

Same Task, Different Place: Developing Novel Simulation Environments with Equivalent Task Difficulties

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Abstract. We introduce a novel framework for creating and evaluating multiple virtual reality environments (VEs) that are naturalistic and similar in navigational complexity. We developed this framework in support of a spatial-learning study using a within-subjects design. We generated three interior environments and used graph-theoretic methods to ensure similar complexity. We then developed a scavenger-hunt task that ensured participants would visit all parts of the environments. Here, we describe VE development and a user study evaluating the relative task difficulty in the environments. Our results showed that our techniques were generally successful: the average time to complete the task was similar across environments. Some participants took longer to complete the task in one of the environments, indicating room for refinement of our framework. The methods described here should be of use for future studies using VEs, especially in within-subjects design.

Keywords: Virtual environments · Graph-theoretic measures · Within-subjects designs · Task development · Floorplan design · Bayesian modelling

1 Introduction

Virtual, augmented, and mixed reality technologies, and the virtual environments (VEs) that run on them, have become increasingly popular tools for research, education, and commercial applications. While some applications, such as gaming, may employ unnatural or fanciful VEs, naturalistic virtual spaces are valued in military

training, architectural modeling, ergonomics, and spatial cognition research. Using VEs for these purposes has the potential to reduce costs and improve flexibility compared to real environments or physical models. Much research has aimed at determining what variables influence the effectiveness of VEs for these purposes [1, 2], or what variables influence spatial cognition [1], ergonomics [3], or task performance [4] in VEs when used to model real world experiences. Such variables may include different task framing, different levels of teamwork, different narrative, different display technologies or display parameters, or a multitude of other possibilities.

Research in these areas commonly proceeds by using a single test environment and changing the variable of interest across subjects. For example, each subject might be assigned to one of three different screen refresh rates or one of three different room temperatures. However, because individuals respond to VEs differently, it is often desirable to control for individual differences by using a within-subjects research design. This introduces new challenges, chief among them the need to manage order effects. Counterbalancing is essential, but even with this precaution, order effects may overwhelm the effects of the test variable if the participant becomes too familiar with the same VE over repeated exposure. In this paper, we describe our development process for generating a set of three different difficulty-balanced naturalistic VEs for use in within-subjects research designs. We then describe the validation of the equivalent task difficulty of the VEs and provide some recommendations.

A need for within-subjects design arose in our research on immersive technologies in spatial learning. We wished to examine the effects of three different levels of immersive display technologies—desktop monitor, mid-grade head-mounted display (HMD), and fully occlusive HMD—on spatial learning in VEs. To control for effects from individual differences, we favored a within-subjects design. This necessitated the development of three unique VEs and a common task, ideally of equivalent difficulty, that we could deploy using the three display technologies. The task we selected was an ordered scavenger hunt task. Within each VE, we attached a numbered flag to each of eight scavenger hunt items, and participants sought the items in serial order. Participants could see the flags of items that were not the current target, allowing them to note the location of a future hunt item as they sought their current target.

In designing the VEs and task, we considered three main challenges to achieving equivalent difficulty. The first challenge was constructing three different floorplans with equivalent navigability. The second challenge was the placement of scavenger hunt items. The third challenge was ordering the scavenger hunt items. In what follows, we describe the processes and principles we employed to address these challenges. Subsequently, we show evidence of the success (*i.e.*, difficulty equivalence) of our design as well as some possibilities for improvements.

2 Approach

The goal of our broader study was to use a within-subjects design to evaluate the effects of different immersive technologies in the context of a spatial navigation task. This goal required the creation of three different VEs that were nonetheless equivalent in navigational difficulty. Within these equivalent VEs, we required a task that ensured

participants engaged with and explored the VEs. Here, we lay out our approach for designing the floorplans of the VEs and the task to perform therein.

