KALKULUS 2

(CTI212)

MODUL 7

REVIEW MATERI UTS

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2020

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REVIEW MATERI UTS

Kemampuan akhir yang diharapkan dengan adanya pembelajaran ini adalah mahasiswa dapat memahami dan menyelesaikan soal-soal UTS yang diberikan. Pada modul 7 ini akan dibahas kembali mengenai soal-soal UTS yaitu turunan parsial dan aplikasinya, serta integral lipat dua baik atas daerah persegi Panjang atau bukan, serta integral lipat dua pada koordinat polar.

TURUNAN PARSIAL DAN APLIKASINYA

Contoh

1. Hitunglah z_x dan z_y dari fungsi-fungsi berikut.
   a. $z = 2x^2 - 5xy + y^2$
   b. $z = x^y$
   c. $z = \sin 3x \cos 4y$
   d. $x^3yz + \sin(xyz) = e^{(x-yz)}$
2. Hitunglah $z_{xx}, z_{yy}, z_{xy}, z_{yx}$ dari fungsi-fungsi berikut.
   a. $z = x^2 - 4xy^2 + 3y$
   b. $z = e^{2x-3y}$
3. Tentukan nilai ekstrim dari fungsi $z = 2x^2 - 2xy + y^2 + 5x - 3y$
4. Dapatkan luas permukaan minimum dari kotak yang mempunyai isi 32 m^3.

Penyelesaian.

1a. $z = 2x^2 - 5xy + y^2$

$z_x = 4x - 5y$

$z_y = -5x + 2y$

1b. $z = x^y$

$z_x = yx^{y-1}$

$z_y = x^y \ln x$

1c. $z = \sin 3x \cos 4y$

$$z_x = \sin 3x \frac{d}{dx}(3x) \cos 4y$$
$$= 3 \sin 3x \cos 4y$$
$$z_y = \sin 3x\,(-\sin 4y) \frac{d}{dy}(4y)$$
$$= -4 \sin 3x \sin 4y$$

1d. $x^3yz + \sin(xyz) = e^{(x-yz)}$

Diturunkan terhadap variabel x.

$$u'v + uv' + \cos(xyz) \frac{\partial}{\partial x}(xyz) = e^{(x-yz)} - ye^{(x-yz)} \frac{\partial z}{\partial x}$$
$$3x^2yz + x^3y\frac{\partial z}{\partial x} + \cos(xyz)\ (u'v + uv') = e^{(x-yz)} - ye^{(x-yz)} \frac{\partial z}{\partial x}$$
$$3x^2yz + x^3y\frac{\partial z}{\partial x} + \cos(xyz)\ \left(yz + xy\frac{\partial z}{\partial x}\right) = e^{(x-yz)} - ye^{(x-yz)} \frac{\partial z}{\partial x}$$
$$3x^2yz + x^3y\frac{\partial z}{\partial x} + yz\cos(xyz) + xy\cos(xyz)\frac{\partial z}{\partial x} = e^{(x-yz)} - ye^{(x-yz)} \frac{\partial z}{\partial x}$$
$$x^3y\frac{\partial z}{\partial x} + xy\cos(xyz)\frac{\partial z}{\partial x} + ye^{(x-yz)}\frac{\partial z}{\partial x} = e^{(x-yz)} - 3x^2yz - yz\cos(xyz)$$
$$\left(x^3y + xy\cos(xyz) + ye^{(x-yz)}\right)\frac{\partial z}{\partial x} = e^{(x-yz)} - 3x^2yz - yz\cos(xyz)$$
$$\frac{\partial z}{\partial x} = \frac{e^{(x-yz)} - 3x^2yz - yz\cos(xyz)}{x^3y + xy\cos(xyz) + ye^{(x-yz)}}$$

Diturunkan terhadap variabel y.

$$u'v + uv' + \cos(xyz) \frac{\partial}{\partial y}(xyz) = -ze^{(x-yz)} - ye^{(x-yz)} \frac{\partial z}{\partial y}$$
$$x^3z + x^3y\frac{\partial z}{\partial y} + \cos(xyz)\ (u'v + uv') = -ze^{(x-yz)} - ye^{(x-yz)} \frac{\partial z}{\partial y}$$
$$x^3z + x^3y\frac{\partial z}{\partial y} + \cos(xyz)\ \left(xz + xy\frac{\partial z}{\partial y}\right) = -ze^{(x-yz)} - ye^{(x-yz)} \frac{\partial z}{\partial y}$$
$$x^3z + x^3y\frac{\partial z}{\partial y} + xz\cos(xyz) + xy\cos(xyz)\frac{\partial z}{\partial y} = -ze^{(x-yz)} - ye^{(x-yz)} \frac{\partial z}{\partial y}$$
$$x^3y\frac{\partial z}{\partial y} + xy\cos(xyz)\frac{\partial z}{\partial y} + ye^{(x-yz)}\frac{\partial z}{\partial y} = -ze^{(x-yz)} - x^3z - xz\cos(xyz)$$

