 1: Generators, Demand Sites, and Wind Farm Parameters[1]

2.1 Generation and Content of Scenarios

Our scenarios comprise 24-hour slices and are based on varying wind production, meaning that the load quantity bids are static across scenarios. The wind data is created from hourly observed wind production data over the entire course of 2019 from Renewable Ninja, where a 6 MW wind farm is scaled to fit the size of our market. When solving the deterministic problems, a single scenario is specifically chosen among the 365 possible with the aim of visually presenting the consequences of price maker behavior. When solving the stochastic problems, only 200 random samples are chosen out of the 365 scenarios for computational purposes. This should result in representative scenarios of seasonality and varying production, but as to test this hypothesis, we leave 20% of the random samples for out of sample analysis. If the scenarios in sample and out of sample are sufficiently alike, we assume that the number of samples chosen is sufficiently large.

2.2 Parameters for production and consumption units

In this project, social welfare is defined as the sum of consumer and producer surplus, excluding any system operator revenue. To analyze changes in social welfare across different market sizes, we will maintain constant total generational capacities and loads. Consequently, the sensitivity of social welfare changes will depend on the total number of participants and their corresponding bids. Parameters for generational or consuming units are created using the following approach:

Capacities: The France energy data provider, RTE is used to create realistic variations in our load profiles. A profile from RTE is normalized and then scaled according to desired simulation preferences. Wind production profiles follow the same approach where Renewable Ninja is used to create realistic variations in wind production units on an hourly basis for each day throughout a year. Thermal generational units has a constant maximum capacity. However, unit G1 is allowed to change its capacity when it acts a price-maker.

Costs: Demand costs are determined using normalized load profiles, scaled with a constant price and adjusted by adding noise. This approach ensures that when a unit's electricity consumption is high relative to its average consumption, its willingness to pay is also elevated. This can also be seen in Appendix B. However, we have designed the loads in such a way that D0 is a small, crucial infrastructure which bids at high prices, whereas the opposite accounts for D2. A marginal production cost of zero is assumed for wind units. Costs for thermal generational units are randomly chosen between an upper and lower bound. G1's costs are calibrated to be the price-setting unit in the system. The price-maker behavior is limited to changes in offered generation, hence G1 is not allowed to change its bidding price.

[1]Icons attribution: Power plant icons created by Flat Icons - Flaticon, Wind icons created by Culmbio - Flaticon, Utilities icons created by Smashicons - Flaticon

2.3 General modeling assumptions

Throughout this project, four optimization problems are examined to address the problem statement, with the following general assumptions applied to all:

1) Copper-plate structure is assumed i.e. transmission capacities are assumed to always be sufficient, and no losses from transmitting electricity are included.

2) There are no ramping constraints on the generational units.

3) Generational units has no lower bound on their production if they are dispatched in the DA market. If this where to be modeled, it would result in having to solve a market dispatch problem which is not linear. Thus, minimum dispatch quantities are neglected as the scope of this project is to investigate the value of social welfare.

An object oriented approach is applied in Python to solve the optimization problems using Gurobi. Furthermore, decomposition methods are applied to the stochastic optimization problems to investigate computational time. A description of the computational power used to process each of the problems is presented in Appendix A. Here, the number of constraints along with the number of linear and integer variables and computation time are used to compare the processing between each of the models.

3 Deterministic market clearing problem

To investigate the influence of a participant acting as a price maker, the deterministic market clearing problem (DMCP) is investigated. In this setup, social welfare can be directly derived from the objective function of this problem. In DMCP, it is assumed that all actors are price takers in a perfectly competitive market, where all price bids are set to the marginal costs. Decision variables include supplied loads l_{dh}, dispatch of thermal units p_{gh} and wind turbines p_{wh}. The following mathematical formulation is applied to solve the DMCP using baseline system parameters:

$$\max_{p_{gh},p_{wh},l_{dh}} \sum_{h\in H}\left(\sum_{d\in D} c_d^D l_{dh} - \left(\sum_{g\in G} c_g^G p_{gh} + \sum_{w\in W} c_w^W p_{wh}\right)\right) \tag{1}$$

s.t.

$$\sum_{d\in D} l_{dh} - \sum_{g\in G} p_{gh} - \sum_{w\in W} p_{wh} = 0 \qquad : \lambda_h \quad \forall h \in H \tag{2}$$

$$0 \le l_{dh} \le \overline{L}_{dh} \qquad : \underline{\sigma}_{dh}, \overline{\sigma}_{dh} \quad \forall d \in D, \forall h \in H \tag{3}$$

$$0 \le p_{gh} \le \overline{P}_{gh} \qquad : \underline{\mu}_{gh}, \overline{\mu}_{gh} \quad \forall g \in G, \forall h \in H \tag{4}$$

$$0 \le p_{wh} \le \overline{P}_{wh} \qquad : \underline{\mu}_{wh}, \overline{\mu}_{wh} \quad \forall w \in W, \forall h \in H \tag{5}$$

The constraint in Equation 2 ensures balance between production and consumption, whereas Equation 3, 4 and 5 ensure operation of generating and consuming units within the physical limits. The dual value of the balancing constraint corresponds to the marginal price for the given hour. The dual formulation of the problem can be found in Appendix C.

3.1 Results from solving the deterministic market clearing problem

Inserting the wind profile for a chosen day, the merit order curve for hour 10 is illustrated as an example in Figure 2. The shaded yellow area illustrates the total social welfare. In this specific hour, the market clears at the load with the lowest offering price.

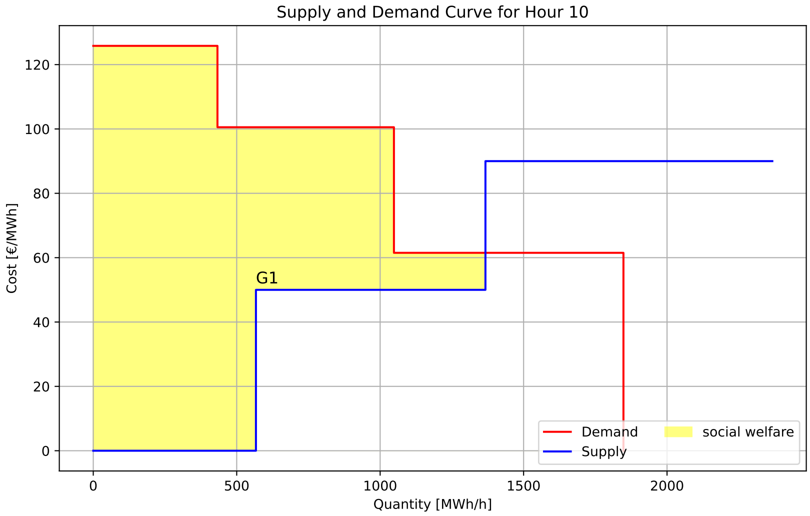


Figure 2: Merit order curve for deterministic market clearing problem.

The point where the supply and demand curve intersect corresponds to the marginal price for this hour. As the generating unit, G2 has a higher production cost than the marginal price, it means that this unit is not activated in the hour. To compare the influence of a participant acting as a price maker, the relevant points include the effects on the social welfare, the average market clearing price, the profit of the price maker unit and the production of this unit. The key results across all 24 hours are summarized in Table 1.

	MCP	Unit
Total social welfare	1,891	k€
Avg. market clearing price	54.17	€/MWh
G1 profit	80.13	k€
Total G1 production	16.1	GWh

Table 1: Results from deterministic market clearing problem

4 Deterministic price maker problem

In this section, we wish to highlight the consequences on social welfare, when one actor tries to bid strategically with her offered quantity. This requires a bi-level optimization problem, where the upper level regarding the price maker is effected by the results of the lower level market clearing. The lower level would therefore be similar to the one shown in section 3, but including the constraint stating the price maker may bid as the upper bound in a separate capacity constraint, excluding the price maker from Equation 4.

$$0 \leq p_{gh} \leq \overline{P_{gh}} \qquad : \underline{\mu_g}, \overline{\mu_g} \quad \forall g \in G \backslash \{G_1\}, \forall h \in H \tag{6}$$

$$0 \leq p_{G1} \leq p^{DA}_{G1h} \qquad : \underline{\mu}_{G1}, \overline{\mu}_{G1} \quad \forall h \in H \tag{7}$$

Where p^{DA}_{G1} is considered fixed. On the upper level, the market actor wishes to maximize profits i.e. the difference between the market price and generator costs multiplied by the production. This is formulated below:

$$\max_{p^{DA}_{G1h}, p_{G1h}, \lambda_h} \sum_{h \in H} \left(\left(\lambda_h - c^G_{G1h} \right) \cdot p^{DA}_{G1h} \right) \tag{8}$$

s.t.

$$0 \leq p^{DA}_{G1h} \leq \overline{P}_{G1} \qquad : \underline{\mu_{G1}}, \overline{\mu}_{G1} \quad \forall h \in H \tag{9}$$

$$\{p_{G1h}, \lambda_h\} = \text{primal and dual solutions of the lower level problem} \tag{10}$$

However, these two optimization problems depend on each other and should be solved simultaneously. Directly solving a bi-level problem is challenging due to its hierarchical nature, which is why the rewriting of the lower level into KKT conditions is necessary. In such, we rewrite the optimization problem of the lower level into a set of equations and inequalities that can be added to the upper level as constraints, resulting in a single level mathematical program with equilibrium constraints (MPEC), which is more tractable to solve.

4.1 Deterministic price maker as single level

To convert the price maker problem into a single level problem, the structure of the market clearing problem is taken into advantage. In order to do so, the Lagrangian function is expressed before formulating the KKT conditions of the market-clearing problem. To streamline the theoretical analysis and practical solution, the market-clearing problem is first reformulated into standard form (minimization problem), where after its Lagrangian function can be expressed:

$$\begin{aligned}\mathcal{L}\left(p_{gh}, p_{wh} l_{dh}, \lambda_h, \overline{\mu}_{wh}, \underline{\mu}_{wh}, \overline{\mu}_{gh}, \underline{\mu}_{gh}, \overline{\sigma}_{dh}, \overline{\sigma}_{dh}\right) = \sum_{h\in H} \Bigg[& \sum_{g\in G} c_g^G p_{gh} + \sum_{w\in W} c_w^W p_{wh} - \sum_{d\in D} c_d^D l_{dh} \\ & + \lambda_h \left(-\sum_{g\in G} p_{gh} - \sum_{w\in W} p_{wh} + \sum_{d\in D} l_{dh} \right) \\ & + \sum_{g\in G} \left[-\underline{\mu}_{gh} p_{gh} + \overline{\mu}_{gh} \left(p_{gh} - \overline{P_{gh}} \right) \right] \\ & + \sum_{w\in W} \left[-\underline{\mu}_{wh} p_{wh} + \overline{\mu}_{wh} \left(p_{wh} - \overline{P_{wh}} \right) \right] \\ & + \sum_{d\in D} \left[-\underline{\sigma}_{dh} l_{dh} + \overline{\sigma}_{dh} \left(l_{dh} - \overline{L_{dh}} \right) \right] \Bigg]\end{aligned} \tag{11}$$

Strong duality holds because the market-clearing optimization problem is convex and satisfies the weak Slaters' condition. Hence, its KKT conditions are sufficient and necessary optimality conditions. The KKT conditions can be categorized into four groups of constraints. The first group is the stationarity conditions which state that the gradient of the Lagrangian function with respect to our primal decision variables is equal to 0 at optimality:

$$\text{Loads} \qquad \frac{\partial \mathcal{L}}{\partial l_{dh}} = -c_d^D + \lambda_h - \underline{\sigma}_{dh} + \overline{\sigma}_{dh} = 0 \qquad \forall d \in D, h \in H \tag{12}$$

$$\text{Generators} \qquad \frac{\partial \mathcal{L}}{\partial p_{gh}} = c_g^G - \lambda_h - \underline{\mu}_{gh} + \overline{\mu}_{gh} = 0 \qquad \forall g \in G, h \in H \tag{13}$$

$$\text{Wind farms} \qquad \frac{\partial \mathcal{L}}{\partial p_{wh}} = c_w^W - \lambda_h - \underline{\mu}_{wh} + \overline{\mu}_{wh} = 0 \qquad \forall w \in W, h \in H \tag{14}$$

