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How inefficient can a market be?

The goal

Characterize the **inefficiency** of the electricity market

$$\eta_{th} = \frac{W_{out}}{Q_{in}} < 1$$

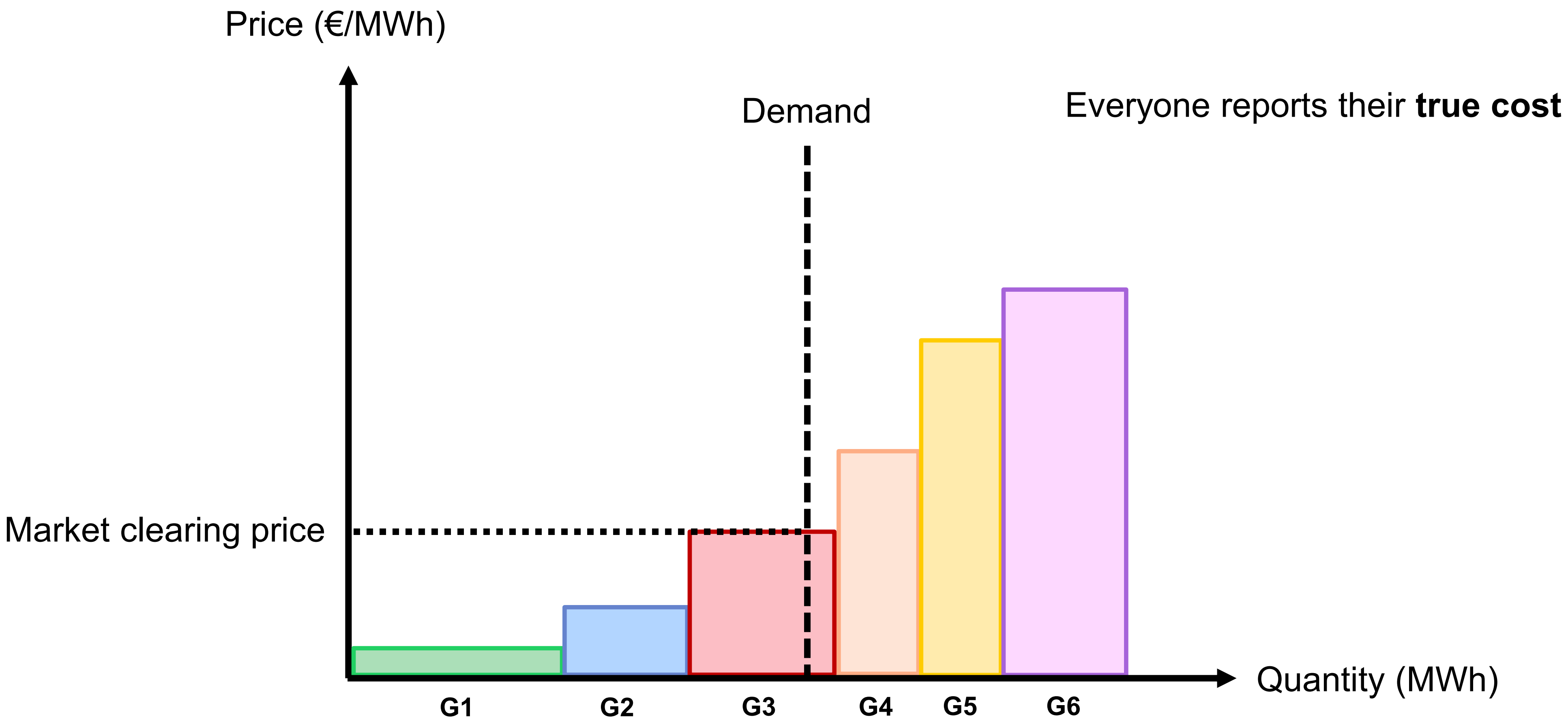
$$\boldsymbol{Price\ of\ Anarchy = \frac{Cost\ at\ equilibrium}{Minimum\ cost} \geq 1}$$

cost = burned fuel

Price of Anarchy **= 1**, we burn the **minimum** fuel to satisfy demand

Price of Anarchy **= 2**, we burn **twice as much** fuel to satisfy demand

How is the power system operated?



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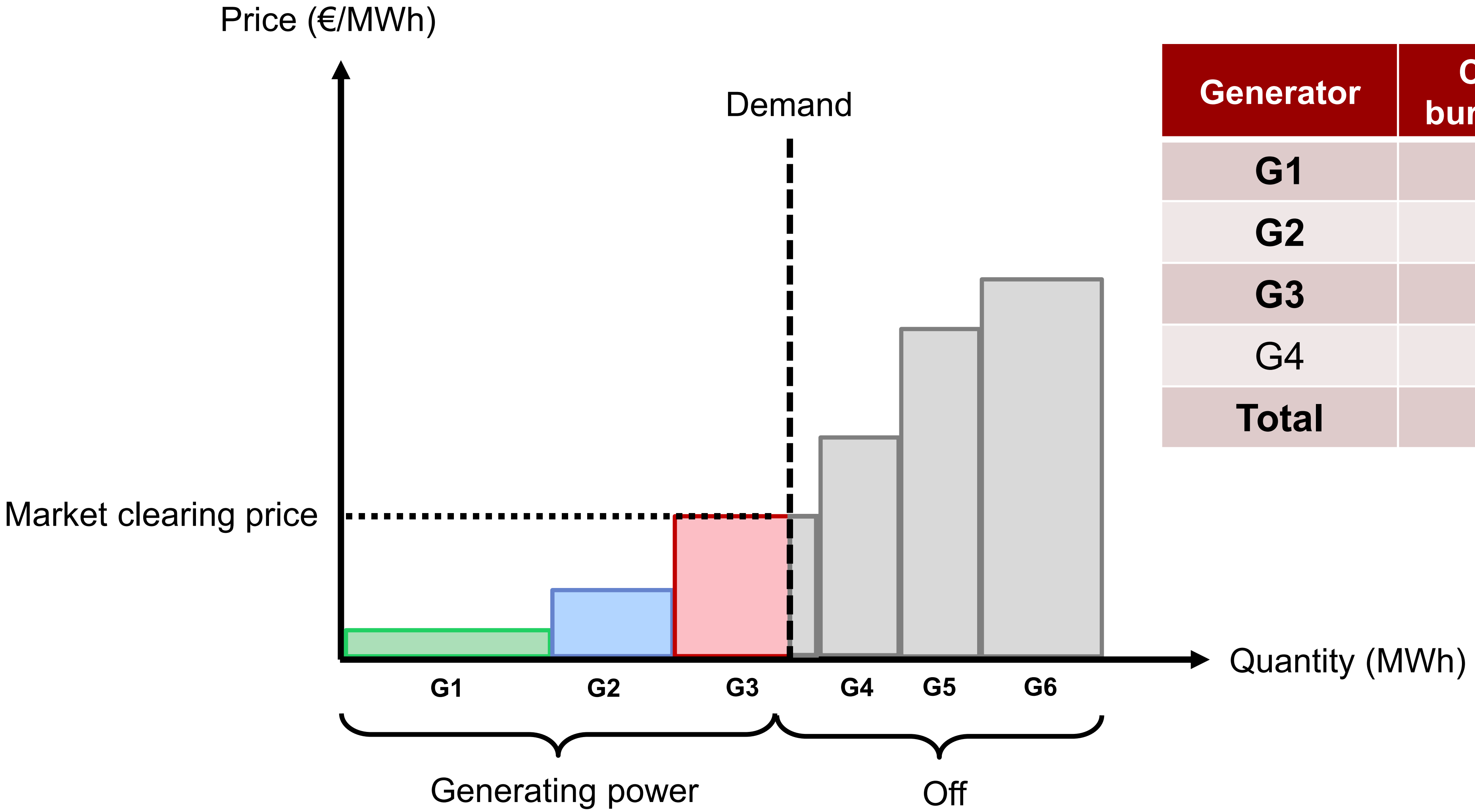
How is the power system operated?