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$$\left(x^3y + xy\cos(xyz) + ye^{(x-yz)}\right)\frac{\partial z}{\partial y} = -ze^{(x-yz)} - x^3z - xz\cos(xyz)$$

$$\frac{\partial z}{\partial y} = \frac{-ze^{(x-yz)} - x^3z - xz\cos(xyz)}{x^3y + xy\cos(xyz) + ye^{(x-yz)}}$$

$$\frac{\partial z}{\partial y} = \frac{-z\left(e^{(x-yz)} + x^3 + x\cos(xyz)\right)}{y(x^3 + x\cos(xyz) + e^{(x-yz)})}$$

$$\frac{\partial z}{\partial y} = \frac{-z\left(e^{(x-yz)} + x^3 + x\cos(xyz)\right)}{y(e^{(x-yz)} + x^3 + x\cos(xyz))}$$

$$\frac{\partial z}{\partial y} = \frac{-z}{y}$$

2a. $z = x^2 - 4xy^2 + 3y$

$z_x = 2x - 4y^2$

$z_y = -4x\ 2y + 2 = -8xy + 2$

$z_{xx} = 2$

$z_{yy} = -8x$

$z_{xy} = -8y$

$z_{yx} = -8y$

2b. $z = e^{2x-3y}$

$z_x = 2e^{2x-3y}$

$z_y = -3e^{2x-3y}$

$z_{xx} = 2(2)e^{2x-3y} = 4e^{2x-3y}$

$z_{yy} = -3(-3)e^{2x-3y} = 9e^{2x-3y}$

$z_{xy} = 2(-3)e^{2x-3y} = -6e^{2x-3y}$

$z_{yx} = -3(2)e^{2x-3y} = -6e^{2x-3y}$

3. Tentukan nilai ekstrim dari fungsi $z = 2x^2 - 2xy + y^2 + 5x - 3y$

Tentukan titik kritis.

- Tidak mempunyai titik batas
- Titik stasioner

$$z_x = 0$$
$$4x - 2y + 5 = 0$$

dan

$$z_y = 0$$
$$-2x + 2y - 3 = 0$$
$$y = \frac{2x+3}{2}$$

Substitusi $y = \frac{2x+3}{2}$ ke $4x - 2y + 5 = 0$, sehingga diperoleh

$$4x - 2y + 5 = 0$$
$$4x - 2\left(\frac{2x+3}{2}\right) + 5 = 0$$
$$4x - 2x - 3 + 5 = 0$$
$$2x + 2 = 0$$
$$x = -\frac{2}{2}$$
$$x = -1$$

Karena $x = -1$ maka

$$y = \frac{2x+3}{2}$$
$$y = \frac{2(-1)+3}{2}$$
$$y = \frac{1}{2}$$

Jadi, titik stasionernya adalah $\left(-1, \frac{1}{2}\right)$.

- Tidak mempunyai titik singular

Jadi, titik kritisnya adalah $\left(-1, \frac{1}{2}\right)$.

Tentukan jenis titik kritisnya.

$z = 2x^2 - 2xy + y^2 + 5x - 3y$

$z_x = 4x - 2y + 5$

$z_y = -2x + 2y - 3$

$z_{xx} = 4$

$z_{yy} = 2$

$z_{xy} = -2$

sehingga

$$
\begin{aligned}
D &= z_{xx}(x_0, y_0) z_{yy}(x_0, y_0) - z_{xy}^2(x_0, y_0) \\
&= 4(2) - (-2)^2 \\
&= 8 - 4 \\
&= 4
\end{aligned}
$$

Jadi, $D\left(-1, \frac{1}{2}\right) > 0$.

Karena $D > 0$ dan $z_{xx} > 0$ maka $\left(-1, \frac{1}{2}\right)$ adalah titik ekstrim minimum lokal dengan nilai minimumnya adalah

$$
\begin{aligned}
z &= 2x^2 - 2xy + y^2 + 5x - 3y \\
&= 2(-1)^2 - 2(-1)\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 + 5(-1) - 3\left(\frac{1}{2}\right) \\
&= 2 + 1 + \frac{1}{4} - 5 - \frac{3}{2} \\
&= -2 + \frac{1}{4} - \frac{3}{2} \\
&= \frac{-8 + 1 - 6}{4} \\
&= \frac{-13}{4}
\end{aligned}
$$

4. Dapatkan luas permukaan minimum dari kotak yang mempunyai isi 32 m^3.

Misal:

$p =$ panjang

$l =$ lebar

$t =$ tinggi

Diketahui:

$$
\begin{aligned}
V &= 32 \\
plt &= 32 \\
t &= \frac{32}{pl}
\end{aligned}
$$

Ditanya:

$$L = 2(pl + pt + lt)$$

Penyelesaian.

$$L = 2(pl + pt + lt)$$

$$L = 2\left(pl + p\frac{32}{pl} + l\frac{32}{pl}\right)$$

$$L = 2\left(pl + \frac{32}{l} + \frac{32}{p}\right)$$

$$L = 2pl + \frac{64}{l} + \frac{64}{p}$$

Tentukan titik kritis.

- Titik batas

$$p > 0$$

$$l > 0$$

$$t > 0$$

- Titik stasioner

$$L_p = 0$$

$$2l - \frac{64}{p^2} = 0$$

$$l = \frac{32}{p^2}$$

dan

$$L_l = 0$$

$$2p - \frac{64}{l^2} = 0$$

Substitusi $l = \frac{32}{p^2}$ ke $2p - \frac{64}{l^2} = 0$, sehingga diperoleh

$$2p - \frac{64}{l^2} = 0$$

$$2p - \frac{64}{\left(\frac{32}{p^2}\right)^2} = 0$$

$$2p - \frac{64}{\frac{32^2}{p^4}} = 0$$

$$2p - \frac{p^4}{16} = 0$$

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$$32p - p^4 = 0$$
$$p(32 - p^3) = 0$$
$$p = 0, p = 2\sqrt[3]{4}$$

Karena $p > 0$ maka $p = 2\sqrt[3]{4}$ dan

$$l = \frac{32}{p^2}$$
$$l = \frac{32}{\left(2\sqrt[3]{4}\right)^2}$$
$$l = \frac{32}{8\sqrt[3]{2}}$$
$$l = 2\sqrt[3]{4}$$

Jadi, titik stasionernya adalah $\left(2\sqrt[3]{4}, 2\sqrt[3]{4}\right)$.

