Singapore

ECONOMIC PERFORMANCE OF THE LITTLE RED DOT 1970-2017

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1. Introduction

Modern Singapore was founded in 1819 by Sir Stamford Raffles and was a colony under the British rule till the Japanese occupation between 1942 to 1945. Singapore gained independence and became a sovereign nation in 1965. Since then, despite the lack of natural resources, the nation developed rapidly and is known to be one of the 4 Asian Tigers alongside South Korea, Hong Kong and Taiwan. Singapore has a small land mass of 721.5km^2 and a total population of 5.61 million in 2017.

In the short span since her independence, Singapore has developed rapidly and has largely transitioned from a manufacturing-oriented economy to a service-oriented economy. Per-capita GDP ranks 3rd in the world in terms of Purchasing Power Parity (PPP). Her main sectors are in banking, biotechnology as well as in energy and infrastructure. Singapore is a global financial hub, being ranked the third most competitive financial centre in the world after London and New York City. Also, Singapore have good multilateral ties with most countries and can be considered to be a neutral country, hosting summits between China and Taiwan leaders in 2015 as well as United States and North Korea leaders in 2018.

Singapore has a highly developed market economy which is reliant on entrepôt trade. It is the premier port of call in Southeast Asia and is the second busiest in the world by cargo tonnage. Its trade to GDP ratio is one of the highest in the world, averaging about 400% between 2008 and 2011. Its main exports are refined petroleum, electronic integrated circuits and computers while its main imports are aircraft, crude oil and petroleum products as well as electronic components. As such, Singapore can be considered to be reliant on intermediary trade, by purchasing raw goods and refining them for re-exportation. Re-exports accounted for 43% of Singapore's trade with other countries in 2000. Its main trading partners include China, Malaysia, European Union as well as the United States.

In this paper, we use a couple of methods to analyse Singapore's economic growth between 1970 to 2017. They are the index number method, econometric approach with both consumer and producer models, nonparametric approach and detailed consumer models. First, the index number method measures total factor productivity and decomposes it into various components. The two-good consumer model measures elasticities of both consumption and leisure. The seven-good producer model measures elasticities of consumption, government output, gross investment, exports, imports and labour. This model allows us to estimate technical progress growth over the sample period. The nonparametric model analyses the efficiency of productivity. Lastly, the detailed consumer models break down consumption into a basket of 15 goods and measures both price and income elasticities of each component.

There were 4 major economic shocks that occurred over this sample period. First, there was the first post-independence local recession in 1985 which was caused by both external and internal factors. Singapore was also experiencing decreased demand for its goods and services because regional countries were trading directly, bypassing Singapore as an entrepôt. Neighbouring countries such as Indonesia, Thailand and the Philippines had implemented exit taxes, and Malaysia introduced a 50 percent tax on goods bought from Singapore by residents. The situation was further aggravated by poor performances from Singapore's key industries in oil refining and shipbuilding and repair due to the entry of new competitors. Internally, the high operating costs attributed to high wages and rentals made Singapore less competitive in the global market, as there was no corresponding increase in productivity. The construction slump, high domestic savings rate and rigidity in the economy further weakened the economy. Also, Singapore was affected by regional and global recessions such as the Asian Financial Crisis in 1997, Dot Com Bubble in 2001 as well as the Global Financial Crisis in 2008. During these 4 specific periods, Singapore's economic performance was affected as we will see in the analysis later on.

2. Data

The primary source of data used in our models come from its national statistical agency: Singapore Department of Statistics (DOS). Also, we relied on a secondary source from the Asian Productivity Organization (APO) that has data from 1970 to 2016. We use APO data mainly for filling in missing labour components from DOS such as hours of work, compensation of employees and employment. Further, both sources allow us to cross-check expenditure components of GDP. Lastly, we extrapolated APO's data to 2017.

2.1 Output Series

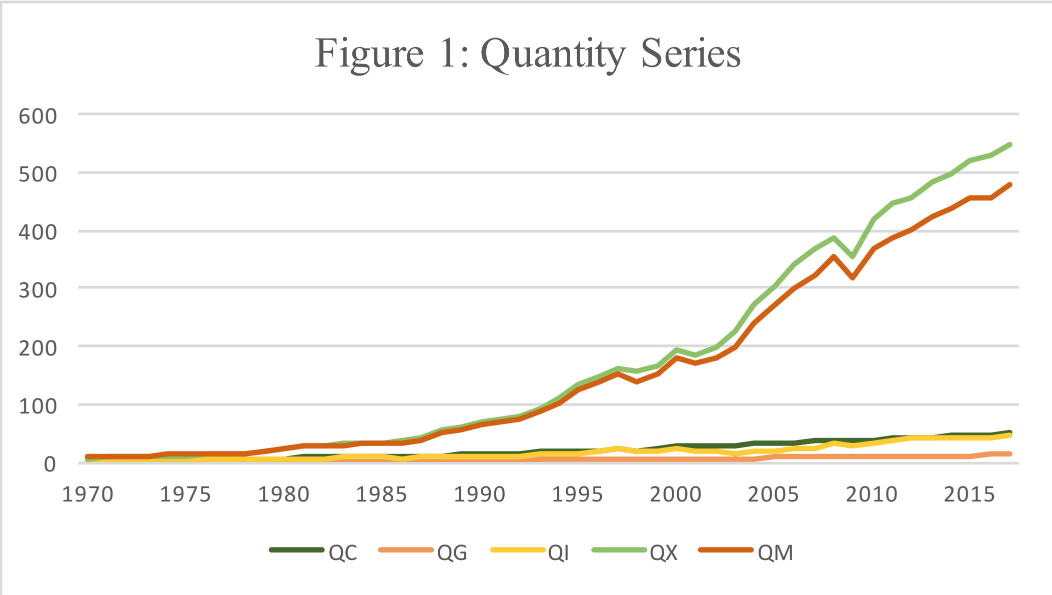
We use two out of the three approaches in measuring GDP: expenditure and income approach to construct our data series.

Through the expenditure approach in measuring GDP, we use output data series at constant and current prices. V represents the value series (current prices) while Q represents the constant prices series. The base year for constant prices series here is in year 2010 and we use DOS data from 1970-2017.

- *QC, VC: Household Consumption Expenditure*
- *QG, VG: Government Consumption Expenditure*
- *QI1, VI1: Gross Fixed Capital Formation*
- *VI2: Change in Inventories*
- *QX, VX: Exports of Goods and Services*
- *QM, VM: Imports of Goods and Services*

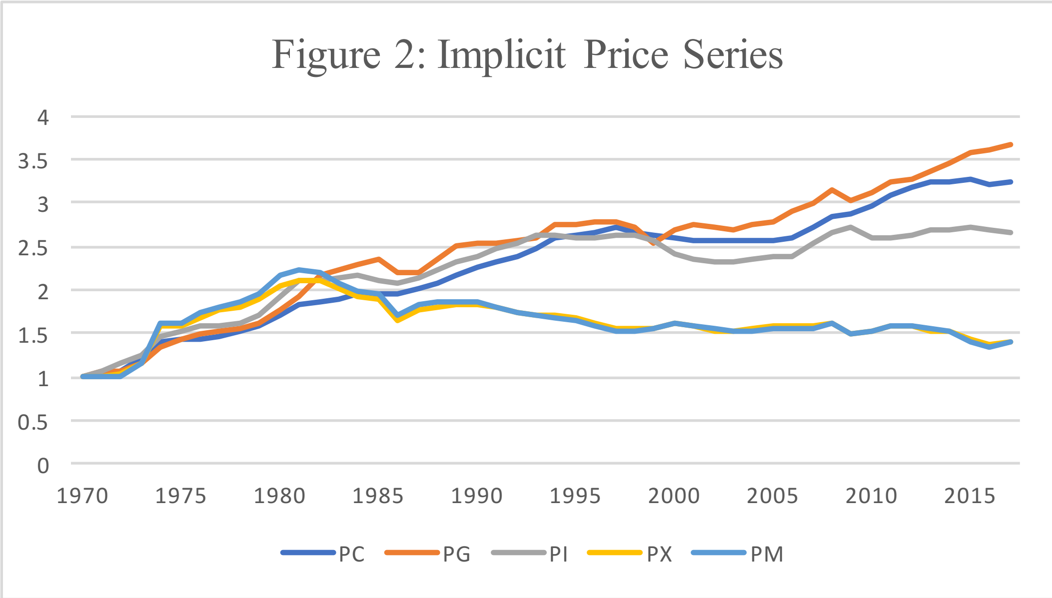
To calculate the implicit price series, we use both the value series and constant price series, where $Price = \frac{Value\ (V)}{Quantity\ (Q)}$.

- $PC = \frac{VC}{QC}, PG = \frac{VG}{QG}, PI = \frac{VI1}{QI1}, PX = \frac{VX}{QX}, PM = \frac{VM}{QM}$
- $QI2 = \frac{VI2}{PI}$ is constructed using PI and total $QI = QI1 + QI2$

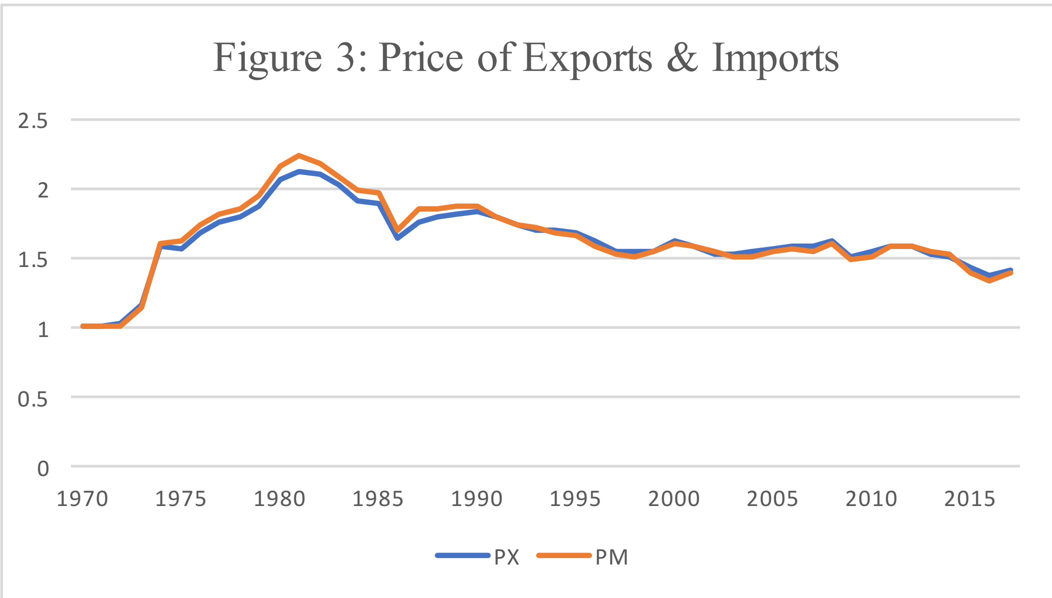


Between 1970 to 2017, the quantity series rose steadily, especially in exports and imports. As mentioned earlier, Singapore has huge trade volume and the increase in X and M verifies this. There are 3 dips in the quantity series. First in 1997, then in 2001 and lastly in 2008. This corresponds with events that impacted Singapore's economy: the Asian Financial Crisis which hit in 1997, the Dot Com Bubble in 2001 and the Global Financial Crisis which struck in 2008. The GFC affected trade the most as seen in the huge decline in both exports and imports but steadily recovered between 2009 and 2010 and has been increasing since then.

Also, the quantity series for consumption and investment have been increasing gradually and in tandem over the years while quantity series for government expenditure experienced a modest increase. Further, it is worth to mention that if this trend continues to persist, we can safely assert that the proportion of exports and imports relative to GDP will continue to increase.



The implicit price series is normalized to one with respect to the starting period 1970. We can see that price of government expenditure rose the most over the sample period, by slightly less than 4-fold, and is continuing to increase. This is followed by the price of consumption expenditure and then price of investment, albeit slowing down in recent years.



Looking closely at the price series for exports and imports, we can observe that they move closely in tandem, with its peak in 1981 by slightly over 2-fold, before declining and staying at about 1.5-fold. Since Singapore has huge volumes of exports and imports coming in and out of the country from its port regularly, it makes sense that the prices of exports and imports will follow each other closely.

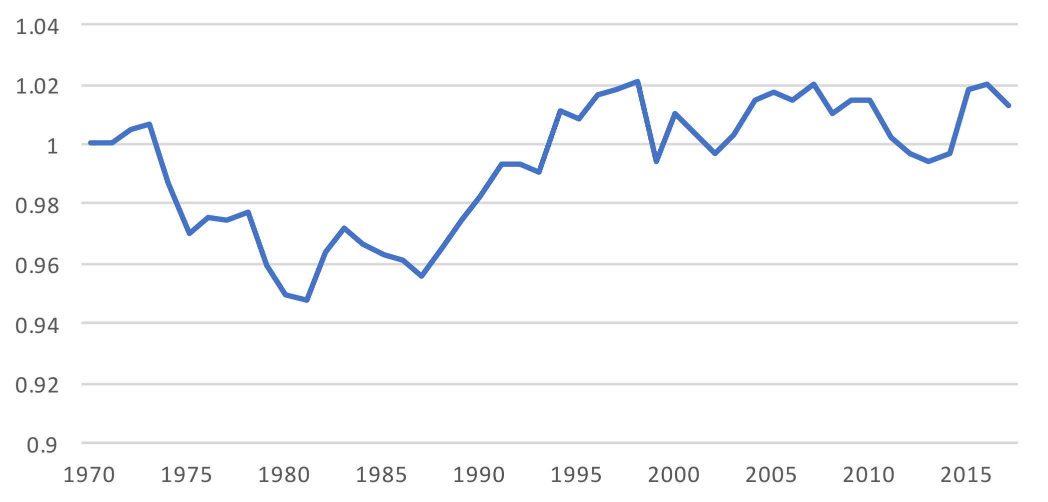


Figure 4: Terms of Trade

By using the price series for exports and imports, we construct terms of trade (*TOFT*) as $TOFT = \frac{PX}{PM}$. Since price series for exports and imports move closely in tandem, Singapore's terms of trade fluctuate about 1, reaching its minimum at about 0.95 in 1981. Due to the 2008 Global Financial Crisis, international trade dampened and the price of exports fell more than the price of imports. We can see a decrease in TOFT before it recovers in 2014.

To generate the output series, we aggregate the price and quantity series to nominal GDP with a chained Divisia index. We will use this GDP series to generate the input series and ratios further in the following sections.

- Price series: $p^t = (PC_t, PG_t, PI_t, PX_t, PM_t)$

- Quantity series: $q^t = (QC_t, QG_t, QI_t, QX_t, -QM_t)$

- The chained Divisia Price Index:
  $PY_T(p^0,\ldots,p^t,q^0,\ldots,q^t) = P_T(p^0,q^0,p^1,p^1) \times P_T(p^1,q^1,p^2,p^2) \times \ldots \times P_T(p^{t-1},q^{t-1},p^t,p^t)$

- Implicit Divisia Quantity Index:

$$QY_T(p^0,\ldots,p^t,q^0,\ldots,q^t) = (p^tq^t/p^0q^0)/PY_T(p^0,\ldots,p^t,q^0,\ldots,q^t)$$

2.2 Input Price Series

We measure two kinds of inputs, labour and capital. We exclude natural resources and land, as data is not available and it is difficult to have accurate measurement of these inputs.

2.2.1 Capital Inputs

Consumption of fixed capital data was unavailable, so we assume the depreciation rate to be 5% in the starting period and growing at a 4% rate which is reasonable for most countries.

$$D_t = D_{t-1} \times 1.004,\ D_0 = 0.05$$

We then apply the geometric model of depreciation to calculate QK.

$$QK_t = (1-D_{t-1}) \times QK_{t-1} + QI_{t-1}$$

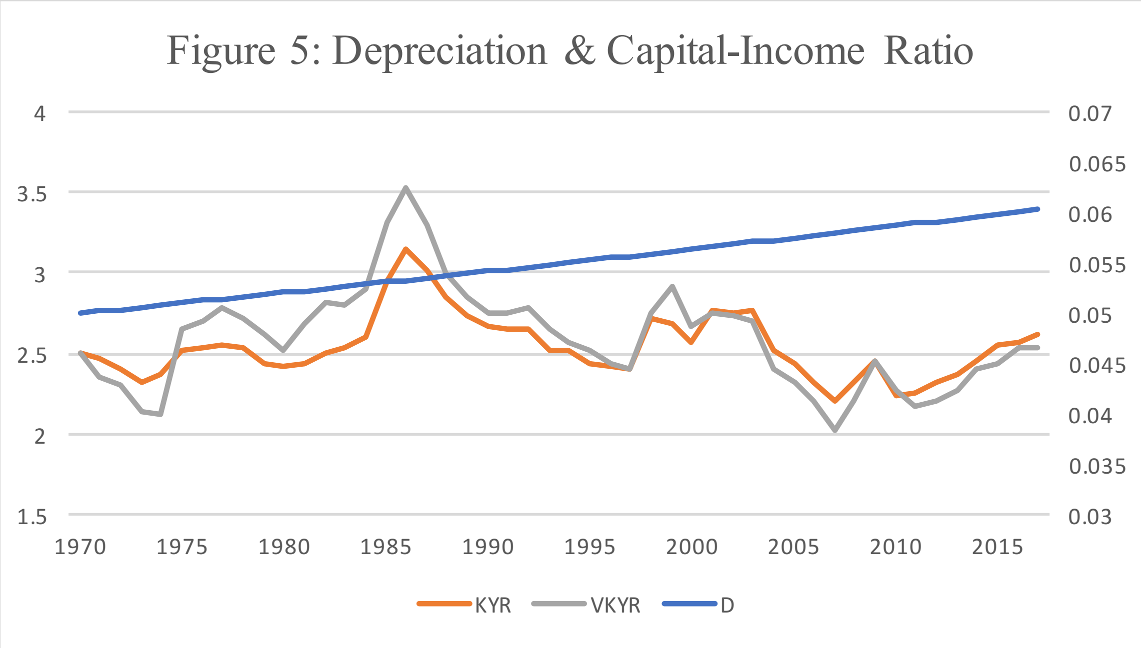
Following which, we set the price of beginning of year capital stock to be equal to the investment price of the previous year, and the price of capital at the start of year 1970 (PK^{1970}) to be equal to the investment price of year 1970 (PI^{1970}).

- *$PK = PI$ for year 1970 and $PK = PI_{t-1}$ thereafter*
- *$VK = PK \times QK$*

We then generate the real (*KYR*) and nominal (*VKYR*) capital output ratios using capital's price and quantity series and the Divisia index. We assume *QK* to be 2.5 times of *QY* at our starting period.

$$KYR = \frac{QK}{QY}$$

$$VKYR = \frac{VK}{VY}$$



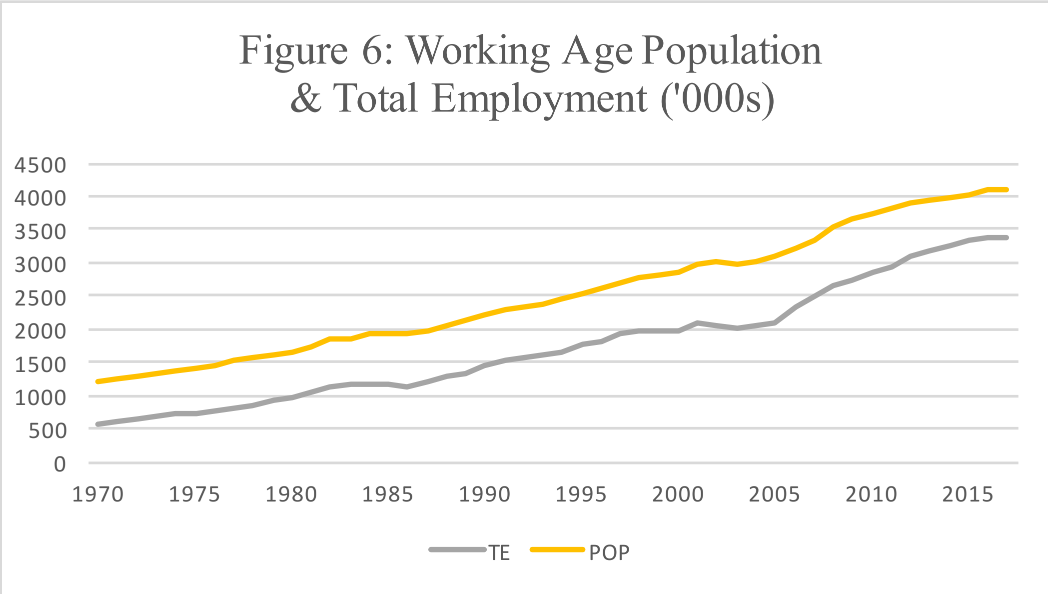
The linear blue straight line corresponds to the right vertical axis where depreciation starts at 0.05 and increases at a 4% rate. Referring to the left vertical axis for capital output ratio, we observe that both nominal and real capital output ratio peaked in 1986 and then declined and fluctuated about 2.5. Between 1970 to 2000, nominal capital output ratio outperformed real capital output ratio which implies that real capital output ratio performed worse in relative terms across this period.

2.2.2 Labour Inputs

We generate the labour input series by considering all types of workers.

- Civilian Employment (*CE*) = Wage Earners (*WE*) + Self Employed (*SE*) + Unpaid Family Workers (*UP*)
- Total Employment (*TE*) = Wage Earners (*WE*) + Armed Forces (*AF*)
- Total Labor Force (*LF*) = Civilian Employment (*CE*) + Unemployed (*UN*) + Armed Forces (*AF*)

We assume AF to be 0 since data description says that total labor force in the economy is equals to the sum of employed workers and unemployed, hence the number of armed forces would be included in civilian employment and inseparable. Further, we have only data of CE and WE across the sample period and we generate $SE + UP$ to be the difference between CE and WE. We classify $SP = SE + UP$ for further aggregation.



From Figure 6, both working age population and total employment rose at the same rate and are smooth over the sample period. The working age population in 2017 is about 4 million people, while total employment is about 3.5 million people.

We calculate total labour quantity of wage earners (QE), total labour quantity of the sum of self-employed and unpaid family workers (QSP) by generating total employment, self-employment and unpaid family workers.

- $QE = \frac{TE \ x \ Hours}{1000000}$

- $QSP = \frac{SP \ x \ Hours}{1000000}$

- $Hourly \ wage \ PE = \frac{VE}{QE}$

Figure 7: Annual Hours

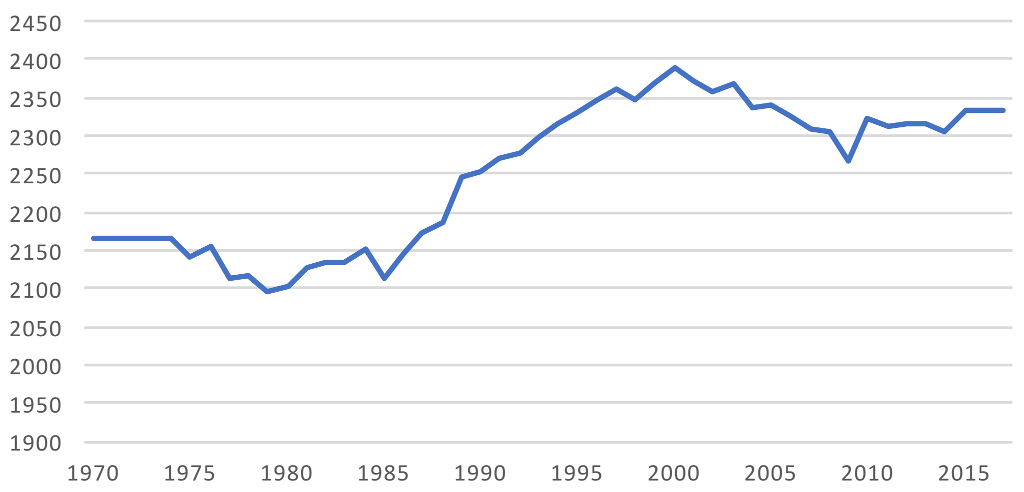
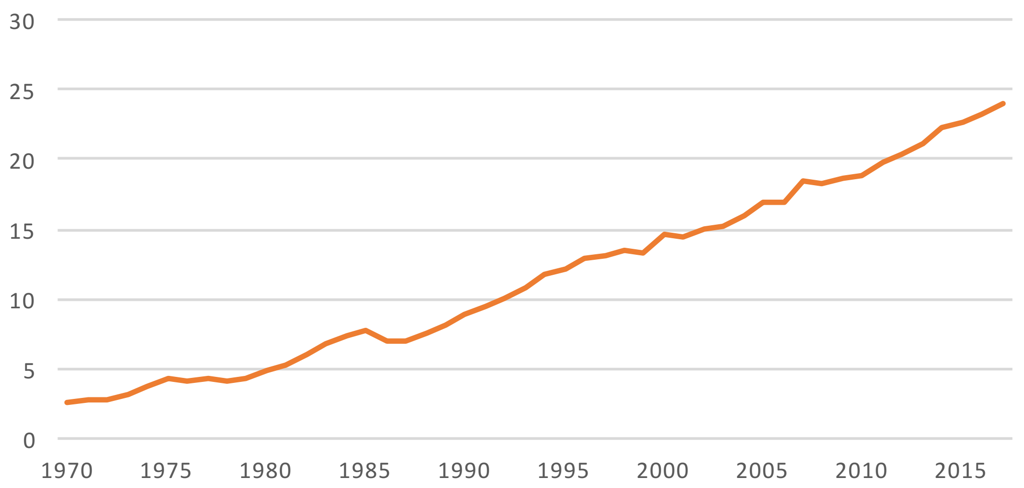


Figure 8: Hourly Wage Rate



Annual working hours was steady at about 2150 hours from 1970 to 1985, before it increased to close to 2400 hours in 2000, before fluctuating steadily at about 2350 thereafter. This corresponds to a 45-hour work week. Nominal hourly wage rate has also increased over the sample period from 3 to 24 by about 8-fold and is continuing to grow.

We assume that self-employed people earn 2/3 of employee wages and unpaid family workers get 1/3 of employee wages since self-employed people earn less than wage earners and unpaid family workers earn the least.

- $PSE = \frac{2}{3}\, PE$
- $PUP = \frac{1}{3}\, PE$

From the chained Fisher index, we can then aggregate our labour series on price and quantity vectors where $p^t = (PE^t, PSP^t)$ *and* $q^t = (QE^t, QSP^t)$.

- Fisher index for period t:

$$P_F(p^{t-1}, p^t, q^{t-1}, q^t) = \sqrt{\frac{\sum_{i=1}^{3} p_i^t q_i^{t-1}}{\sum_{i=1}^{3} p_i^{t-1} q_i^{t-1}}} \sqrt{\frac{\sum_{i=1}^{3} p_i^t q_i^t}{\sum_{i=1}^{3} p_i^{t-1} q_i^t}}$$

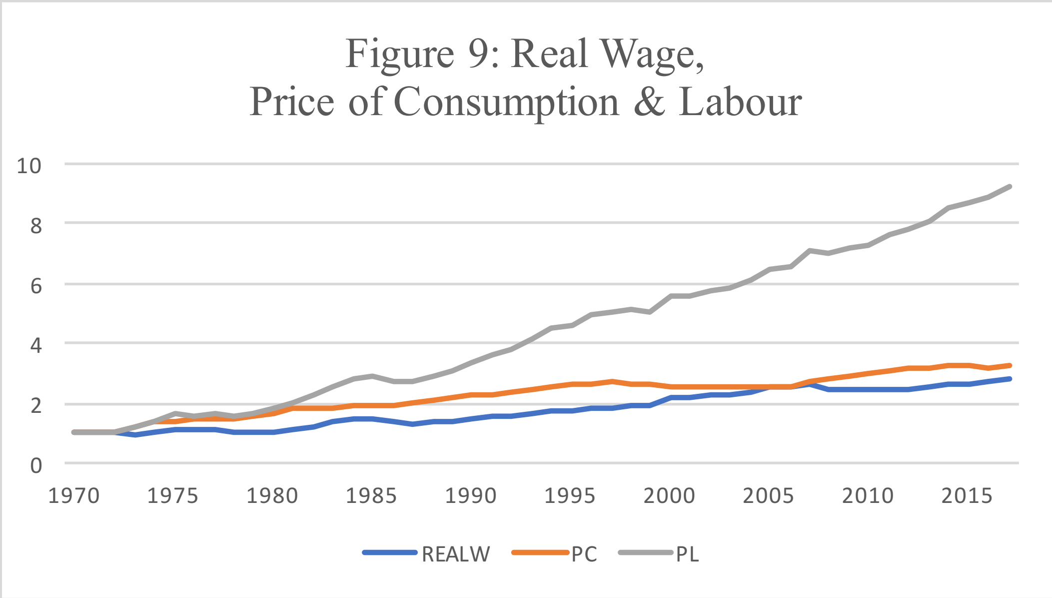
$$Q_F(p^{t-1}, p^t, q^{t-1}, q^t) = \sqrt{\frac{\sum_{i=1}^{3} p_i^{t-1} q_i^t}{\sum_{i=1}^{3} p_i^{t-1} q_i^{t-1}}} \sqrt{\frac{\sum_{i=1}^{3} p_i^t q_i^t}{\sum_{i=1}^{3} p_i^t q_i^{t-1}}}$$

- Chained Fisher index:

$PL_F\ (p^0,\ldots,p^t,q^0,\ldots,q^t) = P_F(p^0, q^0, p^1, q^1) \text{ x } P_F(p^1, q^1, p^2, q^2) \text{ x } \ldots \text{ x } P_F(p^{t-1},q^{t-1},p^t,q^t)$

$QL_F\ (p^0,\ldots,p^t,q^0,\ldots,q^t) = P_F(p^0, q^0, p^1, q^1) \text{ x } P_F(p^1, q^1, p^2, q^2) \text{ x } \ldots \text{ x } P_F(p^{t-1},q^{t-1},p^t,q^t)$

According to Hick's aggregation theorem, PL is proportional to PE since wages of self-employed and unpaid family workers are proportional to wages of wage earners. We calculate real wage as $REALW = \frac{PL}{PC}$.



As shown in Figure 9, real wages grew by 2.85-fold over the sample period. We can observe that price of consumption has remained relatively flat since 1995 while price of labour continues to increase which resulted in a gradual increase in real wage.

2.3 Tax Rates

Taxes and subsidies adjustments for our output data are taken into account in this paper, data on taxes and subsidy levels are taken from DOS.

- Taxes on consumption (*TC*)
- Taxes on capital (*TK*)
- Taxes on exports (*TX*)
- Taxes on imports (*TM*)
- Taxes on labour (*TL*)
- Subsidy on production (*VS*)

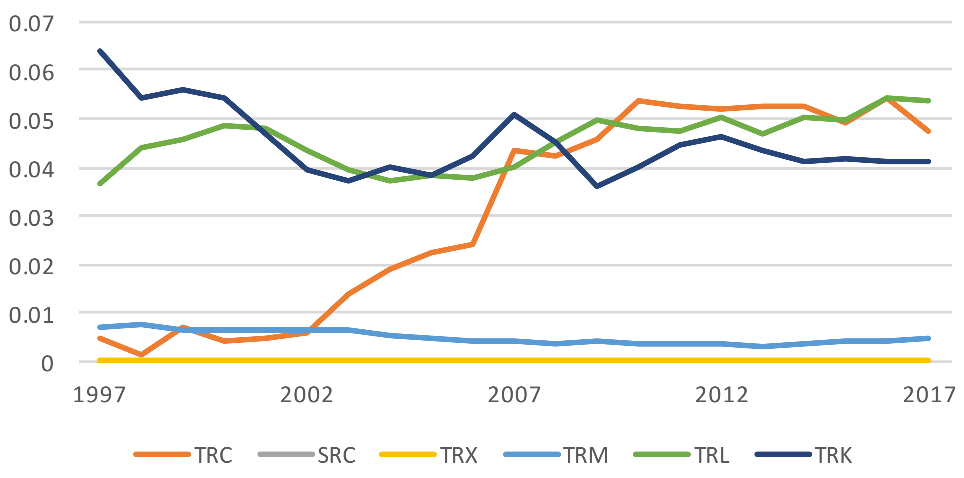
Tax and subsidy rates are then calculated from dividing their levels by their respective value series.

- $TRC = \frac{TC}{VC}$

- $TRK = \frac{TK}{VK}$
- $TRX = \frac{TX}{VX}$
- $TRM = \frac{TM}{VM}$
- $TRL = \frac{TL}{VL}$
- $SRC = \frac{VS}{VC}$

We use tax level data provided by the DOS that starts from 1997, and goes to 2017 in order to calculate the tax and subsidy rates. For the years between 1970 and 1997, we take the tax and subsidy rates to be the same as 1997. We justify this by observing the trends of tax rates for each component as seen from the figure below.

