Average Variance Portfolio Management

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ABSTRACT

There's nothing very interesting here, but the format (achieved using the file `jf.sty`) makes it suitable for publication in the *Journal of Finance* even if the content doesn't. Here's a nice, informative, double-spaced abstract.

JEL classification: XXX, YYY.

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Invented in 1867[1] and in wide spread use starting in the 1870s, the stock ticker provided the first reliable means of conveying up to the minute stock prices over a long distances and market participants have been discusing the relationship between returns, risk, and portfolios for at least as long.(Rutterford and Sotiropoulos, 2016) The fundamental principal that there is a "risk-premium" such that more risky investment must generate greater returns to attack capital existed from the begining. While the fundamental idea that higher risk required higher return came before, Markowitz brought formality to the notion of risk, portfolio construction and optimization in the 1950s with modern portfolio theory and mean-variance analysis. Modern portfolio theory stated that portfolios with higher variance need to generate higher mean returns to attract rational investors.(Markowitz, 1952) However, several challanges have appeared to this foundational mean-variance claim. One of the most basic is commonly refered to as the "low-risk" or "low-volatilty" anomaly. Haugen and A. (1972) found there was little to no evidence for a "risk-premium" for increased portfolio volatility. Indeed, it was easy to construct portfolios with lower volatility which earned greater returns over a span of 20 years. A more mordern and comprehensive treatment of the low-risk issue can be found in Moreira and Muir (2017) who show, across investment strategies and asset classes that simply managing leverage in a portfolio by porfolio volatility produces greater expected returns and performance ratios. As such, these results fundamentally challenge the mean-variance notion of investment risk premium. This risk-return trade off is central to modern financial theory, so naturally I want to address this problem by making it worse. By managing the market portfolio using the average variance of the individual asset returns in the prior time period rather than the variance of the market return, I generate higher expected returns and significantly better performance ratios. Thankfully, this worse problem exposes evidence consistent with the leverage explaination of the low-risk anomaly and inconsistent with the lottery story. Decoupling the idosyncratic variance of individual returns from the variance of the market portfolio returns also sheds light on the risk-return trade-off dynamics following risk shocks. ONE MORE SENTENCE HERE (transmission of shock to AC?)

Since the identification of a low-risk anomaly, or the absence of a risk premium, a large number of researchers have sought to identify a positive relationship between return variance and expected returns. Campbell (1987), French, Schwert, and Stambaugh (1987), Glosten, Jagannathan, and Runkle (1993) and many, many others have found very limited success identifying a positive relationship between return variance and future returns. On the other hand, Moreira and Muir (2017) demonstrate that volatility managed portfolios which decrease leverage when volatility is high produce large alphas, increase Sharpe ratios, and produce large utility gains for mean-variance investors. These results hold across investment styles, e.g. value or momentum, and in different asset classes, e.g. equities and currency portfolios. The results even hold for positions which already seek to exploit the low-risk anomaly like the "betting-against-beta" strategy of Frazzini and Pedersen (2014). Additionally, other researchers have applied similiar variance or volatility management to specific assets or trading styles.[2] Practioners call approaches like this risk-parity and as of 2016 at least $150 and as much as $400 billion dollars was invested in risk-parity funds.(Steward, 2010;

Cao, 2016) The generation of greater expected returns without a equivalent increase in portfolio volatility implies that a representative investor should time asset volatility, demanding less return when its higher meaning the investor's risk appetite must be higher in periods like recession and market downturns when its expected to be lower. In short, it seems risk does not equal reward. However, some volatility is more equal than other volatility.

Pollet and Wilson (2010) decompose market variance into the average correlation between pairs of assets and the average variance of the individual assets. This yeilds a series strongly related to both future volatility and higher returns, average correlation, and a series strongly related to future volatility but unrelated to returns, average variance. Scaling investment in the market portfolio by the inverse of the previous periods average variance should then improve return performance over and above managing investment the previous market variance since it avoids, somewhat, decreasing investment when average correlation is high and higher future returns are expected. Average variance is also a better candidate for portfolio management because it has a better chance to pickup economic information sooner as individual assets respond to informtion about their own risk and expected returns as it's made public while changes in aggregate materalize as information across assets is combined.(Campbell, Lo, MacKinlay, et al., 1997; Campbell, Lettau, Malkiel, and Xu, 2001)

Prior literature on the low-risk anomaly proposes two explainations. Either investors are leverage constrained and are unable to form the positions which generate the abnormal returns or investors have a preference for the extreme right tail, "lottery", returns which are not possible when employing risk managed strategies. Asness, Frazzini, Gormsen, and Pedersen (2018) take a related approach to decomposing the betting-against-beta strategy of Frazzini and Pedersen (2014) into betting-against-correlation, BAC, and betting-against-variance factors, BAV, finding the BAC factor has a significant Fama-French five factor alpha but unrelated to behavior explainations of the low-risk anomally.(Fama and French, 2016) From the decomposition of daily market returns I find similar results about the more general low-risk strategy of volatility management. Management by either market variance or average asset variance actually increases the lottery-like returns of the market portfolio. Both strategies increase the Rachev ratios of the returns.

