Counting, Discrete Probability, Conditional Probability, Monty Hall 6

Computer Science Mentors 70

March 6 to March 10, 2017

1 Intro to Counting

1.1 Introduction

Rules of counting:

1. If your event is composed of different independent events then you can multiply together the probabilities of the independent events.
2. If order does not matter then count with order and then divide by the number of orderings/sorted objects

1.2 When Order Matters

1. (a) You have 15 chairs in a room and there are 9 people. How many different ways can everyone sit down?

   Solution: $\frac{15!}{6!}$ There are 15 places to put the first person, then 14 places to put the second person, 13 places to put the third person, etc all the way to the last person who has 7 places to sit Another way to think about this is like the anagram example above. We have 9 unique letters and 6 repeats (our empty spaces). We divide by the repeats giving us: $15*14*13*12*11*10*9*8*7 = \frac{15!}{6!}$

   (b) How many ways are there to fill 9 of the 15 chairs? (We dont care who sits in them)

Solution: $\binom{15}{9} = \frac{15!}{9!(15-9)!}$ In this example, we dont care about the uniqueness of each person, so we can just count each person as a repeat. So like the anagram example well divide for every repeat. We have 9 human repeats, and 6 empty space repeats. Hence $\frac{15!}{9!6!}$

2. **Identical Digits** The numbers 1447, 1005, and 1231 have something in common. Each of them is a four digit number that begins with 1 and has two identical digits. How many numbers like this are there?

Solution: Case 1: the identical digits are 1 (e.g. $11xy, 1x1y, 1xy1$)
Since there can only be two numbers that are identical, x and y cannot be 1 and $x \neq y$.
So [Possible formats] * [Possible x values] * [Possible y values] $= 3 * 9 * 8 = 216$
Case 2: identical digits are not 1 (e.g. $1xxy, 1xyx, 1yxx$).
So [Possible formats] * [Possible x values] * [Possible y values] $= 3 * 9 * 8 = 216$
Add both cases to arrive at the final result. $216 + 216 = 432$

1.3 More Practice

1. At Starbucks, you can choose either a Tall, a Grande, or a Venti drink. Further, you can choose whether you want an extra shot of espresso or not. Furthermore, you can choose whether you want a Latte, a Cappuccino, an Americano, or a Frappuccino.

   How many different drink combinations can you order?

Solution: $3 \cdot 2 \cdot 4$ (# sizes * espresso or not * # types of coffee)

2. We grab a deck of cards and its poker time. Remember, in poker, order doesnt matter.
   (a) How many ways can we have a hand with exactly one pair? This means a hand with ranks (a, a, b, c, d)

Solution: $= 13 * \binom{4}{2} * \binom{12}{3} * 4^3$ There are 13 value options for a (2, 3, 4, $\ldots$, K, A). We then need to choose 2 out of the 4 possible suits. Now we need to choose b, c, and d. There are 12 values left (must be different from a). Finally, there are 4 suit options for each of the values chosen for b, c, d.

   (b) How many ways can we have a hand with four of a kind? This means a hand with ranks (a, a, a, a, b)

Solution: $= 13 * 12 * 4$

(c) How many ways can we have a straight ? A straight is 5 consecutive cards, that dont all necessarily have the same suit. straight can be (2, 3, 4, 5, 6); (3, 4, 5, 6, 7); ; (10, J, Q, K, A) can start from 2 - 10, which is 9 possibilities each number in hand has 4 possibilities (suits)

Solution: $so = 9 * (4^5)$

(d) How many ways can we have a hand of all of the same suit?

Solution: $4 * \binom{13}{5}$ For each of the 4 suits, there are $\binom{}{135}$ different combinations of 5 cards among 13 to choose from

(e) How many ways can we have a straight flush? This means we have a consecutive-rank hand of the same suit. For examples, (2, 3, 4, 5, 6), all of spades is a straight flush, while (2, 3, 5, 7, 8) of all spades is NOT, as the ranks are not consecutive.