Generator	Cost of burned fuel	Generation
G1	**10**	**100%**
G2	**15**	**100%**
G3	**24**	**80%**
G4	0	0%
Total	**50**	**-**

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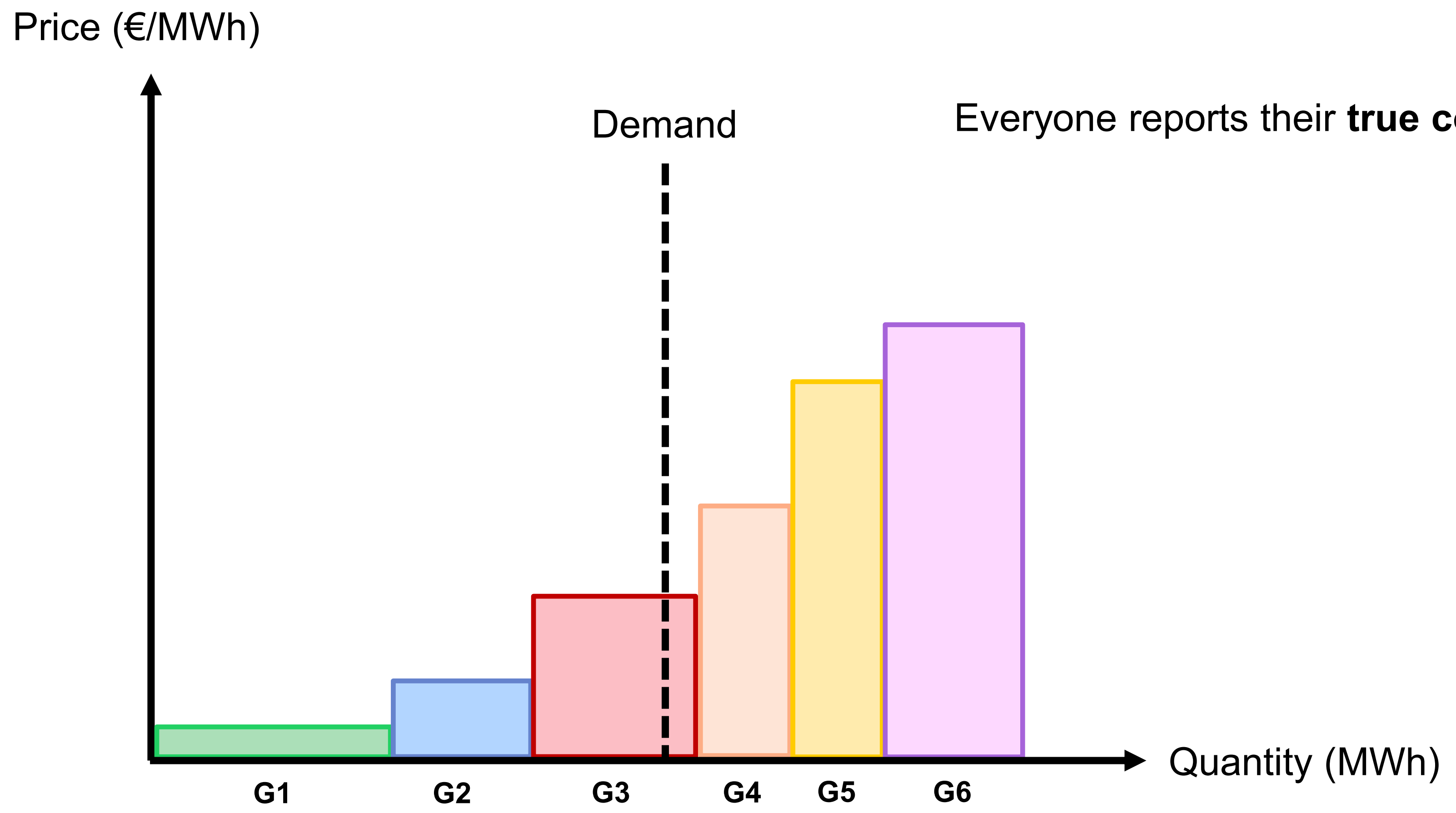
Strategic behaviour



Everyone reports their **true cost**

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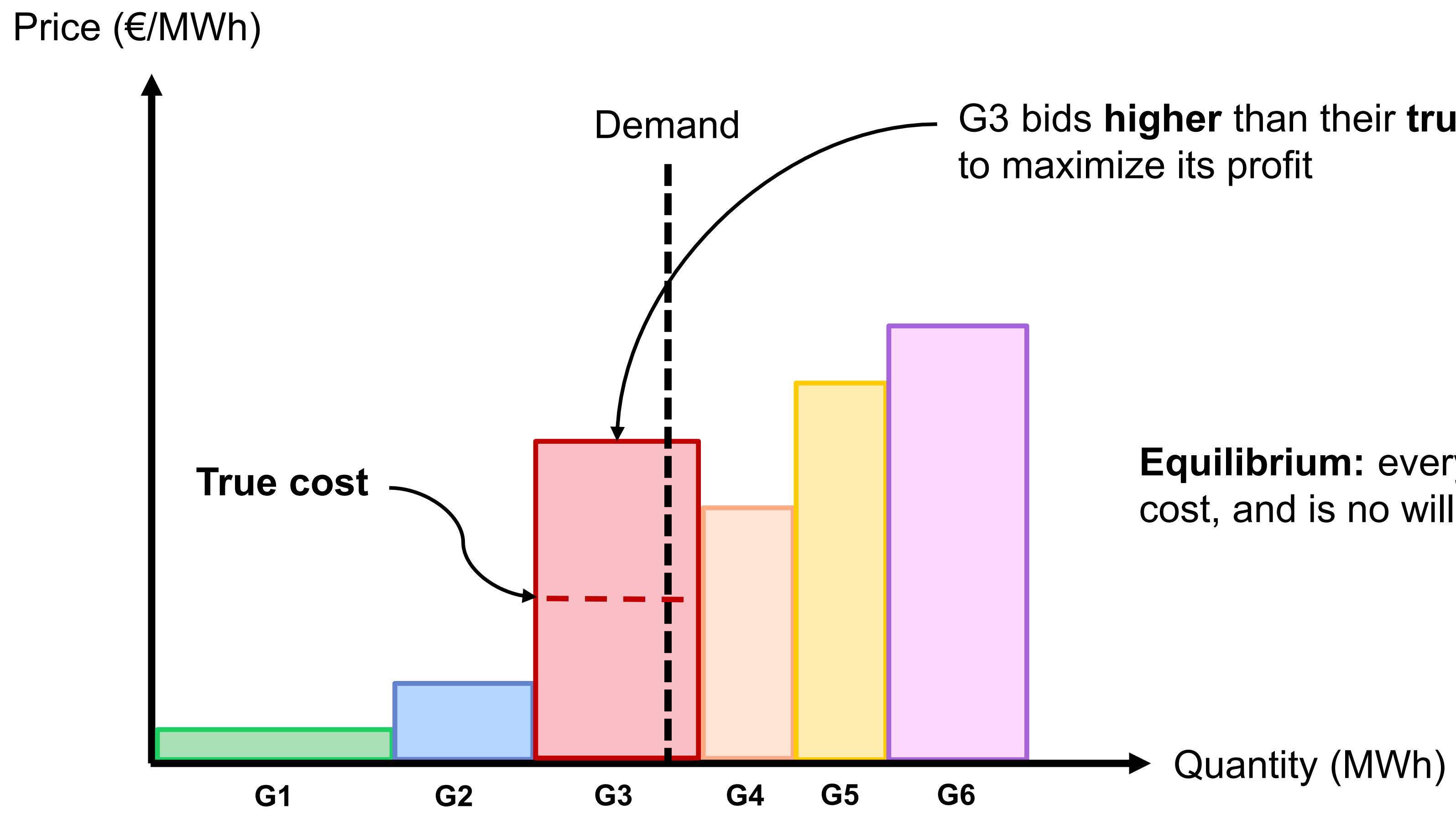
Strategic behaviour



