How Inefficient Can An Electricity Market Be?

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***Abstract*—Imperfectly competitive electricity markets are exposed to strategic bidding behaviors by big players and are thereby threatened by significant inefficiency. The related literature has studied relevant game-theoretic models based on equilibrium problems with equilibrium constraints. However, the papers of this literature consider, each time, a given system (e.g. particular generators and cost functions) and assess the equilibrium inefficiency of that particular system. In this paper, we are interested in assessing the inefficiency of market equilibria, not for a single (arbitrary) set of participants, but *in the worst-case*. Specifically, we formulate the problem of searching in the space of possible generators' true cost functions to identify the worst-case combination that would lead to the most severe market inefficiency at equilibrium. Our main result asserts that the worst-case inefficiency of electricity markets violates previous theoretical bounds that were derived through simplified market models. In particular, our simulations indicate that the worst-case equilibrium cost can as high as 1.5 - 2 times the optimal system cost, depending on the number of market participants.**

***Index Terms*—electricity market, equilibrium, inefficiency, strategic behavior, price of anarchy**

I. Introduction

A. Motivation

Electricity markets are notorious for their tendency to be oligopolies and thereby exposed to issues of market power [1]. Such imperfect competition enables certain participants to manipulate prices unilaterally, by submitting carefully engineered bids, which mislead the operator into making suboptimal dispatch decisions that raise system costs. In the face of this adversity, it is important to quantify and assess the magnitude of the inefficiency to which electricity markets are exposed. Is it insignificant and can be safely overlooked, or is it severe and calls for attention?

B. Related Literature

Modeling the bidding strategy of a strategic electricity market participant has been a popular topic in the literature. The predominant model is based on bi-level optimization: the participant is assumed to be well-informed on the Operator's economic dispatch problem as well as on the other participants' bidding strategies, which enables the focal participant to calculate the bids that will result in a maximization of its own profit. This bi-level bidding optimization problem takes the form of a Mathematical Problem with Equilibrium Constraints (MPEC) and its study goes at least as back as [2].

While the MPEC model considers the perspective of a single strategic participant, the case where multiple participants bid strategically brings the problem into the realm of Game Theory. The relevant solution concept is the Nash Equilibrium where, for electricity markets, its calculation takes the form of an Equilibrium Problem with Equilibrium Constraints (EPEC) [3], [4]. There is a rich literature that uses the EPEC model to calculate electricity market equilibria and assess their inefficiency for different setups, including the case of step-wise bidding curves [5], wind power producers [6], loads with load-shifting capabilities [7], and more.

However, the papers of this literature consider, each time, a given system (e.g. particular generators and cost functions) and assess the equilibrium inefficiency of that particular system. Naturally, a different system setup (i.e. a different set of generators with different cost functions etc) would result in a different equilibrium and respective inefficiency. Towards assessing the vulnerability of the market mechanism itself (and not of a particular system) to market power abuse, the relevant question concerns the inefficiency of the equilibrium *in the worst-case*: i.e. rather than assessing the outcome for a single (arbitrary) set of generators, we are interested in searching in the space of possible generators' combinations to identify the worst-case combination that would lead to the most severe market inefficiency.

The goal described above has attracted interest from economists and computer scientists, with papers providing upper bounds for the welfare loss at equilibrium of electricity markets. In [8], the authors model the electricity market as a so-called Supply Function Game (SFG) and study its equilibrium properties. In [9] the authors provide theoretical upper bounds on the welfare loss at equilibria for the whole class of SFG, thereby bounding the market's inefficiency for any possible combination of generators. The authors in [10], extend those bounds for the case where the limited capacity of the network lines is also considered. For the same type of bound, the recent study [11] provided a tightness guarantee.

The issue with this literature is that the derivation of analytical results necessitates modeling simplifications that diverge from how actual electricity markets operate. For starters, the SFG itself considers that each generator can only submit a single parameter representing a supply function of a given parametric form; whereas in actual markets, participants bid multi-parameter (namely piece-wise linear) cost functions, which enrich the operator's economic dispatch model, but hinder the derivation of analytical bounds on the market's efficiency loss at equilibrium.

