

# How to Look Clever and Have Envious Neighbors: Average Volatility Managed Leverage Timing

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## ABSTRACT

While taking leverage to invest may be seen as risky, timing leverage can produce higher returns and timing by the right signal can produce significantly higher returns. Compared to a portfolio that manages investment in the market by the variance of daily market returns, a portfolio that manages by the average variance (AV) of the individual assets in the market portfolio produces utility gains by generating significantly higher average returns with significantly better Sharpe, Sortino, and Kappa ratios. The average variance managed portfolio is also cheaper and more practically implementable than the variance managed portfolio. This performance gain arises because AV is uncompensated, non-systemic risk using it to time leveraged investment reduces investor exposure when risk does not accompany reward and increases investment when it does. The AV managed strategy works across countries supporting the argument that reducing AV exposure better aligns investment with systemic risk.

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While Warren Buffet may not be an enthusiastic supporter of taking leverage to invest<sup>1</sup>, even at fixed levels of portfolio volatility leveraged investing can be used to increase returns. Moreira and Muir (2017) show across investment strategies and asset classes that merely managing leverage in a portfolio by that portfolio's volatility produces greater expected returns and performance ratios. As Rutterford and Sotiropoulos (2016) document, the discussion of the risk-return tradeoff is at least a century old and central to modern financial theory, however Moreira and Muir (2017) demonstrate that managing investment in the market portfolio by the inverse of the prior period volatility generates greater returns and better performance ratios. Unfortunately, the volatility managed portfolio requires extreme levels of leverage. I show that managing investment in the market portfolio by the inverse of the average variance of individual daily asset returns is not only a better performer, but cheaper and more practically implementable. These improvements arise because returns to the average variance managed portfolio depend on the level of average pairwise correlation between individual asset daily returns which exposes a systemic risk pricing dynamic which is obscured by the use of total market volatility.

Since the early identification of a low-risk anomaly, the absence of a need to take on more risk for higher returns as in Haugen and Heins (1972), a large number of researchers have sought to identify a positive relationship between return variance and expected returns. Campbell (1987), French, Schwert, and Stambaugh (1987), Glosten, Jagannathan, and Runkle (1993) and many, many others have found insufficient evidence identifying a positive relationship between return variance and future returns. Some investors seek to exploit aspects of this disconnection through risk management and risk-parity funds. As of 2016, at least \$150 and as much as \$400 billion sat in these funds.(Steward, 2010; Cao, 2016) The Moreira and Muir (2017) results even hold for positions which already seek to exploit the low-risk anomaly like the "betting-against-beta" strategy of Frazzini and Pedersen (2014). Additionally, other researchers have applied similar variance or volatility management to specific assets or trading styles.<sup>2</sup> The generation of higher expected returns without a commensurate increase in portfolio volatility implies that the representative investor, holding the market, requires less equity premium in time of high volatility. In short, it seems risk does not equal reward. However, the central premiss of modern portfolio theory is that systemic risk is rewarded as investors are not able to avoid it and not all portfolio variance is systemic.

While the total variance of stock market returns (SV) is a function of both the variance of individual asset returns and the covariance between assets in the market, Pollet and Wilson (2010) show that the average pair-wise correlation of portfolio assets is the component of portfolio volatility most related to systemic changes in the economy and is the risk component compensated with higher returns. By managing the market portfolio by the other component of total volatility, the average variance of the individual asset returns (AV), I generate higher average annualized monthly returns and significantly better Sharpe, Sortino, and Kappa ratios. In addition to identifying a better strategy for investors, decoupling AV from the SV sheds light on the risk-return trade-off dynamics of the market and allows leveraging investment into times when higher volatility will be compensated

and pulling out when it will not.