The second group of constraints are the complementarity slackness conditions which ensure that for each inequality constraint $g(x) \leq 0$, either the constraint is active or its corresponding multiplier is zero:

$$\text{Generators lower bound} \qquad 0 \leq \underline{\mu}_{gh} \perp 0 \leq p_{gh} \qquad \forall g \in G, h \in H \tag{15}$$

$$\text{Wind farms lower bound} \qquad 0 \leq \underline{\mu}_{wh} \perp 0 \leq p_{wh} \qquad \forall w \in W, h \in H \tag{16}$$

$$\text{Loads lower bounds} \qquad 0 \leq \underline{\sigma}_{dh} \perp 0 \leq l_{dh} \qquad \forall l \in L, h \in H \tag{17}$$

$$\text{Generators upper bounds} \qquad 0 \leq \overline{\mu}_{gh} \perp 0 \leq \overline{P}_{gh} - p_{gh} \qquad \forall g \in G \setminus \{G_1\}, \forall h \in H \tag{18}$$

$$\text{G1's upper bounds} \qquad 0 \leq \overline{\mu}_{G_1} \perp 0 \leq p_h^{DA} - p_{G1h} \qquad \forall h \in H \tag{19}$$

$$\text{Wind farms upper bounds} \qquad 0 \leq \overline{\mu}_{wh} \perp 0 \leq \overline{P}_{wh} - p_{wh} \qquad \forall w \in W, \forall h \in H \tag{20}$$

$$\text{Loads upper bounds} \qquad 0 \leq \overline{\sigma}_{dh} \perp 0 \leq \overline{L}_{dh} - l_{dh} \qquad \forall d \in D, \forall h \in H \tag{21}$$

The linearization of the complementarity constraints can be accomplished by Fortuny Amat Linearization. This is done in Appendix D.
The third group of constraints is the primal feasibility conditions which is simply the original problem con-

straints:

$$\sum_{d\in D} l_{dh} - \sum_{g\in G} p_{gh} - \sum_{w\in W} p_{wh} = 0 \qquad \forall h\in H \tag{22}$$

$$0 \le l_{dh} \le \overline{L}_{dh} \qquad \forall d\in D, \forall h\in H \tag{23}$$

$$0 \le p_{gh} \le \overline{P}_{gh} \qquad \forall g\in G\backslash\{G_1\}, \forall h\in H \tag{24}$$

$$0 \le p_{G1h} \le p_{G1h}^{DA} \qquad \forall h\in H \tag{25}$$

$$0 \le p_{wh} \le \overline{P}_{wh} \qquad \forall w\in W, \forall h\in H \tag{26}$$

The last group of constraints are the dual feasibility conditions stating that all dual variables must be non-negative:

$$\text{Loads} \qquad \underline{\sigma}_{dh} \ge 0, \quad \overline{\sigma}_{dh} \ge 0 \qquad \forall d\in D, \forall h\in H \tag{27}$$

$$\text{Generators} \qquad \underline{\mu}_{gh} \ge 0, \quad \overline{\mu}_{gh} \ge 0, \qquad \forall g\in G, \forall h\in H \tag{28}$$

$$\text{Wind farms} \qquad \underline{\mu}_{wh} \ge 0, \quad \overline{\mu}_{wh} \ge 0 \qquad \forall w\in W, \forall h\in H \tag{29}$$

Now we are ready to write the deterministic price maker as a single-level problem. The objective function is almost identical to Equation 8 with the addition of the dual variables as secondary decision variables. This objective function will be subject to the capacity constraint of the price maker as well as all the KKT conditions.

$$\max_{\substack{p_h^{G_1}, p_{gh}, p_{wh}, l_{dh}, \lambda_h \\ \underline{\mu}_{gh}, \overline{\mu}_{gh}, \underline{\mu}_{gh}, \overline{\mu}_{gh} \\ \underline{\sigma}_{lh}, \overline{\sigma}_{lh}}} \sum_{h\in H} \left(\left(\lambda_h - c_h^{G_1} \right) p_h^{G_1} \right) \tag{30}$$

s.t.

$$\text{Capacity constraint price-maker} \qquad 0 \le p_h^{G_1} \le \overline{P}_h^{G_1} \qquad \forall h\in H \tag{31}$$

$$\text{KKT conditions, } Equation\ 12-29 \tag{32}$$

The objective function provided for the price maker setup involves multiplication of two decision variables making it non-linear. We are interested in the linearization of this function to significantly improve computational time. According to the strong duality theorem, at optimality the objective of the primal and dual problems are equal. At optimality, this means that the optimal value of the market clearing problem, must be equal to the objective value of the its dual problem. For the price maker scenario, this can be written as:

$$\overbrace{\sum_{h\in H} \left(\sum_{d\in D} c_d^D l_{dh} - \left(\sum_{g\in G} c_g^G p_{gh} + \sum_{w\in W} c_w^W p_{wh} \right) \right)}^{\text{Primal problem}} = \overbrace{\sum_{h\in H} \left(\sum_{d\in D} \overline{L}_{d,h} \overline{\sigma}_{d,h} + \sum_{g\in G\backslash 1} \overline{P}_g \overline{\mu}_{g,h} + \sum_{w\in W} \overline{P}_w \overline{\mu}_{w,h} + p_{G1,h}^{DA} \overline{\mu}_{G1,h} \right)}^{\text{Dual problem}} \tag{33}$$

Isolating the terms including G1 yields:

$$\sum_{h\in H} -c_{G1}^G p_{G1,h} - p_{G1,h}^{DA} \overline{\mu}_{G1,h} = \sum_{h\in H} \left(\sum_{d\in D} \overline{L}_{d,h} \overline{\sigma}_{d,h} - c_d^D l_{dh} + \sum_{g\in G\backslash 1} \overline{P}_g \overline{\mu}_{g,h} + c_g^G p_{gh} + \sum_{w\in W} \overline{P}_w \overline{\mu}_{w,h} + c_w^W p_{wh} \right) \tag{34}$$

Based on the complementarity condition in Equation 19 ($p_{G1,h}^{DA} \overline{\mu}_{G1,h} = p_{G1,h} \overline{\mu}_{G1,h}$) this can be reformulated as:

$$\sum_{h\in H} -\left(c_{G1}^G + \overline{\mu}_{G1,h} \right) p_{G1,h} = \sum_{h\in H} \left(\sum_{d\in D} \overline{L}_{d,h} \overline{\sigma}_{d,h} - c_d^D l_{dh} + \sum_{g\in G\backslash 1} \overline{P}_g \overline{\mu}_{g,h} + c_g^G p_{gh} + \sum_{w\in W} \overline{P}_w \overline{\mu}_{w,h} + c_w^W p_{wh} \right) \tag{35}$$

The stationary conditions for generator G1 in Equation 13 enforces that $c_{G1}^G + \overline{\mu}_{G1,h} = \lambda_h + \underline{\mu}_{G1,h}$, which can be inserted, yielding:

$$\sum_{h\in H} -\left(\lambda_h + \underline{\mu}_{G1,h} \right) p_{G1,h} = \sum_{h\in H} \left(\sum_{d\in D} \overline{L}_{d,h} \overline{\sigma}_{d,h} - c_d^D l_{dh} + \sum_{g\in G\backslash 1} \overline{P}_g \overline{\mu}_{g,h} + c_g^G p_{gh} + \sum_{w\in W} \overline{P}_w \overline{\mu}_{w,h} + c_w^W p_{wh} \right) \tag{36}$$

With the complementarity condition in Equation 15 giving that $\underline{\mu}_{G1,h} p_{G1} = 0$ and multiplying through with -1, this can be reformulated as:

$$\sum_{h \in H} \lambda_h p_{G1,h} = \sum_{h \in H} \left(\sum_{d \in D} \left(-\overline{L}_{d,h} \overline{\sigma}_{d,h} + c_d^D l_{dh} \right) - \sum_{g \in G \setminus 1} \left(\overline{P}_g \overline{\mu}_{g,h} + c_g^G p_{gh} \right) - \sum_{w \in W} \left(\overline{P}_w \overline{\mu}_{w,h} + c_w^W p_{wh} \right) \right) \quad (37)$$

The objective function stated in Equation 30 for the price maker optimization problem can be rewritten as:

$$\max_{\substack{p_h^{G_1}, p_{gh}, p_{wh}, l_{dh}, \lambda_h \\ \underline{\mu}_{gh}, \overline{\mu}_{gh}, \underline{\mu}_{gh}, \overline{\mu}_{gh} \\ \underline{\sigma}_{lh}, \overline{\sigma}_{lh}}} \sum_{h \in H} \lambda_h p_{G1,h}^G - c_{G1,h}^G p_{G1,h}^G \quad (38)$$

Which then can be linearized by:

$$\max_{\substack{p_h^{G_1}, p_{gh}, p_{wh}, l_{dh}, \lambda_h \\ \underline{\mu}_{gh}, \overline{\mu}_{gh}, \underline{\mu}_{gh}, \overline{\mu}_{gh} \\ \underline{\sigma}_{lh}, \overline{\sigma}_{lh}}} \sum_{h \in H} \left(\sum_{d \in D} \left(-\overline{L}_{d,h} \overline{\sigma}_{d,h} + c_d^D l_{dh} \right) - \sum_{g \in G \setminus 1} \left(\overline{P}_g \overline{\mu}_{g,h} + c_g^G p_{gh} \right) - \sum_{w \in W} \left(\overline{P}_w \overline{\mu}_{w,h} + c_w^W p_{wh} \right) - c_{G1,h}^G p_{G1,h}^G \right) \quad (39)$$

This linearized version of the objective function will be used when solving the problem in Python to significantly lower the computational time.

4.2 Results from solving deterministic price maker problem

Looking at the market clearing example in Figure 2, it is apparent that G1 can manipulate the price by lowering its offered capacity such that load 3 is pushed out of the market. Hence, the price will be set by the offering price of G2, increasing G1's profits. The implications of this price-maker behavior, again using hour 10 as an example, are illustrated in Figure 3. In the same manner, social welfare is illustrated with the shaded yellow area. The red shaded area shows the lost social welfare due to price-maker behavior. Table 2 summarizes the key results from clearing the market with and without G1 acting as a price maker across all 24 hours. It is clear that when G1 reduces its production, social welfare decreases by 2%. In doing so, G1's profit increases 4.75 times compared to the profit generated in the MC-problem. These two key numbers tell us that a lot of the social welfare has been allocated from consumer utility to the producer surplus of G1.

	MCP	**PMP**	**PMP/MCP**	**Unit**
Total social welfare	1,891	1,860	0.98	k€
Avg. market clearing price	54.17	85.56	1.58	€/MWh
G1 profit	80.13	380.46	4.75	k€
Total G1 production	16.15	10.32	0.64	GWh

Table 2: Results from deterministic market clearing and price maker problems across all 24 hours.

