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# META

LLSE, Covariance, Conditional Expectation, and Markov Chains

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## 1 General Comments

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1. Focus on
  - LLSE: all parts
  - Covariance: 2.1 and 2.2.1
  - Conditional Expectation: all parts
2. Covariance: Make the connection between the covariance formula and the variance formula, and why it makes sense to define covariance this way.
3. LLSE
  - Make sure to do covariance and conditional expectation first because LLSE relies on it
  - Explain what LLSE is, and why the formula makes sense—intuitively, why. If there is time or interest, prove it.
4. Conditional Expectation
  - Make the connection between normal expectation and conditional expectation, and explain intuitively what conditional expectation means
  - Explain what LLSE is, and why the formula makes sense—intuitively, why. If there is time or interest, prove it.
5. Markov Chains
  - First order Markov property: probability of an object being in any state depends only on the state the object was in right before

- Homogeneity: Regardless of how many steps the chain has run, the transition matrix doesn't change (i.e.  $P(X_{i+1} = a | X_i = b)$  is the same regardless of the value of  $i$ )
- Irreducibility: there is no sink
- A state  $x$  is periodic if by starting in  $x$ , all ways of returning to  $x$  have some GCD greater than 1. If any state is periodic, then the chain is periodic.
- Explain the intuition behind stationary/invariant distributions and irreducibility and aperiodicity

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## 2 Questions

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### 2.1 Covariance

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#### 1. Roll 2 Dice

- This goes back to basic expectation and probability; students who can do 2.1 but get stuck here likely have some gaps in their understanding of previous topics.

#### 2. Fah Kat

- Back to basics; given the actual distribution, calculate expectation (and thus covariance) from scratch.

### 2.2 Conditional Expectation

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#### 1. Expectation of Conditional Expectation

- Using many properties of summations, make sure students understand each step

#### 2. Take Out $h$

- Using many properties of summations, make sure students understand each step

#### 3. Given Probability Distribution

- Plug and chug, going back to basic distributions.

### 2.3 Markov Chains

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#### 1. Life of Alex

- Basic uses of the formulas

- d: Do not forget that any distribution must add up to 1, and explain that using this method of finding the stationary distribution, two of the first-step equations will be linearly dependent.