

Average Variance Portfolio Management

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ABSTRACT

There's nothing very interesting here, but the format (achieved using the file `jf.sty`) makes it suitable for publication in the *Journal of Finance* even if the content doesn't. Here's a nice, informative, double-spaced abstract.

JEL classification: XXX, YYY.

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Invented in 1867¹ and in wide spread use starting in the 1870s, the stock ticker provided the first reliable means of conveying up to the minute stock prices over a long distances and market participants have been discussing the relationship between returns, risk, and portfolios for at least as long. (Rutterford and Sotiropoulos, 2016) The fundamental principal that there is a "risk-premium" such that more risky investment must generate greater returns to attract capital existed from the beginning. While the fundamental idea that higher risk, measured in some way, required higher return. Markowitz brought formality to the notion of risk, portfolio construction and optimization in the 1950s with modern portfolio theory and mean-variance analysis which stated that portfolios with higher variance should have higher returns. (Markowitz, 1952) However, several challenges have appeared to this foundational mean-variance claim. One of the most basic is commonly referred to as the "low-risk" or "low-volatility" anomaly. Haugen and A. (1972) found there was little to no evidence for a "risk-premium" for increased portfolio volatility and it was indeed easy to construct portfolios with lower volatility which earned greater returns than those with higher volatilities over a span of 20 years. A more modern and comprehensive treatment of the low-risk issue can be found in Moreira and Muir (2017) who show, across investment strategies and asset classes that simply managing leverage in a portfolio by its volatility produces greater expected returns and performance ratios than the underlying portfolio. As such, these results fundamentally challenge the mean-variance notion of investment risk premium. This risk-return trade off is central to modern financial theory, so naturally I want to address this problem by making it worse. By managing the market portfolio using the average variance of the individual assets in the prior quarter or month, I generate higher expected returns and significantly better performance ratios than by managing using the market portfolio volatility. Thankfully, at least in the equity market setting, this worse problem exposes evidence consistent with the leverage explanation of the low-risk anomaly and inconsistent with the lottery story, while shedding light on the risk-return trade-off dynamics in structural asset pricing theories.

Moreira and Muir (2017) demonstrate that volatility managed portfolios which decrease leverage when volatility is high produce large alphas, increase Sharpe ratios, and produce large utility gains for mean-variance investors. These results hold across investment styles, e.g. value or momentum, and in different asset classes, e.g. equities and currency portfolios. The results even hold for positions which already seek to exploit the low-risk anomaly like the "betting-against-beta" strategy of Frazzini and Pedersen (2014). Additionally, other researchers have applied similar variance or volatility management to specific assets or trading styles.² Practitioners call approaches like this risk-parity and as of 2016 at least \$150 and as much as \$400 billion dollars was invested in risk-parity funds. (Steward, 2010; Cao, 2016) The generation of greater expected returns without an equivalent increase in portfolio volatility implies that a representative investor should time asset volatility, demanding less return when its higher meaning the investor's risk appetite must be higher in periods like recession and market downturns when its expected to be lower. In short, it seems risk does not equal reward. However, some volatility is more equal than other volatility.

Pollet and Wilson (2010) decompose market variance into the average correlation between pairs

of assets and the average variance of the individual assets yields a series strongly related to both future volatility and higher returns, average correlation, and a series strongly related to future volatility but unrelated to returns, average variance. Scaling investment in the market portfolio by the inverse of previous months average variance should then improve return performance over and above those managing investment by the previous market variance since it avoids, somewhat, decreasing investment when average correlation is high and higher future returns are expected. Average variance is also a better candidate for portfolio management because it has a better chance to pick up economic information sooner as individual assets respond to information about its own risk and expected returns as it's made public while changes in aggregate materialize as information across assets is combined. (Campbell, Lo, MacKinlay, et al., 1997; Campbell, Lettau, Malkiel, and Xu, 2001) Asness, Frazzini, Gormsen, and Pedersen (2018) take a related approach to decomposing the betting-against-beta strategy of Frazzini and Pedersen (2014) into betting-against-correlation, BAC, and betting-against-variance factors, BAV, finding the BAC factor has a significant Fama-French five factor alpha but unrelated to behavior explanations of the low-risk anomaly. (Fama and French, 2016) From the decomposition of daily market returns I find similar results about the more general low-risk strategy of volatility management.