G3 bids **higher** than their **true cost,** to maximize its profit

Equilibrium: everyone modified their cost, and is no willing to move

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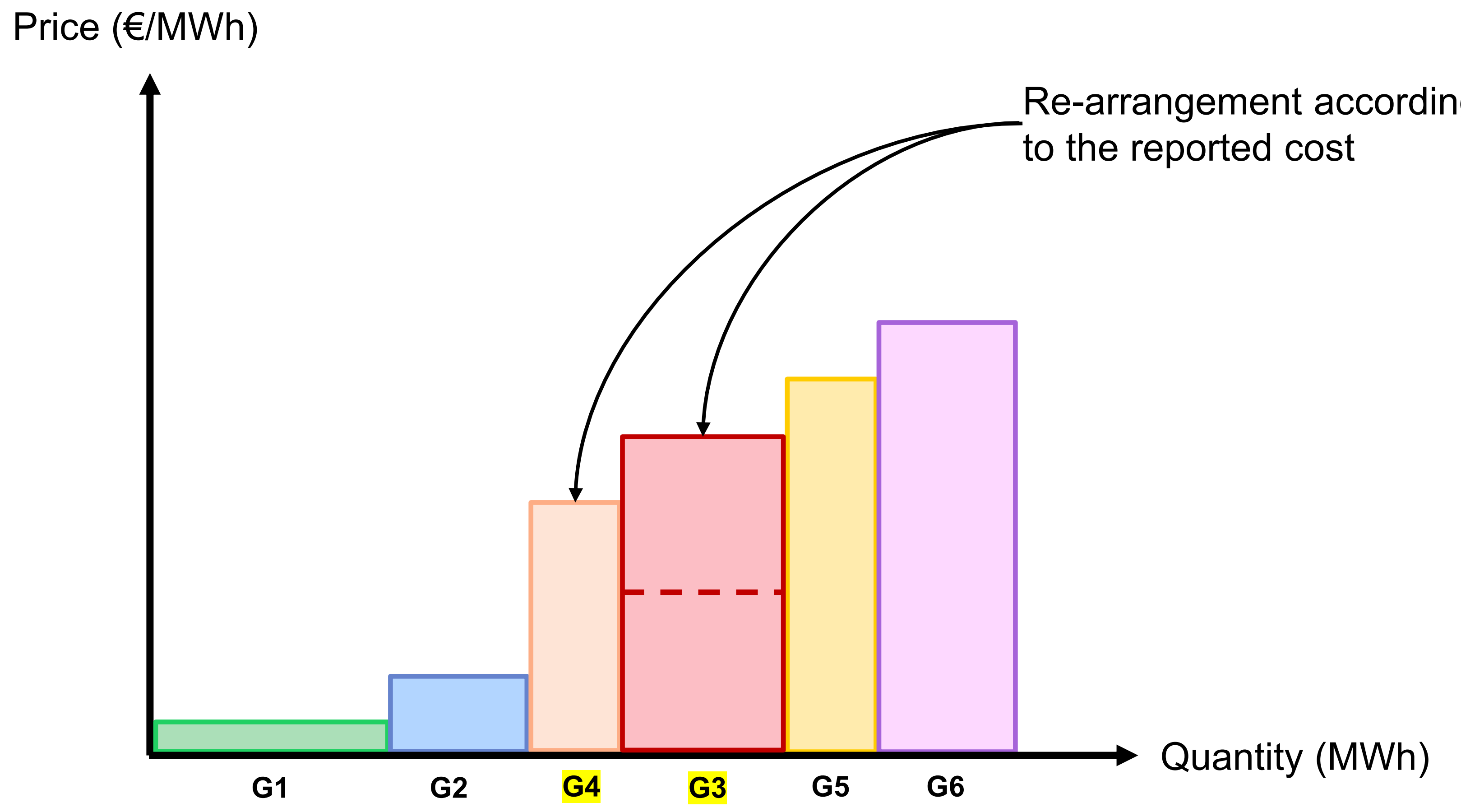
Strategic behaviour



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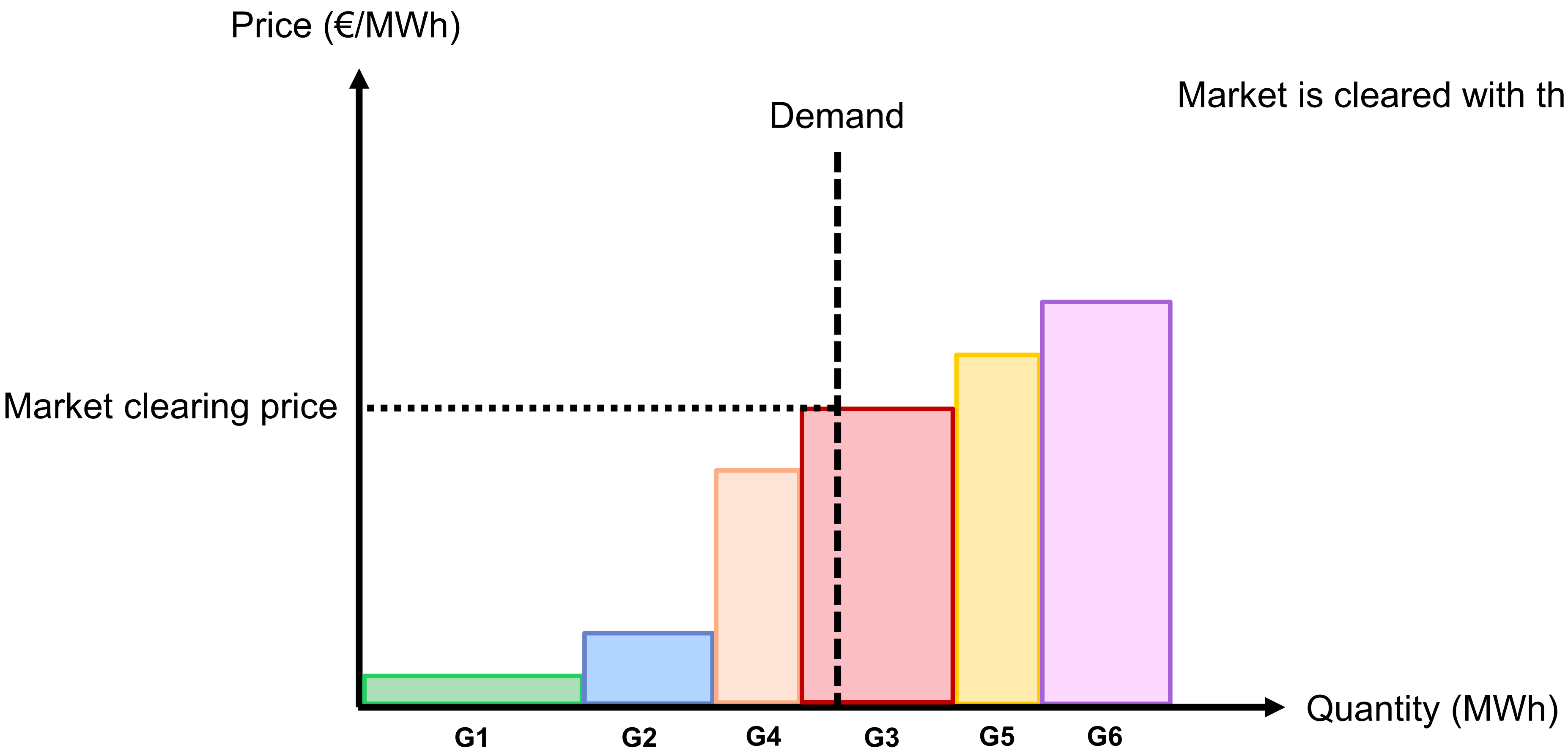
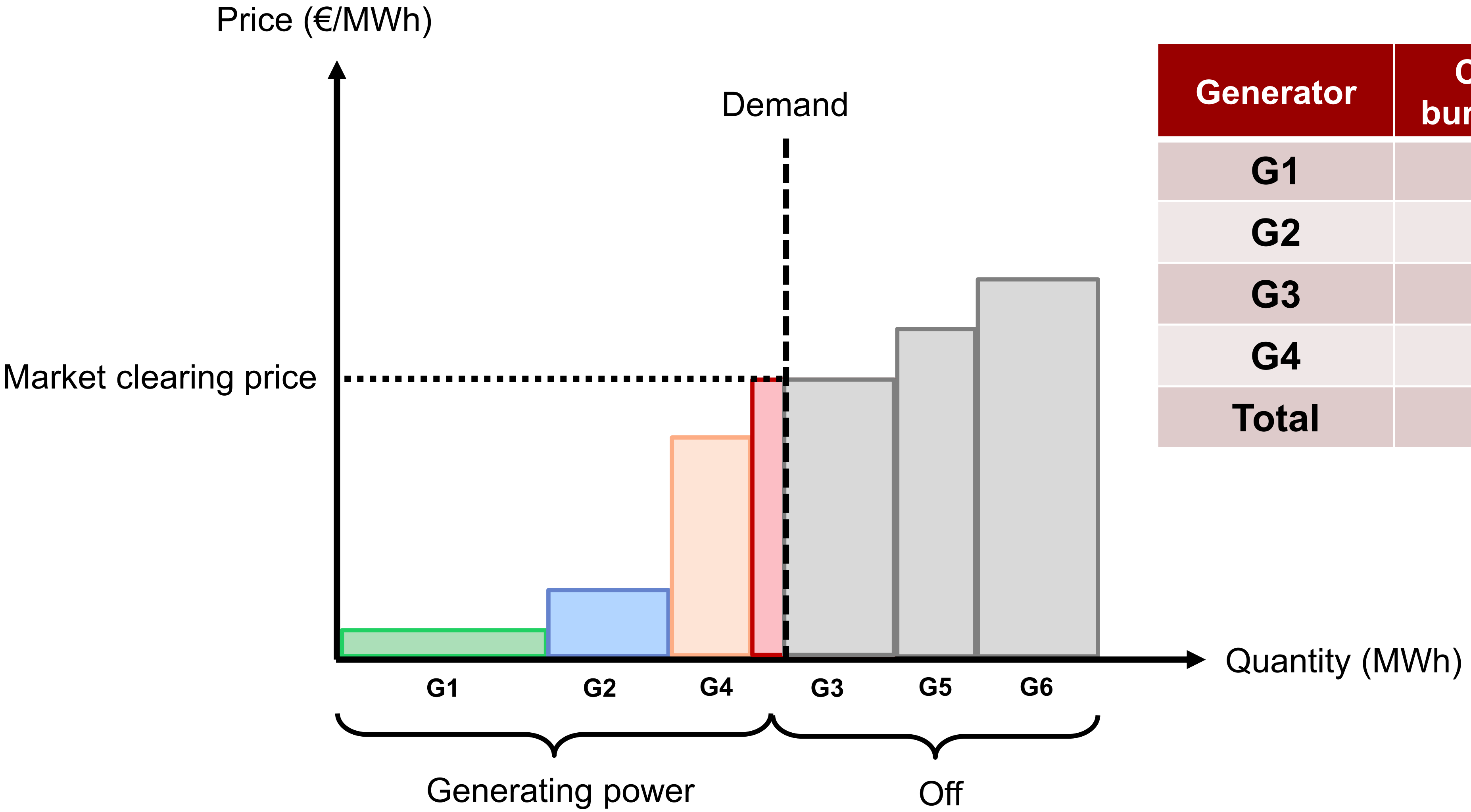
Strategic behaviour