Solution: For each of 4 suits, there are 9 number combinations (as shown in c, starting from 2 to starting from 10). Each number combination is unique, because there is only one number per suit $= 4 * 9 = 36$

3. How many solutions does $x+y+z = 10$ have, if all variables must be positive integers?

Solution: We know no number can be greater than 8, because all are positive. So position: y can take on any value from $1 \to 8$, and z will just be whatever is left (y can only each of 8 values of x ($1 \to 8$), we solve $y + z = 10 - x$ Take example $x = 1$, then $y + z = 9$. Because each value ≥ 1, there are 8 solutions for this equality take on 8 values because $z \geq 1$). So we can see that in general, there are $10 - x - 1$ solutions for each value of x. so when $x = 1$, 8 solutions; when $x = 2$, 7 solutions, etc. for a total of $8 + 7 + 6 + 5 + 4 + 3 + 2 + 1$ (1 happens when $x = 8$ and y and z both must $= 1$) total $= 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 36$ solutions

Easier to think in terms of stars and bars. Bars can't be next to each other since variables are all positive integers. So $n = 10$ stars, $k = 3$ bars. Answer = $\binom{n-1}{k-1} = \binom{9}{2} = 36$

4. How many ways are there to arrange the letters of the word SUPERMAN

(a) On a straight line?

> **Solution:** 8!

(b) On a straight line, such that SUPER occurs as a substring?

> **Solution:** 4! (treat SUPER as one character)

(c) On a straight line, such that SUPER occurs as a subsequence (S U P E R appear in that order, but not necessarily next to each other)?

> **Solution:** 3! * (8 choose 3) This reduces to a stars and bars problem–the S U P E R are bars, and we want to put M A N somewhere in the sequence. Once we do so, there can be any permutation of M A N within the bars. Equivalently, we can arrange the letters of SUPERMAN (8! ways), but divide by 5! because we have arranged SUPER in any of 5! ways, when we only want one way. This gives us 8! / 5!, which is equal to 3! * 8! / (5! 3!) = 3! * (8 choose 3)

(d) On a circle?

> **Solution:** 7! Anchor one element, arrange the other 7 in a line around it

(e) On a circle, such that SUPER occurs as a substring?

> **Solution:** 3! Anchor one element, arrange the other 7 around in a line, but treat SUPER as a single character, so its arranging the other 3 around in a line

(f) On a circle, such that SUPER occurs as a subsequence (S U P E R appear in that order, but not necessarily next to each other)?

> **Solution:** 2! * (7 choose 2) Anchor one element (for simplicity, choose M, A, or N). Then following the same procedure as part c), we have 5 bars and 2 stars, where the two stars can be ordered any way.

2 Counting

2.1 Introduction

Theorem 1 : Distributing k distinguishable balls into n distinguishable boxes, with exclusion, corresponds to forming a permutation of size k, taken from a set of size n. Therefore, there are $P(n, k) = n_k = n * (n-1) * (n-2) \dots (n-k+1)$ different ways to distribute k distinguishable balls into n distinguishable boxes, with exclusion

Theorem 2 : Distributing k distinguishable balls into n distinguishable boxes, without exclusion, corresponds to forming a permutation of size k, with unrestricted repetitions, taken from a set of size n. Therefore, there are n^k different ways to distribute k distinguishable balls into n distinguishable boxes, without exclusion.

Theorem 3 : Distributing k indistinguishable balls into n distinguishable boxes, with exclusion corresponds to forming a combination of size k, taken from a set of size n. Therefore, there are $C(n, k) = \binom{n}{k}$ different ways to distribute k indistinguishable balls into n distinguishable boxes, with exclusion.

2.2 Questions

1. How many 5-digit sequences have the digits in non-decreasing order?

   Solution: This is a stars-and-bars problem with 5 slots and 9 dividers between digits, so the answer is $\binom{14}{9} = 2002$

2. How many ways can you deal 13 cards to each of 4 players so that each player gets one card of each of the 13 values (ace-2-3-. . . -king)?

