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# Abstract

Maybe use the first paragraph below and restructure to including findings. Then start background straight with History.

# Introduction/Problem Statement

The volatile nature of financial markets imposes various sources of uncertainty on an investor's portfolio. The options market provides a way for investors to combat those uncertainties, by hedging their portfolio against the risk of adverse price movements. Thus, pricing option contracts are an important daily activity for banks and other financial institutions. It is also common for retail investors like ourselves to study options for either short term short term gains or to be a beneficiary from a company as another retirement strategy. Therefore, for this paper we will compare the findings from analytical pricing methods and from simulation techniques, to approximate the fair value of the European and American option price today.

# Background and Description

## History Of Options

The history of Option contracts can be traced back to 332 BC, when Thales bought the rights to buy olives prior to a harvest, who went on to amass a fortune from the purchase of these rights. In 1636 options were widely bought to speculate on the soaring prices of tulips in Europe. At the time tulips became a sign of high status, and beauty, which fueled a craze of speculation from all levels in society. The wild speculation caused tulip prices to skyrocket, which also increased the number of growers and dealers looking to get into the trade. Dealers began selling the rights to own tulips in advance of the harvest, for buyers looking to secure a definite buying price. A year later, the price of tulips had gotten so high that no buyers were willing to pay the inflated price, resulting in a selling frenzy. The price of tulips had fallen faster than it rose; and almost all the speculators were wiped out as the price declined lower than the originally agreed upon price to buy. The Dutch economy would collapse after the sharp decline, leading to the notorious reputation for options being a dangerous speculative instrument. Although options gained a bad reputation, they later surfaced in London in the early 1700’s with financiers acknowledging its speculative power through its inherent leverage. It was later declared illegal in 1733, as groups opposed the speculative nature of options trading. It was outlawed for about 100 years, making a return in 1860 almost 100 years later.

In 1872 Russel Sage, a well-known American financier from New York; was the first to create calls and puts for trading in the US. Sage created options that were unstandardized and highly illiquid, resulting in an inefficient market for participants. However, Sage still managed to make millions within a few short years, before losing a fortune in the stock market crash of 1884. These unregulated Options continued to trade until the creation of the SEC after the stock market crash of the 1930’s. In 1973 The Chicago Board of Exchange and Options Clearing Corporation were formed to standardize and allow retail investors to participate in the trade under the performance guarantee of the OCC, and liquidity of the market maker system. By 1977 put options were introduced to the CBOE, which created the options market that we can trade today.

## Definitions and Literature Review

Option contracts refer to an agreement between two parties to facilitate a potential transaction on an underlying security at a predefined strike price prior to, or on a specified expiration date. A single stock option contract gives the option holder the right to 100 shares of the underlying security.

### Vanilla Options

A Vanilla Option is a simple call or put with no special features or observation dates. Call options give the option holder the right to buy 100 shares of an asset at the predefined strike price, prior to or on a defined expiration date. Put Options give the option holder the right to sell 100 shares of an underlying asset, prior to or on a defined expiration date. If an investor is unsure of the future performance of a specific stock, and they have a strong belief that the price will rise in the future, the investor may choose to buy a call option on the stock. This allows the investor to gain exposure from a future price increase, while not having to commit the capital to purchase the hundred shares on the specific asset. The European is the simplest form of a vanilla option, giving the investor the right to buy or sell shares of an asset a specified expiration date. American options differ from the European counterpart, in the fact that they can be exercised at any point until or on the expiration date. Vanilla options are some of the most traded option contacts due to their simplistic nature and lack of exotic characteristics.

### Exotic Options

Exotic Options are options that differ from Vanilla options in their payment structures, expiration dates and predefined strike prices. The underlying asset can vary in an exotic option, thus giving investors alternative opportunities. Exotic options can be customized to meet the risk tolerance and investment objective of the investor. Asian Options an exotic option in that they are dependent on the average price of the underlying asset over a specified time. Bermuda options are also an example of an exotic option, they give the investor the right to buy or sell an asset at many preset expiration dates. Exotic options provide investors more control over when the option is exercised, and they often come with a higher initial cost when compared to a vanilla option contract.