- Titik singular

  Titik singularnya adalah $p = 0$ dan $l = 0$. Karena $p > 0$ dan $l > 0$ maka fungi z tidak mempunyai titik singular.

Jadi, titik kritisnya adalah $\left(2\sqrt[3]{4}, 2\sqrt[3]{4}\right)$.

Tentukan jenis titik kritisnya.

$L = 2pl + \frac{64}{l} + \frac{64}{p}$

$L_p = 2l - \frac{64}{p^2}$

$L_l = 2p - \frac{64}{l^2}$

$L_{pp} = \frac{128}{p^3}$

$L_{ll} = \frac{128}{l^3}$

$z_{pl} = 2$

sehingga

$$D = L_{pp}(x_0, y_0)L_{ll}(x_0, y_0) - L_{pl}^2(x_0, y_0)$$
$$= \frac{128}{p^3}\frac{128}{l^3} - 2$$
$$= \frac{2^7}{(pl)^3} - 2$$

Jadi,

$$D\left(2\sqrt[3]{4}, 2\sqrt[3]{4}\right) = \frac{2^7}{(pl)^3} - 2$$

$$= \frac{2^7}{\left(2\sqrt[3]{4}\,2\sqrt[3]{4}\right)^3} - 2$$

$$= \frac{2^7}{2^5} - 2$$

$$= 4 - 2$$

$$= 2$$

Karena $D > 0$ maka cek nilai dari L_{pp}.

$$L_{pp} = \frac{128}{p^3}$$

$$L_{pp} = \frac{128}{\left(2\sqrt[3]{4}\right)^3}$$

$$L_{pp} = \frac{128}{8(4)}$$

$$L_{pp} = 4$$

Karena $D > 0$ dan $L_{pp} > 0$ maka $\left(2\sqrt[3]{4}, 2\sqrt[3]{4}\right)$ adalah titik ekstrim minimum lokal dengan luas permukaan minimumnya adalah

$$L = 2pl + \frac{64}{l} + \frac{64}{p}$$

$$= 2\left(2\sqrt[3]{4}\right)\left(2\sqrt[3]{4}\right) + \frac{64}{2\sqrt[3]{4}} + \frac{64}{2\sqrt[3]{4}}$$

$$= 8(4)^{\frac{2}{3}} + \frac{32}{\sqrt[3]{4}} + \frac{32}{\sqrt[3]{4}}$$

$$= 8(4)^{\frac{2}{3}} + \frac{64}{\sqrt[3]{4}}$$

$$= 8(4)^{\frac{1}{3}} + \frac{64}{4^{\frac{1}{3}}} \frac{4^{\frac{2}{3}}}{4^{\frac{2}{3}}}$$

$$= 8(4)^{\frac{2}{3}} + \frac{64}{4} 4^{\frac{2}{3}}$$

$$= 8(4)^{\frac{2}{3}} + 16(4)^{\frac{2}{3}}$$

$$= 24\sqrt[3]{16}$$

INTEGRAL LIPAT DUA

Hitunglah:

1. $\int_{-1}^{4}\int_{1}^{2}(x+y^2)\,dy\,dx$

   Jawab.

$$\int_{-1}^{4}\int_{1}^{2}(x+y^2)\,dy\,dx = \int_{-1}^{4}\left(\int_{1}^{2}(x+y^2)\,dy\right)dx$$

$$= \int_{-1}^{4}\left(xy+\frac{1}{3}y^3\right)_1^2 dx$$

$$= \int_{-1}^{4}\left(2x+\frac{1}{3}2^3-\left(x+\frac{1}{3}\right)\right)dx$$

$$= \int_{-1}^{4}\left(2x+\frac{8}{3}-x-\frac{1}{3}\right)dx$$

$$= \int_{-1}^{4}\left(x+\frac{7}{3}\right)dx$$

$$= \left(\frac{1}{2}x^2+\frac{7}{3}x\right)_{-1}^{4}$$

$$= \frac{1}{2}4^2+\frac{7}{3}4-\left(\frac{1}{2}(-1)^2+\frac{7}{3}(-1)\right)$$

$$= 8+\frac{28}{3}-\left(\frac{1}{2}-\frac{7}{3}\right)$$

$$= 8+\frac{28}{3}-\frac{1}{2}+\frac{7}{3}$$

$$= 8+\frac{35}{3}-\frac{1}{2}$$

$$= \frac{48+70-3}{6}$$

$$= \frac{115}{6}$$

2. $\int_0^{\pi} \int_0^{1} x \sin y \, dx \, dy$

Jawab.

$$\int_0^{\pi} \int_0^{1} x \sin y \, dx \, dy = \int_0^{\pi} \left(\int_0^{1} x \sin y \, dx \right) dy$$

$$= \int_0^{\pi} \left(\frac{1}{2} x^2 \sin y \right)_0^1 dy$$

$$= \int_0^{\pi} \left(\frac{1}{2} (1^2 - 0^2) \sin y \right) dy$$

$$= \int_0^{\pi} \left(\frac{1}{2} \sin y \right) dy$$

$$= \left(-\frac{1}{2} \cos y \right)_0^{\pi}$$

$$= -\frac{1}{2} (\cos \pi - \cos 0)$$

$$= -\frac{1}{2} (-1 - 1)$$

$$= 1$$

3. $\int_1^{2} \int_0^{x-1} y \, dy \, dx$

Jawab.

