QUANTIFIERS, METHODS OF PROOF $\mathbf{0}$

COMPUTER SCIENCE MENTORS 70

Independent review

1   Quantifiers

1.1 Questions

1. Let $P(x, y)$ denote some proposition involving x and y. For each statement below, either prove that the statement is correct or provide a counterexample if it is false.

   a. $\forall x \forall y P(x, y) \implies \forall y \forall x P(x, y)$.

   b. $\exists x \exists y P(x, y) \implies \exists y \exists x P(x, y)$.

   c. $\forall x \exists y P(x, y) \implies \exists y \forall x P(x, y)$.

    d. $\exists x \forall y P(x, y) \rightarrow \forall y \exists x P(x, y)$.

2   Contrapositive and Contradiction

2.1   Questions

1. Write the contrapositive of the following statements and, if applicable, the statement in mathematical notation. (Using quantifiers, etc.)

    a If a quadrilateral is not a rectangle, then it does not have two pairs of parallel sides. (Skip mathematical notation for this problem, just write the contrapositive)

    b For all natural numbers a where a^2 is even, a is even.

    c Negate this statement: For all integers x, there exists an integer y such that $x^2 + y = 16$.

2. Prove or disprove: If $P \implies Q$ and $R \implies \neg Q$, then $P \implies \neg R$.

3   Proof by Cases

3.1 Questions

1. For any integer x, x^2 has remainder 1 or 0 when divided by 3.

4   Induction

4.1   Questions

1. What are the three steps of induction?

2. Prove that $\Sigma_{i=0}^{n} i * i! = (n+1)! - 1$ for $n \geq 1$ where $n \in N$.

5   More Practice

Use any method of proof to answer the following questions.

1. Let x be a positive real number. Prove that if x is irrational (i.e., not a rational number), then $\sqrt{x}$ is also irrational.

2. McDonalds sells chicken McNuggets only in 6, 9, and 20 piece packages. This means that you cannot purchase exactly 8 pieces, but can purchase 15. The Chicken Mc-Nugget Theorem states that the largest number of pieces you cannot purchase is 43. Formally state the Chicken McNugget Theorem using quantifiers.

3. Prove or disprove the following statement: If n is a positive integer such that $\frac{n}{3}$ leaves a remainder of 2, then n is not a perfect square.

4. In a large field, n people are standing so that for each person, the distances to every other person is different. At a given signal, each person fires a water pistol and hits the person who is closest to them. When n is odd, prove that there is at least one person who is left dry.