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C. Research Gap and Contributions

In light of the above, this paper identifies a gap in formulating a framework for quantifying the worst-case electricity market efficiency loss at equilibrium. To that end, we formulate this quest as a tri-level optimization problem, where:

- the upper-level problem concerns searching in the space of possible generators' cost function combinations to identify systems that maximize the inefficiency of equilibrium;
- the middle-level problem calculates the equilibrium bids for a given combination provided by the upper-level problem;
- the lower-level problem clears the market for a given bidding profile provided by the middle-level problem.

While each of the two bodies of related literature (i.e. the EPEC and the SFG) comes with certain limitations, as discussed in the previous subsection, the proposed approach aspires to make the best of both worlds: the middle-level and the lower-level problem taken together is essentially the EPEC model of the related literature. Introducing the upper-level problem generalizes this literature towards search in the space of possible EPECs to assess the worst-case inefficiency of electricity market equilibrium. On the other hand, the proposed approach compromises the formal elegance of the SFG literature and takes a numerical optimization approach. This enables the consideration of far richer and more accurate models of electricity markets, which can provide more reliable insight into the inefficiency loss of real electricity markets. In fact, our simulation results indicate that the worst-case inefficiency can be more severe than previously anticipated.

Section II presents the electricity market model including the MPEC and EPEC formulations. Section III formulates the described framework and presents the paper's methodological approach to track the problem's solutions. Section IV presents our simulations' setup and results, while Section V concludes the paper.

II. Preliminaries & System Model

We consider a day-ahead electricity market for a horizon $\mathcal{T} = \{1, 2, ..., t, ..., \mathrm{T}\}$ comprising T discrete timeslots of equal duration. A set $\mathcal{G}$ of generators submit price-quantity bids in the market, and the operator decides the dispatch $X = (x_{g,t})_{g\in\mathcal{G},t\in\mathcal{T}}$ towards satisfying a demand profile $\mathrm{D} = (\mathrm{D}_t)_{t\in\mathcal{T}}$ at the minimum possible cost. The operator's objective therefore reads as:

$$\min_X \left\{ \sum_{g\in\mathcal{G}} \sum_{t\in\mathcal{T}} C(x_{g,t}; \alpha_{g,t}, \beta_{g,t}) \right\} \qquad \text{(Optimal)}$$
$$\text{s.t.} \sum_{g\in\mathcal{G}} x_{g,t} = \mathrm{D}_t, \quad \forall t \in \mathcal{T},$$
$$0 \le x_{g,t} \le \overline{\mathrm{x}}_g \quad \forall g \in \mathcal{G}, t \in \mathcal{T}.$$

The maximum power of a generator $g \in \mathcal{G}$ is denoted by $\overline{\mathrm{x}}_g$, and the cost $C(x_{g,t}; \alpha_{g,t}, \beta_{g,t})$ of a generator $g \in \mathcal{G}$ at $t \in \mathcal{T}$ is modeled as

$$C(x_{g,t}; \alpha_{g,t}, \beta_{g,t}) = \alpha_{g,t} x_{g,t} + \beta_{g,t}(x_{g,t})^2, \quad \forall g \in \mathcal{G}, t \in \mathcal{T}, \tag{1}$$

where $\alpha_{g,t}, \beta_{g,t}$ parameterize the function.