2.1 Floorplan Design

The floorplan design process began with a 13-room template. The template for the three VEs was a floorplan of an actual home, although no VE used the unaltered original. We separated the template into four blocks that contained three or four rooms each. When selecting the blocks, we focused on constructing versatile bounding edges for ease of generating configurations. We rearranged and rotated these four blocks to generate new layouts aiming for compactness, completeness, and naturalistic fidelity. Compact layouts have a roughly square footprint; non-compact layouts might be much wider than they are tall or have rooms protruding into space. Complete layouts have no voids of un-reachable space that were discontinuous with the outer perimeter. Naturalistic fidelity reflects efforts to ensure that the configuration did not violate any norms expected in an environment layout (e.g., a long, narrow room would typically be a hall leading to another room). Using the same blocks ensured that each VE would have the same surface area, number of rooms (13), and room dimensions.

The principles guiding placement of the doors between rooms were traversability, equality, and naturalistic fidelity. The VE had to be completely traversable so that participants could access all of the rooms in the VE. The process began by adding doorways to each edge that joined rooms. No doors were added to the perimeter. We reviewed naturalistic fidelity for each room to evaluate the practicality of door placement. We subsequently adjusted the number of doorways to equate the quantity of doorways across VEs. After this initial layout and door placement, each VE contained 13 doors.

2.2 Graph Representations of Floorplans

After this initial layout was set, we used graph-theoretic summaries to examine the navigational complexity of the VEs [5, 6]. A broader set of tools that include graphtheoretical elements are called Space Syntax, and have been used for similar ends in previous work [7]. We represented each of the three VEs with a binary undirected graph, in which nodes represented rooms and edges represented doors. We selected this representation because goal understand the navigational was to difficulty/memorability of the VE by reflecting the number of choices the participant faced for where to go next rather than how much distance the participant would need to cover to get through a given door.

These graphs were analyzed using graph-theoretic summaries implemented in the Brain Connectivity Toolbox [8]. First, we counted the number of connected edges of each node for the initial layout, which is called the node degree. Node degree is a measure of centrality, meaning that nodes with higher degree are likely more important to the graph. In this setting, a node with higher degree has more doors, so it is likely to entail more navigational options and be involved in more routes compared with a node of lower degree. Other measures of centrality might have better captured the navigability of these VEs, but an advantage of degree distribution is that it is relatively easy to

adjust by adding and deleting edges. We added, subtracted, and moved doors so that each VE had the same degree distribution: four nodes of degree 1, five nodes of degree 2, three nodes of degree 3, and one node of degree 5, while maintaining a naturalistic arrangement of doorways. After this process, each VE had 14 total doors.

After these adjustments, other graph summaries were computed: characteristic path length, which is the average length of the shortest path between two points in the graph; nodal eccentricity, which equals the shortest path length from a given node to the node that is farthest from it; graph radius which is the minimum of nodal eccentricity; and graph diameter, which is the maximum of nodal eccentricity. Characteristic path length, graph radius, and graph diameter for each VE appear in Table 1. Although these measures are not identical, they differed by no more than one unit, and there was no obvious way to equalize them without losing equality of degree distribution.

	-		
Environment		Graph radius	Graph diameter
	path length		
Home	3.06	4	7
Office	2.78	3	6
School	3.09	3	6

Table 1. Graph measures of the environments.

After layout was determined, we built the VEs in a 3D game engine, finalized the theme of each room, and populated the VEs with objects consistent with each room's theme with careful consideration not to repeat objects across VEs. The object selection process and the development of transfer questions about those objects are detailed elsewhere [9]. Next, we faced the second challenge of where to put hunt items in the VE.

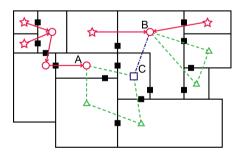


Fig. 1. Target object placement heuristics. Dead-end rooms (*stars*) all required target objects to ensure that they would be visited. Must-visit status propagates (*arrows*) from must-visit rooms with only one connection to a target-optional room into that room (*circles*). All routes from point A to point B pass through a common node (*square*, point C). Parallel chains (*dashed lines*) necessitated targets be added to rooms (*triangles*) to give them must-visit status. Rooms are connected by open doors (*black squares*).