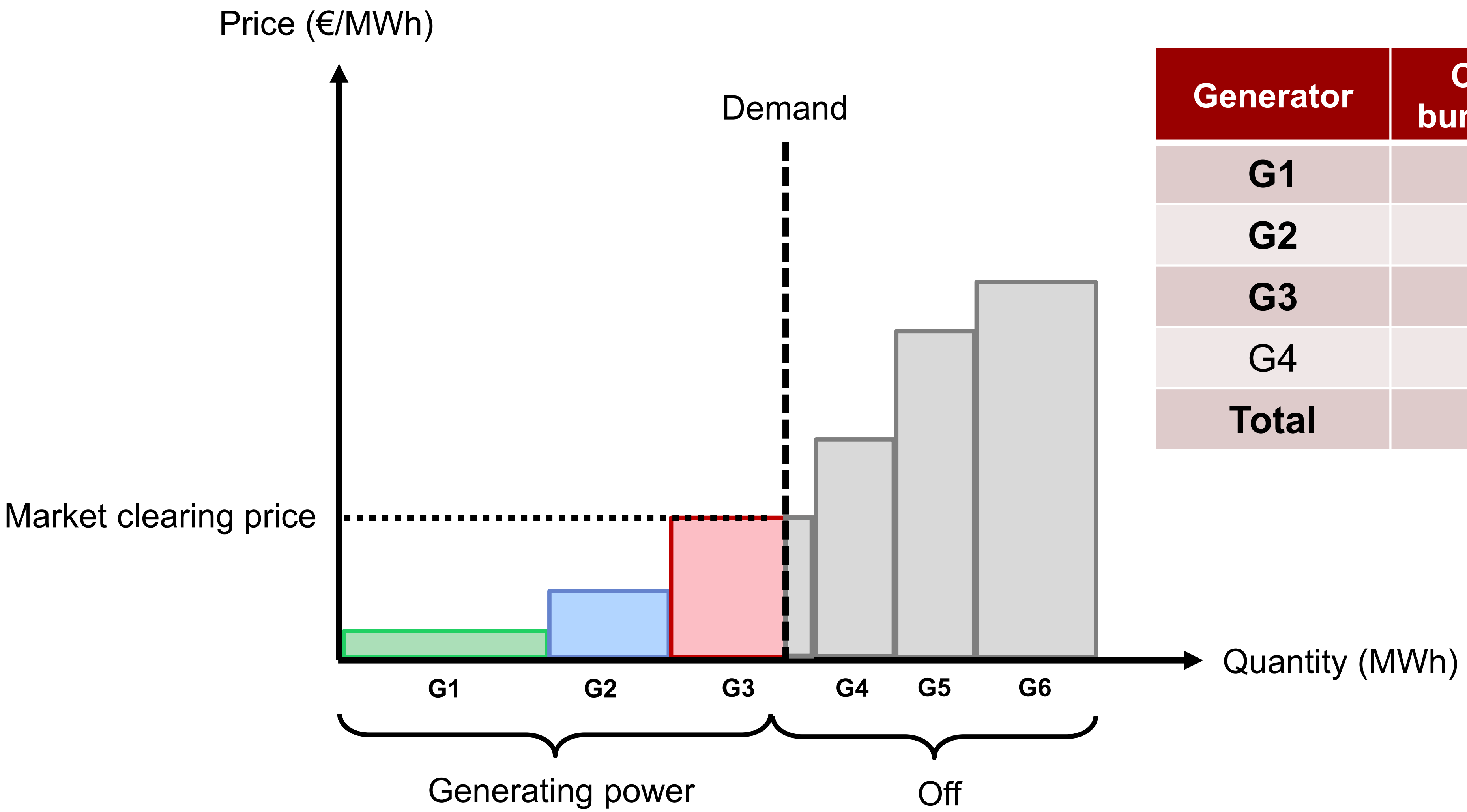
Market is cleared with this information

Strategic behaviour



Generator	Cost of burned fuel	Generation
G1	10	100%
G2	15	100%
G3	6	20%
G4	35	100%
Total	66	-

Original system



Generator	Cost of burned fuel	Generation
G1	**10**	**100%**
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G3	**24**	**80%**
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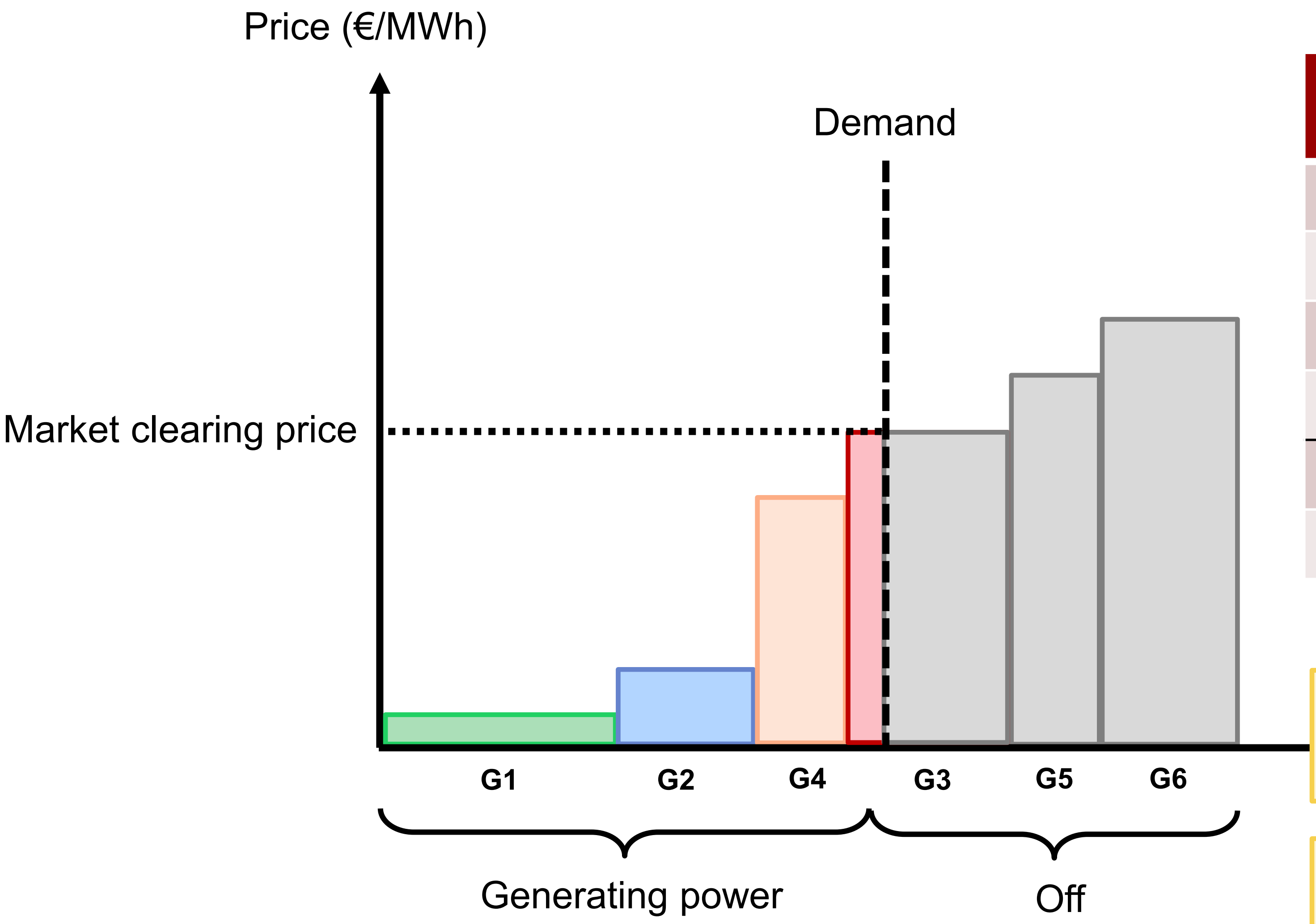
Total	**50**	**-**

