Average Variance Managed Investment Timing

Jeramia Poland*

ABSTRACT

Compared to using the variance of index returns, managing investment by the average of the variance of index components (AV) produces significant return and ratio performance improvements. AV managed investment in the market index takes less extreme leverage making it more practical and cheaper while generating more substantial utility gains. AV management highlights the fundamental risk make up of portfolio return variance. AV management provides key information for optimal investment by signaling changes in the mix of compensated and uncompensated risk. As such, investors can use equity AV as a signal to manage assets across the economy.

JEL classification: G10, G11, G17.

*Indian School of Business, email jeramia_poland@isb.edu, +91 9866656230
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Since Markowitz (1952) diversification has held the position as the main portfolio choice mechanism for risk management. Diversification, through index fund investment, remains quite popular with nearly $7 trillion currently invested in index funds. Collins (2018) Many funds have moved past simple diversification to incorporate risk hedging strategies. Between 400 and 500 billion dollars sits in funds utilizing a risk management, targeting, or risk-parity strategy. (Cao, 2016; Verma, 2018) Many of these funds employ cross-sectional trading strategies but as Moreira and Muir (2017) showmanaging investment weight in a portfolio by its return volatility in time series produces greater expected returns and performance ratios over long periods. This holds for well diversified, low risk portfolios like the market index. The central premise of modern portfolio theory is that the systematic risk demands compensation but, even for diversified portfolios, systematic risk is not the only source of variance. There appears to be a disconnect in the dynamics of the risk-return relationship allowing investors to move out of the market index when risk will not accompany return and leverage in when it will. I find that returning to the fundamental composition of portfolio variance identifies a better investment timing signal. The weighted average of the return variance of individual portfolio or index constituents (AV) is an unsystematic and uncompensated risk which can be managed over time even in well diversified holdings. Zeroing in on this unsystematic uncompensated part better manages dynamic volatility, takes on less extreme leverage, generates higher returns at lower risk, costs less for the investor, benefits fund managers and informs us about changes in the systematic risk across the economy.

The practical benefits of AV management for investors and fund managers will be established first. Using daily market returns from the Center for Research in Securities Prices (CRSP), I extend Pollet and Wilson (2010) generating monthly time series of the variance of daily stock market index returns (SV), the market cap weighted average of pairwise correlation between index component daily returns (AC), and the market cap weighted average of the individual variance of daily index component returns (AV) from August 1926 to December 2016. The AV management strategy takes an investment weight, held for the following month, based on the inverse of this month's AV. Using a constant to keep the volatility of the AV managed portfolio the same as the buy and hold market portfolio, as in Moreira and Muir (2017), an investor without borrowing constraints earns an annualized average return of 9.7%. This return is a statistically significant increase of more than one percentage point over the SV managed portfolio; the difference in annualized average monthly returns grows to more than two percentage points when practical leverage constraints are applied. With unconstrained borrowing, the AV managed portfolio has significantly better performance ratios like the symmetric Sharpe ratio, 0.52, and more asymmetric risk-return measures, e.g., Kappa_3 and Kappa_4 at 0.15 and 0.11 respectively. The advantage of managing with AV grows with risk aversion. The most risk-averse, $\gamma = 5$, constrained mean-variance investor sees a certainty equivalent return (CER) gain of more than two percentage points annualized; this return represents a 27.4% increase in utility nearly as substantial as the utility gains seen in return forecasting strategies. (Campbell, Lo, MacKinlay, et al., 1997) Stochastic dominance tests show the preference for AV management extends beyond mean-variance investors. Investors with a broad class of utility functions, including those with higher order risk and prospect theory preferences, prefer the AV management strategy to SV. The significantly better performance of AV is also robust to other choices of target volatility; AV outperforms SV management when both portfolios seek to have 10% or 12% annual return volatility.

Average returns and ratios are measures over the whole period of performance, but investors and those managing the money of others may also be interested in the intermediate performance. Drawdowns, losses in portfolio value, are an important measure of risk for funds and those with liquidity concerns. Drawdowns are shallower and shorter for the AV managed portfolio at 9% versus 11.2% and 10.5 versus 15 months on average. The drawdown profiles for the AV and SV managed portfolios also reveal that the AV managed portfolio benefits fund managers in addition to fund investors. The SV managed portfolio exposes a fund manager to twice the risk of a drawdown severe enough to threaten their job, or possibly shutter the fund, and would be nearly 91.7% more expensive to insure against such a loss. Thus AV managed returns are better evaluated at the end and over the intermediate period.

AV equity management is also better globally. The AV managed portfolio is a better performer in returns, ratios, drawdowns, and costs in 8 of 9 non-US markets tested. AV management generates significantly better performance in both developed and emerging markets including Brazil, Germany, France, India, Japan, and Great Britain. In China, AV management more than quintuples index returns over the buy and hold strategy. In Japan, AV management transforms negative buy and hold returns into a significant positive 1.375% annualized from 1993 to 2015. AV management of investment in Germany increases annualized returns by almost 3.5 percentage points over SV management and results in a significant improvement in Sharpe ratio. AV index management works not only in the countries individually but for an equity portfolio diversified across the globe. AV management of the MSCI All Countries World Index (ACWI) produces higher annualized returns, 8.6%, with a better Sharpe ratio, 0.551, than either SV management or the buy and hold strategy. AV managed MSCI ACWI also has shallower drawdowns and is robust to much higher trading costs.

Regardless of the volatility target, some of the investments demanded by the SV management strategy are unrealistic. In several time periods SV management requires over 1500% leverage while AV management tops out at 325%. This makes the SV management strategy impractical as there are no easy ways for investors to take leverage levels this large. As Ang, Gorovyy, and Inwegen (2011) document the average level of leverage taken by hedge funds is 36%. The amount of leverage in swaps, derivatives, and fixed income tops out at 400% leverage and it seems unlikely that positions four times as large are easily possible. While the mean weights taken by each strategy, for any volatility target, are the same there is a consistent difference in the variances and maximum values. In addition to taking far more extreme weight in the market more often, the SV management strategy has higher turnover. So, the AV management strategy has lower borrowing costs, 11.411 versus 15.107 basis points per month on average, and generates lower transactions costs with a break-even trading cost more than twice the SV managed portfolio.

The performance of AV managed returns is somewhat of a surprise given portfolio variance is the standard measure of investment risk. However, empirically, the link between the variance and future returns is weak. A large number of researchers have sought to identify a positive relationship between variance and expected returns. From Haugen and Heins (1972) through French, Schwert, and Stambaugh (1987) and Ang, Hodrick, Xing, and Zhang (2006), decades of researchers have found insufficient evidence supporting a positive relationship between variance and future returns. SV is a function of both the variance of individual asset returns and the covariance between assets in the index. Pollet and Wilson (2010) show that the average pair-wise correlation (AC) of assets is the component most related to systematic changes in the economy and is the risk component compensated with higher returns. Moreover, again from Pollet and Wilson (2010), so long as market index returns are a good proxy of returns to aggregate wealth and the observable market is a significant part of investors wealth, high AC will signal covariance of returns across the economy. Further details are provided in section I. Shocks to the economy have a common effect with specific impacts on each part of an investors wealth determined by idiosyncratic deviations from the common level. Intuitively, the common effect is larger in the observable index when, on average, those assets move together, and when aggregate wealth is sensitive to those changes, observable and unobservable returns covary. When AV dominates, the idiosyncratic deviations imply that unobservable returns need not move with the observable index. By responding to total variance, the prior literature only indirectly manages exposure to AV. The performance due to and information in the AV depends on this signal of the changes in the mix of systematic and unsystematic variation behind the variation in index returns.

It is, of course, difficult to test whether AV management better times investment to periods when observable and unobservable returns covary. Still, I find several results consistent with the theory in Pollet and Wilson (2010) and supporting the notion that AV is better than SV at signaling investors to times of better conditions across the economy. A sharp result showing AV management's ability to signal investors to changes in systematic risk would be its ability to manage returns from other asset classes. Moreira and Muir (2017) find that equity SV is not a useful signal for currency investment. However, equity AV works as a management signal not only for currency investments but real estate and bond market investment as well. I test this using four currency indices, the S&P US Real Estate Investment Trust (REIT) index, and the Bloomberg US Universal Bond index. All series are from 2005 to 2015. The buy and hold and SV management strategies have annualized returns of -2.07% and -0.36% for the Deutsche Bank Currency Carry Index. This carry trade index uses one of the oldest currency trade strategies based on the work on uncovered interest rate parity from Bilson (1981); Fama (1984) and studied by Lustig and Verdelhan (2007); Brunnermeier, Nagel, and Pedersen (2009); Burnside, Eichenbaum, and Rebelo (2011) among others. Management using equity AV, generates a 1.44% annualized return from the carry trade index. Equity AV is very successful at managing REIT investment over the sample period nearly doubling the SV managed returns, 26.7% to 15% and the AV management return of 3.95% and Sharpe ratio of 1.17 are significant improvements over SV for the bond index. Additionally, the AV of currency and bond returns works as an investment timing signal for the MSCI World index. The average variance component of equity market variance is better globally and across asset classes supporting the argument that compensated systematic risk is higher when average variance is lower.

The Pollet and Wilson (2010) theory implies that the relationship between AC and returns depends on the proportion of the market observed through the index proxy and the relationship of the stock market return and the aggregate wealth return. This relationship should also be observable in portfolio performance. Using the ratios of market return to return on wealth and market capitalization to GDP per capita[1] as proxies for the relationship of the stock market to aggregate wealth, AV works better both in the time series of US equity returns and in the cross-section of countries when the market is more related to aggregate wealth. AV managed returns are higher with a better Sharpe ratio when the market capitalization to GDP per capita ratio is higher in the US. In high ratio periods the ratio is systematically important, measured by return alpha, to AV returns and it is not in low ratio periods. Investors capture average annualized returns over 5% with positive and significant factor alphas from a portfolio using AV managed weights long in countries with high and short in those with low ratios. The relationship of the stock market to aggregate wealth is systematically important to AV management, as expected.

I run several regression specifications to examine the relationship of AV to future SV, AV, AC, and returns. The examination of regression results also allows for the correction of "volatility-feedback" effects, proposed by Campbell and Hentschel (1992), and makes it possible to judge the stability of the dynamic risk-return relationship mix revealed by AV and AC. AV is a significant in-sample predictor of higher SV, AV, and AC. A one standard deviation increase in annualized AV, from 0.77 to 1.62, is related to an increase in next months annualized SV of 0.627 of a standard deviation. The increase in SV makes it more than double its mean, 0.25 vs 0.56. AV remains a significant predictor of next months

[1]I follow several papers cited in section V.B which use GDP per capita to control for cross-country wealth effects.

SV even when controlling for this month's SV. AV has a slightly higher R^2 value when predicting next month's SV and is better at predicting next month's AV. Controlling for this month's AV, SV is not a significant predictor of next month's AV. Neither SV nor AV is significantly related to future returns. However, as in the Pollet and Wilson (2010) results, AC is positively and significantly related to future returns. This evidence explains the performance of AV as an investment management signal. Scaling investment in the market with the inverse of AV in the current month will pull funds out when the following month will have high SV without sacrificing higher expected returns when AC is high. These results support the existence of a long-run relationship between AC and future returns and support the conclusion that AV is not a compensated risk.

As always, asset pricing and portfolio tests are limited by the indices and proxies used. Thus, in a sub-sample where it is unlikely the index represents the whole market, and the stock market itself is known to be an insignificant part of aggregate wealth, AC should not signal returns to systematic risk. This is the case for the CRSP data before 1962 which covers only the New York Stock Exchange (NYSE) and covers a period when the average investment was meager. (Taylor, 2014) Indeed, in a placebo-like result, before 1962 AC does not predict next month's return even when controlling for AV. The pre-1962 result supports the Pollet and Wilson (2010) argument that the information which AC, and thus AV, provides on the mix of compensated and uncompensated risk depends on the relationships of the index returns being used, the stock market, and aggregate wealth.

The full in-sample results suggest AC positively forecasts next month's return and AV positively forecasts next month's risk without a positive relationship to returns. This relationship appears to manifest in performance depending on relationships of AC and AV to the economy as a whole. However, as Welch and Goyal (2008) argue, forecasting relationships may be unstable and quite sensitive to sample period choice. They may not respond dynamically with the limited information available to investors in real-time and may not explain or support the performance observed for any trading strategy at all. Furthermore, robust out-of-sample results would indicate that there is more information available on the real-time risk-return dynamics in AV management than SV which would support the notion that AC is systematic and compensated while AV is un-systematic and uncompensated explaining the portfolio performance results.

To explain why AV is a better real-time market portfolio leverage signal, I run expanding window out-of-sample regressions using AV on SV, AV, AC, and log excess returns. From July 1939 to December 2016 and using the predictions from SV as a benchmark, AV is a significantly better forecaster of next months AV, AC, and SV. It generates better Diebold and Mariano (1995) test statistics, significantly lower mean squared forecast errors judging by the MSE-F statistic from McCracken (2007) and the encompassing test of Harvey, Leybourne, and Newbold (1998) show that AV contains all of the forecasting information in SV. As with the in-sample results, AV serves investors at least as well as SV and likely better in avoiding risk without giving up return. Out-of-sample testing always raises questions about model specification, recursive expansion versus rolling window parameter estimation, choices of the training period and prediction window and the influence of specific periods. Using the techniques in Rossi and Inoue (2012), the Diebold and Mariano (1995) and Harvey et al. (1998) measures can be calculated robust to concerns on window selection for either an expanding or rolling specification. The Rossi and Inoue (2012) robust statistics show that AV is a significantly better predictor than SV robust to the choice of window or regression specification. AV is a better investment timing signal than SV because it scales investment by future risk without giving up the future return. AV management does this because there is more information in AV on the the risk-return dynamic than in SV.

Using the decomposition of market variance, I identify a better portfolio investment management signal. Weighting investment by the inverse of AV, rather than SV, is a new addition to the portfolio management literature letting investors capture better performance as measured by expected annualized returns, performance ratios, costs, and utility gains. My results complement recent work including the Moreira and Muir (2017) and Hocquard, Ng, and Papageorgiou (2013) who use volatility timing and a constant target to manage portfolio tail risk. Use of less leverage, also makes the AV management strategy more practical, cheaper, and provides utility gains for many classes of investors regardless of risk aversion or borrowing restrictions. The returns to the AV managed portfolio improve our understanding of the risk-return dynamic by showing that the time variation in the mix between the systematic and unsystematic parts of the market index variance is an essential dimension of risk and optimal investment over time. Recent work in the risk-return dynamic literature has also attempted to generate more systematic signals from components of portfolio volatility or across many portfolios. Gonzlez-Urteaga and Rubio (2016) examine the average volatility across many portfolios and the risk premium compensation. Bollerslev, Hood, Huss, and Pedersen (2017) generate a better-realized volatility risk signal from the high-frequency data of many assets demonstrating better forecasting performance and utility gains for investors. Rather than a construction or statistical extraction, AV represents a return to the fundamental composition of portfolio variance. AV management demonstrates a simple signal inside index volatility which signals systematic changes, forecasts performance, and generates utility gains for a broad class of investors.

I. Theoretical Framework

As Samuelson (1967) demonstrates "diversification always pays." This work is not any criticism of diversification which works to reduce risk by minimizing idiosyncratic risk through holding a cross section of many assets. AV management works by limiting the risk investors face to uncompensated risk, as much as possible, over time. AV is non-systematic, uncompensated risk. AC is compensated systematic risk. These assertions are supported algebraically Pollet and Wilson (2010). I highlight some important results below and refer the reader to their excellent article for further details.