2.3 Hunt Item Placement

One of the main goals of having a scavenger hunt task was to ensure participants fully explored the VEs. Therefore, the main criterion for hunt item placement was that finding all the items required visiting each room in the VE at least once. This criterion lead to a few heuristics (Fig. 1) that governed which rooms needed to have hunt items in order for each room to have must-visit status (i.e. the room must be visited to complete the scavenger hunt). All dead-end rooms (nodes of degree 1) needed to have a hunt object, because dead-end rooms are never a necessary step in a path to any room other than that dead-end room. Placing a hunt item in each dead-end room gave them must-visit status. Following this, all rooms directly connected to a dead-end room obtained must-visit status, because one would have to pass through that room to get to the dead-end room. This must-visit status propagates iteratively; at each step any must-visit room that is connected to exactly one room without must-visit status grants that room must-visit status. Following dead-end propagation, any room that was included in all possible paths between any two must-visit rooms also obtained must-visit status. This process covered most of the rooms in our VEs.

The remaining rooms formed parallel chains of rooms (*i.e.*, two different paths can get from point A to point B without including common nodes other than A and B). With parallel chains of rooms, a participant might always choose one of the two routes and miss seeing the rooms along the parallel route. When the dead-end propagation procedure did not already render each room in a parallel chain as must-visit, that room needed a hunt item to ensure the participant visited each step in the chain rather than possibly turning around instead of completing the chain. The dead-end propagation followed by parallel chain resolution dictated the minimum number and placement of hunt items.

Following these heuristics, the required number of hunt items for each VE was seven, eight, and eight for the home, office, and school VEs, respectively. To keep the number of hunt items equal, each hunt consisted of eight items. We could have added additional hunt items, but that might have required longer experimental sessions to complete.

2.4 Hunt Item Order

Having selected the number and location of the hunt items, we needed to specify an order for participants to seek the hunt items. For each VE, we selected a starting position, first hunt item, and last hunt item based on the scientific aims of the main study. For each possible hunt order respecting these constraints, the minimum path length (i.e. smallest number of nodes visited or revisited) to complete the hunt was calculated based on the graph representations of the VEs. The median minimum path lengths were 25, 23, and 27 for home, office, and school VEs, respectively. Therefore, we selected only hunt item orders with minimum path length of 25. Having a consistent minimum path length aimed to keep the variance in search time similar across VEs. There was no expectation that participants would complete the hunt following the shortest path length, but some proportion would have done so due to lucky guessing; we expected that a consistent floor value on the scavenger hunt path would minimize variability across VEs due to this kind of luck.

There remained more than 50 options in each VE that satisfied all criteria including the minimum path length of 25, so we applied an additional constraint that the distribution of minimum path lengths between subsequent hunt items should be the same. Only one distribution of minimum path lengths occurred in all VEs: one with path length 1, two each of lengths 2, 3, and 4, and one with path length 6. The final layout and scavenger hunt items and order appear in Fig. 2.

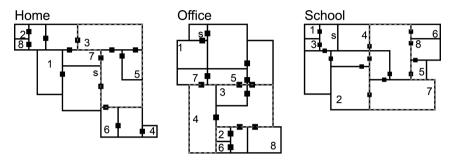


Fig. 2. Environment floorplans and scavenger hunt order. Participants began at the starting point (s) and explored the environment looking for the scavenger hunt items in the order indicated (numerals). Rooms are connected by open doorways (solid black squares). The floorplans are composed of four blocks of rooms taken from an original extant floorplan. One such block is highlighted (dashed outline).