AV is substantially better than SV as a leverage management signal. Targeting the volatility of the buy and hold market portfolio return, as in Moreira and Muir (2017), an investor without borrowing constraints earns an annualized average monthly return of 9.7% from the AV managed portfolio. This return is a statistically significant increase of more than 1% over the SV managed portfolio; the difference in annualized average monthly returns grows to more than 2% when practical leverage constraints are applied. With unconstrained borrowing, the AV managed portfolio has significantly better performance ratios like the symmetric Sharpe ratio, .52, and more asymmetric risk-return measures, e.g.,  $Kappa_3$  and  $Kappa_4$  at .15 and .11 respectively. The advantage of managing with AV grows with risk aversion. The most risk-averse,  $\gamma = 5$ , constrained investor sees a certainty equivalent return (CER) gain of more than 2% annualized; this return represents a 27.4% increase in utility nearly as substantial as the utility gains seen in return timing strategies. (Campbell, Lo, MacKinlay, et al., 1997) Targeting the volatility of the buy and hold return requires seeing into the future and knowing the buy and hold return volatility. However, this look-ahead does not affect performance ratios like the Sharpe ratio, moreover the significantly better performance of AV is robust to other choices of target volatility.

The AV and SV portfolio management strategies each generate a weight in the market portfolio as a function of the daily returns of the market the prior month. Some of the investments demanded by the AV managed portfolio maybe difficult to achieve, however the investments needed to make the SV management strategy work are simply problematic. The SV managed portfolio takes far more extreme leverage far more often. In addition to higher average monthly borrowing costs, 15.107 vs 11.411 basis points, the SV managed portfolio has higher turnover. The AV managed portfolio generates lower transactions costs with a break even transaction cost more than 2.5 times higher than the SV managed portfolio. This makes the AV managed portfolio cheaper for the investor in both borrowing and transaction costs while generating higher returns. The AV managed portfolio is also cheaper to insure. Drawdowns are shallower and shorter for the AV managed portfolio at 9% vs 11.2% and 10.5 vs 15 months on average. The drawdown profiles for the AV and SV managed portfolios also reveal that the AV managed portfolio benefits fund managers in addition to fund investors. The SV managed portfolio exposes a fund manager to twice the risk of a drawdown sever enough to threaten their job, or possibly shutter the fund, and would be nearly 91.7% more expensive to insure.

The identification of a better portfolio management signal is valuable, certainly to investors and fund managers. However, a more significant contribution to our understanding of the market can be made if the reason that AV outperforms SV can be discovered. This requires examining the relationship of AV to future SV, AV, AC and returns.

Using daily market returns from the Center for Research in Securities Prices (CRSP), I extend Pollet and Wilson (2010) generating monthly time series of stock market variance, SV, average correlation, AC, and average variance, AV. From June 1962 to the end of 2016, encompassing the sample of Pollet and Wilson (2010), AV is a significant in-sample predictor of higher SV, AV, AC,

and lower log excess market returns at the monthly frequency. A one standard deviation increase in annualized AV, from .77 to 1.62, is related to an increase in next months annualized SV of .545 of a standard deviation or a .22 increase. This makes next months SV more than double the mean. AV remains a significant predictor of next month's SV even when this month's SV is included. A one standard deviation increase in AV also anticipates a .13 standard deviation, .58 percentage point, lower log excess return. This makes the following month's expected return negative. When both AV and SV are used to predict next month's return, AV is significant but SV is not. However, over the full, 1926 to 2016, AV is a significant predictor of higher SV, SV, AC, but not log excess market returns. This evidence explains the performance of AV as a leverage management signal. Scaling investment in the market by the inverse of AV in the current month will pull funds out when the following month will have high SV without sacrificing higher expected returns. It may, in fact, avoid negative returns. These results support the intuition from the work on volatility management in Moreira and Muir (2017) and portfolio AV and AC in Pollet and Wilson (2010) but in-sample regression use all available information and do not necessarily identify tradeable strategies. (Welch and Goyal, 2008)