To explore the relationship of variance and future returns, we start with the market equilibrium optimal portfolio weights for investors with power utility over end-of-period wealth from Campbell and Viceira (2002):

$$E_t[r_{i,t+1} - r_{f,t+1}] + \frac{\sigma_{i,t}^2}{2} \approxeq \gamma \sigma_{im,t}$$

where i is an asset in the market, $r_{f,t+1}$ is the log return to the risk free rate, and $\sigma_{im,t}$ is the covariance of i and the market, m.. The implication is that the excess returns of an asset are proportional to the conditional covariance of the asset with the market. This log CAPM holds for the stock market observed by an index proxy, asset s, like the S&P 500 or CRSP index. Hence, by replacing the covariance of the index, s, and the market with the level covariance of R_s and R_m and then decomposing this covariance into the weighted sum of the variance of the market index and the covariance of the index with the remainder of the market aggregate wealth not observable through the index, asset u, excess returns are related to observed index variance and return covariance.

$$\begin{aligned} E_t[r_{s,t+1} - r_{f,t+1}] + \frac{\sigma_{s,t}^2}{2} \approxeq & \gamma cov_t(R_{s,t+1}, R_{m,t+1}) \\ & \gamma cov_t(R_{s,t+1}, w_{s,t} R_{s,t} + (1 - w_{s,t} R_{u,t+1})) \\ & \gamma (w_{s,t} Var_t(R_{s,t+1}) + (1 - w_{s,t}) cov_t(R_{s,t+1}, R_{u,t+1})) \end{aligned}$$

Total index, or portfolio, variance is a function of the variances and covariances of the individual assets. The total portfolio variance of holding K assets is given by:

$$SV_t \ : \ \sigma_{s,t}^2 = \sum_{j=1}^{K} \sum_{k=1}^{K} w_{j,t} w_{k,t} cov(R_{j,t}, R_{k,t}) \tag{1}$$

where w_t is the market capitalization weight of the asset in the index or portfolio. Average variance and average correlation are defined as:

$$AV_t \ : \ \bar{\sigma}_t^2 = \sum_{k=1}^{K} w_{k,t}\sigma_t^2 \tag{2}$$

$$AC_t \ : \ \bar{\rho}_t = \sum_{j=1}^{K}\sum_{k=1}^{K} w_{j,t}w_{k,t}\rho_{j,k,t} \tag{3}$$

The return of the stock market index as a function of the return on aggregate wealth, market, is given by the equation:

$$R_{s,t+1} = \beta_t R_{m,t+1} + \epsilon_{t+1}$$

where ϵ_{t+1} is the stock-specific shock component of any shock to aggregate wealth. Any such shock to the stock market, will have a common stock component, $\bar{\epsilon}_{z,t+1}$, with variance $\theta_{t+1}\sigma^2_{z,t+1}$ and an idiosyncratic component with variance (1-θ_{t+1})$\sigma^2_{z,t+1}$. The shock to the stock market will be a component of a shock to aggregate wealth with variance $\sigma^2_{m,t}$. As the number of assets, K, grows the pairwise covariance converges to the variance of the market index and the return on aggregate wealth is the weighted combination of the return to the stock market and the unobserved portfolio. Thus, the return unobservable from the market index, $R_{u,t+1}$, can be written as:

$$R_{u,t+1} = \left(\frac{1 - w_{s,t}\beta_t}{1 - w_{s,t}}\right) R_{m,t+1} - \left(\frac{w_{s,t}}{1 - w_{s,t}}\right) \bar{\epsilon}_{z,t+1}$$

and the covariance of returns to the observable index and the unobservable component of aggregate wealth is:

$$Cov(R_{s,t+1}, R_{u,t+1}) = \left(\frac{1 - w_{s,t}\beta_t}{1 - w_{s,t}}\right) \frac{\bar{\sigma}_t^2}{\beta_t} \left(\frac{\bar{\rho}_t - \theta_t}{1 - \theta_t}\right) - \left(\frac{w_{s,t}}{1 - w_{s,t}}\right) \left(\frac{1 - \bar{\rho}_t}{1 - \theta_t}\right) \bar{\sigma}_t^2.$$

Equation (6) from Pollet and Wilson (2010) gives the relationship of AC ($\bar{\rho}$), AV ($\bar{\sigma}_t^2$) and the risk premium.

$$E_t[r_{s,t}] - r_{f,t} + \frac{\sigma^2_{s,t}}{2} = \frac{\gamma}{\beta_t(1 - \theta_t)}\bar{\rho}_t\bar{\sigma}_t^2 - \frac{\gamma}{\beta_t(1 - \theta_t)}\theta_t\bar{\sigma}_t^2$$

As they explain, ceteris paribus a change in AV has both positive and negative effects on returns with similar magnitudes. So, AV should have no significant relationship with future returns. AC, in contrast, has only a positive effect on future returns.

Equation (8), in Pollet and Wilson (2010), relates AC and AV to the correlation of stock returns and the unobserved component of aggregate wealth, the unconditional β of the stock market on aggregate wealth, and the unconditional proportion of shock which is common, θ:

$$cov(R_{s,t+1}, R_{u,t+1}) = \pi_0 + \frac{(1 - w_{s,t}\beta(1-\theta))E[\bar{\sigma}_t^2]}{(1 - w_{s,t})\beta(1-\theta)}\bar{\rho}_t - \frac{(1 - w_{s,t}\beta(1-\theta))E[\bar{\rho}_t] - \theta}{(1 - w_{s,t})\beta(1-\theta)}\bar{\sigma}_t^2.$$

Again they explain, the denominators for both coefficients of interest are positive if the β of the stock market on aggregate wealth is positive and the proportion of the market which is observed, w_s is greater than zero but less than one. Hence, the coefficient for AC is positive if 1-$w_s\beta \geq 0$ or equivalently if the covariance between the unobservable return and aggregate wealth is positive. For plausible parameter values 1-$w_s\beta(1-\theta)$ is small and $E[\bar{\rho}_t] - \theta$ is close to zero, as θ represents the portion of the shock to stock returns which is common among returns. Thus, the effect of AV will be negative but small.

Although the algebra is dense, the intuition is fairly clear. A shock to the economy will have an average common effect; the affect on the stock market will have a common effect from which each asset will deviate idiosyncratically. The idiosyncratic variance of stocks is orthogonal to the common aggregate effect. Thus when the variance of stock returns is largely from the average variance of individual assets then returns across the market are not covarying, hence returns across the economy need not covary. Managing investment timing by AV will avoid investment when risk is high but future returns are uncertain or possibly negative, as equation (6) implies. Additionally, managing investment by AV will give investors a signal, from equities, for higher returns in other asset classes, as equation (8) implies.

The formation and analysis of the AV signal come from the fundamental understanding of risk, portfolio variance and returns. This work requires a few publicly available data sets and a few considerations for the calculations at the monthly frequency. Most of the work is in the calculations required to show significance in portfolio performance and in the regressions which establish the relationship of AV, risk, and future returns.

II. Data and Methodology

To calculate SV, AV, and AC, for the US equities, I use daily return data from CRSP and calculate the variance of daily returns monthly. To simplify the analysis of individual assets, I require that the asset be traded on each day in the month which mitigates any liquidity effects and ensures consistent variance, covariance and correlation calculations. Since the series of daily returns is very short for many months in the data, Pearsons correlation is not an unbiased estimator of the true correlation, even if the returns are normal. (Hotelling, 1953) To correct this, I use the approximate correction from Olkin and Pratt (1958) such that the monthly correlation between two assets m and n is:

$$\rho_{m,n,t} = \widehat{\rho_{m,n,t}}\left(1 + \frac{1 + \widehat{\rho_{m,n}}^{2}}{2(t-3)}\right) \tag{4}$$

where $\widehat{\rho_{m,n,t}}$ is the Pearson correlation between m and n.[2]

Pollet and Wilson (2010) show that SV $\approx$ AV*AC. To calculate average correlations, they follow a method which makes an assumption on the deviation of individual asset variance from the average variance. This approach has the advantage of not implying that all variance terms vanish as taking the number of assets, N, to infinity implies.[3] However, as Tierens and Anadu (2004) note it makes no practical difference which approximation is used.

[Place Figure 1 about here]

To make the analysis more computationally tractable I use only, at most, the top 500 assets in CRSP by market capitalization for a given month.[4] To test the performance of AV management across international equity markets, I collect daily returns for the Australian (AUS), Brazilian (BRA), Chinese (CHN), German (DEU), French (FRA), Indian (IND), Italian (ITA), Japanese (JPN), and English (UK) markets from Compustat - Capital IQ. For international markets the number of assets is chosen to mimic the number of assets in the broad market index used to calculate SV. Table I shows the names of the indices used, the data time frames and the number of assets used for the calculation of market capitalization weights, AV and AC. In all cases the data series are much shorter. The primary limitations are the availability of daily returns and dividend data for total returns. To test AV and SV management against a globally diversified portfolio, I use the MSCI All Countries World Index (ACWI). To construct AV and SV leverage timing signals for the world index, I use a market capitalization weighted average of the country values. For each month the weight of the country is the index market capitalization in US dollars divided by the total market capitalization of the 10 country market indices, including the US. Globally, ten markets and more than 1700 assets are covered. Figure 1 shows the time series behavior of SV and AV, in percent, as well as AC for the US market. With the easily noticeable exception of October 1987, spikes in both SV and AV concentrate around NBER defined recessions. Panel (b) of figure 1 shows that AV takes a larger weight in the market more often than SV but that SV takes far larger weights in the market in certain times. Figure 2 makes this clearer and shows that changing the target volatility only affects the most extreme investments made by the SV management strategy.

[Place Figure 2 about here]

[2]For details, see the appendix.

[3]This is detailed as approach (b) on Kemp (2013).

[4]The least number of assets which trade every day in a given month is 392 in August of 1932. There are regularly 500 qualifying assets by the end of the 1930s.

Table I panels (a) - (c) show the summary statistics for the calculated variables. Despite differences in approach, the quarterly calculated values are almost identical to those in Pollet and Wilson (2010) over the same sample. Over the expanded the period, the annualized monthly AV has a mean value of 0.88%. The annualized SV mean is much lower at 0.25% monthly. AC is relatively consistent at 0.23 quarterly in the Pollet and Wilson (2010) sample, 0.261 monthly over a similar time period and 0.276 over the full time period. AV is more volatile than SV, more than twice as much. In each sample AV has the highest auto-correlation. While AC is also persistent, the stock market variance is only strongly persistent at the monthly frequency with autocorrelation of 0.61. All three time series are stationary rejecting the unit root null in the tests of Dickey and Fuller (1979), Ng and Perron (2001), and Elliott, Rothenberg, and Stock (1996).

[Place Table I about here]

As my primary interest is in the use of AV versus SV in the management of leverage in the CRSP market portfolio, I test AV and SV against log excess returns. Specifically, I take the difference between the natural log of one plus the CRSP market return and the natural log of one plus the risk-free rate using:

$$r_t = \log R_{m,t} - \log R_{f,t} \tag{5}$$

where $R_{f,t}$ the risk-free return for which I use the 1-month treasury bill, T-bill, rate from Ken French's website[5]. As shown in table II panel (a), over the full data period, each variance and correlation time series are contemporaneously correlated to lower log excess returns. AV is significantly correlated with next month's SV, AV, and AC. Surprisingly, over the full data set, this month's AV is even nominally more correlated with next month's SV than this month's SV is, 0.625 versus 0.612.

[Place Table II about here]

Panels (b) and (c) show international and cross asset class excess log return correlations. China is the market least correlated with the rest of the countries and the global market index. Importantly, as panel (c) shows, the other asset class returns are not all highly correlated with the MSCI ACWI returns. The currency return indices are generally weakly negatively correlated while both real estate and commodity returns are positively correlated. The Bloomberg US Universal Bond index is weakly positively correlated with equities and real estate and weekly negatively correlated with currencies.

[5] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html#Research

For in and out-of-sample tests, I regress market and average variance, and average correlation against these excess log return values next month. Out-of-sample regressions are run with expanding windows so coefficients estimated from a training window are used to forecast values for the first out of sample month. I set the first in-sample training period for the out-of-sample tests at 15% of the available time series for consistent calculation of robust out of sample statistics later in the analysis. This training window means that out of sample regressions, analysis begins in December 1939 and moves forward one month at a time as the training window expands.

III. Investment

To measure portfolio performance, in addition to annualized monthly log excess return, I will calculate each portfolio's Sharpe ratio, Sortino ratio, and two Kappa ratios and factor alphas. The classic Sharpe ratio is a symmetric measure of risk and is defined as the ratio of the expected excess portfolio return over the standard deviation of portfolio returns.

$$\frac{\mathbb{E}[r_x]}{\sigma(r_x)} \tag{6}$$

While Sharpe measures each dollar of expected return for dollar of risk, the Sortino ratio attempts to more directly measure the risk of loss. While the Sharpe ratio uses the full second moment in the denominator, the Sortino uses only the lower partial moment. By using only downside deviation in the denominator, the Sortino quantifies each dollar of expected return for each dollar of loss. This downside is measured relative to a target return. (Sortino and Price, 1994) As the log returns are already excess of the risk free rate, I set the Sortino target to 0 which makes the Sortino formula:

$$\frac{\mathbb{E}[r_x - 0]}{\sqrt{\int_{-\infty}^{0} (0 - r_x)^2 f(r_x)\, dr}} \tag{7}$$

where f is the probability density function of the strategy returns. The Sortino is just a specific instance of a more general risk measurement ratio formula. The Kappa ratio keeps the expected return relative to a target in the numerator but allows any lower partial moment in the denominator. (Kaplan and Knowles, 2004) With the target again set to 0 the general formula is of the form:

$$\frac{\mathbb{E}[r_x - 0]}{\sqrt[n]{LPM_n}} \tag{8}$$

The Sortino ratio is the Kappa$_2$ ratio. I calculate Kappa$_3$ and Kappa$_4$ also to see relative performance of AV and SV management adjusted for negative return skew and kurtosis. All of the measures are annualized, e.g., log returns are multiplied by 12 and ratios, like Sharpe, are multiplied by the square root of 12. Fama-French three and three-factor with momentum alphas are also calculated and annualized.

AV and SV managed returns, like market returns, are weakly auto-correlated, slightly skewed, and have much fatter tails when compared to normally distributed returns meaning the Memmel (2003) method of testing significance will not work. Moreover, current period returns to either the AV or SV managed portfolio depend on the prior period variance of the market return strongly questioning the i.i.d assumption made in most hypothesis testing methodologies. Studentized time series bootstrap sampling preserves the time series properties of the AV and SV managed returns which is critical; for example, Scherer (2004) demonstrates that methods which loose the time dependence in the calculation of differences in Sortino ratios fail to properly estimate the sampling distribution and critical values. Time series bootstrap method preserves the original data structure and allows for efficient robust hypothesis testing. (Politis and Romano, 1994; Davison and Hinkley, 1997) Ledoit and Wolf (2008) show that this method is even more efficient than using Newey and West (1987) or Andrews and Monahan (1992) heteroskedasticity and auto-correlation corrected standard errors for testing the significance of differences between portfolio Sharpe ratios. I follow the p-value estimation method in Ledoit and Wolf (2008) to determine the significance of the difference between the ratios of the AV and SV management strategies. This method uses circular block bootstrapping of the return time series, robust centered studentized statistics computed from the bootstrap samples and is proven to be the most efficient hypothesis testing method.[6] (Politis and Romano, 1992; Ledoit and Wolf, 2008)

As in Moreira and Muir (2017), investment weight in the market portfolio is a function of the inverse of the variance of daily market returns, SV, or the average daily return asset variance, AV, scaled by a constant, c. Moreira and Muir (2017) use a constant that scales the variance of the volatility managed portfolio equal to the buy and hold market portfolio. In the basic portfolio weighting specification, I use the same approach so that the returns of both the SV and AV managed portfolios have the same variance as the buy and hold strategy. This constant is denoted c_{BH} and it takes different values for SV and AV. This scaling requires knowing the full sample buy and hold return variance. While this does not distort the performance ratios, to insure robustness two other specifications for the scaling targets are used. Annual volatilities of 12% and 10% are common in academic literature and fund management so c_{12} and c_{10} approximately target those levels. (Barroso and Santa-Clara, 2015; Morrison and Tadrowski, 2013; Verma, 2018; Fleming, Kirby, and Ostdiek, 2002; Hocquard et al., 2013) Using each of these constant an investor re-balances at the end of month t investing in the market portfolio with weight:

$$w_{x,t} = \frac{c_{target}}{x(t)} \tag{9}$$

[6] I randomize the block sampling size rather than calculate the optimal size. This results in wider standard errors and a more conservative test. (Lahiri, 1999).

where $x(t)$ is either SV_t or AV_t and hold for month t + 1.

[Place Table III about here]

Table III shows summary statistics for the resulting investment weights for AV and SV for the three volatility targets. The investment weight turnover, shown in table VII, mean, and standard deviation for the SV management strategy targeting the buy and hold are nearly identical to the values in Moreira and Muir (2017).

[Place Table VII about here]

A. US Performance

Before examining implications of the leverage demands, I present the results for the SV and AV strategies targeting the buy and hold volatility without leverage constraints in table IV. As in Moreira and Muir (2017) the portfolio performance is measured across the whole CRSP data set, however the relative performance is the same or better across the basis data set.

[Place Table IV about here]

Table IV presents the performance ratios for the SV and AV managed portfolios targeting the buy and hold volatility without investment constraints. The buy and hold market strategy is included for reference. However since Moreira and Muir (2017) establish that the SV managed portfolio out performs the buy and hold, statistical significance results are only presented for the comparison of the SV and AV managed portfolios. The AV managed portfolio generates a statistically significant 1.09 percentage points higher average annualized log excess return. As shown in the bottom panel of figure 3 the AV strategy builds its performance advantage slowly but consistently starting from the early 1950s and from that time the SV managed portfolio is never a better investment. As both strategies are targeting the same volatility, the significant difference in return translates into a significant difference in Sharpe ratio. At 0.520 versus 0.462, for the SV managed portfolio, the AV managed Sharpe ratio is 12.6% higher. There maybe some concern with the use of mean-variance symmetric performance measures like the Sharpe ratio, however AV management outperforms SV on asymmetric performance ratios as well. AV has a higher Sortino ratio. AV also generates significantly higher Kappa_3 and Kappa_4 ratios. These gains in skewness and kurtosis performance show AV management captures tail risk gains, similar to those in Hocquard et al. (2013) over SV management when both target the same volatility. The Fama-French three factor alpha is significantly higher for the AV versus SV managed portfolio. There is no difference in significance either way when the momentum factor is also included, but this is due to one data point as detailed below. So while the overall payment for downside risk, measured by Sortino ratio, may not be significantly higher, the payment for downside skewness and extreme downside return is. Measuring risk-adjusted returns using either the Fama-French three factor or three-factor with momentum model, AV management significantly outperforms AV. The differences are slightly higher than the difference in annualized average return.

[**Place Figure 3 about here**]

Portfolio performance numbers always bring forward questions on performance in subsamples. Panels (b) and (c) in table IV present the performance of the buy and hold, SV and AV managed portfolios in NBER defined expansions and contractions. In business cycle expansion, the AV managed portfolio provides significantly more compensation and significantly more compensation for every measure of risk. The results for NBER contractions go in the other direction. The SV managed portfolio appears to be so much better that it might be a more desirable option given that we cannot know periods of extended contractions before they begin and investors may desire a portfolio that protects value through downturns more than one that maximizes returns during market upswings. However, as panel (d) shows, the better performance of the SV managed portfolio is due to one, albeit a rather important, data point. The significantly better performance of the SV managed portfolio through NBER contractions depends entirely on the 1929 to 1933 Great Depression. Excluding that time period, The AV managed portfolio, again, provides higher average and risk-adjusted returns but the difference in performance by any portfolio risk measure, while better for AV, is insignificant. As a result, its clear that without the 1929 to 1933 depression, the SV managed portfolio is unable to compete with AV management in measures of risk-return performance. So, investors only concerned with end-of-period performance will prefer AV management. However, AV management will also be prefered by those concerned with losses of value regardless of when they occur.