   Solution: Approach this problem value by value: there are 24 ways to distribute the aces to the 4 players $4!$, 24 ways to distribute the twos, and so on, so the number of ways to deal the cards in this manner is $4!^{13} = 24^{13}$.

3. How many ways can you give 10 cookies to 4 friends if each friend gets at least 1 cookie?

Solution: Give out 4 necessary cookies, then count the number of ways to give 6 cookies to 4 friends if some can get no cookies. 6 choices from 4 options with repetition, so the number of ways is $(6+4-\binom{1}{4}-1) = \binom{9}{3} = 84$.

2.3 Extra Practice

1. In Jorge Luis Borges The Library of Babel, the narrator describes a massive library: Every book in the library has 410 pages, each page has 40 lines, and each line has 80 characters. Besides lowercase letters, the only characters appearing in the books are the period, the comma, and the space. In order to catch up with the 21st century, the Library got itself a Twitter account (@LibraryofBabel)! It appears to have been active for only a short time, but while it was running it diligently tweeted one 140-character message per day. Assume that it uses only the 26 letters of the English alphabet (plus the period, comma, and space) and that any 140-character combination is possible (e.g. asdfasdf. . . as and ,,,,,,. . . ,,,, are both perfectly valid).

   (a) How many possible tweets are there?

   Solution: 29^{140}

   (b) How many tweets use no spaces?

   Solution: 28^{140}

   (c) How many tweets consist entirely of whitespace?

   Solution: 1

   (d) Let T be some particular tweet. How many tweets differ from T by exactly one character?

   Solution: $140 * 28 = 3920$

   (e) How many have exactly six spaces and five commas?

   Solution: $\binom{140}{6} * \binom{134}{5} * 27^{129} = \frac{140!}{6!5!128!} * 27^{129}$

3 Combinatorial Proofs

3.1 Questions

1. $n! = \binom{n}{k}k!(n-k)!$

> **Solution:** Arrange n items
> LHS: number of ways to order n items
> RHS: Choose k items without ordering. Order these k items. Order the remaining $n-k$ items.

2. $\sum_{k=0}^{n} k^2 = \binom{n+1}{2} + 2\binom{n+1}{3}$

> **Solution:** Number of ordered triplets of the form (i, j, k) where i and j are less than or equal to k for every k from 0 to n
> LHS: For each k there are k options for i and k options for j so k^2 options for all.
> RHS: Consider the case where $i = j$. Then we must choose two numbers from $\{0, \ldots, n\}$ which amounts to $\binom{n+1}{2}$. If $i \neq j$ then we choose 3 numbers from $n+1$. But i can be less than j or greater than j so we must multiply by 2.

3. Prove $a(n-a)\binom{n}{a} = n(n-1)\binom{n-2}{a-1}$ by a combinatorial proof.

> **Solution:** Suppose that you have a group of n players. The left-hand side is the number of ways to pick a team of a of these players, designate one member of the team as captain, and then pick one reserve player from the remaining $n-a$ people. The right-hand side is the number of ways to pick the captain, then the reserve player, and then the other $a-1$ members of the team.