Using daily market returns from the Center for Research in Securities Prices, I follow Pollet and Wilson (2010) generating quarterly time series of stock market variance, SV, average correlation, AC, and average variance, AV. I then extend this by calculating the time series monthly. As expected, AV is strongly related to next period market variance and unrelated, at best insignificantly negatively, related to next period excess log returns. As is standard, excess log returns are the difference between the log of one plus the CRSP market return and the log of one plus the risk free return. The monthly risk free return is the return on the one month treasury bill while the quarterly risk free rate is the return on the three month treasury bill, each series is collected from the St. Louis Federal Reserve Economic Data (FRED) website. This relationship not only holds both in and out of sample but in both National Bureau of Economic Research (NBER) defined business cycle contractions and expansions. Out of sample forecast accuracy and encompassing tests show that AV is a superior predictor of both SV and log excess returns next period. Out of sample testing always raises questions about model specification, recursive expansion versus rolling window parameter estimation, and choices of training period and prediction window. Using Rossi and Inoue (2012) robust statistics I show that the superior performance of AV, against either the running historical mean or the current time period SV in either a rolling or expanding regression specification regardless of window choice.

The robust performance of AV translates into significant asset allocation gains. A strategy which weights investment in the market portfolio produces average annualized log excess returns of 7.8% quarterly and 8.7% monthly. The AV strategy produces quarterly and monthly Sharpe ratios, .48 and .59, each statistically significantly greater than those of the SV, volatility, management strategy. Additionally, AV generates higher values in more asymmetric risk-return measures like the Sortino, Kappa, and upside potential ratios. Asset allocation performance is not all perfect,

however, and the lower performance of AV in terms of Rachev ratio hints at possible explanation for the low-risk anomaly seen in volatility and average variance management.

”There is no such thing as a free lunch,” is not only a wonderfully pervasive adage, particularly loved by economists, but a provable restriction on optimization problems. (Wolpert and Macready, 1997) Here, AV provides no free lunch. The improved performance measured by expected log excess returns, Sharpe, Sortino, Kappa and upside potential ratios is betrayed by worse performance in Rachev ratio. The Rachev ratio measures the right tail reward value at play relative to the left tail value at risk. In short it measures the ratio of expected lottery rewards and losses. Volatility managed market portfolio investment has higher lottery winning potential for each dollar of potential lottery loss. Average variance management thins both tails of the return distribution but the upper tail slightly more trading the positive shift in the expected value of the log excess returns with a loss in the most extreme possible positive return values. This is inconsistent with the notion that the low-risk anomaly is the result of a behavioral preference of investors for lottery-like returns. (Barberis and Huang, 2008; Brunnermeier, Gollier, and Parker, 2007) Rather, the generation of higher log excess returns by the AV, SV, and betting-against-beta strategies support the notion that the low-risk anomaly arises from leverage constraints first suggested in Jensen, Black, and Scholes (1972). Boguth and Simutin (2018) link capital constraints and the betting-against-beta strategy through mutual fund betas, and more generally Malkhozov, Mueller, Vedolin, and Venter (2017) show that international illiquidity predicts betting-against-beta returns.

Some change in the risk-return dynamic through average variance management was anticipated given the relationship between AV and next period SV and the lack of relationship between AV and next period log excess returns. Given that idiosyncratic events are reflected in individual asset volatility first we should also be able to explain the better performance of AV in terms of timing and response.

I. The Model

There’s not actually a model here as it’s not really a paper, but this is about where a model might go. Note that the first sentence of this section is indented (as required by the JF) using the `indentfirst` package.

A. A Subsection

Note that subsections in JF are just labeled with letters. When referring to them in the text, you need to add the section number back, e.g., Section I.B or I.B.1. This is taken care of in `jf.sty`. Let’s also add some parenthetical citations (see ?, ?, ?).