Investors can only make decisions using the limited information available to them at a given time. For example in June of 2007 investors and investment models could only use historical information up to that month; the effects of November 2008 on the regression coefficients cannot not affect the predictions for July 2007. Moreira and Muir (2017) demonstrate that market volatility is an effective market portfolio management technique across the CRSP data set from 1926 to 2015. To explain why AV is a better real-time market portfolio leverage signal, I run expanding window out-of-sample regressions using AV on SV, AV, AC, and log excess returns. From June 1926 to December 2016 and using the predictions from SV as a benchmark, AV is a significantly better forecaster of next months AV, AC and SV. It generates better Diebold and Mariano (1995) test statistics, significantly lower mean squared forecast errors judging by the MSE-F statistic from McCracken (2007) and the emcompassing test of Harvey, Leybourne, and Newbold (1998) show that AV contains all of the forecasting information in SV. As with the in-sample results, AV serves investors at least as well as SV and likely better in avoiding risk without giving up return. Out-of-sample testing always raises questions about model specification, recursive expansion versus rolling window parameter estimation, choices of the training period and prediction window and the influence of specific periods. Using the techniques in Rossi and Inoue (2012), the Diebold and Mariano (1995) and Harvey et al. (1998) measures can be calculated robust to concerns on window selection for either an expanding or rolling specification. The Rossi and Inoue (2012) robust statistics show that AV is a significantly better predictor than SV robust to the choice of window or regression specification. AV is a better leverage timing portfolio management signal than SV because it scales investment by future risk without giving up future return.

Using the decomposition of market variance, I identify a better portfolio leverage management signal. Weighting investment leverage by the inverse of the average of individual asset variance, AV, rather than SV, is a new addition to the portfolio management literature letting investors

capture better performance as measured by expected annualized returns, performance ratios, costs and utility gains. The returns to the AV managed portfolio serve as another example in the risk-return dynamics literature exposing the returns to correlation risk obscured when risk is measured by SV. These developments come from the formation and analysis of the AV signal. This work requires a few publically available datasets and a few considerations for the calculations at the monthly frequency. Most of the work is in the calculations required to show significance in portfolio performance and in the regression which establish the relationship of AV, risk, and future returns.

## I. Data

To calculate stock market variance (SV), average asset variance (AV), and average asset correlation (AC), I use daily return data from CRSP and calculate the variance of daily returns monthly. To simplify the analysis of individual assets, I require that the asset be traded on each in the month which mitigates any liquidity effects and ensures consistent variance, covariance and correlation calculations. These conditions make the calculation of asset variance:

$$\sigma_{m,t}^2 = \frac{1}{T-1} \sum_{\tau=1}^T \left( R_{m,\tau} - \frac{\sum_{\tau=1}^T R_{m,\tau}}{T} \right)^2. \quad (1)$$

where  $R_{m,\tau}$  is the daily return, including dividends, on an asset for day  $\tau$  in month  $t$ . When the asset is the market portfolio, so  $R_{m,\tau} = R_{s,\tau}$ , the result is the variation of market returns, SV. The standard Pearsons correlation where the correlation of assets  $m$  and  $n$  for month  $t$  is:

$$\rho_{m,n,t} = \frac{\sum_{\tau=1}^T \left( R_{m,\tau} - \frac{\sum_{\tau=1}^T R_{m,\tau}}{t} \right) \left( R_{n,\tau} - \frac{\sum_{\tau=1}^T R_{n,\tau}}{t} \right)}{\sqrt{\left( R_{m,\tau} - \frac{\sum_{\tau=1}^T R_{m,\tau}}{t} \right)^2 \sum_{\tau=1}^T \left( R_{n,\tau} - \frac{\sum_{\tau=1}^T R_{n,\tau}}{t} \right)^2}}. \quad (2)$$