B. Drawdowns

Drawdowns, the peak-to-trough decline in the value of a portfolio, may be the most natural measure of real market risk. (Magdon-Ismail and Atiya, 2006) A drawdown occurs when a portfolio loses value for two or more consecutive months. The drawdown continues until the portfolio regains the value it had at the beginning of the first month of the drawdown even if the portfolio gains back some value in between. Maximum drawdown, the largest peak-to-trough decline in portfolio value, in particular is often used in place of return variance as a portfolio risk measure. (Johansen and Sornette, 2000; Vecer, 2006; Sornette, 2003) Drawdowns play a significant role in the lives of fund managers as deep losses not only rob the fund of capital but motivate investors to withdraw funds making drawdowns a significant determinant of fund survival. (Baba and Goko, 2009; Papaioannou, Park, Pihlman, and Hoorn, 2013; Lang and Prestbo, 2006)

[Place Table VIII about here]

Table VIII panel (a) presents the drawdown statistics for the buy and hold, SV managed, and AV managed portfolios. The AV managed portfolio has more discrete drawdown events, 87, than either the buy and hold or SV managed portfolio. However, the drawdowns are much less severe. The buy and hold strategy losses a maximum of 84.8% of its value at the deepest point of its maximum drawdown, Max DD. AV and SV lose only 60.3% and 63.6% at the bottom of their worst drawdowns. The SV managed portfolio has the worst average loss during a drawdown, Avg DD, at 11.2% of the portfolio value while AV's average loss is only 9%. SV also stays "underwater" the longest both on average, 15 months, and during its longest drawdown, 246 months. Figure 4 shows the time series of drawdowns for the buy and hold, SV, and AV managed returns. However, the deepest and longest sustained losses of value for the SV managed portfolio start in the 1960s and SV does not recover until 1989. The notion that there is a drawdown so severe that it causes the collapse of the fund, or at least a management turnover, is known as the "knockout" drawdown. (Pav, 2016) Given a knockout drawdown value, it is possible to estimate the likelihood of the knockout occurring, the fund or manager not surviving, by fitting a binomial distribution to the drawdown observations using the knockout drawdown level as a cutoff to create binary values indicating a drawdown exceeding that level, 1, or not, 0. (Pav, 2016) Setting the knockout drawdown at 40%, as suggested by Pav (2016), in any given month the SV managed and AV managed portfolios have probabilities of 1.49% and .85% of incurring a knockout drawdown in the next 12 months. The SV managed portfolio if far more likely to fail and 75.7% more expensive, in theory, to insure using the max drawdown insurance of Carr, Zhang, and Hadjiliadis (2011).[7] This holds if the knockout drawdown is taken at either 35% or 45% where SV is 55.2% and 91.7% more expensive. Certainly, US equity investors are better served by the AV managed portfolio than the SV. However clear this conclusion, it raises the question of generalization.

[Place Figure 4 about here]

C. Global Performance

The AV management strategy is a better performer than SV across the world. Table V shows the differences in performance across countries, AV managed portfolio generates

[7]Calculation of actual insurance costs require prices on the zero coupon bond, however given this common price the digital call option on the knockout value of the SV managed portfolio is 1.757 times the price of the AV managed portfolio.

higher annualized average returns and better Sharpe ratios in all countries except Italy. In the fastest growing markets, China and India, the AV managed portfolio increases annualized average return by 2.455 and 2.637 percentage points. Both AV and SV management are improvements over the buy and hold in all markets. Australia shows the best buy and hold Sharpe ratio at 0.614, but the AV management strategy is able to increase it to 0.981. The results for the AV management strategy for the Chinese market are the most alluring, a 27.381% annualized return with a 0.868 Sharpe ratio, but this also the shortest and the most volatile of the international return series. Both AV and SV management generate better returns and Sharpe ratios for the globally diversified world index, but again AV management is an improvement over SV.

Table VIII panel (b) shows the drawdown statistics for each portfolio across countries. Unlike for the prior return measures, it is not always the case that SV management is an improvement over the buy and hold strategy. In Japan, India, and the UK, SV managed drawdowns are deeper, longer, and take longer to recover from on average. In every country but Australia, the AV managed portfolio has a shallower average drawdown. The AV managed portfolio has shorter average drawdowns in every country tested. In every country but China, the AV managed portfolio has a shorter average recovery time. Indeed, in every country but Japan, the AV management strategy is able to recover from a drawdown in less than 10 months on average. SV management of the world index makes the average drawdown statistics worse across the board allowing AV management to draw some sharp distinctions. The AV management strategy has an average drawdown depth nearly 30% shallower than the SV strategy, a 6.982% versus 9.776% loss and the average drawdown length for the AV management strategy is more than 20% shorter than for SV, 9.9 versus 12.5 months.

Across the globe, the AV managed portfolio is a better investment management signal. It results in higher annualized average returns. It captures in better Sharpe ratios. It generates better drawdown statistics and is cheaper, in trading costs, to execute. However, the performance evaluated thus far ignores the realities of implementing the trading strategies.

IV. Leverage

More practical analysis of portfolio performance requires recognition of the borrowing conditions and leverage constraints faced by real-world investors. As table VII shows, when targeting the volatility of the market portfolio, both portfolios are leveraged into the market on average with investment weights of 1.3, indicating 30% leverage. Regardless of the volatility target the SV managed portfolio calls for extreme levels of leverage. Figure 2 shows that the SV strategy calls for investment weights above the maximum AV weight in several periods. More than 500% leverage is needed at the end of the

1920s, throughout the 1960s, and in the 1990s. Given that these levels of leverage are unrealistic for most investors, it will be important to see if there is a difference in performance for the AV and SV strategies under real-world investment constraints and to investigate the associated costs generated by the trading needed for the SV managed portfolio.

[Place Figure 2 about here]

A. Investment Weights

One method of measuring the impact of investment constraints on the AV and SV strategies is to externally impose limits on the weights generated by each. Leverage of 50%, a coefficient of 1.5 on the market, is a common constraint meant to mimic real market leverage constraints for the average investor based in part on the Reg. T margin requirement[8]. (Campbell and Thompson, 2008; Rapach, Strauss, and Zhou, 2010; Rapach and Zhou, 2013; Huang, Jiang, Tu, and Zhou, 2015; Rapach, Ringgenberg, and Zhou, 2016; Moreira and Muir, 2017; Deuskar, Kumar, and Poland, 2017) Ang et al. (2011) find mean hedge fund leverage to be around 36%, an investment weight of 1.36, and at the extreme tops out at 400%. There are at least two exchange traded funds, ETFs, which target three times the return of the SP500.[9] So, I take a market coefficient of three as the maximum feasible investment a typical investor can make in the market portfolio. This represents 200% leverage is easily accessible for all investors.

[Place Table VI about here]

Table VI presents the results from applying investment constraints after calculating the weights for AV and SV targeting the buy and hold volatility. Panel (b) shows the ETF, 200%, leverage constraint. AV management outperforms by 1.71 and 2.07 percentage points when targeting buy and hold volatility. Investors using the leveraged ETFs are rewarded not only with higher returns but significantly better performance ratios across the board. SV management takes a performance hit first as leverage constraints are applied due to its more extreme leverage. The ETF leverage constrained AV strategy even generates better Sharpe and Sortino ratios than the unconstrained strategy. Panel (a) shows the results of applying the 50% leverage limit. The brokerage investment restrictions pull the volatility of the AV

[8]Federal Reserve Board Regulation T (Reg T) establishes a baseline requirement that investors deposit 50% of an investment position in their margin trading accounts, however a brokerage house may set a higher equity requirement.

[9]The Direxion Daily S&P 500 Bull 3x Shares ETF, symbol SPXL, and ProShares Ultra Pro S&P 500 ETF, symbol UPRO, are two such funds.

managed portfolio too far from SV to generate significant differences in performance ratios, but the difference in AV and SV managed returns is always greater than one percentage point and close to two when targeting the buy and hold volatility. Panels (a) and (b) in figure 3 show the effects of the growing separation. While SV is barely able to clear the buy and hold strategy under typical brokerage constraints, returns to the AV managed portfolio remain clearly above. The results in Panels (a) and (b) of table VI demonstrate that better performance of AV is not a result of, or contingent on, looking to the target volatility. AV management is better, regardless of variance target, as leverage constraints are applied.

B. Costs

The differences in investment weight demanded by AV and SV management shown in table III not only generate differences in returns but also in costs. As show in table VII panel (a), for the US market, the AV managed portfolio generates less than half the turnover of the SV managed portfolio. The average monthly absolute change in investment weight is 0.752 for the SV managed portfolio and only 0.317 for AV. Table VII shows the trading costs needed to reduce the annualized average return of the SV and AV managed portfolios to the buy and hold or to reduce the factor alphas to zero. (Frazzini, Israel, and Moskowitz, 2015; Moreira and Muir, 2017) Seen in table VII, the break even transaction costs are more than twice that for the AV managed portfolio. The SV managed portfolio breaks even at 29.422, 60.694 and 35.472 basis points while it takes costs of 254.364, 176.467 and 83.176 basis points to reduce AV returns to the buy and hold or zero out the annualized alphas. However, transaction costs are not the only expense incurred by the leveraged portfolios. To estimate the borrowing costs for each strategy, I assume that any month a strategy requires a position greater than one in the market the difference between the investment weight and one is borrowed. The average monthly cost of borrowing to invest for the AV managed strategy is nearly 25% lower than for the SV managed portfolio. Using the broker call money lending rates available in Bloomberg from September 1988 to October 2016, SV incurs an average monthly cost of 15.107 basis points while AV costs only 11.411 basis points. This ignores the possibility of the investor using saved gains rather than borrowing and the necessity of borrowing when the investor has lost money, however, as the AV managed portfolio has greater returns and better drawdown statistics included gains and losses would only further the separation.

Table VII panel (b) highlights the differences in the trading costs of the strategies across countries. The break even trading costs are calculated in reference to reducing the annualized average returns of the AV and SV managed portfolios to the buy and hold only. The AV managed portfolio is able to tolerate higher trading costs in every country analyzed. In all countries, as in the US market, the AV managed portfolio is able to tolerate higher transaction costs. In a majority of countries, 6 out of 9, the AV managed portfolio is able to tolerate trading costs twice as high as the SV managed portfolio. As for the individual country indices, the AV managed world index is far cheaper than the SV managed portfolio allowing an investor to keep more of the higher returns generated.

C. Investor Utility

One part still missing from the analysis of the difference in performance between AV and SV is a measure of the impact on different investors. Investors with different risk aversion will experience different utility effects to the constrained returns of the AV and SV strategies[10]. I consider a mean-variance investor as in Kandel and Stambaugh (1996), Campbell and Thompson (2008), Ferreira and Santa-Clara (2011), Rapach et al. (2016), and others. To measure changes to investor utility from switching between the SV and AV portfolios and due to leverage constraints, I use the difference in certainty equivalent return, CER. The CER change will measure the change in risk-less return an investor demands, given their risk aversion, to use a given portfolio versus another. This can equivalently be though of as a measure of the management fee an investor, given a specific risk aversion, would be willing to pay to have access to an investment fund using AV rather than SV as a signal. CER change is calculated as the difference in risk adjusted return using:

$$\Delta\ \text{CER}_x = \left(\hat{\mu}_{r_x} - \frac{1}{2}\gamma\hat{\sigma}^2_{r_x}\right) - \left(\hat{\mu}_{r_y} - \frac{1}{2}\gamma\hat{\sigma}^2_{r_y}\right) \tag{10}$$

where $\hat{\mu}_{r_x}$, $\hat{\mu}_{r_y}$, $\hat{\sigma}^2_{r_x}$, and $\hat{\sigma}^2_{r_y}$ are the means and variances of the returns to the x and y portfolios and γ is the investor risk aversion coefficient. I multiply the gains by 12 to annualize them. All investor risk aversion coefficients from 1 to 5 are tested for investors subject to investment constraints from a limit of 1 to 3 on the market portfolio, no leverage to 200% leverage.

[**Place Figure 5 about here**]

Figure 5 shows the decrease in CER, from unconstrained AV or SV management, as leverage constraints are applied to each strategy. CER losses accumulate for the SV managed portfolio sooner than for the AV managed portfolio. This is due to the extreme leverage positions needed for the SV managed portfolio. Across risk aversion coefficients, leverage constraints bite sooner and cut deeper into the SV managed portfolio until both are driven together when no leverage is allowed. The CER of the AV managed strategy starts higher at 7.95%, 8.98%, and 9.33% versus 6.87%, 7.90%, and 8.25% for investors with risk aversion coefficients of 1, 3, and 5. When those investors are subject to 200% leverage constraint the CERs are 7.96%, 8.99%, 9.33%, versus 6.22%, 7.05%, and 7.33% for SV management.

[10]As Moreira and Muir (2017) note, "With no leverage limit, percentage utility gains are the same regardless of risk-aversion because investors can freely adjust their average risk exposure."

The AV manged portfolio provides the same investor utility while the benefit from the SV managed portfolio decreases by 9.46%, 10.76%, and 11.15%. To incorporate the starting difference in utility I look at the CER gains moving from the SV to AV managed portfolio.

[Place Figure 6 about here]

As shown in figure 6, CER gains for the market variance targeting AV managed portfolio are increasing in both risk aversion and leverage use for constrained risk averse mean-variance investors. An investor with a risk aversion coefficient of 2 would capture an annualized CER gain of 1.49 percentage points using 50% leverage and 1.91 implementing the AV strategy through the 200% leveraged ETFs. The most risk averse investors subject to a 20% leverage limit see a CER gain of 1.35 percentage points while the most risk tolerant see a 1.08 percentage points gain. The most risk averse investor, using the highest feasible leverage, realizes a CER gain of over 2 percentage points which translates to a utility increase of 27.4%. This increase is in the neighborhood of those typically seen from return timing strategies. (Campbell et al., 1997; Moreira and Muir, 2017) Risk averse, mean-variance investors see substantial utility gains switching from the SV to AV managed portfolio and these gains increase with leverage usage. AV management is better, regardless of risk aversion level or leverage constraint.

[Place Figure 6 about here]

D. Stochastic Dominance

Again, there may be some concern about using mean-variance investors. However, it seems reasonable that investors still care about the variance of their portfolio return as the AV management strategy relies on the superiority of the AV signal not on investors taking AV as the measure of risk. However for robustness, I test AV management against SV for a larger class of investors by moving away from the assumption of mean-variance utility using stochastic dominance. As Hadar and Russell (1969); Hanoch and Levy (1969); Rothschild and Stiglitz (1970); Levy and Levy (2002) detail, the use of stochastic dominance tests allow us to make conclusions about the preferred strategy of investors whose expected utility functions differ from simple mean-variance optimization. I use the methods detailed in Vinod (2004, 2008) to test the stochastic dominance of AV over SV management to the forth order. Second order stochastic dominance, AV over SV management, means that any risk averse investor would prefer AV. (McFadden, 1989; Valle, Roman, and Mitra, 2017) Third order stochastic dominance means that any expected utility investor regardless of the form of the utility function would prefer AV to SV. (Whitmore, 1970; Chan, Clark, and Wong, 2016) At forth order stochastic dominance, in addition to the concept of risk aversion both prudence and temperance are incorporated and all expected-utility theory (EUT) investors prefer AV management to SV. (Kimball, 1993; Eeckhoudt, Gollier, and Schneider, 1995) However, investors can have non-EUT preferences which, for example, place more weight on the loss of utility from negative returns than the gain from positive as in Kahneman and Tversky (1979). As, Prelec (1998) shows a probability weighting function can be used to incorporate the deviations in preference of non-EUT investors into stochastic dominance analysis.

Following Vinod (2008), I first calculate kernel smoothed density and cumulative distribution functions (CDFs) for the unconstrained AV and SV managed portfolio return distributions. Figure 7 shows the density and CDFs estimated for the US equity returns to the buy and hold targeting AV and SV managed portfolios without investment constraints.

[**Place Figure 7 about here**]

The stochastic dominance tests involve the comparisons of the CDFs of the appropriate order, or power, of the portfolio returns. Generally, the test is a comparison of which distributions CDF sits "to the right" of the other. The CDF "to the right" means the return distribution which has more likely higher values. However, the empirical CDF, observed from the returns, is subject to random variation. This may cause one of the distributions to cross the other over part of the observed range whereas the true distributions would not. To test this statistically, each observation is given equal weight, $\frac{1}{n}$ where n is the number of observations, then the distributions are repeatedly sampled and the areas under the simulated CDFs are compared to check dominance. The portfolio CDFs can now be compared to test stochastic dominance at various orders, but more information on the preferences of investors can be incorporated before performing the tests.

Prelec (1998) defines a flexible function with parameter α controlling the "slope" of the "weight" an investor places on an observed return, ordered from smallest to largest. When α equals zero, investors are EUT-compliant and place equal weight on all observed utilities from returns regardless of size when evaluating two possible portfolios. When α is less than one but greater than zero, investors place greater weight on the lower, negative, realized returns than the higher a result consistent with loss aversion from prospect theory. Figure 8 shows the relationship between α, the ordered cumulative return probability and the weight the investor places on the probability of such a return. Further details on the stochastic dominance tests can be found in the appendix section **??**.