To justify adding a subsection here, from now on, we’ll assume

CONDITION 1: $0 < \hat{\mu} < \gamma\sigma^2$.

This condition might be useful if there was a model.

B. Another Subsection, With a Figure

Figures get put at the end, with a note marking where they should go in the text, like this:

[Place Figure 1 about here]

B.1. A Subsubsection with a Proposition

Let's put a proposition here.

PROPOSITION 1: *If Condition 1 is satisfied, a solution to the central planner's problem, $V(B, D, t) \in C^2(\mathbb{R}_+^2 \times [0, T])$, with control $a : [0, 1] \times [0, T] \rightarrow [-\lambda, \lambda]$ if $\gamma > 1$ is*

$$V(B, D, t) = -\frac{(B + D)^{1-\gamma}}{1 - \gamma} w\left(\frac{B}{B + D}, t\right). \quad (1)$$

Appendix A. An Appendix

Here's an appendix with an equation. Note that equation numbering is quite different in appendices and that the JF wants the word “Appendix” to appear before the letter in the appendix title. This is all handled in `jf.sty`.

$$E = mc^2. \tag{A1}$$

Appendix B. Another Appendix

Here's another appendix with an equation.

$$E = mc^2. \tag{B1}$$

Note that this is quite similar to Equation (A1) in Appendix A.

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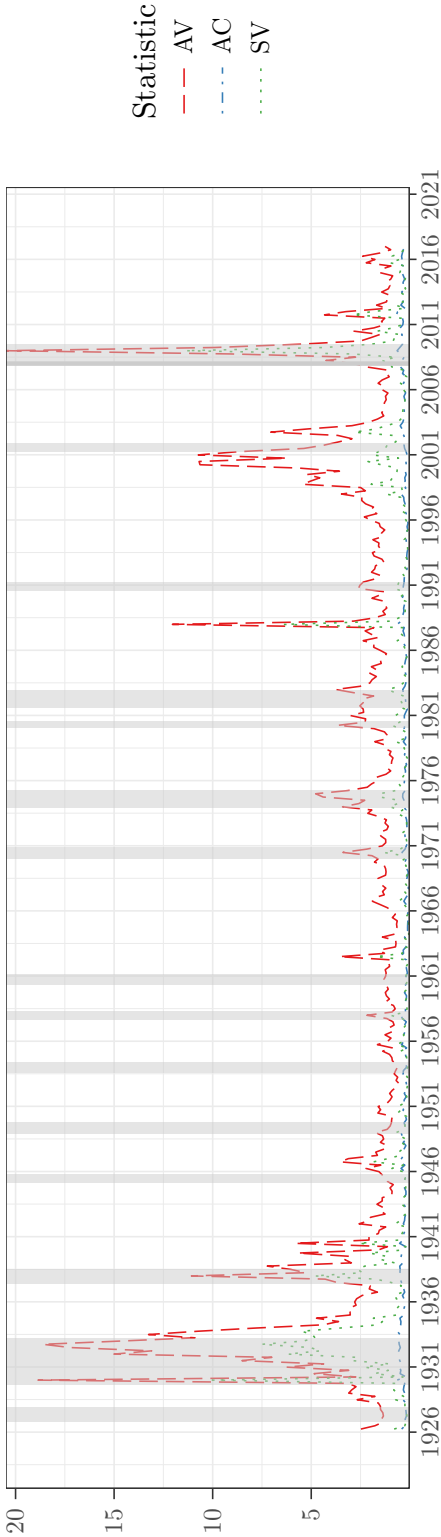
Notes

¹Invented by Edward A. Calahan, an employee of the American Telegraph Company.

²See, for example, Barroso and Santa-Clara (2015) and Kim, Tse, and Wald (2016) for discussions of volatility management of the momentum portfolio.

Figure 1. Time Series of Market Statistics: The average variance of individual asset, average correlation of asset pairs, and the variance of the CRSP portfolio calculated from daily returns.