$$\int_1^{2} \int_0^{x-1} y \, dy \, dx = \int_1^{2} \left(\int_0^{x-1} y \, dy \right) dx$$

$$= \int_1^{2} \left(\frac{1}{2} y^2 \right)_0^{x-1} dx$$

$$= \int_1^{2} \left(\frac{1}{2} ((x-1)^2 - 0^2) \right) dx$$

$$= \int_1^{2} \left(\frac{1}{2} (x^2 - 2x + 1) \right) dx$$

$$= \int_1^{2} \left(\frac{1}{2} x^2 - x + \frac{1}{2} \right) dx$$

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$$= \left(\frac{1}{6}x^3 - \frac{1}{2}x^2 + \frac{1}{2}x\right)_1^2$$

$$= \frac{1}{6}2^3 - \frac{1}{2}2^2 + \frac{1}{2}2 - \left(\frac{1}{6} - \frac{1}{2} + \frac{1}{2}\right)$$

$$= \frac{8}{6} - 2 + 1 - \frac{1}{6}$$

$$= \frac{7}{6} - 1$$

$$= \frac{1}{6}$$

4. $\int_1^3 \int_{-y}^{2y} xe^{y^3}\, dx\, dy$

Jawab.

$$\int_1^3 \int_{-y}^{2y} xe^{y^3}\, dx\, dy = \int_1^3 \left(\int_{-y}^{2y} xe^{y^3}\, dx \right) dy$$

$$= \int_1^3 \left(\frac{1}{2}x^2 e^{y^3}\right)_{-y}^{2y} dy$$

$$= \int_1^3 \left(\frac{1}{2}((2y)^2 - (-y)^2)\right) e^{y^3}\, dy$$

$$= \int_1^3 \left(\frac{1}{2}(4y^2 - y^2)\right) e^{y^3}\, dy$$

$$= \int_1^3 \left(\frac{3}{2}y^2 e^{y^3}\right) dy$$

Dengan menggunakan metode substitusi, Misal:

$u = y^3$

$\frac{du}{dy} = 3y^2$ maka $dy = \frac{du}{3y^2}$

$$\int_1^3 \left(\frac{3}{2}y^2 e^{y^3}\right) dy = \int_1^{27} \frac{3}{2}y^2 e^u \frac{du}{3y^2}$$

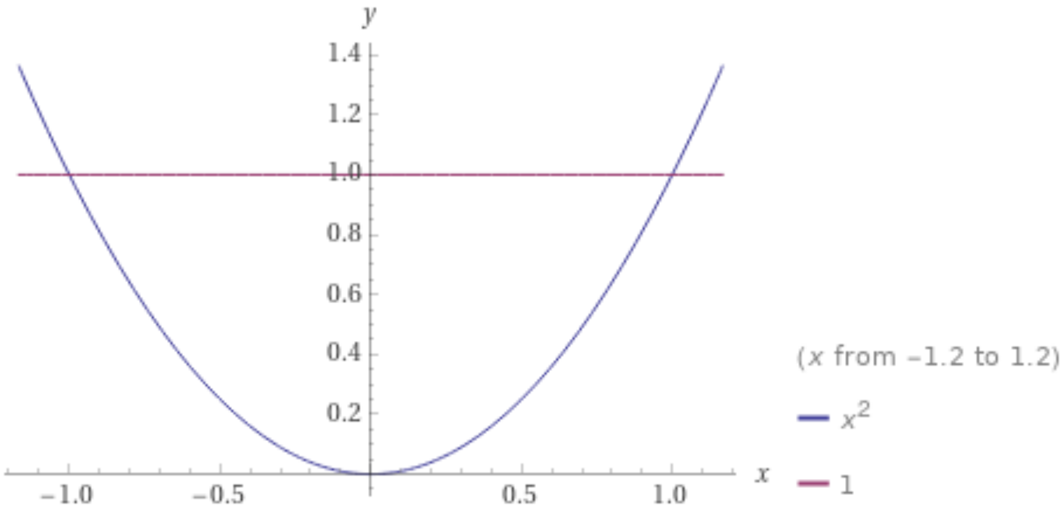
$$= \int_1^{27} \frac{1}{2}e^u du$$

$$= \left(\frac{1}{2}e^u\right)_1^{27}$$

$$= \frac{1}{2}(e^{27} - e^1)$$

5. $\iint_R xydA$, dimana R adalah daerah antara $y = x^2$ dan $y = 1$.

   Jawab.