Figure 10: Taxes and Subsidy Rates



As shown in Figure 10, Singapore has a zero subsidy rate on production (*SRC*). Also, it has a zero tax rate on exports (*TRX*), since it is highly dependent on trade. For the same reason, import tariffs (*TRM*) are low, at less than 1%. Tax rates on capital (*TRK*) and labour (*TRL*) fluctuate about 4%, while tax rates on consumption (*TRC*) grew from less than 1% in 1997 to approximately 5% in 2017. Intuitively, tax rates on consumption will grow over time. Hence, it is reasonable for us to use tax rates in year 1997 and apply it to the years before for our calculations.

We also generate gross profits (*GPROF*) using price and quantity series as well as tax and subsidy rates:

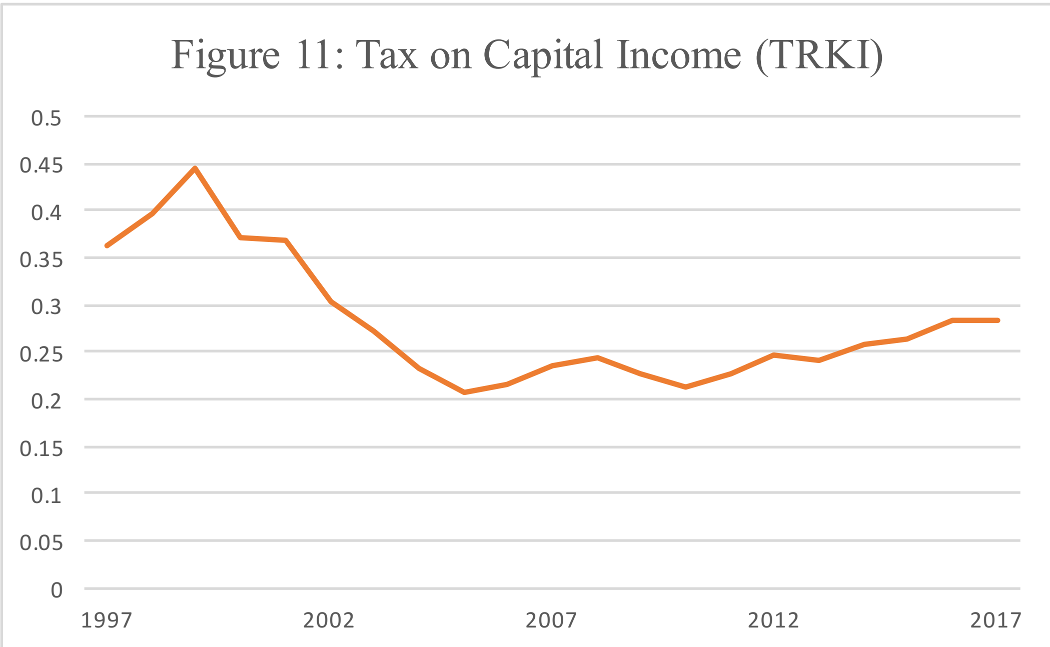
$$GPROF = PC\ x\ QC(1 - TRC + SRC) + PI\ x\ QI + PG\ x\ QG + PX\ x\ QX(1 - TRX) - PM\ x\ QM(1 + TRM) - PL\ x\ QL$$

We then generate net profits (*NPROF*) by subtracting value of depreciation from gross profits:

$$NPROF = GPROF - VD$$

Finally, we generate a tax rate on profits (income), *TRKI*:

$$TRKI = \frac{TK}{NPROF}$$



Tax rate on capital income (*TRKI*) plays the largest component of government revenue as compared to the other components. It reached its peak at 45% in year 1999 before declining to 20% in year 2005 and has then stayed between 25 to 30%.

2.4 Rates of Return

2.4.1 Nominal Rates of Return

First, we calculate the change in value of beginning of period capital stock over the period while it depreciates (DVK). Since the beginning of period value of capital stock is VK and end of period value of initial capital stock is $(1 - D)\ x\ (1 + IK)\ x\ VK$,

$$DVK = VK - (1 - D)\ x\ (1 + IK)\ x\ VK$$

We then set the value of inputs to be equal to the value of outputs such that gross profits will be equal to the value of capital services.

$$GPROF = VK\ x\ [(1 + R) - (1 + IK)\ x\ (1 - D)] = VK\ x\ R + DVK$$

and the ex post nominal rate of return *R* will be:

$$R = \frac{(GPROF - DVK)}{VK}$$

and the ex post measure of asset price inflation will be:

$$IK^t = \frac{PK^{t+1}}{PK^t} - 1$$

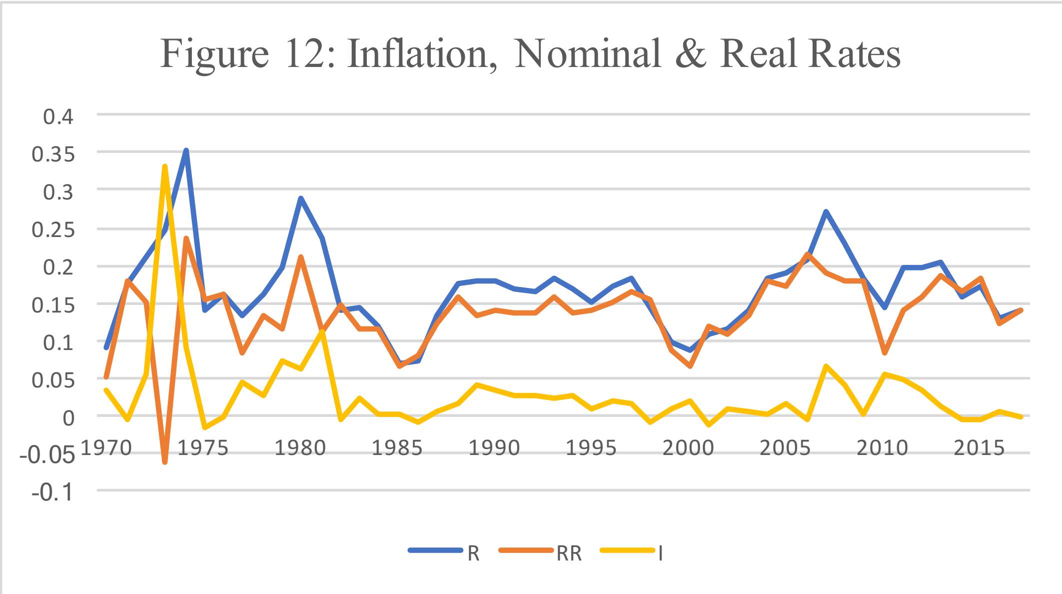
2.4.2 Real Rates of Return

We use data on the consumer price index (*CPI*) from DOS and compare it to price series of consumption. Inflation rate of the CPI is generated based on the CPI from January of the current year to January of the following year:

$$I^t = \frac{CPI^{t+1}}{CPI^t} - 1$$

and the ex post real rate of return RR will be calculated using the Fisher equation:

$$RR = \frac{1+R}{1+I} - 1$$



Singapore faced relatively low positive inflation after 1982, fluctuating about 2%. As such, nominal rates of returns are consistently higher than the real rates. Also, there is a decline in rates of return after the 1997 Asian Financial Crisis due to uncertainty. Further, rates of return recovered since then, and continued to grow before crashing again during the 2008 Global Financial Crisis. After which, rates of return recovered around 2011 and have been at about 15% since then.

3. Index Number Method

3.1 Total Factor Productivity

In this section, we will generate gross and net productivity rates for Singapore and decompose it to the different components.

3.1.1 Gross Real Income Framework

Firstly, we construct Törnqvist-Theil index (chained Divisia indexes) input and output aggregates.

- $p^t = (PL^t, PKS^t)$
- $q^t = (QL^t, QKS^t)$
- Törnqvist-Theil price index for period t:

$$ln\mathrm{P}_{\mathrm{T}}(p^{t-1}, p^t, q^{t-1}, q^t) = \sum_{n=1}^{2} \frac{1}{2}(s_n^{t-1} + s_n^t) ln(\frac{\mathrm{P}_n^t}{p_n^{t-1}})$$

- The chained Divisia price index:

$$PZ_T(p^0,\ldots,p^t,q^0,\ldots,q^t)=P_T(p^0,q^0,p^1,q^1)\times\ldots\times P_T(p^{t-1},q^{t-1},p^t,q^t)$$

- Implicit Divisia quantity index:

$$\mathrm{QZ}_{\mathrm{T}}(p^0,\ldots,p^t,q^0,\ldots,q^t) = \frac{p^t q^t / p^0 q^0}{\mathrm{PY}_{\mathrm{T}}(p^0,\ldots,p^t,q^0,\ldots,q^t)}$$

We construct producer prices where,

- $PPC = PC(1 - TRC + SRC)$
- $PPG = PG$
- $PPI = PI$
- $PPX = PX(1 - TRX)$
- $PPM = PM(1 + TRM)$

We aggregate output series using the chained Divisia indexes with price and quantity vectors,

- $p^t = (PPC^t, PPG^t, PPI^t, PPX^t, PPM^t)$
- $q^t = (QC^t, QG^t, QI^t, QX^t, -QM^t)$

We can then generate the level of total factor productivity (TFP) as:

$$PROD = \frac{QY}{QZ}$$

We calculate TFP as:

$$YG = \frac{QY^t}{QY^{t+1}}$$

$$ZG = \frac{QZ^t}{QZ^{t+1}}$$

$$PRODG = \frac{YG}{ZG}$$

As seen from appendix table 8, the average gross TFP growth was 1.63% each year.

3.1.2 Net Real Income Framework

Next, we generate TFP using the DMK net product model.

- User cost: $PU = PK\ x\ [(1 + R) - (1 + IK)\ x\ (1 - D)]$
- Waiting services: $PK\ x\ R$
- Depreciation: $PK\ x\ D$
- Value of asset appreciation: $-PK\ x\ (1 - D)\ x\ IK$
- Domestic net investment aggregate: $PI\ x\ QI - PK\ x\ D\ x\ QK + PK\ x\ (1 - D)\ x\ IK\ x\ QK$ where $PI\ x\ QI$ is our old gross investment
- New contribution of capital services to nominal income: $R\ x\ PK\ x\ QK$
- New domestic net investment aggregate: $PI\ x\ QI - PK_t\ x\ QK + (1 - D)\ x\ PK_{t+1}\ x\ QK$
- End of period price of capital: $PKE = PK\ x\ (1 + IK)$
- End of period quantity of capital: $QKE = (1 - D)\ x\ QK$

We calculate total net factor productivity in a similar way as how we calculated for total gross factor productivity. As seen from appendix table 9, the average net TFP growth was 1.71% per year.

3.2 Real Income Growth Decomposition

3.2.1 Real Gross Income Growth Decomposition

Diewert Morrison Decomposition

We construct a real income aggregate to generate growth of real income and decompose the growth to its contributory factors.

- Real income aggregate: $RI = PD\ x\ QD - PX\ x\ QX + PM\ x\ QM$
- Real income growth: $RLINK_t = \frac{RI_t}{RI_{t-1}}$

Real income growth can be decomposed into three main factors, namely technology progress, input growth and output growth. As mentioned by Diewert and Morrison (1986), the natural log of real income has the translog function form:

$$\gamma^t \approx \tau^t \alpha^t \beta^t$$

where τ^t is the technology growth factor, α^t is the growth factor of real output prices and β^t is the growth factor of real input prices factor.

$$\gamma^t = \mathrm{RLINK}^t$$

$$ln\alpha^t = \sum_{m=1}^{3} \frac{1}{2} \left(\frac{p_m^{t-1} \times y_m^{t-1}}{p^{t-1} \times y^{t-1}} + \frac{p_m^t \times y_m^t}{p^t \times y^t}\right) ln\left(\frac{p_m^t}{p_m^{t-1}}\right)$$

$$ln\beta^t = \sum_{m=1}^{2} \frac{1}{2} \left(\frac{w_m^{t-1} \times x_m^{t-1} w}{w^{t-1} \times x^{t-1}} + \frac{w_m^t \times x_m^t}{w^t \times x^t}\right) ln\left(\frac{x_m^t}{x_m^{t-1}}\right)$$

$$\tau^t = \frac{\gamma^t}{\alpha^t \times \beta^t}$$

We generate output growth effects of consumption (PC), government output (PG), investment prices (PI), export prices (PX), import prices (PM), labour input (QL) and capital input (QK).

Output price link factors are as follow:

$$PCLINK = exp(\frac{1}{2}(\frac{PC^t \times QC^t}{RI^t} + \frac{PC^{t-1} \times QC^{t-1}}{RI^{t-1}}) \times ln(\frac{PC^t}{PC^{t-1}}))$$

$$PGLINK = exp(\frac{1}{2}(\frac{PG^t \times QG^t}{RI^t} + \frac{PG^{t-1} \times QG^{t-1}}{RI^{t-1}}) \times ln(\frac{PG^t}{PG^{t-1}}))$$

$$PILINK = exp(\frac{1}{2}(\frac{PI^t \times QI^t}{RI^t} + \frac{PI^{t-1} \times QI^{t-1}}{RI^{t-1}}) \times ln(\frac{PI^t}{PI^{t-1}}))$$

$$PXLINK = exp(\frac{1}{2}(\frac{PX^t \times QX^t}{RI^t} + \frac{PX^{t-1} \times QX^{t-1}}{RI^{t-1}}) \times ln(\frac{PI^t}{PI^{t-1}}))$$

$$PMLINK = exp(\frac{1}{2}(\frac{PM^t \times QM^t}{RI^t} + \frac{PM^{t-1} \times QM^{t-1}}{RI^{t-1}}) \times ln(\frac{PM^t}{PM^{t-1}}))$$

Input growth link factors are as follow:

$$QLLINK^t = exp(\frac{1}{2}(\frac{PL^t \times QL^t}{RI^t} + \frac{PL^{t-1} \times QL^{t-1}}{RI^{t-1}}) \times ln(\frac{QL^t}{QL^{t-1}}))$$

$$QKLINK^t = exp(\frac{1}{2}(\frac{PKS^t \times QKS^t}{RI^t} + \frac{PKS^{t-1} \times QKS^{t-1}}{RI^{t-1}}) \times ln(\frac{QKS^t}{QKS^{t-1}}))$$

Lastly, we can generate the productivity link factor:

$$TLINK^t = \frac{RLINK^t}{(PCLINK^t \times PGLINK^t \times PILINK^t \times PXLINK^t \times PMLINK^t \times QLLINK^t \times QKLINK^t}$$

We combine the effects of real import and export price change as terms of trade:

$$PTLINK = PXLINK\ x\ PMLINK$$

Kohli Type Decomposition

We generate a Kohli type decomposition of real income into multiplicative effects:

For t = 1 (year = 1970),

$$RIL = TT = CC = GG = II = XX = MM = LL = KK = TTT = 1$$

For t ≥ 2 (year = 1970),

$$RIL^t = RIL^{t-1} \times RLINK^t$$
$$TT^t = TT^{t-1} \times TLINK^t$$
$$CC^t = CC^{t-1} \times PCLINK^t$$
$$GG^t = GG^{t-1} \times PGLINK^t$$
$$II^t = II^{t-1} \times PILINK^t$$
$$XX^t = XX^{t-1} \times PXLINK^t$$
$$MM^t = MM^{t-1} \times PMLINK^t$$
$$TTT^t = TTT^{t-1} \times PTLINK^t$$
$$LL^t = LL^{t-1} \times QLLINK^t$$
$$KK^t = KK^{t-1} \times QKLINK^t$$

Per Working Age Real Income Growth

We divide real income by population and normalise the result to the starting period to obtain real income growth per working age population.

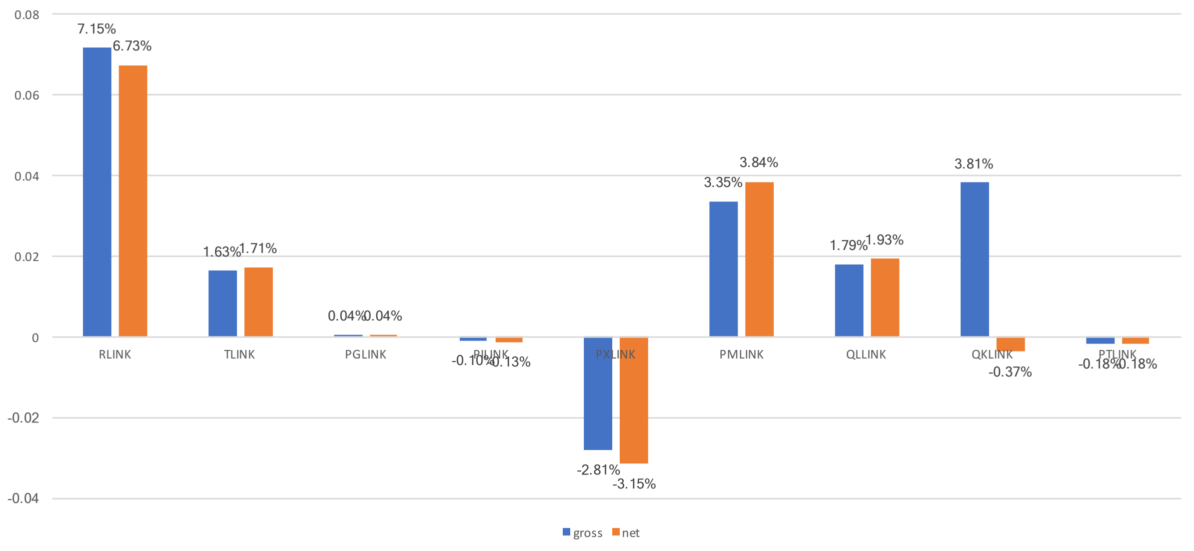
$$RIPERPOP = \frac{RI}{POP}$$

3.2.2 Real Net Income Growth Decomposition

Similar to constructing a real gross income growth decomposition, we now have to include end of period quantity of capital and beginning of period quantity of capital. We also remove the real asset price appreciation from user cost to generate real income.

3.2.3 Results

Figure 13: Diewert - Morrison Decomposition

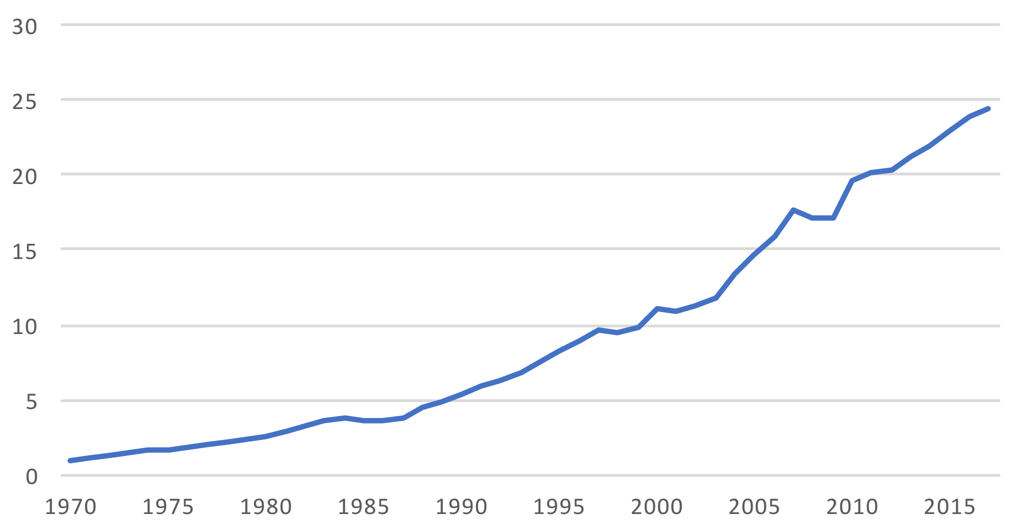


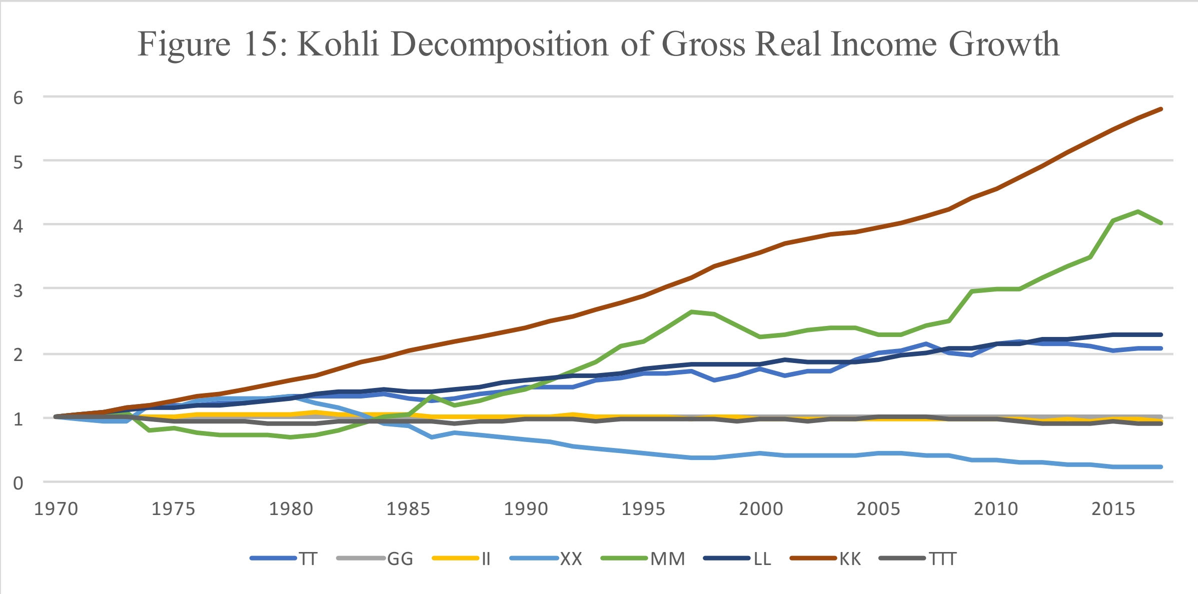
RLINK: Real income growth
TLINK: Productivity growth
PGLINK: Growth in government output
PILINK: Change in real investment prices
PXLINK: Change in real export prices
PMLINK: Change in real import prices
QLLINK: Growth in labour input
QKLINK: Growth in capital input
PTLINK: Terms of trade

Gross		Net	
Name	Average	Name	Average
RLINK	1.071499	RLINK	1.0673
TLINK	1.0163	TLINK	1.0171
PGLINK	1.0004	PGLINK	1.0004
PILINK	0.99903	PILINK	0.99872
PXLINK	0.97192	PXLINK	0.96848
PMLINK	1.0335	PMLINK	1.0384
QLLINK	1.0179	PKLINK	1.0152
QKLINK	1.0381	PKELINK	0.99158
PTLINK	0.99824	QLLINK	1.0193
		QWSLINK	1.0351
		PTLINK	0.99825
		PNKLINK	0.99702

From Figure 13, the Diewert and Morrison (1986) decomposition breaks down the various components of real income growth. We generate the average QKLINK for net real income by taking the average of the real price of the end (KKE) and beginning (KKB) of the period capital stock. Average gross real income growth was 7.15% and average net real income growth was 6.73%. The factors that contributed to gross real income growth was growth in capital input (QKLINK = 3.81%) which takes up more than half of gross real income growth, followed by the change in real import prices. On the other hand, for net real income growth, the main factors are the change in real import prices followed by growth in labour input.

Figure 14: Gross Real Income Growth

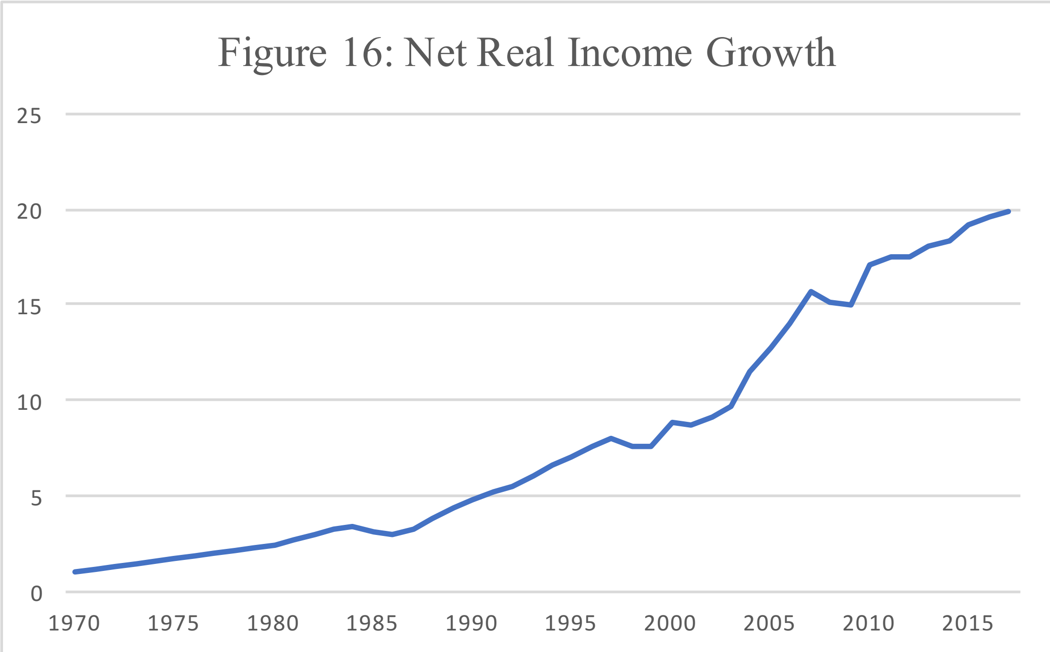


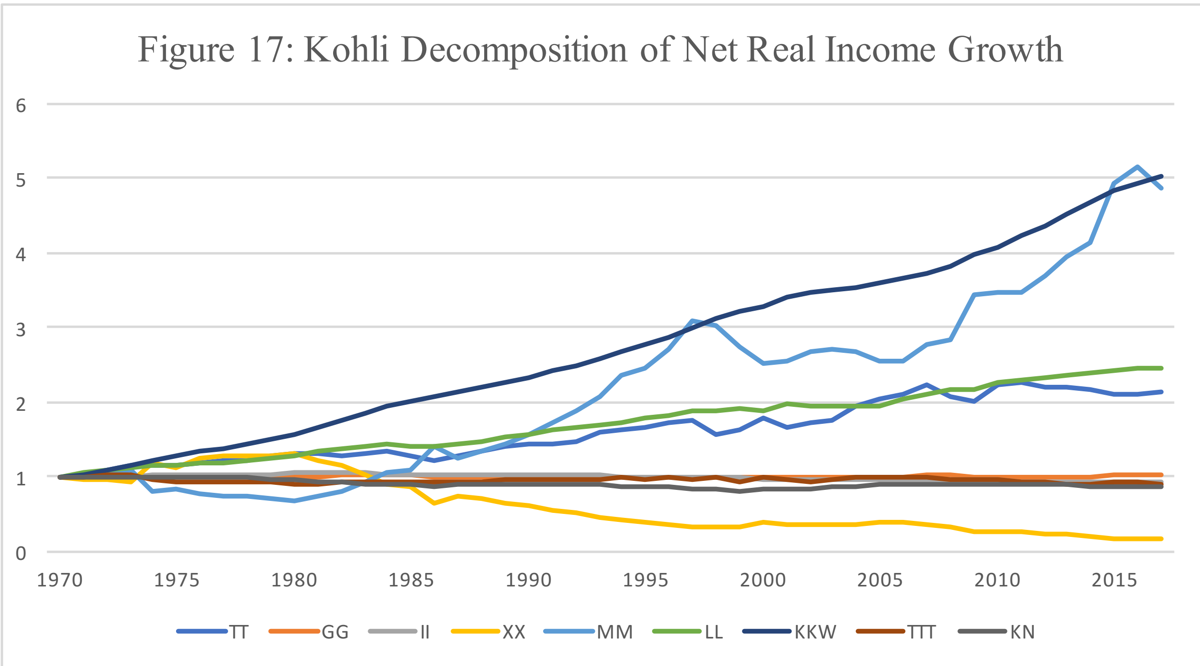


First, we analyse gross real income growth. From Figure 14, gross real income growth grew by 24.3 times over the sample period and it can be decomposed into the various factors in Figure 15. There were several dips in gross real income growth as can be associated with these events: the 1985 post-independence recession, the 1997 Asian Financial Crisis, the 2001 Dot Com Bubble, and the 2008 Global Financial Crisis. The biggest factors that contributed to real gross income growth in Singapore over the sample period were:

- KK; capital services; final cumulated growth factor = 5.79;
- MM: decreases in real import prices; final factor = 4.03
- LL; labour services; final factor = 2.29;
- TT; technical progress; final factor = 2.07;
- GG; increases in real government output prices; final factor = 1.02;

However, the declines in real export prices (XX: the light-blue line) outweighed the declines in real import prices. Hence, the terms of trade growth factor (TTT: the green line) decreased to a final factor of 0.914, which is a drag on real income growth during this period.



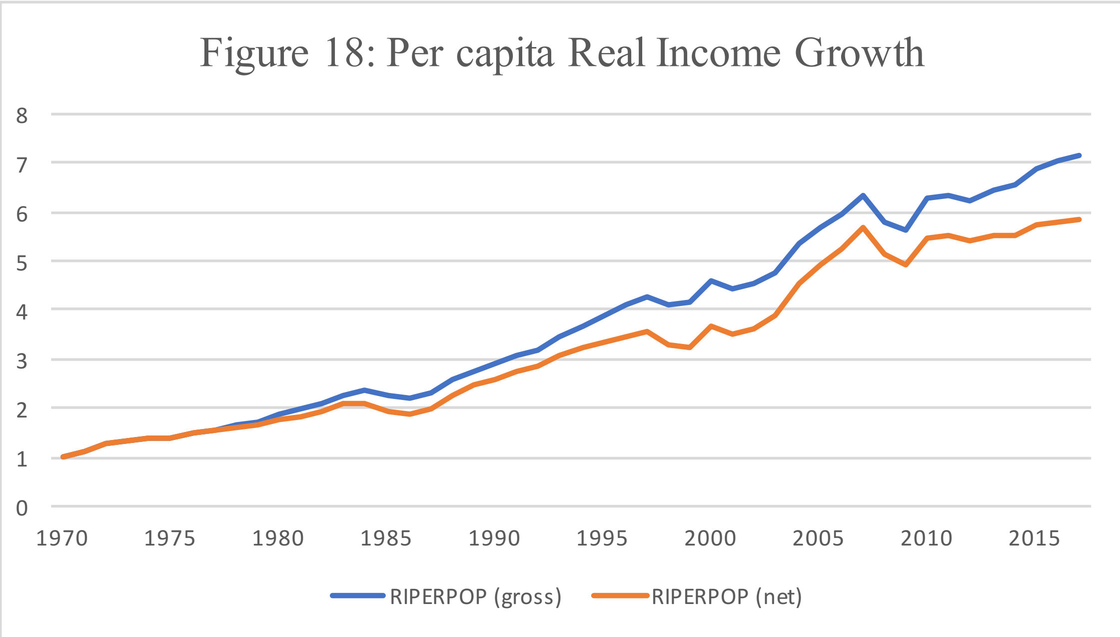


Next, we analyse net real income growth. As seen from Figure 16, net real income growth grew by approximately 20-fold over the sample period. We can see similar dips in periods as mentioned earlier. Referring to Figure 17, the biggest factors that contributed to real net income growth in Singapore over the sample period were:

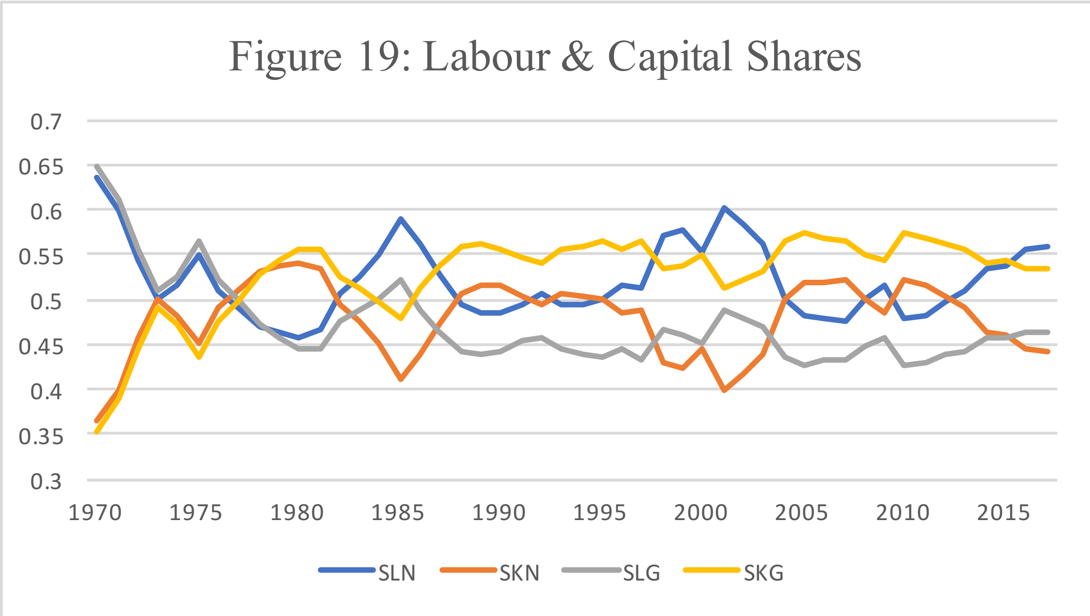
- KKW; capital waiting services; final cumulated growth factor = 5.04;
- MM: decreases in real import prices; final factor = 4.88
- LL; labour services; final factor = 2.44;

- TT; technical progress; final factor = 2.13;
- GG; increases in real government output prices; final factor = 1.02;
- KN; the net effect of changes in the real price of the beginning and end of period capital stocks; final factor = 0.87 which reduced income growth.