3.2 Challenge

1. Prove the Hockey Stick Theorem:

$$\sum_{t=k}^{n} \binom{t}{k} = \binom{n+1}{k+1}$$

where n, t are natural numbers and $n > t$

Solution: $\sum_{t=k}^{n} \binom{t}{k} = \binom{n+1}{k+1}$
where n, t are natural numbers and $n > t$
In the homework you were asked to show the following:
$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$
Plug this in for:
$\binom{t+1}{k+1}$
$\binom{t+1}{k+1} = \binom{t}{k} + \binom{t}{k+1} \implies \binom{t}{k} = \binom{t+1}{k+1} - \binom{t}{k+1}$
Now do the summation:
$\sum_{t=k}^{n} \binom{t}{k} = \sum_{t=k}^{n} \binom{t+1}{k+1} - \sum_{t=k}^{n} \binom{t}{k+1}$
Split apart the two summations as follows: take out the last term in the first summation and the last term in the second summation
$\sum_{t=k}^{n} \binom{t}{k} = \left(\sum_{t=k}^{n-1} \binom{t+1}{k+1} + \binom{n+1}{k+1} \right) - \left(\sum_{t=k+1}^{n} \binom{t}{k+1} + \binom{k}{k+1} \right)$
Look at all subsets of $\{1, 2, 3, \ldots, 2015\}$ that have 1000 elements. Choose the least element from each subset. Find the average of all least elements. Since $\binom{k}{k+1}$ is 0 we can just remove this term:
$\sum_{t=k}^{n} \binom{t}{k} = \binom{n+1}{k+1} + \sum_{t=k}^{n-1} \binom{t+1}{k+1} - \sum_{t=k+1}^{n} \binom{t}{k+1}$
We want the summations to match. So change t such that the new t, t, goes from $k+1$ to $n-1$. So let t = t - 1. Then $t = k+1 \to t = k$ and $t = n \to t = n-1$.
$\sum_{t=k}^{n} \binom{t}{k} = \binom{n+1}{k+1} + \sum_{t=k}^{n-1} \binom{t+1}{k+1} - \sum_{t'=k}^{n-1} \binom{t'+1}{k+1}$
Notice that the summations cancel out. We are left with the statement we were trying to prove.

4 Discrete Probability

4.1 Introduction

1. What is a sample (event, outcome) space?

   Solution: The set of all possible outcomes

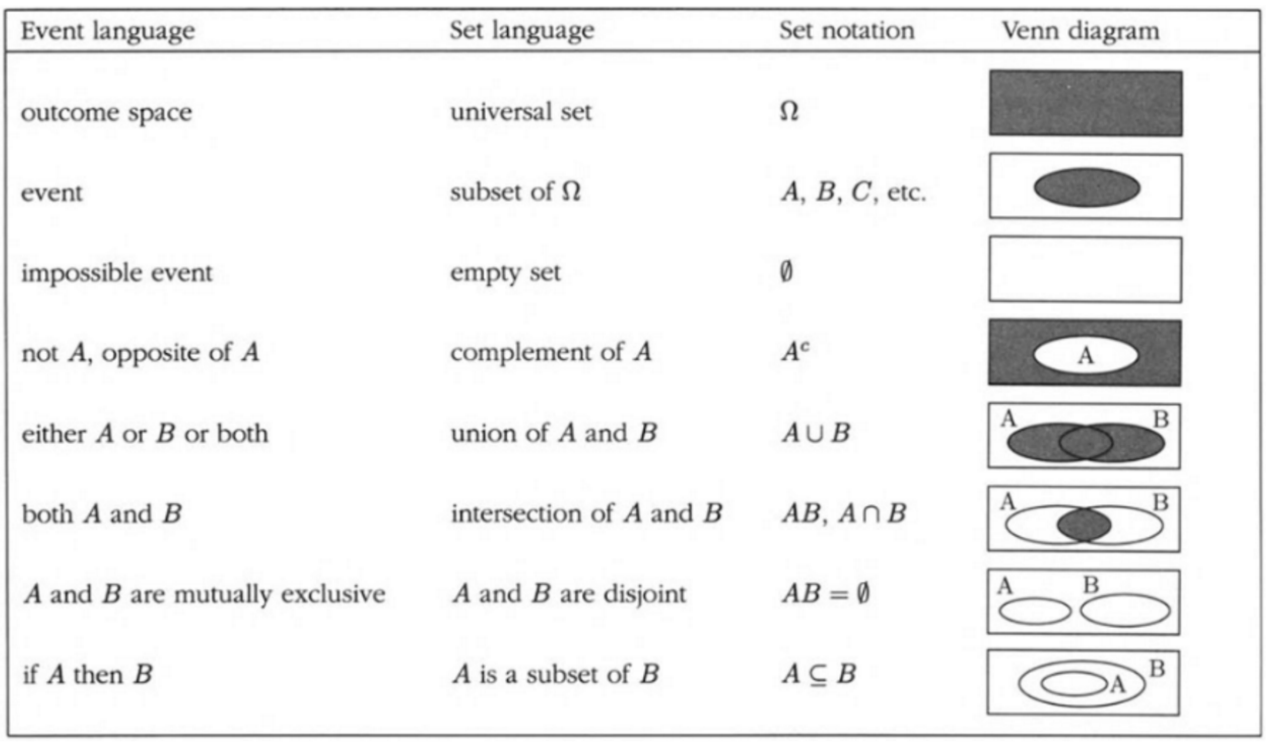
2. What is an event?