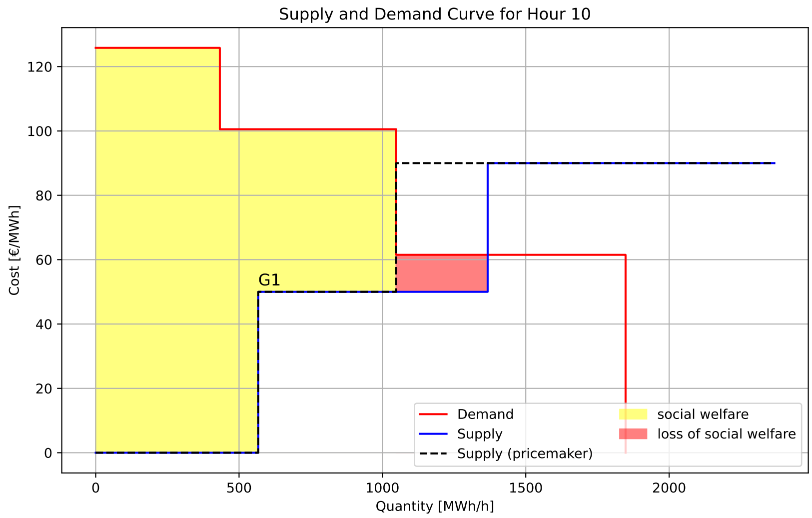
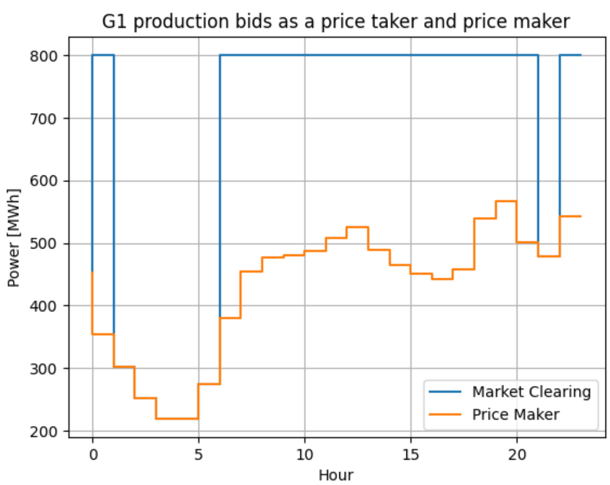
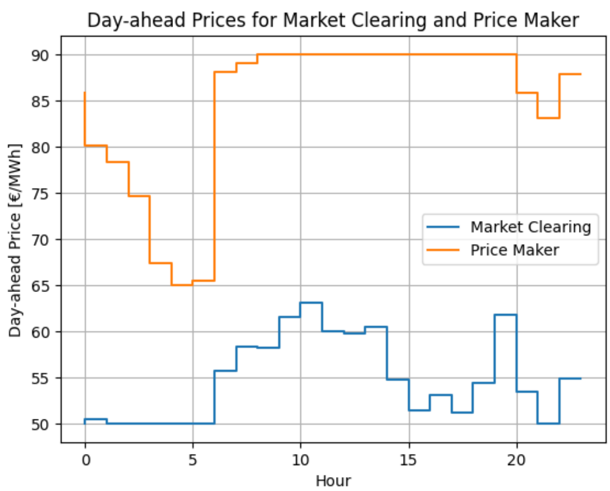


Figure 3: Merit order curve for deterministic price maker problem. G1 reducing generation to achieve clearing price of generator 2 in hour 10.

Figure 4 shows the variation in the cleared day-ahead price and offered capacity by G1 with and without G1 showcasing price maker behavior. In Figure 4a, one can see a large reduction in G1 generation in hours 7-21, which pushes out one of the demands from the market to achieve the price of G2. The day ahead price also varies significantly in hours where there is no large difference in G1 generation. In these hours, G1 reduces its generation minimally to achieve a market price defined by the adjacent load rather than its own marginal price as illustrated in Figure 5. There is not much sense to this second type of price-maker behavior, since this implies that G1 has perfect information of the merit order curve. Hence, uncertainty is included later in the project which eliminates the opportunity for this 'fine-tuned' type of market manipulation.



(a) Generation from G1 in all hours with and without price-maker behavior.



(b) Cleared day-ahead price in all hours with and without price-maker behavior.

Figure 4: Comparison of generation (a) and day-ahead prices (b) with and without generator G1 displaying price-maker behavior.

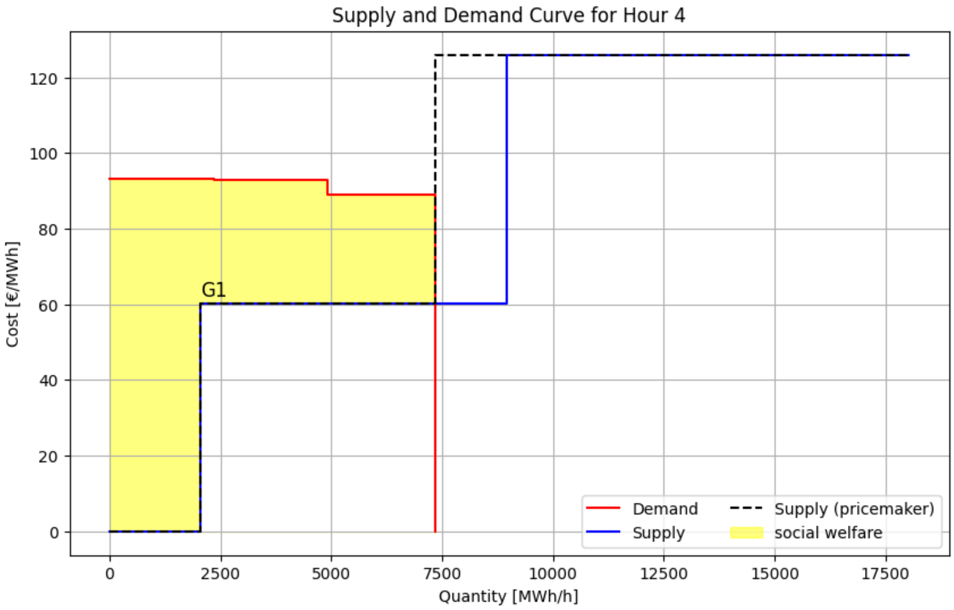


Figure 5: Merit order curve for deterministic price maker problem.G1 reducing generation to achieve clearing price of load 3 in hour 4.

5 Market clearing problem across multiple scenarios

In section 4, the bid made by the participant acting as price maker is done with perfect knowledge of the energy demand and supply for the given day. As mentioned earlier, wind power production is associated with great uncertainty. This means that the bid made in a specific scenario may not be the best bid across multiple scenarios. In order to maximize the bid across multiple scenarios, and investigating how this influences social welfare, the basis for comparison is made by solving the market clearing problem for multiple scenarios. This optimization problem is much alike the one in section 3. However, the social welfare is replaced by expected social welfare, which is simply the probability weighted sum of the social welfare for the included scenarios. The market problem is cleared with perfect information meaning that each decision variable is optimized for each scenario. This enables a meaningful comparison between social welfare in the market clearing problem across multiple scenarios and the stochastic problem with generator one acting as price-maker.

$$ESW = \max_{p_{gh}, p_{wh}, l_{dh}} \sum_{\omega \in \Omega} \pi^{\omega} \left(\sum_{h \in H} \left(\sum_{d \in D} c_d^D l_{dh}^{\omega} - \left(\sum_{g \in G} c_g^G p_{gh}^{\omega} + \sum_{w \in W} c_w^W p_{wh}^{\omega} \right) \right) \right) \tag{40}$$

s.t.

$$\sum_{d \in D} l_{dh}^{\omega} - \sum_{g \in G} p_{gh}^{\omega} - \sum_{w \in W} p_{wh}^{\omega} = 0 \qquad \forall h \in H, \forall \omega \in \Omega \tag{41}$$

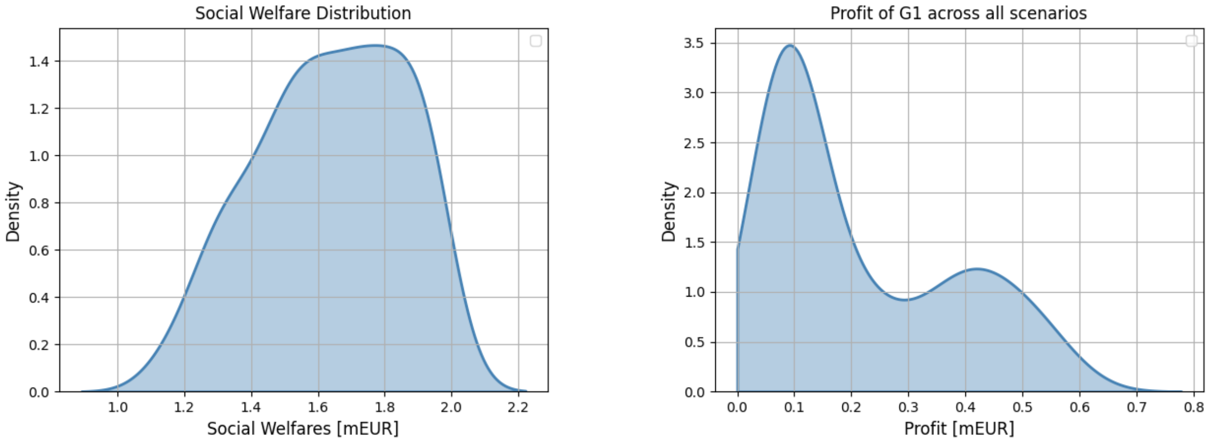
$$0 \leq l_{gh}^{\omega} \leq \overline{L}_{gh} \qquad \forall d \in d, \forall h \in H, \forall \omega \in \Omega \tag{42}$$

$$0 \leq p_{gh}^{\omega} \leq \overline{P}_{gh} \qquad \forall g \in G, \forall h \in H, \forall \omega \in \Omega \tag{43}$$

$$0 \leq p_{wh}^{\omega} \leq \overline{P}_{wh}^{\omega} \qquad \forall w \in W, \forall h \in H, \forall \omega \in \Omega \tag{44}$$

5.1 Results from solving the market clearing problem across multiple scenarios

As the model now optimizes based on many scenarios, the granularity of our results will mostly be across scenarios. Recalling the results from section 3, where G1 profited 80 k€, the profit of G1 now varies greatly among scenarios as depicted in Figure 6b, with a fat tail on the low-profit scenarios, meaning that G1 is often the marginal producer. As all our scenarios are equiprobable, the expected profit is simply the average profit across scenarios, which is 225 k€. This indicates that the chosen scenario of the deterministic market clearing is a low profit scenario, but it can also be noted that a profit of 80 k€ is among the most likely of profits.



(a) Social welfare distribution.

(b) Profit distribution.

Figure 6: Distribution of social welfare (a) and G1's profits (b) across all scenarios as a result of the 24-hour optimal bidding profile.

In order to compare the effects of including scenarios, we provide Table 3, highlighting that the social welfare is lower for the optimization across multiple scenarios, indicating that the chosen scenario in section 3 is above the average scenario with regards to social welfare and below average regarding G1's profits.

	Single scenario	**Multiple scenarios**	**Unit**
Social Welfare	1,891	1,606	k€
G1 Profit	80	224	k€

Table 3: Results from multiple scenario market clearing problem compared to the single deterministic. Note that the values regarding multiple scenarios are the expected values.

6 Stochastic price-maker profit maximization problem

With the uncertainty regarding the wind power production, an offering strategy for the participant acting as price maker could be to investigate the historic scenarios. The strategy could then be to make the bid, that overall yields the greatest expected profit. This can be done in a similar manner as in section 4 by utilizing the KKT conditions, however, in this setup it is desired to generate the best bid across the scenarios, making the bid of G1 independent of scenario, but all other decision variables and their respective dual variables scenario dependent. The objective function for this problem becomes:

$$\max_{\substack{p_h^{G_1}, p_{gh}^{\omega}, p_{wh}^{\omega}, l_{dh}^{\omega}, \lambda_h^{\omega} \\ \underline{\mu}_{wh}^{\omega}, \overline{\mu}_{wh}^{\omega}, \underline{\mu}_{gh}^{\omega}, \overline{\mu}_{gh}^{\omega} \\ \underline{\sigma}_{lh}^{\omega}, \overline{\sigma}_{lh}^{\omega}}} \sum_{\omega \in \Omega} \pi^{\omega} \left(\sum_{h \in H} \left(\left(\lambda_h^{\omega} - c_h^{G_1} \right) p_h^{G_1} \right) \right) \tag{45}$$

s.t.

