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ME 4205

ASSIGNMENT NO. 1

I. SOLVE FOR THE LAPLACE TRANSFORMATION OF THE FOLLOWING:

1. $L[3-e^{-3t}+5\sin 2t]=F(S)$

- $L[3]=F(S)$

  $3L[1]=F(S)$

  $1 \approx u(t)$

  $u(t) \to 1/S$

  $F(S)=3(1/S)$

  $F(S)=3/S$

- $L[e^{-3t}]=F(S)$

  $e^{-3t} \approx e^{-at}u(t)$; $a=3$

  $e^{-at}u(t) \to \dfrac{1}{s+a} = \dfrac{1}{s+3}$

  $F(S) = \dfrac{1}{s+3}$

- $L[5\sin 2t]=F(S)$

  $5L[\sin 2t]=F(S)$

  $\sin 2t \approx \sin \omega t\, u(t)$

  $\sin \omega t\, u(t) = \dfrac{\omega}{s^2+\omega^2}$; $\omega = 2$

  $F(S)=5\left(\dfrac{\omega}{s^2+\omega^2}\right)=5\left(\dfrac{2}{s^2+2^2}\right)$

  $F(S)=\dfrac{10}{s^2+4}$

$$\boxed{F(S)=\frac{3}{s}-\frac{1}{s+3}+\frac{10}{s^2+4}}$$

2. $L[3+12t+42t^3-3e^{2t}]=F(S)$

- $L[3]=F(S)$

  $3L[1]=F(S)$

  $1 \approx u(t)$

  $u(t) \to 1/S$

  $F(S)=3(1/S)$

  $F(S)=3/S$

- $L[12t]=F(S)$

  $t = t\,u(t)$

  $t\,u(t) \to 1/s^2$

  $F(S)=12/s^2$

- $L[42t^3]=F(S)$

  $42L[t^3]=F(S)$

  $t^3 \approx t^n u(t)$

  $t^n u(t) \to \dfrac{n!}{s^{n+1}}$; $n=3$

$$F(S) = 42\left(\frac{n!}{S^{n+1}}\right) = 42\left(\frac{3}{S^{3+1}}\right)$$

$$F(S) = \frac{252}{S^4}$$

- $L[3e^{2t}] = F(S)$

$3L[e^{2t}] = F(S)$

$e^{2t} \approx e^{-at}u(t) ; a = -2$

$$e^{-at} \rightarrow \frac{1}{S+a}$$

$$F(S) = 3\left[\frac{1}{S+a}\right] = 3\left[\frac{1}{S-2}\right]$$

$$F(S) = \frac{3}{S-2}$$

$$\boxed{F(S) = 3/S + 12/S^2 + 252/S^4 - 3/S-2}$$

3. $L[(t+1)(t+2)] = F(S)$

- $L[t^2 + 3t + 2] = F(S)$

$L[t^2] = F(S)$

$t^2 \approx t^n u(t)$

$$t^n u(t) \rightarrow \frac{n!}{S^{n+1}} ; n = 2$$

$$F(S) = \frac{n!}{S^{n+1}} = \frac{2!}{S^{2+1}}$$

$$F(S) = \frac{2}{S^3}$$

- $L[3t] = F(S)$

$3L[t] = F(S)$

$t \approx t\,u(t)$

$$t\,u(t) \rightarrow \frac{1}{S^2}$$

$$F(S) = 3\left(\frac{1}{S^2}\right)$$

$$F(S) = \frac{3}{S^2}$$

- $L[2] = F(S)$

$2L[1] = F(S)$

$1 \approx u(t)$

$u(t) \rightarrow 1/S$

$F(S) = 2(1/S)$

$F(S) = 2/S$

$$F(S) = \frac{2}{S^3} + \frac{3}{S^2} + \frac{2}{S}$$

II. SOLVE FOR THE INVERSE LAPLACE TRANSFORMATION OF THE FOLLOWING:

1. $$L^{-1}\left[\frac{8 - 3S + S^2}{S^3}\right] = f(t)$$

$$L^{-1}\left[\frac{8}{S^3} - \frac{3S}{S^3} + \frac{S^2}{S^3}\right] = f(t)$$

$$L^{-1}\left[\frac{8}{S^3} - \frac{3}{S^3} + \frac{1}{S^3}\right] = f(t)$$

$\bullet\ L^{-1}\left[\frac{8}{s^3}\right] = f(t)$

$4L^{-1}\ \frac{2}{s^2+1} = f(t)$

$\frac{2}{s^{2+1}} \approx \frac{n!}{s^{n+1}}\ ;\ n=2$

$\frac{n!}{s^{n+1}} \rightarrow t^n U(t)$

$f(t) = 4t^n U(t)$

$f(t) = 4t^2 U(t)$

$\bullet\ L^{-1}\left[\frac{3}{s^2}\right] = f(t)$

$3L^{-1}\ \frac{1}{s^2} = f(t)$

$\frac{1}{s^2} \rightarrow tU(t)$

$f(t) = 3tU(t)$

$\bullet\ L^{-1}\left[\frac{1}{s}\right] = f(t)$

$\frac{1}{s} \rightarrow U(t)$

$f(t) = U(t)$

$\boxed{f(t) = 4t^2U(t) - 3tU(t) + U(t)}$

2. $L^{-1}\left[\frac{5}{s-2} - \frac{4s}{s^2+a}\right] = f(t)$

$\bullet\ L^{-}\left[\frac{5}{s-2}\right] = f(t)$

$5L^{-}\left[\frac{1}{s-2}\right] = f(t)$

$\frac{1}{s-2} \approx \frac{1}{s+a}\ ;\ a=-2$

$\frac{1}{s+a} \rightarrow e^{-a\circ}\ U(t)$

$f(t) = 5e^{-(-2)t}\ U(t)$

$f(t) = 5e^{2t}\ U(t)$

$\bullet\ L^{-}\left[\frac{4s}{s^2+a}\right] = f(t)$

$4L^{-}\left[\frac{s}{s^2+a}\right] = f(t)$

$\frac{s}{s^2+a^2} \approx \frac{s}{s^2+w^2}\ ;\ \omega = 3$

$\frac{s}{s^2+\omega^2} \rightarrow \cos\omega t\, U(t)$

$f(t) = 4\cos\omega t\, U(t)$

$f(t) = 4\cos 3t\, U(t)$

$\boxed{f(t) = 5e^{2t}\ U(t) - 4\cos 3t\, U(t)}$

3. $L^{-1}\left[\frac{7}{s^2+6}\right] = f(t)$

$7L^{-1}\left[\frac{1}{s^2+6}\right] = f(t)$

$\frac{1}{s^2+6} \approx \frac{\omega}{s^2+\omega^2}\ ;\ \begin{matrix}\omega^2 = 6\\ \omega = \sqrt{6}\end{matrix} \qquad 1 = \frac{\sqrt{6}}{\sqrt{6}}$

$7L^{-1}\left[\frac{\sqrt{6}/\sqrt{6}}{s^2+6}\right] = \frac{7}{\sqrt{6}}\ L^{-1}\left[\frac{\sqrt{6}}{s^2+6}\right]$

$\frac{\sqrt{6}}{s^2+6} \rightarrow \sin\omega t\, U(t)$

$f(t) = \frac{7}{\sqrt{6}} \sin\omega t\, U(t) = \frac{7}{\sqrt{6}} \sin\sqrt{6}\, t\, U(t)$

$\boxed{f(t) = \frac{7\sqrt{6}}{6} \sin\sqrt{6}\, t\, U(t)}$