Strategic behaviour

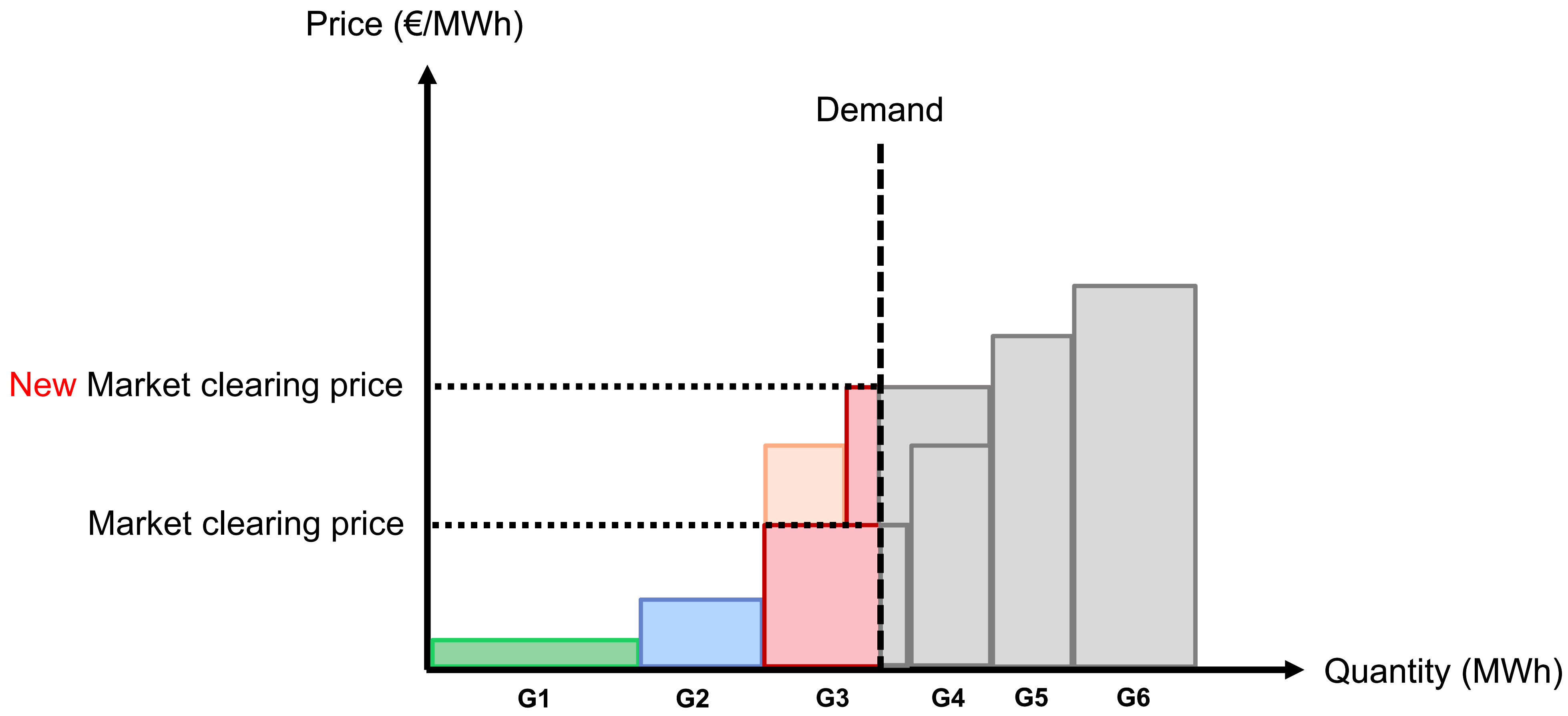


Generator	Cost of burned fuel	Generation
G1	10	100%
G2	15	100%
G3	6	20%
G4	35	100%
Total (old)	50	-
Total (new)	66	-

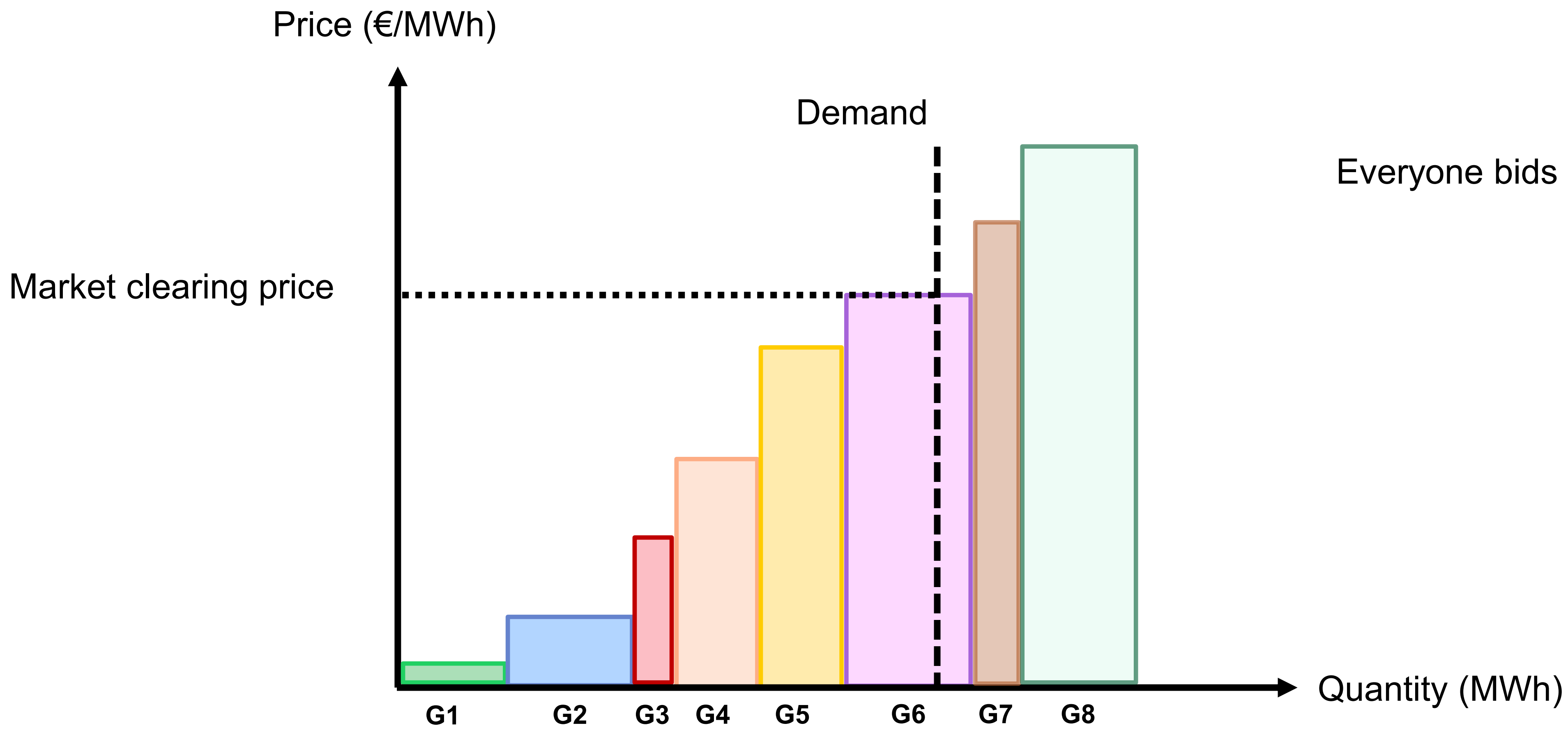
$$\textit{Price of Anarchy} = \frac{66}{50} = 1.32$$