### Time Value of Money

Discount factor incorporates “time value of money” to calculate the options price more accurately. Time value of money represents the idea that value of money changes with time driven by factors such as inflation, changes in interest rates (although the model assumes it is constant), company-specific & country-specific premiums, unforeseen events such as pandemic, among others. Theoretically, it can be explained by the concepts of present value (PV) and future value (FV) where it is believed that the sum of money is worth more now than the sum will be at a future date due to its earning potential in the interim:

## Brownian Motion

Brownian Motion is the random chaotic movement observed in particles as the result of collisions with other surrounding particles. The Standard Brownian Motion generates random points and is defined as the following:

Some important properties of Standard Brownian motion are that it has stationary and independent increments between points. The Stationarity property assumes the change in Brownian motion between two time periods is due to the length of the increment only. The independence property states that the change in motion between two pairs of time periods have no relationship with each other.

### Using Geometric Brownian Motion to Model a Stock Price:

Geometric Brownian Motion is commonly used to model stock prices. Where μ is the drift, 𝝈 is the volatility, and is the current stock price. The standard Brownian motion is denoted , thus the Geometric Brownian process can be shown as follows:

## Input data

We obtained historical data from Yahoo Finance for three major indices to track overall movements in the stock market. The data was obtained over the past year, beginning 2022 and collected twice a day, at the open and close. The stocks chosen are the SPDR S&P 500 ETF (SPY), Invesco QQQ Trust (QQQ), and the iShares Russel 2000 (IWM). The options on these stocks are traded heavily by financial institutions and retail investors alike. The tickers we chose are regular leaders in stock option volume traded each day, so there is a very large interest in trading options on these assets.

# Main findings

### 

### Defining Black-Scholes:

Black-Scholes starts with replacing the value of from Geometric Brownian motion equation from above in the expected value section of the equation of European call option below. It is then multiplied with phi of z and integrated over . The Brownian motion becomes as it is Nor (0,1).

The final equation above can also be written as a ‘closed form’ solution below to help us understand the formula more intuitivelyWe will show it for a European call option, c, as an example:

We pay strike price (K) if the underlying stock price (S) is above the strike price at maturity, thus the probability of that can be defined as KN(d2) where N is the cumulative normal function (a statistical operator) and d1 and d2 are expressed as below. To get the value of this cash flow today, we discount it by factor exp-rt as explained above. Thus, the value of cash to buy option today is KN(d2) *exp-rt.*

By the same logic, SN(d1) is the value of the stock received once the option is bought whose expected value is proportional to S, stock price today, and can be written as SN(d1).

,

## Simulation Methods

Take average of the differences between all stock price paths (St) from the simulation and the strike price (K) which is the predetermined price of the stock. It is then multiplied with the discount factor (e-rt) to find the fair value of the price today. The fair value of the price is the minimum acceptable stock price for the contract to break even or make profit when exercised. Stock price - Strike price is also known as option premium, which is the price that you pay for the option. If r is our continuously compounded interest rate for a period of t years, then the discount factor will be e-rt.

### European Options:

### American optIons

One of the methodologies we have employed for simulating American options is the concept of Least Squared Monte Carlo (LSM). The major factor for consideration of the American option is that the contract holder has the option to sell said contract early if they believe it to be more profitable than at expiration. This is important to understand and what the LSM takes into account. In this methodology, we are starting with basic simulation and use Brownian motion to derive all the different paths a stock can take in the future. There in then an iterative process of keeping track of decisions on whether to sell at the current time, or to keep until expiration. The is done by taking only the paths in the money at time t, and using a regression model to calculate the projected payoff would be at the next discounted time increment t+1 and compare to the current payoff at time t. This loops backwards in time and tracks when it would have been earliest optimal to sell for each path. Then based on those optimal time, the paths are discounted to Net Present Value and averaged to get the average option price for the specific contract. For the regression models, we have used a normal distribution model as well as a polynomial model to evaluate the paths future payoff.