[**Place Figure 8 about here**]

Using many values of α, I test the stochastic dominance of AV relative to SV. The

Vinod (2004, 2008) method bootstrap samples from the return distributions of the compared strategies. Positive test values indicate that AV dominates, the AV values are "to the right", and the differences between the AV and SV CDF are positive. As table IX shows, expected utility investors whose α equals 1 prefer AV management to SV management. The positive numbers in across columns SD2, SD3, and SD4 indicate up to forth order stochastic dominance. Moreover, investors with non-EUT, e.g. prospect theory, preferences still prefer AV management to SV. (Kahneman and Tversky, 1979) For all values of α AV management stochastically dominants to the forth order over the SV portfolio.

[Place Table IX about here]

I use the extensions in Vinod (2016) to show that the preference for AV is robust to deviations from expected utility theory. These extensions incorporate the Prelec (1998) function and α weight the return distributions so that lower return values have greater utility importance. AV is the preferred management strategy for investors who care about utility whether they adhere to strict expected utility or behaviorally stray. Table IX shows the results of the dominance tests for values of α ranging from .99, a nearly completely EUT investor, to .01, a nearly completely non-EUT investor. All investors prefer AV managed returns.

V. Systematic Risk

As detailed in section I, Pollet and Wilson (2010) managing by AV signals periods of higher systematic risk and return covariance across the economy. Investors should benefit from investments not just in the equity market but from other assets as well. AV managed returns should depend systematically on the relationship of the observed index and aggregate wealth and there should be evidence of a dynamic information relationship between AV, AC, and future returns supporting the portfolio performance in section III.A.

A. Additional Investment

Moreira and Muir (2017) show that equity SV does not perform as a currency management signal. If equity AV serves as a signal for other assets, this will demonstrate a stark contrast and show AV signals changes in the mix of compensated and uncompensated risk. To test AV and SV management in other asset classes, I use the world market capitalization weighted AV and SV values constructed before and several asset class indices. These indices include the Bloomberg US Dollar Spot Index, the Deutsche Bank Currency Return Index, The Deutsche Bank Currency Carry Index, the Deutsche Bank Currency Momentum Index, the S&P U.S. Real Estate Investment Trust (REIT), the Bloomberg Commodity Index, and the

Bloomberg US Universal Bond index. All series start in July of 2005 and end in December 2015.

[Place Table X about here]

Table X shows the average annualized average returns and Sharpe ratios for each of the different asset class indices. The results for the currency indices support the conclusion in Moreira and Muir (2017). Management of currency investments using SV does not work. In many cases returns and Sharpe ratios are worse, e.g. the Deutsche Bank Currency and Currency Momentum indices. In contrast, management by equity AV is better than both the buy and hold and SV management strategies for currencies and real estate investments. As shown in table II, this is not driven by high correlation between world equity returns and currency returns. Negative returns on the Bloomberg Dollar Spot, Deutsche Bank Currency, and Deutsche Bank Currency Carry indices become positive when managed by AV. The AV managed real estate investment nearly quintuples returns to the S&P REIT index, 26.7% vs 5.3% and the Sharpe ratio is more than four times as high, .995 vs .198. Equity AV management is able to generate significant improvements over SV management in the bond market. The annualized return of 3.951% is a 2.5 percentage point improvement over SV and the 1.168 Sharpe ratio is more than twice that of the equity SV managed investment. Equity AV management does not appear to improve the return or Sharpe ratio to investment in commodities. [11]

Appendix table E.5 shows the drawdown statistics for the equity AV and SV management strategies applied to the alternative asset class indices. In all cases, even for commodities, AV management results in better drawdown statistics. The average depths are shallower, lengths and recoveries shorter. Equity SV management is not as successful. For currencies, SV management is only successful in making the average drawdown depth less for the Bloomberg Dollar Spot Index, a 10.1% versus 13.6% loss, but still the AV managed average loss is less at 9.7%. AV management cuts the average commodity loss by more than 50%, reducing a 26.6% loss to 10.1%; the average length by nearly 70%, 39.3 to 12.2 months; and the average recovery time by more than 50%, 4.3 to 2.1 months. For all statistics SV management makes the drawdown statistics worse. So, while it may not generate higher returns or better Sharpe ratios, AV management of a commodity investment may allow better sleep at night.

[11] While AV does outperform SV management for the Bloomberg Commodity index, it generates a lower return and worse Sharpe ratio than the buy and hold strategy. This is certainly disappointing, however it may be expected. Gorton and Rouwenhorst (2006); Buyuksahin, Haigh, and Robe (2008); Bhardwaj, Gorton, and Rouwenhorst (2015) document the lack of relationship between equity and commodity returns. This is likely the genesis for the advice suggesting commodities as a portfolio hedge. Additionally, as Erb and Harvey (Forthcoming) show commodity returns are linked to income returns and not prices and link to wealth in a different manner than equities. Thus, it is possible that the systematic risk to investor wealth equity AV management times is different from the wealth risk related to the performance of commodity investments.

Finally, shown in appendix table E.6, equity AV management of the other asset classes is cheaper than SV management. The equity AV and SV management signals carryover the same relationship from prior performance tables, the AV managed investments have less turnover and thus incur lower transaction costs than SV. This means that they are able to tolerate higher trading costs and have higher break even costs. Neither strategy improves commodity investment returns so neither strategy can tolerate even zero trading costs when managing the Bloomberg commodity index. However, the SV managed investments in the Deutsche Bank Currency and Currency Momentum indices cannot even tolerate zero trading costs while the break even costs for all of the AV managed investments are reasonable.

Clearly, the use of equity AV to manage investments in other asset classes is better than the use of equity SV. The difference clearly indicate something qualitatively different in AV management compared to SV. Returns, Sharpe ratios, drawdown statistics and trading costs are better in currency, real estate, and bond investments. There may be no benefit to AV management in commodities over the buy and hold, but SV management appears to be clearly worse than the buy and hold and AV management is a significant improvement over SV. This is likely due to commodity returns stemming from a risk unrelated to the risk identified by equity returns as detailed in the diversification and return generation arguments in Gorton and Rouwenhorst (2006); Buyuksahin et al. (2008); Bhardwaj et al. (2015); Erb and Harvey (Forthcoming).

As the argument is AV signals dynamic change in the relationship of systematic risk and return, it should also be possible to use the AV of returns from other assets to manage equity returns. I test this using the 10 largest, by trading volume, currency pairs and bond returns from Financial Industry Regulatory Authority's (FINRA's) Trade Reporting and Compliance Engine (TRACE) through WRDS, for each AV is the trade volume weighted average variance.[12] As table X shows using either AV calculated from either currency or bond returns equity returns are higher than the buy and hold strategy. The average annualized returns nearly double from 4.914% to 8.416% and 8.582%. The cross asset performance supports the notion that AV signals changes in the amount of systematic risk across the economy. Additionally, AV returns should display dependence on market β and aggregate wealth as implied by Pollet and Wilson (2010).

B. Return Dependence

The importance of the relationship of the stock market and aggregate wealth to the performance of the AV managed strategy should show up in the AV managed returns systematically. By the arguments made in Pollet and Wilson (2010) and implied in the

[12]A full history of currency data was not available. I used volume snapshots found on mos (2013, 2015, 2016); Bradfield (2019). For robustness, I test equally weighted AV from both currency and bond returns and the performance of equity returns is still better.

sub-sample regressions above, AV management will under-perform where the stock market return is a smaller portion and less representative of returns to aggregate wealth.

GDP per capita often appears as a control in cross country studies of wealth and income effects. (Barro, 1989; Levine and Zervos, 1993; Baird, Friedman, and Schady, 2010) Thus, the market capitalization to GDP per capita ratio should serve as a useful, though imperfect, proxy for the proportion of aggregate wealth represented by the stock market. From January 1975 to December 2015, I use periods when the ratio is above the median against when it is below to test the importance of the ratio in the time series of AV managed US equity returns. As seen in table XI panel (a), AV management produces positive returns in both sub-samples. However, returns and the Sharpe ratio are better when the ratio is higher as expected from equation (6) from Pollet and Wilson (2010). Moreover, high ratio periods have positive and significant Fama-French three, five-factor, and five-factor plus Momentum alphas meaning the relationship of the stock market to aggregate wealth is systematically important as expected from equation (8) from Pollet and Wilson (2010).

To test this hypothesis across countries, I use Credit Suisse's annual reports on global wealth.[13] This should be a better proxy than market capitalization to GDP per capita, however this will still not be perfect as it depends on how well Credit Suisse's measure of wealth captures all of aggregate wealth[14]. Each year, starting in 2005, I rank countries on the ratio of annual market return to return on annual wealth. I form portfolios long in the above median ratio countries, short in the below and a long minus short using both. For robustness, I also use market capitalization to GDP per capita. If the relationship of the index returns and aggregate wealth is systematically important to AV returns in the direction expected, the AV managed strategy should do better in countries with above median market to GDP ratio. Thus, a strategy long the high ratio countries and short the low ratio countries should have a positive and significant alpha.

As seen in table XI panel (b), AV management produces positive returns across all countries so the performance of the long-short strategy will depend on the long side producing significantly better performance than the short. The long-short AV strategy performs just as expected. Using the Credit Suisse wealth numbers, investors capture average annualized returns over 12.6% with a Sharpe ratio above 0.74 on the long side. The long-short portfolio nets investors better than 5% annualized return with a Sharpe ratio better than the US buy and hold equity return. Moreover, the portfolio has a positive and significant Fama-French three, five, and five-factor plus Momentum alphas meaning the relationship of the stock market to aggregate wealth is systematically important to AV management as expected from the arguments in Pollet and Wilson (2010). These results hold for the market capitalization to GDP per capita sorting strategy as well.

[13]The reports are available for 2011 through 2017, covering data from 2000 to 2017, at https://www.credit-suisse.com/corporate/en/research/research-institute/global-wealth-report.html

[14]Yet another manifestation of the joint hypothesis problem.

C. In-Sample

Average variance is an auto-correlated time series; this opens the possibility that predictive regressions using AV have estimation bias as highlighted in Stambaugh (1999). Campbell and Hentschel (1992) show that the Stambaugh bias in predictive regressions involving volatility measures and future returns can be particularly severe because of a volatility feedback effect. This may also be compromise the direct conclusion that the performance of AV management means AV and AC tell us about changes in risk. To eliminate the Stambaugh bias in the estimated coefficients on AV, I follow the methodology in Amihud and Hurvich (2004) and further make the p-values used for coefficient significance robust through wild-bootstrapping as detailed in MacKinnon (2002). Robust coefficients and p-values are presented in all in-sample regression tables.

In each in-sample regression, all of the information available in the sample is used to estimate the parameters. In general, the regressions take this form:

$$y_{t+1} = \alpha + \beta x_t + \epsilon_t. \tag{11}$$

The contemporaneous regressions decomposing market variance are left unreported. The results show the same relationships found in Pollet and Wilson (2010) table 2. For all in-sample regressions, the series are standardized to a mean of zero and standard deviation of one.

Appendix table C.1 contains the results of regressions run on the full sample from 1926 to 2016. The results largely support the quarterly regressions in Pollet and Wilson (2010). AV is a significant predictor of next months SV in all specifications and slightly better in terms of R^2. This months AV even remains significant in the specification including this months SV and the inclusion of SV appears to be of little to no help as the adjusted R^2 increases only slightly. there is a definite advantage to using this months AV in the prediction of next months average variance. The adjusted R^2 of this month's AV is 51.5% versus 36.7%. When both AV and SV are included in the predictive regression, AV retains significance and SV does not. Investors are certainly no worse off using AV in the prediction of next month's SV and are better when predicting next months AV. Hence, investors have a signal at least as good for overall risk and better for uncompensated risk. AC is a significant predictor of higher returns in the next month. So, investors lose out on slightly higher returns by divesting when SV is high because AC is high. AV management avoids this. AV is not significantly related to returns in any specification. When included with AC, AV is insignificant. While no return specification is promising as a return timing strategy, the ability to manage risk without giving up return makes AV management a better strategy than SV.

D. Regression Sub-sampling

The CRSP daily return data covers only returns for assets traded on the New York Stock Exchange (NYSE) prior to 1962. This makes the pre-1962 data very different from the post-1962 data. The earlier data is much shallower having months with fewer than 400 assets total that meet my data requirements. Given the value of the assets traded outside the NYSE, as much as 13% of traded securities by market capitalization are missing from the CRSP data as of the 1950s. (NYSE, 2016; Investopedia, 2003) Twice as many firms covering twice as many industries are available in July 1962 versus June. As shown in Taylor (2014) the NYSE market was not a significant part of marginal wealth in the US following the Great Depression and before the late 1950s. And, as documented in Jones (2002) the pre-1962 period is significantly and persistently more illiquid. Merrill Lynch and the NYSE began its first monthly investment plan and "Own Your Share" advertising push in 1954 with the goal of linking Wall Street and Main Street; the program included commission discounts and automatic dividend reinvestment and the push more than doubled the number of investors in the US from 1954 to 1968. (Staff, 1964; Traflet, 2003) Thus, AC calculated from daily returns prior to 1962 is not likely to be a good proxy for systematic risk. SV and AV will either be measures of the same risk or SV may outperform AV. This means regressions on the relationship of AC and future returns in the 1926 to 1962 sub-sample can provide us a placebo-like test suggesting that AV management works when systematic risk is better proxied.

Table XII presents return prediction regressions for the 1926 to 1962, pre-1962, and post-1962 CRSP samples. In the post-1962 sample AC is positively and significantly related to next month's return. Both AV and SV are both significantly related to future SV with nearly the same coefficient, .550 and .556, and similar R^2 values, 29.6% and 30.3%. As in the full sample, AV is a better predictor than SV for next month's AV. Investors are served as well by AV as SV in timing risk and as AC is related to higher future returns, investor have the opportunity to stay invested in good times using AV management. In contrast, in the pre-1962 period, AC is not significantly related to future returns. The coefficient on AC is actually negative but insignificant. The relationships between AV, SV and future risk remain the same but as suspected AC no longer signals compensated risk. This supports the understanding that the ability of AC, and thus AV, to signal changes in the risk mix depends on the relationship of the used index returns and aggregate wealth, the Pollet and Wilson (2010) argument.

The full in-sample regression monthly results support the conclusions reached by Pollet and Wilson (2010). Using the intuition in the argument for AC as a signal of systematic risk which depends on the relationship of the market index and aggregate wealth, I demonstrate a placebo-like sub-sample with an expected lack of results, a difference in US equity performance, and a systematic difference in AV managed returns across countries. Each of these results suggest AV management works by allowing investors to time changes in the mix of systematic risk in market index variance. However, in-sample relationships are not sufficient to know that investors have access to the dynamics of this relationship in real-time. It is well documented that many in-sample strategies do not work out-of-sample, and there is no real-time information.

E. *Out-of-Sample*

As Welch and Goyal (2008) definitively show, out-of-sample performance is not guaranteed by in-sample performance and is essential to understanding any investment strategy which generates positive returns. To determine the out-of-sample relationships between market and average variance, average correlation and returns, I run regressions of the standard form

$$y_{t+1} = \alpha_t + \beta_t x_t + \epsilon_t \tag{12}$$

where α_t and β_t are estimated with from the data available only until time t. That is, I estimate α_t and β_t by regressing $\{y_{s+1}\}_{s=1}^{t-1}$ on a constant and $\{x_s\}_{s=1}^{t-1}$. In all the reported results, I follow an expanding window approach so that for the next period $t+2$, y_{t+2} is estimated as $\alpha_{t+1} + \beta_{t+1}x_{t+1}$, where α_{t+1} and β_{t+1} by regressing $\{y_{s+1}\}_{s=1}^{t}$ on a constant and $\{x_s\}_{s=1}^{t}$. I follow this process for all subsequent months. However, as part of a test on the robustness of the out-of-sample results, I demonstrate that the results do not depend on the use of an expanding window. Most critically, equation (12) prevents any look-ahead bias. The out-of-sample prediction tests use the same set of variables as the in sample tests.

Each out-of-sample test requires an in-sample training period in which parameters are estimated using all the data up to the time period before the first out-of-sample month. For consistency, the first 15% of the data is used as the initial parameter estimation period and then I move through the remaining observations recursively generating out-of-sample predictions. Three measures of out-of-sample performance are estimated. I use the Diebold and Mariano (1995) statistic and McCracken (2007) MSE-F as measures of the increased accuracy of AV based forecasts compared to forecasts from SV as a benchmark. The DM statistic is defined as:

$$DM = \frac{\bar{d}}{\sqrt{\frac{2\pi f_d(0)}{T}}} \tag{13}$$

where $\bar{d}$ is the mean difference in the loss differential. The loss differential is the function used to measure the difference between the forecast and actual values. I use the squared forecast error, $(y_t - \hat{y}_t)^2$. So, $\bar{d}$ is the mean value of the difference between the squared error using AV and the squared error using the benchmark forecast from SV.

$$\bar{d} = \frac{1}{T}\sum_{\tau=1}^{T}((y_t - \hat{y}_{AV,t})^2 - (y_t - \hat{y}_{SV,t})^2) \tag{14}$$

I use the same consistent estimator for the mean loss differential, $f_d(0)$ as in Diebold and Mariano (1995). The statistic is normally distributed under the null hypothesis of no difference in accuracy between the benchmark and proposed model. The standard positive critical values from the normal Z-table serve as cutoffs to establish a significant improvement in forecast accuracy. MSFE is mean squared forecast error when a benchmark model is used to generate out-of-sample predictions. Mean squared forecast error is defined as

$$MSFE_x = \frac{1}{T}\sum_{\tau=t}^{T}(y_\tau - \hat{y}_\tau^x)^2 \tag{15}$$

where $\hat{y}_t^x$ is the out-of-sample prediction of y_t generated from the a model using variable x, t is the first out-of-sample prediction time period, and T is the total number of out-of-sample time periods. The F-statistic in McCracken (2007) is calculated by

$$MSE - F = T\frac{MSFE_x - MSFE_b}{MSFE_b}. \tag{16}$$

The significance of the F-statistic is determined from bootstrapped values provided in McCracken (2007). Each of these two tests depends on the reduction of average squared error by the predictor x relative to a benchmark, b, model. The final measure is a forecast encompassing statistic.