Using daily market returns from the Center for Research in Securities Prices, I follow Pollet and Wilson (2010) generating quarterly time series of stock market variance, SV, average correlation, AC, and average variance, AV. I then extend this by calculating the time series monthly. As expected, AV is strongly related to next period market variance and unrelated, at best insignificantly negatively, related to next period excess log returns. This relationship not only holds both in and out of sample but in both National Bureau of Economic Research (NBER) defined business cycle contractions and expansions. Out of sample forecast accuracy and encompassing tests show that AV is a superior predictor of both SV and log excess returns next period. Out of sample testing always raises questions about model specification, recursive expansion versus rolling window parameter estimation, and choices of training period and prediction window. Using Rossi and Inoue (2012) robust statistics I show that the superior performance of AV, against either the running historical mean or the current time period SV in either a rolling or expanding regression specification regardless of window choice.

The robust performance of AV translates into significant asset allocation gains. A strategy which weights investment in the market portfolio produces average annualized log excess returns of 7.8% quarterly and 8.7% monthly. The AV strategy produces quarterly and monthly Sharpe ratios, .48 and .59, each statistically significantly greater than those of the SV, volatility, management strategy. Additionally, AV generates higher values in more asymetric risk-return measures like the Sortino, Kappa, and upside potential ratios. Asset allocation performance is not all perfect, however, and the lower performance of AV in terms of Rachev ratio hints at possible explaination for the low-risk anomaly seen in volatility and average variance management.

"There is no such thing as a free lunch," is not only a wonderfully prevasive adage, particullary loved by economists, but a provable restriction on optimization problems.(Wolpert and Macready, 1997) Here, AV provides no free lunch. The improved performance measured by expected log excess returns, Sharpe, Sortino, Kappa and upside potential ratios is betrayed by worse performance in Rachev ratio. The Rachev ratio measures the right tail reward value at play relative to the left tail value at risk. In short it measures the ratio of expected lottery rewards and losses. Volatility managed market portfolio investment has higher lottery winning potential for each dollar of potenial lottery loss. Average variance management thins both tails of the return distribution but the upper tail slightly more trading the positive shift in the expected value of the log excess returns with a loss in the most extreme possible positive return values.[3] This is inconsistent with the notion that the low-risk anomaly is the result of a behavioral preference of investors for lottery-like returns.(Barberis and Huang, 2008; Brunnermeier, Gollier, and Parker, 2007) Rather, the generation of higher log excess returns by the AV, SV, and betting-against-beta strategies support the notion that the low-risk anomaly arises from leverage constraints first suggested in Jensen, Black, and Scholes (1972). Boguth and Simutin (2018) link capital constraints and the betting-against-beta strategy through mutual fund betas, and more generally Malkhozov, Mueller, Vedolin, and Venter (2017) show that international illiquidity predicts betting-against-beta returns.

Some change in the risk-return dynamic through average variance management was anticipated given the relationship between AV and next period SV and the lack of relationship between AV and next period log excess returns. Given that idosyncratic events are reflected in individual asset volatility first we should also be able to explain the better performance of AV in terms of timing and response. Using impulse-response functions from vector auto-regression (VAR) analysis its clear that AV responds more quickly and too a larger degree to shocks than SV. A one standard deviation positive shock to AV remains significant for at least five months. Consistent with the lack of relationship show in the regression analysis the response of excess log returns to the shock in AV is negative but insignificant in the first month turning and staying positive, but in significant there after. In response, AV driven investment weight drops the first month by more than 11%. It recovers sharply in the second month but then slowly drifts back to its initial level over the course of the next 24 months. Similar to AV, a one standard deviation shock to SV remains significant for at least five months. Excess log return reaction is also similiar to the reaction to AV but less pronounced, again consistent with the return regression results. Investment weight reaction is different however, the SV strategy sheds up to 22% of its investment in the market before recovering. This leaves the SV strategy further out of the market during the recovery to the shock than the AV strategy which results in lost returns.

By using the decomposition of market variance I am able to identify a better portfolio leverage management signal. Weighting investment leverage by the inverse of the average of individual asset variance rather than overall return variance investors are able to capture better performance as measured by expected annualized returns. Investors are also rewarded with better investment ratios with the exception of the Rachev ratio. This exposes the trade-off which investors must accept when managing risk by manipulating leverage conditional on average variance. This contributes evidence against the lottery explaination of the low-risk anomaly in mean-variance analysis and in support of the leverage constraints explaination. By scarificing some potential extremely positive returns, investors using AV have access that better responds to risk shocks and participates more in the subsequent recovery. This supports prior literature which argues that individual assets should signal changes due to economic events before aggregate signals. All of this, of course, depends on the data and the construction of the individual and aggreagate signals.

I. Data

To calculate stock market variance, average asset variance and average asset correlation, I use daily return data from CRSP. Starting second quarter of 1926[4] the variance of returns to the CRSP market portfolio is caluclated monthly and quarterly. Where t is the number of trading days in the period, month or quarter, the realized stock market variance is

$$SV_t = \frac{1}{t-1}\sum_{\tau=1}^{t}\left(R_\tau^m - \frac{\sum_{\tau=1}^{t} R_\tau^m}{t}\right)^2 \tag{1}$$

where R_t^m is the return on the CRSP market portfolio at time t. To simplify the analysis of individual assets I require that the asset be traded on each day in the time period begin analyzed, the month or quarter. This mitigates any liquidity affects and insures consistent variance, covariance and correlation calculations. This makes the calculation of individual asset variance as straight forward as the calculation of the variance of the market portfolio.

