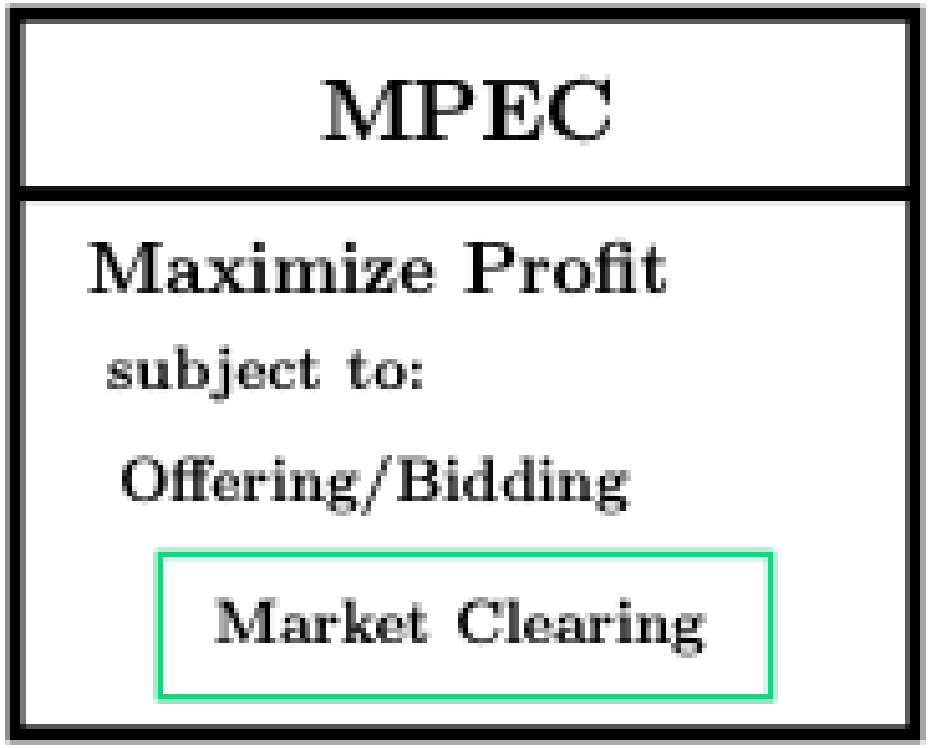
Marcel Solé Àvila, Georgios Tsaousoglou

The Best Response Algorithm Part 2

Technical University of Denmark

The MPEC



https://corporatefinanceinstitute.com/resources/economics/prisoners-dilemma/

The MPEC

$$\min_{X_{\text{UL}},\Xi_{\text{UL}}} \left\{ \sum_{i\in\Omega_j^{\text{I}}} \left(-\lambda P_i^{\text{G}} + c_i^{\text{G}} P_i^{\text{G}} \right) \right\}$$

subject to:

$$\alpha^{\text{min}} \leq \alpha_i^{\text{G}} \leq \alpha^{\text{max}} \quad \forall i \in \Omega_j^{\text{I}},$$

$$\min_{X_{\text{LL}}} \left\{ \sum_{i\in\mathcal{I}} \alpha_i^{\text{G}} P_i^{\text{G}} \right\}$$

subject to:

$$\text{D} - \sum_i P_i^{\text{G}} = 0 : \lambda,$$

$$\text{P}_i^{\text{Gmin}} \leq P_i^{\text{G}} \leq \text{P}_i^{\text{Gmax}} : \mu_i^{\text{G}_{\text{min}}}, \mu_i^{\text{G}_{\text{max}}}, \qquad \forall i \in \mathcal{I}$$

Where:

$$X_{\text{UL}} = \left\{ \alpha_{i\in\Omega_j^{\text{I}}}^{\text{G}}, P_{i\in\mathcal{I}}^{\text{G}} \right\}$$

$$\Xi_{\text{UL}} = \left\{ \lambda_{n,t}, \mu_i^{\text{G}_{\text{min}}}, \mu_i^{\text{G}_{\text{max}}} \right\}$$