The third statistic is an encompassing test statistic. Encompassing tests the more stringent requirement that the benchmark forecasts contain no useful information absent in the forecasts of variable x. Forecast encompassing tests come from the literature on optimal forecast combination. (Chong and Hendry, 1986; Fair and Shiller, 1990) An optimal forecast as a convex combination of two forecasts for time period t +1 defined as

$$\hat{y}_t^* = (1-\lambda)\hat{y}_t^b + \lambda\hat{y}_t^x \tag{17}$$

where $\hat{y}_t^x$ are predicted values generated from the model using variable x and $\hat{y}_t^b$ are forecasts from the benchmark model. I use the forecast encompassing test of Harvey et al. (1998), ENC-HLN. The encompassing test of Harvey et al. (1998) directly tests the value and significance of the forecast combination λ. The test procedure rests on the calculation of a modification to the Diebold and Mariano (1995) test statistic and the consistent estimation of the long-run covariance between the difference in forecast error between the benchmark model and a model based on a competing variable, x. As such there is no one line equation that sums up the statistic used to judge the significance of λ. However, intuitively λ must be significantly different from zero for AV to have information above and beyond the forecasting information in SV and values close to one indicate that AV has all of the relevant information in SV and is optimal by itself.

[Place Table XIII about here]

The results in table XIII panels (a) and (b) show that AV is a significantly better out-of-sample predictor of AV and SV. For both the variables, all three measures of out-of-sample performance show significant improvement in both the out-of-sample period starting in 1970, corresponding to the post-1962 in-sample period, and starting in 1939, the full sample. The forecast encompassing tests also show that AV contains all the forecasting information in SV and is optimal on its own. This means that investors concerned about the variance in the returns on their investment in the market are better off using this month's level of average asset variance to hedge next month's stock market variance than using this month's stock market variance. AV is also a significantly better predictor of next month's log excess return for the out-of-sample period starting in 1970. The DM statistic, 1.278, is nearly significant at the 10% level and both the MSE-F test and encompassing tests show significant improvement for AV. Again the encompassing test shows that AV is optimal alone and no weight needs to be given to SV in the prediction of log excess returns. Over the period starting July 1970, investors attempting to predict next month's returns would have been better off using this month's average asset variance rather than total market variance and investors deleveraging based on high values of AV would have done better avoiding negative returns rather than deleveraging based on high values of SV. Supporting the results seen in the in-sample tests, across the longer out-of-sample period, starting in 1939, AV is not a significant improvement over SV in the prediction of next month's log excess return.

It is important to remember that these out-of-sample statistics are measures relative to the SV benchmark. As with the in-sample results neither AV nor SV predict future returns well. There are many other variables better suited. (Campbell and Thompson, 2008; Rapach et al., 2016; Deuskar et al., 2017) The results for the single specification of a 15% training window and expanding window regressions starting from 1939 shown in XIII panel (b) allow for the possibility that AV is no improvement in real-time information on risk-return dynamics over SV; AV is most likely an improvement on the prediction of risk given the significant MSE-F and ENC-HLN results in panel (b) but may actually misinform investors about future returns. To address this issue, I calculate robust out-of-sample statistics.

F. Robust Out-of-Sample

Out of sample estimation always raises issues with the choices made in the specification of the model and how to split the data into in and out of sample windows. Bluntly speaking, there are no good answers. The standard practice as in Rapach and Zhou (2013), Rapach et al. (2010), Rapach et al. (2016), and Huang et al. (2015), and many others, is to show performance in a few sub-samples split by dates that the authors choose. One of the concerns with sub-sample selection is that the window may be "ad-hoc" and the selection may mask significant results that would appear if the sub-samples had been constructed differently. A second, more cynical, concern is that the presented sub-sample may represent significant performance that has been found either by chance or as the result of analyzing many sub-sample and only presenting the significant results. In any case, evaluation of the differences in performance across sub-samples is often left to the imagination of the reader and whatever importance they place on the first half of the sample versus the second, the middle third versus the first and last thirds or however the data has been separated. While the selection of 1962 is not arbitrary, we have seen already a difference in return prediction performance for AV between the period after 1962 and the whole data set starting in 1926 which raises the question of the robustness of the out-of-sample results. Is the lack of significance in the pre-1962 data indicative of instability, or worse, is the relationship between AC, AV and future risk and returns or is this simply type II error, a lack of result in a specific sub-sample for a specific regression specification? Are the strong out-of-sample results for the AV predictions of future risk and return type I error, the result of a favorable data and regression specification leading to significance where none actually exists? To resolve these issues we need out-of-sample results robust to the data window specification.

To address the robustness of the out of sample results, I use out-of-sample statistics robust to both the specification of the prediction model, either expanding or rolling, and the choice of prediction window. Rossi and Inoue (2012) formulate out of sample statistics robust to the choice of split between in and out of sample periods and regression specification. The paper presents the calculation of the Diebold and Mariano (1995) statistic and the Harvey et al. (1998) encompassing test such that the choice of out-of-sample starting period is eliminated as a nuisance parameter and the asymptotic behavior of the statistics can be used to measure their observed significance. Fundamentally, this involves the calculation of each of the statistics for all feasible out-of-sample windows and in the case of rolling regression specification all feasible window sizes. The modifications are different for each of the statistics and the calculation of the robust statistic is different depending both on which statistic and which concern is being addressed. When the concern is that the chosen window could be overly optimal, perhaps the best results of many tests, then it is possible that the null of no improvement is rejected based on the calculated statistics when in general it is true. To eliminate this possibility, Rossi and Inoue (2012) provide the R_T measure which essentially insures that the highest calculated statistic is so extreme that it could not occur without a underlying significant improvement in forecast accuracy over the benchmark model. The A_T measure insures that the average calculated statistic is large enough that an arbitrarily selected out-of-sample starting period would not lead to the failure to reject the null of no accuracy improvement when it was indeed false. This two measures tackle the type I and II error questions. Given the results thus far, we will be looking for significant R_T values to support the significant ability of AV to predict stock market returns, SV and AV across the data set, and significant A_T values to tell us that the significant ability of AV to predict log excess returns, seen from 1970 forward, is indicative of a real accuracy improvement while the lack of performance when including predictions from 1939 forward is simply a noisy period of poor performance obfuscating the superiority of the AV based model.

Table XIII panel (c) shows the robust out-of-sample statistics. The R_T and A_T statistics for the comparison of all possible expanding window forecasting models using AV with in-sample training windows of at least 15% of the data and out-of-sample forecasting periods of at least 15% of the data against a benchmark model using SV with the same specifications. The proportional data cut offs are necessary to use the critical values provided in the Rossi and Inoue (2012) paper, 15% is the smallest, and mean that the first feasible specification starts forecasting in December 1939 as in the out-of-sample results shown above and forecasts for at least April 2003 to December 2016 are made. Every DM statistic is significant indicating that the AV model is an improvement in forecasting accuracy for all variable of interest. Every encompassing test statistic is significant, however these are not λ values directly so while we know that AV contains information over and above SV they do not directly indicate it is optimal alone. The results for comparisons of rolling window specification tests are shown in the appendix. Overall, the out-of-sample results show that AV is a better signal of the dynamic risk-return trade-off. The relationships between AC and AV, compensated systematic risk and uncompensated, are robust to model specification and testing construction through out the full sample.

Regression results, both in-sample and out-of-sample, support the performance results detailed before. There is a robust dynamic relationship between AV and future risk. AV is more informative than SV of changes in future risk and return. The AV managed return performance, in US equities, in global equities, and across asset classes is backed by signals of changes in the dynamic mix of compensated and uncompensated risk.

VI. Conclusion

Weighting investment by the inverse of AV rather than SV increases market investment when total variance is expected to be low and decreases when higher total variance and lower returns are expected. The results are better Sharpe, Sortino and Kappa ratios with better Fama-French alphas. With better access to investment timing on compensated risk, investors capture more utility with lower costs. AV managed portfolio investors, as well as fund managers, are more protected agains drawdowns. I analyze the relationship between

AV, the average variance of individual asset returns, and future risk and future returns to reveal the mechanism that makes AV a better timing signal. AV is a significant predictor of future portfolio variance, controlling for current variance, both in and out of sample. In contrast, AV is not significantly related to future returns and thus serves as an investment timing signal. By using the decomposition of market variance, I add a better portfolio management signal to the literature, because AV management better aligns investment to times of compensated risk. These investment benefits manifest because the AV managed portfolio takes advantage of the correlation risk and return dynamics rather than the portfolio variance and return dynamics.

AV management outperforms across the globe. Returns, Sharpe ratios, drawdown statistics and costs are better in 9 of 10 countries studied. Performance is also better for a globally diversified equity investment. Not only is the average variance of equity returns better for the management of equity investments but it is better across asset classes. AV generates better performance in currency, real estate, and bond index investment supporting the argument that AV management better aligns investment to times that compensated systematic risk is higher. SV management fails to perform across asset classes, in many cases its worse than the buy and hold. Hence, the AV managed portfolio adds another dimension to the risk-return literature.

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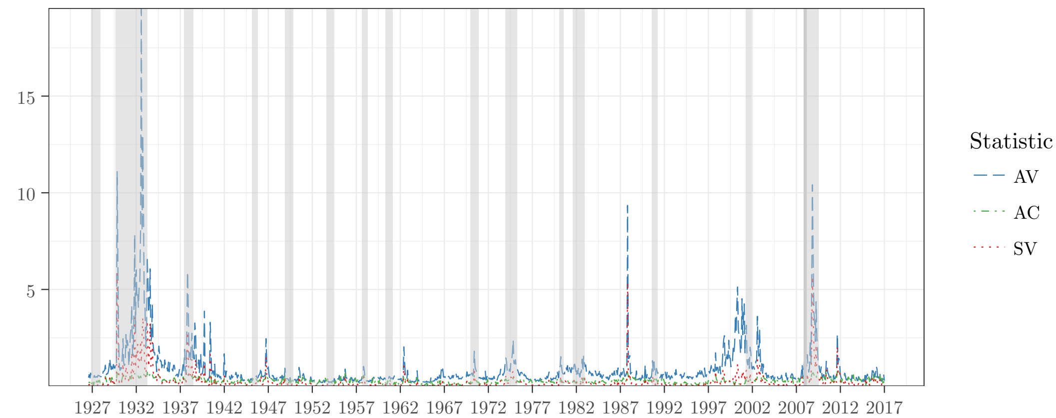
Figure 1. Time Series of Market Statistics: Panel (a) the time series of the total variance of market returns, AV is the average variance of the daily returns of individual assets in percentage; SV is the total variance of the daily market return in percentage, and AC is the average pairwise correlation of daily asset returns in the market. Panel (b) shows the ratio of AV to SV.

(a) Monthly Measures of Daily Return Statistics



(b) AV Investment Weight Minus SV Investment Weight

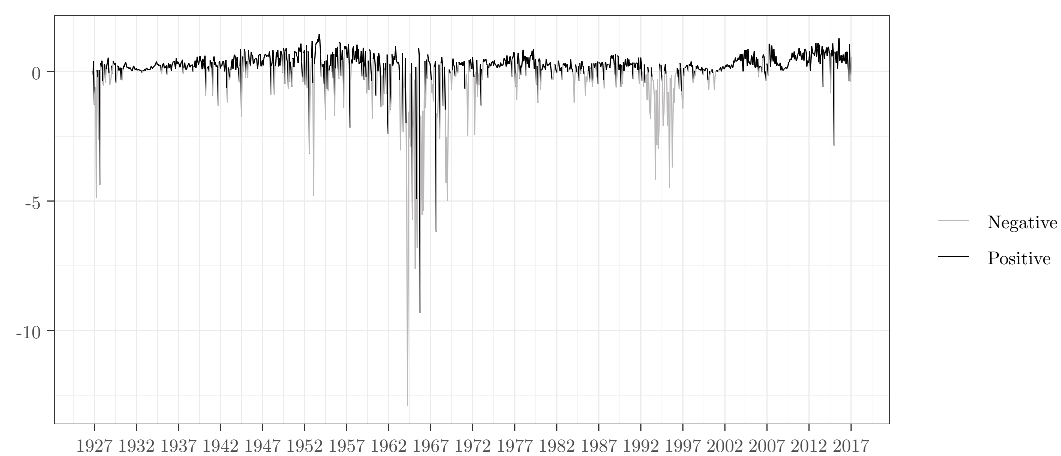


Figure 2. Time Series of Investment Weights: The time series of the investment weight into the market portfolio for SV and AV managed portfolios. c_{10} and c_{12} represents the weights when the SV and AV strategies target 10% and 12% annual volatilities while c_{BH} represent targeting the buy and hold annual volatility over the 1926-2016 holding period.

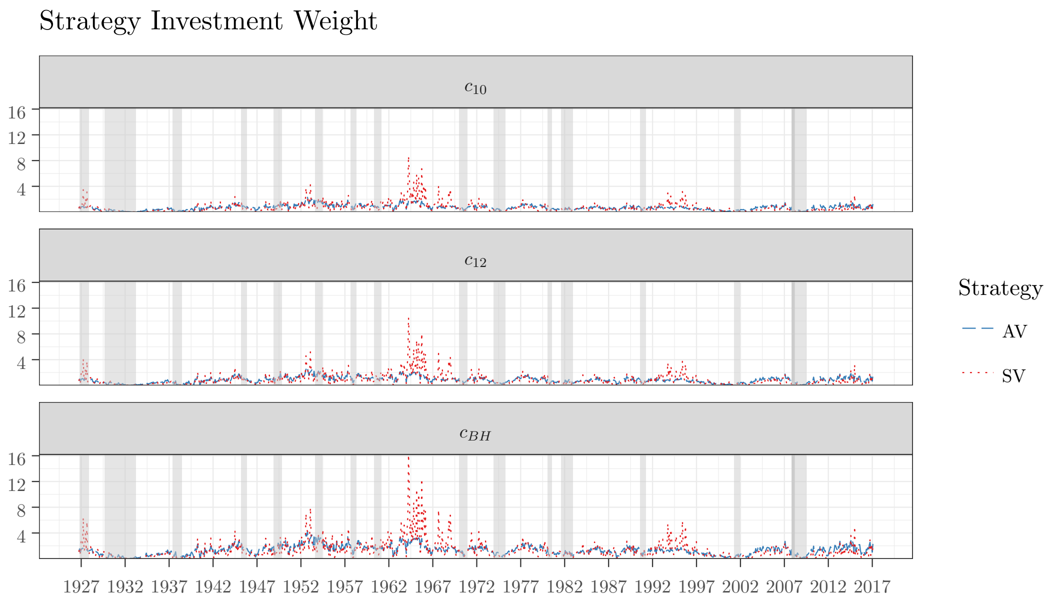


Figure 3. Cumulative Log Excess Returns: The time series of cumulative log excess returns for the buy and hold market investment as well as the AV and SV managed portfolios. Panel (a) limits the coefficient on the market portfolio between 0 and 1.5 for the AV and SV strategies; panel (b) limit them to weights from 0 to 3 and they are unconstrained in panel (c).

Figure 4. Portfolio Drawdowns: The time series of cumulative drawdowns, loss in portfolio value, for the buy and hold market investment as well as the AV and SV managed portfolios. SV and AV managed portfolios are targeting the buy and hold market volatility and have no investment constraints. See section III.B for details.

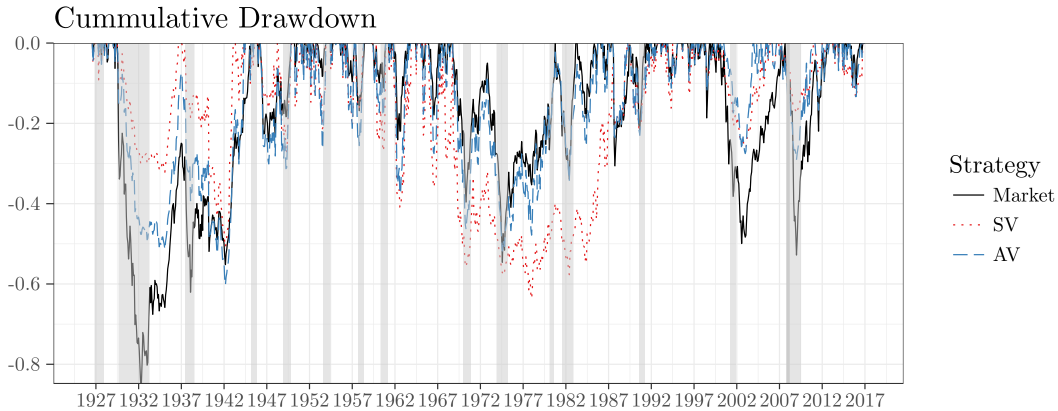


Figure 5. CER Losses: This figure displays the loss of certainty equivalent return, for a mean-variance investor, as leverage constraints are applied to the AV and SV managed portfolios. The losses are expressed in percentage point differences of the constrained returns for investors with γ risk aversion coefficients of 1, 2.5, and 5 against the baseline of an unconstrained investor employing the AV or SV strategy. See section IV.C for details.