$$\sigma_{a,t}^2 = \frac{1}{t-1}\sum_{\tau=1}^{t}\left(R_\tau^a - \frac{\sum_{\tau=1}^{t} R_\tau^a}{t}\right)^2. \tag{2}$$

where R_t^a is the return, including dividends, on asset a at time t. With 63 to 66 trading days in a typical period, quarterly pair-wise asset correlation is calculated using the standard Pearson's correlation where the correlation of assets a and b is

$$\rho_t^{a,b} = \frac{\sum_{\tau=1}^{t}\left(R_\tau^a - \frac{\sum_{\tau=1}^{t} R_\tau^a}{t}\right)\left(R_\tau^b - \frac{\sum_{\tau=1}^{t} R_\tau^b}{t}\right)}{\sqrt{\left(R_\tau^a - \frac{\sum_{\tau=1}^{t} R_\tau^a}{t}\right)^2 \sum_{\tau=1}^{t}\left(R_\tau^b - \frac{\sum_{\tau=1}^{t} R_\tau^b}{t}\right)^2}}. \tag{3}$$

Unfortunately, for samples as small as the monthly series of daily returns Pearson's correlation is not an unbiased estimator of the true correlation, even if the returns are normal.Hotelling (1953) The average month in my sample has 22 trading days however the number commonly drops into the teens during the later part of the year.[5] For samples of these sizes the bias causes an underestimation of the correlation which is worse the lower the true correlation is between the two assets. As a parital correction, I employ an approximate correction from Olkin and Pratt (1958) such that the monthly correlation between two assets a and b is

$$\rho_t^{a,b} = \widehat{\rho_t^{a,b}}\left(1 + \frac{1 + \widehat{\rho^{a,b}}^2}{2(t-3)}\right) \tag{4}$$

where $\widehat{\rho_t^{a,b}}$ is the Pearson correlation between a and b.[6] Average variance and average correlation are value-weighted so each month so I calculate market capitalization for all of the assets available in CRSP. The capitalization used in month t for asset a is the product of the end of month price (PRC) and common shares outstanding (SHROUT) values for asset a in month t-1.

$$MCAP_t^a = PRC_{t-1}^a \times SHROUT_{t-1}^a \tag{5}$$

To make the analysis more computationally trackable I use only, at most, the top 500 assets in CRSP by market capitalization for a given month. There are 500 assets available which trade every trading day in a given month consistently starting in October of 1926.[7] Given this restriction an assets market capitalization weight is defined by

$$w_t^a = \frac{MCAP_t^a}{\sum_{n=1}^{N} MCAP_t^n} \tag{6}$$

with n $\leq$ 500. An assets quarterly market capitalization values are just the mean value of the monthly market capitalization which make up the quarter and the quarterly weight is calculated from the top assets trading all quarter in the same way as monthly.[8] Thus, the two other series of interest average variance, AV, and average correlation, AC, are defined by

$$AV_t = \sum_{n=1}^{N} w_t^n \sigma_{n,t}^2 \tag{7}$$

$$AC_t = \sum_{n=1}^{N} \sum_{m \neq n}^{N} w_t^n w_t^m \rho_t^{n,m} \tag{8}$$

Figure 1 shows the time series behavior of market and average variance, in percent, as well as average correlation. With the easily noticable exception of October 1987, spikes in both average market and average variance are concetrated around NBER defined recessions.

[Place Figure 1 about here]

Table I shows the summary statistics for the calculated variables. Dispite the use of the actual

number of time period trading days, versus the use of the average of 22, and the restriction to assets that trade every trading day, the calculated values are almost identical to those in Pollet and Wilson (2010) over the same sample. Expanding the time period, average variance has a mean value of 2.53% quarterly and .64% monthly. The stock market variance numbers are much lower at .74% and .25% a quarter and month respectively. Average correlation is remarkablely consistent at .23 in the Pollet and Wilson (2010) sample, .282 quarterly and .276 monthly in the full sample. Average variance is more volatile than stock market variance, more than twice as much quarterly. In each sample average variance has the strongest autocorrelation. While average correlation is also persistant, stock market variance is only strongly persistant at the monthly frequency with an autocorrelation of .61. All three time series are stationary rejecting the unit root null in the tests of Dickey and Fuller (1979), Ng and Perron (2001), and Elliott, Rothenberg, and Stock (1996).

[Place Table I about here]

As my primary interest is in the use of average variance versus market variance in the management of leverage in the CRSP market portfolio, I test AV and SV against CRSP log excess returns. Specifically, I take the difference between the natural log of one plus the CRSP market return[9] and the natural log of one plus the risk free rate using

$$r_t = \log R_t^m - \log R_t^f \tag{9}$$

where R_t^f is the return on the 3-month or 1-month treasury bill. For the quarterly frequency I take the return on the 3-month treasury bill from FRED and the return on the 1-month treasury bill for the monthly frequency, for simplicity I get this from Ken French's website. All analysis, as in the prior literature, starts in July 1963.[10] For in and out of sample market and average variance, and average correlation are regressed against these excess log return values. Out of sample regressions require an "in sample" training period which is set at 15% of the available time seires for consistent calculation of robust out of sample statistics later in the analysis. This means that out of sample regressions, asset allocation results and VAR analyses all start in the fourth quarter of 1948 when the analysis is quarterly and January 1949 when its monthly.