3 Assessing Task Difficulty

3.1 Data Collection

Participants took part in three experimental sessions, each separated by at least two weeks to minimize any carryover effects. In each session, a participant filled out questionnaires (details and results discussed elsewhere), and was then set up with one of the three display technologies and a game controller. This was followed by a brief training session and practice to familiarize them with the game controller and display technology and to give them exposure to the types of spatial tasks they would encounter and questions that would later be asked. Participants then entered one of the three naturalistic VEs and were asked to complete a scavenger hunt, following on screen instructions. Participants spent a total of 15 min in each VE. Scavenger hunt items were marked with numbered flags, and participants were instructed to find each number in order and to press a button to mark each one as found. The intention behind the scavenger hunt approach was to make sure the participant was motivated to traverse the entire VE and see all the rooms. After the participant found all the scavenger hunt objects, they were instructed to freely explore the VE for the rest of their 15 min. The experiment software automatically recorded the participant's virtual location and scavenger hunt progress at 100 ms intervals. After exiting the VE and removing the display technology, the participants answered spatial memory and navigation questions to demonstrate spatial learning of the VE. The nature of these questions, their development, and results of these measures will be reported elsewhere. Participants returned after a break of at least two weeks and then performed the same tasks with a new display technology and new VE. The technology, VE, and the order of presentation were counterbalanced.

Fifty participants (37 F, 13 M) completed the scavenger hunt in each of the three sessions of the experiment. Data from two participants were excluded, because they did not finish the scavenger hunt by the time limit in the home VE. The mean age was 21 years (range 19–29). All participants were recruited through various online platforms that were associated with the community at the University of California, Santa Barbara including the Psychology Department's paid participant subject pool. Inclusion criteria were at least 18 years of age, normal hearing, color vision, and normal vision (or corrected-to-normal provided that participants would wear contact lenses to the experiment). Exclusion criteria included heart problems, pregnancy, easily getting motion sick, or 11 or more hours of experience with virtual reality equipment. The voluntary, fully informed, written consent of participants in this research was obtained as required by Title 32, Part 219 of the CFR and Army Regulation 70-25. All human subjects testing was approved by the Institutional Review Board of the U.S. Army Research Laboratory.

3.2 Model Fitting

The primary question of interest is whether the three different VEs and their respective hunt orders and locations had similar effects on task difficulty. Time to complete the hunt served as a measure for the overall difficulty of the scavenger hunt task. To answer this question, we fit two Bayesian models to the data. One model had separate effects for each of the three VEs, and the other had one common effect for all three VEs. In both models, we modeled participant effects as random additive effects. Time (all units in seconds) to complete the scavenger hunt for subject i in VE j was modeled with a truncated generalized Student's t distribution:

$$t_{i,j} \sim Student's \ t(v, \mu_{i,j}, \sigma)T[0, 900].$$
 (1)

$$\mu_{i,j} = s_i + e_j. \tag{2}$$

Priors were $v \sim \Gamma(2, \frac{1}{10})$, $\sigma \sim \mathcal{N}^+(0, 100)$, $s_i \sim \mathcal{N}(0, 200)T[-900, 900]$, and $e_j \sim \mathcal{N}(450, 200)T[0, 900]$. We selected a generalized Student's t distribution so the model would be robust to extreme values. Truncation at 0 and 900 s reflects that the scavenger hunt ended if the participant had not found the last item after 900 s. The common-mean model was similar to the three-mean model:

$$t_{i,j} \sim Student's \ t(v, \mu_i, \sigma)T[0, 900].$$
 (3)

$$\mu_i = s_i + e_{common}. \tag{4}$$

Priors were $e_{common} \sim \mathcal{N}(450, 200)T[0, 900]$ and the same priors from the three-mean model for v, σ , and s_i . The only distinction between the two models is that in the three-mean model, e_j represents one parameter per VE, while e_{common} represents one parameter for all three VEs. The priors for these models are vaguely informative, in that they indicate that solutions with values near the middle of the time range are more likely than values near the extrema, but the large standard deviations represent low certainty about the expected locations of effects.

These models were implemented in Stan [10] using the RStan interface [11]. The default No U-Turn Sampler was used in 12 independent chains each using 2,000 warmup and 2,000 post-warmup iterations. Convergence diagnostics were used with the criteria that effective sampling ratio was over 0.1 and \hat{R} was less than 1.1 for all parameters, and all chains ran with no divergences.