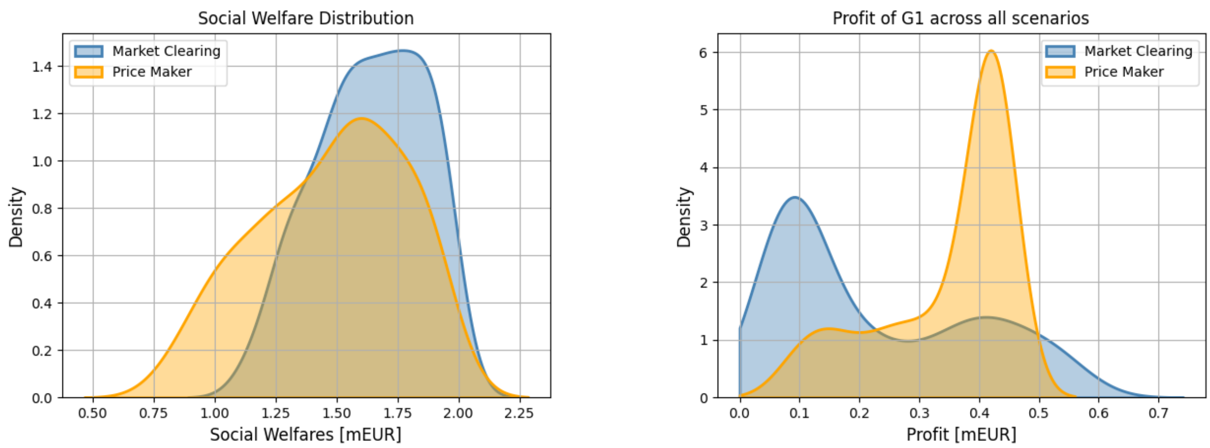
$$\text{Capacity constraint price-maker} \qquad 0 \leq p_h^{G_1} \leq \overline{P}_h^{G_1} \qquad \forall h \in H \tag{46}$$

Equation 45 shows the objective function for pricer-maker G1 where π^{ω} is the probability of each wind production scenario and λ^{ω} is the associated market clearing prices. Note that G1 faces decisions for DA dispatch quantities which do not depend on scenarios ω. Thus, the stochastic price-maker profit maximization problem is a bi-level optimization problem as for the case of the deterministic version in section 4. Again, the lower level optimization problem needs to rewritten into a set of constraints that can be used to formulate a single level optimization problem in this section. To do this, the KKT constraints are formulated for the problem presented in section 5. Also, the objective function is linearized to make sure that strong duality theorem can be applied and reduce computational time. The Lagrangian function, KKT conditions and total number of variables and constraints can be seen in Appendix E. When linearizing Equation 45 the objective function it becomes:

$$\max_{\substack{p_h^{G1}, p_{gh}^{\omega}, p_{wh}^{\omega}, l_{dh}^{\omega} \\ \lambda_h^{\omega}, \underline{\mu}_{wh}^{\omega}, \overline{\mu}_{wh}^{\omega}, \underline{\mu}_{gh}^{\omega} \\ \overline{\mu}_{gh}^{\omega}, \underline{\sigma}_{lh}^{\omega}, \overline{\sigma}_{lh}^{\omega}}} \sum_{\omega \in \Omega} \pi^{\omega} \sum_{h \in H} \Bigg(\sum_{d \in D} \left(-\overline{L}_{d,h} \overline{\sigma}_{d,h}^{\omega} + c_d^D l_{dh}^{\omega} \right) - \sum_{g \in G \setminus 1} \left(\overline{P}_g \overline{\mu}_{g,h}^{\omega} + c_g^G p_{gh}^{\omega} \right) - \sum_{w \in W} \left(\overline{P}_w^{\omega} \overline{\mu}_{w,h}^{\omega} + c_w^W p_{wh}^{\omega} \right)$$
$$- C_{G1,h}^{G} p_{G1,h}^{G} \Bigg) \tag{47}$$

6.1 Results from solving stochastic price maker problem

In this section, we wish to compare the expected profits of G1 and the expected social welfare of the market clearing problem across multiple scenarios and the price maker problem. To do so, we start by presenting the distribution of social welfare when G1 acts strategically compared with the market clearing in Figure 7a. Here, it can be seen that likeliness of achieving a high social welfare is lower in the case where G1 acts strategically. Figure 7b show the profit distributions for market clearing and the price maker problem. As also seen in Figure 4, the profits of G1 increase, as it bids strategically. In scenarios where G1 is the marginal price setter, it will decrease its generation bid and hence greatly reduce the likeliness of receiving 0 profit. This results in G1 having a minimum profit of 81 k€ as computed in our 200 scenarios. This minimum value is however also stochastic, and so we chose to visualize this using a smoothing kernel plot instead of bars.



(a) Comparing the SW distributions of MCP and PMP.

(b) Comparing the profit distributions of MCP and PMP

Figure 7: Distribution of social welfare (a) and G1's profits (b) across all scenarios as a result of the 24-hour bidding profile.

It can be derived from the figures that the expected social welfare drops, while the expected profits of G1 increases. The specific numbers are summarized in Table 4.

	MCP	**PMP**	**PMP/MCP** [Unitless]	**Unit**
Social Welfare	1,606	1,468	91%	k€
G1 Profit	225	347	154%	k€

Table 4: Results from multiple scenario market clearing problem compared to the stochastic price maker problem.

7 Stochastic price maker problem with perfect information

Staying in the perspective of the profit maximizing price maker, it could be interesting to evaluate the potential benefits of having better understanding of market dynamics and forecasts of e.g. load and wind production instead of using solely historic data as the input for all future bid profiles. The way we set up our optimization problem makes it difficult for the price maker to take into account the forecasted wind speeds, in which case the implementation of machine learning e.g. linear regression could be beneficial. However, what we can calculate is the upper limit to the benefit of perfect information. As an average over all our scenarios, we call this the expected value of perfect information (EVPI). This is calculated by solving each scenario as a deterministic scenario, allowing specific bidding strategies for each scenario instead of developing a single optimal bidding strategy for all scenarios. In this way, the task resembles section 4 on deterministic market clearing quite a lot.

7.1 Results from solving stochastic price maker problem with perfect information

When solving the scenarios individually as deterministic price maker problems, we achieve the results presented in Table 5:

	Stochastic PMP	**Stochastic PMP with PI**	**EVPI**	**Relative Change [%]**
Profit of G1	347 k€	534 k€	187 k€	54%

Table 5: Results of G1 obtaining perfect information, when acting as price maker.

These results can be used as benchmark of the optimal average profit of G1 across scenarios. We find that if G1 did have perfect insight in every merit order curve, she could have earned 54% more, which in the case of G1 would encourage the development of more precise models.

8 Stochastic price-maker and CVaR optimization

In the case of a risk averse producer acting as price maker, we now investigate the consequences of optimizing not only for the expected profit, but also the conditional value at risk (CVaR). This introduces two new decision variables. The value at risk(VaR) ξ and an auxiliary variable η^{ω}. Furthermore, the objective function will include β which is used to decide the weighting between expected profit and CVaR. The objective function for the price maker problem including a risk-averse strategy can be formulated as follows:

$$\max_{\substack{p_h^{G_1}, p_{gh}^{\omega}, p_{wh}^{\omega}, l_{dh}^{\omega} \\ \lambda_h^{\omega}, \underline{\mu}_{wh}^{\omega}, \overline{\mu}_{wh}^{\omega}, \underline{\mu}_{gh}^{\omega} \\ \overline{\mu}_{gh}^{\omega}, \underline{\sigma}_{lh}^{\omega}, \overline{\sigma}_{lh}^{\omega}, \eta^{\omega}}} (1-\beta)\left(\sum_{\omega\in\Omega}\pi^{\omega}\sum_{h\in H}\left(\sum_{d\in D}\left(-\overline{L}_{d,h}\overline{\sigma}_{d,h}^{\omega}+c_d^D l_{dh}^{\omega}\right)-\sum_{g\in G\setminus 1}\left(\overline{P}_g\overline{\mu}_{g,h}^{\omega}+c_g^G p_{gh}^{\omega}\right)\right.\right.$$
$$\left.\left.-\sum_{w\in W}\left(\overline{P}_w^{\omega}\overline{\mu}_{w,h}^{\omega}+c_w^W p_{wh}^{\omega}\right)-c_{G1,h}^G p_{G1,h}^G\right)\right)+\beta\left(\xi-\frac{1}{1-\alpha}\sum_{\omega\in\Omega}\pi^{\omega}\eta^{\omega}\right) \tag{48}$$

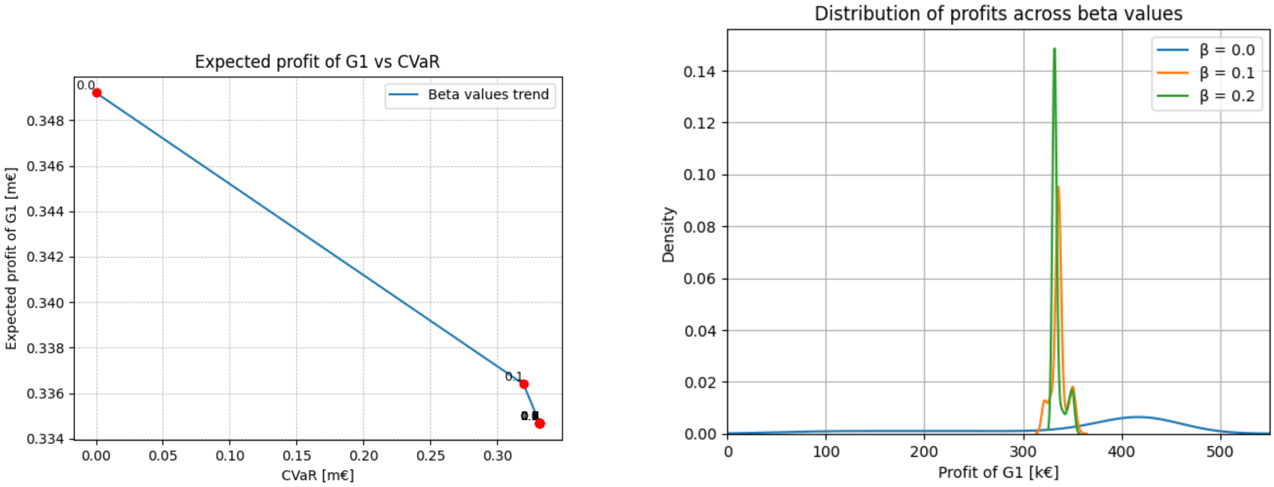
An α value of 0.95 is chosen to look at the 5% worst scenarios. In addition to changes in the objective function the problem is expanded with constraints enforcing that the auxiliary variable η^{ω} is equal to profit per scenario if lower than VaR_{α} and 0 otherwise. st.

$$\eta^{\omega} \geq 0 \qquad \forall\omega\in\Omega \tag{49}$$

$$\eta^{\omega} \geq \xi-\sum_{h\in H}\left(\sum_{d\in D}\left(-\overline{L}_{d,h}\overline{\sigma}_{d,h}^{\omega}+c_d^D l_{dh}^{\omega}\right)-\sum_{g\in G\setminus 1}\left(\overline{P}_g\overline{\mu}_{g,h}^{\omega}+c_g^G p_{gh}^{\omega}\right)\right.$$
$$\left.-\sum_{w\in W}\left(\overline{P}_w^{\omega}\overline{\mu}_{w,h}^{\omega}+c_w^W p_{wh}^{\omega}\right)-c_{G1,h}^G p_{G1,h}^G\right) \qquad \forall\omega\in\Omega \tag{50}$$

8.1 Results from solving stochastic price-maker profit and CVaR maximization problem

In Figure 8a, it can be seen that with increasing CVaR values and thereby decreasing risk, the expected profit falls. However, the difference in expected profits between β=0 (349 k€) and β=1 (335 k€) is only 4%. Furthermore, it stands clear that there is close to no difference in expected profit nor CVaR between β values of 0.2-1. The instance of the profit distribution decreasing with increasing β values is further visualized in the kernel-smoothed Figure 8b. Here, the worst case scenarios are eliminated with increasing β values, which could lead one to project an increase in the expected profit. However, as the worst case scenarios become of greater importance, the best case scenarios are also eliminated, placing both the expected profit and most likely profit at lower values. Nevertheless, it could be argued in the cases of companies with little liquidity that decreasing the expected profit by 3.7% to 336 k€ as seen with β=0.1 could be favorable.