II. Regression Analysis

A. In Sample

In order to get an understanding of the relationship between stock market or average variance and returns, I begin with in sample regressions. In each of these regressions all of the information available in the sample is used to estimate the parameters. In general the regressions take this form

$$y_{t+h} = \alpha + \beta x_t + \epsilon_t. \tag{10}$$

Both monthly and quarterly analysis is done with non-overlaping time periods which effectily sets h equal to 1 in both. In the decomposition of current market variance, it will be the y variable and each of average correlation, average variance, and both AC and AV will serve as x. In the prediction of future market variance, SV_{t+1} will be the y variable and the x variables tested will include current SV as well as SV + AV in addition to the x variables tested in the variance decomposition regressions. In the prediction of next period returns, r_{t+1} will be the y variable tested against the same x variables as in the predition of future market variance.

Table **??** shows a replication of the in sample variance decomposition found in Pollet and Wilson (2010) table 2. The cotemporanious relationships between average correlation, average variance and stock market variance are the same, indeed the calculated values are almost identical. This confirms that average variance is a significant determinant of stock market variance and a significant measure of risk in the mean-variance sense. This verifies that stock market variance, SV, can be decomposed into average correlation, AC, and average variance, AV, but does not indicate any relationship to subsequent realized stock market variance, SV_{t+1} or future returns, r_{t+1}. As such, the extension of this analysis to the full sample, either quarterly or monthly, is skipped.

The effectiveness of either market variance or average variance as an investment management signal will be driven primarily by their relationship with future risk and return. Its the trade-off which is key to the leverage management strategy. Assuming that investors hold a portfolio with whose risk-return ratio they are indifferent. When risk increases but expected returns do not, the risk-return ratio become more unattractive and any risk averse investor would like to decrease there position. In table **??**, panel A is a replication of the relationships of current quarter stock market variance, average variance and average correlation and next quarter stock market variance. AV is a significant predictor of SV next period. This holds even when current period SV is included in the regression. In the full samples, average variance accounts for 38% and 39% of the variation in next periods market variance. This is even greater than the amount expalained by this periods market variance. Table **??** shows that with full information average variance is the dominant predictor of future market variance. When predicting next period market variance, it generates higher R^2 and t-statistics in horse races and remains significant when current market variance is included in all samples.

Table **??** panel A shows the results of my replication of table 3, panel A, from Pollet and Wilson (2010). Confirming their results, AC is a strong predictor of next quarters excess log returns, but neither AV nor SV are predictive. When appearing alone,in the full quarterly sample the coefficient on AV is negative, -.371, but insignificant so decreasing investment in the market portfolio does not mean a decrease in expected future returns if anything decreasing investment means avoiding approaching losses. However, when controlling for market variance, the coefficient on AV is not only still negative but significant. This supports evidence in Pollet and Wilson (2010) suggesting that when controlling for average correlation an increase in average variance is a negative economic signal representing risk that investors are not compensated for taking. In the full monthly sample, AV is a significantly negative predictor of next month's excess log return with a coefficent of . In the full quarterly and monthly samples SV behaves like AV with smaller coefficients and t-statics indicating a weaker relationship to future returns than AV. In either case investors are unlikely to be punished for pulling back on investment given higher values of AV or SV so long as the relationships hold with the limited information that investors have available at the time they make investment decisions.

B. Out of Sample

While the in sample dominance of AV is clear, the out of sample performance remains in doubt. As Welch and Goyal (2008) definitively show, out of sample performance is not guaranteed by in sample performance and is essential to any investment strategy which hopes to generate positive returns. To determine the out of sample relationships between market and average variance, average correlation and returns, I run regressions of the standard form

$$y_{t+1} = \alpha_t + \beta_t x_t + \epsilon_t \tag{11}$$

where α_t and β_t are estimated with from the data available only until time t. That is, I estimate α_t and β_t by regressing $\{y_{s+1}\}_{s=1}^{t-1}$ on a constant and $\{x_s\}_{s=1}^{t-1}$. In all the reported results, I follow an expanding window approach so that for the next period $t+2$, y_{t+2} is estimated as $\alpha_{t+1} + \beta_{t+1}x_{t+1}$, where α_{t+1} and β_{t+1} by regressing $\{y_{s+1}\}_{s=1}^{t}$ on a constant and $\{x_s\}_{s=1}^{t}$. I follow this process for all subsequent months. However, as part of a test on the robustness of the out of sample results, I demonstrate that the results do not depend on the use of an expanding window. Most critically, equation (11) prevents any look-ahead bias. The out of sample prediction tests use the same set of variables as the in sample tests. Each out of sample test requires an "in sample" training period in which parameters are estimated using all the data up to the time period before the first out of sample quarter or month.

For consistency, the first one-fourth of data, either quarterly or monthly, is used as the initial parameter estimation period with the remaining three-fourths of observations moved through recursively generating out of sample predictions. Four measures of out of sample performance are estimated.[11] I use the R^2_{oos} statistic Campbell and Thompson (2008), the mean squared error F-statistic of Clark and McCracken (2001) to evaluate the accuracy of the out-of-sample predictions.

R^2_{oos} is defined as

$$R^2_{oos} = 1 - \frac{MSFE_x}{MSFE_b} \tag{12}$$

where $MSFE_x$ is the mean squared forecast error when the variable x is used to generate out-of-sample predictions. An $R^2_{OS} > 0$ suggests that $MSFE$ based on variable x is less than that based on the benchmark, b. We evaluate the statistical significance of R^2_{OS} using Clark and West (2007) statistic. This statistic tests the null hypothesis that $H_0 : R^2_{OS} \leq 0$ against the alternative $H_A : R^2_{OS} > 0$. $MSFE_b$ is mean squared forecast error when a benchmark model is used to generate out-of-sample predictions. Mean squared forecast error is defined as

$$MSFE_x = \frac{1}{T}\sum_{\tau=t}^{T}(y_\tau - \hat{y}^x_\tau)^2 \tag{13}$$

where $\hat{y}^x_t$ is the out of sample prediction of y_t generated from the a model using variable x, t is the first out of sample prediction time period, and T is the total number of out of sample time periods. The F-statistic in McCracken (2007) is calculated by

$$MSE - F = T\frac{MSFE_x - MSFE_b}{MSFE_b}. \tag{14}$$

The significance of the F-statistic is determined from bootstrapped values provided in McCracken (2007). Each of these two tests depends on the reduction of average squared error by the predictor x relative to a benchmark model. The next two measures used are forecast encompassing statistics.