$$X_{\text{LL}} = \left\{ P_{i\in\mathcal{I}}^{\text{G}} \right\}$$

→

Other players bids are parameters.
Only our bids are optimized over

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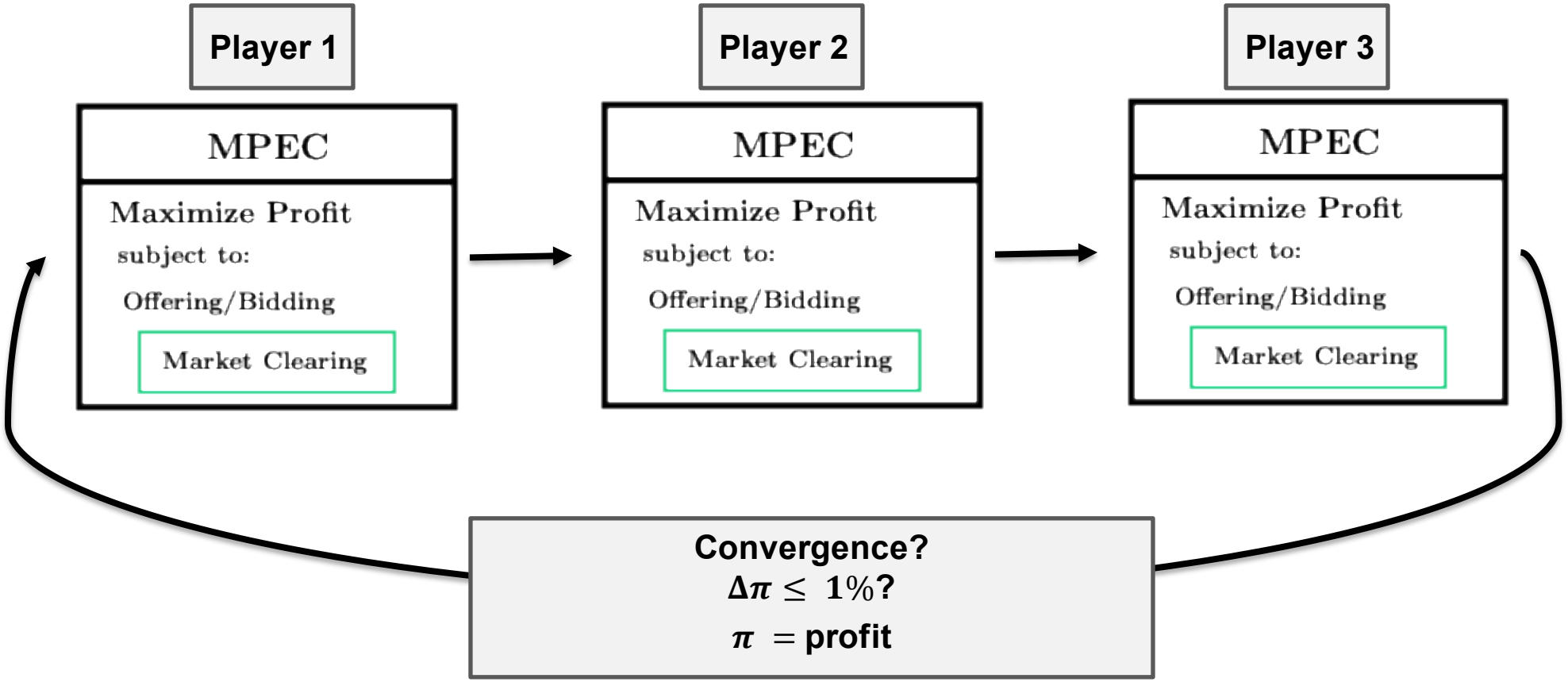
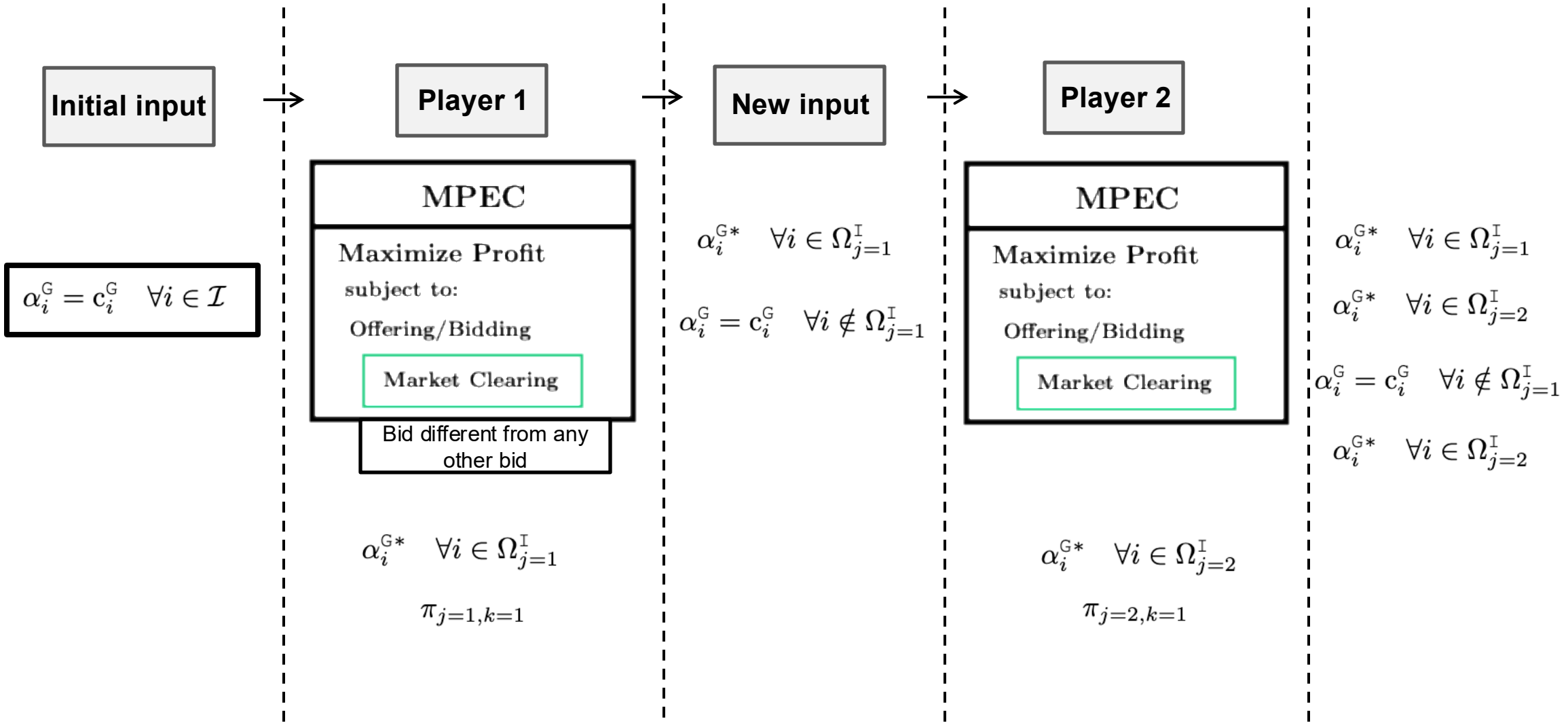
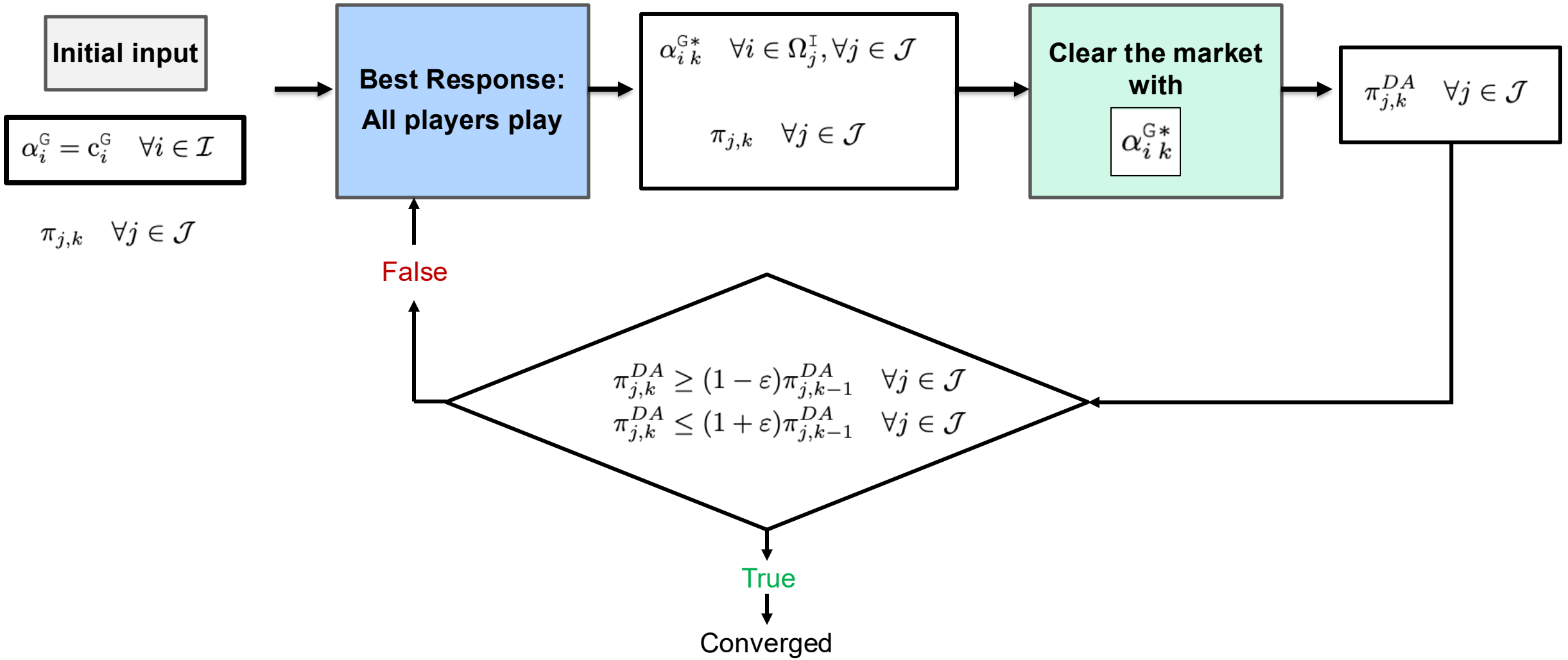
The Best Response Algorithm



