

QUANTIFIERS, METHODS OF PROOF 0

COMPUTER SCIENCE MENTORS 70

Independent review

1 Quantifiers

1.1 Questions

1. Let $P(x, y)$ denote some proposition involving x and y . For each statement below, either prove that the statement is correct or provide a counterexample if it is false.

a. $\forall x \forall y P(x, y) \implies \forall y \forall x P(x, y)$.

b. $\exists x \exists y P(x, y) \implies \exists y \exists x P(x, y)$.

c. $\forall x \exists y P(x, y) \implies \exists y \forall x P(x, y)$.

d. $\exists x \forall y P(x, y) \rightarrow \forall y \exists x P(x, y)$.

2 Contrapositive and Contradiction

2.1 Questions

1. Write the contrapositive of the following statements and, if applicable, the statement in mathematical notation. (Using quantifiers, etc.)

a If a quadrilateral is not a rectangle, then it does not have two pairs of parallel sides. (Skip mathematical notation for this problem, just write the contrapositive)

b For all natural numbers a where a^2 is even, a is even.

c Negate this statement: For all integers x , there exists an integer y such that $x^2 + y = 16$.

2. Prove or disprove: If $P \implies Q$ and $R \implies \neg Q$, then $P \implies \neg R$.

3 Proof by Cases

3.1 Questions

1. For any integer x , x^2 has remainder 1 or 0 when divided by 3.

4 Induction

4.1 Questions

1. What are the three steps of induction?

2. Prove that $\sum_{i=0}^n i * i! = (n + 1)! - 1$ for $n \geq 1$ where $n \in \mathbb{N}$.

3. Prove or disprove the following statement: If n is a positive integer such that $\frac{n}{3}$ leaves a remainder of 2, then n is not a perfect square.

4. In a large field, n people are standing so that for each person, the distances to every other person is different. At a given signal, each person fires a water pistol and hits the person who is closest to them. When n is odd, prove that there is at least one person who is left dry.