   Solution: A partition of the sample space.

3. Given a uniform probability space Ω such that $|\Omega| = N$, how many events are possible?

   Solution: 2^N. Each point is either included or excluded from any particular subset, and the number of subsets is the number of events.

Figure 1: From Pitman



Event language	Set language	Set notation	Venn diagram
outcome space	universal set	Ω	
event	subset of Ω	A, B, C, etc.	
impossible event	empty set	$\emptyset$	
not A, opposite of A	complement of A	A^c	A
either A or B or both	union of A and B	$A \cup B$	A B
both A and B	intersection of A and B	AB, $A \cap B$	A B
A and B are mutually exclusive	A and B are disjoint	$AB = \emptyset$	A B
if A then B	A is a subset of B	$A \subseteq B$	A B

4.2 Questions

1. **Probably Poker**

   (a) What is the probability of drawing a hand with a pair?

   Solution: $\frac{13*\binom{4}{2}*\binom{12}{3}*4^3}{\binom{52}{5}}$

   (b) What is the probability of drawing a hand with four of a kind?

   Solution: $\frac{13*12*4}{\binom{52}{5}}$

   (c) What is the probability of drawing a straight?

   Solution: $\frac{9*4^5}{\binom{52}{5}}$

   (d) What is the probability of drawing a hand of all of the same suit?

   Solution: $\frac{4*\binom{13}{5}}{\binom{52}{5}}$

   (e) What is the probability of drawing a straight house?

   Solution: $\frac{4*9}{\binom{52}{5}}$

2. Suppose you arrange 12 different cars in a parking lot, uniformly at random. Three of the cars are Priuses, four of the cars are Teslas, and the other five are Nissan Leaves. What is the probability that the three Prius's are all together?

   Solution: There are $12!$ possible ways to arrange the 12 cars. Now, there are $12 - 3 + 1 = 10$ different places the Priuss could go (positions 1,2,3, positions 2,3,4, all the way until positions 10,11,12). For each of those 10 places, there are 3! ways to arrange the Priuss and 9! ways to arrange the other 9 cars in the 9 remaining spots. So, in total, there are $10 * 3! * 9!$ ways of arranging the cars so that the 3 Priuss are together. So the probability we get what we want is $\frac{10*3!*9!}{12!}.0455$

5 Conditional Probability

5.1 Introduction

Bayes' Rule

$$\mathrm{P}[A|B] = \frac{\mathrm{P}[A \cap B]}{\mathrm{P}[B]}$$

Total Probability Rule

$$\mathrm{P}[B] = \mathrm{P}[A \cap B] + \mathrm{P}[\bar{A} \cap B] = \mathrm{P}[B|A] * \mathrm{P}[A] + \mathrm{P}[B|\bar{A}] * (1 - \mathrm{P}[A])$$

Independence
Two events A, B in the same probability space are independent if

$$\mathrm{P}[A \cap B] = \mathrm{P}[A] * \mathrm{P}[B]$$

5.2 Questions

1. You have a deck of 52 cards. What is the probability of:

   (a) Drawing 2 Kings with replacement?