(a) (b)

Figure 8: Results regarding the CVaR optimization.

8.2 Out-of-sample analysis

The optimal bid produced from the CVaR optimization has been computed over the in-sample scenarios. 80% of the available scenarios have been used for in-sample analysis to compute the optimal bids, where the remaining 20% are used for out-of-sample analysis. In the out-of-sample analysis, the optimal bidding strategy for the price maker at different β-values is constrained to the market clearing problem. The profit of the price maker can be calculated according to the prices for each scenario. If the distribution of profits in the out-of-sample calculation is significantly different to the distribution of the in-sample, this suggests that the sample size needs to be increased to accommodate for diversity in the realizations. With the assumption that the wind realizations are based on a certain distribution, and with the assumption of normal distributions for both in- and out-of-sample profits, the mean μ and variance σ^2 can be tested using the Welch's t-test and F-test respectively. This is done at a significance level α at 5%.

	μ	σ^2	**T-test**	**F-test**
$\beta = 0.0$	326191.12	16482866074.77	0.3082	0.8824
$\beta = 0.1$	336371.42	76043117.19	0.9898	0.9227
$\beta = 0.2$	335270.93	44828552.93	0.6094	0.8765
$\beta = 0.3$	335292.10	44914833.17	0.5926	0.8886
$\beta = 0.4$	335292.10	44914833.17	0.5926	0.8886
$\beta = 0.5$	335292.10	44914833.17	0.5926	0.8886
$\beta = 0.6$	335292.10	44914833.17	0.5926	0.8886
$\beta = 0.7$	335292.10	44914833.17	0.5926	0.8886
$\beta = 0.8$	335292.10	44914833.17	0.5926	0.8886
$\beta = 0.9$	335292.10	44914833.17	0.5926	0.8886
$\beta = 1.0$	335292.10	44914833.17	0.5926	0.8886

Table 6: The parameters of the normal distributions for the out-of-sample analysis, and the test results compared to the parameters written in Appendix F

The results from the out-of-sample analysis showcase that the sample size for every β-value are sufficiently large, as the test results for both tests compute p-values well above the confidence level. As all bids made for $\beta \geq 0.2$ are equal, the test results are exactly the same for these values. Investigating the mean of the out-of-sample results shows, that the lowest mean is obtained at $\beta = 0.0$. As seen in Figure 8b the distribution of profits is spanning over a wide range of profits. This suggests that the offer made is targeting high-risk scenarios that yield a low average in the out-of-sample analysis. Having a slight risk-averse approach improves the average for the out of sample, but lowers the variance dramatically. This highlights the benefits of using a more risk-averse approach as this optimization prioritizes stability over chasing extreme events.

9 Increasing market size

The system presented in Figure 1 with only 3 loads, 2 thermal generators, and 1 wind producer, significantly amplifies the potential of price-maker behavior by Generator 1. In a small system, the merit order curve consists of a few discrete steps. Small reduction in G1's generation shift the clearing price dramatically, as we have seen previously.

In a realistically larger market with more participants, the merit order curve would be smoother due to the higher granularity of supply and demand. This larger market dilutes the influence of a single generator, making it harder for G1 to act as a marginal price setter. This would decrease the profitability of strategic bidding and in turn, minimize social welfare losses. While the small market size in our model provides clear insights into the mechanisms of price-making behavior, it likely overestimates the real-world impact of such strategies. A natural extension is made, quantifying how market size mitigates the effects of strategic bidding in larger systems. Figure 9 shows a deterministic market clearing for hour 1, where the number of loads, wind generators and thermal generators is increased to 30, 10 and 20 respectively. G1's share of the total market supply is kept constant to make the results more comparable. It shows that with a smoother merit order curve, G1's ability to maximize its profit is significantly lowered compared to the smaller market. This results in a smaller reduction in social welfare as compared to the smaller market in section 6 and now only reduces social welfare by 2%, and the amount of consumer utility allocated to G1's producer surplus lowered to 154 % of G1's profits vs. 475 % in the smaller market. Main results are summarized in table Table 7.

	MCP	PMP	PMP/MCP [Unitless]	Unit
Total Social Welfare	2,026	1,983	0.98	k€
Avg. Market Clearing Price	56.10	63.40	1.13	€/MWh
G1 Profit	116.65	180.00	1.54	k€
Total G1 Production	18.23	12.73	0.70	GWh

Table 7: Results from increased market size deterministic market clearing and price maker problems across all 24 hours.

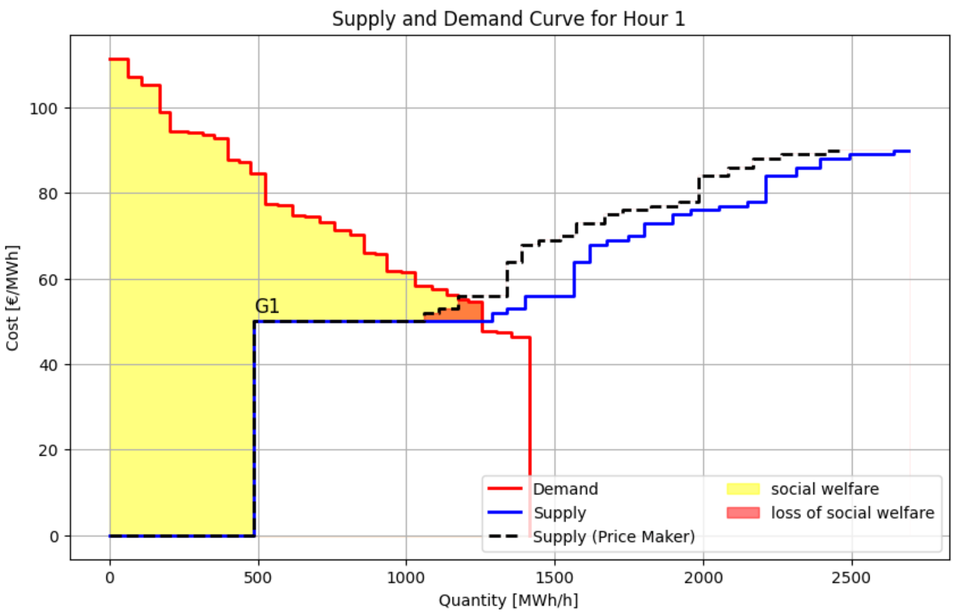


Figure 9: Merit order curve showing impact of G1 price maker behavior when market size is increased, but G1 market share is held constant.

Figure 10 shows a deterministic clearing of market clearing for hour 1 where the number of loads and generators is the same as previously but in addition, G1's market share is reduced. As a result of these changes, G1 is no longer able to significantly affect the market price and social welfare of the system.

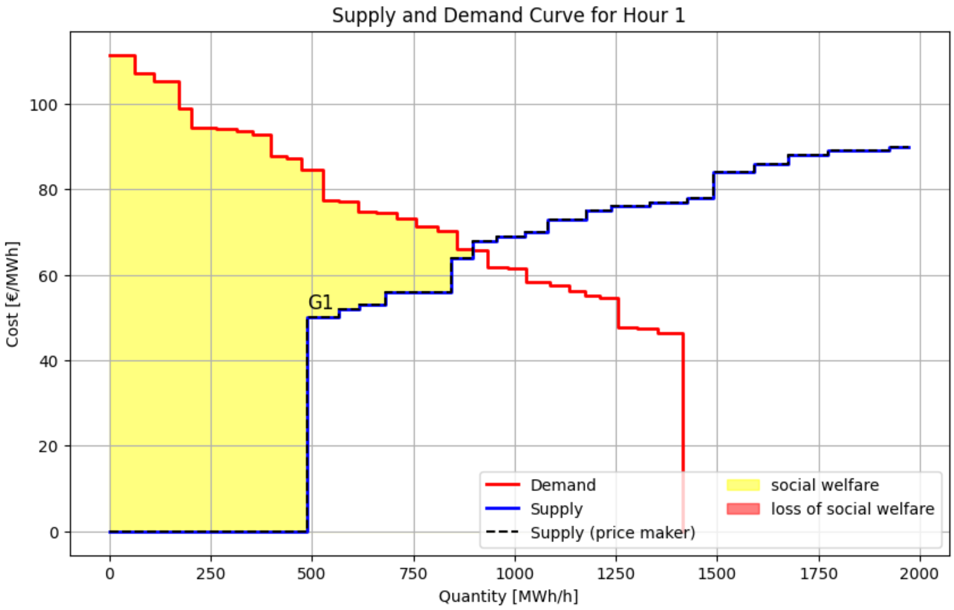


Figure 10: Merit order curve showing impact of G1 price maker behavior when market size is increased and G1 market share is decreased.

This extended analysis of market size shows that there is no need for regulation in a large market, where no individual market participant has a very large share of the market. It is not apparent in the figure, but G1 does manage to increase its profits by 2% over the scenario average. Main results are summarized in Table 8.

	MCP	PMP	PMP/MCP [-]	Unit
Total Social Welfare	1,792	1,792	1.00	k€
Avg. Market Clearing Price	69.80	70.26	1.01	€/MWh
G1 Profit	38.00	38.74	1.02	k€
Total G1 Production	1.91	1.92	1.01	GWh

Table 8: Results from increased market size and reduced market share deterministic market clearing and price maker problems across all 24 hours.

10 Computational Challenges

For the energy system presented in Figure 1 with 200 scenarios investigated, the Stochastic Price Maker setup takes approximately 63 seconds to solve. This section of the report will discuss which measures can be taken into use in order to reduce the computational time of the different optimization problems presented in the report.

10.1 Decomposing the market clearing problem

Looking at the market clearing problem solved across multiple scenarios presented in section 5, this optimization problem can be decomposed in several ways depending.
Firstly, there are no inter-temporal constraints involved. This makes the scenarios and hours independent of each other meaning the optimization problem has a decomposable structure in two regards. Each scenario and each hour of a given scenario can be optimized independently. This leads to $|\Omega| \cdot |H| = 200 \cdot 24 = 4,800$ subproblems, where the solution of each subproblem can be combined into a solution for the overall problem.