Quarterly Measures of Daily Return Statistics



Monthly Measures of Daily Return Statistics

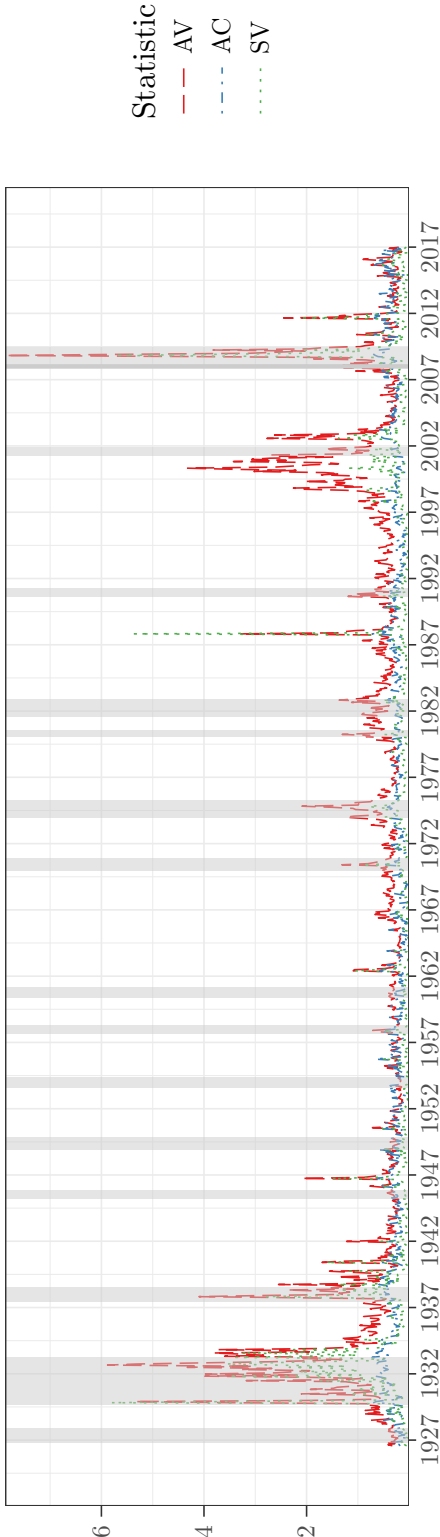


Table I Summary Statistics

Pollet and Wilson Sample 1963Q1:2006Q4

Statistic	N	Mean	St. Dev.	Min	Max	Autocorrelation
RET	176	1.163	8.369	−30.072	19.956	0.000
AC	176	0.230	0.090	0.034	0.648	0.572
AV	176	2.218	1.828	0.634	12.044	0.696
SV	176	0.483	0.616	0.029	6.397	0.311

Quarterly 1926Q1:2016Q4

Statistic	N	Mean	St. Dev.	Min	Max	Autocorrelation
RET	331	1.694	8.591	−30.072	30.956	0.097
AC	364	0.282	0.121	0.034	0.678	0.578
AV	364	2.533	2.839	0.539	20.485	0.668
SV	364	0.741	1.258	0.029	11.378	0.469

Monthly 1926M1:2016M12

Statistic	N	Mean	St. Dev.	Min	Max	Autocorrelation
RET	1,085	0.494	5.371	−34.553	33.258	0.106
AC	1,086	0.276	0.134	0.019	0.762	0.610
AV	1,086	0.642	0.729	0.129	7.867	0.794
SV	1,086	0.248	0.502	0.006	5.808	0.612

Table IIReplication - cotemp

<i>Dependent variable:</i>				
	SV			
	(1)	(2)	(3)	(4)
AC	0.042*** (0.004)		0.027*** (0.002)	
AV		0.281*** (0.014)	0.238*** (0.011)	
AC * AV				0.842*** (0.009)
Constant	-0.005*** (0.001)	-0.001*** (0.0004)	-0.007*** (0.001)	0.0001 (0.0001)
Observations	176	176	176	176
R ²	0.387	0.698	0.832	0.982
Adjusted R ²	0.383	0.696	0.830	0.982
Residual Std. Error	0.005 (df = 174)	0.003 (df = 174)	0.003 (df = 173)	0.001 (df = 174)
F Statistic	109.623*** (df = 1; 174)	402.497*** (df = 1; 174)	429.338*** (df = 2; 173)	9,491.243*** (df = 1; 174)