Model comparisons were done by computing expected log predictive density using leave-one-participant-out cross-validation [12]. We re-fit the models with N-1 participants' data and computed predictive density of the left-out data at each iteration. Repeated N times, leaving a different participant out each time, this provides the expected log predictive density (ELPD) of a new participant's results. We calculated model ELPD difference and report the mean and standard error of within-subjects differences.

4 Results

Mean completion times (in seconds) and bias-corrected, accelerated bootstrapped 95% within-subjects confidence intervals were 399.4 [355.8, 453.2], 400.2 [362.8, 437.3], and 433.0 [399.8, 467.9] for home, office and school VEs, respectively (Fig. 3). Consistent with the overlapping confidence intervals, a repeated-measures ANOVA using the Greenhouse-Geisser correction for non-sphericity indicated no strong incompatibility between the data and a null hypothesis of no difference in VE means, F(1.75, 85.5) = 1.06, $\varepsilon = .87$, p = 0.342, $\eta^2 = .021$.

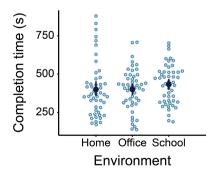


Fig. 3. Scavenger hunt completion times in three virtual environments. Individual results (*light circles*) are summarized with the mean (*large dark circles*) and bootstrapped 95% within-subjects confidence intervals (*dark whiskers*).

In the Bayesian model fitting, all convergence criteria were satisfied. The main variables of interest, the VE means in the three-VE model and the common mean in the common-mean model appear in Fig. 4. In the three-VE model, posterior distribution mean and central 90% uncertainty intervals, conditional on the priors and data, were 375.3 [321.8, 429.1], 399.1 [346.9, 451.0], and 440.1 [388.0, 491.5] for home, office, and school VEs, respectively. In the common-mean model, the posterior distribution for the common mean was 407.6 [358.5 456.6].

Participant effects were similar across the two models. Posterior summaries for each participant appear in Fig. 5. For the three-mean model, mean participant effect posteriors ranged from -212.1 to 168.0 with an inter-quartile range of 115.4. In the common-mean model, mean participant effect posteriors ranged from -209.3 to 172.1 with an inter-quartile range of 110.9.

Posterior distributions for degrees of freedom v and standard deviation σ for the generalized Student's t distribution were obtained for both models. In the three-VE model, mean and central 90% uncertainty intervals for v were 5.7 [2.3, 13.7] and for σ were 92.8 [70.9, 117.7]. In the common-mean model, they were 12.2 [3.6, 31.7] and 109.7 [88.3, 130.2]. These values are similar, although the lower degrees of freedom in the three-VE model, indicating heavier tails than in the common-mean model, suggests that extreme values may be more apparent in the three-VE model.

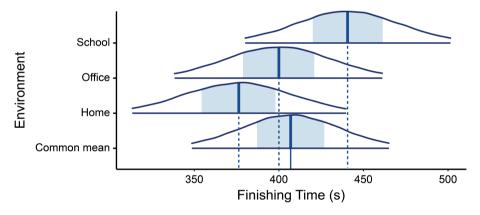


Fig. 4. Posterior distributions on independent environment effects in the three-mean model overlap substantially with the common environment effect in the common-mean model. The posterior distributions are summarized with mean (*heavy vertical bar*), central 50% uncertainty intervals (*shaded region*), and density estimates covering the central 90% uncertainty intervals (*lines*).

The ELPD was -974.6, SE 11.6 for the three-VE model and -976.9, SE 10.3 for the common-mean model. The ELPD difference between the three-VE model and the common-mean model was 2.4, SE 3.0, reflecting a predictive advantage of the three-VE model of less than one standard error. This shows that between-subjects variability in ELPD is larger than the systematic difference between the two models.