10.1.1 Lagrangian relaxation

Analyzing the market clearing problem it is noted that this problem includes a complicating constraint, namely the balancing constraint presented in Equation 41. This constraint is linking all of the decision variables. Because the objective function is linear, Augmented Lagrangian Relaxation (ALR) method can be used. In our specific case, we can relax the complicating constraint by fixing the value of the Lagrange multiplier λ as $\hat{\lambda}$. Note that the h and ω indexes are removed from decision variables in the equivalent augmented market clearing problem EA_{MCP} Equation 51 since the problem will be solved independently for all hours and scenarios. The upper bounds for each constraint are still indexed by h and ω where relevant:

$$EA_{MCP} = \max_{p_g, p_w, l_d} \left(\sum_{d \in D} c_d^D l_d - \left(\sum_{g \in G} c_g^G p_g + \sum_{w \in W} c_w^W p_w \right) \right) - \hat{\lambda} \left(\sum_{d \in D} l_d - \sum_{g \in G} p_g - \sum_{w \in W} p_w \right) \tag{51}$$

$$-\frac{\gamma}{2} \left\| \left(\sum_{d \in D} l_d - \sum_{g \in G} p_g - \sum_{w \in W} p_w \right) \right\|^2 \tag{52}$$

st.

$$0 \leq l_g \leq \overline{L}_{gh} \qquad \forall d \in D \tag{53}$$

$$0 \leq p_g \leq \overline{P}_g \qquad \forall g \in G \tag{54}$$

$$0 \leq p_w \leq \overline{P}_{wh}^{\omega} \qquad \forall w \in W \tag{55}$$

The last penalty term includes products of the decision variables, making it inseparable. To deal with this and allow for decomposition, the optimal values of this relaxation can be solved by using the Alternating Direction of Multipliers Method (ADMM). The algorithm has an iterative approach with the main idea being to fix all but one of the decision variables in each iteration to make the problem decomposable. We have three groups of subproblems; loads, thermal- and wind generators. These will be referred to as $SubP_{ld}, SubP_{pg}$ and $SubP_{pw}$ respectively. The ADMM algorithm can be summarized in four steps:

- **Step 1:** Initialize values of variables $l_d^{(-1)}, p_g^{(-1)}, p_w^{(-1)}$, and the Lagrange multipliers $\lambda^{(0)}$.
- **Step 2: At each iteration** $v \geq 0$**:** Solve subproblems with fixed Lagrange multiplier $\lambda^{(v)}$. All decision variables except one should be kept constant to the solution of the previous iteration. Solve $SubP_{ld}$ for $d \in \{1, \ldots, D\}$ with values $l_j^{(v-1)} \neq d$, $SubP_{pg}$ for $g \in \{1, \ldots, G\}$ with values $p_j^{(v-1)} \neq g$, $SubP_{pw}$ for $w \in \{1, \ldots, W\}$ with values $p_j^{(v-1)} \neq w$ and get solutions $l_d^{(v)}, p_g^{(v)}$ and $p_w^{(v)}$.
- **Step 3: At iteration** $v \geq 0$**:** Update the values of Lagrange multipliers in master problem such that:

$$\lambda^{(v+1)} = \lambda^{(v)} + \gamma \left(\sum_{d \in D} l_d^{(v)} - \sum_{g \in G} p_g^{(v)} - \sum_{w \in W} p_w^{(v)} \right)$$

- **Step 4: Convergence check:**

  If: $\frac{\|\lambda^{(v+1)} - \lambda^{(v)}\|}{\|\lambda^{(v)}\|} \leq \epsilon$, optimal solution with accuracy level ϵ is achieved.

  Else, $v \leftarrow v + 1$

The subproblems for each iteration $v \geq 0$ consist of individual producers or loads. The formulation of the three subproblem groups follow below:

$$SubP_{ld} = \max_{l_d} \quad c_d^D l_d - \hat{\lambda}^{(v)} l_d - \frac{\gamma}{2} \left(l_d + \sum_{j \in J \backslash d} l_j^{(v-1)} - \sum_{g \in G} p_g^{(v-1)} - \sum_{w \in W} p_w^{(v-1)} \right)^2 \tag{56}$$

st.

$$0 \leq l_d \leq \overline{L}_{d,h} \qquad \forall d \in D \tag{57}$$

$$SubP_{pg} = \max_{p_g} \quad c_g^G \cdot (-p_g) + \hat{\lambda}^{(v)} \cdot p_g - \frac{\gamma}{2} \left(\sum_{d \in D} l_d^{(v-1)} - p_g - \sum_{j \in J \backslash g} p_j^{(v-1)} - \sum_{w \in W} p_w^{(v-1)} \right)^2 \tag{58}$$

st.

$$0 \leq p_g \leq \overline{P}_{g,h} \qquad \forall g \in G \tag{59}$$

$$SubP_{pw} = \max_{p_w} \quad c_w^W \cdot (-p_w) + \hat{\lambda}^{(v)} \cdot p_w - \frac{\gamma}{2} \left(\sum_{d \in D} l_d^{(v-1)} - \sum_{g \in G} p_g^{(v-1)} - p_w - \sum_{j \in J \setminus w} p_j^{(v-1)} \right)^2 \tag{60}$$

st.

$$0 \leq p_w \leq \overline{P}_{w,h} \qquad \forall w \in W \tag{61}$$

In summation, the implementation of Lagrangian relaxation allows the decomposition of the stochastic market clearing problem by eliminating the complicating balance constraint which instead is penalized in the objective function. This results in the total of $|\Omega| \cdot |H| \cdot |D| \cdot |G| \cdot |W| = 200 \cdot 24 \cdot 3 \cdot 2 \cdot 1 = 28,800$ subproblems in stochastic market clearing problem presented in section 5 with a basis in the network presented in Figure 1.

10.2 Decomposing the price maker problem

The optimization problem for solving the stochastic price maker problem presented in section 6, on the other hand, is aiming to generate the best offer across multiple scenarios. The bid made from the price-maker is dependent on the outcome of the scenarios, and hence it is not directly possible to split this optimization problem into sub-problems for each scenario. The hours in each scenario are still independent of each other, enabling decomposition of the problem into hourly subproblems.

The stochastic price maker problem involves the complicating variable λ_h^w. Benders decomposition method can be used to decompose problems with a complicating variable. The method uses an iterative approach that converges by utilizing the fact that tangents of a convex function are lower bounds of the function. If Benders decomposition were to be implemented successfully, the price maker problem could further be decomposed with Lagrangian relaxation similarly to the market clearing problem. The issue is the introduction of binary variables in the lower level complementary slackness constraints. Benders decomposition needs convexity for the assumptions to comply. The binary variables can be made continuous and thereby making the function convex, but will plausibly still hamper the algorithm in reaching convergence correctly. For these complicating reasons, the possibility of decomposing the stochastic price maker problem more than what the original structure allows is not applied in this project.

10.3 Results from decomposition

Results from implementing the decomposition strategies discussed in the preceding sections are summarized in Table 9 as the change in computational time. In the case of price maker problem, the computational time is reduced, but in market clearing problem across multiple scenarios it increased underlining the fact that optimization problems require a large complexity for decompositioning to be beneficial. In the case of market clearing problem, setting up a lot of sub-problems is thus more time consuming than the gain from being able to solve problems in parallel.

	Time Taken [s]	**Time Saved [s]**	**Relative Gain [%]**
MCP Baseline	1.07	-	-
MCP Hour/Scenario	2.30	-1.23	-115
MCP Lagrange	1,583.05	-1,582.98	-147,913
SPMP Baseline	63.38	-	-
SPMP Hour	24.67	38.71	61

Table 9: Performance comparison of baseline and two decomposition methods for market clearing problem across multiple scenarios and the price maker problem. Negative values in Time Saved and Relative Gain indicate a performance loss compared to the baseline[2].

The way longer computation time for the ADMM algorithm compared to the non-decomposed setup underlines the efficiency of the solvers in gurobipy. The problems are solved in a for-loop for each scenario and each hour. This means that the structure of the decomposed setup is not fully utilized as it can be solved by parallel processing. Moreover, the choice of hyperparameters for the ADMM algorithm has great influence on the rate of convergence. The ϵ value and maximum iteration number is chosen to be 10^{-6} and 1000 respectively. The

[2]For a fair comparison, between the different setups in the market clearing problem and the price maker problem respectively, they have been incorporated in the same Python class, thus the baseline takes longer to be computed than in Appendix A

initial values for $l_d^{(-1)}, p_g^{(-1)}, p_w^{(-1)}$ are all 0, and the Lagrange multipliers $\lambda^{(0)}$ is 10. The γ-value indicates how much the deviation of the complicated constraint is penalized, and thereby what λ^{v+1} becomes. The value for γ is chosen to be 0.015. For simplicity, the initial values are chosen to be the same for all of the hours and scenarios inspected, however as shown in Figure 4b, the prices fluctuate over the span of 24 hours. With this knowledge, appropriate initial values could be assigned. An illustration of how λ converges for the ADMM algorithm for different hours is presented in Figure 11.

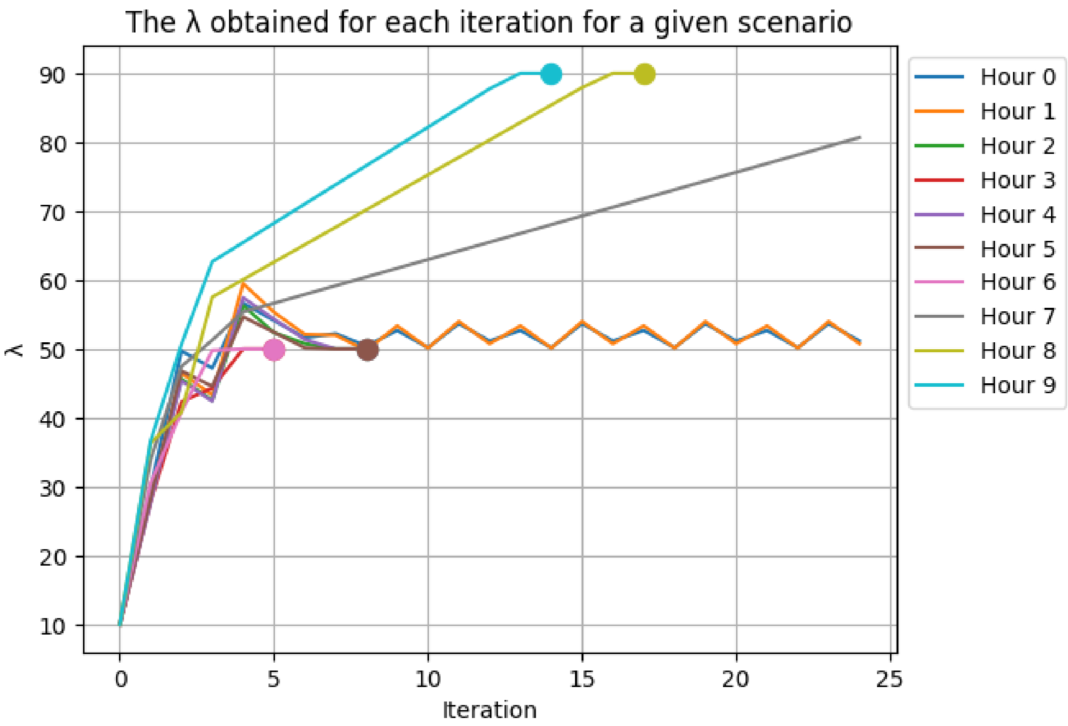


Figure 11: Convergence of λ for the first 10 hours

The λ-value for most of the hours converges within the first 25 iterations, however for some hours in different scenarios the maximum iteration number is reached. Combining values for $l_d^{(v)}, p_g^{(v)}, p_w^{(v)}$ and λ^v can then be inserted into the initial market clearing problem, yielding an optimal solution at 1,737 k€, which is slightly different than the optimal solution provided by the non-decomposed setup (1,792 k€) and also compared to the other decomposition methods. This is due to the fact that the last λ value found at the maximum iteration number is used for calculation of the optimal solution.