*p<0.1; **p<0.05; ***p<0.01

Table III Replication - next

	Dependent variable:				
	(1)	(2)	(3)	(4)	(5)
AC	0.014*** (0.005)		SV _{t+1} 0.005 (0.005)		
AV		0.145*** (0.023)	0.136*** (0.024)		
AC * AV				0.265*** (0.061)	
SV					0.312*** (0.072)
Constant	0.002 (0.001)	0.002** (0.001)	0.001 (0.001)	0.003*** (0.001)	0.003*** (0.001)
Observations	175	175	175	175	175
R ²	0.044	0.184	0.190	0.098	0.097
Adjusted R ²	0.039	0.180	0.180	0.092	0.092
Residual Std. Error	0.006 (df = 173)	0.006 (df = 173)	0.006 (df = 172)	0.006 (df = 173)	0.006 (df = 173)
F Statistic	7.997*** (df = 1; 173)	39.136*** (df = 1; 173)	20.152*** (df = 2; 172)	18.694*** (df = 1; 173)	18.604*** (df = 1; 173)

*p<0.1; **p<0.05; ***p<0.01

Note:

Table IV Quarterly

	Dependent variable:				
	(1)	(2)	(3)	(4)	(5)
	SV _{t+1}				
AC	0.044*** (0.005)		0.010* (0.005)		
AV		0.276*** (0.018)	0.252*** (0.022)		
AC * AV				0.459*** (0.033)	
SV					0.555*** (0.044)
Constant	-0.005*** (0.002)	0.0004 (0.001)	-0.002 (0.001)	0.003*** (0.001)	0.003*** (0.001)
Observations	363	363	363	363	363
R ²	0.176	0.387	0.393	0.352	0.308
Adjusted R ²	0.173	0.386	0.390	0.350	0.306
Residual Std. Error	0.011 (df = 361)	0.010 (df = 361)	0.010 (df = 360)	0.010 (df = 361)	0.010 (df = 361)
F Statistic	76.897*** (df = 1; 361)	228.220*** (df = 1; 361)	116.725*** (df = 2; 360)	195.707*** (df = 1; 361)	160.807*** (df = 1; 361)

*p<0.1; **p<0.05; ***p<0.01

Note:

Table VMonthly

	Dependent variable:				
	(1)	(2)	SV _{t+1} (3)	(4)	(5)
AC	0.016*** (0.001)		0.005*** (0.001)		
AV		0.432*** (0.016)	0.386*** (0.019)		
AC * AV				0.767*** (0.028)	
SV					0.612*** (0.024)
Constant	-0.002*** (0.0003)	-0.0003* (0.0002)	-0.001*** (0.0003)	0.001*** (0.0001)	0.001*** (0.0001)
Observations	1,085	1,085	1,085	1,085	1,085
R ²	0.173	0.393	0.406	0.411	0.375
Adjusted R ²	0.173	0.393	0.405	0.410	0.374
Residual Std. Error	0.005 (df = 1083)	0.004 (df = 1083)	0.004 (df = 1082)	0.004 (df = 1083)	0.004 (df = 1083)
F Statistic	227.171*** (df = 1; 1083)	702.324*** (df = 1; 1083)	370.196*** (df = 2; 1082)	754.567*** (df = 1; 1083)	648.537*** (df = 1; 1083)

Note:

*p<0.1; **p<0.05; ***p<0.01

Table VIReplication - Next Ret

	<i>Dependent variable:</i>				
	(1)	(2)	RET _{t+1}	(4)	(5)
AC	0.213*** (0.069)		0.248*** (0.072)		
AV		-0.117 (0.347)	-0.520 (0.357)		
AC * AV				1.356 (0.870)	
SV					1.454 (1.025)
Constant	-0.037** (0.017)	0.014 (0.010)	-0.034** (0.017)	0.004 (0.008)	0.005 (0.008)
Observations	176	176	176	176	176
R ²	0.053	0.001	0.064	0.014	0.011
Adjusted R ²	0.047	-0.005	0.053	0.008	0.006
Residual Std. Error	0.082 (df = 174)	0.084 (df = 174)	0.081 (df = 173)	0.083 (df = 174)	0.083 (df = 174)
F Statistic	9.649*** (df = 1; 174)	0.113 (df = 1; 174)	5.918*** (df = 2; 173)	2.431 (df = 1; 174)	2.013 (df = 1; 174)