Titik potong antara garis dan kurva adalah

$$y = y$$

$$x^2 = 1$$

$$x = \pm 1$$

Berdasarkan gambar diketahui

Batas kanan $x = 1$

Batas kiri $x = -1$

Batas atas $y = 1$

Batas bawah $y = x^2$

Sehingga

$$\int_{-1}^{1}\int_{y=x^2}^{y=1} xy\,dy\,dx = \int_{-1}^{1}\left(\frac{1}{2}xy^2\right)_{x^2}^{1} dx$$

$$= \int_{-1}^{1}\frac{1}{2}x(1^2 - (x^2)^2)\,dx$$

$$= \int_{-1}^{1} \frac{1}{2} x(1 - x^4)\, dx$$

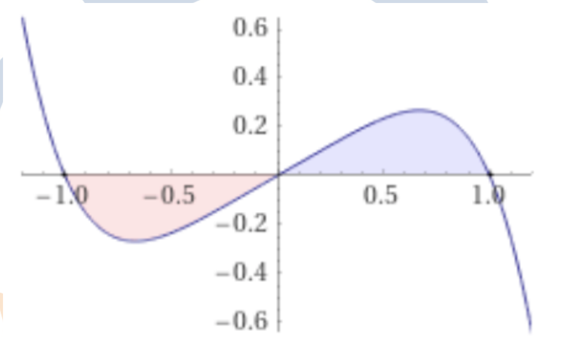
$$= \int_{-1}^{1} \left(\frac{1}{2}x - \frac{1}{2}x^5\right) dx$$

$$= \left(\frac{1}{4}x^4 - \frac{1}{12}x^6\right)_{-1}^{1}$$

$$= \frac{1}{4} - \frac{1}{12} - \left(\frac{1}{4} - \frac{1}{12}\right)$$

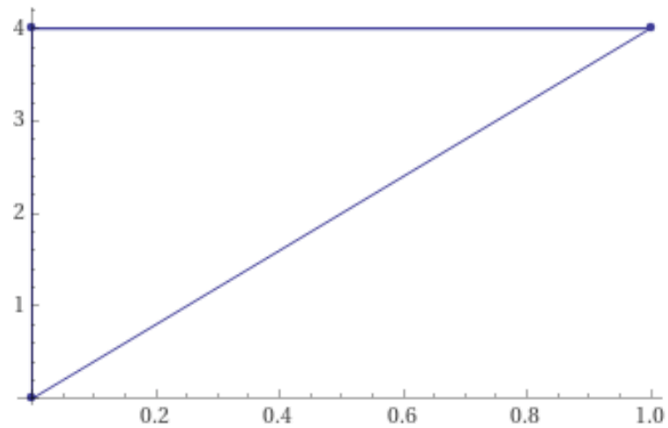
$$= 0$$

Berikut diberikan visualisasi dari $\int_{-1}^{1} \left(\frac{1}{2}x - \frac{1}{2}x^5\right) dx$.



6. $\iint_R (x + y)dA$, dimana R adalah daerah segitiga dengan titik-titik sudutnya adalah $(0,0)$, $(0,4)$, and $(1,4)$.

Jawab.



Berdasarkan gambar diketahui

Batas kanan $x = 1$

Batas kiri $x = 0$

Batas atas $y = 4$

Batas bawah $y = 4x$

Sehingga

$$\int_0^1 \int_{y=4x}^{y=4} (x+y)\,dy\,dx = \int_0^1 \left(xy + \frac{1}{2}y^2\right)_{4x}^{4} dx$$

$$= \int_0^1 \left(4x + \frac{1}{2}4^2 - \left(x(4x) + \frac{1}{2}(4x)^2\right)\right) dx$$

$$= \int_0^1 \left(4x + 8 - (4x^2 + 8x^2)\right) dx$$

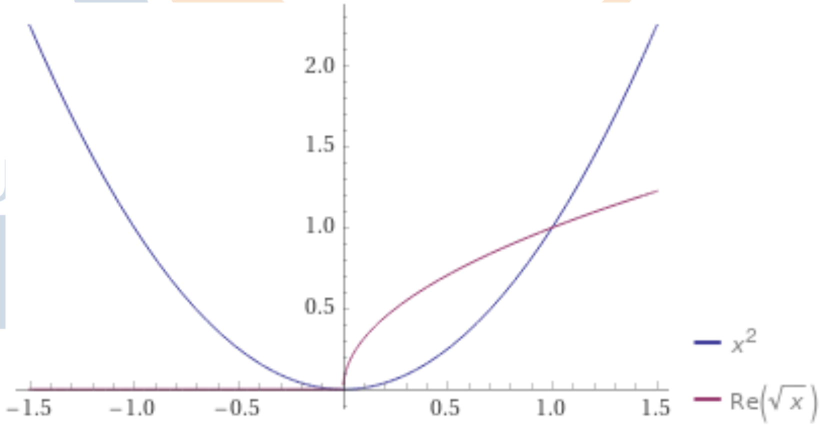
$$= \int_0^1 (-12x^2 + 4x + 8)\,dx$$

$$= (-4x^3 + 2x^2 + 8x)_0^1$$

$$= -4 + 2 + 8$$

$$= 6$$

7. $\iint_R (x^2 + 2y)dA$, dimana R adalah daerah antara $y = x^2$ dan $y = \sqrt{x}$



Titik potong antara garis dan kurva adalah

$$y = y$$

$$x^2 = \sqrt{x}$$

$$x^4 = x$$

$$x^4 - x = 0$$

$$x(x^3 - 1) = 0$$

$$x = 0, x = 1$$

Berdasarkan gambar diketahui

Batas kanan $x = 1$

Batas kiri $x = 0$

Batas atas $y = \sqrt{x}$

Batas bawah $y = x^2$

Sehingga

$$\int_0^1 \int_{y=x^2}^{y=\sqrt{x}} (x^2 + 2y)\, dy\, dx = \int_0^1 (x^2 y + y^2)_{x^2}^{\sqrt{x}}\, dx$$

$$= \int_0^1 \left(x^2(\sqrt{x}) + \sqrt{x}^2 - (x^2(x^2) + (x^2)^2) \right) dx$$

$$= \int_0^1 \left(x^{\frac{5}{2}} + x - (x^4 + x^4) \right) dx$$

$$= \int_0^1 \left(x^{\frac{5}{2}} + x - 2x^4 \right) dx$$

$$= \int_0^1 \left(-2x^4 + x^{\frac{5}{2}} + x \right) dx$$

$$= \left(-\frac{2}{5}x^5 + \frac{2}{7}x^{\frac{7}{2}} + \frac{1}{2}x^2 \right)_0^1$$