Similar to the decomposition of gross real income growth, the declines in real export prices outweighed the declines in real import prices. As such, the terms of trade growth factor decreased to a final factor of 0.913, which is a drag on real income growth as well.



Singapore's real income growth is mainly attributed to capital services, as seen from its transition to a service oriented economy and its huge trade with the rest of the world. Gross per capita real income growth grew by 7.2-fold while net per capita real income growth grew by 5.8-fold.



From Figure 19, labour share of income in GDP (SLG) has a downward trend from year 1970 to 1990 before remaining at about 45%, while capital share of income in GDP (SKG) moved in the opposite direction and fluctuated about 55% in recent years. Moreover, labour share of income in GNP (SLN) followed the same pattern as SLG till around 1980, before diverging. SLN remained higher than SLG and fluctuated about 50%. The opposite goes for capital share of income in GNP (SKN), where it is consistently higher than SKG after 1980.

4. Econometric Approach

In this section, we attempt to find a good functional form to estimate both consumer and producer demand functions and producer supply functions. The function form that we choose have to be flexible enough so that it does not restrict the elasticities of supply and demand. Flexible function forms have a second order approximation property, hence allowing elasticities of supply and demand to not be restricted.

4.1 Two Goods Consumer Model

A two-goods consumer regression model is used in this section, where Good 1 is per capita consumption and Good 2 is per capita labour supply converted into leisure demand. We use consumer data generated from Section 2 to convert it into leisure demand. The effective hours are assumed to be equal to twice the average hours of labour supplied and the maximum number of hours worked is arbitrary.

Firstly, we generate an after-tax value of labour supply and then find the difference between the value of consumption and the after-tax value of labour supply. Next, we calculate per capita utility and lastly, we will get the functional form for indirect utility U, the fitted expenditure *FE* and expenditure function *E(U,P).* The index and indirect utility we get from our demand model will be the same at the start of the sample period but they will diverge through time.

- Consumer expenditure minimizing problem:

$$E(u,p) = \min_{x}\{p^T x{:}f(x)\geq u\}$$

- Hicksian demand function as the derivative of expenditure function with respect to the price vector:

$$x = \nabla_p E(g(u,p)$$

We cannot observe the utility level so we use the expenditure cost and the price vector to represent it. Utility level $U = g(Y,p)$ and expenditure $Y = E(u,p)$. As such, the final demand function will be:

$$x = \nabla_p E(g(Y,p),p)$$

In the following consumer models, we follow the method from Diewert (1980) and will assume non-homothetic preferences.

4.1.1 CMOD1

CMOD1 is based on the normalized quadratic expenditure function and curvature conditions are not imposed.

$$E(u,p) = a^T p + (b^T p + \frac{1}{2}\frac{p^T Cp}{a^T p})u$$

- According to $x = \nabla_p E(u,p)$, the corresponding Hicksian demand function is:

$$x(u,p) = a + \left[b + \frac{1}{2}\frac{Cp(\alpha^T p) + p^T Cp\alpha}{(\alpha^T p)^2}\right] u$$

$$= a + \left[b + \left(\frac{1}{2}\frac{Cp}{\alpha^T p} + \frac{p^T Cp\alpha}{(\alpha^T p)^2}\right)\right] u$$

$$= a + \left[b + \frac{1}{2}\left(C\frac{p}{\alpha^T p} + \left(\frac{p}{\alpha^T p}\right)^T C\left(\frac{p}{\alpha^T p}\right)\alpha\right)\right] u$$

- To solve for U, we let expenditure Y to be equal to the expenditure function:

$$Y = E(u,p)$$

$$= a^T p + \left(b^T p + \frac{1}{2}\frac{p^T Cp}{a^T p}\right) u$$

- As such, we get

$$u = \frac{Y - a^T p}{b^T p + \frac{1}{2}\frac{p^T Cp}{a^T p}}$$

- The estimating equation is as follows:

$$x^t(u,p) = a + \frac{\left[b + Cv^t - \frac{1}{2}v^{tT} Cv^t \alpha\right]\left(\frac{Y^t - \alpha^T p^t}{\alpha^T p^t}\right)}{b^T v^t + \frac{1}{2}v^{tT} Cv^t} + e^t, \text{ where } v = \frac{p}{\alpha^T p}$$

- The expenditure share of i^th^ commodity is:

$$s_i^t = \frac{p_i^t x_i^t}{y} \alpha$$

$$= \frac{p_i^t}{Y}\left[a + \frac{\left(b + Cv^t - \frac{1}{2}v^{tT} Cv^t \alpha\right)\left(\frac{Y^t - \alpha^T p^t}{\alpha^T p^t}\right)}{b^T v^t + \frac{1}{2}v^{tT} Cv^t} + e^t\right]_i$$

- $PC_i = \sum_{j=1}^{N} C^*_{ij} v_j$
- $\text{PCP} = \sum_{i=1}^{\text{N}} p_i v_i$
- $u^* = \frac{Y - a^{*T} p}{b^{*T} p + \frac{1}{2} \frac{p^T C^* p}{a^{*T} p}}$
- The price elasticities of demand EC and income elasticity ECU are:

$$\text{EC}_{ij} = \frac{(\text{C}^*_{ij} - \alpha_i \text{CP}_j - \alpha_j \text{CP}_i + \alpha_i \alpha_j \text{PCP}) v_j u^*}{x^*_i}$$

$$\text{ECU}_i = \frac{(b^*_i + \text{CP}_i - \frac{1}{2} \alpha_i \text{PCP}) u^*}{x^*_i}$$

4.1.2 CMOD2

In CMOD2, we substitute the C matrix by AA^T matrix, where A is a lower triangular matrix and $Ap^*=0_N$ so as to impose the curvature conditions because we want the expenditure function to be concave in price vector p. Should CMOD1 satisfy the curvature conditions, then it will have the same results as CMOD2. The result of all price elasticities of demand in both CMOD1 and CMOD2 are identical since CMOD1 satisfies the curvature conditions.

4.1.3 CMOD3

In CMOD3, we calculate utility using the exact index number formula for the normalized quadratic functional form from Diewert (2013; 21,22).

- $$\begin{aligned} Q_{NQ}(p^0, p^1, q^0, q^1; \alpha) &= [\rho^0 + \rho^1] \cdot \frac{q^1}{[\rho^0 + \rho^1]} \cdot q^0 \\ &= [\tfrac{1}{2}\rho^0 + \tfrac{1}{2}\rho^1] \cdot q^1 / [\tfrac{1}{2}\rho^0 + \tfrac{1}{2}\rho^1] \cdot q^0 \end{aligned}$$
- The expenditure function is:

$$E(u,p) = a^T p + (b^T p + \frac{1}{2}\frac{p^T C p}{a^T p})u$$

$$= a^T p + \frac{1}{2}u\frac{p^T A A^T p}{a^T p} + d(u,p)$$

We define G1 and G2 to be equal to the sample means of X1 and X2 respectively, where X1 is per capita consumption and X2 is per capita labour supply. After which, we normalize G1 and G2 to sum to one.

- $PG = PC_1 \; x \; G_1 + PC_2 \; x \; G_2$

- Income or expenditure normalized prices:

  $PY_1 = \frac{PC_1}{Y}, PY_2 = \frac{PC_2}{Y}$

- Expenditure shares:

  $s_1 = PY_1 \; x \; X_1, s_2 = PY_2 \; x \; X_2$

- Normalize prices as v_1 and v_2:
- $v_1 = \frac{PC_1}{PG}, v_2 = \frac{PC_2}{PG}$

- The exact index number formula for normalized quadratic preferences using a fixed base:

$$\frac{u^t}{u^1} = \frac{[(v_1^t+v_1^1)x_1^t+(v_2^t+v_2^1)x_2^t]}{[(v_1^t+v_1^1)x_1^1+(v_2^t+v_2^1)x_2^1]}$$

We normalize the first period (*year* = 1970) index utility and UNQCH[1] to 1.

- The chained normalized quadratic exact indexes:

$$\text{NQLINK} = \frac{[(v_1^t+v_1^{t-1})x_1^t+(v_2^t+v_2^{t-1})x_2^t]}{[(v_1^t+v_1^{t-1})x_1^{t-1}+(v_2^t+v_2^{t-1})x_2^{t-1}]}$$

- The implied indirect utility UI:

$$\mathrm{UI} = \frac{[\mathrm{Y} - (\alpha_1 \times pc_1 + \alpha_2 \times pc_2)]}{\left[\beta_1 \times pc_1 + \beta_2 \times pc_2 + \frac{1}{2} v^{\mathrm{T}} \mathrm{C} v \times pg\right]}$$

- $\mathrm{UNQCH}^{t} = \mathrm{UNQCH}^{t-1} \times \mathrm{NQLINK}$

$$= \mathrm{UNQCH}^{t-1} \times \frac{[(v_1^t + v_1^{t-1}) x_1^t + (v_2^t + v_2^{t-1}) x_2^t]}{[(v_1^t + v_1^{t-1}) x_1^{t-1} + (v_2^t + v_2^{t-1}) x_2^{t-1}]}$$

- Fitted X's and expenditure shares:

$$FX1 = \alpha_1 + \beta_1 \ x \ U + (c_{11} \ x \ v_1 + c_{12} \ x \ v_2) \ x \ U - \frac{1}{2} g_1 \ x \ v^T C v \ x \ U$$

$$FX2 = \alpha_2 + \beta_2 \ x \ U + (c_{21} \ x \ v_1 + c_{22} \ x \ v_2) \ x \ U - \frac{1}{2} g_2 \ x \ v^T C v \ x \ U$$

4.1.4 CMOD4

Similar to CMOD3, we calculate utility using the exact index number formula for the normalized quadratic functional form. We use break points in CMOD3 to impose splines on utility to improve the fit of our equations. The break points we use are at observations 4 (*year* = 1973), 6 (*year* = 1975), 11 (*year* = 1980), 16 (*year* = 1985), 28 (*year* = 1997), 36 (*year* = 2005).

We then plug the break points into the implicit utility:

$$\mathrm{UI} = \frac{(\mathrm{Y} - \mathrm{AP})}{(\mathrm{BP1} + \frac{1}{2} v^{\mathrm{T}} \mathrm{C} v \times \mathrm{PG})}$$

In CMOD4, the correlation between U and UI is 0.99958. This is lower than the corresponding correlation in CMOD3, where it is 0.99986. However, we have a better fit of estimating equation in CMOD4, as such CMOD4 is the most preferred amongst all of the 4 consumer models.

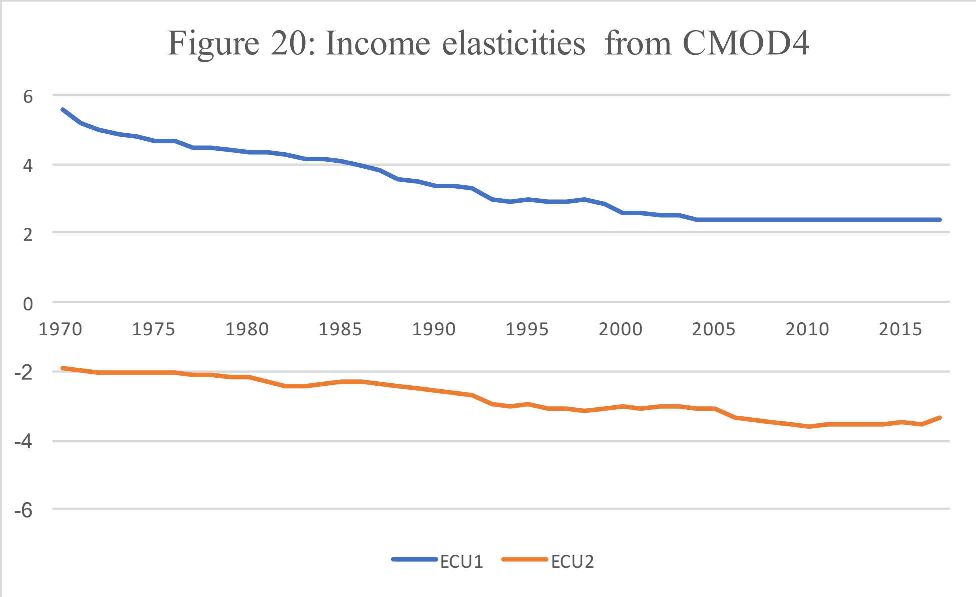
4.1.5 CMOD Results

The curvature conditions are satisfied for all CMOD1, CMOD2, CMOD3 and CMOD4 models. In CMOD4, we impose splines and allow time trend to change and this increases the log-likelihood by 52 points. Average R-square is the highest in CMOD4.

Though curvature conditions are not imposed in CMOD1, it is actually satisfied since price elasticities EC11 and EC22 are both positive while the cross elasticities EC12 and EC21 are both negative. Theoretically, price elasticity is the measure of percentage change in quantity demanded with respect to a percentage change in price and should be non-positive. Further, cross elasticity is the measure of percentage change in quantity demanded of the good with respect to a percentage change in price of another good and should be non-negative. Hence, both price and cross elasticities across CMOD1 to CMOD4 have the correct signs.

Table 1: Regression Results for Consumer Models

	CMOD1	**CMOD2**	**CMOD3**	**CMOD4**
Description	Consumer regression model based on normalized quadratic expenditure function	Consumer regression model based on normalized quadratic expenditure function	Consumer regression model based on normalized quadratic expenditure function (utility calculated from index numbers)	Consumer regression model based on normalized quadratic expenditure function (utility calculated from index numbers & splined imposed)
Curvature Conditions	Satisfied	Imposed	Imposed	Imposed
Final Log-Likelihood	111.2248	111.2248	48.86103	100.9022
EQ1 R^2	0.8099	0.8099	0.9858	0.9892
EQ2 R^2			0.8943	0.9607
EC11 (-)	-0.58822	-0.58822	-0.068659	-0.020157
EC12 (+)	0.58822	0.58822	0.068659	0.020157
EC21 (+)	0.59689	0.59689	0.067921	0.019882
EC22 (-)	-0.59689	-0.59689	-0.067921	-0.019882
ECU1	1.3639	1.3639	1.1478	3.3823
ECU2	0.4247	0.4247	0.79778	-2.7769
Corr(U,UI)			0.99986	0.99958



Income elasticity of consumption, ECU1, is positive for all the consumer models. Income elasticity of leisure, ECU2, is negative from CMOD1 to CMOD3 and it turns negative in CMOD4. Since CMOD4 is the most reasonable with the largest R-squared and log-likelihood, we can say that consumption is a normal good while leisure is an inferior good. This indicates that as income increases, consumption will increase while leisure will decline which implies that people choose to work more. From Figure 20, the magnitude of income elasticity of consumption has declined to 2.35 while the magnitude of income elasticity of leisure has increased to 3.36 in year 2017.

4.2 Seven Goods Producer Model

The 7 goods we use in our producer models are consumption, government output, investment, exports, imports, labour as well as fixed capital services. We use data generated from Section 2 to implement in our producer models.

The profit function, $V(p,k)$:

$$V(p,k) = \max_{y}\{p^{T}y : k=F(y)\}\ \forall\ p>>0_N$$

We will assume the firm's capital requirement function, $F(y)$, to be homogenous and choose the functional form $v(p)$ to be various flexible function forms. We implement three types of profit functional forms, namely translog, Leontief as well as normalized quadratic forms. PMOD1 and PMOD2 use the translog functional form, PMOD3 uses the Leontief functional form while PMOD4 to PMOD12 use the normalized quadratic functional form.

4.2.1 PMOD1 to PMOD2

PMOD1 is using the constant returns to scale translog variable profit function. The translog unit profit function, *v(p,t)* is:

$$lnv(p,t) = \alpha_0 + \beta_0 t + \sum_{i=1}^{6} \alpha_i lnp_i^t + \frac{1}{2}\sum_{i=1}^{6}\sum_{j=1}^{6} \gamma_{ij} lnp_i^t lnp_j^t + \sum_{i=1}^{6} \beta_i lnp_i^t$$

We have a couple of restrictions:

$\gamma_{ij} = \gamma_{ji};$	$1 \le i<j \le N$	15 symmetry restriction
$\sum_{i=1}^{6} \alpha_i = 1$		1 restriction
$\sum_{i=1}^{6} \gamma_{ij} = 0$	i = 1, …, 6	6 restrictions

The period t observed variable profit or gross return to capital is:

$$V^t \equiv p^{tT} y^t \equiv \sum_{i=1}^{N} p_i^t y_i^t$$

Since we assume *F(y)* to be homogenous,

$$V(k,p,t) \equiv kv(p,t)$$

The estimating equation for *V(k,p,t)* is:

For t = 1,…,48,

$$lnV^t = lnk^t + lnv^t + e_0^t$$

$$= lnk^t + \alpha_0 + \beta_0 t + \sum_{i=1}^{6} \alpha_i lnp_i^t + \frac{1}{2}\sum_{i=1}^{6}\sum_{j=1}^{6} \gamma_{ij} lnp_i^t lnp_j^t + \sum_{i=1}^{6} \beta_i lnp_i^t + e_0^t$$

So,

$$ln\left[\frac{V^t}{k^t}\right] = \alpha_0 + \beta_0 t + \sum_{i=1}^{6}\alpha_i lnp_i^t + \frac{1}{2}\sum_{i=1}^{6}\sum_{j=1}^{6}\gamma_{ij} lnp_i^t lnp_j^t + \sum_{i=1}^{6}\beta_i lnp_i^t + e_0^t$$

Estimating equations for the output share function $s_i^t(k,p)$ for t=1,…,48 and i=1,…,6

$$\begin{aligned} s_i^t(k,p) &= \frac{p_i^t y_i^t}{V(k,p,t)} \\ &= p_i^t \frac{k\partial v(p,t)}{\partial p_i}\frac{1}{V(k,p,t)} \qquad \text{using Hotelling's lemma } y_i^t = \frac{k\partial v(p,t)}{\partial p_i} \\ &= p_i^t \frac{k\partial v(p,t)}{\partial p_i}\frac{1}{kv(p,t)} \\ &= p_i^t \frac{\partial v(p,t)}{\partial p_i}\frac{1}{v(p,t)} \\ &= \frac{\partial lnv(p,t)}{\partial lnp_i} \end{aligned}$$

As such,

$$s_i^t(k,p) = \alpha_i + \sum_{j=1}^{5}\gamma_{ij} lnp_j + \beta_i^t + e_i^t$$

Due to the restrictions on the translog unit profit function and $\sum_{i=1}^{5} s_i^t = 1$, for t=1,…,48 and i=1,…,5:

$$ln\left[\frac{V^t}{p_6^t k^t}\right] = \alpha_0 + \beta_0 t + \sum_{i=1}^{5}\alpha_i ln(\frac{p_i^t}{p_6^t}) + \sum_{i=1}^{5}\beta_i t ln(\frac{p_i^t}{p_6^t}) + \frac{1}{2}\sum_{i=1}^{5}\gamma_{ij}\left[ln\left(\frac{p_i^t}{p_6^t}\right)\right]^2$$

$$+\sum_{i=1}^{5}\sum_{j=1, i<j}^{5}\gamma_{ij} ln\left(\frac{p_i^t}{p_6^t}\right) ln\left(\frac{p_j^t}{p_6^t}\right) + e_0^t$$

And

$$s_i^t(k,p) = \alpha_i + \sum_{i=1}^{5}\gamma_{ij}\, ln\left(\frac{p_i^t}{p_6^t}\right) + \beta_i t + e_i^t$$

The estimates for the period t price elasticity:

$$e_{ij}^t = \frac{\partial lny_i(k^t,p^t)}{\partial p_j} = [s_i^{t*}]^{-1}\gamma_{ij}^* + s_j^{t*} \quad \text{for cross elasticities of net supply}$$

$$e_{ii}^t = \frac{\partial lny_i(k^t,p^t)}{\partial p_i} = [s_i^{t*}]^{-1}\gamma_{ii}^* + s_i^{t*} \quad \text{for own elasticities of net supply}$$

For us to measure the technical progress, we differentiate *V(k,p,t)* with respect to time and evaluate at the estimate values of β_i for i=1,…,6.

$$\mathrm{TECH}^t \equiv \frac{\partial ln\mathrm{V}(k,p,t)}{\partial t} = \beta_0^* + \sum_{i=1}^{6} \beta_i^* lnp_i^* = \left[\beta_0^* + \sum_{i=1}^{6} \beta_i^* lnp_i^t\right]\left[\frac{\mathrm{V}^{t*}}{\mathrm{V}^{t*} - p_6^t y_6^t}\right]$$

PMOD2 is similar to PMOD1 but we impose linear splines to model technical progress, by using breakpoints from each of the six equations in PMOD1.

The time trend $d_i(t)$ now becomes:

$$\begin{cases} \beta_{i1} t & 1 \le t \le t_1 \\ \beta_{i1} t_1 + \beta_{i2}(t - t_1) & t_1 < t \le t_2 \\ \beta_{i1} t_1 + \beta_{i2}(t_2 - t_1) + \beta_{i3}(t - t_2) & t_2 < t \le t_3 \\ \beta_{i1} t_1 + \beta_{i2}(t_2 - t_1) + \beta_{i3}(t_3 - t_2) + \beta_{i4}(t - t_3) & t_3 < t \le t_4 \\ \beta_{i1} t_1 + \beta_{i2}(t_2 - t_1) + \beta_{i3}(t_3 - t_2) + \beta_{i4}(t_4 - t_3) + \beta_{i5}(t - t_4) & t_4 < t \le t_5 \\ \beta_{i1} t_1 + \beta_{i2}(t_2 - t_1) + \beta_{i3}(t_3 - t_2) + \beta_{i4}(t_4 - t_3) + \beta_{i5}(t_5 - t_4) + \beta_{i6}(t - t_5) & t_5 < t \le 48 \end{cases}$$

The translog unit function with the multiple time trend is:

$$lnv(p,t) = \alpha_0 + d_0(t) + \sum_{i=1}^{6} \alpha_i lnp_i^t + \frac{1}{2}\sum_{i=1}^{6}\sum_{j=1}^{6} \gamma_{ij} lnp_i^t lnp_j^t + \sum_{i=1}^{6} d_i(t) lnp_i^t$$

The estimation equations for t=1,…,48 and i=1,…,5:

$$ln\left[\frac{V^t}{p_6^t k^t}\right] = \alpha_0 + d_0(t) + \sum_{i=1}^{5} \alpha_i ln\left(\frac{p_i^t}{p_6^t}\right) + \sum_{i=1}^{5} d_i(t) ln\left(\frac{p_i^t}{p_6^t}\right) + \frac{1}{2}\sum_{i=1}^{5} \gamma_{ij}\left[ln\left(\frac{p_i^t}{p_6^t}\right)\right]^2$$

$$+ \sum_{i=1}^{5} \sum_{j=1.i<j}^{5} \gamma_{ij} ln\left(\frac{p_i^t}{p_6^t}\right) ln\left(\frac{p_j^t}{p_6^t}\right) + e_0^t$$

$$s_i^t(k,p) = \alpha_i + \sum_{i=1}^{5} \gamma_{ij} ln\left(\frac{p_i^t}{p_6^t}\right) + d_i(t) + e_i^t$$

We use the same price elasticity formula as in PMOD1 and there is some change on the measure of technical progress. We multiply TFP growth rate from the index number model by $\frac{V^{t*}}{V^{t*} - p_6^t y_6^t}$.

$$\text{TECH}^t \equiv \frac{\partial ln V(k,p,t)}{\partial t}$$

$$= \frac{\partial d_0^*(t)}{\partial t} + \sum_{i=1}^{6} \frac{\partial d_i^*(t)}{\partial t} lnp_i^t = \left[\frac{\partial d_0^*(t)}{\partial t} + \sum_{i=1}^{6} \frac{\partial d_i^*(t)}{\partial t} lnp_i^t\right]\left[\frac{V^{t*}}{V^{t*} - p_6^t y_6^t}\right]$$

4.2.2 PMOD3

In PMOD3, we estimate a Leontief no substitution profit function. We have two different models here, first being a fixed costs and non-CRS model and second being a CRS model and no substitution between inputs or outputs model.

In the non-CRS Leontief model, the variable profit function is:

$$V(p,k,t) = a^T p + b^T pk + c^T pkt + d^T pt$$

Returns to scale:

$$RS = \frac{b^{*T} pk + c^{*T} pkt}{V^{t*}}$$

To make the measure of technical progress comparable to estimates of TFP growth, we replace fitted profits in the denominator of technical progress by the sum of gross profits plus the fitted value of labour input.

$$\mathrm{TECH}^t = \frac{\partial ln\mathrm{V}(p,k,t)}{\partial t}\frac{\mathrm{V}^{t*}}{\mathrm{V}^{t*} - p_6^t y_6^t}$$
$$= \frac{c^{*\mathrm{T}}pk}{\mathrm{V}^{t*} - p_6^t y_6^t}$$

In the CRS Leontief model, the variable profit function is:

$$V(p,t) = b^T pk + c^T pkt$$

The technical progress is:

$$\mathrm{TECH}^t = \frac{\partial ln\mathrm{V}(p,k,t)}{\partial t}\frac{\mathrm{V}^{t*}}{\mathrm{V}^{t*} - p_6^t y_6^t} = \frac{c^{*\mathrm{T}}pk}{\mathrm{V}^{t*} - p_6^t y_6^t}$$

4.2.3 PMOD4 to PMOD7

From PMOD4 to PMOD7, we will use the normalized quadratic profit function. We define production unit's period t variable profit function *V(k,p,t)* as:

$$V(k,p,t) \equiv b^T pk + \left(\frac{1}{2}\right)\left(\frac{p^T Bp}{\alpha^T p}\right)k + c^T ptk$$

where $\alpha^T = (\frac{\mu_i}{\sum_{i=1}^{6}\mu_i})_{i=1,\ldots,6}$, where μ_i is the mean of x_i and b, c and B are estimated objects.

In PMOD4, cc^T is added, where cc^T is a rank 1 substitution matrix and the unit profit function is:

$$V(k,p,t) = b^T pk + \left(\frac{1}{2}\right)\left(\frac{p^T cc^T p}{\alpha^T p}\right)k + c^T ptk$$

In PMOD5, dd^T is added, where dd^T is a rank 2 substitution matrix and the unit profit function is:

$$V(k,p,t) = b^T pk + \left(\frac{1}{2}\right)\left(\frac{p^T(cc^T + dd^T)p}{\alpha^T p}\right)k + c^T ptk$$

In PMOD6, ff^T is added, where ff^T is a rank 3 substitution matrix and the unit profit function is:

$$V(k,p,t) = b^T pk + \left(\frac{1}{2}\right)\left(\frac{p^T(cc^T + dd^T + ff^T)p}{\alpha^T p}\right)k + c^T ptk$$

In PMOD7, hh^T is added, where hh^T is a rank 4 substitution matrix and the unit profit function is:

$$V(k,p,t) = b^T pk + \left(\frac{1}{2}\right)\left(\frac{p^T(cc^T + dd^T + ff^T + hh^T)p}{\alpha^T p}\right)k + c^T ptk$$

Technical progress is:

$$TECH^t = \frac{\beta^{*T} pk}{V^{t*} - p_6^t y_6^t}$$

4.2.4 PMOD8 to PMOD12

We differentiate the normalized quadratic variable profit function without the use of splines from PMOD8 to PMOD11.

$$y(k,p,t) = \nabla_p V(k,p,t) = bk + [(\alpha^T p)^{-1} Bp - (1/2)(\alpha^T p)^{-2} p^T Bp\alpha]k + \nabla_p d(k,p,t).$$

In PMOD8, we estimate a normalized quadratic cost function with a rank 4 substitution matrix (from PMOD7) with single time trends for technical progress. First, we allow for a linear time trend for the c vector in the substitution matrix. The unit profit function is:

$$V(k,p,t) = b^T pk + \left(\frac{1}{2}\right)\left(\frac{p^T\left(\frac{1-t}{T}\right)cc^T + \left(\frac{t}{T}\right)c'c'^T + dd^T + ff^T + hh^T)p}{\alpha^T p}\right)k + c^T ptk$$

In PMOD9, we allow for a linear time trend for two vectors c and d in the substitution matrix. The unit profit function is as follows:

$$V(k,p,t) = b^T pk + \left(\frac{1}{2}\right)\left(\frac{p^T\left(\frac{1-t}{T}\right)(cc^T + dd^T) + \left(\frac{t}{T}\right)(c'c'^T + dd^T) + ff^T + hh^T)p}{\alpha^T p}\right)k + c^T ptk$$

In PMOD10, we allow for a linear time trend for three vectors c, d and f in the substitution matrix. The unit profit function is as follows:

$$V(k,p,t) = b^T pk + \left(\frac{1}{2}\right)\left(\frac{p^T\left(\frac{1-t}{T}\right)(cc^T + dd^T + ff^T) + \left(\frac{t}{T}\right)(c'c'^T + dd^T + ff^T) + hh^T)p}{\alpha^T p}\right)k + c^T ptk$$

In PMOD11, we allow for a linear time trend for four vectors c, d, f and h in the substitution matrix. The unit profit function is as follows:

$$V(k,p,t) = b^T pk + \left(\frac{1}{2}\right)\left(\frac{p^T\left(\frac{1-t}{T}\right)(cc^T + dd^T + ff^T + hh^T) + \left(\frac{t}{T}\right)(c'c'^T + dd^T + ff^T + hh^T))p}{\alpha^T p}\right)k + c^T ptk$$

Finally, in PMOD12, we use splined time trends instead of single time trends and change the normalized quadratic profit function into a splined model. The time trend *d(t)* is:

$$
\begin{cases}
\beta_{i1}t & 1 \le t \le t_1 \\
\beta_{i1}t_1 + \beta_{i2}(t - t_1) & t_1 < t \le t_2 \\
\beta_{i1}t_1 + \beta_{i2}(t_2 - t_1) + \beta_{i3}(t - t_2) & t_2 < t \le t_3 \\
\beta_{i1}t_1 + \beta_{i2}(t_2 - t_1) + \beta_{i3}(t_3 - t_2) + \beta_{i4}(t - t_3) & t_3 < t \le t_4 \\
\beta_{i1}t_1 + \beta_{i2}(t_2 - t_1) + \beta_{i3}(t_3 - t_2) + \beta_{i4}(t_4 - t_3) + \beta_{i5}(t - t_4) & t_4 < t \le t_5 \\
\beta_{i1}t_1 + \beta_{i2}(t_2 - t_1) + \beta_{i3}(t_3 - t_2) + \beta_{i4}(t_4 - t_3) + \beta_{i5}(t_5 - t_4) + \beta_{i6}(t - t_5) & t_5 < t \le 48
\end{cases}
$$

The periods t* and t** are break points that the rate of technological change shifts from one to another.

For our consumption equation (EQ1), our break points are at observations 11(*year*=1980), 17(*year*=1986), 24(*year*=1993), 29(*year*=1998), 38(*year*=2007), 43(*year*=2012).

For government equation (EQ2), our break points are at observations 3(*year*=1972), 6(*year*=1975), 20(*year*=1989), 31(*year*=2000), 35(*year*=2004), 41(*year*=2010).

For investment equation (EQ3), our break points are at observations 15(*year*=1984), 17(*year*=1986), 23(*year*=1992), 31(*year*=2000), 34(*year*=2003), 43(*year*=2012).

For exports equation (EQ4), our break points are at observations 9(*year*=1978), 12(*year*=1981), 22(*year*=1991), 27(*year*=1996), 32(*year*=2001), 38(*year*=2007).

For imports equation (EQ5), our break points are at observations 9(*year*=1978), 11(*year*=1980) ,22(*year*=1991), 28(*year*=1997), 32(*year*=2001), 39(*year*=2008).

For labour equation (EQ6), our break points are at observations 9(*year*=1978), 15(*year*=1984), 19(*year*=1988), 28(*year*=1997), 35(*year*=2004), 38(*year*=2007).

4.2.5 PMOD Results

In PMOD1 and PMOD2, since we are using the translog functional form, the value of R-squared and the absolute value of log-likelihood are low as compared to the other producer models, since *k* and *t* in the translog functional form could be highly multicollinear. Also, curvature conditions for both PMODs fail as the elasticity for government is negative in PMOD1 and elasticity for consumption is negative in PMOD2. As such, the level of technical progress for both PMODs are lower than our ideal value of 1.62% and the translog functional form is not satisfactory.

In PMOD3, the first model allows for non-constant returns to scale and returns to scale was estimated at 0.4033. Ideally, returns to scale should be between 0.9 and 1.2 so our estimate is way lower than ideal. Estimate for technical progress is 18% which is much higher than our ideal. In the second Leontief model, the estimate for technical progress was 1.31% which is lower than what we want. As such, the Leontief model applied here is also not satisfactory.