We refer to the minimizer $X^{\mathrm{opt}} = (x^{\mathrm{opt}}_{g,t})_{g\in\mathcal{G},t\in\mathcal{T}}$ of problem (Optimal) as the *optimal dispatch*, and to the corresponding system cost

$$C^{\mathrm{opt}} = \sum_{g\in\mathcal{G}} \sum_{t\in\mathcal{T}} C(x^{\mathrm{opt}}_{g,t}; \alpha_{g,t}, \beta_{g,t}) \tag{2}$$

as the *minimum system cost*. This outcome could be realized if the operator was informed of the generators' true cost parameters $\alpha_{g,t}, \beta_{g,t}$. In an electricity market, the operator asks each generator to *bid* its cost function. In this paper, we assume bids of the form $(\widehat{\alpha}_{g,t}, \widehat{\beta}_{g,t})$, representing the cost function $C(x_{g,t}; \widehat{\alpha}_{g,t}, \widehat{\beta}_{g,t})$. For a generator $g \in \mathcal{G}$, a bid $(\widehat{\alpha}_{g,t}, \widehat{\beta}_{g,t}) = (\alpha_{g,t}, \beta_{g,t})$ is called a *truthful* bid. After gathering all bids, the operator clears the market by solving

$$\min_X \left\{ \sum_{g\in\mathcal{G}} \sum_{t\in\mathcal{T}} C(x_{g,t}; \widehat{\alpha}_{g,t}, \widehat{\beta}_{g,t}) \right\} \qquad \text{(Market-clearing)}$$
$$\text{s.t.} \sum_{g\in\mathcal{G}} x_{g,t} = \mathrm{D}_t, \quad \forall t \in \mathcal{T} \quad : (\lambda_t),$$
$$0 \le x_{g,t} \le \overline{\mathrm{x}}_g \quad \forall g \in \mathcal{G}, t \in \mathcal{T}.$$

where λ_t is the optimal dual variable of the supply-demand constraint and is interpreted as the market-clearing price for timeslot t. Accordingly, each generator receives a payment of:

$$\pi_g = \sum_{t\in\mathcal{T}} \lambda_t x^*_{g,t}, \quad \forall g \in \mathcal{G}, \tag{3}$$

where $x^*_{g,t}$ is the minimizer of problem (Market-clearing).

Since each generator is a profit-maximizing player, it is incentivized to strategize over its bids $(\widehat{\alpha}_{g,t}, \widehat{\beta}_{g,t})_{t\in\mathcal{T}}$ so that it steers the market outcome towards solutions $(\lambda_t, x^*_{g,t})_{t\in\mathcal{T}}$ that maximize its own profit. Thereby, the generator's bidding optimization problem takes a bi-level form, formalized as

$$\max_{(\widehat{\alpha}_{g,t}, \widehat{\beta}_{g,t})_{t\in\mathcal{T}}} \left\{ \sum_{t\in\mathcal{T}} \lambda_t x^*_{g,t} - C(x^*_{g,t}; \alpha_{g,t}, \beta_{g,t}) \right\} \quad (\mathrm{MPEC}_g)$$
$$\text{s.t. } (x^*_{g,t}, \lambda_t)_{t\in\mathcal{T}} \in \text{(Market-clearing)},$$

where the upper-level problem refers to the generator's bidding decision with the objective to maximize its own profit and the lower-level problem specifies that the prices and generator's dispatch result from the operator's market-clearing problem (Market-clearing). This bi-level problem of generator g is commonly known as a Mathematical Program with Equilibrium Constraints, hence we refer to it as (MPEC$_g$).

Notice that the solution to problem (MPEC$_g$) for a focal generator g, depends on the bids of other generators. On that note, when more (or all) generators bid strategically, we have a collection $(\mathrm{MPEC}_g)_{g\in\mathcal{G}}$ of interdependent problems. This models the electricity market as a game $\mathcal{E}$ defined by the generators (as players), their bids (as actions/strategies) and their profits, resulting from the market-clearing problem (as their payoffs). A solution

$$(\widehat{\alpha}^{\text{eq}}_{g,t}, \widehat{\beta}^{\text{eq}}_{g,t})_{g\in\mathcal{G},t\in\mathcal{T}} \in (\text{MPEC}_g)_{g\in\mathcal{G}}, \qquad \text{(EPEC)}$$