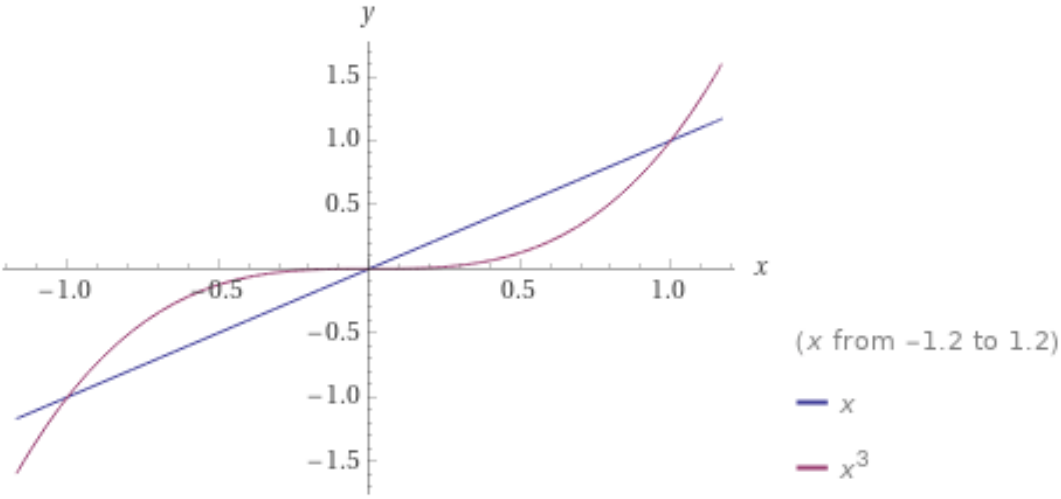
$$= -\frac{2}{5} + \frac{2}{7} + \frac{1}{2} - 0$$

$$= \frac{-28 + 20 + 35}{70}$$

$$= \frac{27}{70}$$

8. $\iint_R x\, dA$, dimana R adalah daerah antara $y = x$ dan $y = x^3$.

Jawab.



Titik potong antara garis dan kurva adalah

$$y = y$$
$$x^3 = x$$
$$x^3 - x = 0$$
$$x(x^2 - 1) = 0$$
$$x(x+1)(x-1) = 0$$
$$x = 0, x = -1, x = 1$$

Karena ada 2 daerah penyelesaian, maka daerahnya dibagi menjadi 2 bagian. Berdasarkan gambar diketahui batasan-batasan daerah 1 sebagai berikut.

Batas kanan $x = -1$

Batas kiri $x = 0$

Batas atas $y = x^3$

Batas bawah $y = x$

Dan batasan-batasan daerah 2 sebagai berikut.

Batas kanan $x = 0$

Batas kiri $x = 1$

Batas atas $y = x$

Batas bawah $y = x^3$

Sehingga:

$$\int_{-1}^{0}\int_{y=x}^{y=x^3} x\,dy\,dx + \int_{0}^{1}\int_{y=x^3}^{y=x} x\,dy\,dx = \int_{-1}^{0} (xy)_{x}^{x^3}\,dx + \int_{0}^{1} (xy)_{x^3}^{x}\,dx$$

$$= \int_{-1}^{0} \big(x(x^3 - x)\big)\,dx + \int_{0}^{1} \big(x(x - x^3)\big)\,dx$$

$$= \int_{-1}^{0} (x^4 - x^2)\,dx + \int_{0}^{1} (x^2 - x^4)\,dx$$

$$= \left(\frac{1}{5}x^5 - \frac{1}{3}x^3\right)_{-1}^{0} + \left(\frac{1}{3}x^3 - \frac{1}{5}x^5\right)_{0}^{1}$$

$$= 0 - \left(-\frac{1}{5} + \frac{1}{3}\right) + \frac{1}{3} - \frac{1}{5} - 0$$

$$= \frac{1}{5} - \frac{1}{3} + \frac{1}{3} - \frac{1}{5}$$

$$= 0$$

INTEGRAL LIPAT DUA PADA KOORDINAT POLAR

Hitunglah

1. $\int_0^{\pi}\int_0^{1-\cos\theta} r\sin\theta\,dr\,d\theta$
2. $\iint_R \sqrt{4 - x^2 - y^2}\,dA$, Dengan R adalah daerah yang dibatasi oleh $x^2 + y^2 = 4$, $\mathrm{y} = 0$, dan $y = x$ di kuadran I.
3. $\iint_R y\,dA$, dimana R adalah daerah yang berada di kuadran polar pertama yang dibatasi oleh $x^2 + y^2 = 4$ dan $x^2 + y^2 = 1$

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Penyelesaian.