From PMOD4 to PMOD7, the curvature conditions are imposed and the sign for the elasticities of each component are good. There is no significant jump in log-likelihood (by about 4 points) from PMOD4 to PMOD7. There is also no difference in estimates between PMOD6 and PMOD7, hence adding an additional rank matrix from 3 to 4 does not improve our estimates. In PMOD7, our estimate of technical progress is 2.01% which is higher than ideal.

From PMOD8 to PMOD11, the log-likelihood jumped in absolute value by about 64 points. The average R-squared is also high, implying a better fit of the model. Also, between PMOD10 and 11, there is no difference in estimates as the h-vector coefficients are 0. In PMOD11, curvature conditions are imposed and estimate of technical progress is 1.79% which is still higher than the ideal 1.62%.

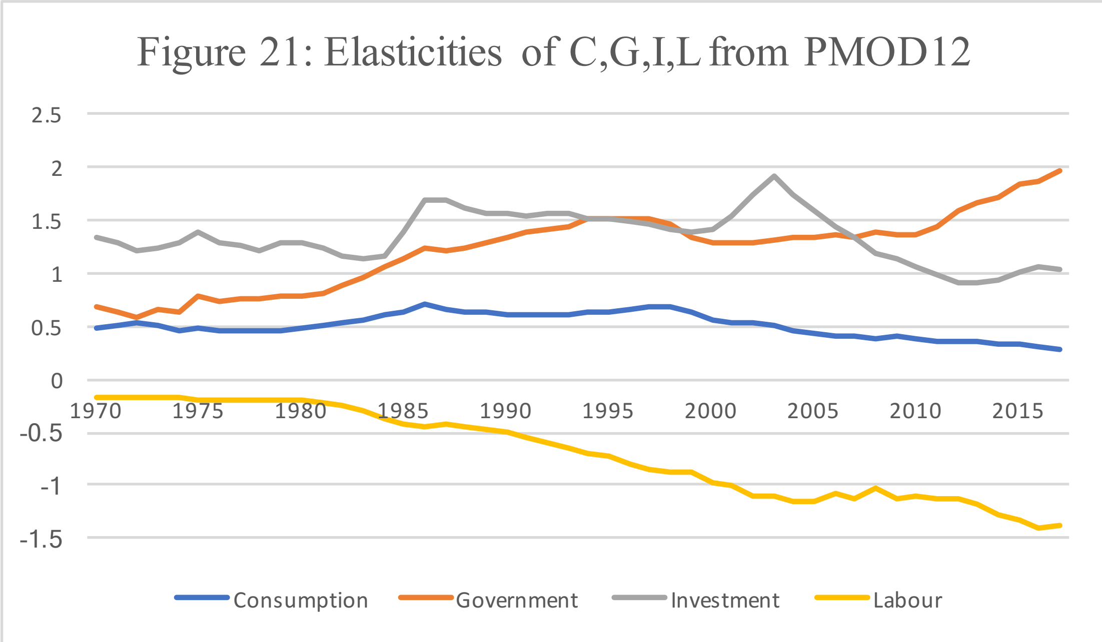
Finally, in PMOD12, we implement splines into our model and there is a major improvement in log-likelihood by 234 points. Average R-squared is also the highest amongst the rest of the PMODs. Final estimate of technical progress is 1.5% which is just slightly lower than our ideal 1.62%.

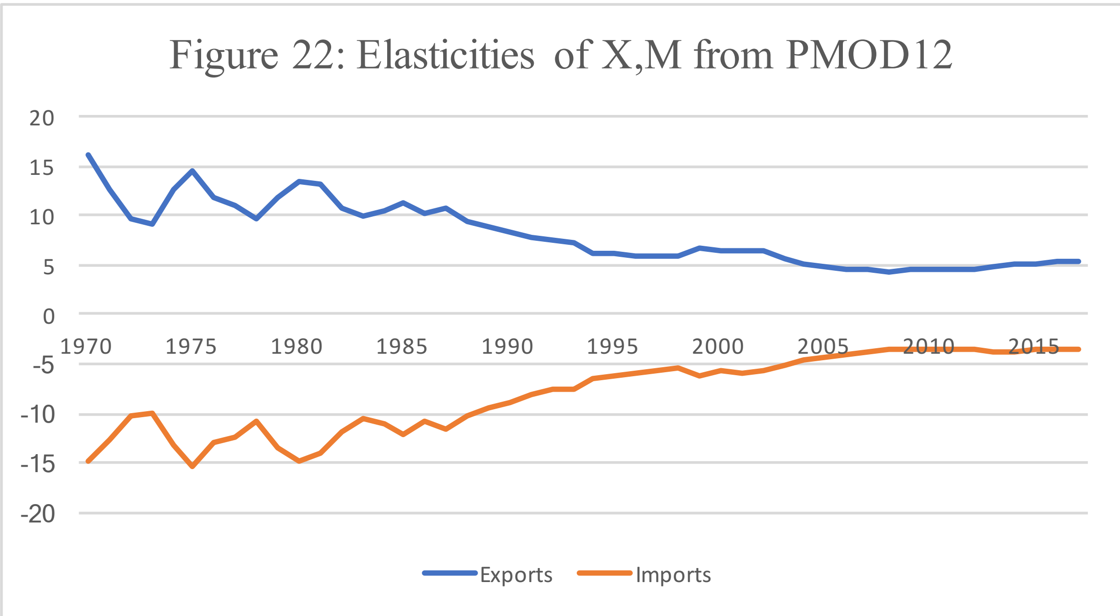
Table 2: Regression Results for Producer Models

	Ideal	PMOD1	PMOD2	PMOD3(1)	PMOD3(2)	PMOD4	PMOD5	PMOD6
Description		Translog variable profit function with CRS	Translog variable profit function with CRS & linear splines	Leontief profit function with fixed costs (Non-CRS: RS=0.40330)	Leontief profit function with CRS & no substitution between inputs/outputs	NQ cost function with a rank 1 substitution matrix	NQ cost function with a rank 2 substitution matrix	NQ cost function with a rank 3 substitution matrix
Curvature Conditions		Failed	Failed	Imposed	Imposed	Imposed	Imposed	Imposed
Final Log-Likelihood		426.6352	594.7247	-511.1719	-573.9909	-568.7289	-564.8827	-564.3768
EQ1 R^2		0.8615	0.8412	0.9902	0.9899	0.9888	0.9915	0.9909
EQ2 R^2		0.8202	0.8849	0.9863	0.9861	0.9844	0.9883	0.9885
EQ3 R^2		0.4547	0.95	0.9556	0.9494	0.9542	0.9533	0.9534
EQ4 R^2		0.6085	0.8806	0.9783	0.9734	0.9732	0.9853	0.9841
EQ5 R^2		0.5549	0.9387	0.976	0.9711	0.97	0.982	0.9805
EQ6 R^2		0.3551	0.9675	0.9911	0.9737	0.9742	0.9696	0.9707
E11 (Consumption)	(+)	0.81454	-0.08297	-	-	0.11285	0.099708	0.14575
E22 (Government)	(+)	-0.41378	0.67553	-	-	0.060914	0.07376	0.071543
E33 (Investment)	(+)	1.6028	0.22716	-	-	1.481	1.5466	1.5524
E44 (Exports)	(+)	1.2679	1.542	-	-	0.058458	5.8274	5.4961
E55 (Imports)	(-)	-2.2515	-1.855	-	-	-0.030941	-4.4516	-4.3978
E66 (Labour)	(-)	-0.43772	-0.32447	-	-	-0.17788	-0.453	-0.40894
Technical Progress (TECH)	1.62%	1.58%	1.47%	18%	1.31%	1.55%	2.06%	2.01%

Table 3: Regression Results for Producer Models

	Ideal	PMOD7	PMOD8	PMOD9	PMOD10	PMOD11	PMOD12
Description		NQ cost function with a rank 4 substitution matrix	NQ cost function with a rank 4 substitution matrix (linear time trend C vector)	NQ cost function with a rank 4 substitution matrix (linear time trend C,D vectors)	NQ cost function with a rank 4 substitution matrix (linear time trend C,D,F vectors)	NQ cost function with a rank 4 substitution matrix (linear time trend C,D,F,H vectors)	NQ cost function with a rank 4 substitution matrix (splined; linear time trend C,D,F,H vectors)
Curvature Conditions		Imposed	-	-	-	Imposed	Imposed
Final Log-Likelihood		-564.3768	-510.8968	-502.7129	-500.8563	-500.8563	-268.5354
EQ1 R^2		0.9909	0.9937	0.9941	0.9947	0.9947	0.9992
EQ2 R^2		0.9885	0.9938	0.996	0.9957	0.9957	0.9979
EQ3 R^2		0.9534	0.9557	0.9519	0.9505	0.9505	0.9855
EQ4 R^2		0.9841	0.989	0.9893	0.9893	0.9893	0.998
EQ5 R^2		0.9805	0.9879	0.988	0.9881	0.9881	0.9967
EQ6 R^2		0.9707	0.9832	0.9841	0.9848	0.9848	0.9981
E11 (Consumption)	(+)	0.14575	-	-	-	0.38911	0.5049
E22 (Government)	(+)	0.071543	-	-	-	0.40941	1.2189
E33 (Investment)	(+)	1.5524	-	-	-	2.3285	1.3423
E44 (Exports)	(+)	5.4961	-	-	-	12.377	8.0313
E55 (Imports)	(-)	-4.3978	-	-	-	-8.48	-8.0068
E66 (Labour)	(-)	-0.40894	-	-	-	-1.1107	-0.69424
Technical Progress (TECH)	1.62%	2.01%	-	-	-	1.79%	1.50%

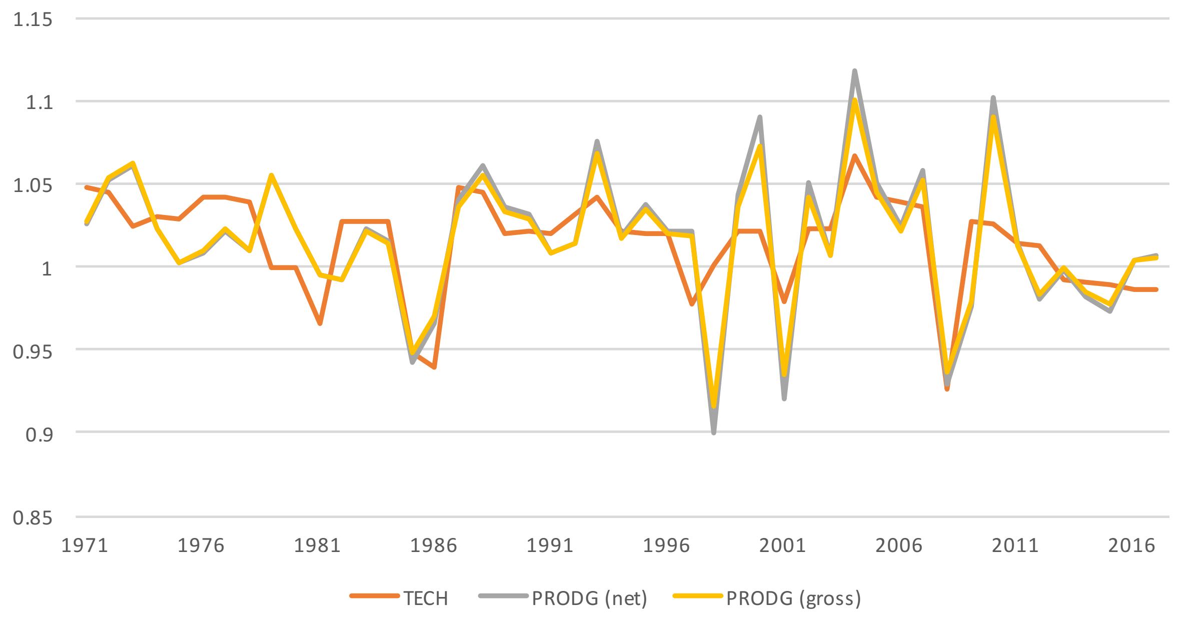




Name	Consumption	Government Output	Investment	Exports	Imports	Labour
Average	0.5049	1.2189	1.3423	8.0313	-8.0068	-0.6942

As seen in Figure 21, the elasticities of consumption and investment were relatively constant at about 0.5 and 1.2 respectively. Elasticity of government output has an upward trend and elasticity of labour has been decreasing. In Figure 22, the magnitude of elasticities of both exports and imports are really high at the start of the sample period but have decreased over the years, ending at 5.34 for exports and -3.64 for imports in year 2017. This is because of the huge trade volume that comes in and out of Singapore regularly which results in the high magnitude in elasticities of both exports and imports.

Figure 23: Index number & Econometric estimates of TFP growth



In Figure 23, the results from index number method and technical progress (TECH) from PMOD12 are shown. We can observe that econometric estimates move in tandem to a large extent with index number method. This is because by implementing splines in PMOD12, technical progress should smoothen PROD.

5. Nonparametric Approach

5.1 Nonparametric Model

In this section, we will construct nonparametric models to estimate the efficiency of production for Singapore by following closely to the theory in Diewert (2009). We use the last 25 years of quantity data from year 1993 to year 2017 in these models. The producer data on five variables are used: labour and capital as input variables as well as domestic output, import and export as output variables.

5.1.1 NONPAR1

In NONPAR1, we compute technical efficiency for each observation assuming a convex technology. An approximation to the basic technology is defined to be the convex free disposal hull of the observed quantity data, $\{(y_1^k, y_2^k, x_1^k, x_2^k): k = 1, \ldots, K\}$. This technology assumption is consistent with decreasing returns to scale and constant returns to scale. We will restrict ourselves to the methodology used by Debreu (1951) – Farrell (1957). We measure the inefficiency of observation i by the smallest positive fraction δ_i^* of the i^{th} input vector (x_1^i, x_2^i), which is such that $(y_1^k, y_2^k, \delta_i^*, x_1^k, \delta_i^*, x_2^k)$ is on the efficient frontier spanned by the convex free disposal hull of the K observations. If $\delta_i^*=1$, the i^{th} observation is efficient. The smaller the δ_i^*, the lower the efficiency of the i^{th} observation.

$$\delta_i^* = \min_{\delta_i \geq 0, \lambda_1 \geq 0, \ldots, \lambda_K \geq 0} \left\{ \delta_i : \sum_{k=1}^{K} y_1^k \lambda_k \geq y_1^i; \sum_{k=1}^{K} y_2^k \lambda_k \geq y_2^i; \sum_{k=1}^{K} y_3^k \lambda_k \geq y_3^i; \sum_{k=1}^{K} x_1^k \lambda_k \leq \delta_i x_1^i; \sum_{k=1}^{K} x_2^k \lambda_k \leq x_2^i; \sum_{k=1}^{K} \lambda_k = 1 \right\}$$

5.1.2 NONPAR2

In NONPAR2, we compute technical efficiency for each observation assuming a convex, constant returns to scale technology. The efficiency of observation I is measured by the positive fraction δ_i^{**} of the i^{th} input vector (x_1^i, x_2^i), such that $(y_1^k, y_2^k, \delta_i^{**}, x_1^k, \delta_i^{**}, x_2^k)$ is on the efficient frontier spanned by the conical convex free disposal hull of the K observations.

$$\delta_i^{**} = \min_{\delta_i \geq 0, \lambda_1 \geq 0, \ldots, \lambda_K \geq 0} \left\{ \delta_i : \sum_{k=1}^{K} y_1^k \lambda_k \geq y_1^i; \sum_{k=1}^{K} y_2^k \lambda_k \geq y_2^i; \sum_{k=1}^{K} y_3^k \lambda_k \geq y_3^i; \sum_{k=1}^{K} x_1^k \lambda_k \leq \delta_i x_1^i; \sum_{k=1}^{K} x_2^k \lambda_k \leq x_2^i \right\}$$

5.1.3 NONPAR3

In NONPAR3, we compute technical efficiency for each observation assuming a convex technology and cost minimization. We define overall efficiency measure ε_i^* for observation i. The linear programming function that minimizes cost subject to the convex assumption on technology is:

$$\varepsilon_i^*\left[\sum_{n=1}^{2} w_n^i x_n^i\right] \equiv \min_{\lambda_1 \geq 0,\ldots,\lambda_K \geq 0} \left\{ w_1^i \sum_{k=1}^{K} x_1^K \lambda_K + w_2^i \sum_{k=1}^{K} x_2^K \lambda_K : \sum_{k=1}^{K} y_1^k \lambda_k \geq y_1^i; \sum_{k=1}^{K} y_2^k \lambda_k \geq y_2^i \sum_{k=1}^{K} y_3^k \lambda_k \geq y_3^i \sum_{k=1}^{K} \lambda_k = 1 \right\}$$

5.1.4 NONPAR4

In NONPAR4, we compute technical efficiency for each observation assuming a convex, constant returns to scale technology and cost minimization. Since the new linear programming function has one less constraint than the function in NONPAR3, it will get a smaller optimized objective function and so $\varepsilon_i^{**} \leq \varepsilon_i^*$. Hence, if we compare the new linear programming function to the function in NONPAR2, $\delta_i^{**} \geq \varepsilon_i^{**}$. The assumption of cost minimization will tend to decrease the efficiency of observation I, ε_i^{**}, relative to the measure of technical efficiency in previous NONPARs.

$$\varepsilon_i^*\left[\sum_{n=1}^{2} w_n^i x_n^i\right] \equiv \min_{\lambda_1 \geq 0,\ldots,\lambda_K \geq 0} \left\{ w_1^i \sum_{k=1}^{K} x_1^K \lambda_K + w_2^i \sum_{k=1}^{K} x_2^K \lambda_K : \sum_{k=1}^{K} y_1^k \lambda_k \geq y_1^i; \sum_{k=1}^{K} y_2^k \lambda_k \geq y_2^i \sum_{k=1}^{K} y_3^k \lambda_k \geq y_3^i \right\}$$

5.1.5 NONPAR5

In NONPAR5, we compute technical efficiency for each observation assuming a convex technology and variable profit maximization. In order to obtain an operational approach, we consider a variable profit maximization instead of the full profit maximization problem.

$$\left[\sum_{n=1}^{3} p_m^i y_m^i\right] - \alpha_i^* \left[\sum_{n=1}^{2} w_n^i x_n^i\right] \equiv \max_{\lambda_1 \geq 0, \ldots, \lambda_K \geq 0} \left\{ \sum_{m=1}^{3} p_m^i \sum_{k=1}^{K} y_m^k \lambda_k - \sum_{n=1}^{2} w_n^i \sum_{k=1}^{K} x_n^k \lambda_k ; \sum_{k=1}^{K} \lambda_k = 1 \right\}$$

5.1.6 NONPAR6

In NONPAR6, we compute technical efficiency for each observation assuming a convex, constant returns to scale technology and variable profit maximization, holding capital fixed.

$$\left[\sum_{n=1}^{3} p_m^i y_m^i\right] - \alpha_i^{**} \left[\sum_{n=1}^{2} w_n^i x_n^i\right] \equiv \max_{\lambda_1 \geq 0, \ldots, \lambda_K \geq 0} \left\{ \sum_{m=1}^{3} p_m^i \sum_{k=1}^{K} y_m^k \lambda_k - \sum_{n=1}^{2} w_n^i \sum_{k=1}^{K} x_n^k \lambda_k : \sum_{k=1}^{K} x_2^k \lambda_k \leq x_2^i \right\}$$

5.1.7 NONPAR

In this model, we decompose productivity growth into the product of technical progress and cost efficiency components for all 48 observations between year 1970 to year 2017. We use the 7 commodity data series from Section 2, where 1=consumption, 2=government, 3=gross investment, 4=exports, 5=-imports, 6=-labour, 7= capital services.

This model is adopted from Diewert and Fox (2016) discussion paper while the nonparametric approach is based partially on Farrell's (1957) model. In NONPAR, we deal with many outputs and inputs instead of just one output. We also make use of the cost constrained value-added function. The reference technology used is the conical free disposal hull of the observed past production observations and we also use all price information.

- The period t best practice unit cost function if the producer faces the nonnegative, nonzero vector of input prices $w>0_N$:

$$
\begin{aligned}
c(w,t) &\equiv \min_{\lambda_1,\dots,\lambda_t} \{w\cdot(\Sigma_{s=1}^{t} x^s\lambda_s) \; ; \; \Sigma_{s=1}^{t} y^s\lambda_s \geq 1 \; ; \; \lambda_1 \geq 0 ,\dots, \lambda_t \geq 0\} \\
&= \min \{w\cdot x^s/y^s : s = 1,2,\dots,t\} \\
&= \min \{w\cdot z^s : s = 1,2,\dots,t\}
\end{aligned}
$$

- The cost efficiency of the production unit during period t, e^t:

$$e^t \equiv \frac{c(w^t,t)}{w^t}.z^t \leq 1 \, for \; t = 1, \dots, T$$

- The change in the production unit's cost efficiency going from period t-1:

$$\varepsilon^t \equiv \frac{e^t}{e^{t-1}} = \frac{\left[\frac{c(w^t,t)}{c^t}\right]}{\left[\frac{c(w^{t-1},t-1)}{c^{t-1}}\right]}$$

If $\varepsilon^t > 1$, then the cost efficiency of the production unit has improved going from period t-1 to t. The opposite holds true.

- The family of measures of technical progress:

$$\tau(w,t) \equiv \frac{c(w,t-1)}{c(w,t)}$$

- Laspeyres and Paasches type indexes of technical progress in period t:

$$\tau_L^t \equiv \tau(w^{t-1},t) = \frac{c(w^{t-1},t-1)}{c(w^{t-1},t)} \; for \; t = 2,3, \dots, T$$

$$\tau_P^t \equiv \tau(w^t,t) = \frac{c(w^t,t-1)}{c(w^t,t)} \; for \; t = 2,3, \dots, T$$

- Fisher type index of technical progress in period t:

$$\tau^t \equiv \left[\tau_L^t \tau_p^t\right]^{0.5} \; for \; t = 2,3, \dots, T$$

- The family of input price indexes going from period t-1 to period t:

$$\beta(w^{t-1}, w^t, s) \equiv \frac{c(w^t, s)}{c(w^{t-1}, s)}$$

- Laspeyres and Paasches type indexes of input prices in period t:

$$\beta_L^t \equiv \beta(w^{t-1}, w^t, t-1) = \frac{c(w^t, t-1)}{c(w^{t-1}, t-1)} \; for \; t = 2,3, \dots, T$$

$$\beta_P^t \equiv \beta(w^{t-1}, w^t, t) = \frac{c(w^t, t)}{c(w^{t-1}, t)} \; for \; t = 2,3, \dots, T$$

- Fisher type index of input price in period t:

$$\beta^t \equiv [\beta_L^t \beta_p^t]^{0.5} \; for \; t = 2,3, \dots, T$$

- Implicit input quantity index going from period t-1 to period t:

$$\gamma^t \equiv [\frac{C^t}{C^{t-1}}]/\beta^t \; for \; t = 2,3, \dots, T$$

- Total Factor Productivity Growth:

$$TFPG^t \equiv [\frac{y^t}{y^{t-1}}]/\gamma^t \; for \; t = 2,3, \dots, T$$

- TFP growth is set equal to the product of cost efficiency multiplied by technical progress:

$$TFPG^t = \varepsilon^t \tau^t \ for\ t = 2,3, \dots, T$$

We set $X^1=y^1=C^1$ and $TFP^1=E^1=T^1\equiv 1$, while $X_t \equiv \gamma^t X^{t-1}$, $TFP^t \equiv y^t/X^t$, $E^t \equiv \varepsilon^t E^{t-1}$ and $T^t \equiv \tau^t T^{t-1}$ for t=2,3,...,T. Thus,

$$TFP^t = \frac{y^t}{X^t} = E^t T^t \ for\ t = 1, \dots, T$$

Further, we construct the cost constrained value-added function as the explanatory function instead of the unit cost function which was used above. This model is based on the theory behind Diewert and Fox (2017). Here, we do not have to aggregate outputs using Fisher's formula, hence the advantage of this model since we can deal adequately with the case of multiple inputs and outputs without having to aggregate outputs using an index number formula.

According to Balk (1998;82), we define value added or net revenue efficiency of the sector during period t, e^t:

$$e^t \equiv p^t . y^t / R^t(p^t, w^t, x^t) \leq 1$$

As such, if $e^t = 1$, then production is allocative efficient in period t. If $e^t < 1$, then production is allocative inefficient in period t.

We define period t cost constrained value added function, $R^t(p,w,x)$ for the production unit as:

$$R^t(p,w,x) \equiv \max_{y,z} \ \{p \cdot y : (y,z) \in S^t;\ w \cdot z \leq w \cdot x\}$$

$R^t(p,w,x)$ can be rewritten as:

$$R^t(p,w,x) = w \cdot x \max_s \{p \cdot y^s / w \cdot x^s : s = 1,2,\ldots,t\}$$

$$= w \cdot x / \min_s \{w \cdot x^s / p \cdot y^s : s = 1,2,\ldots,t\}$$

$$= w \cdot x / c^t(w,p)$$

- Period t nonparametric unit cost function $c^t(w,p)$:

$$c^t(w,p) \equiv \min_s \{w \cdot x^s / p \cdot y^s : s = 1,2,\ldots,t\}$$

- Period t TFP growth rate:

$$TFPG^t \equiv \{[p^t \cdot y^t / p^{t-1} \cdot y^{t-1}] / \alpha^t\} / \beta^t = \varepsilon^t \gamma^t \delta^t \tau^t$$

$$TFP^t = [p^t \cdot y^t / p^1 \cdot y^1] / [A^t B^t] = C^t D^t E^t T^t$$

5.1.8 Nonparametric Model Results

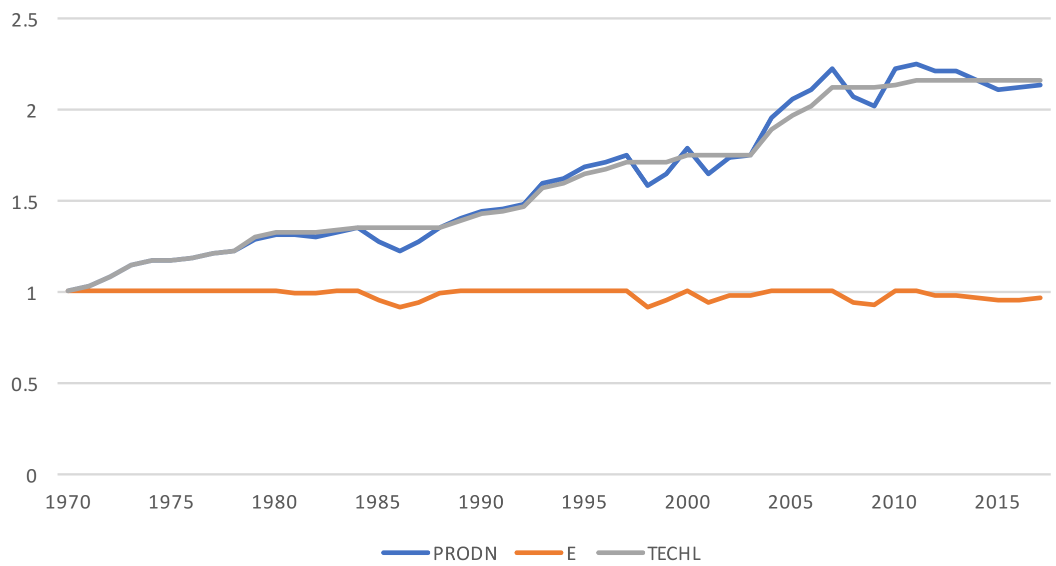
Table 4 below shows the results for the six nonparametric models from NONPAR1 to NONPAR6.

Table 4: Efficiency Estimation for Nonparametric Models

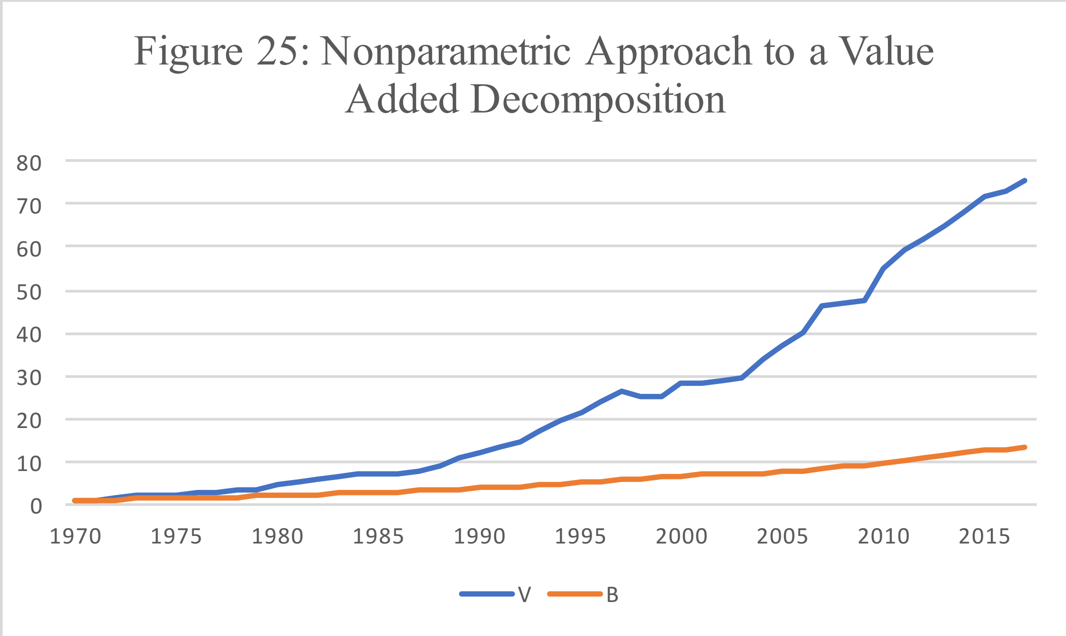
	NONPAR1	NONPAR2	NONPAR3	NONPAR4	NONPAR5	NONPAR6
Convexity	Imposed	Imposed	Imposed	Imposed	Imposed	Imposed
CRS	-	Imposed	-	Imposed	-	Imposed
Cost Min / Profit Max	-	-	Cost Min	Cost Min	Profit Max	Profit Max
1993	1	1	1	1	0.1866	0.7116
1994	1	0.9972	1	0.9724	0.273	0.7213
1995	1	0.9878	0.999	0.9506	0.4191	0.7613
1996	0.999	0.9935	0.9899	0.9676	0.5017	0.7785
1997	1	1	0.9985	0.9866	0.5979	0.8057
1998	1	1	1	1	0.3919	0.6838
1999	0.9367	0.9364	0.936	0.9272	0.5036	0.724
2000	1	1	1	1	0.6286	0.7792
2001	1	0.9686	1	0.946	0.5123	0.6982

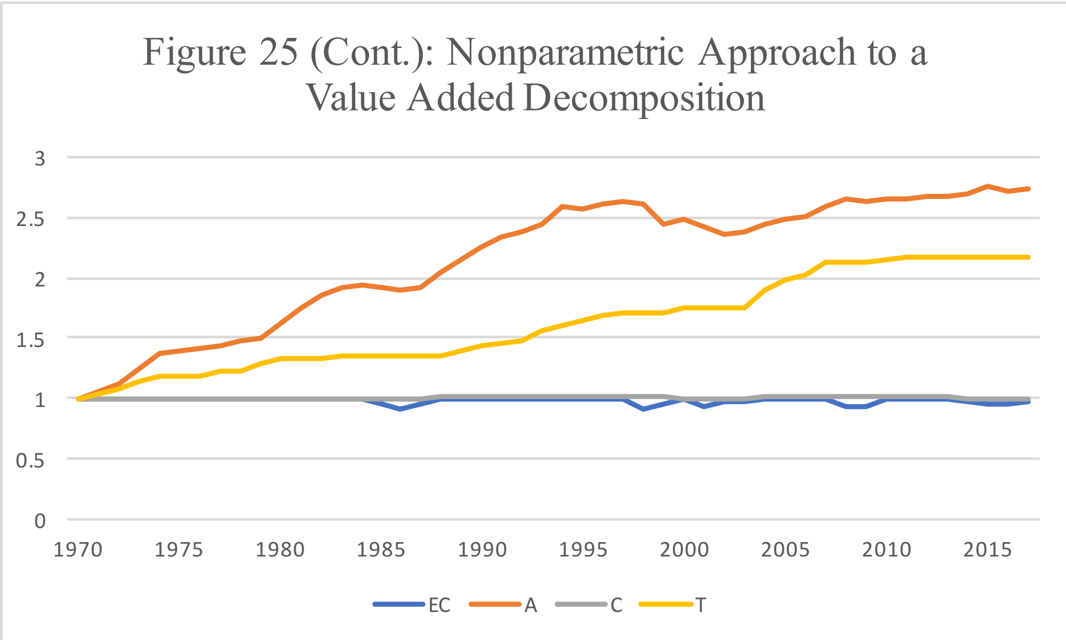
2002	1	1	1	0.9654	0.6187	0.7457
2003	1	1	1	1	0.6355	0.7476
2004	0.9949	0.9231	0.9639	0.9205	0.8001	0.8596
2005	1	0.9404	0.9958	0.9359	0.8717	0.9067
2006	0.9992	0.9693	0.9977	0.9672	0.9098	0.9321
2007	1	1	1	1	0.9809	0.9851
2008	1	1	1	1	0.8992	0.9168
2009	0.9586	0.9423	0.9476	0.9406	0.8826	0.8961
2010	1	1	1	1	0.9865	0.9874
2011	1	1	1	1	1	1
2012	1	1	1	1	0.9846	0.9835
2013	1	1	1	0.9981	0.9862	0.9841
2014	0.9946	0.9943	0.9938	0.9814	0.9758	0.9702
2015	1	0.9907	0.9978	0.9559	0.9594	0.9468
2016	1	1	1	1	0.9659	0.9531
2017	1	1	1	0.9785	0.9739	0.9623
Efficient years (#)	19	14	15	10	1	1

Figure 24: Nonparametric Decomposition - Productivity



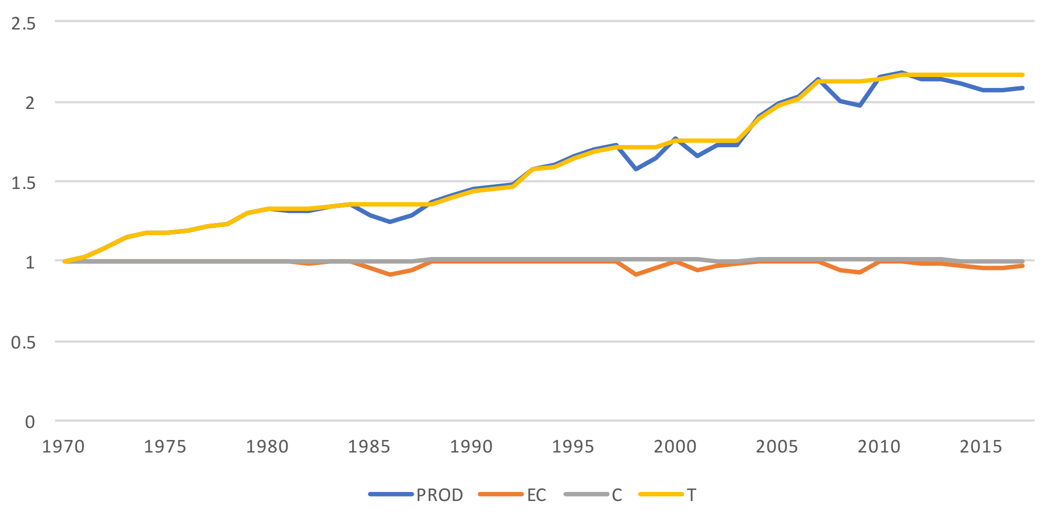
The efficiency factor E was below 1 in 1985-1987, 1997-1999, 2001-2002 and 2008-2009. This means that productivity was below the index of technical progress and Singapore's economy was inefficient during those years. This result coincides with the historical events that affected Singapore as mentioned earlier, 1985 first post-independence recession, 1997 Asian Financial Crisis, 2001 Dot Com Bubble and 2008 Global Financial Crisis.