11 Discussion

When modeling large systems, a crucial part is to choose when, where and how simplifications and assumptions are necessary. It is essential to be attentive to how these choices affect the results and adjust the conclusions appropriately. This section will briefly elaborate on the effects of the modeling choices, assumptions, and simplifications made in this project. Further, the discussion will describe how the analysis of price maker behavior could be developed with further work.

The assumption of static loads across scenarios simplifies the analysis and probably makes price-maker behavior more pronounced. Uncertainty on loads would be a realistic feature of the project, but its exclusion does not significantly impact our conclusions. The inclusion of wind production uncertainty already captures the broader effects of system uncertainty, effectively disrupting the more 'fine-tuned' market manipulation observed in the deterministic case.

Transmission capacity constraints are not included in this analysis. This aligns with the assumption used by TSO's when clearing day-ahead markets within a single price zone, where a copper plate model is adopted to represent unconstrained transmission. It could be reasonable to argue that in hours of line congestion, the gain of bidding strategically could be higher.

A key simplification in this study is the assumption that all generators and loads can operate at a minimum value of zero. This decision was made to ensure the convexity of the optimization problem, which simplifies its computational resolution. Realistically, thermal generators have non-zero minimum generation limits due to technical and economic constraints. Similarly, certain loads might have baseline demand.

Accounting for these minimum limits would require the introduction of binary variables to model on/off states or enforce minimum production constraints. Another approach could also be to firstly solve the Unit Commitment problem, and then assign the status of the generators. These adjustment increases the complexity of the problem unnecessarily when keeping the goal of the study in mind.

In this project, all scenarios for the wind production profile are weighted equally. The scenarios for realized wind production consist of historical data on wind production on different days of the year. The effect of this is that for each of the 24 hours, it is equally likely that the realized wind production matches a windy autumn day as a quiet summer day. A more sophisticated way to predict wind production could be developed e.g. by using machine learning. With a better forecast, the distribution of scenarios would possibly have a smaller standard deviation. Lowering the uncertainty would increase the room to maneuver for the price maker as we see in the comparison of stochastic and deterministic cases. However, this simplification does not interfere with our main conclusion and the goal of the project, namely a comparison of social welfare with and without price-maker behavior.

As can be seen in the optimization with perfect information there is a relative change of 54%. The results throughout the report has only used historic data to generate the bid in the day ahead market, and based on the different scenarios optimize the expected profit. It could also be investigated predicting scenarios and place the bids according to the predicted scenario. If the predictions perform well, it could lower the relative change in having perfect information. The prediction of a scenario could be based on machine learning tools, such as regression models for predicting wind power production. If the predicted scenarios aligns with the realization it would be possible to solve the optimization problems deterministically.

The assignment only investigates the profits by participation in the day-ahead market. However, when optimizing the bidding strategy to generate the highest profit, and not just within the day-ahead market a very different strategy could arise. It would also introduce more complexity as participation in the other markets would enforce making a two-stage or even multi-stage stochastic optimization, but may yield a higher profit. In the setup provided the generating unit acting as price maker can only adjust the quantity of the, and the price. This setup was deemed more interesting, because if it was possible to change the selling price for the generating unit, the price would never resemble it's true cost. Instead the selling price would be slightly less than the price setter.

The setup assumes that only one of the market participants acts as price maker, however, a more realistic setup would be that multiple independent market actors would show this behavior. Each participant adjust the placed bids to maximize their profits, but also in response to other participants decisions. Then the problem becomes a game theoretic problem, where each participant must solve an optimization problem dependent on how the other participants place bids. This setup is considered out of scope for this report.

12 Conclusion

In this project, we analyzed how a thermal power plant, G1, acting as a price-maker influences social welfare in the day-ahead electricity market. The primary hypothesis was that social welfare decreases in imperfect market conditions, particularly when a thermal power plant exercises market power through strategic bidding. To test this hypothesis, several optimization problems with varying objectives, scales, and complexities were formulated and solved to estimate the resulting social welfare.

The optimization problems were categorized into two main types:

1. **Deterministic problems**, which yield a single solution for a specific set of parameters.
   - A deterministic market clearing problem (DMCP) and a deterministic price-maker problem (DPMP) were studied to examine how strategic behavior impacts outcomes in the day-ahead market.
2. **Stochastic problems**, which account for the uncertainty of renewable energy production faced by a price-maker during strategic bidding.
   - A one-stage stochastic market clearing problem (SMCP) and a stochastic price-maker profit maximization problem (SPMP) were analyzed to evaluate how uncertainty in wind production affects expected social welfare and G1's profits across various scenarios. To enable fast computational time, decomposition techniques such as Augmented Lagrangian Relaxation and the Alternating Direction of Multipliers Method ADMM have been applied .

The deterministic analysis revealed that social welfare decreases when G1 acts as a price-maker. By strategically reducing its quantity bid, G1 raises the average market clearing price by 58%, limiting supply and preventing some demand from being fulfilled, but increasing its own profits with 375%. Social welfare is reduced by 2%. Additionally, it was observed that linearizing the DPMP's objective function is a critical step in proving that the strong duality theorem holds and drastically reducing computational time. The DPMP is solved as a single-level problem with KKT constraints derived from the DMCP.

The stochastic analysis substantiated to the conclusions from the deterministic analysis. Results from stochastic analysis when G1 acts as a price maker with 200 wind scenarios demonstrated that the distribution of social welfare becomes flatter and shifts to the left, reducing expected social welfare by 9% compared to the optimal market clearing. Conversely, the expected value G1's profits could increase by 54% from 224.9 k€ to 347.3 k€ if bidding strategically as a price maker.

Furthermore, an analysis was carried out for the price-maker with the objective of optimizing not only expected profits but also the conditional value at risk (CVaR). Since this algorithm is optimizing both the expected profits as well as the CVaR found at worst 5% scenarios, we found a decrease in expected profits with increasing weight (β) on optimizing the CVaR. However, most effect was observed when increasing the β from 0 to 0.1, which reduced the expected profit by 3.7%, while increasing the CVaR. This strategy may be beneficial for risk-averse producers.

Finally, an analysis was conducted to evaluate G1's ability to exercise market power in a market with a large number of participants, while assuming no changes in total generation and load capacities compared to the baseline system used in the previous optimization problems. Two scenarios were examined: one in which G1 retained its original generation market share, and another where G1's generation market share was reduced. The results showed that the 9% loss of social welfare seen in section 6 was reduced to 2% simply when increasing the number of market participants without changing the market share of G1. This is due to the smoother supply and demand curve. However, it did not eliminate G1's chances of obtaining more profit nor did we see close to perfect market conditions. However, when decreasing the market share of G1 by simply decreasing its capacity, we observed that G1 could no longer effect social welfare. On the other hand, G1 was still able to increase its profits, but also production, by 2% and 1% respectively.

In conclusion, this study demonstrates that price-maker behavior can decrease social welfare in a predictable market with imperfect market conditions, emphasizing the importance of regulatory actions by market authorities. However, a market with increasing uncertainty and more market participants will limit a price-maker's influence on social welfare and its ability to manipulate market clearing prices.

Appendices

A Variables, constraints and computation time for optimization models

All the final outcomes were generated by using Gurobi Optimizer version 11.0.0 build v11.0.0rc2 (win64 - Windows 11+.0 (22631.2)), with the following computer specifications: CPU model: Intel(R) Core(TM) i7-10510U CPU @ 1.80GHz, instruction set [SSE2—AVX—AVX2] Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Section	Constraints	Linear Variables	Integer Variables	Computational Time
MC (3)	168	144	0	0.000945 sec
PM (4)	912	480	576	0.003000 sec
MC across scenarios (5)	26880	23040	0	0.110000 sec
Stochastic PM (6)	165144	72984	92160	63.526000 sec
EVPI (7.1)	168960	76800	92160	1.078000 sec
CVaR $\beta = 0.0$ **(8)**	165304	73145	92160	44.008000 sec
CVaR $\beta = 0.1$ **(8)**	165304	73145	92160	3327.055000 sec
CVaR $\beta = 0.2$ **(8)**	165304	73145	92160	246.413000 sec
CVaR $\beta = 0.3$ **(8)**	165304	73145	92160	729.622000 sec
CVaR $\beta = 0.4$ **(8)**	165304	73145	92160	348.811000 sec
CVaR $\beta = 0.5$ **(8)**	165304	73145	92160	272.468000 sec
CVaR $\beta = 0.6$ **(8)**	165304	73145	92160	215.370000 sec
CVaR $\beta = 0.7$ **(8)**	165304	73145	92160	196.880000 sec
CVaR $\beta = 0.8$ **(8)**	165304	73145	92160	55.917000 sec
CVaR $\beta = 0.9$ **(8)**	165304	73145	92160	34.445000 sec
CVaR $\beta = 1.0$ **(8)**	165304	73145	92160	25.226000 sec
Increase no. participants MC (9)	1464	1440	0	0.001979 sec
Increase no. participants PM (9)	8688	4368	5760	0.193000 sec

Table 10: Overview of constraints and variables for all steps and models.

B Demand profiles

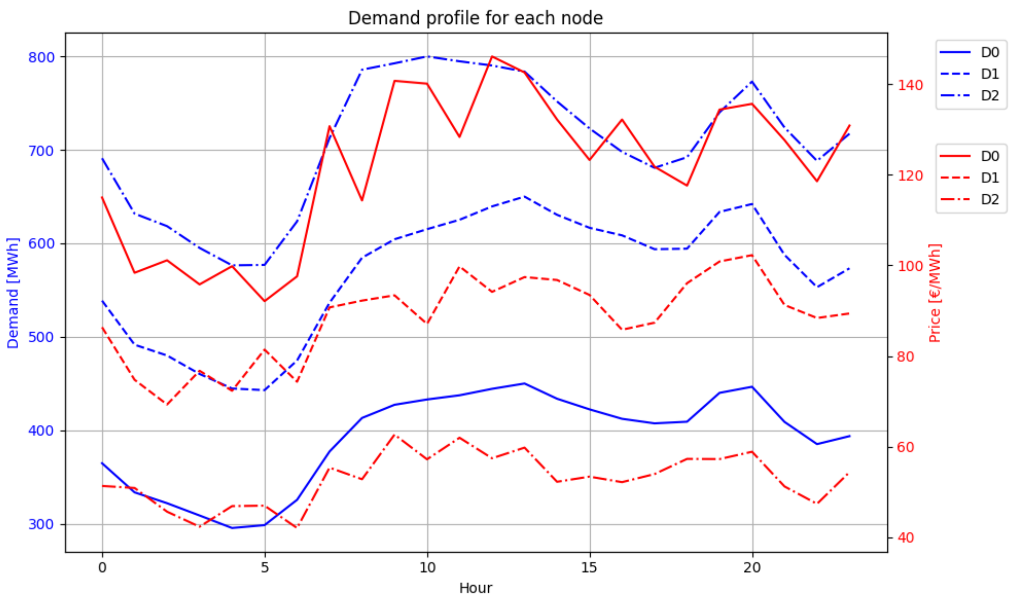


Figure 12: Visualization of the price and quantity profiles of the three demand profiles.