   Solution: Uniform Probability space enumerates the $52 * 52$ combinations of two cards. There are $4 * 4$ that are 2 Kings, so the probability is $\frac{1}{13^2}$

   (b) Drawing 2 Kings without replacement?

   Solution: We need to change our uniform probability space to the $\binom{52}{2} = 52 * 51$ pairs of cards possible without replacement (with order mattering, but this is not important in this case, as long as you keep it consistent). Of these $4 * 3$ represent pairs of kings. Therefore the probability is $\frac{4*3}{52*51}$. We can also use conditional probability (which we will have to use in the next part):

   $$P(K \text{ on 2nd and } K \text{ on 1st}) = P(\text{K on 2nd}|\text{K on 1st})P(\text{K on 1st}) = \frac{3}{51} * \frac{4}{52}$$

   (c) The second card is a King without replacement?

Solution:

$$\begin{aligned} P(Kon2nd) &= P(K \text{ on 2nd } | K \text{ on 1st}) P(K \text{ on 1st}) \\ &+ P(K \text{ on 2nd } | \text{ no } K \text{ on 1st}) P(\text{no } K \text{ on 1st}) \\ &= (3/51)(4/52) + (4/51)(48/52) \\ &= (4/52) * ((48+3)/51) = 4/52 \end{aligned} \quad (1)$$

Note that this is the same as the $P(K \text{ on 1st})$, because a K is equally likely to be anywhere in the deck.

(d) **[EXTRA]** The nth card is a King without replacement ($n < 52$)?

Solution: The last problem is a hint that we can argue this by symmetry. Since we have no information about what any of the preceding cards were before it, it is equally likely that the nth card is any of the 52 possible cards, so the probability that it is a King is $\frac{4}{52}$.

2. Find an example of 3 events A, B, and C such that each pair of them are independent, but they are not mutually independent.

Solution: Consider a fair 4-sided die. Let A be the event that 1 or 2 appears in a die roll, B be the event that 1 or 3 appears, and C be the event that 1 or 4 appears.Then,

$$Pr(A) = Pr(B) = Pr(C) = \frac{1}{2}$$

Furthermore,

$$Pr(A \cap B) = Pr(1 \text{ appears}) = \frac{1}{4} = Pr(A)Pr(B).$$

So A and B are pairwise independent. Similarly (A, C) and (B, C) are pairwise independent. However,

$$Pr(A \cap B \cap C) = Pr(1 \text{ appears}) = \frac{1}{4} = Pr(A)Pr(B)Pr(C) = \frac{1}{8}$$

So these 3 events are not mutually independent.The answer is not unique; any other valid answer is acceptable.

3. A lie detector is known to be 80% reliable when the person is guilty and 95% reliable when the person is innocent. If a suspect is chosen from a group of suspects where only 1% have ever committed a crime, and the test indicates that the person is guilty, what is the probability they are innocent?

> **Solution:** Let I and G be the events that the person is innocent and guilty respectively, and let L_I and L_G be the events that the test says innocent or guilty.
>
> $$P(I|L_G) = \frac{P(L_G|I) * P(I)}{P(L_G|I) * P(I) + P(L_G|G) * P(G)} = \frac{0.05 * 0.99}{0.05 * 0.99 + 0.8 * 0.01} = 0.86$$

4. Jim and George are setting up venture capital portfolios. Suppose that Jim picks $n+1$ startups to fund and George picks n startups to fund. Suppose that the probability of any startup succeeding is $\frac{1}{2}$ and all of the startups succeed or fail independently. What is the probability that Jim picks more successful startups than George?

> **Solution:** Since Jim picks one more startup than George, it is impossible that they pick both the same number of successful startups and the same number of unsuccessful startups. So Jim picks either more successful startups than George or more unsuccessful startups than George (but not both). Since the probability of succeeding is $\frac{1}{2}$, these events are equally likely by symmetry, so both events have probability $\frac{1}{2}$.