Referring to Figure 25, gross profits (V) grew over the sample period, with dips or being relatively stagnant during the recession periods, to a final factor of 75.4. The input quantity growth factor (B) increased smoothly, to a final factor of 13.2. Output price growth factor (A) increased till around year 1994 at 2.6, before remaining relatively flat since then. Technical progress growth (T) steadily increased till around 2.1 in year 2006 and have remained flat since then. The efficiency factor (EC) corresponds to the historical events as mentioned previously when EC was below 1.

Figure 26: Diewert and Fox Nonparametric Decomposition of Productivity



As seen in Figure 26, the movement of productivity (PROD) and the efficiency factor (EC) coincided for years that are inefficient. The nonparametric level of productivity ended up at 2.087. The cumulated efficiency growth factor ended up at 0.968, which is a negative contribution to productivity growth. The cumulated mixed factor ended up at 0.9996, which is a small negative contribution to productivity growth. Level of technology ended up at 2.165 which of course was the major contributor to productivity growth. The technology level peaked at 2011, which corresponds to the efficient year in NONPAR5 and NONPAR6, at 2.179.

6. Detailed Consumption Model

6.1 Fifteen Goods Consumer Model

In this section, we construct a 15 goods consumer model data base. We read Singapore data for 15 classes of per population consumption from consumer data obtained from DOS for the year 1971 to year 2017.

	LIST OF 15 GOOD CONSUMER MODEL
1	Food & Non-alcoholic Beverages
2	Alcoholic Beverages & Tobacco
3	Clothing & Footwear
4	Housing & Utilities
5	Furnishings, Household Equipment & Routine Household Maintenance
6	Health
7	Transport
8	Communication
9	Recreation & Culture
10	Education
11	Food Serving Services
12	Accommodation Services
13	Miscellaneous Goods & Services
14	Residents' Expenditure Abroad
15	Non-residents' Expenditure Locally

6.1.1 CON1 to CON2

CON1 estimates non-homothetic Leontief preferences and we do not impose money metric utility scaling in this model. In CON2, we improve on CON1 by splining utility in this model. Our break points are at observations 24(*year*=1994) and 36(*year*=2006).

6.1.2 CON3 to CON11

In CON3, we further improve on CON2 by estimating a normalized quadratic rank 1 substitution matrix. Here, we add an additional 14 parameters in the model. We repeat this process in CON4 by estimating a normalized quadratic rank 2 substitution matrix and adding an additional 13 parameters into the model. This goes on till CON11 where we estimate a normalized quadratic rank 9 substitution matrix (C, D, E, F, H, I, J, K, M) and adding an additional 6 parameters from CON10 to CON11.

In CON11,

$$\# \, of \, observations = 15 \, goods \, x \, 47 \, years = 705 \, observations$$

$$\# \, of \, coefficients = 150$$

$$Degress \, of \, freedom = \frac{705}{150} = 4.7$$

Ideally, we would like to have 10 degrees of freedom for each parameter, so CON11 will be our final model.

6.1.3 CON Results for Price Elasticities of Demand

Tables 5 and 6 below show the average price elasticities of demand (EP) for the 15 goods. All of them are negative which implies that as prices increase for these goods, quantity demanded falls. In CON 3, 12 out of the 15 goods have price elasticities of less than 0.2 in magnitude. This means that these 12 goods are very inelastic. Good 8 (Communication) has price elasticity of more than 1 in magnitude, implying that it is price elastic. As we add more ranks of substitution matrices, all the way to rank 9 substitution matrix in CON11, we observe that the log-likelihood increased by about 214 points after adding 76 more parameters. In our final CON11, 8 out of the 15 goods are price elastic, especially good 8 (Communication) which has a large price elasticity of magnitude 6.67. The other 7 goods are price inelastic, with good 4 (Housing & Utilities) to be the most inelastic amongst them all. This is intuitive since having a roof over one's head is one of the basic essential needs of a person.

6.1.4 CON Results for Income Elasticities of Demand

Tables 7 and 8 below show the average income elasticities of demand (EU) for the 15 goods. Referring to CON1, we observe that Good 8 (Communication) and 14 (Residents' Expenditure Abroad) have elasticities of large magnitude. However, these elasticities become more reasonable after we spline utility in CON2. Similarly, as we add more rank substitution matrices, log-likelihood increases and 14 out of the 15 goods have positive income elasticities. The only good which has consistently negative income elasticity is Good 2 (Alcoholic Beverages & Tobacco). This is reasonable since as income increases, people will value health outcomes more and change their preferences due to the adverse consequences of smoking and alcohol, so demand will fall. In our final CON11, we have 6 out of 15 goods that have income elasticities of more than 1 in magnitude, namely Goods 6 (Health), 7 (Transport), 10 (Education), 12 (Accommodation Services), 13 (Miscellaneous Goods & Services) and 14 (Residents' Expenditure Abroad), with Good 14 being the largest in magnitude of 2.93. Naturally, as income increases, people make plans for travelling and it is reasonable that this good has a large income elasticity of demand.

Table 5: Regression Results for Detailed Consumer Models

Description	CON3	CON4	CON5	CON6	CON7
	NQ Rank 1 substitution matrix	NQ Rank 2 substitution matrix	NQ Rank 3 substitution matrix	NQ Rank 4 substitution matrix	NQ Rank 5 substitution matrix
# of coefficients	74	87	99	110	120
Final Log-Likelihood	1853.439	1911.278	1945.843	2017.111	2042.932
E1P1	-0.17885	-0.38057	-0.39325	-0.29803	-0.49066
E2P2	-0.000058374	-0.21882	-0.22933	-0.19343	-0.21611
E3P3	-0.32524	-0.90681	-0.94368	-0.50035	-0.80059
E4P4	-0.0010192	-0.017448	-0.022989	-0.16015	-0.13398
E5P5	-0.14778	-0.16412	-0.19533	-0.27302	-0.4416
E6P6	-0.03598	-0.082698	-1.1779	-1.4034	-1.3767
E7P7	-0.000044288	-0.0010612	-0.0056321	-0.68642	-0.75286
E8P8	-1.3707	-4.1683	-4.3824	-4.7353	-5.9073
E9P9	-0.0016153	-0.026593	-0.21996	-0.17455	-0.33212
E10P10	-0.31596	-0.22302	-0.59878	-0.73555	-0.72395
E11P11	-0.13344	-0.012278	-0.26136	-0.5936	-1.2412
E12P12	-0.095848	-0.088987	-0.15483	-0.1446	-0.15832
E13P13	-0.023046	-0.035777	-0.061417	-0.089227	-0.18263
E14P14	-0.080742	-0.1433	-0.19584	-0.40318	-0.44756
E15P15	-0.40417	-0.39618	-0.44225	-0.52766	-0.50898

Table 6: Regression Results for Detailed Consumer Models

Description	CON8	CON9	CON10	CON11
	NQ Rank 6 substitution matrix	NQ Rank 7 substitution matrix	NQ Rank 8 substitution matrix	NQ Rank 9 substitution matrix
# of coefficients	129	137	144	150
Final Log-Likelihood	2057.019	2062.044	2067.283	2067.377
E1P1	-0.8687	-0.95784	-0.96548	-1.0047
E2P2	-0.22296	-0.22424	-0.32989	-0.4338
E3P3	-1.8056	-2.164	-2.2797	-2.2936
E4P4	-0.11637	-0.15505	-0.15134	-0.15405
E5P5	-0.51261	-0.63464	-0.907	-0.92157
E6P6	-1.4545	-1.4629	-1.6326	-1.6339
E7P7	-0.81611	-0.91834	-0.91376	-0.93619
E8P8	-6.107	-6.4147	-6.6235	-6.6771
E9P9	-0.92114	-0.8529	-1.2395	-1.3372
E10P10	-0.73863	-0.74069	-0.77026	-0.77427
E11P11	-1.8744	-1.9412	-1.9314	-1.9639
E12P12	-0.17552	-0.19534	-0.19192	-0.19057
E13P13	-0.24374	-0.45847	-0.67468	-0.6901
E14P14	-0.55582	-1.0438	-1.5489	-1.5544
E15P15	-0.78238	-0.81413	-1.0607	-1.1574

Table 7: Regression Results for Detailed Consumer Models

Description	CON1	CON2	CON3	CON4	CON5	CON6
	Non-homothetic Leontief preferences	Non-homothetic Leontief preferences with splined utility	NQ Rank 1 substitution matrix	NQ Rank 2 substitution matrix	NQ Rank 3 substitution matrix	NQ Rank 4 substitution matrix
# of coefficients			74	87	99	110
Final Log-Likelihood	1645.556	1776.46	1853.439	1911.278	1945.843	2017.111
EU1	0.2588	0.13655	0.27035	0.23581	0.18794	0.19167
EU2	-0.80283	-0.47127	-0.47734	0.091731	0.13824	-0.17065
EU3	0.18795	0.3112	0.023379	0.15231	0.11288	0.44223
EU4	0.84613	0.68263	0.68858	0.75293	0.77098	0.98487
EU5	0.50014	0.26939	0.39756	0.38042	0.39073	0.21399
EU6	1.4109	1.3871	1.444	1.2182	1.4863	1.6127
EU7	1.1233	0.9086	0.90847	0.91286	0.91299	1.5715
EU8	40.412	3.7585	3.4556	0.62205	0.16882	0.64888
EU9	1.2765	1.3623	1.36994	1.4356	1.5257	0.99142
EU10	2.211	2.07	1.8572	1.5522	1.2821	1.0474
EU11	0.56614	0.59233	0.70833	0.5508	0.44912	0.73254
EU12	0.99324	1.0787	1.2447	1.22	1.1901	1.0118
EU13	1.2018	1.2794	1.3063	1.361	1.337	1.121
EU14	-195.61	3.2231	3.2547	3.4607	3.4385	2.8341
EU15	0.68114	0.84061	0.69862	0.75748	0.76379	0.7307

Table 8: Regression Results for Detailed Consumer Models

Description	CON7	CON8	CON9	CON10	CON11
	NQ Rank 5 substitution matrix	NQ Rank 6 substitution matrix	NQ Rank 7 substitution matrix	NQ Rank 8 substitution matrix	NQ Rank 9 substitution matrix
# of coefficients	120	129	137	144	150
Final Log-Likelihood	2042.932	2057.019	2062.044	2067.283	2067.377
EU1	0.26855	0.22898	0.20834	0.22175	0.23442
EU2	-0.14269	-0.18979	-0.27949	-0.35206	-0.31707
EU3	0.56816	0.67124	0.62262	0.68606	0.69664
EU4	0.962	0.95373	0.95681	0.95672	0.96166
EU5	0.27217	0.31404	0.29818	0.27221	0.28125
EU6	1.6265	1.5259	1.5528	1.5754	1.5708
EU7	1.5058	1.4829	1.5354	1.5237	1.5429
EU8	0.52365	0.59972	0.66507	0.57091	0.61597
EU9	1.0335	1.0251	1.027	0.97784	0.94007
EU10	1.0418	1.1878	1.1543	1.1748	1.1605
EU11	0.64249	0.7251	0.70978	0.68757	0.70417
EU12	1.0178	1.0896	1.0748	1.0627	1.0523
EU13	1.1153	1.1081	1.0831	1.1193	1.1289
EU14	2.7866	2.7282	2.8823	2.9452	2.9305
EU15	0.76099	0.74254	0.73197	0.71744	0.69073

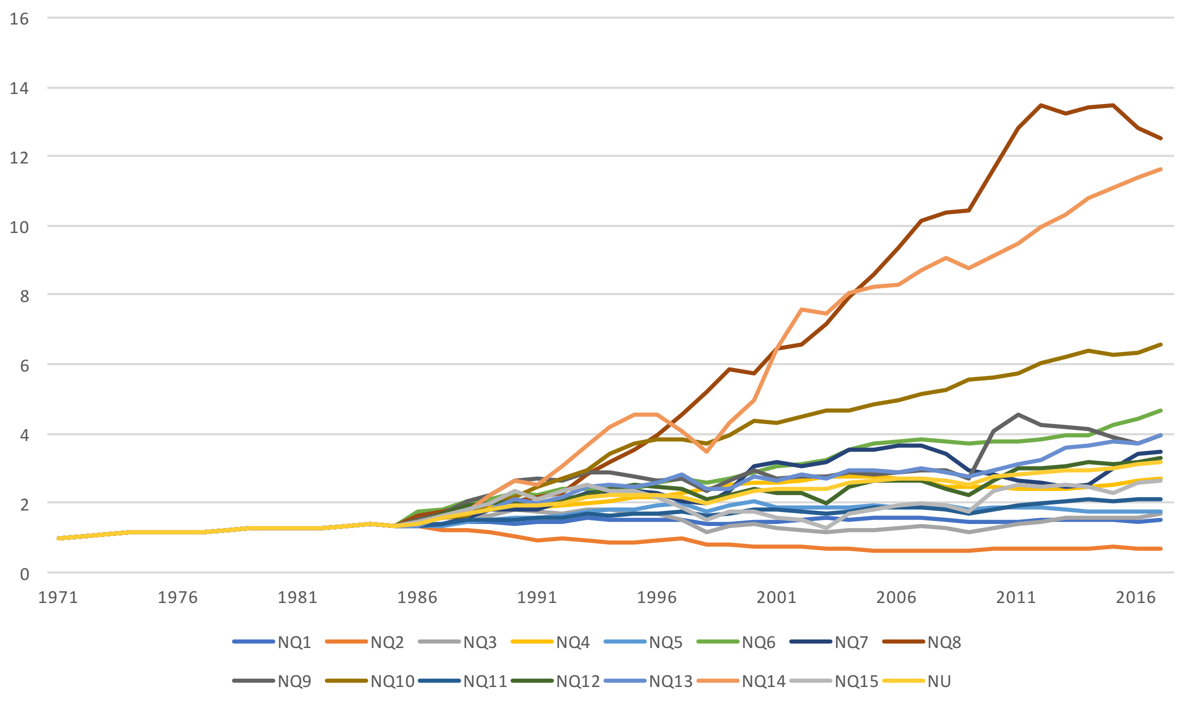
6.1.5 Per Capita Real Consumption

We also generated per capita real consumption of each good in order to better see which categories of per capita household demand have grown faster. First, in our final model CON11, we normalize utility to the first period as well as normalize quantities by the first period too.

$$NU = \frac{U}{U(1)} \text{ where } U(1) \text{is the generated utility in period 1 (year 1971)}$$

$$NQ_i = \frac{Q}{Q(1)} \text{where } i = 1, \ldots, 15 \text{ and } Q(1) \text{ is the quantity in period 1 (year 1971)}$$

Figure 27: Per Capita Real Consumption in CON11



YEAR	NQ1	NQ2	NQ3	NQ4	NQ5	NQ6	NQ7	NQ8
2017	1.5014	0.6879	1.679	2.7302	1.7461	4.6425	3.4712	12.4914
	NQ9	**NQ10**	**NQ11**	**NQ12**	**NQ13**	**NQ14**	**NQ15**	**NU**
	3.9368	6.5576	2.1046	3.3144	3.9653	11.6161	2.6778	3.1974

From Figure 27, we can see that Good 8 (Communication) has increased the most over the sample period but has declined slightly since year 2011. Further, Good 14 (Residents'

Expenditure Abroad) has grown steadily over the sample period and is still increasing in recent years and is likely to surpass Good 8 if the trend persists. Good 10 (Education) has also been increasing over the years while Good 2 (Alcoholic Beverages & Tobacco) has been declining. Singapore's per capita real consumption (NU) grew to a final factor of 3.2 in year 2017.

We also generated geometric rates of growth of per capita consumption.

$$GR_i = NQ_i(47)^{\frac{1}{46}} \; for \; i = 1, \ldots, 15$$
$$and \; NQ(47) \; is \; the \; per \; capita \; real \; consumption \; in \; the \; last \; period \; (year = 2017)$$

GR1	GR2	GR3	GR4	GR5	GR6	GR7	GR8
1.00887	0.9919	1.01133	1.02207	1.01219	1.03394	1.02742	1.05643
GR9	**GR10**	**GR11**	**GR12**	**GR13**	**GR14**	**GR15**	**GRU**
1.03024	1.04173	1.01631	1.02639	1.0304	1.05476	1.02164	1.02559

Average geometric growth rate of per capita real consumption is 2.56% per year. 10 out of the 15 categories grew more than 2% annually, 3 out of 15 grew between 1-2% annually. Good 1 (Food & Non-Alcoholic Beverages) grew between 0-1% annually while Good 2 (Alcoholic Beverages & Tobacco) has negative growth rates.

7. Conclusion

In this paper, we analyzed the economic performance of Singapore's economy by constructing economic models using data provided by the Singapore Department of Statistics as well as the Asian Productivity Organization. We focus on three approaches: index number method, econometric approach as well as nonparametric approach. The results from these three approaches coincide and we can conclude that Singapore's economic performance was overall good from 1970 to 2017.

All in all, the transformation of the Singapore economy over the past five decades has been impressive. During this period, Singapore has evolved into a developed economy with multiple engines of growth including globally competitive manufacturing clusters, one of the world's pre-eminent financial and transportation centers, and the location for regional or global headquarters of major corporations. Today, the Singapore economy faces significant challenges such as changing structures of international competitiveness and disruptions caused by new technologies. This has led to slowdowns in GDP growth and productivity growth. The changing structure of the economy with manufacturing share of GDP declining whilst services share of GDP increasing could be a possible reason why there is slowdown in productivity growth rates since overall labour productivity growth will be affected. Also, competition with low cost producers from emerging giant economies such as China and India will slowly erode Singapore's competitive edge in manufacturing and weigh on export performance. As such, it is imperative for the Singapore government to implement policies to boost productivity growth.

References

Diewert,W.E. (1980). “*Symmetry Conditions for Market Demand Functions*.” The Review of Economic Studies, 47(3), 595-601.

Diewert, W.E. (2013). “*Chapter 5: Index Number Theory: Part II: Economic Approach to Index Number Theory*”.

Diewert, W.E. & Fox, K.J. (Jun 29, 2016). “*A Decomposition of US Business Sector TFP Growth into Technical Progress and Cost Efficiency Components*.”

Diewert, W.E. & Fox, K.J. (Dec.29, 2016). “*Decomposing Value Added Growth over Sectors into Explanatory Factors*.”

Diewert, W.E. & Fox, K.J.(Jan.20, 2017). “*Decomposing Value Added Growth into Explanatory Factors*.”

Diewert, W.E. & Morrison, C.J. (1986), “*Adjusting Output and Productivity Indexes for Changes in the Terms of Trade*.” The Economic Journal 96, 659-679.

Appendix

List of Tables

Table 1: Output Quantity Series

YEAR	QC	QG	QI	QX	QM
1970	3.948	0.696	2.244	7.41	8.527
1971	4.415	0.814	2.606	8.181	9.576
1972	4.818	0.923	2.959	8.673	9.988
1973	5.21	0.975	3.281	10.613	11.503
1974	5.569	0.978	3.886	12.14	13.189
1975	5.748	1.005	3.521	11.98	12.412
1976	6.051	1.053	3.777	13.299	13.58
1977	6.367	1.146	3.633	15.127	14.878
1978	6.785	1.275	4.283	16.84	16.82
1979	7.236	1.272	5.241	20.802	20.564
1980	7.645	1.405	6.066	25.389	24.946
1981	8.016	1.482	6.503	28.421	27.409
1982	8.338	1.678	7.449	29.888	29.088
1983	8.812	1.84	8.225	31.514	30.518
1984	9.328	1.94	9.102	34.089	32.985
1985	9.337	2.332	7.977	32.957	31.662
1986	9.705	2.353	7.225	37.195	35.117
1987	10.692	2.382	7.838	42.421	39.798
1988	11.934	2.24	8.032	55.077	50.433
1989	12.929	2.376	9.022	60.434	55.267
1990	13.892	2.632	10.497	68.243	63.267
1991	14.743	2.847	10.848	74.105	67.812
1992	15.595	2.917	11.924	79.454	72.966
1993	17.404	3.375	13.941	93.043	86.535
1994	18.711	3.316	14.067	110.433	100.8
1995	19.396	3.683	16.275	134.957	123.601
1996	20.638	4.373	18.318	147.909	136.45
1997	21.853	4.677	21.72	163.195	152.069
1998	21.195	5.095	17.17	156.036	139.023
1999	23.117	5.459	18.722	168.248	151.445
2000	26.512	6.598	24.041	192.526	181.919
2001	28.278	6.89	18.91	185.715	169.878
2002	29.676	7.259	18.129	199.642	180.084
2003	29.892	7.286	12.841	228.017	198.057
2004	31.392	7.393	18.98	271.546	242.01
2005	32.462	7.766	19.16	305.474	269.838
2006	33.518	8.371	21.93	339.541	299.911
2007	35.579	8.599	24.83	368.726	322.262
2008	36.821	9.108	31.375	386.554	355.552
2009	36.405	9.488	28.665	356.96	318.013
2010	38.55	10.506	35.185	419.211	369.848
2011	40.22	10.319	36.442	447.184	389.101
2012	41.809	10.168	41.603	454.784	399.622
2013	43.308	11.325	43.575	481.829	423.63
2014	44.789	11.351	44.403	497.998	436.106
2015	47.006	12.241	41.598	521.598	454.01
2016	47.813	12.673	43.197	527.361	454.518
2017	49.306	13.191	46.44	548.963	478.362

Table 2: Implicit Price Series

YEAR	PC	PG	PI	PX	PM
1970	1	1	1	1	1
1971	1.03388	1.06185	1.06611	1.01192	1.01189
1972	1.0608	1.07769	1.1466	1.02179	1.01668
1973	1.22841	1.15392	1.23274	1.15916	1.15127
1974	1.39333	1.33978	1.46919	1.57982	1.60031
1975	1.43488	1.43105	1.5129	1.57134	1.6198
1976	1.44359	1.47849	1.5729	1.68633	1.72815
1977	1.47427	1.51024	1.58618	1.75956	1.80505
1978	1.51329	1.55339	1.6208	1.80152	1.84424
1979	1.57993	1.61653	1.69858	1.8769	1.95678
1980	1.70887	1.77668	1.91998	2.05832	2.16669
1981	1.81466	1.92334	2.08993	2.11647	2.23245
1982	1.86425	2.17067	2.11067	2.10737	2.18671
1983	1.88801	2.21561	2.14898	2.01513	2.07367
1984	1.94777	2.28272	2.15129	1.91538	1.98196
1985	1.96205	2.34733	2.10515	1.88735	1.9607
1986	1.96642	2.20218	2.06886	1.63515	1.70122
1987	2.0106	2.20157	2.12193	1.76098	1.8431
1988	2.08672	2.35212	2.222	1.79199	1.8557
1989	2.18054	2.51151	2.31406	1.8189	1.86672
1990	2.27149	2.5468	2.39392	1.83032	1.86226
1991	2.31293	2.54867	2.45997	1.78639	1.79924
1992	2.36779	2.56503	2.52901	1.72853	1.74094
1993	2.4611	2.60977	2.61271	1.70129	1.71738
1994	2.59613	2.73623	2.6327	1.69387	1.67518
1995	2.63048	2.75446	2.59033	1.6728	1.6582
1996	2.66351	2.76812	2.59879	1.61812	1.5925
1997	2.71687	2.78083	2.61595	1.5484	1.52066
1998	2.66257	2.70329	2.63737	1.54067	1.50939
1999	2.61382	2.54666	2.55462	1.54019	1.54857
2000	2.58889	2.68705	2.39835	1.62347	1.6065
2001	2.57358	2.73587	2.34848	1.58909	1.5846
2002	2.56104	2.71153	2.31382	1.53356	1.53876
2003	2.55901	2.68014	2.32441	1.52067	1.51596
2004	2.56032	2.74335	2.35134	1.53822	1.51572
2005	2.55678	2.7811	2.36746	1.57057	1.54374
2006	2.59774	2.8891	2.39327	1.59147	1.56826
2007	2.71175	3.00037	2.53055	1.57989	1.54859
2008	2.84082	3.14999	2.64688	1.62669	1.60953
2009	2.88106	3.03463	2.71318	1.5066	1.48516
2010	2.97062	3.12561	2.58713	1.536	1.51395
2011	3.09418	3.25246	2.58322	1.57756	1.57405
2012	3.18811	3.27959	2.61066	1.57398	1.57828
2013	3.22513	3.37012	2.68185	1.53454	1.54327
2014	3.25357	3.45605	2.68067	1.51618	1.52052
2015	3.25743	3.56652	2.72602	1.42184	1.39652
2016	3.22215	3.60916	2.67775	1.36483	1.33757
2017	3.23326	3.68095	2.66213	1.41238	1.39397

Table 3: Capital Series

YEAR	QI	QK	D
1970	2.2445	14.42625	0.05
1971	2.60582	15.94944	0.0502
1972	2.95893	17.7546	0.0504
1973	3.28148	19.81868	0.0506
1974	3.88635	22.09728	0.0508
1975	3.52119	24.86098	0.05101
1976	3.77659	27.11406	0.05121
1977	3.63288	29.50209	0.05142
1978	4.28282	31.61806	0.05162
1979	5.24139	34.26868	0.05183
1980	6.06633	37.73395	0.05204
1981	6.50313	41.83674	0.05224
1982	7.44854	46.15413	0.05245
1983	8.22545	51.18172	0.05266
1984	9.10192	56.71177	0.05287
1985	7.97676	62.81512	0.05309
1986	7.22493	67.45731	0.0533
1987	7.83805	71.08691	0.05351
1988	8.03225	75.12103	0.05373
1989	9.02249	79.1174	0.05394
1990	10.49704	83.87231	0.05416
1991	10.84765	89.82718	0.05437
1992	11.92378	95.79072	0.05459
1993	13.94083	102.4853	0.05481
1994	14.06741	110.8091	0.05503
1995	16.27483	118.77897	0.05525
1996	18.31792	128.49155	0.05547
1997	21.7198	139.68223	0.05569
1998	17.16969	153.62308	0.05591
1999	18.72234	162.20321	0.05614
2000	24.04101	171.81998	0.05636
2001	18.90979	186.17698	0.05659
2002	18.12892	194.55162	0.05681
2003	12.84101	201.62744	0.05704
2004	18.98012	202.96754	0.05727
2005	19.1599	210.324	0.0575
2006	21.92975	217.39076	0.05773
2007	24.83024	226.77106	0.05796
2008	31.37534	238.45799	0.05819
2009	28.66506	255.95736	0.05842
2010	35.18501	269.66859	0.05866
2011	36.44249	289.0357	0.05889
2012	41.60272	308.45646	0.05913
2013	43.57464	331.82108	0.05936
2014	44.40311	355.69765	0.0596
2015	41.59758	378.90083	0.05984
2016	43.1967	397.82522	0.06008
2017	46.44004	417.12109	0.06032

Table 4: Labour and Capital Service Inputs

YEAR	QL	QKS	PL	PKS
1970	3.682	2.009	1	1
1971	3.962	2.221	1.035	1.173
1972	4.243	2.472	1.06	1.461
1973	4.523	2.76	1.183	1.867
1974	4.663	3.077	1.429	1.95
1975	4.674	3.462	1.636	1.706
1976	4.91	3.776	1.596	1.892
1977	5.04	4.108	1.632	1.991
1978	5.379	4.403	1.603	2.184
1979	5.69	4.772	1.677	2.371
1980	6.017	5.255	1.848	2.636
1981	6.594	5.826	1.988	2.805
1982	7.015	6.427	2.29	2.753
1983	7.218	7.127	2.577	2.734
1984	7.398	7.897	2.816	2.623
1985	7.098	8.747	2.953	2.197
1986	7.121	9.394	2.719	2.173
1987	7.553	9.899	2.699	2.396
1988	8.015	10.461	2.884	2.794
1989	8.628	11.017	3.103	3.1
1990	9.092	11.679	3.389	3.312
1991	9.723	12.509	3.607	3.383
1992	10.026	13.339	3.834	3.4
1993	10.31	14.271	4.161	3.755
1994	10.863	15.43	4.514	4.045
1995	11.537	16.54	4.63	4.172
1996	12.081	17.893	4.964	4.188
1997	12.773	19.451	5.05	4.329
1998	13.049	21.392	5.155	3.605
1999	13.143	22.587	5.079	3.454
2000	13.034	23.926	5.619	3.719
2001	14	25.926	5.579	3.152
2002	13.657	27.092	5.763	3.168
2003	13.624	28.077	5.829	3.205
2004	13.624	28.264	6.159	3.849
2005	13.818	29.288	6.472	4.095
2006	15.191	30.272	6.537	4.31
2007	16.066	31.578	7.08	4.709
2008	17.148	33.206	6.979	4.418
2009	17.162	35.643	7.197	4.111
2010	18.441	37.552	7.27	4.796
2011	18.975	40.249	7.635	4.76
2012	19.759	42.953	7.791	4.595
2013	20.219	46.207	8.119	4.472
2014	20.671	49.532	8.558	4.216
2015	21.362	52.763	8.73	4.187
2016	21.73	55.398	8.903	4.021
2017	21.636	58.085	9.226	3.955