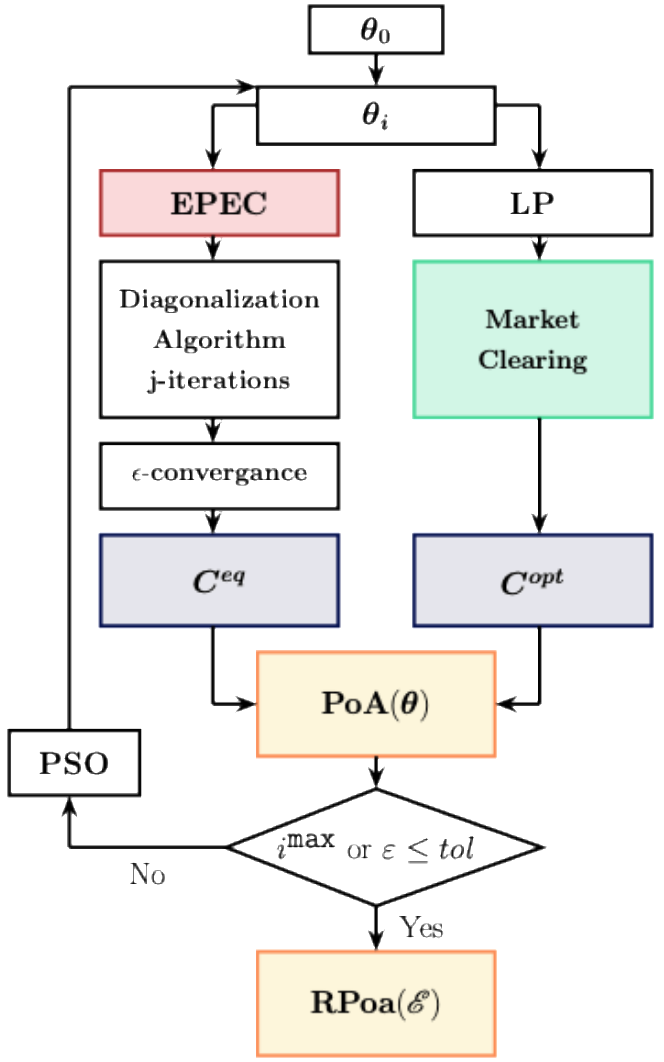
The Best Response Algorithm



The Best Response Algorithm



RPoA over the discretized search space



Diagonalization Algorithm == Best Response

Exercise

- Consider 3 players, with 1 generator each.
- Consider a range of possible cost range(minFuelCost, maxFuelCost) for each player.
- Discretize each range in S segments. Keep it to 2 segments so the search space is not to large.
- For every combination SxSxS run the BR algorithm. Use convergence based on profit or a number of max iterations.
- For every combination clear the market at true costs.
- Calculate the PoA of the SxSxS space.
- Generalize the previous in a python function where you can pass the number of players, the range of fuel cost of each player, and the number of segments for discretatization.
- What do you see when you increase the number of players?
- What about when the discretization is finer?
- Can you even compute all the combinations?

Related literature

- **Complementarity, Not Optimization, is the Language of Markets,** Antonio J. Conejo, Carlos Ruiz
- https://doi.org/10.1109/oajpe.2020.3029134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