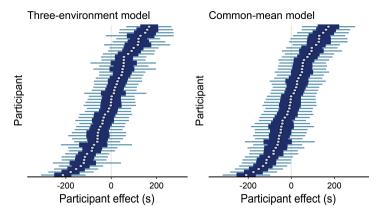


Fig. 5. Participant effect posteriors were similar for the three-environment and common-mean models. Participant data (N = 50) is sorted by posterior mean (*light central dot*). Central 50% (*dark horizontal bar*) and 90% (*whiskers*) uncertainty intervals summarize the posterior distributions.

5 Discussion

In general, the approach we followed was successful. An initial frequentist estimate showed no strong incompatibility between the data and a hypothesis of no VE-specific effects. Bayesian estimates of independent VE effects all had means falling within 65 s of each other, and the posterior distributions of the three VE effects overlapped substantially with the common VE in the common-mean model. Fitting with the common-mean model resulted in a negligible loss in predictive power relative to the three-mean model, indicating that the common-mean model was not appreciably worse at accounting for the data than the three-VE model. Taken together, these results support the claim that the difficulty of the task was similar across the three different VEs.

Of course, the difficulties were not identical and the three-VE model did estimate different, albeit very similar, VE effects. With this in mind, future analyses should attempt to control for environment effects, even though the evidence available indicates that the environment effects here are small.

The variabilities in the participant effects for both models were large relative to the VE effects. This variability occurred despite the participant sample representing a narrow demographic range. Studies recruiting from a more general population could expect even higher between-participant variability. This observation reinforces the value of within-subjects designs in VE studies, because these substantial between-participant differences can be isolated from the experimental effects of interest.

5.1 Extreme Values

The robust error term in the Bayesian models prevented extreme values from having a substantial impact on the effect distributions. However, extreme values were concentrated in the Home VE. Moreover, two participants who failed to complete the

scavenger hunt in a session both failed to complete the Home VE. This suggests that our design process failed to account for something in the Home VE that, on rare occasion, lead to relatively slow searches. Without further experimentation we can only speculate on what might have caused these slow searches, but the Home VE did have a larger graph radius and diameter than the other two VEs. This relatively larger graph might have afforded more opportunities for costly navigation errors, leading to the small number of long hunt completion times. Another possible explanation is that specific hunt items might have been harder to spot within a room, or that some other factor accounts for these extreme values.

6 Conclusions

The approach described here to designing the VE layouts and the specific design of the scavenger hunt lead to a generally successful outcome. Typical completion times for the scavenger hunt in all VEs were within a range of 65 s, which is a considerably smaller range than the variability between individuals, regardless of VE.

Graph-theoretic measures are an important tool for understanding spatial layouts. Free, open source tools for computing such measures are available. Although advanced measures of environment graphs might be more informative, an advantage of the relatively simple measures we used was that they lend themselves to easy adjustments with a fair amount of leeway to account for other experimental or design constraints.

Our overall approach was not entirely sufficient to overcome all design challenges. In particular, although all our VEs had similar typical completion times, one VE induced a small number of participants to take an unusually long time to complete the hunt.

The greatest manual effort was in ensuring the hunt item placement satisfied the completeness criterion. The outlined heuristics were sufficient for our VEs, but they might not cover all possible environment layouts. Automated solutions would be needed when designing a relatively large number of environments or generating them algorithmically.

All these VEs were interiors. Different design principles may be needed for less path-constrained exterior and mixed indoor-outdoor VEs. In particular, applying graph theory when nodes and edges are not conveniently defined by rooms and doors (or streets and well-defined intersections) might be challenging. Past work suggests that space syntax tools are a viable complement to the graph-theoretic approach [7]. Isovists [13] can give an indication of environment inter-complexity for large scale spaces and even indoor environments such as ours.

Designing naturalistic VEs for use in within-subjects designs raises the problem of how to design VEs that are different but equivalent. Here, we laid out our approach to this conundrum and showed that the approach seems to have worked well. It is our hope this approach will be helpful for future experimental designs, unlocking the potential advantages of within-subjects designs for VE research and application.

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