that is consistent across all problems $(\text{MPEC}_g)_{g\in\mathcal{G}}$, i.e. a bidding profile that is simultaneously a maximizer of each and every generator's bidding optimization problem, defines a Nash Equilibrium of the game $\mathcal{E}$ and it features the property that, once discovered, no generator would be incentivized to change its bid unilaterally. Calculating such a solution is referred to as an Equilibrium Problem with Equilibrium Constraints (EPEC). Such an EPEC model constitutes an archetypical prediction of where the bids in an electricity market would end up. Given a solution $(\widehat{\alpha}^{\text{eq}}_{g,t}, \widehat{\beta}^{\text{eq}}_{g,t})_{g\in\mathcal{G},t\in\mathcal{T}}$ to the EPEC problem, a corresponding equilibrium dispatch

$$(x^{\text{eq}}_{g,t})_{g\in\mathcal{G},t\in\mathcal{T}} = \min_{X} \left\{ \sum_{g\in\mathcal{G}} \sum_{t\in\mathcal{T}} C(x_{g,t}; \widehat{\alpha}^{\text{eq}}_{g,t}, \widehat{\beta}^{\text{eq}}_{g,t}) \right\} \qquad (4)$$

$$\text{s.t.} \sum_{g\in\mathcal{G}} x_{g,t} = \mathrm{D}_t, \quad \forall t \in \mathcal{T} \quad : (\lambda^{\text{eq}}_t),$$

$$0 \le x_{g,t} \le \overline{\mathrm{x}}_g \quad \forall g \in \mathcal{G}, t \in \mathcal{T}.$$

and equilibrium cost

$$C^{\text{eq}} = \sum_{g\in\mathcal{G}} \sum_{t\in\mathcal{T}} C(x^{\text{eq}}_{g,t}; \alpha_{g,t}, \beta_{g,t}) \qquad (5)$$

follow directly. It is important to note that C^{eq} is the *true* system cost at equilibrium and is therefore measured based on the true parameters $\alpha_{g,t}, \beta_{g,t}$ of generators and not over their bids $\widehat{\alpha}^{\text{eq}}_{g,t}, \widehat{\beta}^{\text{eq}}_{g,t}$; the latter influence the equilibrium cost C^{eq} by affecting the equilibrium dispatch $x^{\text{eq}}_{g,t}$ through problem (4).

III. Problem Formulation & Methodology

A. Problem Formulation

Given the definitions above, the inefficiency of a market is defined as the game's *Price of Anarchy* (PoA), i.e. the ratio between the system cost at equilibrium and the minimum system cost. Naturally, it is $PoA \geq 1$. Given a game $\mathcal{E}$, the PoA captures the market inefficiency stemming from the generators' strategic behavior and its value depends on the true costs $(\alpha_{g,t}, \beta_{g,t})_{g\in\mathcal{G}}$ of generators, which define the game at hand. Let us define a generator's type $\theta_g = (\alpha_{g,t}, \beta_{g,t})$ to succinctly capture its true cost function which, in this model, fully characterizes the generator. The vector $\boldsymbol{\theta} = (\theta_g)_{g\in\mathcal{G}}$ contains the types of all generators and we denote the game's PoA as $PoA(\boldsymbol{\theta})$ to highlight the PoA dependence on the generator types' combination:

$$PoA(\boldsymbol{\theta}) = \frac{C^{\text{eq}}(\boldsymbol{\theta})}{C^{\text{opt}}(\boldsymbol{\theta})}, \qquad (6)$$

While the EPEC literature on electricity markets typically calculates the PoA for a given setup/game, in this paper we are interested in quantifying the worst-case (or, robust) PoA for a *class* of games $\mathscr{E} = (\mathcal{E}_{\boldsymbol{\theta}})_{\boldsymbol{\theta}\in\Theta(\mathscr{E})}$, where $\Theta(\mathscr{E})$ is the set of all possible generator types considered in the game class. In other words, we are after discovering the worst-case combination of generators that would maximize the electricity market's inefficiency, so as to stress-test the market mechanism's performance. The Robust PoA (RPoA) for a game class $\mathscr{E}$ is thus defined as:

$$RPoA(\mathscr{E}) = \max_{\boldsymbol{\theta}\in\Theta(\mathscr{E})} \{PoA(\boldsymbol{\theta})\} \qquad (7)$$