*p<0.1; **p<0.05; ***p<0.01

Note:

Table VII Quarterly

	Dependent variable:				
	(1)	(2)	RET _{t+1} (3)	(4)	(5)
AC	0.109** (0.042)		0.170*** (0.046)		
AV		-0.371 (0.233)	-0.777*** (0.254)		
AC * AV				-0.198 (0.444)	
SV					-0.130 (0.524)
Constant	-0.012 (0.012)	0.025*** (0.007)	-0.012 (0.012)	0.018*** (0.006)	0.018*** (0.006)
Observations	331	331	331	331	331
R ²	0.020	0.008	0.047	0.001	0.0002
Adjusted R ²	0.017	0.005	0.041	-0.002	-0.003
Residual Std. Error	0.085 (df = 329)	0.086 (df = 329)	0.084 (df = 328)	0.086 (df = 329)	0.086 (df = 329)
F Statistic	6.640** (df = 1; 329)	2.519 (df = 1; 329)	8.064*** (df = 2; 328)	0.199 (df = 1; 329)	0.062 (df = 1; 329)

*p<0.1; **p<0.05; ***p<0.01

Note:

Table VIII Monthly

	<i>Dependent variable:</i>				
	(1)	(2)	(3)	(4)	(5)
			RET _{t+1}		
AC	0.004 (0.012)		0.024* (0.014)		
AV		-0.484** (0.223)	-0.706*** (0.259)		
AC * AV				-0.771** (0.389)	
SV					-0.604* (0.325)
Constant	0.004 (0.004)	0.008*** (0.002)	0.003 (0.004)	0.007*** (0.002)	0.006*** (0.002)
Observations	1,085	1,085	1,085	1,085	1,085
R ²	0.0001	0.004	0.007	0.004	0.003
Adjusted R ²	-0.001	0.003	0.005	0.003	0.002
Residual Std. Error	0.054 (df = 1083)	0.054 (df = 1083)	0.054 (df = 1082)	0.054 (df = 1083)	0.054 (df = 1083)
F Statistic	0.125 (df = 1; 1083)	4.687** (df = 1; 1083)	3.774** (df = 2; 1082)	3.940** (df = 1; 1083)	3.461* (df = 1; 1083)

*p<0.1; **p<0.05; ***p<0.01

Note:

Table IX

	Sample	R^2_{oos}	MSE-F	ENC-NEW	ENC-HLN
SV_{t+1}	1963Q2:2007Q1	11.912**	12.846***	17.937**	0.779** *
SV_{t+1}	Quarterly	35.041** *	147.267***	163.112**	0.911** *
SV_{t+1}	Monthly	37.858**	496.516***	582.268**	0.872** *
RET_{t+1}	1963Q2:2007Q1	-7.311	-6.472	0.302	0.043
RET_{t+1}	Quarterly	-0.358	-0.975	0.898**	0.324
RET_{t+1}	Monthly	0.636	5.220***	3.706**	1*
variable	Sample	R^2_{oos}	MSE-F	ENC-NEW	ENC-HLN
SV_{t+1}	1963Q2:2007Q1	15.135**	16.942***	12.563**	1**
SV_{t+1}	Quarterly	8.769**	26.239***	29.585**	0.898** *
SV_{t+1}	Monthly	2.793**	23.417***	109.745**	0.56**
RET_{t+1}	1963Q2:2007Q1	15.307	17.170***	15.152**	1
RET_{t+1}	Quarterly	0.746	2.052**	1.866**	1
RET_{t+1}	Monthly	0.178	1.454*	0.9**	1