The cost is 32% higher than needed

Our first system



A new system, different generators



Everyone bids their **true cost**

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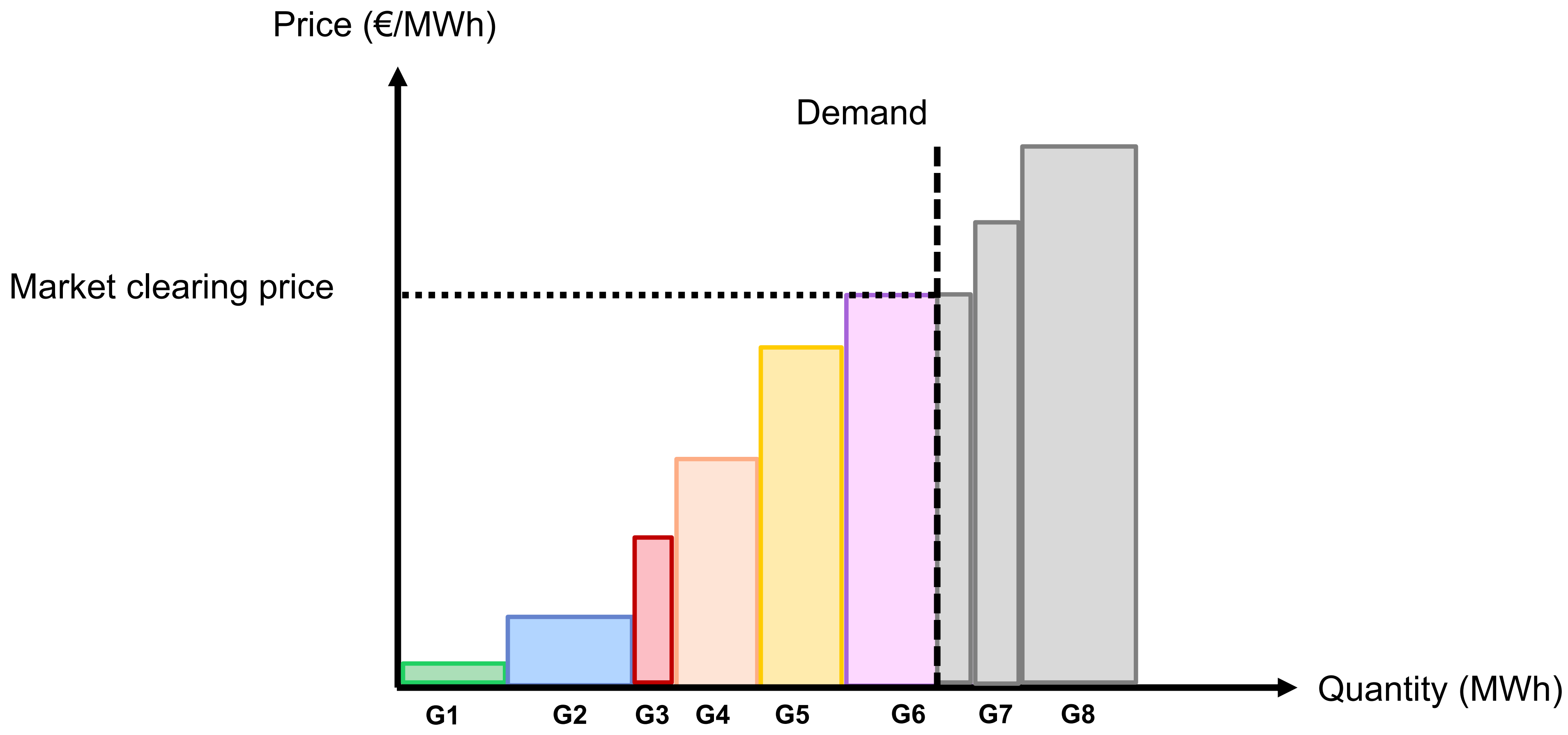
A new system, different generators



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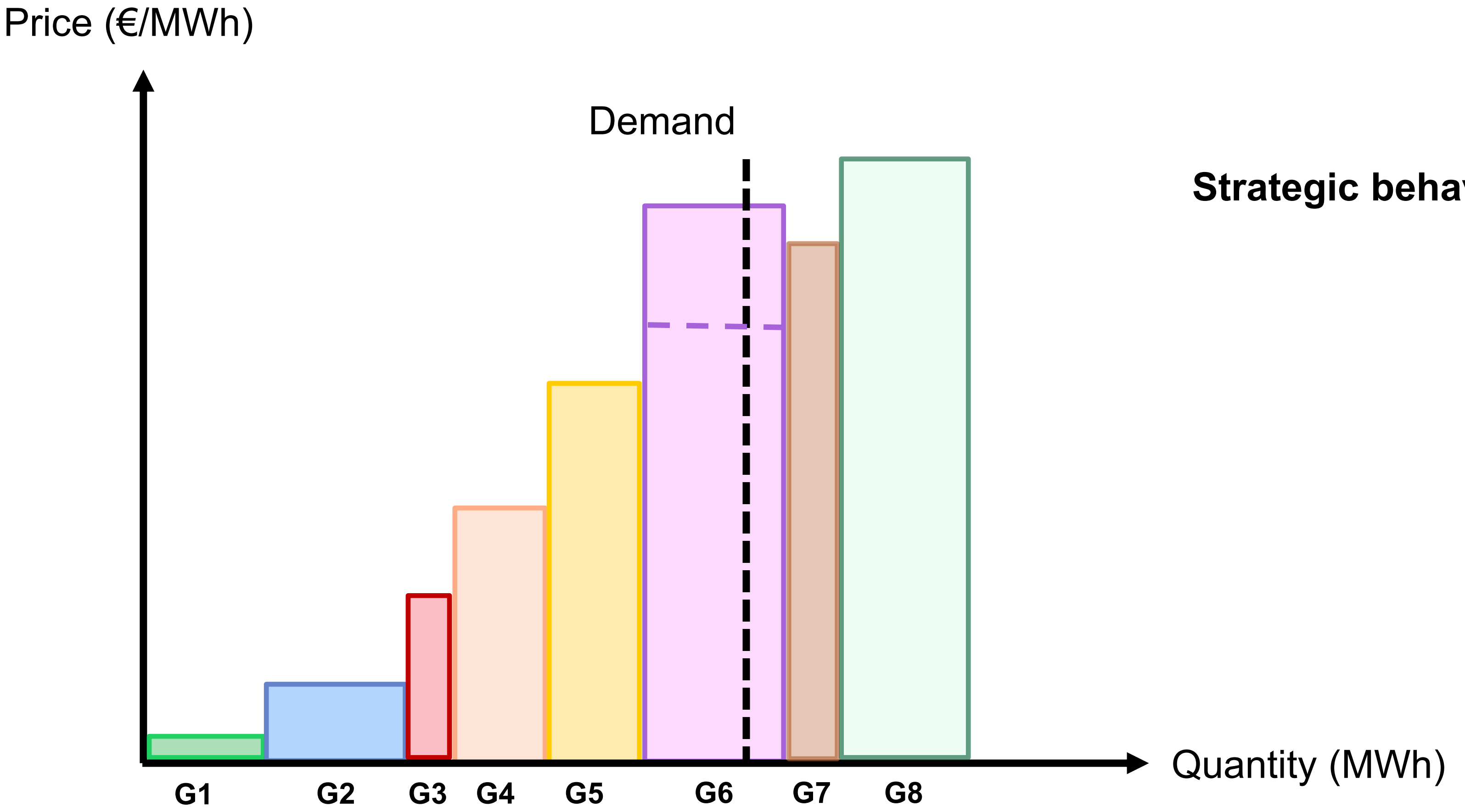
A new system, different generators