Calculating the RPoA is remarkably challenging as it entails searching in the space of generator types' combinations where, to evaluate the PoA of a single combination ($\boldsymbol{\theta}$), we need to solve problems (Optimal) and (EPEC) to calculate the combination's optimal and equilibrium costs respectively. Problem (EPEC) in particular (for a given combination ($\boldsymbol{\theta}$)) is a computationally challenging problem just in itself, with no closed-form solutions and, worse still, without explicit derivatives' information.

B. Solution Methodology

Let us first consider a fixed $\boldsymbol{\theta}$ and focus on calculating the PoA, as defined in (6). The denominator (the optimal system cost) is given by solving problem (Optimal), which is a convex problem. The numerator is the cost at equilibrium which entails solving problem (EPEC). Problem (EPEC), in turn, is a collection of the bi-level problem (MPEC_g). To calculate an equilibrium (for a given $\boldsymbol{\theta}$), we deploy the well-known best-response algorithm in which we solve the MPEC problem (MPEC_g) of each player sequentially, each time using the latest update of the other players' bids. Towards solving the bi-level problem (MPEC_g), we convert its (convex) lower-level problem (4) to the equivalent set of Karush-Kuhn-Tucker (KKT) conditions (following the well-documented formula, refer e.g. to [12]), which results in a single-level mixed-integer linear program. Such a reformulation creates bilinear terms which, however, do not need explicit treatment since they are handled automatically by many commercial solvers by creating an outer linear approximation that is refined iteratively, making the problem solvable given certain tolerances.

Using this standard reformulation, problem (MPEC_g) can be brought to a form that can be solved by commercial solvers. Thus, the best-response algorithm can be implemented. Algorithm 1 formalizes the best-response procedure that calculates the cost at equilibrium $C^{\text{eq}}(\boldsymbol{\theta})$ for a given $\boldsymbol{\theta}$. The algorithm's convergence properties will be discussed in the next section.

We now turn to the overall problem (7) of searching over the space $\Theta(\mathscr{E})$ of possible vectors $\boldsymbol{\theta}$. For starters, the objective function (depending on the middle-level problem (EPEC)) is inherently non-differentiable, which necessitates the use of a zero-order method. Then, the problem's special nature makes it particularly prone to bearing narrow valleys, which hinders the effectiveness of local search methods. This characteristic, in combination with the problem's high dimensionality, motivates the use of population-based methods. Moreover, observe how even evaluating the problem's objective function

Algorithm 1 Best-Response algorithm for calculating $C^{\mathrm{eq}}(\boldsymbol{\theta})$ for a given $\boldsymbol{\theta}$

Initialize iteration number $k = 1$ and $\delta^{(1)} = \infty$.
Initialize $\widehat{\boldsymbol{\theta}}^{(\boldsymbol{k})} = \boldsymbol{\theta}$.
while $\delta^{(k)} \geq \varepsilon$:
  for $g \in \mathcal{G}$:
    set $(\widehat{\alpha}_{g,t}^{(k)}, \widehat{\beta}_{g,t}^{(k)})_{t\in\mathcal{T}} = \arg\max\{(\mathrm{MPEC}_g)\}$
    set $\delta_g^{(k)} = \max_{t\in\mathcal{T}}\{\max\{|\widehat{\alpha}_{g,t}^{(k)} - \widehat{\alpha}_{g,t}^{(k-1)}|, |\widehat{\beta}_{g,t}^{(k)} - \widehat{\beta}_{g,t}^{(k-1)}|\}\}$
  set $\delta^{(k)} = \max_{g\in\mathcal{G}}\{\delta_g^{(k)}\}$
set $\widehat{\alpha}_{g,t}^{\mathrm{eq}} = \widehat{\alpha}_{g,t}^{(k)}, \widehat{\beta}_{g,t}^{\mathrm{eq}} = \widehat{\beta}_{g,t}^{(k)}$
set $x_{g,t}^{\mathrm{eq}}$ as in (4)
set $C^{\mathrm{eq}}(\boldsymbol{\theta})$ as in (5)
return $C^{\mathrm{eq}}(\boldsymbol{\theta})$