Encompassing tests the more stringent requirement that the benchmark forecasts contain no useful information absent in the forecasts of variable x. Forecast encompassing tests come from the literature on optimal forecast combination. (Chong and Hendry, 1986; Fair and Shiller, 1990) An optimal forecast as a convex combination of two forecasts for time period t +1 defined as

$$\hat{y}^*_t = (1-\lambda)\hat{y}^b_t + \lambda\hat{y}^x_t \tag{15}$$

where $\hat{y}^x_t$ are predicted values generated from the model using variable x and $\hat{y}^b_t$ are forecasts from the benchmark model. I use the forecast encompassing tests of Harvey, Leybourne, and Newbold (1998), ENC-HLN, and the ENC-NEW statistic of Clark and McCracken (2001). The ENC-NEW statistic is defined as

$$ENC - NEW = T\frac{\sum_{\tau=t}^{T}\left[(y_\tau - \hat{y}^b_\tau)^2 - (y_\tau - \hat{y}^b_\tau)(y_\tau - \hat{y}^x_\tau)\right]}{\sum_{\tau=t}^{T}(y_\tau - \hat{y}^x_\tau)^2}. \tag{16}$$

The significance of the ENC-NEW is determined from bootstrapped values provided in the same paper. The encompassing test of Harvey et al. (1998) directly tests the value and significance of the forecast combination λ. The test procedure rests on the calculation of a modification to the Diebold and Mariano (1995) test statistic and the consistent estimation of the long-run covariance between the difference in forecast error between the benchmark model and a model based on a competing variable, x. As such there is no one line equation that sums up the statistic used to judge the significance of λ.

Table **??** shows the results of the out of sample tests. Panel A contains the results of running out of sample expanding window regression using AV as the predictor versus using only a constant. The use of only a constant in the rolling regression is effectively using the running historical mean as a benchmark in the tests of out of sample performance. It is clear that AV is a significant improvement in the prediction of next period market variance. It generates postive and significant R^2_{oos} values in all samples. The values generated for MSE-F and ENC-NEW are very large and statistically significant; and the near 1 λ values for the ENC-HLN tests, in all samples, indicate that AV provides all the information available in the historical mean. Its also clear that AV is not a predictor of future returns. R^2_{oos} values are negative or insignificant indicating. Some positive and significant values appear in the MSE-F, ENC-NEW and ENC-HLN tests at the monthly level which is the result of how poor a predictor of monthly returns is the historical mean. Particularly around market downturns use of the historical mean generates massive prediction errors.

Panel B shows the results of running the same test with forecasts generated by SV as the benchmark model. The results for the same but the magnitudes are very different. AV is a significantly better predictor of next period's market variance in all samples. However, the performance over and above SV is much more muted than in the historical mean in panel A. At the monthly frequency, the frequency used later for asset allocation, AV is a significant improvement over SV as indicated by the positive R^2_{oos}, MSE-F, and ENC-NEW values. However, the ENC-HLN λ value is on .56. While this is statistically significant, it indicates that for the best out of sample prediction of monthly stock market variance both AV and SV should be included and at nearly equal weights. The results for the prediction of future excess log returns are even, slightly, better than those in panel A. This is again owning as much to the terrible performance of the benchmark, here SV, as to any positive performance of AV.

B.1. Robustness

Out of sample estimation always raises issues with the choices made in the specification of the model and how to split the data into in and out of sample windows. Bluntly speaking, there are no good answers. The standard practice as in Rapach and Zhou (2013),Rapach, Strauss, and Zhou (2010), Rapach, Ringgenberg, and Zhou (2016), and Huang, Jiang, Tu, and Zhou (2015), and many others, is to show performance in a few subsamples split by dates that the authors choose for unknown reasons. The concerns with subsample selection are that the window may either be "ad-hoc" and the selection may mask significant results that would appear if the subsamples had been constructed differently. A second, more cynical, concern is that the presented subsample represent significant performance that has been found either by chance or as the result of analyzing many subsample and only presenting the significant results. In any case, evaluation of the differences in performance across subsamples is often left to the imagination of the reader and whatever importance they place on the first half of the sample versus the second, the middle third versus the first and last thirds or however the data has been separated.

To avoid these issues first I present subsample results splitting the out of sample prediction window between NBER defined business cycle contractions and expansion. The purpose of this is to contrast the performance of the AV and SV based forecasts when returns are generally expected to be positive and when they are negative. These are particularly meaningful subsamples for AV, AC and SV. As Forbes and Rigobon (2002), Hartmand, Straetmans, and de Vries (2004), and Ang and Chen (2002) and many others have demonstrated the correlation between equity returns increases during crises and market downturns and decreases during upturns. As average correlation, average variance and market variance are all related by definition, contractions are likely to affect the predictive performance of AV and SV differently than expansions.