Unfortunately, for samples as small as the monthly series of daily returns Pearsons correlation is not an unbiased estimator of the true correlation, even if the returns are normal. Hotelling (1953) The average month in my sample has 22 trading days however the number commonly drops into the teens.<sup>3</sup> For samples of these sizes, the bias causes an underestimation of the correlation which is worse the lower the true correlation. I employ an approximate correction from Olkin and Pratt (1958) such that the monthly correlation between two assets  $m$  and  $n$  is:

$$\rho_{m,n,t} = \widehat{\rho_{m,n,t}} \left( 1 + \frac{1 + \widehat{\rho_{m,n,t}}^2}{2(t-3)} \right) \quad (3)$$

where  $\widehat{\rho_{m,n,t}}$  is the Pearson correlation between  $a$  and  $b$ .<sup>4</sup> AV and AC are value-weighted so each month I calculate market capitalization for all of the assets available in CRSP. The capitalization used in month  $t$  for asset  $m$  is the product of the end of month price (PRC) and common shares outstanding (SHROUT) values for asset  $m$  in month  $t-1$ . To make the analysis more computation-

ally trackable I use only, at most, the top 500 assets in CRSP by market capitalization for a given month.<sup>5</sup> Given this restriction, an assets market capitalization weight is defined by:

$$w_{m,t} = \frac{MCAP_{m,t}}{\sum_{j=1}^J MCAP_{n,t}} \quad (4)$$

with  $j \leq 500$ . Thus, the series of interest, SV, AV, and AC are defined by:

$$SV_t = \frac{1}{t-1} \sum_{\tau=1}^T \left( R_{s,\tau} - \frac{\sum_{\tau=1}^T R_{s,\tau}}{t} \right)^2 \quad (5)$$

$$AV_t = \sum_{m=1}^M w_{m,t} \sigma_{m,t}^2 \quad (6)$$

$$AC_t = \sum_{m=1}^M \sum_{n \neq m}^N w_{m,t} w_{n,t} \rho_{m,n,t} \quad (7)$$

Figure 1 shows the time series behavior of SV and AV, in percent, as well as AC. With the easily noticeable exception of October 1987, spikes in both SV and AV concentrate around NBER defined recessions.

**[Place Figure 1 about here]**

Table I shows the summary statistics for the calculated variables. Despite the use of the actual number of trading days, the restriction to assets that trade every trading day, and the adjustment to the calculation of correlation, the quarterly calculated values are almost identical to those in Pollet and Wilson (2010) over the same sample. Over the expanded the period, the annualized monthly AV has a mean value of .88%. The annualized SV mean is much lower at .25% monthly. AC is relatively consistent at .23 quarterly in the Pollet and Wilson (2010) sample, .261 monthly in the same sample and .276 over the full time period. AV is more volatile than SV, more than twice as much. In each sample AV has the highest autocorrelation. While AC is also persistent, the stock market variance is only strongly persistent at the monthly frequency with autocorrelation of .61. All three time series are stationary rejecting the unit root null in the tests of Dickey and Fuller (1979), Ng and Perron (2001), and Elliott, Rothenberg, and Stock (1996).

**[Place Table I about here]**

As my primary interest is in the use of AV versus SV in the management of leverage in the CRSP market portfolio, I test AV and SV against CRSP log excess returns. Specifically, I take the difference between the natural log of one plus the CRSP market return and the natural log of one plus the risk-free rate using:

$$r_t = \log R_{m,t} - \log R_{f,t} \quad (8)$$

where  $R_{f,t}$  the risk-free return for which I use the 1-month treasury bill rate from Ken French's website<sup>6</sup>. As shown in table II, over the full data period, each variance and correlation time

series are contemporaneously correlated to lower log excess returns. AV is significantly correlated with next month's SV, AV, and AC. Surprisingly, over the full data set, this month's AV is even nominally more correlated with next month's SV than this month's SV is, 0.625 versus .612. Over the basis period, AV is the time series most negatively correlated with next months log excess return at -.129, while it is entirely unrelated to next month's return over the whole data set.

**[Place Table II about here]**

As in Pollet and Wilson (2010), July 1963 serves as the start of the basis data period for the regression analysis.<sup>7</sup> For in and out-of-sample tests, I regress market and average variance, and average correlation against these excess log return values. Out-of-sample regressions require an in-sample training period which, I set this at 15% of the available time series for consistent calculation of robust out of sample statistics later in the analysis. This training window means that out of sample regressions, analysis begins at the end of July 1970 in the basis sample and December 1939 in the full sample.