Table 5: Tax Rates

YEAR	TRC	SRC	TRX	TRM	TRL	TRKI	TRK
1970	0.00495	0	0	0.00706	0.03638	0.6907	0.06165
1971	0.00495	0	0	0.00706	0.03638	0.54492	0.06165
1972	0.00495	0	0	0.00706	0.03638	0.43896	0.06165
1973	0.00495	0	0	0.00706	0.03638	0.34993	0.06165
1974	0.00495	0	0	0.00706	0.03638	0.3638	0.06165
1975	0.00495	0	0	0.00706	0.03638	0.55717	0.06165
1976	0.00495	0	0	0.00706	0.03638	0.50138	0.06165
1977	0.00495	0	0	0.00706	0.03638	0.49385	0.06165
1978	0.00495	0	0	0.00706	0.03638	0.43998	0.06165
1979	0.00495	0	0	0.00706	0.03638	0.40585	0.06165
1980	0.00495	0	0	0.00706	0.03638	0.37576	0.06165
1981	0.00495	0	0	0.00706	0.03638	0.40777	0.06165
1982	0.00495	0	0	0.00706	0.03638	0.47057	0.06165
1983	0.00495	0	0	0.00706	0.03638	0.4828	0.06165
1984	0.00495	0	0	0.00706	0.03638	0.52653	0.06165
1985	0.00495	0	0	0.00706	0.03638	0.69178	0.06165
1986	0.00495	0	0	0.00706	0.03638	0.68139	0.06165
1987	0.00495	0	0	0.00706	0.03638	0.57212	0.06165
1988	0.00495	0	0	0.00706	0.03638	0.47548	0.06165
1989	0.00495	0	0	0.00706	0.03638	0.43933	0.06165
1990	0.00495	0	0	0.00706	0.03638	0.42478	0.06165
1991	0.00495	0	0	0.00706	0.03638	0.43293	0.06165
1992	0.00495	0	0	0.00706	0.03638	0.4471	0.06165
1993	0.00495	0	0	0.00706	0.03638	0.40574	0.06165
1994	0.00495	0	0	0.00706	0.03638	0.38391	0.06165
1995	0.00495	0	0	0.00706	0.03638	0.37264	0.06165
1996	0.00495	0	0	0.00706	0.03638	0.36327	0.06165
1997	0.00495	0	0	0.00706	0.03638	0.36368	0.06411
1998	0.00162	0	0	0.00746	0.044	0.39758	0.05407
1999	0.00697	0	0	0.00671	0.04554	0.44507	0.05619
2000	0.00399	0	0	0.00632	0.04838	0.37252	0.05451
2001	0.00455	0	0	0.0067	0.04778	0.36884	0.04664
2002	0.00573	0	0	0.00624	0.04349	0.30322	0.03973
2003	0.0138	0	0	0.00633	0.03945	0.27333	0.03714
2004	0.01923	0	0	0.00524	0.03732	0.23171	0.04015
2005	0.02219	0	0	0.00474	0.0383	0.2072	0.03833
2006	0.02402	0	0	0.00401	0.03769	0.21515	0.04213
2007	0.04332	0	0	0.00398	0.03989	0.23615	0.05102
2008	0.04227	0	0	0.00361	0.04524	0.24302	0.04494
2009	0.04565	0	0	0.0045	0.0495	0.22812	0.03601
2010	0.0537	0	0	0.00366	0.04826	0.2139	0.04011
2011	0.05267	0	0	0.00348	0.04743	0.22627	0.04465
2012	0.05174	0	0	0.0034	0.05011	0.24571	0.04634
2013	0.05243	0	0	0.00335	0.04684	0.24243	0.04344
2014	0.05267	0	0	0.00383	0.05046	0.25769	0.04105
2015	0.04906	0	0	0.00447	0.04952	0.2648	0.04175
2016	0.05419	0	0	0.00449	0.05441	0.28322	0.04115
2017	0.04747	0	0	0.0048	0.0536	0.28395	0.04127

Table 6: Rates of Return

YEAR	RR	R	I
1970	0.0518	0.08925	0.03561
1971	0.17969	0.17592	-0.00319
1972	0.14977	0.21213	0.05423
1973	-0.06309	0.24749	0.33149
1974	0.2375	0.35152	0.09214
1975	0.15516	0.13887	-0.0141
1976	0.16277	0.16059	-0.00188
1977	0.08537	0.13284	0.04373
1978	0.13157	0.16081	0.02584
1979	0.11526	0.1974	0.07365
1980	0.2125	0.28762	0.06195
1981	0.11166	0.23507	0.11101
1982	0.14582	0.14041	-0.00473
1983	0.11708	0.14488	0.02489
1984	0.11417	0.1181	0.00353
1985	0.06678	0.0688	0.0019
1986	0.08174	0.07415	-0.00702
1987	0.12374	0.13203	0.00737
1988	0.15682	0.17428	0.0151
1989	0.13478	0.17951	0.03942
1990	0.1397	0.17777	0.0334
1991	0.13702	0.16849	0.02767
1992	0.13523	0.16441	0.0257
1993	0.15674	0.18322	0.02289
1994	0.13844	0.1678	0.0258
1995	0.13994	0.15023	0.00902
1996	0.15055	0.17279	0.01933
1997	0.16483	0.18252	0.01519
1998	0.15431	0.14372	-0.00918
1999	0.08672	0.09663	0.00913
2000	0.06739	0.08861	0.01989
2001	0.11914	0.10684	-0.01099
2002	0.10742	0.1171	0.00874
2003	0.13257	0.14018	0.00672
2004	0.17959	0.18421	0.00391
2005	0.17117	0.19146	0.01733
2006	0.21315	0.20608	-0.00583
2007	0.1911	0.27007	0.06629
2008	0.17789	0.22823	0.04273
2009	0.17901	0.18143	0.00205
2010	0.08401	0.14377	0.05513
2011	0.14108	0.19589	0.04804
2012	0.15738	0.19858	0.0356
2013	0.18806	0.20483	0.01412
2014	0.16346	0.15888	-0.00394
2015	0.1812	0.17358	-0.00644
2016	0.12239	0.12866	0.00559
2017	0.13977	0.13988	0.00009

Table 7: Total Factor Productivity Growth

YEAR	PRODN	YG	ZG	PRODG
1971	1.02611	1.11572	1.08734	1.02611
1972	1.079	1.14486	1.08873	1.05155
1973	1.14425	1.15567	1.08978	1.06047
1974	1.16979	1.09533	1.07141	1.02233
1975	1.17208	1.05995	1.05788	1.00195
1976	1.18205	1.0783	1.06921	1.00851
1977	1.20774	1.07972	1.05675	1.02174
1978	1.21937	1.07991	1.06961	1.00962
1979	1.28625	1.13042	1.07164	1.05485
1980	1.31632	1.10606	1.08079	1.02337
1981	1.30959	1.09715	1.10279	0.99488
1982	1.29864	1.07482	1.08389	0.99164
1983	1.32761	1.09084	1.06704	1.02231
1984	1.34738	1.07844	1.06261	1.01489
1985	1.26937	0.96165	1.02076	0.9421
1986	1.22597	0.99737	1.03267	0.96581
1987	1.27618	1.10085	1.05754	1.04095
1988	1.35405	1.1236	1.05899	1.06102
1989	1.40249	1.10261	1.06453	1.03577
1990	1.44597	1.0898	1.05702	1.031
1991	1.45737	1.07868	1.07025	1.00788
1992	1.47877	1.06396	1.04857	1.01468
1993	1.58963	1.12751	1.04888	1.07497
1994	1.61792	1.08649	1.06749	1.0178
1995	1.67958	1.1077	1.06704	1.03811
1996	1.71652	1.08745	1.06404	1.022
1997	1.75247	1.09411	1.07167	1.02094
1998	1.57686	0.95087	1.05676	0.89979
1999	1.64514	1.07206	1.02757	1.0433
2000	1.79251	1.11194	1.02052	1.08958
2001	1.65054	0.99268	1.07806	0.9208
2002	1.73508	1.05465	1.00326	1.05122
2003	1.74786	1.02146	1.01398	1.00737
2004	1.95427	1.1216	1.00314	1.11809
2005	2.05293	1.07713	1.02536	1.05049
2006	2.10212	1.09024	1.06473	1.02396
2007	2.2231	1.11049	1.05005	1.05756
2008	2.06584	0.9843	1.05923	0.92926
2009	2.01542	1.01063	1.03591	0.97559
2010	2.22018	1.17203	1.06393	1.1016
2011	2.25205	1.06608	1.051	1.01436
2012	2.20881	1.03419	1.05443	0.9808
2013	2.20504	1.04726	1.04905	0.9983
2014	2.16439	1.02651	1.04579	0.98156
2015	2.10774	1.0206	1.04802	0.97383
2016	2.11611	1.03601	1.03192	1.00397
2017	2.1298	1.02532	1.01873	1.00647
AVG	1.623792766	1.0731	1.0551	1.0171

Table 8: Real Gross Income Decomposition

YEAR	RLINK	TLINK	PGLINK	PILINK	PXLINK	PMLINK	QLLINK	QKLINK	PTLINK
1971	1.13951	1.02717	1.00336	1.01249	0.97314	1.03237	1.04727	1.03792	1.00464
1972	1.17875	1.05313	0.99863	1.01979	0.98154	1.02893	1.04067	1.04573	1.00994
1973	1.11854	1.06215	0.99104	0.97061	0.97701	1.02871	1.03463	1.05283	1.00505
1974	1.06309	1.02321	1.00246	1.02091	1.27967	0.74099	1.0159	1.05388	0.94822
1975	1.03895	1.00259	1.00383	0.99997	0.95077	1.02778	1.00132	1.05505	0.97719
1976	1.09907	1.00931	1.00279	1.01305	1.09766	0.91381	1.02717	1.04038	1.00305
1977	1.07211	1.02293	1.00002	0.99529	1.03408	0.96433	1.01338	1.04202	0.99719
1978	1.08301	1.01037	1.00022	0.99834	0.99579	1.00785	1.03225	1.03617	1.00361
1979	1.09534	1.05535	0.99966	1.00153	0.9963	0.97092	1.02649	1.044	0.96733
1980	1.10694	1.02348	1.00159	1.01988	1.02777	0.9528	1.02554	1.05427	0.97926
1981	1.11056	0.9956	1.0019	1.01155	0.93557	1.06659	1.04162	1.05893	0.99786
1982	1.116	0.99264	1.00967	0.99211	0.94072	1.09976	1.02888	1.05442	1.03456
1983	1.11389	1.02149	1.00084	1.00248	0.90354	1.12453	1.0139	1.055	1.01606
1984	1.05738	1.01435	0.99986	0.98603	0.87574	1.13253	1.01225	1.05319	0.9918
1985	0.9601	0.94828	1.00251	0.98719	0.96619	1.02881	0.97905	1.05119	0.99403
1986	0.98775	0.97066	0.99124	0.99225	0.79922	1.24759	1.0016	1.03598	0.9971
1987	1.08442	1.03641	0.9972	1.00117	1.08726	0.91188	1.02835	1.02792	0.99146
1988	1.14355	1.05466	1.00319	1.00321	0.96534	1.05415	1.02718	1.03072	1.01761
1989	1.11392	1.03286	1.00215	0.99885	0.94778	1.06899	1.03301	1.02942	1.01317
1990	1.09485	1.02889	0.99739	0.99756	0.93958	1.07659	1.02338	1.03314	1.01155
1991	1.09363	1.00781	0.99835	1.00323	0.92837	1.09037	1.03057	1.03857	1.01226
1992	1.05777	1.014	0.99844	1.00149	0.9099	1.09179	1.01406	1.03559	0.99342
1993	1.10779	1.0681	0.99807	0.99776	0.91445	1.08377	1.01269	1.03774	0.99106
1994	1.09504	1.01649	0.99947	0.98388	0.90856	1.128	1.02338	1.04452	1.02486
1995	1.08414	1.03411	0.99947	0.99011	0.95582	1.03818	1.02676	1.0398	0.99231
1996	1.08832	1.01963	0.99935	0.99679	0.92063	1.09145	1.02048	1.04496	1.00482
1997	1.08069	1.01843	0.99865	0.99514	0.895	1.11094	1.02475	1.04795	0.99429
1998	0.98741	0.91634	0.99896	1.00874	1.02012	0.98525	1.00968	1.05373	1.00508
1999	1.02683	1.0354	0.99658	0.99742	1.0415	0.92736	1.0033	1.02959	0.96585
2000	1.12788	1.07229	1.00621	0.9808	1.11641	0.92874	0.99623	1.03181	1.03686
2001	0.99174	0.93568	1.00279	0.99541	0.97225	1.0121	1.03416	1.04344	0.98402
2002	1.03562	1.04224	0.99966	0.99766	0.94679	1.04106	0.9881	1.02299	0.98566
2003	1.03894	1.00674	0.99968	1.00292	1.001	1.01031	0.99883	1.01898	1.01132
2004	1.14316	1.10065	1.00313	1.00337	1.03533	0.99306	1.00002	1.00364	1.02814
2005	1.09126	1.04481	1.00189	1.00252	1.05785	0.95746	1.0061	1.02045	1.01285
2006	1.08234	1.02193	1.00251	0.99929	0.99812	0.99796	1.04155	1.01903	0.99608
2007	1.11616	1.05242	1.00151	1.0077	0.93315	1.07316	1.02454	1.02425	1.00142
2008	0.96782	0.93631	1.00011	0.99927	0.95872	1.01922	1.02918	1.02847	0.97715
2009	1.00308	0.97868	0.99489	1.00427	0.82713	1.19268	1.00036	1.03947	0.9865
2010	1.13795	1.09055	1.00078	0.98006	0.99432	1.00673	1.03229	1.02954	1.00101
2011	1.02708	1.01333	0.99979	0.98773	0.96909	1.00559	1.01231	1.04042	0.97451
2012	1.01241	0.98349	0.9978	0.99403	0.93375	1.05267	1.01774	1.03747	0.98293
2013	1.0441	0.99904	1.00163	1.00504	0.92965	1.06123	1.01019	1.0417	0.98657
2014	1.0314	0.98461	1.0017	0.99721	0.96018	1.04084	1.01002	1.03891	0.99939
2015	1.05102	0.9782	1.00277	1.00347	0.87742	1.1584	1.01517	1.03483	1.01641
2016	1.03834	1.00395	1.00306	0.99956	0.95724	1.04146	1.00791	1.0266	0.99693
2017	1.02081	1.00602	1.00102	0.99538	1.04283	0.95402	0.998	1.02567	0.99488
AVG	1.07150	1.0163	1.0004	0.99903	0.97192	1.0335	1.0179	1.0381	0.99824

Table 9: Diewert – Morrison Decomposition

YEAR	RLINK	TLINK	PGLINK	PILINK	PXLINK	PMLINK
1971	1.13959	1.02611	1.00329	1.01225	0.97366	1.03173
1972	1.17841	1.05155	0.99865	1.0194	0.98189	1.02836
1973	1.11675	1.06047	0.99121	0.97115	0.97743	1.02817
1974	1.06351	1.02233	1.00242	1.02053	1.27403	0.74498
1975	1.04623	1.00195	1.00375	0.99997	0.9518	1.02718
1976	1.10141	1.00851	1.00272	1.01271	1.095	0.91596
1977	1.06695	1.02174	1.00002	0.99541	1.03323	0.96519
1978	1.06707	1.00962	1.00022	0.99836	0.99586	1.00773
1979	1.07442	1.05485	0.99966	1.00153	0.99629	0.97085
1980	1.0876	1.02337	1.00162	1.0203	1.02837	0.95183
1981	1.09314	0.99488	1.00197	1.01199	0.93322	1.06918
1982	1.0995	0.99164	1.0102	0.99169	0.93765	1.10536
1983	1.10008	1.02231	1.0009	1.00265	0.89733	1.13354
1984	1.03903	1.01489	0.99985	0.98485	0.866	1.14447
1985	0.93058	0.9421	1.00279	0.98579	0.9625	1.03207
1986	0.96798	0.96581	0.99001	0.99117	0.7745	1.28686
1987	1.0954	1.04095	0.99679	1.00134	1.1006	0.89971
1988	1.16677	1.06102	1.0036	1.00362	0.96099	1.06131
1989	1.13042	1.03577	1.00238	0.99872	0.94223	1.07684
1990	1.10365	1.031	0.99713	0.99733	0.93393	1.08431
1991	1.09278	1.00788	0.9982	1.00353	0.92198	1.09916
1992	1.0481	1.01468	0.99829	1.00164	0.9015	1.10125
1993	1.09925	1.07497	0.99787	0.99751	0.90569	1.0932
1994	1.08243	1.0178	0.99941	0.98199	0.89829	1.1442
1995	1.06643	1.03811	0.99939	0.98879	0.95002	1.04344
1996	1.07245	1.022	0.99925	0.9963	0.90914	1.10605
1997	1.06353	1.02094	0.99842	0.99432	0.87831	1.13095
1998	0.94873	0.89979	0.99874	1.0105	1.02426	0.9823
1999	1.00549	1.0433	0.99575	0.9968	1.05175	0.91066
2000	1.15185	1.08958	1.0077	0.97625	1.14629	0.91243
2001	0.98806	0.9208	1.00343	0.99436	0.96599	1.0149
2002	1.04682	1.05122	0.99958	0.99713	0.9352	1.05054
2003	1.05857	1.00737	0.99962	1.00353	1.0012	1.01246
2004	1.18968	1.11809	1.00367	1.00395	1.04153	0.99187
2005	1.11288	1.05049	1.00215	1.00287	1.06608	0.95174
2006	1.09706	1.02396	1.00281	0.99921	0.9979	0.99771
2007	1.12723	1.05756	1.00167	1.00853	0.92629	1.08127
2008	0.95821	0.92926	1.00012	0.99919	0.95442	1.0213
2009	0.98959	0.97559	0.99428	1.00478	0.80867	1.21791
2010	1.14463	1.1016	1.00088	0.97762	0.99362	1.00756
2011	1.02572	1.01436	0.99976	0.98625	0.9654	1.00627
2012	1.00155	0.9808	0.99752	0.99327	0.92559	1.05961
2013	1.03148	0.9983	1.00186	1.00575	0.92013	1.07017
2014	1.01456	0.98156	1.00197	0.99677	0.95405	1.04742
2015	1.04217	0.97383	1.00325	1.00406	0.85791	1.18805
2016	1.02278	1.00397	1.00363	0.99948	0.9495	1.04935
2017	1.01401	1.00647	1.00123	0.99446	1.05157	0.94512
AVG	1.0673	1.0171	1.0004	0.99872	0.96848	1.0384

Table 9: Diewert – Morrison Decomposition (continued)

YEAR	PKLINK	PKELINK	QLLINK	QWSLINK	PTLINK	PNKLINK
1971	1.08357	0.92396	1.04632	1.0392	1.00455	1.00117
1972	0.9153	1.09397	1.03986	1.047	1.00974	1.00131
1973	1.17718	0.84855	1.03398	1.05397	1.00496	0.9989
1974	1.1201	0.89277	1.01561	1.05494	0.94913	0.99999
1975	0.70734	1.42202	1.00129	1.05652	0.97767	1.00585
1976	0.9403	1.06657	1.02645	1.04165	1.00297	1.00289
1977	0.95265	1.0449	1.01305	1.04314	0.99726	0.99543
1978	1.0499	0.93915	1.03175	1.0367	1.00356	0.98601
1979	1.06002	0.92589	1.02654	1.04393	0.96725	0.98147
1980	1.08792	0.90355	1.02607	1.05333	0.97883	0.98299
1981	0.84229	1.1692	1.04322	1.0571	0.99778	0.9848
1982	0.84342	1.16813	1.03047	1.05184	1.03644	0.98523
1983	1.00852	0.9796	1.01486	1.05141	1.01715	0.98795
1984	1.04208	0.94734	1.0133	1.04867	0.99111	0.9872
1985	1.02206	0.96416	0.97674	1.04507	0.99337	0.98543
1986	1.09914	0.90285	1.00183	1.03078	0.99667	0.99236
1987	1.16969	0.86072	1.03256	1.0242	0.99022	1.00677
1988	1.04356	0.96864	1.03074	1.0274	1.01991	1.01084
1989	0.9931	1.01635	1.03669	1.02686	1.01463	1.00934
1990	1.00083	1.00477	1.02568	1.03056	1.01267	1.00561
1991	0.95288	1.04729	1.03345	1.03561	1.01341	0.99794
1992	0.98849	1.00389	1.01546	1.03261	0.99278	0.99233
1993	1.03398	0.95674	1.01406	1.03434	0.9901	0.98925
1994	1.06355	0.92863	1.02618	1.04025	1.02783	0.98764
1995	1.01625	0.96709	1.0304	1.03556	0.99128	0.98281
1996	1.08656	0.90666	1.02363	1.03948	1.00555	0.98514
1997	1.04874	0.93993	1.02902	1.04145	0.99332	0.98574
1998	0.92891	1.0578	1.01167	1.04457	1.00612	0.98261
1999	0.89232	1.10563	1.0041	1.02338	0.95778	0.98658
2000	1.09287	0.92122	0.99533	1.02531	1.04591	1.00677
2001	1.21106	0.84019	1.04217	1.03444	0.98038	1.01753
2002	1.05231	0.96324	0.98543	1.01809	0.98246	1.01362
2003	1.01996	0.99921	0.99859	1.01541	1.01368	1.01915
2004	0.9712	1.04918	1.00002	1.00311	1.03306	1.01897
2005	0.95732	1.05837	1.00694	1.01829	1.01463	1.0132
2006	1.01871	0.99012	1.04663	1.0173	0.99562	1.00865
2007	1.02933	0.97465	1.02719	1.02226	1.00157	1.00324
2008	0.98058	1.01919	1.03234	1.02604	0.97475	0.9994
2009	0.91209	1.09109	1.00041	1.03549	0.98489	0.99517
2010	0.99297	1.00404	1.03636	1.0266	1.00113	0.99698
2011	1.25629	0.79954	1.01381	1.03668	0.97146	1.00445
2012	1.08553	0.91807	1.02004	1.03372	0.98077	0.99659
2013	1.00064	0.99205	1.01163	1.03698	0.98469	0.99268
2014	0.95013	1.04229	1.01161	1.03379	0.99929	0.99031
2015	1.01592	0.97899	1.01781	1.02969	1.01925	0.99457
2016	0.90482	1.09167	1.00939	1.02232	0.99636	0.98776
2017	1.09265	0.91464	0.9976	1.02118	0.99386	0.99939
AVG	1.0152	0.99158	1.0193	1.0351	0.99825	0.99702

Table 10: Kohli Decomposition of Gross Real Income

YEAR	RIL	TT	GG	II	XX	MM	LL	KK	TTT
1970	1	1	1	1	1	1	1	1	1
1971	1.13951	1.02717	1.00336	1.01249	0.97314	1.03237	1.04727	1.03792	1.00464
1972	1.3432	1.08174	1.00198	1.03253	0.95517	1.06224	1.08986	1.08538	1.01462
1973	1.50242	1.14897	0.993	1.00218	0.93321	1.09273	1.1276	1.14272	1.01975
1974	1.59721	1.17564	0.99545	1.02314	1.1942	0.8097	1.14552	1.20428	0.96695
1975	1.65942	1.17868	0.99927	1.0231	1.13541	0.8322	1.14703	1.27058	0.94489
1976	1.82382	1.18966	1.00206	1.03646	1.2463	0.76047	1.17819	1.32189	0.94778
1977	1.95534	1.21694	1.00208	1.03158	1.28877	0.73335	1.19396	1.37744	0.94512
1978	2.11765	1.22956	1.0023	1.02986	1.28335	0.7391	1.23246	1.42725	0.94853
1979	2.31954	1.29762	1.00196	1.03144	1.2786	0.71761	1.26512	1.49006	0.91754
1980	2.5676	1.32809	1.00356	1.05195	1.31411	0.68374	1.29743	1.57092	0.89851
1981	2.85147	1.32224	1.00546	1.0641	1.22944	0.72927	1.35143	1.6635	0.89659
1982	3.18223	1.31251	1.01518	1.0557	1.15656	0.80202	1.39046	1.75403	0.92758
1983	3.54466	1.34071	1.01604	1.05831	1.04499	0.9019	1.40979	1.85051	0.94247
1984	3.74805	1.35995	1.01589	1.04353	0.91514	1.02142	1.42706	1.94893	0.93474
1985	3.5985	1.28962	1.01844	1.03016	0.8842	1.05085	1.39717	2.04869	0.92917
1986	3.55441	1.25177	1.00951	1.02218	0.70667	1.31103	1.39941	2.12241	0.92647
1987	3.85449	1.29736	1.00668	1.02337	0.76834	1.19551	1.43908	2.18167	0.91856
1988	4.40779	1.36827	1.00989	1.02666	0.74171	1.26024	1.47819	2.24869	0.93474
1989	4.90992	1.41324	1.01206	1.02547	0.70298	1.34719	1.52698	2.31485	0.94705
1990	5.37562	1.45407	1.00942	1.02297	0.66051	1.45037	1.56269	2.39156	0.95798
1991	5.87893	1.46543	1.00775	1.02628	0.6132	1.58144	1.61045	2.4838	0.96973
1992	6.21856	1.48594	1.00618	1.02781	0.55795	1.72659	1.6331	2.57219	0.96335
1993	6.88888	1.58713	1.00424	1.0255	0.51022	1.87123	1.65382	2.66926	0.95473
1994	7.54362	1.6133	1.00371	1.00897	0.46356	2.11075	1.69248	2.78808	0.97846
1995	8.17833	1.66834	1.00318	0.999	0.44308	2.19134	1.73777	2.89904	0.97094
1996	8.90065	1.70108	1.00253	0.99579	0.40791	2.39173	1.77336	3.02939	0.97562
1997	9.6188	1.73243	1.00117	0.99095	0.36508	2.65706	1.81725	3.17465	0.97005
1998	9.49774	1.5875	1.00013	0.99961	0.37243	2.61788	1.83485	3.34524	0.97498
1999	9.75258	1.64369	0.9967	0.99703	0.38788	2.42773	1.84091	3.44422	0.94168
2000	10.9997	1.76252	1.00289	0.97789	0.43304	2.25473	1.83398	3.55378	0.97639
2001	10.9088	1.64916	1.00568	0.9734	0.42102	2.28202	1.89662	3.70817	0.96078
2002	11.29733	1.71882	1.00534	0.97112	0.39862	2.37572	1.87404	3.79341	0.947
2003	11.7373	1.7304	1.00502	0.97396	0.39902	2.40021	1.87186	3.86542	0.95772
2004	13.41762	1.90457	1.00816	0.97724	0.41311	2.38355	1.8719	3.87948	0.98467
2005	14.64211	1.98991	1.01006	0.97971	0.43701	2.28215	1.88331	3.95882	0.99732
2006	15.84778	2.03355	1.0126	0.97901	0.43619	2.27749	1.96156	4.03415	0.99342
2007	17.68873	2.14015	1.01413	0.98655	0.40703	2.44411	2.0097	4.13198	0.99483
2008	17.11958	2.00384	1.01424	0.98583	0.39023	2.4911	2.06835	4.24963	0.9721
2009	17.17237	1.96112	1.00906	0.99004	0.32277	2.97109	2.0691	4.41737	0.95898
2010	19.54124	2.13871	1.00984	0.9703	0.32094	2.99107	2.13591	4.54787	0.95995
2011	20.07038	2.16721	1.00963	0.9584	0.31102	3.0078	2.1622	4.73171	0.93547
2012	20.31936	2.13144	1.00741	0.95268	0.29041	3.16621	2.20057	4.90899	0.91951
2013	21.21552	2.12939	1.00905	0.95748	0.26998	3.36008	2.22299	5.11371	0.90716
2014	21.88178	2.09661	1.01077	0.9548	0.25923	3.49729	2.24526	5.31267	0.90661
2015	22.99808	2.0509	1.01357	0.95811	0.22745	4.05128	2.27933	5.49772	0.92148
2016	23.87982	2.059	1.01668	0.95769	0.21773	4.21925	2.29736	5.64398	0.91865
2017	24.37677	2.07139	1.01772	0.95327	0.22705	4.02525	2.29276	5.78886	0.91395

Table 11: Kohli Decomposition of Net Real Income

YEAR	RIL	TT	GG	II	XX	MM	KKB	KKE	LL	KKW	TTT	KN
1970	1	1	1	1	1	1	1	1	1	1	1	1
1971	1.1396	1.0261	1.0033	1.0122	0.9737	1.0317	1.0836	0.924	1.0463	1.0392	1.0045	1.0012
1972	1.3429	1.079	1.0019	1.0319	0.956	1.061	0.9918	1.0108	1.088	1.088	1.0143	1.0025
1973	1.4997	1.1442	0.9931	1.0021	0.9344	1.0909	1.1675	0.8577	1.125	1.1468	1.0194	1.0014
1974	1.5949	1.1698	0.9955	1.0227	1.1905	0.8127	1.3077	0.7657	1.1426	1.2098	0.9675	1.0014
1975	1.6687	1.1721	0.9993	1.0227	1.1331	0.8348	0.925	1.0889	1.144	1.2781	0.9459	1.0072
1976	1.8379	1.182	1.002	1.0356	1.2408	0.7646	0.8698	1.1614	1.1743	1.3314	0.9487	1.0102
1977	1.9609	1.2077	1.002	1.0309	1.282	0.738	0.8286	1.2135	1.1896	1.3888	0.9461	1.0055
1978	2.0924	1.2194	1.0022	1.0292	1.2767	0.7437	0.87	1.1397	1.2274	1.4398	0.9495	0.9915
1979	2.2481	1.2863	1.0019	1.0308	1.272	0.722	0.9222	1.0552	1.26	1.503	0.9184	0.9731
1980	2.4451	1.3163	1.0035	1.0517	1.308	0.6872	1.0032	0.9534	1.2928	1.5832	0.8989	0.9565
1981	2.6728	1.3096	1.0055	1.0643	1.2207	0.7348	0.845	1.1148	1.3487	1.6736	0.8969	0.942
1982	2.9388	1.2986	1.0157	1.0555	1.1446	0.8122	0.7127	1.3022	1.3898	1.7603	0.9296	0.9281
1983	3.2329	1.3276	1.0167	1.0583	1.0271	0.9207	0.7188	1.2756	1.4104	1.8508	0.9456	0.9169
1984	3.3591	1.3474	1.0165	1.0422	0.8894	1.0537	0.749	1.2085	1.4292	1.9409	0.9372	0.9052
1985	3.1259	1.2694	1.0193	1.0274	0.8561	1.0875	0.7656	1.1651	1.396	2.0284	0.931	0.892
1986	3.0258	1.226	1.0092	1.0184	0.663	1.3994	0.8415	1.0519	1.3985	2.0908	0.9279	0.8852
1987	3.3144	1.2762	1.0059	1.0197	0.7297	1.2591	0.9842	0.9054	1.444	2.1414	0.9188	0.8912
1988	3.8672	1.3541	1.0095	1.0234	0.7013	1.3363	1.0271	0.877	1.4884	2.2001	0.9371	0.9008
1989	4.3716	1.4025	1.0119	1.0221	0.6608	1.4389	1.02	0.8914	1.543	2.2592	0.9508	0.9092
1990	4.8247	1.446	1.009	1.0194	0.6171	1.5603	1.0209	0.8956	1.5827	2.3282	0.9628	0.9143
1991	5.2723	1.4574	1.0072	1.023	0.569	1.715	0.9728	0.938	1.6356	2.4111	0.9757	0.9124
1992	5.5259	1.4788	1.0055	1.0247	0.5129	1.8886	0.9616	0.9416	1.6609	2.4897	0.9687	0.9054
1993	6.0744	1.5896	1.0034	1.0221	0.4645	2.0647	0.9943	0.9009	1.6842	2.5752	0.9591	0.8957
1994	6.5751	1.6179	1.0028	1.0037	0.4173	2.3624	1.0574	0.8366	1.7283	2.6789	0.9858	0.8846
1995	7.0119	1.6796	1.0022	0.9925	0.3964	2.465	1.0746	0.8091	1.7809	2.7741	0.9772	0.8694
1996	7.5199	1.7165	1.0014	0.9888	0.3604	2.7264	1.1676	0.7335	1.823	2.8837	0.9826	0.8565
1997	7.9976	1.7525	0.9998	0.9832	0.3166	3.0834	1.2246	0.6895	1.8759	3.0032	0.9761	0.8443
1998	7.5876	1.5769	0.9986	0.9935	0.3242	3.0288	1.1375	0.7293	1.8978	3.137	0.982	0.8296
1999	7.6292	1.6451	0.9943	0.9903	0.341	2.7582	1.015	0.8064	1.9055	3.2104	0.9406	0.8185
2000	8.7878	1.7925	1.002	0.9668	0.3909	2.5167	1.1093	0.7428	1.8966	3.2916	0.9838	0.824
2001	8.6828	1.6505	1.0054	0.9613	0.3776	2.5542	1.3434	0.6241	1.9766	3.405	0.9645	0.8385
2002	9.0893	1.7351	1.005	0.9586	0.3531	2.6833	1.4137	0.6012	1.9478	3.4666	0.9476	0.8499
2003	9.6217	1.7479	1.0046	0.962	0.3536	2.7167	1.4419	0.6007	1.9451	3.52	0.9605	0.8662
2004	11.4467	1.9543	1.0083	0.9658	0.3682	2.6946	1.4004	0.6303	1.9451	3.531	0.9923	0.8826
2005	12.7388	2.0529	1.0105	0.9685	0.3926	2.5646	1.3406	0.667	1.9586	3.5956	1.0068	0.8942
2006	13.9752	2.1021	1.0133	0.9678	0.3918	2.5587	1.3657	0.6605	2.0499	3.6578	1.0024	0.902
2007	15.7532	2.2231	1.015	0.976	0.3629	2.7667	1.4057	0.6437	2.1057	3.7392	1.004	0.9049
2008	15.0949	2.0658	1.0151	0.9752	0.3463	2.8256	1.3785	0.6561	2.1738	3.8366	0.9786	0.9043
2009	14.9378	2.0154	1.0093	0.9799	0.2801	3.4413	1.2573	0.7158	2.1747	3.9727	0.9638	0.9
2010	17.0983	2.2202	1.0102	0.958	0.2783	3.4674	1.2484	0.7187	2.2537	4.0784	0.9649	0.8973
2011	17.538	2.2521	1.01	0.9448	0.2687	3.4891	1.5684	0.5746	2.2849	4.228	0.9374	0.9013
2012	17.5653	2.2088	1.0075	0.9384	0.2487	3.6971	1.7026	0.5276	2.3307	4.3706	0.9193	0.8982
2013	18.1183	2.205	1.0093	0.9438	0.2288	3.9565	1.7036	0.5234	2.3578	4.5322	0.9053	0.8916
2014	18.382	2.1644	1.0113	0.9408	0.2183	4.1441	1.6187	0.5455	2.3852	4.6853	0.9046	0.883
2015	19.1571	2.1077	1.0146	0.9446	0.1873	4.9234	1.6445	0.534	2.4276	4.8244	0.922	0.8782
2016	19.5936	2.1161	1.0183	0.9441	0.1778	5.1664	1.4879	0.583	2.4504	4.9321	0.9187	0.8674
2017	19.8681	2.1298	1.0195	0.9389	0.187	4.8829	1.6258	0.5332	2.4445	5.0366	0.913	0.8669