### Volatility

* Variance Reduction from latest lesson by using same randomly variables, etc*.*

## Fundamental results

* *Describe some of the* ***fundamental results*** *(e.g., this paper gives an explicit expression for an estimator’s expected value in terms of the covariance function).*
  + *Compare analytical way vs. simulation*

The simulation methods seemed to perform better for the Put contracts than the calls with the data we have provided. The Black Scholes model seemed to give a close approximate to the observed price of the option; however as seen below, the LSMC method seems to be more accurate. The errors for the Put contracts (Right) are much closer to 0 than compared to the Calls (Left). We can note that the LSMC with a Normal Basis function performed the worst for all contracts, having a very high variability in its fair value approximation. We note that the LSMC method can estimate the value of the option contract very well especially with a shorter time until expiration. Simulations methods proved to be more accurate with less days until the expiration, with the LSMC Normal Basis Function performing the worst across all time horizons. We can see that the Simulation methods give a noticeably lower standard error for short term contracts than longer term, with simulated results differing within 1 cent of the average simulated fair value.

Since we can get the European component of an option price from the Black-Scholes Model, we already know the price of the option at expiration. The early exercise component of an American option is the difference between the European price and the simulated American option price we have obtained from the methods above. The early exercise value can be thought of the excess value of the option compared to if you held it to expiration. So, if we notice that the early exercise is high, we might want to exercise the option early, to get the most value out of the option, and vice versa if the early exercise value is negative. Thus, the table above shows the difference between the simulated methods and the Black Scholes method, and it clearly shows that the LSMC Normal Basis Function is the worst at estimating the early exercise value. The Polynomial Basis Function performed the best, and the finite difference simulation had similar results to the polynomial basis function. The difference between the Black Scholes method and the simulated methods is well within the bid-ask spread for these option contracts, which suggest that the simulated methods can approximate the early exercise value of the option contracts very well.

## Opportunities

* *Formulate (but you don’t have to solve)* ***a couple of research problems*** *that might be derived from this work.  (Jerald added some in the conclusion*

# Conclusion

* *What did you find/learn from the project?* 
  + *Challenges encountered*
* *Any ideas for future work, if appropriate*
* Determining accuracy of the simulation by using Stand Error

In conclusion, we can see that the LSMC method is able to estimate the fair value of an American option reasonably well and can be extended to various types of options such as equity, commodities, mortgage, and swaps. We have found these methods to show better results with a shorter time until expiration, with the polynomial basis function preforming the best. Choosing a strong basis function should be a priority when using the LSMC method. In future research one should give care in choosing a basis function, to ensure that the choice of basis functions is not highly correlated with one another. Another interesting direction to explore would be to implement the LSMC method with an underlying asset that follows a jump diffusion process.

# Appendix

Option Contract: an agreement between two parties to facilitate a potential transaction involving an asset at a preset price and date.

Payoff: For a Put Option Stock - Strike, for a Call Option Strike - Stock; This is the value of the option contract.

Continuation Value: The value of the option contract at the next time step, ie. You choose to continue to hold the option.

In the money: For a Put Option, the stock has fallen below the chosen strike price. For a Call Option, the stock has risen above the chosen strike price.

Volatility tells us how likely the price deviates from true mean price in each period. Stocks with higher volatility are more likely to have a very high or very low value at maturity which positively impacts both put and call options. Although stocks with higher volatility result in higher returns, it makes the predicting returns challenging which is not preferred by investors while exercising call options.