Nomor 1.

$$\int_0^{\pi}\int_0^{1-\cos\theta} r\sin\theta\, dr\, d\theta = \int_0^{\pi}\left(\int_0^{1-\cos\theta} r\sin\theta\, dr\right)d\theta$$

$$= \int_0^{\pi}\left(\frac{1}{2}r^2\sin\theta\right)_0^{1-\cos\theta} d\theta$$

$$= \int_0^{\pi}\left(\frac{1}{2}\sin\theta\,((1-\cos\theta)^2-0^2)\right)d\theta$$

$$= \int_0^{\pi}\left(\frac{1}{2}\sin\theta\,(1-2\cos\theta+\cos^2\theta)\right)d\theta$$

$$= \int_0^{\pi}\left(\frac{1}{2}\sin\theta-\sin\theta\cos\theta+\frac{1}{2}\sin\theta\cos^2\theta\right)d\theta$$

$$= \left(-\frac{1}{2}\cos\theta-\frac{1}{2}\sin^2\theta-\frac{1}{6}\cos^3\theta\right)_0^{\pi}$$

$$= -\frac{1}{2}\cos\pi-\frac{1}{2}\sin^2\pi-\frac{1}{6}\cos^3\pi-\left(-\frac{1}{2}\cos 0-\frac{1}{2}\sin^2 0-\frac{1}{6}\cos^3 0\right)$$

$$= -\frac{1}{2}(-1)-\frac{1}{2}(0)-\frac{1}{6}(-1)^3-\left(-\frac{1}{2}(1)-\frac{1}{2}(0)-\frac{1}{6}(1)^3\right)$$

$$= \frac{1}{2}-0+\frac{1}{6}-\left(-\frac{1}{2}-0-\frac{1}{6}\right)$$

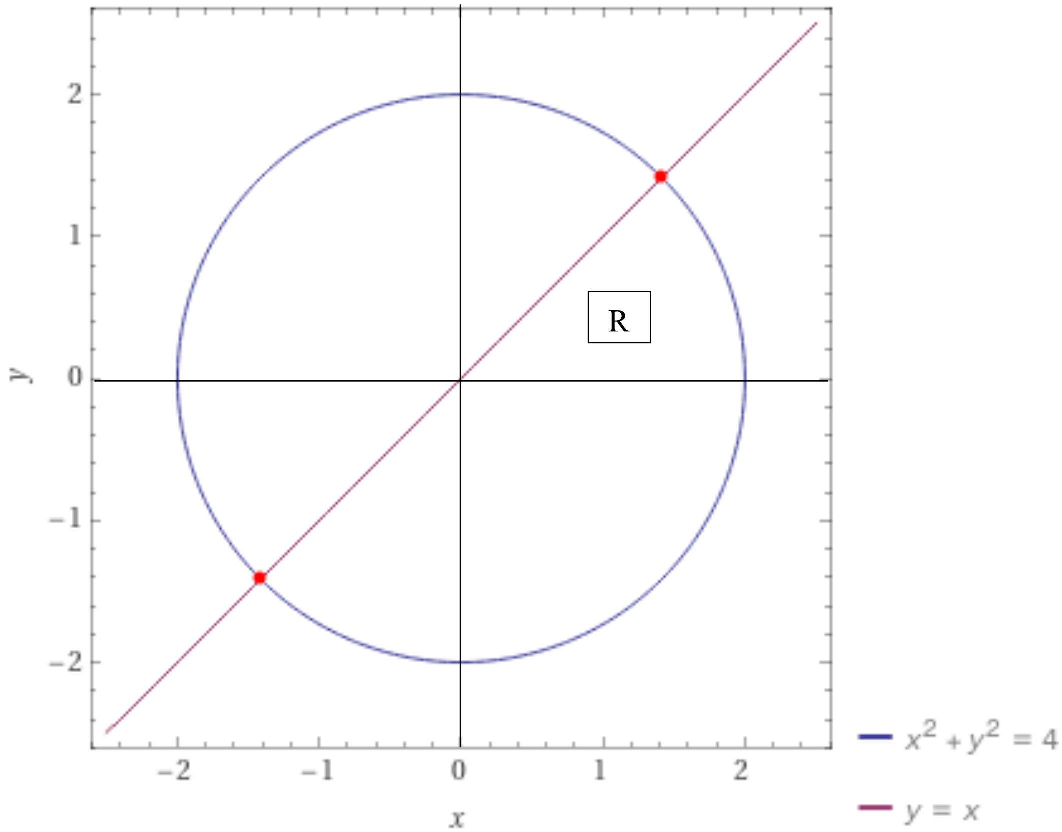
$$= \frac{1}{2}+\frac{1}{6}+\frac{1}{2}+\frac{1}{6}$$

$$= 1+\frac{2}{6}$$

$$= 1+\frac{1}{3}$$

$$= \frac{4}{3}$$

Nomor 2.



Berdasarkan gambar diketahui batasan-batasan daerah R pada koordinat polar sebagai berikut.