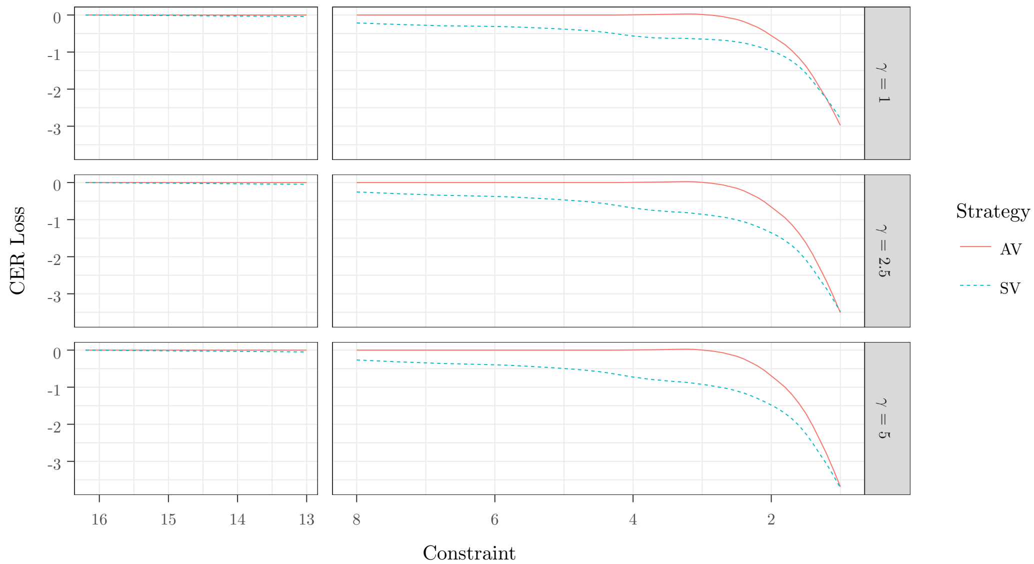


Figure 6. CER Gains: Certainty Equivalent Return gains for mean variance investors with risk aversion coefficients ranging from 1 to 5 and subject to investment constraints ranging from 1 to 3. These gains are calculated as the CER increase for the investor changing from the SV baseline to the AV investment strategy. See section IV.C for details.

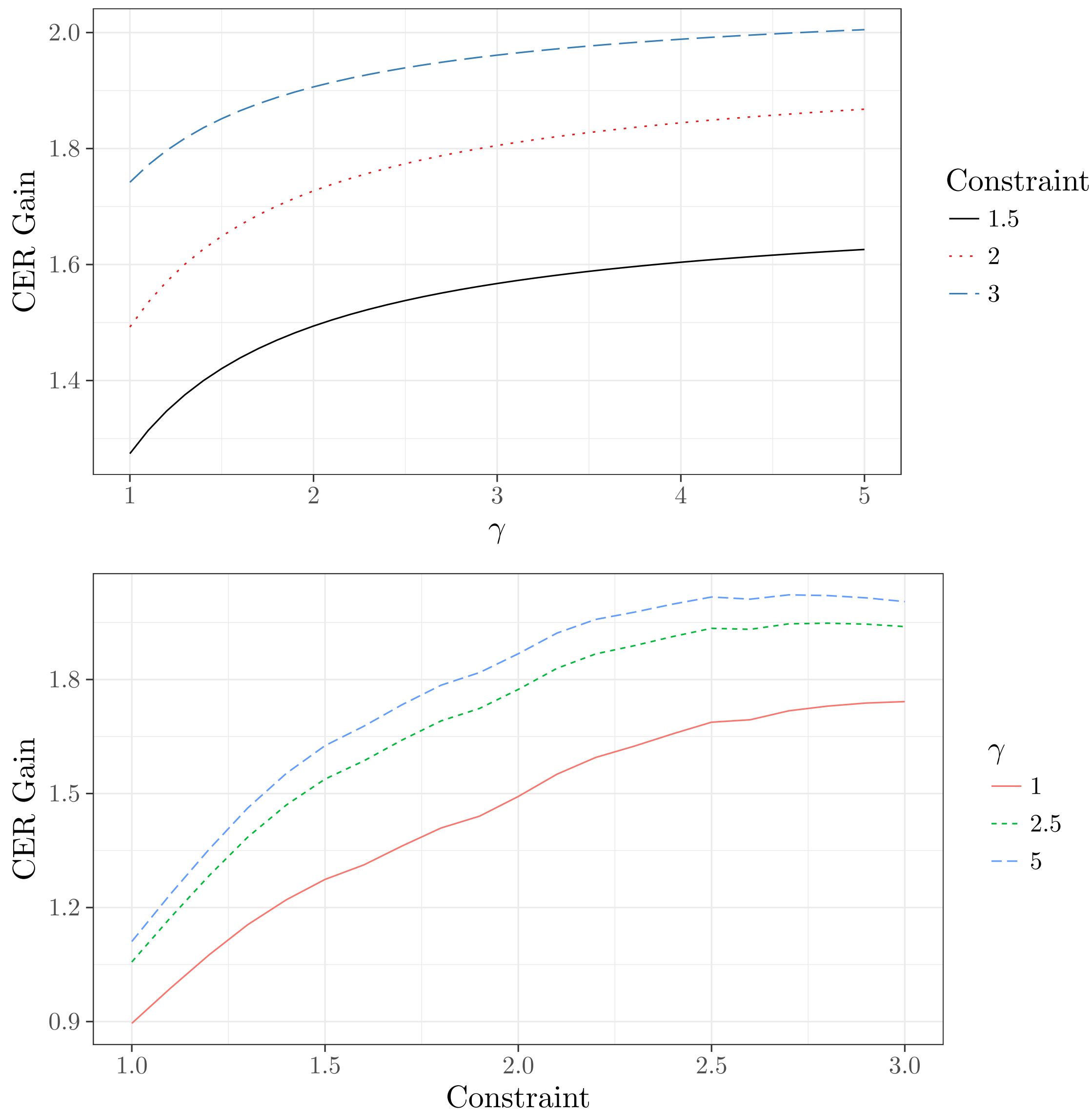


Figure 7. These figures show the smoothed kernel density PDF estimates and empirical CDFs calculated for the returns to the unconstrained AV and SV managed US equity portfolios. The CDF functions are shown to illuminate the stochastic dominance tests which follow. See section IV.D

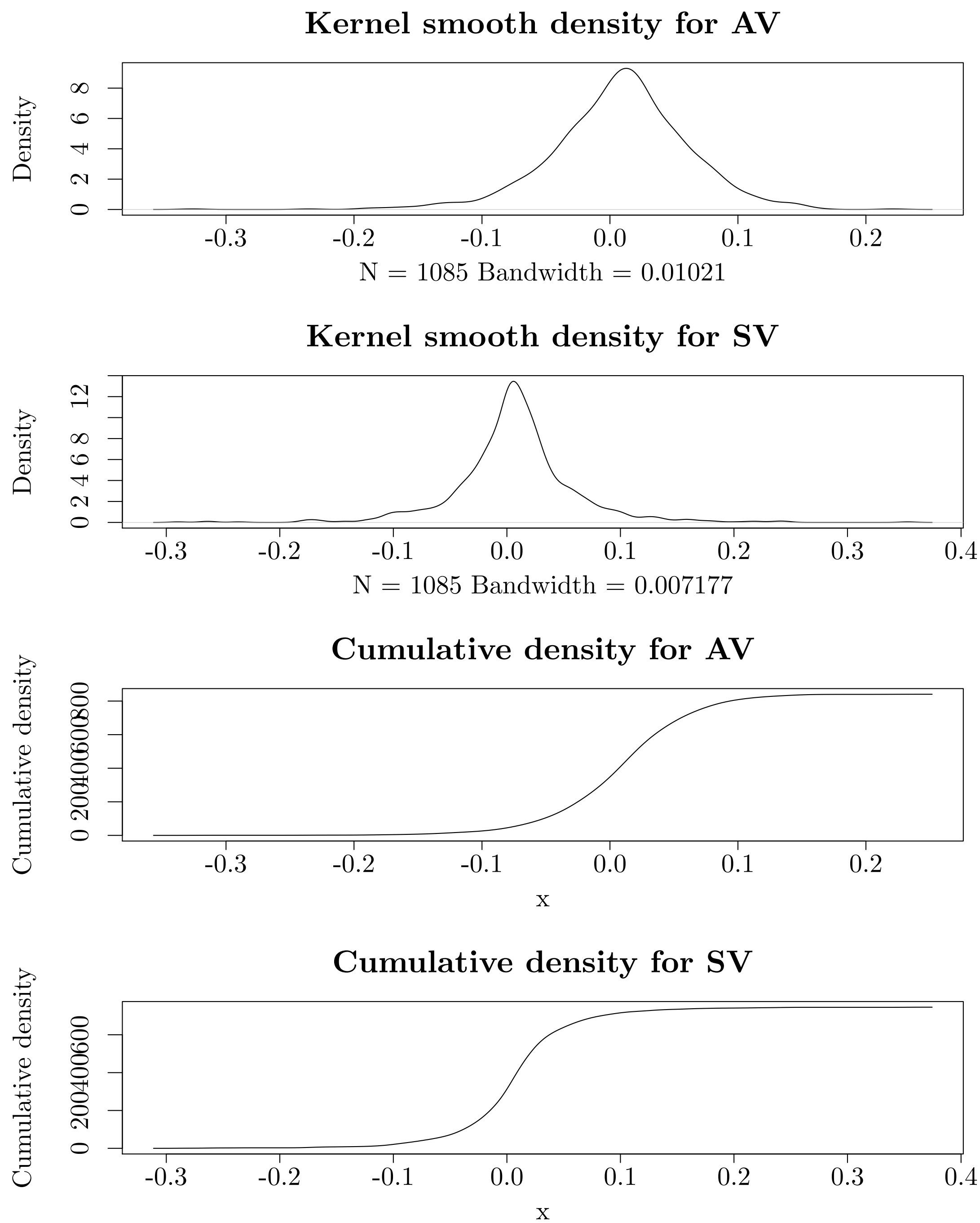


Figure 8. Non-EUT Investor Decision Weights: This figure illustrates the incorporation of non-EUT investor preferences in the decision between two investment portfolios used in the stochastic dominance analysis. The cumulative probability, p, of the rank order statistics of the portfolio returns are on the x-axis. The perceived utility weight, W(p,α), or importance placed on the possibility of incurring a return with probability p. The parameter α measures EUT compliance, $\alpha = 1$ when investors care equally about all returns and $\alpha < 1$ when investors care about extreme loses "more than they should". See section IV.D

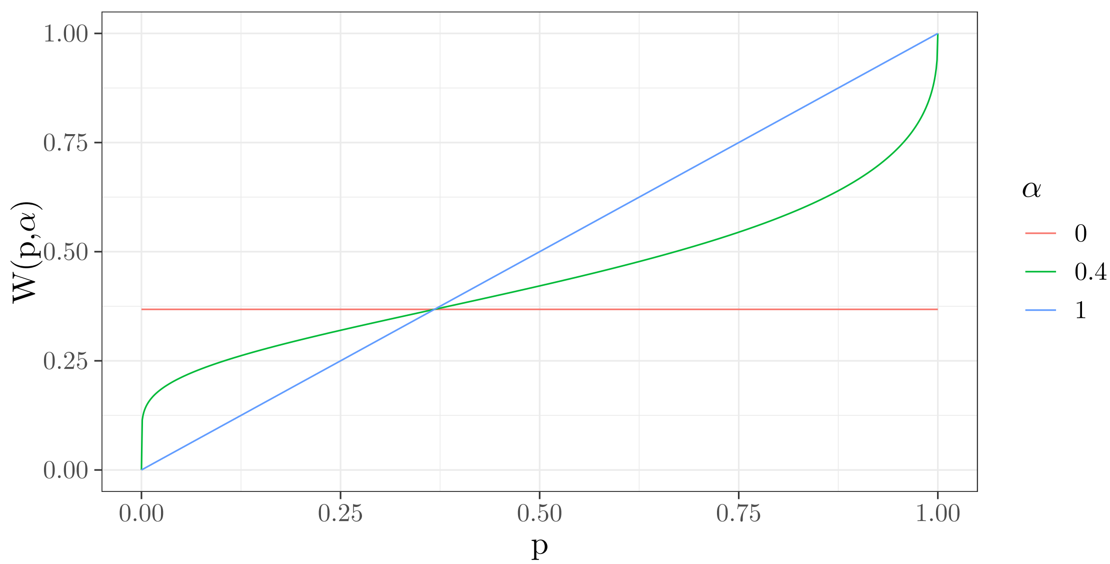


Table I:Summary statistics

Panels (a) - (c) display summary statistics for the market variance and correlation statistics, and returns. RET is the log excess return of the CRSP market portfolio. AC is the average pairwise correlation of the daily returns of the 500 largest firms in the CRSP data set over the month or quarter. AV is the average of the individual variances of daily returns for the 500 largest firms in the CRSP data set. SV is the variance of daily CRSP market returns. See section II for details on construction. Panel (d) displays summary statistics for international data used. The start year and month, number of months, name of the market proxy index, and the average number of assets meeting the trading and liquidity requirements for each country where the performance of AV and SV managed equity portfolios.

(a) Pollet and Wilson Sample 1963Q1:2006Q4

Statistic	N	Mean	St. Dev.	Min	Max	Autocorrelation
RET	176	1.169	8.372	−30.072	19.956	0.000
AC	176	0.231	0.091	0.034	0.648	0.556
AV	176	2.221	1.827	0.634	12.044	0.695
SV	176	0.484	0.615	0.029	6.397	0.310

(b) Sample 1962M6:2016M12

Statistic	N	Mean	St. Dev.	Min	Max	Autocorrelation
RET	655	0.409	4.453	−25.985	14.515	0.081
AC	655	0.261	0.129	0.019	0.762	0.620
AV	655	0.770	0.849	0.198	10.416	0.667
SV	655	0.200	0.406	0.006	5.664	0.551

(c) Full Sample 1926M7:2016M12

Statistic	N	Mean	St. Dev.	Min	Max	Autocorrelation
RET	1,085	0.495	5.371	−34.523	33.188	0.106
AC	1,085	0.276	0.134	0.019	0.762	0.610
AV	1,085	0.881	1.281	0.154	19.540	0.718
SV	1,085	0.248	0.502	0.006	5.808	0.612

(d) Country Indices - Summary

Country	Start	Months	Index	Assests
AUS	2000 - 5	212	ASX	200
BRA	1995 - 2	275	iShares MSCI Brazil ETF	60
CHN	2005 - 5	152	CSI 300	300
DEU	1993 - 11	290	HDAX	110
FRA	1993 - 9	292	SBF 120	120
IND	2000 - 5	212	Nifty 50	50
ITA	2003 - 8	173	FTSE MIB	40
JPN	1993 - 6	295	Nikkei	255
UK	1993 - 6	295	FTSE	100
World	1995 - 3	274	MSCI ACWI	1735

Table II:Correlations

Panel (a) displays Pearson correlation statistics for the market variance and correlation statistics, and returns. RET is the log excess return of the CRSP market portfolio. AC is the average pairwise correlation of the daily returns of the 500 largest firms in the CRSP data set over the month or quarter. AV is the average of the individual variances of daily returns for the 500 largest firms in the CRSP data set. SV is the variance of daily CRSP market returns. See section II for details on construction. Panel (b) shows the Pearson correlation statistics for the excess log returns for each country. Panel (c) shows the correlation statistics for the excess returns of the other asset classes with the US and World excess log returns. See section V.A for details.

(a) US 1926M7:2016M12

	RET	AC	AV	SV	RET_{t+1}	AC_{t+1}	AV_{t+1}	SV_{t+1}
RET	1	0	0	0	0	0	0	0
AC	-0.295	1	0	0	0	0	0	0
AV	-0.136	0.467	1	0	0	0	0	0
SV	-0.279	0.619	0.857	1	0	0	0	0
RET_{t+1}	0.106	0.011	0	-0.057	1	0	0	0
AC_{t+1}	-0.229	0.610	0.383	0.453	-0.295	1	0	0
AV_{t+1}	-0.191	0.358	0.718	0.607	-0.136	0.467	1	0
SV_{t+1}	-0.259	0.416	0.625	0.612	-0.279	0.619	0.857	1

(b) Global Equities

	AUS	BRA	CHN	DEU	FRA	IND	ITA	JPN	UK	USA
AUS	1									
BRA	0.590	1								
CHN	0.381	0.390	1							
DEU	0.663	0.574	0.336	1						
FRA	0.714	0.539	0.292	0.899	1					
IND	0.575	0.576	0.363	0.538	0.538	1				
ITA	0.665	0.448	0.288	0.823	0.885	0.547	1			
JPN	0.616	0.427	0.339	0.543	0.547	0.555	0.608	1		
UK	0.713	0.614	0.279	0.769	0.812	0.552	0.735	0.489	1	
USA	0.744	0.604	0.365	0.771	0.774	0.573	0.719	0.568	0.802	1
World	0.781	0.686	0.381	0.801	0.831	0.635	0.786	0.657	0.862	0.956

(c) Global and Other Asset Classes

	USA	World	$Dollar_{BB}$	$Curr_{DB}$	$Carry_{DB}$	Mom_{DB}	$REIT_{SP}$	$Comm_{BB}$	$Bonds_{Univ}$
USA	1								
World	0.956	1							
$Dollar_{BB}$	-0.557	-0.669	1						
$Curr_{DB}$	0.221	0.205	0.047	1					
$Carry_{DB}$	-0.044	-0.107	0.033	0.216	1				
Mom_{DB}	-0.188	-0.193	0.234	0.557	0.289	1			
$REIT_{SP}$	0.763	0.736	-0.448	0.032	0.004	-0.173	1		
$Comm_{BB}$	0.518	0.611	-0.712	0.017	-0.116	-0.177	0.345	1	
$Bonds_{Univ}$	0.231	0.283	-0.358	-0.068	-0.140	-0.225	0.380	0.197	1

Table III:Investment Weights

This table displays summary statistics for the time series of investment weights taken in the market index by both the AV and SV managed portfolio strategies with different volatility targets. c_{BH} represents targeting the annual volatility of the buy and hold market portfolio over the whole data set, 1926 to 2016. c_{10} and c_{12} target, approximately, 10% and 12% annual return volatility for the AV and SV managed portfolios.