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|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Actual Vs. Simulated Option Price Errors | | | | | | | | | |
| Term | 90 Days | | | 60 Days | | | 10 Days | | |
|  | BS error | LSMC Poly error | MC error | BS error | LSMC Poly error | MC error | BS error | LSMC Poly error | MC error |
| Option |  |  |  |  |  |  |  |  |  |
| IWM $185 Call | 1.615 | 1.642 | 1.572 | 0.813 | 0.847 | 0.829 | 0.211 | 0.214 | 0.207 |
| IWM $185 Put | 0.430 | 0.444 | 0.427 | 0.379 | 0.423 | 0.361 | 0.231 | 0.337 | 0.199 |
| QQQ $307 Call | 1.794 | 1.685 | 1.758 | 1.047 | 1.000 | 1.036 | 0.312 | 0.301 | 0.295 |
| QQQ $305 Put | 1.048 | 0.638 | 1.078 | 0.335 | 0.600 | 0.385 | 0.378 | 0.522 | 0.447 |
| SPY $398 Call | 3.155 | 3.150 | 3.155 | 1.455 | 1.467 | 1.473 | 0.425 | 0.422 | 0.396 |
| SPY $398 Put | 0.644 | 0.680 | 0.674 | 0.573 | 0.687 | 0.553 | 0.304 | 0.421 | 0.265 |

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Standard Errors From Independent Replications | | | | | | | | | |
| Term | 90 Days | | | 60 Days | | | 10 Days | | |
|  | LSMC Normal SE | LSMC Poly SE | MC SE | LSMC Normal SE | LSMC Poly SE | MC SE | LSMC Normal SE | LSMC Poly SE | MC SE |
| Option |  |  |  |  |  |  |  |  |  |
| IWM $185 Call | 0.0686 | 0.1333 | 0.1543 | 0.0355 | 0.0895 | 0.0979 | 0.0147 | 0.0257 | 0.0218 |
| IWM $185 Put | 0.0373 | 0.0909 | 0.1085 | 0.0343 | 0.0535 | 0.0669 | 0.0416 | 0.0495 | 0.0574 |
| QQQ $307 Call | 0.0806 | 0.1421 | 0.1643 | 0.0627 | 0.0900 | 0.1061 | 0.0359 | 0.0401 | 0.0480 |
| QQQ $305 Put | 0.0495 | 0.1369 | 0.1934 | 0.0479 | 0.1008 | 0.1166 | 0.0312 | 0.0486 | 0.0662 |
| SPY $398 Call | 0.1066 | 0.2541 | 0.2719 | 0.0587 | 0.1483 | 0.1553 | 0.0342 | 0.0613 | 0.0571 |
| SPY $398 Put | 0.0696 | 0.1791 | 0.2114 | 0.0763 | 0.1262 | 0.1397 | 0.1036 | 0.1192 | 0.1265 |

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Early Exercise Compare | | | | | | | | | |
| Term | 90 Days | | | 60 Days | | | 10 Days | | |
|  | LSMC Normal | LSMC Poly | MC | LSMC Normal | LSMC Poly | MC | LSMC Normal | LSMC Poly | MC |
| Option |  |  |  |  |  |  |  |  |  |
| IWM $185 Call | 3.708 | 0.194 | 0.176 | 2.298 | 0.100 | 0.080 | 0.376 | 0.015 | 0.029 |
| IWM $185 Put | 1.936 | 0.182 | 0.113 | 0.966 | 0.069 | 0.056 | 0.407 | 0.121 | 0.063 |
| QQQ $307 Call | 3.267 | 0.164 | 0.160 | 1.742 | 0.085 | 0.100 | 0.316 | 0.038 | 0.038 |
| QQQ $305 Put | 2.534 | 1.129 | 0.183 | 1.937 | 0.267 | 0.166 | 0.499 | 0.149 | 0.085 |
| SPY $398 Call | 7.176 | 0.268 | 0.201 | 3.915 | 0.120 | 0.115 | 0.832 | 0.042 | 0.079 |
| SPY $398 Put | 4.375 | 0.349 | 0.188 | 2.820 | 0.161 | 0.137 | 1.435 | 0.185 | 0.153 |

Chart

Description automatically generated

Chart

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