$PoA(\boldsymbol{\theta})$ (for a given $\boldsymbol{\theta}$) entails simulating the best-response algorithm. Thus, the problem bears an objective function that is computationally costly to evaluate. This last characteristic precludes methods that are inherently sequential and calls for a method that features massive parallelization. These considerations motivate us to adopt a Particle Swarm Optimization (PSO) algorithm towards approaching problem (7).

The PSO algorithm is inspired by the social behavior present in flocks of birds or schools of fish. It works by initializing a swarm of particles through the solution space, where each particle is seen as a candidate of the optimal solution. Particles are characterized by their associated position and velocity, and the globally optimal solution is pursued using a position-velocity update method (refer to [13] for more details).

For our purposes, the PSO algorithm was implemented using the Python package PySwarms [14]. Each particle evaluates the objective function in the current position (by executing Algorithm 1) to determine its quality. Notice how this calculation is independent of other particles. Thereby the calculations for different particles can be run in parallel.

IV. Simulation Results

In this Section, we simulate the game $\mathcal{E}$ for the quadratic cost function generator class and evaluate the worst-case PoA by applying the PSO algorithm. The results are compared to the theoretical upper bounds of the SFG literature.

A. Simulation Setup

We consider the class of generators featuring the quadratic cost function of Eq. (1) with $\alpha_{g,t} \in [0.008, 0.070]$, and $\beta_{g,t} \in [9, 12]$, following the setup used in [15]. The capacity of each generator is set to 700 MW, and the demand is set to 1148.4 MW, which corresponded to the demand of the Danish bidding zone DK1 between 9h-10h on August 27, 2023. This demand represents 54.7% of the installed capacity of the system when there are 3 generators. For the cases when more generators are considered, the demand is increased so that the same ratio is maintained.

The tolerance of Algorithm 1 is set to $\varepsilon = 2\%$. If an ε-Equilibrium is not reached within 200 iterations, it is assumed that the average of the last 10 iterations is the $\varepsilon-$equilibrium. This is necessary since there are no convergence guarantees for Algorithm 1. Additionally, in each iteration, a generator can only bid up to 10 times the upper bound of $\alpha_{g,t}, \beta_{g,t}$, i.e., $\widehat{\alpha}_{g,t} \leq 0.7$ and $\widehat{\beta}_{g,t} \leq 120$.

The PSO algorithm is configured as follows: the inertia term, ω, decreases linearly as a function of the iteration number, between 0.9 and 0.7. The cognitive component, c_1, has the same behavior between an initial value of 2 and a final value of 1. The same linear change is applied to the social component, but this time from a value of 1 until a value of 2. The combination of these parameters and their adaptation across iterations is effective to balance the exploration and exploitation capabilities of the PSO algorithm. We used 24 particles and 24 iterations. The PSO is deemed convergent when the best known solution does not change more than 1% for 5 consecutive iterations.

B. Main Result

Our central simulation result concerns evaluating the worst-case market inefficiency $RPoA$, as defined in Eq. (7), for different numbers of generators and comparing it with the upper bound given in [9]. That upper bound was calculated analytically, considering a significantly simpler market model. This simulation tests whether that theoretical upper bound holds true for the more realistic (albeit still simple) model considered here.