Strategic behaviour occurs

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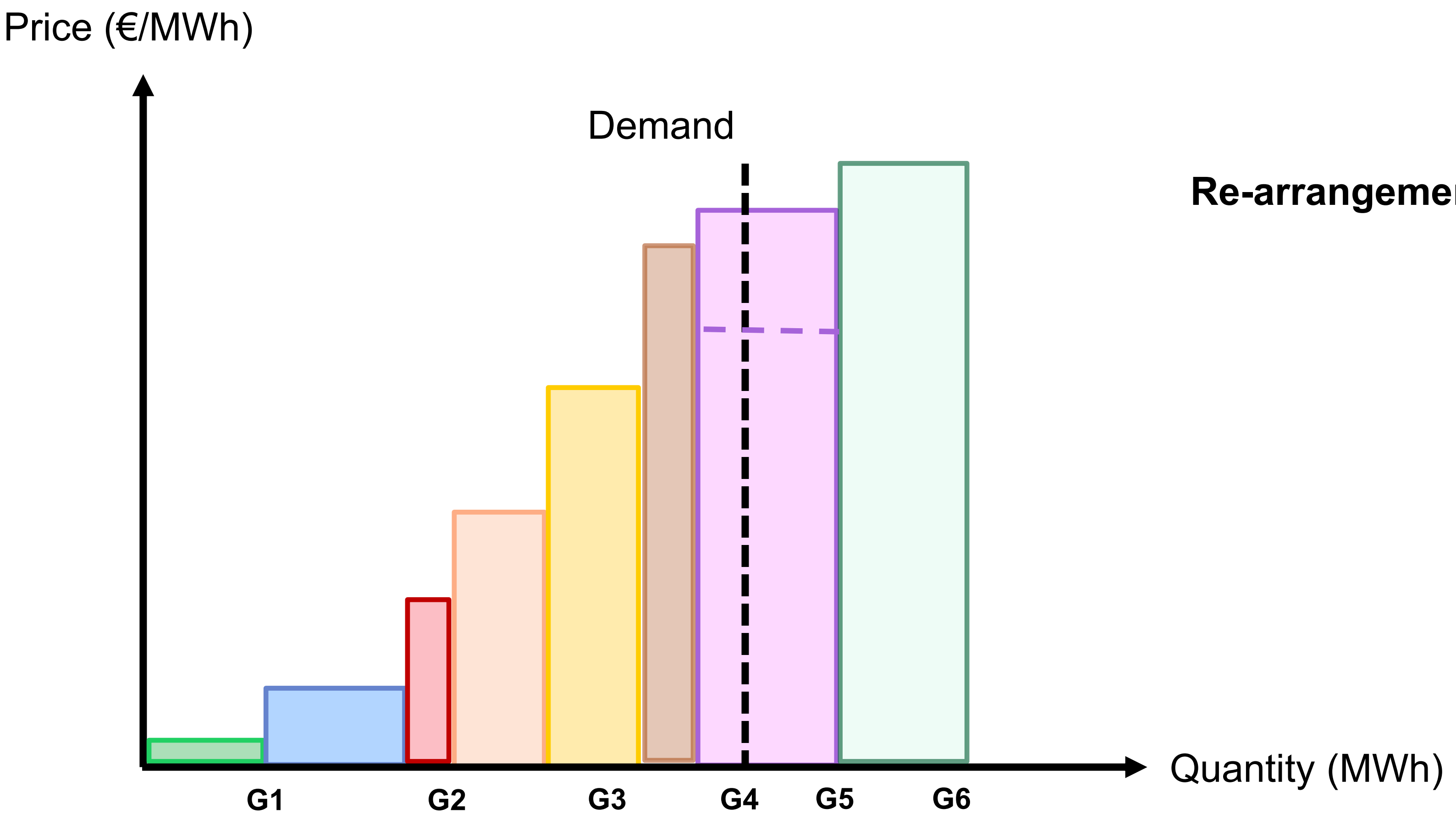
A new system, different generators



Re-arrangement

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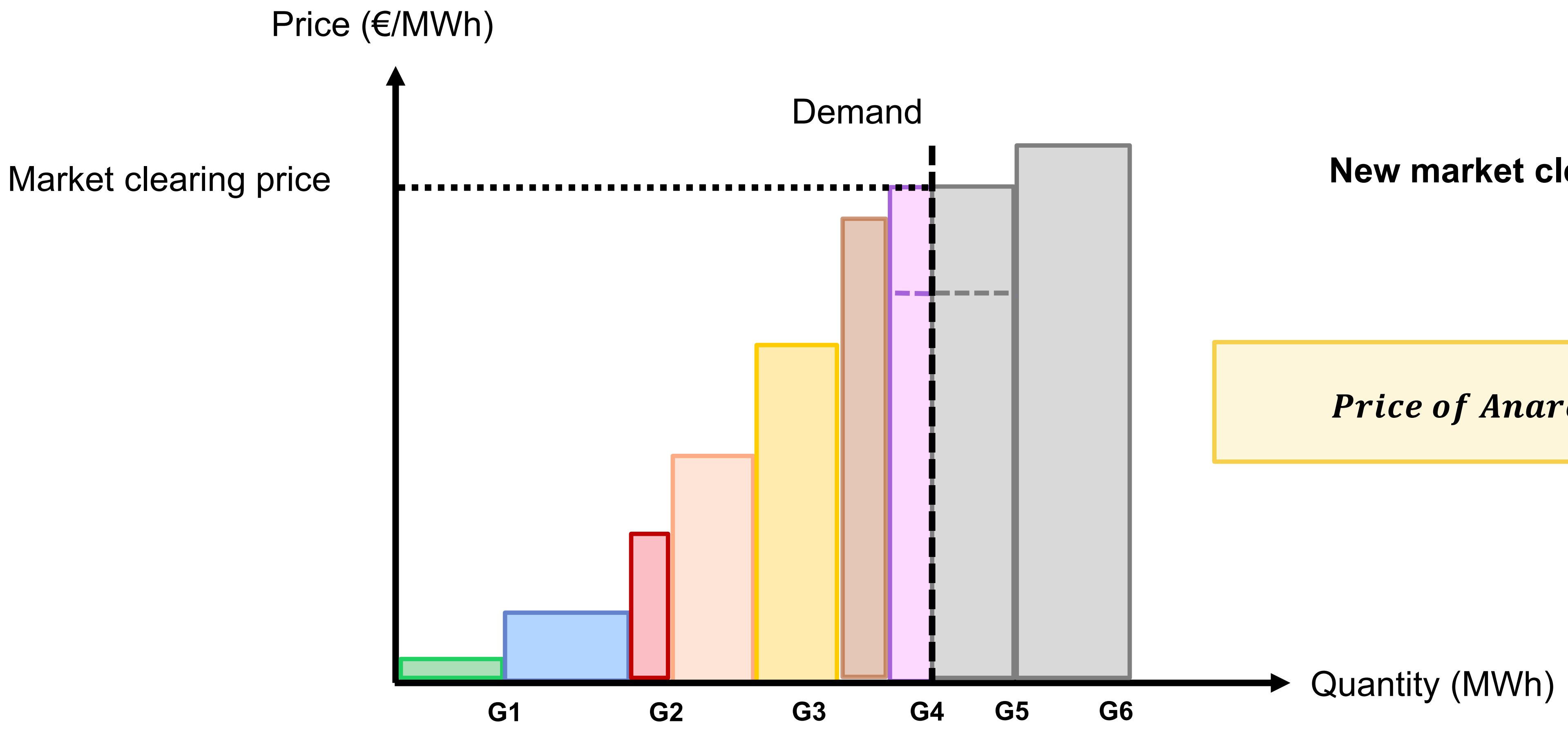
A new system, different generators



New market clearing

$$\boldsymbol{Price\ of\ Anarchy} = \frac{\mathbf{150}}{\mathbf{75}} = \mathbf{2}$$

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The Robust PoA

Robust PoA: worst possible combination of generators that exists

$$\max_{cost\ functions} \{\boldsymbol{Price\ of\ Anarchy}\}$$

$$\left\{ \begin{array}{l} s.t.\ \max_{\theta_n} \{ \sum_t [u_{n,t}\lambda_{n,t} - c_n(u_{n,t})] \} \\ \quad s.t.\, u_{n,t} \in argmax\{ \sum_{m,t} \hat{g}(u_{m,t}; \boldsymbol{\theta}_{m,t}) \} \end{array} \right\} \forall n$$

"Search in the space of possible generators to find the one that **maximizes the inefficiency**"

Johari's upper bound

If enough assumptions are made, there exists a close form **upper bound** for the PoA.

$$\boldsymbol{Price\ of\ Anarchy \leq 1 + \frac{1}{N-2}}$$

$\boldsymbol{N}$: number of generators

When: $\boldsymbol{N} \rightarrow \infty, \ \boldsymbol{PoA} \rightarrow \mathbf{1}$ (perfect competition)