Table 12: Per Capita Real Income Growth

YEAR	RIPERPOP(GROSS)	RIPERPOP(NET)
1970	1	1
1971	1.09905	1.09913
1972	1.2506	1.25032
1973	1.35066	1.3482
1974	1.38836	1.38638
1975	1.39644	1.40421
1976	1.49232	1.50381
1977	1.55068	1.55511
1978	1.62915	1.60975
1979	1.73392	1.68055
1980	1.8718	1.78248
1981	1.95828	1.83559
1982	2.07697	1.91807
1983	2.27457	2.0745
1984	2.3508	2.10683
1985	2.24213	1.94764
1986	2.20277	1.87515
1987	2.33274	2.0059
1988	2.57694	2.26089
1989	2.76766	2.4642
1990	2.90366	2.60608
1991	3.08908	2.77035
1992	3.18448	2.82979
1993	3.46189	3.05257
1994	3.69593	3.22139
1995	3.9015	3.34504
1996	4.09948	3.46353
1997	4.29099	3.56778
1998	4.09061	3.26792
1999	4.15904	3.25353
2000	4.60485	3.67885
2001	4.42037	3.51837
2002	4.51457	3.63222
2003	4.74168	3.88702
2004	5.33576	4.552
2005	5.67404	4.93648
2006	5.94626	5.24364
2007	6.35651	5.66098
2008	5.81518	5.12743
2009	5.63603	4.90262
2010	6.27506	5.49058
2011	6.31327	5.51671
2012	6.24659	5.39993
2013	6.4346	5.49522
2014	6.5734	5.52206
2015	6.85455	5.70976
2016	7.02561	5.76457
2017	7.1655	5.84019

Table 13: 2 Goods Consumer Model: Consumption and After Tax Labour

YEAR	X1	X2	PC1	PC2
1970	3.28791	2.955	1	1
1971	3.54641	3.06714	1.03388	1.03462
1972	3.73658	3.17055	1.0608	1.0598
1973	3.90088	3.26365	1.22841	1.18272
1974	4.03217	3.25335	1.39333	1.42927
1975	4.02905	3.15704	1.43488	1.63571
1976	4.12419	3.22491	1.44359	1.5963
1977	4.20585	3.20777	1.47427	1.63229
1978	4.34799	3.32132	1.51329	1.60273
1979	4.50519	3.41386	1.57993	1.67656
1980	4.64183	3.52057	1.70887	1.84797
1981	4.58499	3.63463	1.81466	1.9876
1982	4.53296	3.67472	1.86425	2.2901
1983	4.71005	3.71771	1.88801	2.57698
1984	4.87298	3.72441	1.94777	2.81575
1985	4.84559	3.54977	1.96205	2.95326
1986	5.00955	3.54197	1.96642	2.71861
1987	5.38977	3.66882	2.0106	2.69898
1988	5.81117	3.76081	2.08672	2.88368
1989	6.07019	3.90346	2.18054	3.1034
1990	6.2502	3.94164	2.27149	3.38873
1991	6.45206	4.10062	2.31293	3.6065
1992	6.6515	4.1207	2.36779	3.83382
1993	7.2846	4.15824	2.4611	4.16085
1994	7.6353	4.2715	2.59613	4.51367
1995	7.70712	4.41761	2.63048	4.62979
1996	7.91742	4.46596	2.66351	4.96375
1997	8.12013	4.57333	2.71687	5.0504
1998	7.60347	4.51098	2.66257	5.11428
1999	8.21135	4.49844	2.61382	5.03069
2000	9.24445	4.3796	2.58889	5.54899
2001	9.54401	4.5531	2.57358	5.51291
2002	9.87759	4.38041	2.56104	5.72009
2003	10.0583	4.41739	2.55901	5.81026
2004	10.39777	4.34852	2.56032	6.15346
2005	10.47784	4.29762	2.55678	6.45905
2006	10.47508	4.57479	2.59774	6.52823
2007	10.64905	4.63379	2.71175	7.05394
2008	10.41758	4.67514	2.84082	6.91462
2009	9.95175	4.5208	2.88106	7.09916
2010	10.31081	4.75284	2.97062	7.18015
2011	10.53784	4.79063	3.09418	7.54743
2012	10.70555	4.87542	3.18811	7.6797
2013	10.94043	4.9219	3.22513	8.03069
2014	11.2068	4.98404	3.25357	8.43342
2015	11.66934	5.1102	3.25743	8.61074
2016	11.7166	5.13122	3.22215	8.7367
2017	12.0719	5.10463	3.23326	9.06151

Table 14: 2 Goods Consumer Model: Price Elasticities

	CMOD3				CMOD4			
YEAR	EC11	EC12	EC21	EC22	EC11	EC12	EC21	EC22
1970	-0.08976	0.08976	0.06737	-0.06737	-0.02934	0.02934	0.01825	-0.01825
1971	-0.08788	0.08788	0.06644	-0.06644	-0.02678	0.02678	0.01887	-0.01887
1972	-0.08676	0.08676	0.06584	-0.06584	-0.02542	0.02542	0.01924	-0.01924
1973	-0.08526	0.08526	0.06414	-0.06414	-0.02374	0.02374	0.0195	-0.0195
1974	-0.08454	0.08454	0.06537	-0.06537	-0.02474	0.02474	0.01914	-0.01914
1975	-0.08445	0.08445	0.06813	-0.06813	-0.0264	0.0264	0.01891	-0.01891
1976	-0.08403	0.08403	0.06707	-0.06707	-0.02564	0.02564	0.01899	-0.01899
1977	-0.08298	0.08298	0.06653	-0.06653	-0.02433	0.02433	0.01951	-0.01951
1978	-0.08235	0.08235	0.06502	-0.06502	-0.02339	0.02339	0.01957	-0.01957
1979	-0.08175	0.08175	0.06475	-0.06475	-0.02272	0.02272	0.0199	-0.0199
1980	-0.08171	0.08171	0.06521	-0.06521	-0.02275	0.02275	0.02005	-0.02005
1981	-0.08373	0.08373	0.06665	-0.06665	-0.02299	0.02299	0.0206	-0.0206
1982	-0.08535	0.08535	0.07075	-0.07075	-0.0247	0.0247	0.02093	-0.02093
1983	-0.08399	0.08399	0.07309	-0.07309	-0.02603	0.02603	0.02044	-0.02044
1984	-0.08227	0.08227	0.07379	-0.07379	-0.02664	0.02664	0.0199	-0.0199
1985	-0.08003	0.08003	0.07362	-0.07362	-0.02698	0.02698	0.01921	-0.01921
1986	-0.07922	0.07922	0.07058	-0.07058	-0.02457	0.02457	0.01975	-0.01975
1987	-0.07766	0.07766	0.06876	-0.06876	-0.02267	0.02267	0.02018	-0.02018
1988	-0.07529	0.07529	0.0681	-0.0681	-0.02106	0.02106	0.02067	-0.02067
1989	-0.07473	0.07473	0.06858	-0.06858	-0.02102	0.02102	0.02072	-0.02072
1990	-0.07356	0.07356	0.06916	-0.06916	-0.02077	0.02077	0.02084	-0.02084
1991	-0.07344	0.07344	0.07037	-0.07037	-0.02138	0.02138	0.02061	-0.02061
1992	-0.07198	0.07198	0.07046	-0.07046	-0.02082	0.02082	0.02078	-0.02078
1993	-0.06814	0.06814	0.069	-0.069	-0.01868	0.01868	0.02174	-0.02174
1994	-0.06695	0.06695	0.06893	-0.06893	-0.01829	0.01829	0.02194	-0.02194
1995	-0.06776	0.06776	0.06987	-0.06987	-0.01903	0.01903	0.02148	-0.02148
1996	-0.06628	0.06628	0.0704	-0.0704	-0.0189	0.0189	0.02141	-0.02141
1997	-0.0663	0.0663	0.07034	-0.07034	-0.01888	0.01888	0.02143	-0.02143
1998	-0.0678	0.0678	0.07238	-0.07238	-0.01992	0.01992	0.02142	-0.02142
1999	-0.06464	0.06464	0.07005	-0.07005	-0.01858	0.01858	0.02109	-0.02109
2000	-0.05751	0.05751	0.06725	-0.06725	-0.01678	0.01678	0.01972	-0.01972
2001	-0.05779	0.05779	0.06746	-0.06746	-0.01689	0.01689	0.01975	-0.01975
2002	-0.05459	0.05459	0.06581	-0.06581	-0.01599	0.01599	0.01913	-0.01913
2003	-0.05398	0.05398	0.06569	-0.06569	-0.0159	0.0159	0.01896	-0.01896
2004	-0.05143	0.05143	0.06481	-0.06481	-0.01539	0.01539	0.01829	-0.01829
2005	-0.04992	0.04992	0.06458	-0.06458	-0.01521	0.01521	0.01777	-0.01777
2006	-0.05197	0.05197	0.06632	-0.06632	-0.01532	0.01532	0.01908	-0.01908
2007	-0.0512	0.0512	0.0664	-0.0664	-0.01534	0.01534	0.01869	-0.01869
2008	-0.05353	0.05353	0.06697	-0.06697	-0.01533	0.01533	0.02001	-0.02001
2009	-0.05355	0.05355	0.06728	-0.06728	-0.01536	0.01536	0.02007	-0.02007
2010	-0.05459	0.05459	0.06774	-0.06774	-0.01536	0.01536	0.02071	-0.02071
2011	-0.05398	0.05398	0.06742	-0.06742	-0.01535	0.01535	0.02033	-0.02033
2012	-0.05435	0.05435	0.06745	-0.06745	-0.01534	0.01534	0.02053	-0.02053
2013	-0.05331	0.05331	0.0673	-0.0673	-0.01537	0.01537	0.01995	-0.01995
2014	-0.05215	0.05215	0.06718	-0.06718	-0.01539	0.01539	0.01932	-0.01932
2015	-0.05136	0.05136	0.06689	-0.06689	-0.01538	0.01538	0.01886	-0.01886
2016	-0.05085	0.05085	0.06696	-0.06696	-0.01539	0.01539	0.01861	-0.01861
2017	-0.049	0.049	0.06593	-0.06593	-0.01532	0.01532	0.0175	-0.0175
AVG	0.068659	0.068659	0.067921	0.067921	0.020157	0.020157	0.019882	0.019882

Table 15: 2 Goods Consumer Model: Income Elasticities

	CMOD1		CMOD2		CMOD3		CMOD4	
YEAR	ECU1	ECU2	ECU1	ECU2	ECU1	ECU2	ECU1	ECU2
1970	1.63516	0.52851	1.63516	0.52851	1.25668	0.8336	5.60256	-1.87993
1971	1.61749	0.5326	1.61749	0.5326	1.25111	0.83588	5.19753	-1.97646
1972	1.60829	0.53544	1.60829	0.53544	1.24832	0.8374	4.99112	-2.03578
1973	1.61369	0.54555	1.61369	0.54555	1.25267	0.84191	4.85992	-2.06925
1974	1.57615	0.53551	1.57615	0.53551	1.23613	0.83823	4.79056	-2.02546
1975	1.5292	0.51385	1.5292	0.51385	1.21505	0.82916	4.68775	-2.02898
1976	1.53833	0.52154	1.53833	0.52154	1.21958	0.83259	4.67635	-2.02818
1977	1.52914	0.52391	1.52914	0.52391	1.2166	0.83391	4.48395	-2.1116
1978	1.54217	0.53482	1.54217	0.53482	1.22348	0.83858	4.48289	-2.10318
1979	1.53634	0.53593	1.53634	0.53593	1.22146	0.83923	4.37602	-2.15585
1980	1.52867	0.53201	1.52867	0.53201	1.2177	0.83771	4.32486	-2.18683
1981	1.54057	0.5241	1.54057	0.5241	1.22061	0.83386	4.32376	-2.28022
1982	1.51273	0.49232	1.51273	0.49232	1.20454	0.82009	4.24689	-2.39997
1983	1.46991	0.46687	1.46991	0.46687	1.18605	0.80912	4.16347	-2.41049
1984	1.44208	0.45429	1.44208	0.45429	1.17518	0.8038	4.11234	-2.37918
1985	1.41637	0.44801	1.41637	0.44801	1.16599	0.80148	4.06966	-2.30903
1986	1.43153	0.47644	1.43153	0.47644	1.17392	0.81497	3.9597	-2.30722
1987	1.42768	0.48961	1.42768	0.48961	1.17409	0.82128	3.78798	-2.34981
1988	1.40424	0.48812	1.40424	0.48812	1.16534	0.8215	3.53752	-2.46161
1989	1.39429	0.48073	1.39429	0.48073	1.1608	0.81859	3.48106	-2.50181
1990	1.37766	0.46908	1.37766	0.46908	1.15344	0.81409	3.37747	-2.57757
1991	1.36969	0.4542	1.36969	0.4542	1.14902	0.8077	3.37944	-2.59734
1992	1.35453	0.44543	1.35453	0.44543	1.14281	0.80448	3.26639	-2.68275
1993	1.32454	0.44245	1.32454	0.44245	1.13193	0.8049	2.98417	-2.92686
1994	1.31418	0.43573	1.31418	0.43573	1.12769	0.80257	2.91331	-3.01424
1995	1.31806	0.42778	1.31806	0.42778	1.1284	0.79871	2.97827	-2.95355
1996	1.30423	0.40917	1.30423	0.40917	1.12208	0.79129	2.90985	-3.05419
1997	1.30457	0.41017	1.30457	0.41017	1.12227	0.79172	2.90981	-3.05297
1998	1.31161	0.39029	1.31161	0.39029	1.12314	0.78189	2.98482	-3.12346
1999	1.29194	0.4013	1.29194	0.4013	1.11729	0.78868	2.84962	-3.0629
2000	1.24454	0.3804	1.24454	0.3804	1.09984	0.78389	2.59828	-3.04768
2001	1.24612	0.37949	1.24612	0.37949	1.10033	0.78329	2.60883	-3.05333
2002	1.22678	0.37293	1.22678	0.37293	1.09337	0.78283	2.50077	-3.03317
2003	1.22293	0.36732	1.22293	0.36732	1.09183	0.78084	2.48406	-3.04079
2004	1.20762	0.34942	1.20762	0.34942	1.08597	0.7751	2.41115	-3.05515
2005	1.19849	0.32947	1.19849	0.32947	1.08234	0.76756	2.37639	-3.08627
2006	1.20963	0.3225	1.20963	0.3225	1.08583	0.76224	2.38769	-3.36857
2007	1.20515	0.30671	1.20515	0.30671	1.08386	0.75565	2.37835	-3.39362
2008	1.21851	0.33288	1.21851	0.33288	1.08916	0.76547	2.40044	-3.47559
2009	1.21848	0.32609	1.21848	0.32609	1.08891	0.76228	2.39851	-3.53055
2010	1.22452	0.33182	1.22452	0.33182	1.09111	0.76398	2.40646	-3.61024
2011	1.22103	0.32961	1.22103	0.32961	1.08986	0.76354	2.40216	-3.55391
2012	1.22316	0.33479	1.22316	0.33479	1.0908	0.76556	2.40594	-3.55834
2013	1.2173	0.32121	1.2173	0.32121	1.08825	0.76036	2.39564	-3.53659
2014	1.21108	0.30373	1.21108	0.30373	1.08549	0.75346	2.3837	-3.5288
2015	1.20692	0.29599	1.20692	0.29599	1.08383	0.75074	2.37672	-3.4888
2016	1.20449	0.2834	1.20449	0.2834	1.08255	0.74543	2.3703	-3.51905
2017	1.19491	0.27216	1.19491	0.27216	1.07906	0.74251	2.35559	-3.36646
AVG	1.3639	0.4247	1.3639	0.4247	1.1478	0.79778	3.3823	-2.7769

Table 16: 7 Goods Producer Model: Technical Progress Estimates

YEAR	PMOD1	PMOD2	PMOD3	PMOD4	PMOD5	PMOD6	PMOD7	PMOD11	PMOD12
1970	0.00522		0.01249	0.01841	0.03704	0.03485	0.03485		
1971	0.00493	0.02037	0.01192	0.01783	0.03434	0.03242	0.03242	0.03334	0.04811
1972	0.00469	0.01913	0.01152	0.01756	0.03248	0.03072	0.03072	0.03145	0.04485
1973	0.0046	0.01979	0.01149	0.01684	0.03149	0.02976	0.02976	0.0311	0.0239
1974	0.00713	0.02632	0.01363	0.0201	0.03775	0.03603	0.03603	0.03345	0.03051
1975	0.00853	0.02957	0.01335	0.01968	0.03431	0.03309	0.03309	0.02991	0.02941
1976	0.00857	0.01742	0.01383	0.02034	0.03585	0.03454	0.03454	0.03093	0.04117
1977	0.00902	0.01904	0.01411	0.02039	0.0357	0.03446	0.03446	0.03069	0.04141
1978	0.00847	0.01812	0.01374	0.01977	0.03433	0.03315	0.03315	0.02973	0.03959
1979	0.00867	0.01824	0.01336	0.01968	0.03259	0.03174	0.03174	0.02676	-0.00083
1980	0.00907	0.01926	0.01316	0.01989	0.03138	0.03074	0.03074	0.02487	-0.00038
1981	0.009	0.02095	0.0125	0.01909	0.02891	0.02841	0.02841	0.0231	-0.03379
1982	0.01056	0.02129	0.01289	0.01833	0.02781	0.02723	0.02723	0.02442	0.02718
1983	0.01181	0.02158	0.0129	0.01776	0.02601	0.02546	0.02546	0.02409	0.02721
1984	0.01224	0.02067	0.01247	0.01682	0.02349	0.02309	0.02309	0.02228	0.02727
1985	0.01289	-0.03143	0.01258	0.0165	0.02254	0.02222	0.02222	0.02165	-0.05152
1986	0.01089	-0.03792	0.01067	0.01442	0.01894	0.01867	0.01867	0.01841	-0.06032
1987	0.01124	0.02655	0.01084	0.01474	0.01934	0.01912	0.01912	0.01828	0.04724
1988	0.01164	0.02671	0.01087	0.01447	0.01902	0.01874	0.01874	0.01845	0.04493
1989	0.01205	0.01214	0.01088	0.01415	0.01852	0.01821	0.01821	0.01843	0.02039
1990	0.01285	0.02441	0.01107	0.01404	0.01814	0.01781	0.01781	0.01842	0.02177
1991	0.01342	0.02391	0.01113	0.01388	0.01762	0.01727	0.01727	0.01815	0.02008
1992	0.01374	0.0227	0.011	0.01361	0.0167	0.01639	0.01639	0.01715	0.03134
1993	0.01419	0.02155	0.01101	0.01343	0.01589	0.01562	0.01562	0.01623	0.04181
1994	0.0146	0.03959	0.01118	0.01307	0.01566	0.01531	0.01531	0.0166	0.02122
1995	0.01497	0.0367	0.01123	0.01288	0.01523	0.01491	0.01491	0.01615	0.02041
1996	0.01568	0.03768	0.01156	0.01293	0.01495	0.01463	0.01463	0.01579	0.0195
1997	0.01548	0.03828	0.01125	0.01249	0.01417	0.01386	0.01386	0.0148	-0.02222
1998	0.01638	-0.02817	0.01159	0.01281	0.01429	0.01401	0.01401	0.01465	0.0006
1999	0.01711	-0.02692	0.01179	0.01302	0.01397	0.01381	0.01381	0.0139	0.02077
2000	0.01942	-0.02237	0.01342	0.01394	0.01529	0.0151	0.0151	0.01541	0.02158
2001	0.01966	0.00588	0.01334	0.01373	0.01479	0.01466	0.01466	0.01462	-0.02044
2002	0.02018	0.05862	0.01368	0.01388	0.01449	0.01441	0.01441	0.01377	0.02258
2003	0.02084	0.06408	0.01384	0.01399	0.01453	0.01444	0.01444	0.01363	0.02248
2004	0.02201	0.0158	0.01448	0.01445	0.01498	0.01488	0.01488	0.01377	0.06732
2005	0.02316	0.01563	0.01514	0.01496	0.01533	0.01526	0.01526	0.0137	0.04182
2006	0.02363	0.03549	0.01503	0.01477	0.01504	0.01499	0.01499	0.01325	0.03926
2007	0.02419	0.03654	0.01543	0.01509	0.015	0.01497	0.01497	0.01239	0.0366
2008	0.02406	0.01765	0.01457	0.01433	0.01415	0.01414	0.01414	0.01181	-0.07322
2009	0.02388	0.02278	0.01453	0.01425	0.01361	0.01361	0.01361	0.01032	0.02708
2010	0.0246	-0.00607	0.01465	0.01406	0.01352	0.01351	0.01351	0.01019	0.02571
2011	0.02514	-0.0059	0.01496	0.0141	0.01326	0.01331	0.01331	0.00924	0.01347
2012	0.02538	-0.00604	0.01486	0.0139	0.01284	0.01291	0.01291	0.00845	0.01279
2013	0.02571	-0.00643	0.0151	0.01407	0.01262	0.01273	0.01273	0.00721	-0.00804
2014	0.02646	-0.00681	0.0157	0.01446	0.01276	0.0129	0.0129	0.00604	-0.00936
2015	0.02659	0.01173	0.01551	0.01424	0.01258	0.01265	0.01265	0.0051	-0.01138
2016	0.02723	0.01168	0.01595	0.01453	0.01264	0.01273	0.01273	0.00374	-0.01316
2017	0.02836	-0.02933	0.01653	0.01491	0.01281	0.01296	0.01296	0.00313	-0.01326
AVG	0.015836	0.014691	0.013098	0.015554	0.020598	0.020133	0.020133	0.01785	0.014967

Table 17: 7 Goods Producer Model: Price Elasticities of Net Supply from PMOD12

YEAR	E11	E22	E33	E44	E55	E66
1970	0.4773	0.6828	1.3206	15.9538	-14.9317	-0.1699
1971	0.4987	0.6344	1.2757	12.5685	-12.5478	-0.1756
1972	0.5185	0.5755	1.2117	9.6895	-10.1791	-0.1799
1973	0.5085	0.6522	1.2305	9.2129	-9.8728	-0.1827
1974	0.4441	0.6276	1.2836	12.4629	-13.2195	-0.1728
1975	0.4664	0.7829	1.3924	14.416	-15.3348	-0.2073
1976	0.4509	0.7307	1.2828	11.7467	-12.8886	-0.1932
1977	0.4447	0.7428	1.2634	11.0081	-12.282	-0.1964
1978	0.4502	0.7529	1.2129	9.564	-10.8384	-0.1932
1979	0.4592	0.7801	1.2836	11.8527	-13.3326	-0.2009
1980	0.4696	0.7777	1.2804	13.2649	-14.7844	-0.2073
1981	0.4969	0.8056	1.2431	13.0876	-14.117	-0.2201
1982	0.5217	0.8832	1.154	10.8199	-11.7159	-0.2585
1983	0.5589	0.9609	1.1246	9.8356	-10.6005	-0.3116
1984	0.607	1.0673	1.1553	10.4588	-11.1725	-0.3734
1985	0.6258	1.1283	1.3924	11.2358	-11.9838	-0.4173
1986	0.7109	1.2327	1.6714	10.1135	-10.7493	-0.4434
1987	0.649	1.202	1.6926	10.6701	-11.5018	-0.4269
1988	0.6386	1.2349	1.6111	9.4804	-10.1916	-0.4577
1989	0.6291	1.2831	1.5689	8.7423	-9.3578	-0.4735
1990	0.6134	1.3326	1.5624	8.2526	-8.7759	-0.5072
1991	0.6079	1.3737	1.5363	7.6929	-8.0956	-0.5413
1992	0.6122	1.4013	1.5554	7.3656	-7.6714	-0.5907
1993	0.6121	1.4265	1.5447	7.2936	-7.5017	-0.6546
1994	0.6225	1.5121	1.5081	6.2591	-6.3844	-0.6964
1995	0.6275	1.4993	1.5034	6.1785	-6.2778	-0.7242
1996	0.6474	1.5101	1.4915	5.9463	-5.9294	-0.8058
1997	0.6768	1.5042	1.4659	5.8486	-5.6187	-0.842
1998	0.6757	1.4563	1.4082	5.7749	-5.4526	-0.8639
1999	0.6328	1.3428	1.3896	6.5546	-6.1152	-0.8866
2000	0.5591	1.2873	1.396	6.2952	-5.7793	-0.9701
2001	0.5402	1.2808	1.5285	6.5299	-5.8776	-0.9943
2002	0.5275	1.2753	1.7329	6.4316	-5.697	-1.1058
2003	0.4972	1.308	1.9029	5.7335	-5.0807	-1.1102
2004	0.4631	1.3307	1.7326	5.1594	-4.5352	-1.1584
2005	0.4285	1.3363	1.584	4.8565	-4.2367	-1.1467
2006	0.4028	1.3463	1.4302	4.5686	-3.9762	-1.0734
2007	0.3924	1.3365	1.337	4.4314	-3.748	-1.1347
2008	0.3837	1.3689	1.1872	4.3839	-3.5581	-1.0234
2009	0.3999	1.3672	1.1333	4.5221	-3.5495	-1.1283
2010	0.3748	1.3572	1.0578	4.4006	-3.4763	-1.09
2011	0.365	1.4257	0.9713	4.6009	-3.5927	-1.1197
2012	0.357	1.5713	0.8925	4.6551	-3.605	-1.115
2013	0.3514	1.6495	0.909	4.908	-3.6889	-1.1881
2014	0.3357	1.7101	0.9318	5.0694	-3.7106	-1.2688
2015	0.3236	1.8447	1.0116	5.0125	-3.5462	-1.3288
2016	0.3062	1.853	1.0513	5.2556	-3.5988	-1.411
2017	0.273	1.9628	1.0259	5.3382	-3.6452	-1.3824
AVG	0.5049	1.2189	1.3423	8.0313	-8.0068	-0.69424

Table 18: Nonparametric Model: Productivity Growth

YEAR	PRODN	E	TL
1970	1	1	1
1971	1.02611	1	1.02699
1972	1.079	1	1.08334
1973	1.14425	1	1.15116
1974	1.16979	1	1.17636
1975	1.17208	1	1.17636
1976	1.18205	1	1.1873
1977	1.20774	1	1.21373
1978	1.21937	1	1.22632
1979	1.28625	1	1.29587
1980	1.31632	1	1.32693
1981	1.30959	0.99482	1.32693
1982	1.29864	0.98751	1.32693
1983	1.32761	1	1.33779
1984	1.34738	1	1.35555
1985	1.26937	0.95295	1.35555
1986	1.22597	0.91852	1.35555
1987	1.27618	0.94729	1.35555
1988	1.35405	0.99544	1.35555
1989	1.40249	1	1.39375
1990	1.44597	1	1.43309
1991	1.45737	1	1.44518
1992	1.47877	1	1.46478
1993	1.58963	1	1.56428
1994	1.61792	1	1.58908
1995	1.67958	1	1.64423
1996	1.71652	1	1.67562
1997	1.75247	1	1.70602
1998	1.57686	0.91743	1.70602
1999	1.64514	0.95418	1.70602
2000	1.79251	1	1.74933
2001	1.65054	0.93659	1.74933
2002	1.73508	0.9761	1.74933
2003	1.74786	0.98042	1.74933
2004	1.95427	1	1.88538
2005	2.05293	1	1.96915
2006	2.10212	1	2.01185
2007	2.2231	1	2.11804
2008	2.06584	0.93599	2.11804
2009	2.01542	0.92254	2.11804
2010	2.22018	1	2.1315
2011	2.25205	1	2.15894
2012	2.20881	0.98386	2.15894
2013	2.20504	0.98497	2.15894
2014	2.16439	0.97257	2.15894
2015	2.10774	0.95279	2.15894
2016	2.11611	0.95889	2.15894
2017	2.1298	0.96756	2.15894
AVG	1.610797083	0.98418	1.612994375

Table 19: Nonparametric Model: Value Added Decomposition

YEAR	V	EC	A	B	C	T
1970	1	1	1	1	1	1
1971	1.17812	1	1.05519	1.08697	1	1.02717
1972	1.42488	1	1.11352	1.1829	1	1.08177
1973	1.84559	1	1.24663	1.28851	1	1.14898
1974	2.22543	1	1.37307	1.37952	1	1.17488
1975	2.38108	1	1.38704	1.4574	0.99931	1.17871
1976	2.63286	1	1.42193	1.55745	0.99931	1.1897
1977	2.88269	1	1.44127	1.64461	0.99931	1.217
1978	3.20464	1	1.48263	1.75905	0.99931	1.22961
1979	3.66472	1	1.49921	1.88511	0.99931	1.2976
1980	4.38769	1	1.62209	2.03817	0.99931	1.32807
1981	5.17445	0.99482	1.74334	2.24813	0.99929	1.32807
1982	5.93251	0.98751	1.85823	2.43893	0.9981	1.32807
1983	6.69237	1	1.9194	2.60885	0.99735	1.34004
1984	7.30034	1	1.9362	2.78125	0.99735	1.35926
1985	7.06044	0.95295	1.91703	2.8624	0.99334	1.35926
1986	6.98949	0.91852	1.89143	2.97015	0.99652	1.35926
1987	7.74984	0.94729	1.91328	3.13963	1.00196	1.35926
1988	9.19787	0.99544	2.03201	3.32403	1.00642	1.35926
1989	10.70629	1	2.15249	3.53479	1.00718	1.39709
1990	12.21068	1	2.25676	3.7373	1.00718	1.43743
1991	13.59754	1	2.32971	4.00008	1.00718	1.44871
1992	14.72431	1	2.36913	4.2007	1.00718	1.46898
1993	16.95432	1	2.43031	4.41452	1.00718	1.56901
1994	19.58429	1	2.58371	4.71882	1.00718	1.59486
1995	21.51295	1	2.57071	5.0379	1.00718	1.64926
1996	23.707	1	2.60546	5.37224	1.00718	1.68162
1997	26.1331	1	2.62612	5.76921	1.00718	1.71259
1998	25.37307	0.91743	2.61709	6.13803	1.00532	1.71259
1999	25.43982	0.95418	2.44398	6.34054	1.00464	1.71259
2000	28.50436	1	2.47629	6.51758	1.00422	1.75871
2001	28.08607	0.93659	2.41462	7.03303	1.00405	1.75871
2002	28.91028	0.9761	2.36008	7.10903	1.00375	1.75871
2003	29.76855	0.98042	2.37711	7.23556	1.00376	1.75871
2004	33.86046	1	2.448	7.26201	1.00561	1.89408
2005	36.78795	1	2.47943	7.45573	1.00561	1.97895
2006	40.37972	1	2.50913	7.91328	1.00561	2.02235
2007	46.11763	1	2.59475	8.3041	1.00561	2.12839
2008	46.80962	0.93599	2.65807	8.78976	1.0057	2.12839
2009	47.45084	0.92254	2.63037	9.14003	1.00518	2.12839
2010	55.20564	1	2.6433	9.71388	1.00617	2.13685
2011	59.12335	1	2.65247	10.23094	1.00617	2.16532
2012	61.73446	0.98386	2.66651	10.8026	1.006	2.16532
2013	65.15779	0.98497	2.67368	11.36773	1.00516	2.16532
2014	67.77974	0.97257	2.6917	11.92832	1.00242	2.16532
2015	71.59369	0.95279	2.7651	12.53111	1.00151	2.16532
2016	73.13673	0.95889	2.7194	12.96623	0.99898	2.16532
2017	75.44831	0.96756	2.72349	13.27242	0.99626	2.16532