Fig. 1 shows the results, where the solid line corresponds to the results of problem (7), while the dashed line corresponds to the upper bound introduced by [9]. The obtained RPoA is consistently higher than the upper bound given in [9] for all cases of generators' number. The result interpretation is that, for satisfying the same demand level, the system cost under strategic generators' behavior can be more than 1.5 times the minimum system cost (for 10 generators, even higher for less generators). The RPoA is generally decreasing as the number of generators increases, since increased competition decreases market inefficiency. This trend is generally preserved in our simulation results, although some noisy non-monotonicities are present; these are attributed to the PSO being stuck at local optima. More particles and/or iterations could discover even worse cases i.e. even higher RPoA. Recall however that the problem's complexity does not leave room for theoretical guarantees of global optimality. Nonetheless, even the local optima discovered are already well above (up to 65.4%) the theoretical upper bound, which constitutes a strong indication that, in practice, electricity markets can be more inefficient than previously anticipated.

Furthermore, as shown in Fig. 2, this result appears relatively insensitive to increased values of ε (i.e. the convergence/equilibrium tolerance parameter of Algorithm 1), especially for higher numbers of generators. A similar insensitivity is observed when the range of the cost parameters is increased by 10% and 20%.

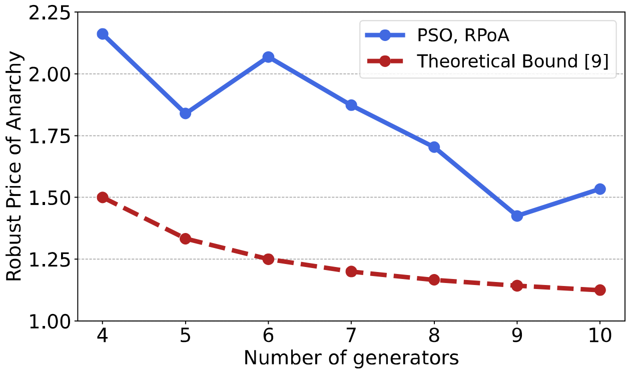

Fig. 1: Robust price of anarchy as a function of the number of strategic market participants.

C. Empirical Distribution of PoA

In light of the evidence that the RPoA exceeds initial expectations, it is relevant to assess how frequently such adverse cases occur within the generator search space. In this subsection, we test whether the RPoA of Fig. 1 is an outlier, or whether there are many possible combinations of generators that can yield such high levels of inefficiency. We assess this by sampling the space of possible generators' true cost functions (i.e. true parameters $\alpha_{g,t}, \beta_{g,t}$) and evaluating the PoA for each sample. For this test, in order to ensure an unbiased sampling[1], we use the full factorial algorithm (cf [16] - Chapter 13.1) where the search space defined by the range of $\alpha_{g,t}$ and $\beta_{g,t}$ is discretized in three values, namely, the lower and upper bounds introduced in the simulation setup, and the average of the two.

Fig. 3 shows the frequency of each PoA-level for the case of three generators. Cases of more players are not analyzed since the number of combinations to analyze grows exponentially large. The results show that PoA values greater than 2 are scarce[2]. However, there are plenty of combinations introducing an inefficiency between 1.1 and 1.3 which, although not as extreme as the RPoA, is still a very significant amount (10-30%) of additional, unnecessary system costs attributed to strategic participants' behavior.

Finally, Fig. 3 provides an insight on the skewness of the market's PoA distribution. This motivates the consideration of a novel metric, which we would call *quantile-PoA*: assessing the probability of an electricity market resulting in inefficiency higher than a level of choice. We leave this for future work.

V. Conclusions

This paper considered the problem of quantifying the vulnerability of the standard, marginal-pricing electricity market mechanism to inefficiency resulting from strategic bidding behavior. Adopting the notion of the Robust Price of Anarchy from Computer Science, we formulated the problem of searching in the space of possible generators' true cost functions to identify the worst-case combination that would lead to the most severe market inefficiency at equilibrium. Our main result asserts that the worst-case inefficiency of electricity markets violates previous theoretical bounds that were derived through simplified market models, indicating that the worst-case equilibrium cost can be 1.5-2 times the optimal system cost (depending on the number of market participants). This result suggests that real electricity markets' exposure to inefficiency, as a result of strategic participants' behavior, is higher than previously anticipated.

