META

Graphs, Trees, Hypercubes

1 General Comments

1. Make sure to take attendance!
2. Emphasize that the worksheet this week is extremely long, it is not meant to be completely covered in 1.5 hours – they should practice the problems that you didnt get to
3. Make sure that you are comfortable with the subtle differences between a walk and path or a tour and cycle.
4. Especially emphasize #4, #5, #6 in Graph Theory
5. If you are teaching in the beginning of the week, this will most likely be their first exposure to proving things about graphs. Go slow and make sure they understand why each step is necessary and how you would think of it
6. If low on time, skip Eulerian tours
7. Have students take special note of the 4 properties of trees we list. Some good general tree advice is to have those 4 written out exactly, which makes for a much easier time coming up with proofs
8. Especially emphasize #1, #5, #6
9. #2, #3 could potentially get removed. If they do not then replace #5 wtih #2 on the required questions list
10. Skip #7 if short on time
11. You may want to take some time to prove the theorem

12. Hypercubes; M/T/W: just talk them through the definition and ask if they need anything clarified from lecture. Take a look at the lecture notes before teaching to see what they covered
13. Remember that there are two major definitions of a hypercube.
14. TH/F: they will probably be more comfortable with hypercubes. Do these 2 questions as time permits.
15. The 2 questions are nice since you can use the recursive definition to make one pretty easy and the binary definition to do the other one, driving home this point of both representations being imortant.

2 Questions

2.1 Graph Theory

1. **3 Cycle**

   For solving, it is easiest to proceed directly. When drawing out the graph on the board, draw something with 8 vertices, but have it be in two halves, 4 lined up on the left and 4 lined up on the right. Use the top left node as your example, dont draw edges for every node, that make too much clutter. The way to proceed is to show that since each node connects with 5 nodes, the top left node has to be connected to nodes from both sides. Then, take a second node, one that the first connected too, and show that there is no way to draw 5 edges without connecting to another node that the first node connected to. This alone is not the full formal solution but it should create a strong intuition.

2. **Build Up Error**

   (*) For #6, many students do not know what buildup error is, so be ready to explain that thoroughly. If you want another good example of how buildup error can happen, the chocolate bar example from week 1 works well too.

   (*) Buildup error isnt inherently wrong; correct solutions can be written by creating an instance of an n+1 case from an n case. However, this is usually a very slippery slope as doing so lends itself to overly strong assumptions and non-generalizable conclusions.

2.2 Hypercubes

1. **Introduction**

   * The first definition is a recursive one, the second one is the labeling of vertices with binary numbers

   * The recursive definition is usually easier to see, so start with that one. Draw a single vertex, pointing out that this is a 0-dimensional hypercube. Draw a second 0-dimensional hypercube and connect them. Now you have a 1-dimensional hypercube. Draw a second 1-dimensional hypercube, match up the vertices and you have a 2-dimensional hypercube, etc. Have students right out the general definition after seeing some of these–that a n-dimensional hypercube is made up of two n-1-dimensional hypercubes that are connected at each vertex. Next, bring up the binary definition of hypercubes. A good way to introduce this would be as follows: draw a 1-dimensional hypercube, label the two vertices 0 and 1. Now draw a second 1-dimensional hypercube labeled the same way.

Now, when you connect the two hypercubes, take all the labels on 1 of them, and put a 0 in front of each one, and put a 1 in front of each label on the second one. This results in a square with vertices spanning from 00 to 11. This is a good time to show that to get from one vertex to a neighbor, you only need to flip one bit. You can also point out that this kind of looks like cartesian coordinates, with 00 being next to 01 and 10, and so forth. Maybe extend this two 3-dimensional hypercube to be sure everyone follows the process, adnthat should be all youll need to do for definitions. Mention that knowing both definitions is very useful since they each have advantages when doing proofs. In general, the recursive definition works really well with inductive proofs, while the binary definition works better with direct proofs.