Table 20: Nonparametric Model: Diewert and Fox Decomposition of Productivity

YEAR	PROD	EC	C	T
1970	1	1	1	1
1971	1.02717	1	1	1.02717
1972	1.08177	1	1	1.08177
1973	1.14898	1	1	1.14898
1974	1.17488	1	1	1.17488
1975	1.17789	1	0.99931	1.17871
1976	1.18887	1	0.99931	1.1897
1977	1.21615	1	0.99931	1.217
1978	1.22876	1	0.99931	1.22961
1979	1.2967	1	0.99931	1.2976
1980	1.32715	1	0.99931	1.32807
1981	1.32026	0.99482	0.99929	1.32807
1982	1.309	0.98751	0.9981	1.32807
1983	1.33649	1	0.99735	1.34004
1984	1.35566	1	0.99735	1.35926
1985	1.28669	0.95295	0.99334	1.35926
1986	1.24416	0.91852	0.99652	1.35926
1987	1.29014	0.94729	1.00196	1.35926
1988	1.36174	0.99544	1.00642	1.35926
1989	1.40713	1	1.00718	1.39709
1990	1.44776	1	1.00718	1.43743
1991	1.45912	1	1.00718	1.44871
1992	1.47953	1	1.00718	1.46898
1993	1.58028	1	1.00718	1.56901
1994	1.60631	1	1.00718	1.59486
1995	1.6611	1	1.00718	1.64926
1996	1.6937	1	1.00718	1.68162
1997	1.72489	1	1.00718	1.71259
1998	1.57952	0.91743	1.00532	1.71259
1999	1.64169	0.95418	1.00464	1.71259
2000	1.76614	1	1.00422	1.75871
2001	1.65386	0.93659	1.00405	1.75871
2002	1.72312	0.9761	1.00375	1.75871
2003	1.73076	0.98042	1.00376	1.75871
2004	1.90469	1	1.00561	1.89408
2005	1.99005	1	1.00561	1.97895
2006	2.03369	1	1.00561	2.02235
2007	2.14032	1	1.00561	2.12839
2008	2.00351	0.93599	1.0057	2.12839
2009	1.9737	0.92254	1.00518	2.12839
2010	2.15003	1	1.00617	2.13685
2011	2.17868	1	1.00617	2.16532
2012	2.14316	0.98386	1.006	2.16532
2013	2.14379	0.98497	1.00516	2.16532
2014	2.11102	0.97257	1.00242	2.16532
2015	2.06621	0.95279	1.00151	2.16532
2016	2.07419	0.95889	0.99898	2.16532
2017	2.08725	0.96756	0.99626	2.16532

Table 21: 15 Goods Consumer Model: Price Elasticities from CON11

YEAR	E1P1	E2P2	E3P3	E4P4	E5P5	E6P6	E7P7	E8P8
1971	-0.6951	-0.0913	-1.6781	-0.1486	-0.7400	-1.9284	-1.2822	-12.1921
1972	-0.7217	-0.0970	-1.7188	-0.1446	-0.7601	-1.8475	-1.1649	-12.2778
1973	-0.7440	-0.1020	-1.7520	-0.1416	-0.7765	-1.7895	-1.0866	-12.3927
1974	-0.7616	-0.1061	-1.7770	-0.1395	-0.7892	-1.7482	-1.0352	-12.4855
1975	-0.7612	-0.1060	-1.7768	-0.1395	-0.7891	-1.7497	-1.0361	-12.4461
1976	-0.7737	-0.1089	-1.7939	-0.1381	-0.7981	-1.7211	-1.0036	-12.5925
1977	-0.7843	-0.1115	-1.8091	-0.1370	-0.8055	-1.7001	-0.9783	-12.6410
1978	-0.8024	-0.1160	-1.8341	-0.1351	-0.8181	-1.6655	-0.9392	-12.7893
1979	-0.8219	-0.1211	-1.8610	-0.1332	-0.8317	-1.6312	-0.9019	-12.8488
1980	-0.8387	-0.1256	-1.8827	-0.1317	-0.8431	-1.6027	-0.8737	-13.0691
1981	-0.8318	-0.1237	-1.8737	-0.1323	-0.8384	-1.6143	-0.8850	-12.9723
1982	-0.8254	-0.1220	-1.8649	-0.1329	-0.8340	-1.6247	-0.8960	-12.9609
1983	-0.8469	-0.1278	-1.8936	-0.1310	-0.8486	-1.5910	-0.8609	-13.0579
1984	-0.8661	-0.1332	-1.9180	-0.1295	-0.8613	-1.5630	-0.8330	-13.2493
1985	-0.8629	-0.1323	-1.9144	-0.1297	-0.8593	-1.5675	-0.8374	-13.1843
1986	-0.8546	-0.1388	-2.0199	-0.1317	-0.9177	-1.5936	-0.7886	-7.8836
1987	-0.8687	-0.1663	-2.1657	-0.1295	-0.9117	-1.4729	-0.7905	-7.8089
1988	-0.9237	-0.1886	-2.3242	-0.1255	-0.9129	-1.3668	-0.8871	-7.4940
1989	-0.9440	-0.2128	-2.1303	-0.1252	-0.9140	-1.3196	-0.9009	-8.2841
1990	-1.0013	-0.2462	-1.9904	-0.1296	-0.9700	-1.1853	-0.8635	-7.9275
1991	-0.9699	-0.2619	-2.0129	-0.1322	-0.9547	-1.2874	-0.9198	-7.9997
1992	-0.9814	-0.2978	-2.1180	-0.1338	-0.9503	-1.3635	-0.9570	-7.1010
1993	-0.9859	-0.3622	-2.2414	-0.1353	-0.9326	-1.3465	-0.9946	-5.9983
1994	-0.9697	-0.3585	-2.2044	-0.1403	-0.8892	-1.4547	-1.1303	-5.1989
1995	-0.9277	-0.3582	-2.1698	-0.1506	-0.8768	-1.5691	-1.1201	-4.8440
1996	-0.9347	-0.3630	-2.1955	-0.1580	-0.8718	-1.5641	-1.0960	-4.4810
1997	-0.9103	-0.3543	-2.1262	-0.1570	-0.8275	-1.6456	-1.2393	-4.1393
1998	-0.8287	-0.3016	-2.1652	-0.1518	-0.8086	-2.0703	-1.0993	-3.6595
1999	-0.9542	-0.3756	-2.2296	-0.1448	-0.8963	-1.8120	-1.0375	-3.9777
2000	-1.0595	-0.4745	-2.4462	-0.1502	-0.9549	-1.7088	-0.9825	-4.0123
2001	-1.0277	-0.5058	-2.6473	-0.1540	-0.9614	-1.8164	-0.8994	-3.6017
2002	-1.0355	-0.5414	-2.8351	-0.1510	-0.9658	-1.8542	-0.8717	-3.5581
2003	-1.0082	-0.6076	-3.0734	-0.1451	-0.9571	-1.9065	-0.8745	-3.0159
2004	-1.1004	-0.7975	-3.1673	-0.1455	-1.0359	-1.7650	-0.7929	-2.7742
2005	-1.1177	-0.9160	-3.2430	-0.1457	-1.0166	-1.6601	-0.7214	-2.4858
2006	-1.1360	-0.9485	-3.2472	-0.1503	-1.0245	-1.5682	-0.6647	-2.0120
2007	-1.1477	-0.9291	-3.0975	-0.1612	-0.9577	-1.5607	-0.7103	-1.8718
2008	-1.2107	-0.9657	-2.9488	-0.1842	-0.9467	-1.5468	-0.7575	-1.6581
2009	-1.2350	-0.8879	-3.2481	-0.1821	-0.9253	-1.6658	-0.7313	-1.5110
2010	-1.2952	-0.8961	-2.8758	-0.1902	-0.9899	-1.6357	-0.8239	-1.3387
2011	-1.3561	-0.9104	-2.5622	-0.2051	-1.0380	-1.5578	-0.9347	-1.2282
2012	-1.3386	-0.8713	-2.5689	-0.2144	-1.0366	-1.6232	-0.9880	-1.1465
2013	-1.3653	-0.8416	-2.6191	-0.2207	-1.0493	-1.7142	-1.0050	-1.0865
2014	-1.3833	-0.8986	-2.4978	-0.2218	-1.0843	-1.7226	-0.9923	-1.0664
2015	-1.4190	-0.9014	-2.4223	-0.2160	-1.1084	-1.6692	-0.9661	-1.1172
2016	-1.4824	-0.9038	-2.4223	-0.2088	-1.2155	-1.6793	-0.9187	-1.1739
2017	-1.5082	-0.8813	-2.4349	-0.2040	-1.2190	-1.7410	-0.9274	-1.2089
AVG	-1.0047	-0.4338	-2.2936	-0.1541	-0.9216	-1.6339	-0.9362	-6.6771

Table 21: 15 Goods Consumer Model: Price Elasticities from CON11 (continued)

YEAR	E9P9	E10P10	E11P11	E12P12	E13P13	E14P14	E15P15
1971	-1.5672	-0.9584	-1.5271	-0.1812	-0.7834	-4.2665	-0.9742
1972	-1.5754	-0.9183	-1.5722	-0.1831	-0.7806	-3.6122	-0.9886
1973	-1.5829	-0.8895	-1.6086	-0.1844	-0.7786	-3.2173	-1.0006
1974	-1.5881	-0.8704	-1.6366	-0.1854	-0.7770	-2.9787	-1.0096
1975	-1.5882	-0.8713	-1.6360	-0.1854	-0.7770	-2.9827	-1.0094
1976	-1.5916	-0.8592	-1.6556	-0.1861	-0.7759	-2.8381	-1.0155
1977	-1.5945	-0.8478	-1.6728	-0.1867	-0.7751	-2.7282	-1.0207
1978	-1.5994	-0.8322	-1.7008	-0.1876	-0.7735	-2.5643	-1.0294
1979	-1.6043	-0.8152	-1.7316	-0.1887	-0.7721	-2.4165	-1.0384
1980	-1.6083	-0.8040	-1.7565	-0.1895	-0.7709	-2.3057	-1.0459
1981	-1.6066	-0.8082	-1.7464	-0.1892	-0.7713	-2.3503	-1.0428
1982	-1.6051	-0.8132	-1.7364	-0.1888	-0.7718	-2.3918	-1.0400
1983	-1.6104	-0.7972	-1.7691	-0.1899	-0.7703	-2.2578	-1.0496
1984	-1.6149	-0.7844	-1.7980	-0.1908	-0.7689	-2.1541	-1.0579
1985	-1.6142	-0.7864	-1.7933	-0.1907	-0.7692	-2.1703	-1.0565
1986	-1.7125	-0.7384	-1.8406	-0.1672	-0.7873	-1.9487	-1.0516
1987	-1.6502	-0.7213	-1.9079	-0.1580	-0.7936	-1.7270	-1.0697
1988	-1.4756	-0.7044	-1.9590	-0.1599	-0.7663	-1.5765	-1.0572
1989	-1.5262	-0.6664	-1.9318	-0.2024	-0.7282	-1.4704	-1.0867
1990	-1.4630	-0.7566	-1.9510	-0.2636	-0.7173	-1.4121	-1.0823
1991	-1.4188	-0.8027	-1.9134	-0.2530	-0.6874	-1.3418	-1.0954
1992	-1.3524	-0.7764	-1.9536	-0.2300	-0.6543	-1.3008	-1.1003
1993	-1.3055	-0.7363	-1.9520	-0.2061	-0.6586	-1.1759	-1.1067
1994	-1.2424	-0.6500	-1.9322	-0.1950	-0.6556	-1.0263	-1.0887
1995	-1.2294	-0.6383	-1.9290	-0.1788	-0.6468	-0.9849	-1.1089
1996	-1.2005	-0.6664	-1.8731	-0.1784	-0.6313	-1.0404	-1.0972
1997	-1.1394	-0.6519	-1.8208	-0.1709	-0.6098	-1.1041	-1.0584
1998	-1.2009	-0.7629	-1.7661	-0.1499	-0.6332	-2.0871	-0.9360
1999	-1.2277	-0.7978	-1.8861	-0.1417	-0.6556	-1.2302	-1.0249
2000	-1.2004	-0.7505	-2.0170	-0.1500	-0.6710	-0.8458	-1.1543
2001	-1.2387	-0.7684	-2.0027	-0.1573	-0.6453	-0.8834	-1.2048
2002	-1.2312	-0.7716	-2.1421	-0.1557	-0.6398	-0.8921	-1.2203
2003	-1.2179	-0.7770	-2.2172	-0.1446	-0.6254	-0.9696	-1.1998
2004	-1.2924	-0.7891	-2.2247	-0.1519	-0.6665	-0.7534	-1.3654
2005	-1.3063	-0.8282	-2.2243	-0.1692	-0.6948	-0.7296	-1.4833
2006	-1.2916	-0.8331	-2.2163	-0.1922	-0.7418	-0.7157	-1.5379
2007	-1.1428	-0.7933	-2.2595	-0.2147	-0.7299	-0.7078	-1.5026
2008	-1.0459	-0.7407	-2.2670	-0.2459	-0.6250	-0.6689	-1.5417
2009	-1.0510	-0.6943	-2.4121	-0.2082	-0.6795	-0.6512	-1.4733
2010	-1.0461	-0.7059	-2.3008	-0.2156	-0.6145	-0.6541	-1.3798
2011	-0.9797	-0.6974	-2.2157	-0.2191	-0.5748	-0.6070	-1.3249
2012	-0.9496	-0.6776	-2.1724	-0.2190	-0.5531	-0.5752	-1.3198
2013	-0.9447	-0.6893	-2.1721	-0.2097	-0.5293	-0.5674	-1.2912
2014	-0.9474	-0.6988	-2.2408	-0.2131	-0.5189	-0.5607	-1.2904
2015	-0.9422	-0.7558	-2.3584	-0.2115	-0.5528	-0.5446	-1.2680
2016	-0.9674	-0.8302	-2.4187	-0.2101	-0.5640	-0.5334	-1.2607
2017	-0.9590	-0.8639	-2.4829	-0.2070	-0.5673	-0.5390	-1.2355
AVG	-1.3372	-0.7743	-1.9639	-0.1906	-0.6901	-1.5544	-1.1574

Table 22: 15 Goods Consumer Model: Income Elasticities from CON11

YEAR	EU1	EU2	EU3	EU4	EU5	EU6	EU7	EU8
1971	0.2743	-0.1563	0.5418	1.5444	0.483	1.8596	2.9808	0.6603
1972	0.2848	-0.1661	0.5545	1.5029	0.496	1.782	2.7081	0.6751
1973	0.2938	-0.1747	0.565	1.4717	0.5067	1.7255	2.526	0.6858
1974	0.3006	-0.1816	0.5731	1.4495	0.515	1.6857	2.4066	0.6936
1975	0.3005	-0.1815	0.5729	1.4499	0.5147	1.6867	2.4088	0.6943
1976	0.3053	-0.1866	0.5787	1.4351	0.5205	1.66	2.3332	0.6979
1977	0.3096	-0.191	0.5834	1.423	0.5255	1.6393	2.2743	0.7027
1978	0.3167	-0.1987	0.5915	1.4036	0.5338	1.6057	2.1834	0.709
1979	0.3245	-0.2073	0.5999	1.3841	0.5426	1.5726	2.0968	0.7178
1980	0.331	-0.2151	0.6072	1.3686	0.55	1.5461	2.0312	0.7215
1981	0.3283	-0.2119	0.6042	1.3749	0.5469	1.5568	2.0575	0.7201
1982	0.3258	-0.2089	0.6015	1.3809	0.5441	1.5669	2.0829	0.7172
1983	0.3343	-0.2189	0.6106	1.3614	0.5536	1.5342	2.0013	0.7257
1984	0.3419	-0.2282	0.6188	1.3451	0.562	1.5073	1.9366	0.7311
1985	0.3406	-0.2266	0.6174	1.3478	0.5606	1.5116	1.9468	0.7308
1986	0.3581	-0.2283	0.6124	1.3294	0.5596	1.4979	1.8516	0.8397
1987	0.3846	-0.2736	0.6198	1.2985	0.5918	1.4299	1.7565	0.8522
1988	0.3898	-0.3448	0.6243	1.2785	0.6191	1.3728	1.7259	0.8647
1989	0.4023	-0.4019	0.6696	1.2571	0.6332	1.3393	1.6719	0.8594
1990	0.3944	-0.4933	0.7027	1.236	0.638	1.2839	1.606	0.8715
1991	0.3977	-0.5032	0.691	1.2433	0.6371	1.2958	1.6466	0.866
1992	0.4164	-0.5655	0.6885	1.2302	0.6584	1.2848	1.6213	0.8846
1993	0.4403	-0.6721	0.6926	1.2138	0.6851	1.2596	1.5774	0.9062
1994	0.4441	-0.6615	0.6938	1.2183	0.6976	1.2707	1.6214	0.9158
1995	-0.0995	-1.0679	0.4515	0.5109	-0.0617	1.9319	1.5819	0.8961
1996	-0.1163	-1.0748	0.4376	0.5092	-0.0653	1.9265	1.5851	0.9025
1997	-0.1186	-1.0576	0.4322	0.4926	-0.0481	1.9836	1.6553	0.9042
1998	-0.0941	-0.7983	0.3758	0.4792	-0.0992	2.2775	1.6763	0.9067
1999	-0.1307	-0.9746	0.4096	0.5085	-0.1007	2.0085	1.5735	0.9077
2000	-0.1398	-1.1823	0.4097	0.5327	-0.0705	1.8462	1.4997	0.9156
2001	-0.1024	-1.1888	0.3678	0.5429	-0.0751	1.8689	1.4754	0.9244
2002	-0.086	-1.2366	0.3415	0.5552	-0.0529	1.8504	1.4523	0.9276
2003	-0.0619	-1.3027	0.2836	0.5555	-0.0572	1.868	1.4573	0.9356
2004	-0.0645	-1.5932	0.3081	0.5762	-0.0813	1.7396	1.3972	0.9431
2005	-0.0466	-1.7674	0.3009	0.5879	-0.0708	1.6829	1.3694	0.9482
2006	-0.0538	-1.8258	0.2962	0.5934	-0.0733	1.6445	1.3495	0.9558
2007	0.3913	0.6578	1.2469	0.5896	0.1103	1.3432	0.565	-0.2628
2008	0.359	0.6212	1.2523	0.5668	0.074	1.3512	0.5108	-0.2115
2009	0.3221	0.6365	1.2915	0.5459	0.0576	1.391	0.4949	-0.1717
2010	0.3412	0.6525	1.2438	0.564	0.055	1.3557	0.5036	-0.0576
2011	0.325	0.6466	1.2188	0.5574	0.0378	1.332	0.4713	-0.0164
2012	0.3283	0.6553	1.2222	0.5494	0.042	1.3396	0.4473	0.0152
2013	0.331	0.67	1.2253	0.5501	0.0373	1.3452	0.4436	0.0464
2014	0.3386	0.6653	1.2158	0.5556	0.0166	1.3379	0.4506	0.057
2015	0.3394	0.6723	1.2065	0.5619	0.0102	1.3213	0.4717	0.025
2016	0.3502	0.6853	1.1972	0.5772	-0.0317	1.3067	0.5083	0.0114
2017	0.366	0.7024	1.1918	0.5879	-0.0087	1.302	0.5232	0.0053
AVG	0.23442	-0.31707	0.69664	0.96166	0.28125	1.5708	1.5429	0.61597

Table 22: 15 Goods Consumer Model: Income Elasticities from CON11 (continued)

YEAR	EU9	EU10	EU11	EU12	EU13	EU14	EU15
1971	0.8917	1.8021	0.448	0.8166	1.0695	4.6104	0.709
1972	0.8967	1.7302	0.4607	0.8242	1.0657	3.9009	0.7197
1973	0.9006	1.6778	0.4714	0.8304	1.0628	3.4751	0.7283
1974	0.9035	1.6415	0.4797	0.835	1.0606	3.217	0.7348
1975	0.9034	1.6421	0.4795	0.8349	1.0607	3.2217	0.7346
1976	0.9054	1.6188	0.4854	0.8381	1.0592	3.0651	0.7392
1977	0.9071	1.5991	0.4902	0.8407	1.058	2.9466	0.7429
1978	0.9099	1.5686	0.4986	0.8451	1.056	2.77	0.7492
1979	0.9127	1.5376	0.5073	0.8497	1.0539	2.6097	0.7558
1980	0.9151	1.5145	0.5149	0.8535	1.0522	2.4907	0.7613
1981	0.9141	1.5238	0.5118	0.8519	1.0529	2.5382	0.7591
1982	0.9132	1.5331	0.509	0.8505	1.0536	2.5835	0.757
1983	0.9162	1.5028	0.5185	0.8553	1.0514	2.4385	0.7639
1984	0.9188	1.4782	0.527	0.8595	1.0496	2.3266	0.77
1985	0.9184	1.4821	0.5256	0.8588	1.0499	2.3441	0.769
1986	0.9184	1.4214	0.535	0.8662	1.0483	2.1601	0.7751
1987	0.928	1.3709	0.5513	0.8786	1.0442	1.949	0.788
1988	0.9399	1.326	0.5642	0.8872	1.0394	1.8064	0.8058
1989	0.9427	1.2891	0.5887	0.8925	1.0357	1.714	0.8127
1990	0.9477	1.277	0.6072	0.8947	1.033	1.6483	0.8278
1991	0.9478	1.2741	0.6075	0.8967	1.0325	1.6373	0.8234
1992	0.9525	1.24	0.619	0.9032	1.0298	1.5767	0.8317
1993	0.9576	1.2076	0.6398	0.9135	1.0273	1.4881	0.8413
1994	0.9599	1.1834	0.6397	0.9186	1.0266	1.442	0.8441
1995	0.4573	1.4483	0.8145	0.9613	0.9776	5.6101	-0.4787
1996	0.4609	1.4585	0.8178	0.9609	0.9776	5.7967	-0.4679
1997	0.4733	1.4522	0.8163	0.9609	0.9777	6.3018	-0.4496
1998	0.3967	1.5352	0.8102	0.9585	0.9748	12.3152	-0.4234
1999	0.4343	1.4871	0.8172	0.9596	0.9765	6.7935	-0.4334
2000	0.481	1.4181	0.8219	0.9623	0.9783	4.5797	-0.4582
2001	0.4621	1.4139	0.8247	0.9621	0.9787	4.6359	-0.5001
2002	0.4771	1.3952	0.8192	0.9614	0.9792	4.4527	-0.4907
2003	0.4784	1.3921	0.8131	0.9603	0.9793	4.7321	-0.4864
2004	0.4788	1.3636	0.8261	0.9631	0.98	3.775	-0.5653
2005	0.4726	1.3631	0.8307	0.9635	0.9799	3.6347	-0.6249
2006	0.4698	1.3572	0.8325	0.9632	0.9793	3.5565	-0.6306
2007	1.5784	0.0579	0.9496	1.6148	1.5384	1.0603	1.939
2008	1.5696	0.0872	0.9487	1.6452	1.5229	1.0605	1.9648
2009	1.6027	0.0953	0.9435	1.7049	1.5896	1.0631	2.0079
2010	1.5334	0.1607	0.9498	1.6162	1.4978	1.0576	1.8635
2011	1.4883	0.1934	0.9527	1.5687	1.4588	1.0534	1.7965
2012	1.4776	0.2241	0.9534	1.5476	1.447	1.0517	1.7876
2013	1.4669	0.2456	0.9545	1.5301	1.4318	1.0505	1.7619
2014	1.4636	0.2548	0.9545	1.5245	1.4224	1.0498	1.7474
2015	1.4552	0.2379	0.9538	1.5136	1.4266	1.0488	1.7215
2016	1.4509	0.2234	0.9553	1.4903	1.414	1.0472	1.6878
2017	1.4327	0.238	0.9559	1.4709	1.3992	1.0461	1.6519
AVG	0.94007	1.1605	0.70417	1.0523	1.1289	2.9305	0.69073

Table 23: 15 Goods Consumer Model: Per Capita Real Consumption from CON11

YEAR	NQ1	NQ2	NQ3	NQ4	NQ5	NQ6	NQ7	NQ8
1971	1	1	1	1	1	1	1	1
1972	1.0535	1.0535	1.0536	1.0536	1.0536	1.0536	1.0535	1.0543
1973	1.0999	1.0998	1.0999	1.0999	1.0999	1.1001	1.0999	1.1009
1974	1.1368	1.1368	1.1369	1.1369	1.1369	1.1369	1.1369	1.1372
1975	1.136	1.136	1.1361	1.1361	1.136	1.1362	1.1361	1.1368
1976	1.1628	1.1628	1.1629	1.1629	1.1628	1.1629	1.1629	1.1632
1977	1.1859	1.1859	1.186	1.1859	1.1859	1.186	1.1859	1.1864
1978	1.226	1.2259	1.226	1.226	1.2259	1.226	1.226	1.2263
1979	1.2702	1.2702	1.2703	1.2703	1.2703	1.2704	1.2703	1.2711
1980	1.3088	1.3088	1.3089	1.3089	1.3088	1.309	1.3088	1.3093
1981	1.2928	1.2928	1.2929	1.2929	1.2928	1.2929	1.2928	1.2935
1982	1.2781	1.278	1.2781	1.2781	1.2781	1.2783	1.2781	1.2786
1983	1.328	1.328	1.3281	1.3281	1.3281	1.3281	1.3281	1.329
1984	1.374	1.3739	1.374	1.374	1.374	1.3741	1.374	1.3746
1985	1.3663	1.3662	1.3663	1.3663	1.3663	1.3664	1.3663	1.367
1986	1.3898	1.3376	1.442	1.5742	1.3484	1.7461	1.3163	1.6566
1987	1.4254	1.2084	1.5517	1.7832	1.3437	1.8287	1.3885	1.7829
1988	1.4862	1.243	1.6114	1.8828	1.4547	2.0519	1.5931	1.9488
1989	1.4715	1.1633	1.636	1.8767	1.4975	2.0866	1.8057	2.024
1990	1.4206	1.0617	1.8049	1.7872	1.5496	2.2897	1.8428	2.0633
1991	1.4615	0.9468	1.7766	1.8714	1.5902	2.2614	1.8332	2.1795
1992	1.4742	1.0117	1.7227	1.9099	1.6649	2.3868	2.0296	2.3093
1993	1.5672	0.9041	1.8274	1.9827	1.7758	2.4036	2.3054	2.8007
1994	1.5241	0.8916	1.7937	2.0632	1.8409	2.3971	2.3295	3.1957
1995	1.5273	0.8782	1.7168	2.1469	1.842	2.4251	2.3495	3.5562
1996	1.5189	0.9095	1.693	2.197	1.9167	2.5647	2.2653	3.9728
1997	1.5013	0.9559	1.5252	2.2657	1.9656	2.694	2.0407	4.5237
1998	1.398	0.8323	1.1745	2.3813	1.7391	2.5786	2.0156	5.2112
1999	1.4124	0.8291	1.3594	2.5048	1.9167	2.7273	2.35	5.8574
2000	1.4487	0.7712	1.3878	2.6034	2.0546	2.8945	3.0497	5.7416
2001	1.4711	0.7568	1.2829	2.6155	1.8863	3.0569	3.1849	6.4666
2002	1.517	0.7654	1.2482	2.6509	1.8563	3.1484	3.0456	6.5538
2003	1.5513	0.7118	1.1409	2.7493	1.8513	3.215	3.2034	7.1434
2004	1.5319	0.6578	1.1928	2.7711	1.8779	3.5408	3.5145	7.9615
2005	1.5737	0.6524	1.2375	2.7425	1.9065	3.694	3.526	8.5879
2006	1.5899	0.6176	1.2999	2.7039	1.8809	3.7796	3.662	9.3863
2007	1.5984	0.6131	1.3636	2.6215	1.9575	3.867	3.634	10.1196
2008	1.5055	0.6082	1.2847	2.4954	1.9224	3.7782	3.417	10.3978
2009	1.4716	0.6328	1.1897	2.4483	1.8164	3.6977	2.9349	10.4509
2010	1.4409	0.6738	1.2719	2.4427	1.8907	3.7961	2.7976	11.6133
2011	1.456	0.6879	1.42	2.4098	1.891	3.7879	2.6291	12.796
2012	1.4908	0.7024	1.4841	2.4091	1.8667	3.8434	2.5854	13.4634
2013	1.5191	0.7044	1.5506	2.4288	1.8079	3.9353	2.4536	13.2548
2014	1.4973	0.7092	1.5808	2.4755	1.751	3.9821	2.5428	13.4073
2015	1.5052	0.7214	1.5864	2.5592	1.7504	4.2249	2.9966	13.4491
2016	1.4697	0.7128	1.6013	2.6453	1.7355	4.4337	3.3946	12.8279
2017	1.5014	0.6879	1.679	2.7302	1.7461	4.6425	3.4712	12.4914
AVG	1.4029	0.95274	1.397	1.9777	1.5982	2.5102	2.1644	5.2643

Table 23: 15 Goods Consumer Model: Per Capita Real Consumption from CON11 (continued)

YEAR	NQ9	NQ10	NQ11	NQ12	NQ13	NQ14	NQ15	NU
1971	1	1	1	1	1	1	1	1
1972	1.0536	1.0537	1.0535	1.0535	1.0536	1.0535	1.0536	1.0536
1973	1.0999	1.1001	1.0999	1.1	1.0999	1.0999	1.0999	1.0999
1974	1.1369	1.1369	1.1369	1.1369	1.1369	1.1368	1.1369	1.1369
1975	1.1361	1.1362	1.1361	1.1361	1.1361	1.1361	1.1361	1.1361
1976	1.1629	1.1629	1.1628	1.163	1.1628	1.1628	1.1629	1.1628
1977	1.186	1.1862	1.1859	1.1859	1.1859	1.1859	1.1859	1.1859
1978	1.226	1.2261	1.2259	1.226	1.226	1.226	1.226	1.226
1979	1.2703	1.2705	1.2702	1.2703	1.2703	1.2702	1.2703	1.2703
1980	1.3089	1.3091	1.3088	1.3088	1.3088	1.3089	1.3089	1.3089
1981	1.2929	1.293	1.2928	1.2928	1.2928	1.2928	1.2929	1.2928
1982	1.2782	1.2785	1.2781	1.2782	1.2781	1.2781	1.2781	1.2781
1983	1.3281	1.3283	1.328	1.328	1.3281	1.3281	1.3281	1.3281
1984	1.374	1.3742	1.3739	1.374	1.374	1.3739	1.374	1.374
1985	1.3663	1.3664	1.3663	1.3664	1.3663	1.3663	1.3663	1.3663
1986	1.5033	1.4803	1.3589	1.448	1.3592	1.4324	1.4617	1.4274
1987	1.7	1.5843	1.4013	1.6711	1.6444	1.6469	1.638	1.5557
1988	2.0703	1.6172	1.4932	1.9101	1.6871	1.7368	1.798	1.6871
1989	2.2571	1.9676	1.5216	2.008	1.8349	2.2555	2.0048	1.8015
1990	2.6201	2.1507	1.523	2.059	2.104	2.6312	2.3544	1.9384
1991	2.7364	2.4847	1.566	1.995	1.9932	2.5122	2.1157	1.9098
1992	2.6486	2.7137	1.6004	2.138	2.1704	3.0403	2.3658	2.0229
1993	2.8622	2.9499	1.6753	2.2568	2.4808	3.6497	2.5154	2.1917
1994	2.8763	3.4401	1.6673	2.4361	2.5511	4.1971	2.375	2.2211
1995	2.7758	3.6989	1.6817	2.5043	2.467	4.5456	2.3429	2.2278
1996	2.6773	3.8299	1.7204	2.4935	2.5935	4.5383	2.166	2.2206
1997	2.6859	3.8413	1.7621	2.4293	2.83	4.0506	1.8488	2.1713
1998	2.351	3.7124	1.6146	2.092	2.3839	3.4584	1.5202	1.9771
1999	2.6264	3.9292	1.7005	2.2157	2.417	4.3132	1.7358	2.1597
2000	2.9443	4.3631	1.8192	2.435	2.7379	4.9754	1.7598	2.3735
2001	2.724	4.2866	1.8138	2.2767	2.6218	6.4779	1.5719	2.3825
2002	2.7459	4.4728	1.7528	2.2768	2.8426	7.5969	1.5151	2.4372
2003	2.7481	4.6434	1.6988	2.0127	2.7172	7.4657	1.2946	2.4106
2004	2.8584	4.6872	1.7806	2.4994	2.9255	8.0581	1.6731	2.585
2005	2.8462	4.8636	1.878	2.6696	2.9293	8.2623	1.8022	2.6398
2006	2.8958	4.9621	1.8809	2.6464	2.8762	8.3225	1.9373	2.6785
2007	2.9551	5.1253	1.8656	2.6553	3.0193	8.6919	1.9916	2.7289
2008	2.9335	5.2866	1.7939	2.3965	2.9095	9.0674	1.9133	2.6558
2009	2.6969	5.5731	1.7079	2.2281	2.7636	8.7698	1.7286	2.5105
2010	4.0762	5.6382	1.8087	2.6495	2.9376	9.1574	2.333	2.7462
2011	4.5466	5.7622	1.9305	3.0146	3.1362	9.4919	2.5445	2.8528
2012	4.2783	6.0127	1.996	2.9965	3.2481	9.9577	2.4987	2.8703
2013	4.1932	6.2191	2.0556	3.0926	3.5909	10.3228	2.5081	2.9162
2014	4.1312	6.423	2.0989	3.1603	3.6818	10.8172	2.4595	2.9539
2015	3.895	6.2573	2.0819	3.1356	3.7616	11.1098	2.2984	3.0003
2016	3.7173	6.3511	2.105	3.1939	3.7437	11.3827	2.5891	3.0997
2017	3.9368	6.5576	2.1046	3.3144	3.9653	11.6161	2.6778	3.1974
AVG	2.4199	3.3002	1.5889	2.0538	2.2371	4.7611	1.7778	2.0164