Panel A of table **??**

This indicates that average correlation is a better performing predictor in times of crises as suggested by the its best performance in Pollet and Wilson (2010) in subsamples dominated by recessions.[12]

To further address the robustness of the out of sample results and avoid the use of subsampling completely, I present encompassing statistics robust to both the specification of the prediction model, either expanding or rolling, and the choice of prediction window. Rossi and Inoue (2012) presents out of sample statistics robust to the choice of split between in and out of sample periods.

III. Preferences

general: clearly AV provides a higher return than either SV or the buy and hold strategy for the same level of risk. This return increases with both investment constraint and risk aversion. This presents a low-risk anomaly like puzzle. Why are higher risks not being compensated for with higher returns. Prior literature presents two possible explainations, lottery and leverage. The lottery explaination relies on investors getting utility, outside of return, from the lottery-like nature of the investment. This bulstered by work of CITE1 and CITE2.

When there are no externalities at work the return of the AV portfolio is a direct reflection of its market beta with is simply the investment weight, leverage in the market. However, as an externality is incorporated the return changes. When the externality is the disutility of leverage costs the return on the levered portfolio decreases. In CITE(borrowing and CML paper) the externality cause the CML line to flatten and the return to AV should decrease. When the externality is positive, leverage excitment, then the CML steepens as the return to AV increases by more than the function of the increase in variance with an additional term multiplying the increased market holding by the utility of holding each of the lottery-like assets.

The average variance managed portfolio produces significantly higher returns than the buy and hold market portfolio for the same level of total variance. As long as investors have access to either portfolio, there must be another investor untility generating aspect to the holding the market portfolio in order for this risk-return relationship to not violate the modern portfolio theory. A reasonable explaination can be taken from the literature on lottery stocks and lottery-like investments. Investors could simply be passing on the higher returns offered by either the average variance or volatility managed portfolio for the buy and hold because the buy and hold offers more lottery-like payoffs.

Assuming the market investment is prefered in times of high investor lottery preference. Barberis and Huang (2008) make an extensive study of the market equilibrium implications of investor preferences for lottery stocks, specifically the preference for skewness in returns. The main findings relevent to test this explaination in the context of average variance or volatility managed portfolios are the their findings that the market equilibrium which emerges is not homogeneous, the lottery stocks are overvalued and provide lower future returns, and that mean-variance investors without lottery prefrences are not able to arbitrage away the lottery misspricing if shorting is costly. As such, the value-weighted market protfolio which emerges in conditions of high market lottery preference will have a lower return than the CAPM mean-variance optimal market portfolio would. The result is a lower Sharpe ratio for the market portfolio and a flatter capital market line. The intuitive effect of this is illustrated in **??**; a flatter capital market line means a smaller difference in excess return between the market portfolio and the managed portfolio. This effect can be measured econometrically.

The other possible explaination for the higher returns of the managed portfolios is that investors may not be able to take the levered position necessary to construct those portfolios. When the managed portfolio requires an investor to borrow to take larger asset positions the investor is subject to the credit conditions of the market. When the investor's broker can meet this demand the investor is subject to the broker's conditions, e.g. interest rates. The leverage constraint explaination goes back to Black (1972), who shows theoretically that borrowing restrictions of the type represented by higher interest rates flatten the capital market line. The same theoretical framework is in Frazzini and Pedersen (2014). Jylh (2018) uses changes in margin requirements to identify the flattening of the captial market line to increases in borrowing constraints. As lending conditions tighten, and the cost of leverage increases, the returns to the managed portfolio are closer to the market return, as illustrated in **??**.

In either the case of lottery preference or leverage constraints the effect to be measured econometrically is the factor flattening the capital market line. Despite each moving the market from one equilbirium to another, this effect can be measured by the interaction term of the market return and a proxy for the factor being tested. This is analogus to what was demonstrated by

We have some reason to suspect leverage already, the constraints drive returns toward BH and some reason to doubt lottery Rachev.

However, the fundamental argument that investors prefer the buy and hold market over the average variance or volatility managed investment because the market is more lottery-like. This itself is unclear.

Bali, Cakici, and Whitelaw (2011) show that the maximum daily return over the past one month, MAX, is a good measure of the lottery-like payoffs of a stock and a significant indicator of lower future returns robust to size, book-to-market, momentum, short-term reversals, liquidity, and skewness. This means that for lottery seeking investors to prefer the buy and hold market its MAX measures must be significantly different from the average variance and volatility managed portfolios. Yet, as seen in table **??** the mean and median values of the highest one day returns, MAX1, and the average of the five highest daily returns within the month, MAX5, are higher for the average variance managed portfolio than either volatility or the buy and hold market portfolio. Using daily return values scaled by the prior months portfolio volatility, as in Asness et al. (2018), the volatility managed portfolio is the most lottery like with the highest mean and median values of scaled MAX1, SMAX1, and scaled MAX5, SMAX5. Notably, the average variance managed portfolio still has higher mean SMAX1 and SMAX5 values than the buy and hold portfolio. This does not mean that the buy and hold portfolio is not viewed as a lottery and investors do not take some additional utility from holding it, however it seems very unlikely that this is even greater than the lottery utility provided by the average variance or volatility managed portfolio let alone large enough to compensate for the difference in return or CER gain.

It remains possible that on some other yet unknown measure the buy and hold strategy is more lottery like. To form a more direct test of the lottery preference explaination

To justify adding a subsection here, from now on, we'll assume

CONDITION 1: $0 < \hat{\mu} < \gamma\sigma^2$.