Batas r dimulai dari $r = 0$ sampai $r = 2$

Batas θ dimulai dari $\theta = 0$ sampai $\theta = \frac{\pi}{4}$

Sehingga

$$\iint_R \sqrt{4 - x^2 - y^2}\, dA = \int_0^{\frac{\pi}{4}} \int_0^2 \sqrt{4 - r^2}\, r\, dr\, d\theta$$

$$= \int_0^{\frac{\pi}{4}} \left(\int_0^2 \sqrt{4 - r^2}\, r\, dr \right) d\theta$$

Misal:

$u = 4 - r^2$

$\frac{du}{dr} = -2r$ sehingga $dr = \frac{du}{-2r}$

$$\int_0^{\frac{\pi}{4}} \left(\int_0^2 \sqrt{4 - r^2}\, r\, dr \right) d\theta = \int_0^{\frac{\pi}{4}} \left(\int_4^0 \sqrt{u}\, r \frac{du}{-2r} \right) d\theta$$

$$= \int_0^{\frac{\pi}{4}} \left(\int_4^0 -\frac{1}{2}\sqrt{u}\, du \right) d\theta$$

$$= \int_0^{\frac{\pi}{4}} \left(-\frac{1}{2}\frac{2}{3}u^{\frac{3}{2}} \right)_4^0 d\theta$$

$$= \int_0^{\frac{\pi}{4}} \left(-\frac{1}{3}u^{\frac{3}{2}} \right)_4^0 d\theta$$

$$= \int_0^{\frac{\pi}{4}} \left(-\frac{1}{3}\left(0 - 4^{\frac{3}{2}}\right) \right) d\theta$$

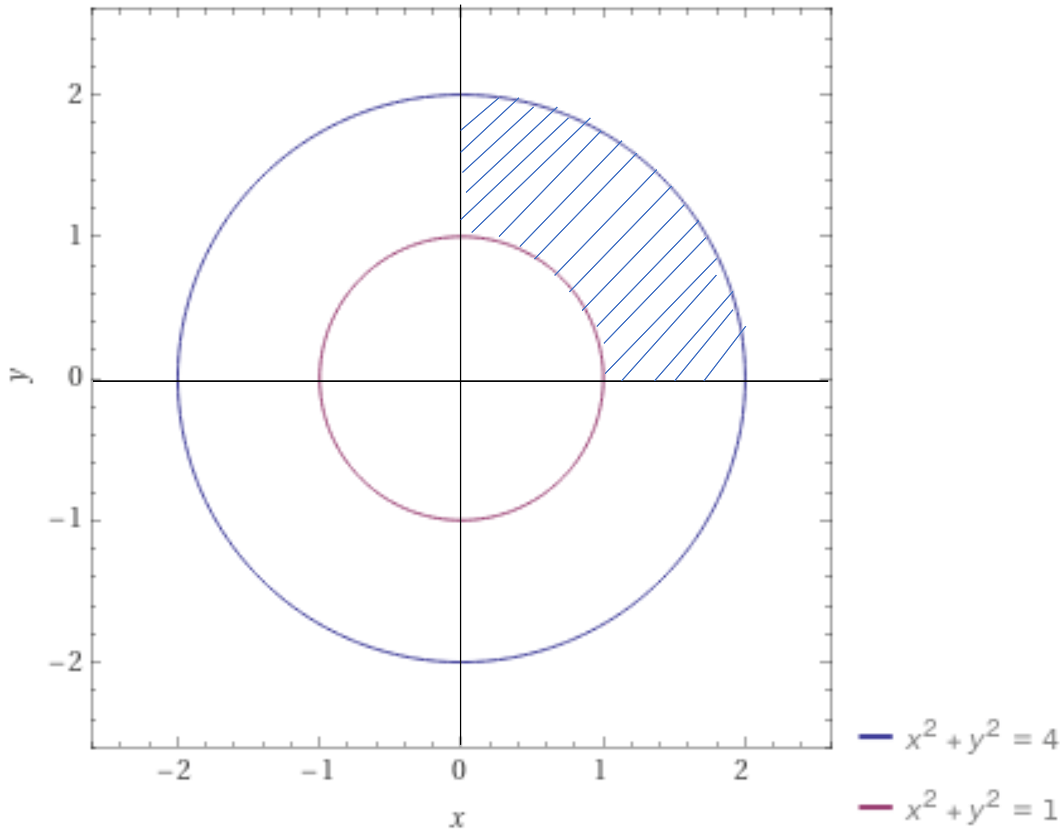
$$= \int_0^{\frac{\pi}{4}} \frac{8}{3} d\theta$$

$$= \left(\frac{8}{3}\theta \right)_0^{\frac{\pi}{4}}$$

$$= \frac{8}{3}\left(\frac{\pi}{4} - 0 \right)$$

$$= \frac{2}{3}\pi$$

Nomor 3.



Berdasarkan gambar diketahui batasan-batasan daerah R pada koordinat polar sebagai berikut.

Batas r dimulai dari $r = 1$ sampai $r = 2$

Batas θ dimulai dari $\theta = 0$ sampai $\theta = \frac{\pi}{2}$

Sehingga

$$\iint_R y\,dA = \int_0^{\frac{\pi}{2}} \int_1^2 r \sin\theta \, r \, dr \, d\theta$$

$$= \int_0^{\frac{\pi}{2}} \left(\int_1^2 r^2 \sin\theta \; dr \right) d\theta$$

$$= \int_0^{\frac{\pi}{2}} \left(\frac{1}{3} r^3 \sin\theta \right)_1^2 d\theta$$

$$= \int_0^{\frac{\pi}{2}} \left(\frac{1}{3} (2^3 - 1^3) \sin\theta \right) d\theta$$

$$= \int_0^{\frac{\pi}{2}} \frac{7}{3} \sin\theta \, d\theta$$

$$= \left(-\frac{7}{3} \cos\theta \right)_0^{\frac{\pi}{2}}$$

$$= -\frac{7}{3} \left(\cos\frac{\pi}{2} - \cos 0 \right)$$

$$= -\frac{7}{3} (0 - 1)$$

$$= \frac{7}{3}$$

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