QUANTIFIERS, METHODS OF PROOF

COMPUTER SCIENCE MENTORS 70

Independent review

1 Quantifiers

1. Let P(x, y) denote some proposition involving x and y. For each statement below, either prove that the statement is correct or provide a counterexample if it is false.

   a $\forall x \forall y P(x, y) \implies \forall y \forall x P(x, y)$.

   > **Solution:** hi

   b $\exists x \exists y P(x, y) \implies \exists y \exists x P(x, y)$.

   c $\forall x \exists y P(x, y) \implies \exists y \forall x P(x, y)$.

   d $\exists x \forall y P(x, y) \to \forall y \exists x P(x, y)$.

2 Methods of Proof

2.1 Contradiction and Contraposition

1. Write the contrapositive of the following statements and, if applicable, the statement in mathematical notation. (Using quantifiers, etc.)

   a If a quadrilateral is not a rectangle, then it does not have two pairs of parallel sides. (Skip mathematical notation for this problem, just write the contrapositive)

   b For all natural numbers a where a^2 is even, a is even.

   c Negate this statement: For all integers x, there exists an integer y such that $x^2+y = 16$.

2. Prove or disprove: If $P \implies Q$ and $R \implies \neg Q$, then $P \implies \neg R$.

3. For any integer x, x^2 has remainder 1 or 0 when divided by 3.

3 Induction

3.1 Questions

1. What are the three simple steps of induction?

   1. ______________________________

   2. ______________________________

   3. ______________________________

2. Prove that $\Sigma_{i=0}^{n} i! * i = (n+1)! - 1$ for $n \geq 1$ where $n \in N$.

4 More Practice

Use any method of proof to answer the following questions.

1. Let x be a positive real number. Prove that if x is irrational (i.e., not a rational number), then $\sqrt{x}$ is also irrational.

2. McDonalds sells chicken McNuggets only in 6, 9, and 20 piece packages. This means that you cannot purchase exactly 8 pieces, but can purchase 15. The Chicken McNugget Theorem states that the largest number of pieces you cannot purchase is 43. Formally state the Chicken McNugget Theorem using quantifiers.

3. Prove or disprove the following statement: If n is a positive integer such that $\frac{n}{3}$ leaves a remainder of 2, then n is not a perfect square.

4. In a large field, n people are standing so that for each person, the distances to every other person is different. At a given signal, each person fires a water pistol and hits the person who is closest to them. When n is odd, prove that there is at least one person who is left dry.