

$$b \frac{\partial c}{\partial t} = \nabla \cdot (D \nabla c) - \nabla \cdot (\vec{v} c)$$

$$b \frac{\partial c}{\partial t} = \frac{D \frac{\partial c}{\partial x}}{\partial x} + \frac{D \frac{\partial c}{\partial y}}{\partial y} - \frac{v_x c}{\partial x} - \frac{v_y c}{\partial y}$$

FDM

$$b \frac{\partial c}{\partial t} = D \frac{c_{i+1,j} - 2c_{i,j} + c_{i-1,j}}{\Delta x^2} + D \frac{c_{i,j+1} - 2c_{i,j} + c_{i,j-1}}{\Delta y^2} - v_x \frac{c_{i+1,j} - c_{i-1,j}}{2\Delta x} - v_y \frac{c_{i,j+1} - c_{i,j-1}}{2\Delta y}$$

FDM + implicit euler

$$b \frac{c_{i,j,t+1} - c_{i,j,t}}{\Delta t} = D \frac{c_{i+1,j,t+1} - 2c_{i,j,t+1} + c_{i-1,j,t+1}}{\Delta x^2} + D \frac{c_{i,j+1,t+1} - 2c_{i,j,t+1} + c_{i,j-1,t+1}}{\Delta y^2} - v_x \frac{c_{i+1,j,t+1} - c_{i-1,j,t+1}}{2\Delta x} - v_y \frac{c_{i,j+1,t+1} - c_{i,j-1,t+1}}{2\Delta y} \quad (1)$$

$$b \frac{c_{i,j,t+1}}{\Delta t} - b \frac{c_{i,j,t}}{\Delta t} = D \frac{c_{i+1,j,t+1}}{\Delta x^2} - 2D \frac{c_{i,j,t+1}}{\Delta x^2} + D \frac{c_{i-1,j,t+1}}{\Delta x^2} + D \frac{c_{i,j+1,t+1}}{\Delta y^2} - 2D \frac{c_{i,j,t+1}}{\Delta y^2} + D \frac{c_{i,j-1,t+1}}{\Delta y^2} - v_x \frac{c_{i+1,j,t+1}}{2\Delta x} + v_x \frac{c_{i-1,j,t+1}}{2\Delta x} - v_y \frac{c_{i,j+1,t+1}}{2\Delta y} + v_y \frac{c_{i,j-1,t+1}}{2\Delta y} \quad (2)$$

$$- D \frac{c_{i+1,j,t+1}}{\Delta x^2} + v_x \frac{c_{i+1,j,t+1}}{2\Delta x} - D \frac{c_{i,j+1,t+1}}{\Delta y^2} + v_y \frac{c_{i,j+1,t+1}}{2\Delta y} + b \frac{c_{i,j,t+1}}{\Delta t} + 2D \frac{c_{i,j,t+1}}{\Delta x^2} + 2D \frac{c_{i,j,t+1}}{\Delta y^2} - D \frac{c_{i-1,j,t+1}}{\Delta x^2} - v_x \frac{c_{i-1,j,t+1}}{2\Delta x} - D \frac{c_{i,j-1,t+1}}{\Delta y^2} - v_y \frac{c_{i,j-1,t+1}}{2\Delta y} = +b \frac{c_{i,j,t}}{\Delta t} \quad (3)$$

$$\left(-\frac{D_x}{\Delta x^2} + \frac{v_x}{2\Delta x}\right)c_{i+1,j,t+1} + \left(-\frac{D_y}{\Delta y^2} + \frac{v_y}{2\Delta y}\right)c_{i,j+1,t+1} + \left(\frac{b}{\Delta t} + \frac{2D_x}{\Delta x^2} + \frac{2D_y}{\Delta y^2}\right)c_{i,j,t+1} + \left(-\frac{D_x}{\Delta x^2} - \frac{v_x}{2\Delta x}\right)c_{i-1,j,t+1} + \left(-\frac{D_y}{\Delta y^2} - \frac{v_y}{2\Delta y}\right)c_{i,j-1,t+1} = b \frac{c_{i,j,t}}{\Delta t} \quad (4)$$