[1]Note that taking each particle-iteration of the PSO algorithm as a realization of the PoA is not unbiased, since the algorithm performs a guided search.

[2]As a side note, this skewness of the PoA distribution is an indication of the "narrow valleys" landscape, which motivated the use of PSO for searching the RPoA.

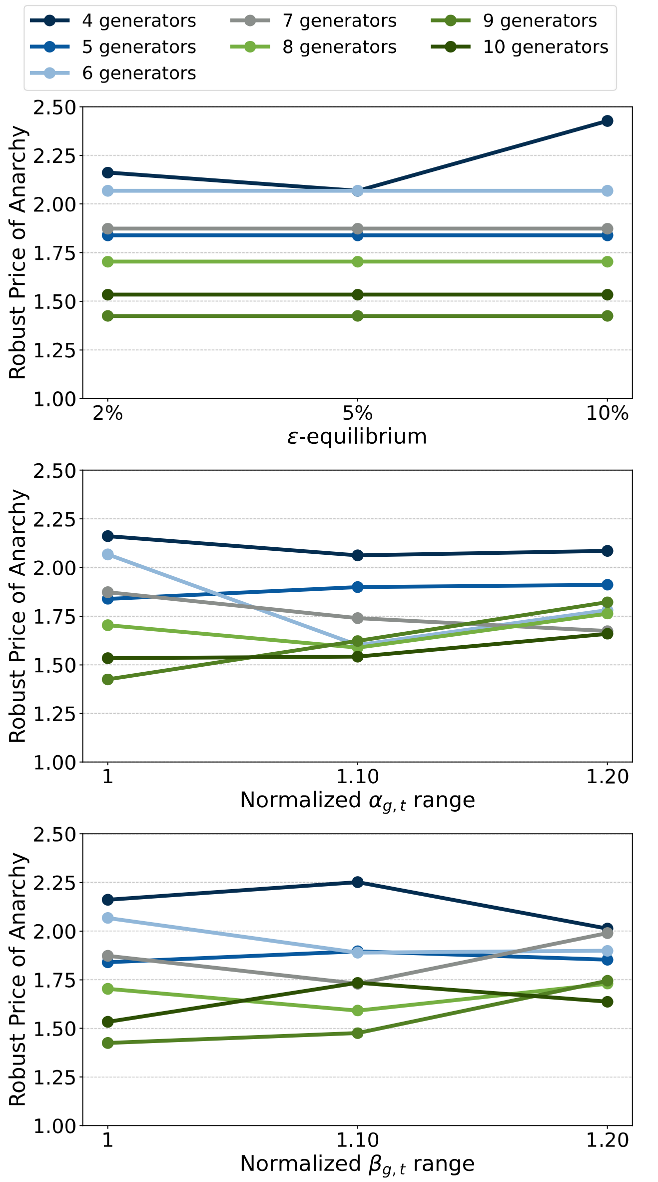

Fig. 2: Sensitivity of RPoA on the tolerance level of the ϵ-equilibrium, the range of $\alpha_{g,t}$, and the range of $\beta_{g,t}$.

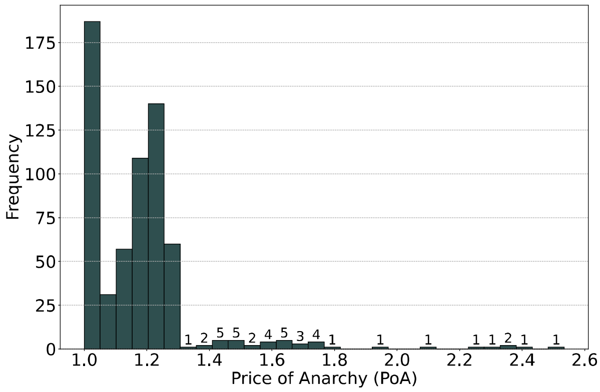

Fig. 3: Empirical frequency of the price of anarchy for three generators (discretized grid of cost function parameters).

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