This condition might be useful if there was a model.

.1. A Subsubsection with a Proposition

Let's put a proposition here.

PROPOSITION 1: *If Condition 1 is satisfied, a solution to the central planner's problem,* $V(B, D, t) \in C^2\left(\mathbb{R}_+^2 \times [0, T]\right)$*, with control* $a : [0, 1] \times [0, T] \to [-\lambda, \lambda]$ *if* $\gamma > 1$ *is*

$$V(B, D, t) = -\frac{(B + D)^{1-\gamma}}{1 - \gamma} w\left(\frac{B}{B + D}, t\right). \tag{17}$$

Appendix A. An Appendix

Here's an appendix with an equation. Note that equation numbering is quite different in appendices and that the JF wants the word "Appendix" to appear before the letter in the appendix title. This is all handled in `jf.sty`.

$$E = mc^2. \tag{A1}$$

Appendix B. Another Appendix

Here's another appendix with an equation.

$$E = mc^2. \tag{B1}$$

Note that this is quite similar to Equation (A1) in Appendix A.

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Notes

[1]Invented by Edward A. Calahan, an employee of the American Telegraph Company.

[2]See, for example, Barroso and Santa-Clara (2015) and Kim, Tse, and Wald (2016) for discussions of volatility managment of the momentum portfolio.

[3]Both SV and AV generate better Rachev ratios than the buy and hold return.

[4]The monthly sample starts in July 1926.

[5]The shortest trading month in the sample is September 2001 with 15 trading days while 17 is a common number in the holiday months.

[6]The exact correction suggested in Olkin and Pratt (1958) is too computationally taxing for the equiptment to which I have access.

[7]The least number of assets in the top 500 which trade every day in a given month is 392 in August of 1932

[8]There are 500 assets trading everyday quarterly starting in the first quarter of 1928.

[9]CRSP monthly returns are compounded into quarterly returns.

[10]CRSP has, as of 2005, backfilled NYSE daily returns to 1926, however the pre-1962 data is very different from the post-1962 data. The earlier data is much shallower having months with fewer than 400 assets total that meet the data requirements. Twice as many firms covering twice as many industries are available at the end of 1962 as compared to the end of 1961. And, as documented in Jones (2002) the pre-1962 period is significantly and persistantly more illiquid.

[11]Additionally, in unreported results, I find the same results if we use the Diebold and Mariano (1995) test.

[12]Average correlation predicts excess returns best in the 1974 to 1985 and 1996 to 2007 subsamples. These 80 quarters contain all of the 18 contraction quarters from 1974 to 2007. Average correlation is insignificant in the 1986 to 1995 subsample.

Figure 1. Time Series of Market Statistics: The average variance of individual asset, average correlation of asset pairs, and the variance of the CRSP portfolio calculated from daily returns.