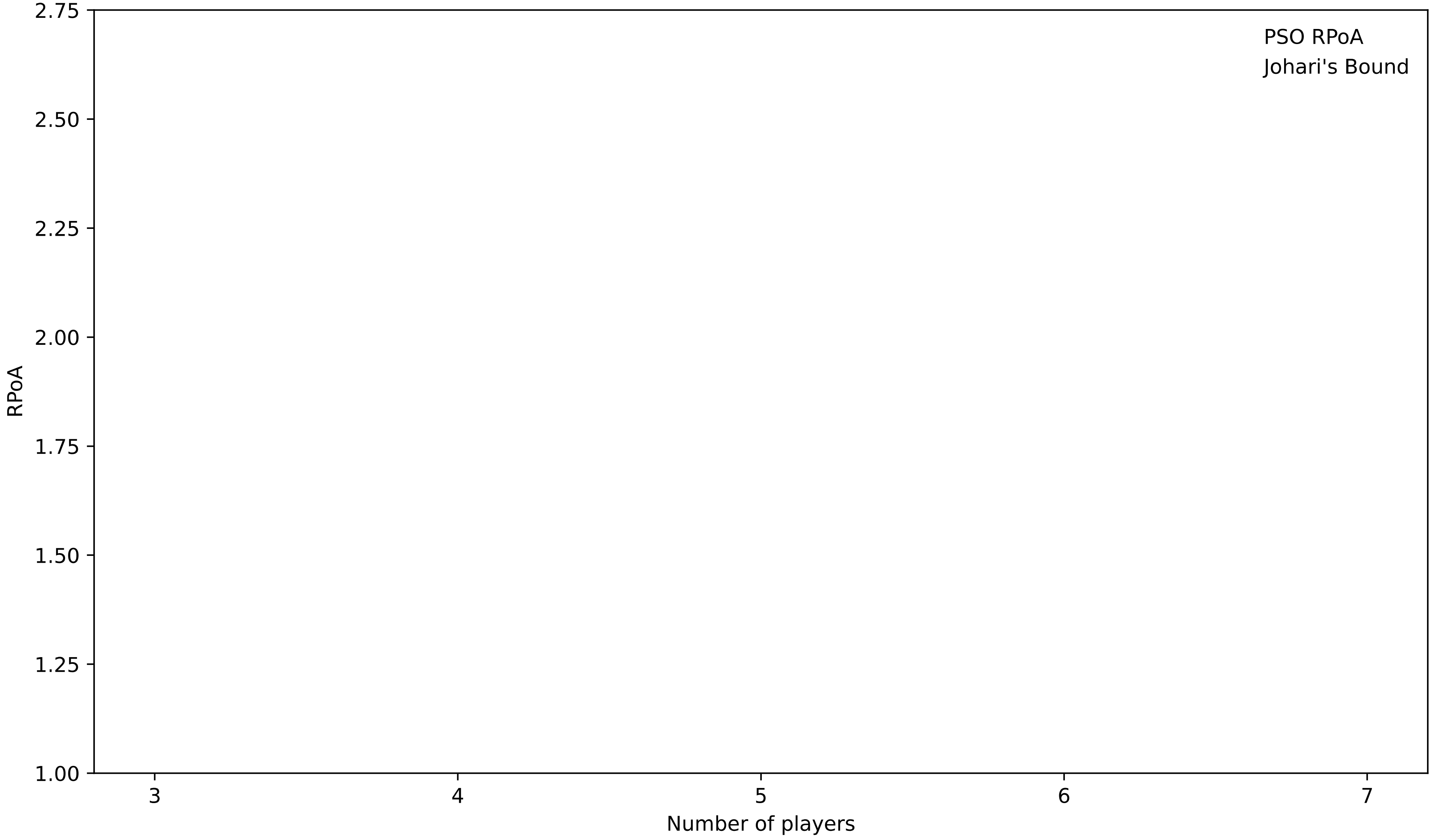
Our contribution

We drop the assumptions to:

- Have a more realistic cost curve for generators

- Approximate better the market behavior

Our results



PSO: **P**article **S**warm **O**ptimization

Exercises

Exercise 1

1. Construct the market clearing problem (merit order curve) as a LP.
   1. Demand is inelastic.
   2. There needs to be at least 3 generators, bid = (price, quantity).
   3. Generators have a minimum and maximum power dispatch
   4. The cost of a generator depends linearly on the power dispatch. Assume: Cost = cx. Where *C* is the cost, *c* the per unit cost (EUR/MWh), and *x* the production (MWh).
   5. There are no start-up costs, no ramp constraints, no minimum up or down time. This keeps the model linear.
   6. Assume the grid to be a copper plate. I.e., there are no power lines.

Exercise 2

1. Construct the dual of exercise 1.
2. Solve the dual.
3. Check that strong duality holds.

Exercise 3

1. Construct the market clearing problem (merit order curve) as a LP.
   1. **Demand bids into the market, bid = (price, quantity).**
   2. **At least 3 consumers.**
   3. There needs to be at least 3 generators, bid = (price, quantity).
   4. Generators have a minimum and maximum power dispatch
   5. The cost of a generator depends linearly on the power dispatch. Assume: Cost = cx. Where *C* is the cost, *c* the per unit cost (EUR/MWh), and *x* the production (MWh).
   6. There are no start-up costs, no ramp constraints, no minimum up or down time. This keeps the model linear.
   7. Assume the grid to be a copper plate. I.e., there are no power lines.

Exercise 4

1. Take the model of exercise 3 and add a model for the power network. Use a linear approximation, e.g., DC-OPF.
2. Keep the amount of network nodes low (e.g., 3 nodes).
3. Populate the grid with enough generators and demands so the market can be cleared (i.e., willingness to pay >= willingness to produce).
4. Report the local marginal price of each bus, i.e., the dual variable of the power balance of each bus.

Duality

Primal (dummy LP with mixed constraints)

$$
\begin{aligned}
&\text{minimize} 2x_1 + x_2 \\
\text{subject to} x_1 + x_2 &\geq 1 && \text{(inequality ')} \\
-x_1 + 2x_2 &\leq 3 && \text{(inequality ')} \\
2x_1 - x_2 &= 0 && \text{(equality)} \\
[2pt] x_1 \geq 0, &\quad x_2 \text{ free.}
\end{aligned}
$$

We'll convert everything to the Lagrangian-friendly "≤ 0" form and add a multiplier for **each** such constraint (including the variable bound for x_1).

- $x_1 + x_2 \geq 1 \iff 1 - x_1 - x_2 \leq 0$ → multiplier $y_1 \geq 0$.
- $-x_1 + 2x_2 \leq 3$ → multiplier $y_2 \geq 0$.
- $2x_1 - x_2 = 0$ → multiplier y_3 **free** (can be any real).
- $x_1 \geq 0 \iff -x_1 \leq 0$ → multiplier $s_1 \geq 0$.
- x_2 is free → **no bound constraint**, hence no s_2.

Duality

Lagrangian

$$\mathcal{L}(x_1, x_2, y_1, y_2, y_3, s_1) = (2x_1 + x_2) \bullet y_1(1 - x_1 - x_2) \bullet y_2(-x_1 + 2x_2 - 3) \bullet y_3(2x_1 - x_2) \bullet s_1(-x_1)$$
$$= \underbrace{(y_1 - 3y_2)}_{\text{constant}} \bullet \underbrace{\left(2 - y_1 - y_2 + 2y_3 - s_1\right)}_{\text{coeff of } x_1} x_1 \bullet \underbrace{\left(1 - y_1 + 2y_2 - y_3\right)}_{\text{coeff of } x_2} x_2.$$

Dual function: $g(y, s) = \inf_{x_1, x_2} \mathcal{L}$

Because we dualized all constraints, the infimum is over **free** x_1, x_2. For the infimum to be finite (not $-\infty$), the coefficients of x_1 and x_2 must vanish (otherwise we could drive them to $\pm\infty$):

$$\begin{aligned} &(\text{stationarity w.r.t. } x_1): && 2 - y_1 - y_2 + 2y_3 - s_1 = 0, \quad s_1 \geq 0, \\ &(\text{stationarity w.r.t. } x_2): && 1 - y_1 + 2y_2 - y_3 = 0 \qquad (\text{no } s_2 \text{ since } x_2 \text{ is free}). \end{aligned}$$

When these hold, the x-terms drop out and
$g(y, s) \;=\; y_1 - 3y_2.$

Duality

Dual problem

Maximize the dual function subject to the multiplier sign rules and the stationarity equations:

- Variables and signs:

$$y_1 \geq 0, \quad y_2 \geq 0, \quad y_3 \in \mathbb{R} \text{ (free)}, \quad s_1 \geq 0.$$

- Equalities/inequalities from stationarity:

$$\begin{aligned} 1 - y_1 + 2y_2 - y_3 = 0 \quad &\iff \quad y_1 - 2y_2 + y_3 = 1, \\ 2 - y_1 - y_2 + 2y_3 - s_1 = 0 \quad &\iff \quad s_1 = 2 - y_1 - y_2 + 2y_3 \geq 0 \\ &\iff \quad y_1 + y_2 - 2y_3 \leq 2. \end{aligned}$$

Putting it all together, the **dual** is:

$$\begin{aligned} &\text{maximize} y_1 - 3y_2 \\ &\text{subject to} y_1 - 2y_2 + y_3 = 1, \\ &\qquad y_1 + y_2 - 2y_3 \leq 2, \\ &\qquad y_1 \geq 0, \ \ y_2 \geq 0, \ \ y_3 \text{ free.} \end{aligned}$$

That's the full Lagrangian-based derivation for a primal with a mix of '$\geq$', '$\leq$', and '$=$' constraints, plus a nonnegative variable and a free variable.

DTU