C Dual formulation of the market clearing problem

The dual problem of this market clearing problem can be formulated by introducing dual variables for each of the constraints as denoted in the primal formulation.
The dual objective function is then:

$$\min_{\lambda_h,\underline{\sigma}_{d,h},\overline{\sigma}_{d,h},\underline{\mu}_{g,h},\overline{\mu}_{g,h},\underline{\mu}_{w,h},\overline{\mu}_{w,h}} \sum_{h\in H}\left(\sum_{d\in D}\overline{\sigma}_{d,h}\overline{L}_{d,h}-\underline{\sigma}_{d,h}\cdot 0+\sum_{g\in G}\overline{\mu}_{g,h}\overline{P}_g-\underline{\mu}_{g,h}\cdot 0+\sum_{w\in W}\overline{\mu}_{w,h}\overline{P}_W-\underline{\mu}_{w,h}\cdot 0\right) \tag{62}$$

Subject to:

$$\lambda_h = c^D_{d,h}+\underline{\sigma}_{d,h}-\overline{\sigma}_{d,h} \qquad \forall d\in D,\forall h\in H \tag{63}$$

$$\lambda_h = -c^G_g+\underline{\mu}_{g,h}-\overline{\mu}_{g,h} \qquad \forall g\in G,\forall h\in H \tag{64}$$

$$\lambda_h = -c^W_{w,h}+\underline{\mu}_{w,h}-\overline{\mu}_{w,h} \qquad \forall w\in W,\forall h\in H \tag{65}$$

D Linearization of complementarity slackness constraints

The complementarity conditions imply that only one of the two constraints can be active at the same time. For the condition:

$$0\le \underline{\mu}_{gh} \quad \perp \quad 0\le p_{gh} \qquad \forall g\in G,h\in H \tag{66}$$

This means that:

$$0\le \underline{\mu}_{gh},\ 0\le p_{gh},\ \underline{\mu}_{gh}\cdot p_{gh}=0 \qquad \forall g\in G,h\in H \tag{67}$$

This formulation converts the constraint into a bilinear constraint introducing nonlinearity, which can significantly increase the difficulty of finding optimal solution. To avoid this, the formulation can be written using the BigM-method as:

$$\underline{\mu}_{gh}\le M\cdot y^1_{gh} \qquad \forall g\in G,h\in H \tag{68}$$

$$p_{gh}\le M\left(1-y^1_{gh}\right) \qquad \forall g\in G,h\in H \tag{69}$$

$$y^1_{gh}\in\{0,1\} \qquad \forall g\in G,h\in H \tag{70}$$

This linearizes the constraint by converting the constraints into a mixed-integer linear program, by introducing binary variables for each lower and upper bound condition. This is done for all of the complementarity slackness constraints. The linearized formulation for the remaining constraints yields:

$$\underline{\mu}_{wh}\le M\cdot y^1_{wh} \qquad \forall w\in W,h\in H \tag{71}$$

$$p_{wh}\le M\left(1-y^1_{wh}\right) \qquad \forall w\in W,h\in H \tag{72}$$

$$\underline{\sigma}_{dh}\le M\cdot y^1_{dh} \qquad \forall d\in D,h\in H \tag{73}$$

$$l_{dh}\le M\left(1-y^1_{dh}\right) \qquad \forall d\in D,h\in H \tag{74}$$

$$\overline{\mu}_{gh}\le M\cdot y^2_{gh} \qquad \forall g\in G,h\in H \tag{75}$$

$$\overline{P}_{gh}-p_{gh}\le M\cdot\left(1-y^2_{gh}\right) \qquad \forall g\in G\setminus\{G_1\},\forall h\in H \tag{76}$$

$$p^{DA}_h-p_{G1h}\le M\cdot\left(1-y^2_{gh}\right) \qquad \forall h\in H \tag{77}$$

$$\overline{\mu}_{wh}\le M\cdot y^2_{wh} \qquad \forall w\in W,h\in H \tag{78}$$

$$\overline{P}_{wh}-p_{wh}\le M\cdot\left(1-y^2_{wh}\right) \qquad \forall w\in W,h\in H \tag{79}$$

$$\overline{\sigma}_{dh}\le M\cdot y^2_{dh} \qquad \forall d\in D,h\in H \tag{80}$$

$$\overline{L}_{dh}-l_{dh}\le M\cdot\left(1-y^2_{dh}\right) \qquad \forall d\in D,h\in H \tag{81}$$

$$y^1_{wh}\in\{0,1\} \qquad \forall w\in W,h\in H \tag{82}$$

$$y^1_{dh}\in\{0,1\} \qquad \forall d\in D,h\in H \tag{83}$$

$$y^2_{gh}\in\{0,1\} \qquad \forall w\in W,h\in H \tag{84}$$

$$y^2_{wh}\in\{0,1\} \qquad \forall d\in D,h\in H \tag{85}$$

$$y^2_{dh}\in\{0,1\} \qquad \forall d\in D,h\in H \tag{86}$$

To improve the run time of the optimization tighter bounds for each M-could be calculated. For this matter, a sufficiently large M-value of 10000 has been chosen.

E Stochastic price-maker profit maximization problem

Compared to the Lagrangian function in Equation 11, decision variables related to wind farms and market clearing prices depend on the specific scenario.

$$\begin{aligned}\mathcal{L}\left(p_{gh}^{\omega}, p_{wh}^{\omega}, l_{dh}^{\omega}, \lambda_h^{\omega}, \overline{\mu}_{wh}^{\omega}, \underline{\mu}_{wh}^{\omega}, \overline{\mu}_{gh}^{\omega}, \underline{\mu}_{gh}^{\omega}, \overline{\sigma}_{dh}^{\omega}, \overline{\sigma}_{dh}^{\omega}\right) = \sum_{\omega\in\Omega}\pi^{\omega}\sum_{h\in H}\Bigg[&\sum_{g\in G} c_g^G p_{gh}^{\omega} + \sum_{w\in W} c_w^W p_{wh}^{\omega} - \sum_{d\in D} c_d^D l_{dh}^{\omega} \\ &+ \lambda_h^{\omega}\left(-\sum_{g\in G} p_{gh}^{\omega} - \sum_{w\in W} p_{wh}^{\omega} + \sum_{d\in D} l_{dh}^{\omega}\right) \\ &+ \sum_{g\in G}\left[-\underline{\mu}_{gh}^{\omega} p_{gh}^{\omega} + \overline{\mu}_{gh}^{\omega}\left(p_{gh}^{\omega} - \overline{P}_{gh}\right)\right] \\ &+ \sum_{w\in W}\left[-\underline{\mu}_{wh}^{\omega} p_{wh}^{\omega} + \overline{\mu}_{wh}^{\omega}\left(p_{wh}^{\omega} - \overline{P}_{wh}^{\omega}\right)\right] \\ &+ \sum_{d\in D}\left[-\underline{\sigma}_{dh}^{\omega} l_{dh}^{\omega} + \overline{\sigma}_{dh}^{\omega}\left(l_{dh}^{\omega} - \overline{L}_{dh}\right)\right]\Bigg]\end{aligned} \tag{87}$$

Stationarity constraint:

$$\begin{aligned}
&\text{Loads} && \frac{\partial\mathcal{L}}{\partial l_{dh}} = -c_d^D + \lambda_h^{\omega} - \underline{\sigma}_{dh}^{\omega} + \overline{\sigma}_{dh}^{\omega} = 0 && \forall d\in D, h\in H, \forall\omega\in\Omega && (88)\\
&\text{Generators} && \frac{\partial\mathcal{L}}{\partial p_{gh}} = c_g^G - \lambda_h^{\omega} - \underline{\mu}_{gh}^{\omega} + \overline{\mu}_{gh}^{\omega} = 0 && \forall g\in G, h\in H, \forall\omega\in\Omega && (89)\\
&\text{Wind farms} && \frac{\partial\mathcal{L}}{\partial p_{wh}^{\omega}} = c_w^W - \lambda_h^{\omega} - \underline{\mu}_{wh}^{\omega} + \overline{\mu}_{wh}^{\omega} = 0 && \forall w\in W, h\in H, \forall\omega\in\Omega && (90)
\end{aligned}$$

Complementarity conditions:

$$\begin{aligned}
&\text{Generators lower bound} && 0\le\underline{\mu}_{gh}^{\omega} \perp 0\le p_{gh}^{\omega} && \forall g\in G, h\in H, \forall\omega\in\Omega && (91)\\
&\text{Wind farms lower bound} && 0\le\underline{\mu}_{wh}^{\omega} \perp 0\le p_{wh}^{\omega} && \forall w\in W, h\in H, \forall\omega\in\Omega && (92)\\
&\text{Loads lower bounds} && 0\le\underline{\sigma}_{gh}^{\omega} \perp 0\le l_{dh}^{\omega} && \forall l\in L, h\in H, \forall\omega\in\Omega && (93)\\
&\text{Generators upper bounds} && 0\le\overline{\mu}_{gh}^{\omega} \perp 0\le\overline{P}_{gh} - p_{gh}^{\omega} && \forall g\in G\backslash\{G_1\}, \forall h\in H, \forall\omega\in\Omega && (94)\\
&\text{G1's upper bounds} && 0\le\overline{\mu}_{G_1} \perp 0\le\overline{P}_h^{G_1} - p_h^{G_1} && \forall h\in H && (95)\\
&\text{Wind farms upper bounds} && 0\le\overline{\mu}_{wh}^{\omega} \perp 0\le\overline{P}_{wh}^{\omega} - p_{wh}^{\omega} && \forall w\in W, \forall h\in H, \forall\omega\in\Omega && (96)\\
&\text{Loads upper bounds} && 0\le\overline{\sigma}_{dh}^{\omega} \perp 0\le\overline{L}_{dh} - l_{dh}^{\omega} && \forall d\in D, \forall h\in H, \forall\omega\in\Omega && (97)
\end{aligned}$$

Primal feasibility conditions:

$$\begin{aligned}
&\sum_{d\in D} l_{dh}^{\omega} - \sum_{g\in G} p_{gh}^{\omega} - \sum_{w\in W} p_{wh}^{\omega} = 0 && \forall h\in H, \forall\omega\in\Omega && (98)\\
&0\le l_{dh}^{\omega}\le\overline{L}_{dh} && \forall d\in D, \forall h\in H, \forall\omega\in\Omega && (99)\\
&0\le p_{gh}^{\omega}\le\overline{P}_{gh} && \forall g\in G\backslash\{G_1\}, \forall h\in H, \forall\omega\in\Omega && (100)\\
&0\le p_{G1}\le p_{G1h}^{DA} && \forall h\in H && (101)\\
&0\le p_{wh}^{\omega}\le\overline{P}_{wh}^{\omega} && \forall w\in W, \forall h\in H, \forall\omega\in\Omega && (102)
\end{aligned}$$

Dual feasibility conditions:

$$\text{Loads} \qquad \underline{\sigma}_{dh}^{\omega} \geq 0, \quad \overline{\sigma}_{dh}^{\omega} \geq 0 \qquad \forall d \in D, \forall h \in H, \forall \omega \in \Omega \tag{103}$$

$$\text{Generators} \qquad \underline{\mu}_{gh}^{\omega} \geq 0, \quad \overline{\mu}_{gh}^{\omega} \geq 0, \qquad \forall g \in G, \forall h \in H, \forall \omega \in \Omega \tag{104}$$

$$\text{Wind farms} \qquad \underline{\mu}_{wh}^{\omega} \geq 0, \quad \overline{\mu}_{wh}^{\omega} \geq 0 \qquad \forall w \in W, \forall h \in H, \forall \omega \in \Omega \tag{105}$$

F In-sample parameters

	μ	σ^2
$\beta = 0.0$	349214.83	12455088734.75
$\beta = 0.1$	336390.87	54304735.29
$\beta = 0.2$	334670.69	34115873.44
$\beta = 0.3$	334663.64	33682453.96
$\beta = 0.4$	334663.64	33682453.95
$\beta = 0.5$	334663.64	33682453.95
$\beta = 0.6$	334663.64	33682453.95
$\beta = 0.7$	334663.64	33682453.95
$\beta = 0.8$	334663.64	33682453.95
$\beta = 0.9$	334663.64	33682453.95
$\beta = 1.0$	334663.64	33682453.95

Table 11: The parameters of the normal distributions for the in-sample analysis