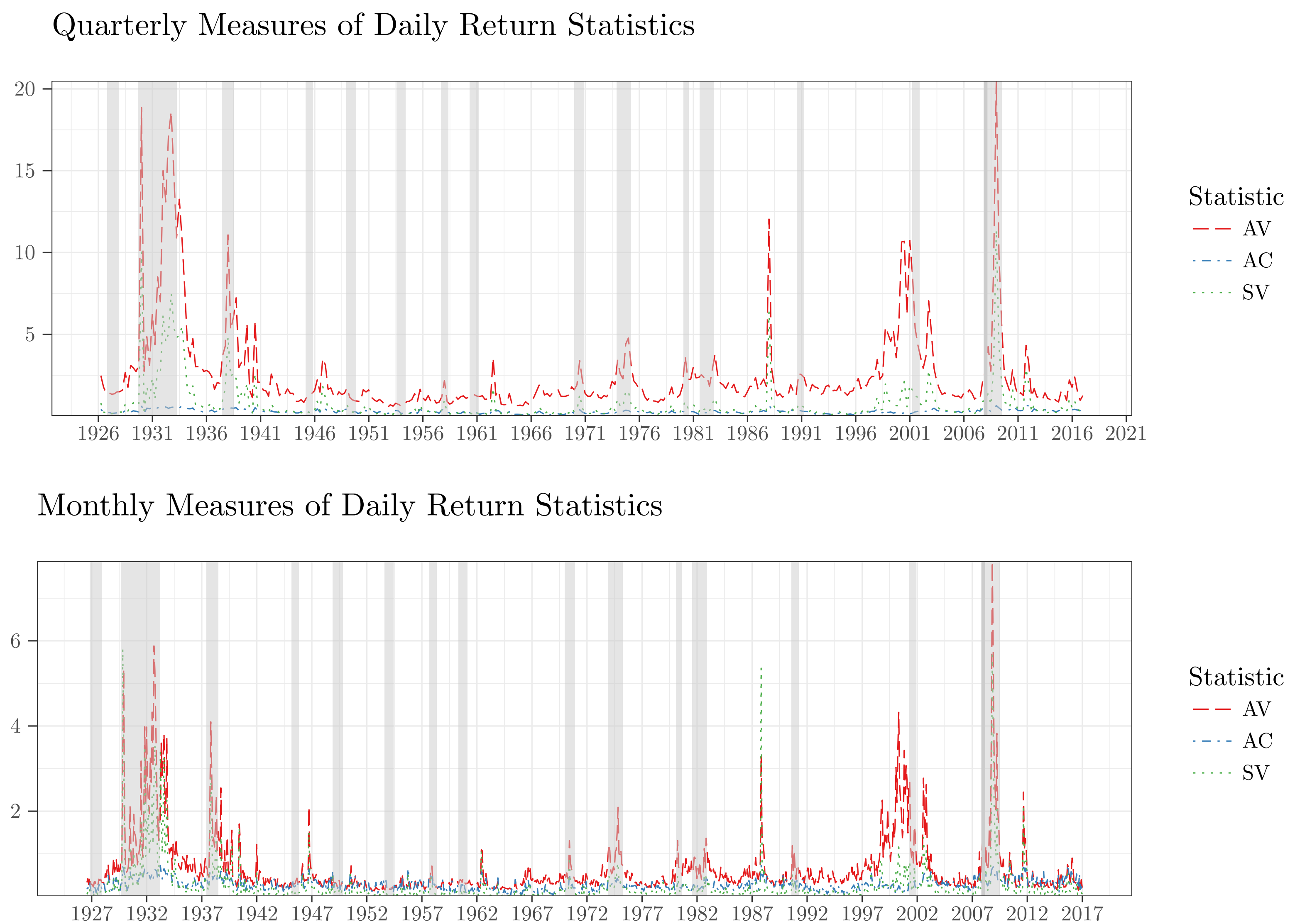

Table I Summary Statistics

Pollet and Wilson Sample 1963Q1:2006Q4

Statistic	N	Mean	St. Dev.	Min	Max	Autocorrelation
RET	176	1.163	8.369	−30.072	19.956	0.000
AC	176	0.230	0.090	0.034	0.648	0.572
AV	176	2.218	1.828	0.634	12.044	0.696
SV	176	0.483	0.616	0.029	6.397	0.311

Quarterly 1926Q1:2016Q4

Statistic	N	Mean	St. Dev.	Min	Max	Autocorrelation
RET	331	1.694	8.591	−30.072	30.956	0.097
AC	364	0.282	0.121	0.034	0.678	0.578
AV	364	2.533	2.839	0.539	20.485	0.668
SV	364	0.741	1.258	0.029	11.378	0.469

Monthly 1962M6:2016M12

Statistic	N	Mean	St. Dev.	Min	Max	Autocorrelation
RET	655	0.410	4.460	−26.134	14.814	0.081
AC	655	0.261	0.129	0.019	0.762	0.620
AV	655	0.770	0.849	0.198	10.416	0.667
SV	655	0.200	0.406	0.006	5.664	0.551

Monthly 1926M7:2016M12

Statistic	N	Mean	St. Dev.	Min	Max	Autocorrelation
RET	1,085	0.495	5.371	−34.523	33.188	0.106
AC	1,085	0.276	0.134	0.019	0.762	0.610
AV	1,085	0.881	1.281	0.154	19.540	0.718
SV	1,085	0.248	0.502	0.006	5.808	0.612

Table II

	Dependent variable:			
	SV			
	(1)	(2)	(3)	
AC	0.042***		0.026***	
	(0.004)		(0.002)	
AV		0.281***	0.239***	
		(0.014)	(0.011)	
AC * AV				
Constant	−0.005***	−0.001***	−0.007***	
	(0.001)	(0.0004)	(0.001)	
Observations	176	176	176	
R^2	0.379	0.697	0.831	
Adjusted R^2	0.375	0.695	0.829	
Residual Std. Error	0.005 (df = 174)	0.003 (df = 174)	0.003 (df = 173)	0.0
F Statistic	106.073*** (df = 1; 174)	400.713*** (df = 1; 174)	426.371*** (df = 2; 173)	9,473.9
Note:				*p<0.1; **

Table III

	Dependent variable:			
	SV_{t+1}			
	(1)	(2)	(3)	(4
AC_t	0.014***		0.005	
	(0.005)		(0.005)	
AV_t		0.144***	0.136***	
		(0.023)	(0.024)	
SV_t				0.31
				(0.0
Constant	0.002	0.002**	0.001	0.00
	(0.001)	(0.001)	(0.001)	(0.0
Observations	176	176	176	17
R^2	0.042	0.184	0.189	0.0
Adjusted R^2	0.037	0.179	0.179	0.0
Residual Std. Error	0.006 (df = 174)	0.006 (df = 174)	0.006 (df = 173)	0.006 (df
F Statistic	7.643*** (df = 1; 174)	39.124*** (df = 1; 174)	20.109*** (df = 2; 173)	18.577*** (d

Note:

	RET$_{t+1}$				
AC$_t$	0.215***		0.248***		
	(0.068)		(0.072)		
AV$_t$		−0.116	−0.512		−1.746***
		(0.347)	(0.356)		(0.615)
SV$_t$				1.466	5.795***
				(1.026)	(1.828)
Constant	−0.038**	0.014	−0.034**	0.005	0.022**
	(0.017)	(0.010)	(0.017)	(0.008)	(0.010)
Observations	176	176	176	176	176
R^2	0.054	0.001	0.065	0.012	0.056
Adjusted R^2	0.049	−0.005	0.054	0.006	0.045

Note: *p<0.1; **p<0.05; ***p<0.01

Benchmark:

	Sample	R^2_{oos}
SV$_{t+1}$	Monthly	25.414*
AV$_{t+1}$	Monthly	38.11**
RET$_{t+1}$	Monthly	-0.059

Bench

	Sample	R^2_{oos}
SV$_{t+1}$	Monthly	4.041
AV$_{t+1}$	Monthly	26.853
RET$_{t+1}$	Monthly	2.116