5. Oski the bear has lost his dog in either forest A (with a priori probability 0.4) or in forest B (with a priori probability 0.6).
   On any given day, if the dog is in A and Oski spends a day searching for it in A, the conditional probability that he will find the dog that day is 0.25. Similarly, if the dog is in B and Oski spends a day looking for it there, the conditional probability that he will find the dog that day is 0.15.
   The dog cannot go from one forest to the other. Oski can search only in the daytime, and he can travel from one forest to the other only at night.

(a) In which forest should Oski look to maximize the probability he finds his dog on the first day of the search?

> **Solution:**
>
> $$P(\text{Finding In A}) = P(\text{Dog In A}) * P(\text{Search Successful}) = \frac{4}{10} * \frac{2}{8} = \frac{1}{10}$$
> $$P(\text{Finding In B}) = P(\text{Dog In B}) * P(\text{Search Successful}) = \frac{6}{10} * \frac{3}{20} = \frac{9}{100}$$
> $$\frac{1}{10} > \frac{9}{100}$$
>
> so Oski should search in **Forest A**

(b) Given that Oski looked in A on the first day but didnt find his dog, what is the probability that the dog is in A?

> **Solution:**
>
> $$P(\text{Dog In A} \mid \text{Searched A and Failed})$$
> $$= \frac{P(\text{Dog In A} \cap \text{Searched A and Failed})}{P(\text{Dog In A} \cap \text{Searched A and Failed}) + P(\text{Dog Not In A})}$$
> $$= \frac{\frac{4}{10} * \frac{3}{4}}{\frac{4}{10} * \frac{3}{4} + \frac{6}{10}} = \frac{1}{3}$$

(c) If Oski flips a fair coin to determine where to look on the first day and finds the dog on the first day, what is the probability that he looked in A?

> **Solution:** $\frac{P(\text{Looks In A}) \cap P(\text{Dog In A}) \cap P(\text{Finds Dog})}{P(\text{Finds Dog})} = \frac{\frac{1}{2} * \frac{4}{10} * \frac{1}{4}}{\frac{1}{2} * \frac{4}{10} * \frac{1}{4} + \frac{1}{2} * \frac{6}{10} * \frac{3}{20}}$

(d) If the dog is alive and not found by the Nth day of the search, it will die that evening with probability $\frac{N}{N+2}$. Oski has decided to look in A for the first two days. What is the probability that he will find a live dog for the first time on the second day?

> **Solution:**
>
> $$P(\text{Dog In A}) \cap P(\text{Find Dog First Day})^C \cap P(\text{Dog Dies})^C \cap P(\text{Finds Dog Second Day})$$

$$= \frac{4}{10} * \frac{3}{4} * \frac{2}{3} * \frac{1}{4}$$

6 Monty Hall

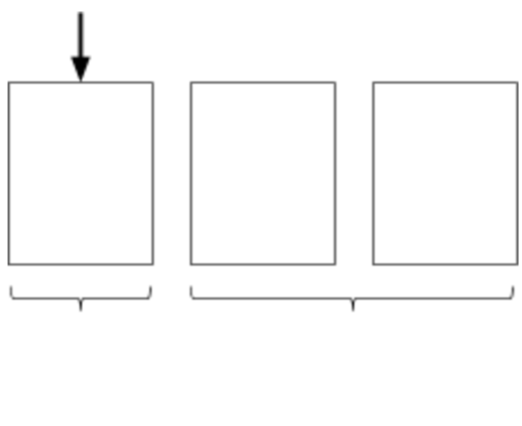
6.1 Introduction

The Problem :
Suppose a contestant is shown 3 doors. There is a car behind one of them and goats behind the rest. Then they do the following:

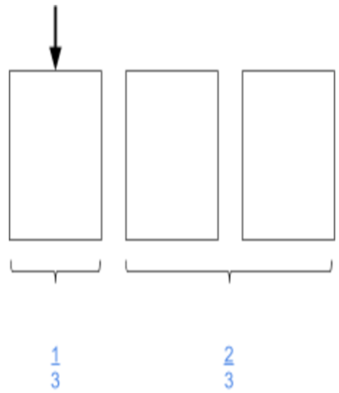
1. Contestant chooses a door.
2. Host opens a door with a goat behind it.
3. Contestant can choose to switch or stick to original choice

Is the contestant more likely to win if they switch?

