## **COMPUTER SCIENCE MENTORS 70**

Independent review

## Quantifiers

### 1.1 Questions

1. Let P(x,y) denote some proposition involving x and y. For each statement below, either prove that the statement is correct or provide a counterexample if it is false.

a. 
$$\forall x \forall y P(x, y) \implies \forall y \forall x P(x, y)$$
.

b. 
$$\exists x \exists y P(x,y) \implies \exists y \exists x P(x,y)$$
.

c. 
$$\forall x \exists y P(x,y) \implies \exists y \forall x P(x,y)$$
.

## 2 Contrapositive and Contradiction

#### 2.1 Questions

- 1. Write the contrapositive of the following statements and, if applicable, the statement in mathematical notation. (Using quantifiers, etc.)
  - a If a quadrilateral is not a rectangle, then it does not have two pairs of parallel sides. (Skip mathematical notation for this problem, just write the contrapositive)

b For all natural numbers a where  $a^2$  is even, a is even.

c Negate this statement: For all integers x, there exists an integer y such that  $x^2+y=16$ .

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2. Prove or disprove: If  $P \implies Q$  and  $R \implies \neg Q$ , then  $P \implies \neg R$ .

# 3 Proof by Cases

### 3.1 Questions

1. For any integer x,  $x^2$  has remainder 1 or 0 when divided by 3.

# 4 Induction

## 4.1 Questions

1. What are the three steps of induction?

2. Prove that  $\sum_{i=0}^{n} i * i! = (n+1)! - 1$  for  $n \ge 1$  where  $n \in N$ .

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Use any method of proof to answer the following questions.

1. Let x be a positive real number. Prove that if x is irrational (i.e., not a rational number), then  $\sqrt{x}$  is also irrational.

2. McDonalds sells chicken McNuggets only in 6, 9, and 20 piece packages. This means that you cannot purchase exactly 8 pieces, but can purchase 15. The Chicken McNugget Theorem states that the largest number of pieces you cannot purchase is 43. Formally state the Chicken McNugget Theorem using quantifiers.

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3. Prove or disprove the following statement: If n is a positive integer such that  $\frac{n}{3}$  leaves a remainder of 2, then n is not a perfect square.

4. In a large field, n people are standing so that for each person, the distances to every other person is different. At a given signal, each person fires a water pistol and hits the person who is closest to them. When n is odd, prove that there is at least one person who is left dry.