Portfolio	Target	Mean	St. Dev.	Min	Pctl(25)	Median	Pctl(75)	Max
SV	c_{10}	0.697	0.762	0.009	0.246	0.512	0.874	8.743
AV	c_{10}	0.702	0.383	0.018	0.425	0.667	0.915	2.296
SV	c_{12}	0.841	0.920	0.011	0.297	0.618	1.055	10.552
AV	c_{12}	0.848	0.463	0.022	0.513	0.805	1.104	2.772
SV	c_{BH}	1.290	1.412	0.017	0.455	0.948	1.619	16.193
AV	c_{BH}	1.301	0.710	0.033	0.787	1.235	1.694	4.253

Table IV:Portfolio Performance - Unconstrained

This table displays portfolio performance measures for the AV and SV managed portfolio strategies using c_{BH} to target the annual volatility of the buy and hold market portfolio over the whole data set, 1926 to 2016, and over NBER business cycle expansions, contractions and contractions excluding the Great Depression. RET is the average annualized monthly log excess return. Sharpe and Sortino are the Sharpe and Sortino ratios respectively; Kappa_3 and Kappa_4 are the lower partial skewness and lower partial kurtosis Kappa measures. See section III for details. No constraints are placed on the level of investment in the market portfolio for the AV and SV managed portfolio; the buy and hold strategy always has an investment weight of one in the market. Stars on the lines for the AV and SV managed portfolios indicate a significant positive performance difference between those two portfolios.

(a) Full Sample

	Return	Sharpe	Sortino	Kappa_3	Kappa_4	α_{FF3}	$\alpha_{FF3+Mom}$
BH	5.934	0.319	0.129	0.082	0.061		
SV	8.589	0.462	0.208	0.132	0.097	5.477	3.201
AV	9.676***	0.520*	0.225	0.150*	0.112*	5.594***	3.164

(b) NBER Expansions

	Return	Sharpe	Sortino	Kappa_3	Kappa_4	α_{FF3}	$\alpha_{FF3+Mom}$
BH	9.598	0.626	0.278	0.179	0.130		
SV	10.05	0.521	0.236	0.149	0.110	2.342	1.192
AV	11.882***	0.640***	0.283**	0.184**	0.136**	2.410**	1.318***

(c) NBER Contractions

	Return	Sharpe	Sortino	Kappa_3	Kappa_4	α_{FF3}	$\alpha_{FF3+Mom}$
BH	-12.246	-0.416	-0.150	-0.108	-0.087		
SV	1.344***	0.092*	0.039***	0.028***	0.023***	5.158***	5.595*
AV	-1.272	-0.069	-0.027	-0.019	-0.016	4.582	5.462

(d) NBER Contractions x1929:09-1933:03

	Return	Sharpe	Sortino	Kappa_3	Kappa_4	α_{FF3}	$\alpha_{FF3+Mom}$
BH	5.205	0.233	0.091	0.062	0.048		
SV	12.542	0.799	0.414	0.287	0.230	10.518	9.683
AV	13.951***	0.752	0.349	0.244	0.194	10.603	10.063***

Table V:International Portfolio Performance - this table shows annualized average log excess returns and Sharpe ratios for the AV and SV managed portfolios compared to the market buy and hold for various international markets. See table I and section III.A for details on the international markets.

	AV		SV		BH	
Country	RET	Sharpe	RET	Sharpe	RET	Sharpe
AUS	12.477***	0.981	11.993	0.943	7.805	0.614
BRA	11.000***	0.291	9.037	0.240	6.163	0.164
CHN	27.381	0.868	24.926	0.790	12.286	0.390
DEU	11.064***	0.537*	7.633	0.371	5.399	0.262
FRA	7.243***	0.404	6.128	0.341	4.904	0.273
IND	14.893***	0.633	12.256	0.521	11.460	0.487
ITA	3.838	0.194	3.912	0.198	1.451	0.073
JPN	1.375***	0.068	0.129	0.006	-0.775	-0.038
UK	6.591***	0.485	5.984	0.441	5.111	0.376
World	8.603***	0.551	8.306	0.536	4.484	0.290

Table VI:Portfolio Performance - Constrained

This table displays portfolio performance measures for the AV and SV managed portfolio strategies using c_{10}, c_{12}, and c_{BH} to target the annual volatilities of 10%, 12% and equal to the buy hold market portfolio over the whole data set, 1926 to 2016. Performance ratios are calculated for investment constraints of a maximum of 1.5 and 3, 50% and 200% leverage. RET is the average annualized monthly log excess return. Sharpe and Sortino are the Sharpe and Sortino ratios respectively; Kappa$_3$ and Kappa$_4$ are the lower partial skewness and lower partial kurtosis Kappa measures. See section III for details. No constraints are placed on the level of investment in the market portfolio for the AV and SV managed portfolio; the buy and hold strategy always has an investment weight of one in the market. Stars on the lines for the AV and SV managed portfolios indicate a significant positive performance difference between those two portfolios.

(a) Constraint - 1.5

	Portfolio	Return	Sharpe	Sortino	Kappa$_3$	Kappa$_4$
c_{10}	SV	4.065	0.461	0.201	0.130	0.097
c_{10}	AV	5.196***	0.522**	0.225**	0.150**	0.966
c_{12}	SV	4.735	0.470	0.205	0.133	0.098
c_{12}	AV	6.081***	0.516*	0.221*	0.147*	0.945
c_{BH}	SV	6.171	0.467	0.200	0.128	0.091
c_{BH}	AV	7.885***	0.486	0.204	0.133	0.097

(b) Constraint - 3

	Portfolio	Return	Sharpe	Sortino	Kappa$_3$	Kappa$_4$
c_{10}	SV	4.396	0.454	0.200	0.127	0.094
c_{10}	AV	5.225***	0.520*	0.225*	0.150**	0.112**
c_{12}	SV	5.219	0.452	0.198	0.127	0.094
c_{12}	AV	6.306***	0.520**	0.225**	0.150**	0.112**
c_{BH}	SV	7.606	0.456	0.199	0.129	0.096
c_{BH}	AV	9.677***	0.522**	0.226**	0.150**	0.112**

Notes: ***,**, and * Significant at the 1, 5, and 10 percent levels.

Table VII:Costs

Panel (a) displays the average change in absolute market investment weight for the buy and hold volatility targeting unconstrained SV and AV managed portfolio strategies and the costs associated with trading and borrowing. Fama-French three factor and three factor with Momentum alphas are calculated using the factor portfolio return data from Ken French's website. Strategy break-even points are calculated in basis points as the cost to investment weight turnover which drives the alphas to zero or average annualized return equal to the buy and hold. Borrowing costs are calculated in basis points as the average monthly cost incurred borrowing to take a position in the market greater than one for the SV and AV manged portfolios at the Bloomberg broker call money rate, 1984-2014. See section IV for details. Panel (b) shows the break even transaction costs for the AV and SV managed portfolios globally. These are the trading costs which reduced the average return to the AV and SV managed portfolios to the buy and hold market return in each country.

(a) US Equity 1926M8:2016M12

				FF-3		FF-3 + Mom		
Strategy	$\lvert\Delta\omega\rvert$	RET	Break Even	α	Break Even	α	Break Even	Borrowing
SV	0.752	8.589	29.422	5.477	60.694	3.201	35.472	15.107
AV	0.317	9.676	254.364	5.594	176.467	3.164	83.176	11.411

(b) Global Equities

	AV			SV			
Country	RET	$\lvert\Delta\omega\rvert$	Break Even	RET	$\lvert\Delta\omega\rvert$	Break Even	RET_{BH}
AUS	12.477	0.486	80.139	11.993	0.466	74.914	7.805
BRA	11.000	0.253	159.118	9.037	0.623	38.462	6.163
CHN	27.381	0.305	412.715	24.926	0.538	195.972	12.286
DEU	11.064	0.499	94.545	7.633	0.581	32.052	5.399
FRA	7.243	0.468	41.656	6.128	0.536	19.041	4.904
IND	14.893	0.710	40.316	12.256	0.507	13.097	11.460
ITA	3.838	0.448	44.366	3.912	0.603	33.991	1.451
JPN	1.375	0.442	40.518	0.129	0.551	13.675	-0.775
UK	6.591	0.473	26.113	5.984	0.509	14.287	5.111
World	8.603	0.439	78.113	8.306	0.642	49.586	4.484

Table VIII:Portfolio Drawdowns

This table displays drawdown statistics for the buy and hold, AV, and SV managed portfolio strategies targeting the buy and hold strategy volatility without investment constraints. N represents the number of drawdowns longer than one month. Max DD and Avg DD are the maximum and mean drawdown in return percentage terms. Max Length and Avg Length are the lengths in months of the maximum drawdown and mean drawdown. Max Recovery and Avg Recovery are the maximum and mean times to recover back to the peak portfolio value at the start of the drawdown. See section III for details. Panel (a) shows the statistics for the US equity market and panel (b) shows the average drawdown statistics for each country globally.

(a) US Equities 1926M8:2016M12

Strategy	N	Max DD	Avg DD	Max Length	Avg Length	Max Recovery	Avg Recovery
BH	82	-84.803	-8.069	188	11.549	154	7.207
SV	65	-63.637	-11.196	246	14.954	135	7.446
AV	87	-60.264	-9.026	205	10.851	135	5.034

(b) Global Equities

	AV			SV			BH		
Country	Avg DD	Avg Length	Avg Recovery	Avg DD	Avg Length	Avg Recovery	Avg DD	Avg Length	Avg Recovery
AUS	-6.302	7.174	3.348	-5.322	9.263	5.421	-6.318	8.600	4.550
BRA	-8.059	9.560	4.208	-17.469	15.235	5.500	-15.064	17.067	4.286
CHN	-9.511	10.333	5.917	-10.074	10.583	3.727	-19.374	27.400	2.000
DEU	-11.051	10.625	5.783	-12.587	16.812	9.933	-10.706	17.125	12.333
FRA	-10.263	14.111	5.941	-15.260	18.267	10.214	-11.590	19.071	15.077
IND	-8.170	6.500	2.885	-12.545	12.467	5.733	-10.862	8.318	4.500
ITA	-14.625	19.500	2.143	-18.174	22.571	2.333	-8.919	15.400	1.667
JPN	-30.655	72.750	41.750	-78.514	294.000	175.000	-40.792	148.00	2.000
UK	-6.060	11.609	4.652	-7.872	14.158	8.158	-6.018	10.560	7.240
World	-6.982	9.909	7.333	-9.776	12.500	7.059	-8.209	10.091	6.429

Table IX:Stocastic Dominance- This table presents the results for tests of stochastic dominance of the AV managed portfolio over the SV managed portfolio. Values of α indicate investor compliance, $\alpha = 1$, or deviation from expected utility theory (EUT). SD1−SD4 are cumulative differences in bootstrapped samples from the observed returns to the AV and SV strategy. Positive values indicate stochastic dominance of AV over SV. See sections IV.D for details.

α	SD1	SD2	SD3	SD4
0.010	238.626	55,366.640	9,760,554.000	1,385,264,265.000
0.100	191.504	44,487.770	7,849,769.000	1,114,596,326.000
0.200	141.808	33,021.230	5,836,320.000	829,429,477.000
0.300	97.646	22,844.570	4,050,563.000	576,605,891.000
0.400	61.450	14,523.020	2,592,250.000	370,297,258.000
0.500	34.620	8,381.752	1,518,794.000	218,664,964.000
0.600	17.052	4,396.376	825,963.300	121,119,656.000
0.700	7.118	2,188.643	447,116.200	68,211,642.000
0.800	2.234	1,158.376	276,458.800	44,930,567.000
0.900	-0.196	702.314	207,620.100	36,179,509.000
0.990	-1.799	430.734	170,608.100	31,907,367.000
1	-1.978	400.635	166,533.500	31,441,514.000

Table X:Alternative Asset Class Performance - Panel (a) of this table shows the annualized average monthly returns and Sharpe ratios to managing investment in the Bloomberg US Dollar, Deutsche Bank Currency, Deutsche Bank Currency Carry, Deutsche Bank Currency Momentum, S&P US Real Estate Investment Trust, Bloomberg Commodity and Bloomberg US Universal Bond asset class indices using the global market capitalization weighted values of equity AV and SV, see section V.A for details. Panel (b) shows the annualized average monthly returns and Sharpe ratio for managing investment in the MSCI World index using the average variance calculated from currency and bond market returns against the strategy of buying and holding the MSCI World index. For both panles, investment begin in July of 2005 and ends in December 2015

(a) Other Assets Managed by Equity AV

	AV		SV		BH	
Index	RET	Sharpe	RET	Sharpe	RET	Sharpe
Dollar_{BB}	1.324***	0.170	0.606	0.078	−0.296	−0.038
Curr_{DB}	1.195***	0.272*	−0.668	−0.152	−0.244	−0.056
Carry_{DB}	1.440***	0.134	−0.361	−0.033	−2.071	−0.192
Mom_{DB}	1.942***	0.214	0.413	0.045	1.095	0.120
$\text{REIT}_{S\&P}$	26.706***	0.995	14.980	0.558	5.302	0.198
Comm_{BB}	−5.579***	−0.303	−6.431	−0.349	−5.279	−0.286
Bond_{Univ}	3.951***	1.168***	1.436	0.425	3.276	0.969

(b) Equity Managed by Other Assets AV

	Currency		Bonds		BH	
Index	RET	Sharpe	RET	Sharpe	RET	Sharpe
World	8.416***	0.515	8.582***	0.525	4.914	0.301

Table XI:Systemic Risk - Performance

Panel (a) displays the effect of Market Capitalization-to-GDP per capita ratio on the time series of AV managed US equity returns from August 1926 to December 2016. High denotes periods when the ratio is above median and Low when it is below. Panel (b) displays the effect of sorting AV managed country returns on Market Return to Credit Suisse Weath Return and Market Capitalization-to-GDP per capita ratio and forming a strategy long on countries with high ratios and shorting low ratio countries. High ratio countries have above median values and low ratio countries are below median. The investments run from June 2005 to December 2015 See V.B for details.

(a) Long - Short Strategies

Market to GDP					
	RET	Sharpe	α_{FF3}	α_{FF5}	$\alpha_{FF5+Mom}$
High	10.482***	0.819	6.331***	4.249**	3.914**
Low	6.800*	0.377	0.319	-0.519	-0.525

(b) Long - Short Strategies

Market RET to Wealth RET					
	RET	Sharpe	α_{FF3}	α_{FF5}	$\alpha_{FF5+Mom}$
Long	12.601**	0.747	9.484**	7.909*	7.725*
Short	7.537*	0.562	5.038*	5.422*	5.318*
Long Short	5.065***	0.405	4.446***	2.488***	2.407**
Market to GDP					
	RET	Sharpe	α_{FF3}	α_{FF5}	$\alpha_{FF5+Mom}$
Long	13.896***	1.021	11.443***	11.322***	11.181**
Shot	5.851	0.326	2.409	1.298	1.082
Long Short	8.045***	0.601	9.034**	10.024**	10.100**

Table XII:Systemic Risk - Regression

This table displays in sample regression results for monthly market variance, correlation and return statistics. SV is the annualized monthly variance of daily CRSP market returns. AV and AC are the monthly average variance and average pairwise correlation of daily returns for the top 500 assets in the CRSP market, as in Pollet and Wilson (2010). RET is the log return of the CRSP market portfolio minus the log return on the 1 month treasury bill. The coefficients and p-values bootstrapped for robustness, see section V.C for details.

			RET_{t+1} - 1926M7:2016M12		
AV	-0.0002			-0.006	0.192
AC		0.010^{**}		0.012^{**}	
SV			-0.057		-0.204
R^2	0.00000	0.0001	0.003	0.0001	0.012
Adjusted R^2	-0.001	-0.001	0.002	-0.002	0.010
			RET_{t+1} - 1926M7:1962M6		
AV	0.061			0.121	0.315
AC		-0.032		-0.099	
SV			-0.028		-0.264
R^2	0.004	0.001	0.001	0.010	0.026
Adjusted R^2	0.002	-0.002	-0.002	0.005	0.021
			RET_{t+1} - 1962M6:2016M12		
AV	-0.131			-0.168^{**}	0.016
AC		0.047^{***}		0.106^{***}	
SV			-0.109		0.254
R^2	0.017	0.002	0.012	0.027	0.017
Adjusted R^2	0.015	0.001	0.010	0.024	0.014

Table XIII:Full Out-of-Sample Results

Panels (a) and (b) displays out-of-sample expanding window regression results for monthly market variance, correlation and return statistics. SV is the annualized monthly variance of daily CRSP market returns. AV and AC are the monthly average variance and average pairwise correlation of daily returns for the top 500 assets in the CRSP market, as in Pollet and Wilson (2010). RET is the log return of the CRSP market portfolio minus the log return on the 1 month treasury bill. DM is the Diebold and Mariano (1995) statistic measuring for cast accuracy. MSE-F is the mean squared error improvement F-test of in McCracken (2007) and ENC-HLN is the forecast encompassing test of Harvey et al. (1998). In each panel the benchmark forecasts come from a model which uses SV_t to predict the independent variable. Panel (c) displays out-of-sample regression results of forecasts using AV_{t+1} as a predictor. Rossi and Inoue (2012) provides the methodology to make the calculations of the out-of-sample accuracy improvements of Diebold and Mariano (1995) and McCracken (2007) and the encompassing test of Harvey et al. (1998). In each panel the benchmark forecasts come from a model which uses SV_t to predict the independent variable.