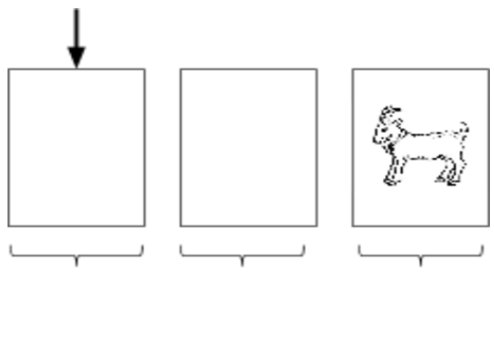
At step 1, what is the probability that the car is behind the door the contestant chose? What is the probability that the car is behind the other two doors?



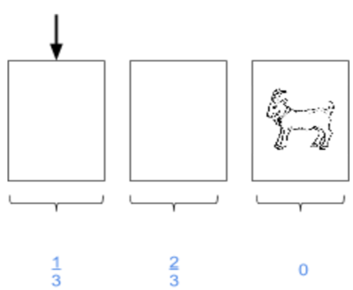
Solution:



After the host opens a door with a goat, what are the probabilities of the car being behind each door?



Solution:



6.2 Questions

1. **Grouping Doors**
   Now we have 6 doors. You pick 1 and the other 5 doors are divided into two groups:

one with 2 doors and the other with 3 doors. He removes doors until each group has 1 door left. Do you switch? What do you switch to?

Solution: Choose the door that was part of the group of three doors. When the doors were split up into three groups, those groups each had a probability of $\frac{1}{6}, \frac{2}{6},$ and $\frac{3}{6}$. These probabilities do not change when doors are removed, so three remaining doors each have probability $\frac{1}{6}, \frac{2}{6},$ and $\frac{3}{6}$.

2. **Macs and Monty**

Suppose instead of the normal Monty Hall scenario in which we have two empty doors and a car residing behind the third we have a car behind one door, a Mac behind another, and nothing behind the third.

Let us assume that the contestant makes an initial pick at his/her discretion (random) and the host proceeds to ALWAYS open the empty door. When the contestant's initial choice corresponds to the empty door, the host will say so and the contestant must switch.

Does the typical Monty Hall paradox of $\frac{2}{3}$ chance of obtaining the car by switching versus a $\frac{1}{3}$ chance of obtaining the car by staying apply in this particular case?

Solution: No. The normal Monty Hall paradox holds because when another door is opened, you learn nothing about your door, so the chance that you picked the right door from the beginning remains $\frac{1}{n}$ (which means the chance of the other door must be $\frac{n-1}{n}$). However, in this case, you may learn something about your door–you may be told that it must be wrong for example–thus shattering the paradox. These are all of the possible scenarios:

Initial Pick	Switch?	Win?
Car	Yes	Lose
Car	No	Win
Mac	Yes	Win
Mac	No	Lose
Empty	Yes (to car)	Win
Empty	Yes (to car)	Lose

Switching results in a win 50% of the time, so there is a 50% chance of winning regardless of strategy.

3. **Generalizing Monty**

Now say we have n doors and there is a car behind one of them. Monty opens k doors, where $0 \leq k \leq n-2$. Should you switch?

Solution: Yes.

$$\frac{n-1}{n} * \frac{1}{n-k-1} = \frac{1}{n} * \frac{n-1}{n-k-1} \geq \frac{1}{n}$$