(a) Sample 1970M7:2016M12

	DM	MSE-F	ENC-HLN
AC_{t+1}	1.074	109.736***	1
SV_{t+1}	1.53*	29.252***	1**
AV_{t+1}	2.286**	109.333***	1***
RET_{t+1}	1.278	11.801***	1*

(b) Sample 1939M12:2016M12

	DM	MSE-F	ENC-HLN
AC_{t+1}	1.604*	46.251***	1**
SV_{t+1}	1.041	21.57***	0.956**
AV_{t+1}	3.104***	198.267***	1***
RET_{t+1}	-2.027	-8.702	0

(c) Robust Expanding Window Results

Stat	Variable	DM	ENC-HLN
R_T	AC_{t+1}	28.532***	6.769***
R_T	SV_{t+1}	8.874***	1.838***
R_T	AV_{t+1}	34.347***	18.197***
R_T	RET_{t+1}	29.124***	4.871***
A_T	AC_{t+1}	19.867***	1.828***
A_T	SV_{t+1}	2.647***	0.949***
A_T	AV_{t+1}	21.751***	10.7***
A_T	RET_{t+1}	13.347***	1.68***

Notes: ***,**, and * Significant at the 1, 5, and 10 percent levels.

VII. Appendix

A. data

The data liquidity restrictions make the calculation of asset variance:

$$\sigma^2_{m,t} = \frac{1}{T-1} \sum_{\tau=1}^{T} \left(R_{m,\tau} - \frac{\sum_{\tau=1}^{T} R_{m,\tau}}{T} \right)^2 . \tag{18}$$

where $R_{m,\tau}$ is the daily return, including dividends, on an asset for day τ in month t. When the asset is the market portfolio, so $R_{m,\tau} = R_{s,\tau}$, the result is the variation of market returns, SV. The standard Pearsons correlation where the correlation of assets m and n for month t is:

$$\rho_{m,n,t} = \frac{\sum_{\tau=1}^{T} \left(R_{m,\tau} - \frac{\sum_{\tau=1}^{T} R_{m,\tau}}{t} \right) \left(R_{n,\tau} - \frac{\sum_{\tau=1}^{T} R_{n,\tau}}{t} \right)}{\sqrt{\left(R_{m,\tau} - \frac{\sum_{\tau=1}^{T} R_{m,\tau}}{t} \right)^2 \sum_{\tau=1}^{T} \left(R_{n,\tau} - \frac{\sum_{\tau=1}^{T} R_{n,\tau}}{t} \right)^2}}. \tag{19}$$

Unfortunately, for samples as small as the monthly series of daily returns Pearsons correlation is not an unbiased estimator of the true correlation, even if the returns are normal. (Hotelling, 1953) The average month in my sample has 22 trading days however the number commonly drops into the teens.[15] For samples of these sizes, the bias causes an underestimation of the correlation which is worse the lower the true correlation. I employ an approximate correction from Olkin and Pratt (1958) such that the monthly correlation between two assets m and n is:

$$\rho_{m,n,t} = \widehat{\rho_{m,n,t}} \left(1 + \frac{1 + \widehat{\rho_{m,n}}^2}{2(t-3)} \right) \tag{20}$$

where $\widehat{\rho_{m,n,t}}$ is the Pearson correlation between a and b.[16] AV and AC are value-weighted so each month I calculate market capitalization for all of the assets available in CRSP. The capitalization used in month t for asset m is the product of the end of month price (PRC) and common shares outstanding (SHROUT) values for asset m in month t-1.

B. Stochastic Dominance

B.1. Kernel Estimation

To estimate f, an unknown distribution, using i.i.d observations x = $x_1, x_2, \ldots, x_n$ the kernel density estimator:

[15]The shortest trading month in the sample is September 2001 with 15 trading days while 17 is a common number in the months with holidays.

[16]The exact correction suggested in Olkin and Pratt (1958) is too computationally taxing for the equipment to which I have access.

$\widehat{f}_h(x) = \frac{1}{n}\sum_{i=1}^{n} K_h(x - x_i) = \frac{1}{nh}\sum_{i=1}^{n} K\left(\frac{x-x_i}{h}\right)$ is used where K is a non-negative function, the kernel, and h is a non-negative smoothing parameter, often called the bandwidth. K_h is the scaled kernel, $\frac{1}{h}K(\frac{x}{h})$. The Gaussian kernel is used, $K(x^*, x_i) = exp\left(-\frac{(x^*-x_i)^2}{2b^2}\right)$ along with Silverman's, Silverman (2018), "rule of thumb" bandwidth which is 0.9 times the minimum of the standard deviation and the interquartile range of x divided by 1.34 times the sample size to the negative one-fifth power.

B.2. Vinod's Test

Repeated simulated return series are taken from the observed returns to the AV and SV management strategies. These the empirical CDFs from these simulated series are tested for stochastic dominance to the forth order while incorporating deviations from EUT. For example, the test for second-order stochastic dominance checks if $\int_{x*}^{x} F_{ab}(y)dy \leq 0$ for $x \in [x_*, x*]$ comparing distributions a and b. This involves the integral of F_{ab} itself the integral of the empirical probability distribution defined by the simulated samples. The method of Anderson (1996) is used to convert this integration into matrix pre-multiplication on the vectors of observed return probabilities. For a detailed walk-through of the calculations with supporting code see Vinod (2008).

C. In Sample Regressions

Full in-sample regression results covering the full sample 1926 to 2016, the pre-1962 period and the post-1962 period encompassing the sample of Pollet and Wilson (2010).

Table C.1:Full In Sample Results

The table displays in sample regression results for monthly market variance, correlation and return statistics. SV is the annualized monthly variance of daily CRSP market returns. AV and AC are the monthly average variance and average pairwise correlation of daily returns for the top 500 assets in the CRSP market, as in Pollet and Wilson (2010). RET is the log return of the CRSP market portfolio minus the log return on the 1 month treasury bill. The sample period is from 1926:07 to 2016:12. The coefficients and p-values are robust, see section V.C for details.

(a) Market Return Variance - SV_{t+1}

AV	0.627***			0.553***	0.368***
	p = 0.000			p = 0.000	p = 0.000
AC		0.418***		0.160***	
		p = 0.000		p = 0.000	
SV			0.615***		0.278***
			p = 0.000		p = 0.000
Constant	−0.0003	−0.0001	−0.0002	−0.0003	−0.0003
	p = 0.991	p = 0.998	p = 0.993	p = 0.991	p = 0.991
R^2	0.391	0.173	0.375	0.410	0.413
Adjusted R^2	0.390	0.173	0.374	0.409	0.412

(b) Average Asset Return Variance - AV_{t+1}

AV	0.721***			0.709***	0.759***
	p = 0.000			p = 0.000	p = 0.000
AC		0.359***		0.029***	
		p = 0.000		p = 0.000	
SV			0.609***		−0.019
			p = 0.000		p = 0.991
Constant	−0.0003	−0.0001	−0.0002	−0.0003	−0.0003
	p = 0.989	p = 0.998	p = 0.993	p = 0.989	p = 0.989
R^2	0.515	0.128	0.368	0.516	0.516
Adjusted R^2	0.515	0.127	0.367	0.515	0.515

(c) Average Asset Return Correlation - AC_{t+1}

AV	0.384***			0.125***	−0.029
	p = 0.000			p = 0.00001	p = 0.641
AC		0.613***		0.554***	
		p = 0.000		p = 0.000	
SV			0.454***		0.458***
			p = 0.000		p = 0.000
Constant	−0.0002	−0.0001	−0.0002	−0.0002	−0.0002
	p = 0.996	p = 0.996	p = 0.996	p = 0.995	p = 0.996
R^2	0.147	0.372	0.205	0.385	0.205
Adjusted R^2	0.146	0.372	0.204	0.384	0.204

(d) Log Excess Market Return - RET_{t+1}

AV	-0.0002			−0.006	0.192
	p = 0.562			p = 0.423	p = 0.833
AC		0.010**		0.012**	
		p = 0.0.052		p = 0.068	
SV			−0.057		−0.204
			p = 0.612		p = 0.865
R^2	0.00000	0.0001	0.003	0.0001	0.012
Adjusted R^2	−0.001	−0.001	0.002	−0.002	0.010

Table C.2:In Sample Results - Pre 1962

The table displays in sample regression results for monthly market variance, correlation and return statistics. SV is the annualized monthly variance of daily CRSP market returns. AV and AC are the monthly average variance and average pairwise correlation of daily returns for the top 500 assets in the CRSP market, as in Pollet and Wilson (2010). RET is the log return of the CRSP market portfolio minus the log return on the 1 month treasury bill. The sample period is from 1926:07 to 2016:1962:06. The series are standardized to a mean of zero and standard deviation of one. The coefficients and p-values are robust, see section V.C for details.

(a) Market Return Variance - SV_{t+1}

AV	0.672***			0.583***	0.414***
	p = 0.000			p = 0.000	p = 0.000
AC		0.492***		0.160***	
		p = 0.000		p = 0.0004	
SV			0.650***		0.266***
			p = 0.000		p = 0.00004
Constant	−0.000	−0.000	0.000	−0.000	0.000
	p = 1.000	p = 1.000	p = 1.000	p = 1.000	p = 1.000
R^2	0.444	0.236	0.413	0.460	0.466
Adjusted R^2	0.443	0.235	0.412	0.458	0.463

(b) Average Asset Return Variance - AV_{t+1}

AV	0.739***			0.702***	0.665***
	p = 0.000			p = 0.000	p = 0.000
AC		0.473***		0.078***	
		p = 0.000		p = 0.000	
SV			0.655***		0.128***
			p = 0.000		p = 0.000
Constant	−0.000	−0.000	0.000	−0.000	−0.000
	p = 1.000	p = 1.000	p = 1.000	p = 1.000	p = 1.000
R^2	0.535	0.218	0.422	0.539	0.539
Adjusted R^2	0.534	0.216	0.420	0.537	0.537

(c) Average Asset Return Correlation - AC_{t+1}

AV	0.503***			0.250***	0.150**
	p = 0.000			p = 0.000	p = 0.030
AC		0.580***		0.432***	
		p = 0.000		p = 0.000	
SV			0.539***		0.384***
			p = 0.000		p = 0.000
Constant	0.000	0.000	0.000	0.000	0.000
	p = 1.000	p = 1.000	p = 1.000	p = 1.000	p = 1.000
R^2	0.249	0.331	0.286	0.374	0.294
Adjusted R^2	0.248	0.329	0.285	0.371	0.291

(d) Log Excess Market Return - RET_{t+1}

AV	0.061			0.121	0.315
	p = 0.609			p = 0.741	p = 0.954
AC		−0.032		−0.099	
		p = 0.520		p = 0.862	
SV			−0.028		−0.264
			p = 0.418		p = 0.948
R^2	0.004	0.001	0.001	0.010	0.026
Adjusted R^2	0.002	−0.002	−0.002	0.005	0.021

Table C.3:In Sample Results - Post 1962

The table displays in sample regression results for monthly market variance, correlation and return statistics. SV is the annualized monthly variance of daily CRSP market returns. AV and AC are the monthly average variance and average pairwise correlation of daily returns for the top 500 assets in the CRSP market, as in Pollet and Wilson (2010). RET is the log return of the CRSP market portfolio minus the log return on the 1 month treasury bill. The sample period is from 1962:06 to 2016:12. The series are standardized to a mean of zero and standard deviation of one. The coefficients and p-values are robust, see section V.C for details.

(a) Market Return Variance - SV_{t+1}

AV	0.550***			0.494***	0.135***
	p = 0.000			p = 0.000	p = 0.001
AC		0.334***		0.162***	
		p = 0.000		p = 0.00001	
SV			0.556***		0.187***
			p = 0.000		p = 0.00002
Constant	−0.0005	−0.0001	−0.0003	−0.0005	−0.0004
	p = 0.989	p = 0.999	p = 0.993	p = 0.989	p = 0.991
R^2	0.297	0.110	0.304	0.320	0.317
Adjusted R^2	0.296	0.109	0.303	0.318	0.315

(b) Average Asset Return Variance - AV_{t+1}

AV	0.672***			0.680***	1.158***
	p = 0.000			p = 0.000	p = 0.000
AC		0.219***		−0.019	
		p = 0.003		p = 0.544	
SV			0.526***		−0.265
			p = 0.000		p = 0.999
Constant	−0.001	−0.00004	−0.0003	−0.001	−0.001
	p = 0.985	p = 1.000	p = 0.994	p = 0.984	p = 0.981
R^2	0.445	0.048	0.273	0.446	0.477
Adjusted R^2	0.445	0.046	0.272	0.444	0.475

(c) Average Asset Return Correlation - AC_{t+1}

AV	0.241***			0.023***	−0.634
	p = 0.000			p = 0.000	p = 0.999
AC		0.626***		0.619***	
		p = 0.000		p = 0.000	
SV			0.362***		0.539***
			p = 0.000		p = 0.008
Constant	−0.0002	−0.0001	−0.0002	−0.0001	−0.00003
	p = 0.996	p = 0.998	p = 0.996	p = 0.997	p = 1.000
R^2	0.057	0.387	0.130	0.387	0.167
Adjusted R^2	0.056	0.386	0.128	0.385	0.164

(d) Log Excess Market Return - RET_{t+1}

AV	−0.131			−0.168**	0.016
	p = 0.161			p = 0.020	p = 0.739
AC		0.047***		0.106***	
		p = 0.001		p = 0.000	
SV			−0.109		0.254
			p = 0.746		p = 0.107
R^2	0.017	0.002	0.012	0.027	0.017
Adjusted R^2	0.015	0.001	0.010	0.024	0.014

Table D.4:Out of Sample Robust Results
The table displays out-of-sample regression results of forecasts using AV_{t+1} as a predictor. Rossi and Inoue (2012) provides the methodology to make the calculations of the out-of-sample accuracy improvements of Diebold and Mariano (1995) and McCracken (2007) and the encompassing test of Harvey et al. (1998). In each panel the benchmark forecasts come from a model which uses SV_t to predict the independent variable.

(a) Robust Expanding Window Results

Stat	Variable	DM	ENC-HLN
R_T	AC_{t+1}	28.532***	6.769***
R_T	SV_{t+1}	8.874***	1.838***
R_T	AV_{t+1}	34.347***	18.197***
R_T	RET_{t+1}	29.124***	4.871***
A_T	AC_{t+1}	19.867***	1.828***
A_T	SV_{t+1}	2.647***	0.949***
A_T	AV_{t+1}	21.751***	10.7***
A_T	RET_{t+1}	13.347***	1.68***

(b) Robust Rolling Window Results

Stat	Variable	DM	ENC-HLN
R_T	AC_{t+1}	27.398***	8.706**
R_T	SV_{t+1}	21.92***	3.973
R_T	AV_{t+1}	34.292***	29.804***
R_T	RET_{t+1}	15.964***	3.884
A_T	AC_{t+1}	8.08***	1.542
A_T	SV_{t+1}	8.218***	2.062
A_T	AV_{t+1}	21.631***	19.449***
A_T	RET_{t+1}	9.209***	1.78

Notes: ***,**, and * Significant at the 1, 5, and 10 percent levels.

E. Alternate Asset Classes

The following tables detail the drawdown, turnover, and break even trading cost statistics for alternate asset classes managed by equity AV.

Table E.5:Alternative Asset Class Drawdowns - This table shows the average drawdown depth, length and recovery period to managing investment in the asset class index using the global market capitalization weighted values of equity AV and SV, see section V.A for details.

	AV			SV			BH		
Index	Avg DD	Avg Length	Avg Recovery	Avg DD	Avg Length	Avg Recovery	Avg DD	Avg Length	Avg Recovery
Bloomberg Dollar	-8.393	29.000	12.750	-10.632	39.333	21.333	-13.565	60.000	27.000
DB Currency	-2.236	9.750	2.667	-10.471	59.500	20.500	-8.839	59.500	41.500
DB Carry	-7.336	14.250	7.375	-33.972	121.000	98.000	-30.332	60.000	21.000
DB Mom	-4.748	11.900	3.300	-14.679	59.000	17.000	-12.278	38.333	18.333
S&P REIT	-7.692	4.400	1.800	-15.016	9.455	5.000	-17.004	15.143	9.286
Bloomberg Commodity	-9.784	12.222	2.111	-31.116	39.000	12.333	-26.638	39.333	4.333
Bloomberg Bond$_{Univ}$	-1.105	4.706	2.118	-1.674	8.083	5.667	-1.660	6.538	3.462

Table E.6:Alternative Asset Class Costs - This table shows the impact of trading costs on managing investment in the asset class index using the global market capitalization weighted values of equity AV and SV, see section V.A for details. Break even transaction costs are in basis points and represent the transaction costs which would reduced either the AV or SV managed strategies to the buy and hold index annualized average monthly return.

	AV			SV			
Index	RET	$\lvert\Delta\omega\rvert$	Break Even	RET	$\lvert\Delta\omega\rvert$	Break Even	RET_{BH}
Bloomberg Dollar	1.324	0.411	32.846	0.606	0.620	12.126	-0.296
DB Currency	1.195	0.430	27.851	-0.668	0.482	-7.339	-0.244
DB Carry	1.440	0.427	68.600	-0.361	0.510	27.947	-2.071
DB Mom	1.942	0.441	16.010	0.413	0.599	-9.501	1.095
S&P REIT	26.706	0.592	301.254	14.980	0.807	99.908	5.302
Bloomberg Commodity	-5.579	0.460	-5.430	-6.431	0.555	-17.285	-5.279
Bloomberg $Bond_{Univ}$	3.951	0.418	13.458	1.436	0.620	-24.712	3.276