## II. Investment

The most direct and practically relevant measure of AV as a portfolio leverage management tool is whether or not it generates portfolio gains. It is also important to make adjustments for the riskiness of the managed portfolio and compare performance across dimensions for a full investment picture. The AV managed portfolio may well generate higher annualized returns, but will it have a better Sharpe ratio than the SV managed portfolio?

To measure portfolio performance, in addition to annualized monthly log excess return, I will calculate each portfolios Sharpe ratio, Sortino ratio, and two Kappa ratios. The classic Sharpe ratio is a symmetric measure of risk and is defined as the ratio of the expected excess portfolio return over the standard deviation of portfolio returns.

$$\frac{\mathbb{E}[r_x]}{\sigma(r_x)} \quad (9)$$

While Sharpe measures each dollar of expected return for dollar of risk, the Sortino ratio attempts to more directly measure the risk most investors worry about. By using only downside deviation in the denominator, the Sortino quantifies each dollar of expected return for each dollar of loss. This downside is measured relative to a target return. (Sortino and Price, 1994) As the log returns are already excess of the risk free rate, I set the Sortino target to 0 which makes the Sortino formula:

$$\frac{\mathbb{E}[r_x - 0]}{\sqrt{\int_{-\infty}^0 (0 - r_x)^2 f(r_x) dr}} \quad (10)$$

The Sortino is just a specific instance of a more general risk measurement ratio formula. The

Kappa ratio keeps the expected return relative to a target in the numerator but allows any lower partial moment in the denominator. (Kaplan and Knowles, 2004) With the target again set to 0 the general formula is of the form:

$$\frac{\mathbb{E}[r_x - 0]}{\sqrt[n]{LPM_n}} \quad (11)$$

The Sortino ratio is the Kappa<sub>2</sub> ratio. I calculate Kappa<sub>3</sub> and Kappa<sub>4</sub> also to see relative performance of AV and SV management adjusted for negative return skew and kurtosis. All of the measures are annualized, e.g., log returns are multiplied by 12 and raitos, like Sharpe, are multiplied by the square root of 12.

Measuring a difference in each of these measures for a pair of portfolios is not difficult; measuring a significant difference is. When evaluating the difference in Sharpe ratios, the method in Memmel (2003) seems to be popular. However, when returns are not normally distributed or autocorrelated this method is not valid. AV and SV managed returns, like market returns, are weakly autocorrelated, slightly skewed, and have much fatter tails when compared to normally distributed returns. Moreover, current period returns to either the AV or SV managed portfolio depend on the prior period variance of the market return strongly questioning the i.i.d assumption made in most hyptohesis testing methodologies. Studentized time series bootstrap sampling preserves the time series properties of the AV and SV managed returns which is critical; for example, Scherer (2004) demonstrates that methods which loose the time dependence in the calculation of differences in Sortino ratios fail to properly estimate the sampling distribution and critical values. Time seires bootstrap methods preserve the structure from the original data and allow for efficient robust hypothesis testing. (Politis and Romano, 1994; Davison and Hinkley, 1997) Ledoit and Wolf (2008) show that this method is even more efficient than using Newey and West (1987) or Andrews and Monahan (1992) heteroskedasticity and autocorrelation corrected standard errors for testing the significance of differences between two portfolio Sharpe ratios. I follow the p-value estimation method in Ledoit and Wolf (2008) to determine the significance of the difference between the portfolio performance ratio measures of the AV and SV mangement strategies. This uses circular block bootstrapping of the return time series, robust centered studentized statistics computed from the bootstrap samples and is proven to be the most efficient hypothesis testing method. Politis and Romano (1992); Ledoit and Wolf (2008)

As in Moreira and Muir (2017), investment weight in the market portfolio is a function of the variance of daily market returns, SV, or the average daily return asset variance, AV, scaled by a constant,  $c$ . Moreira and Muir (2017) use a constant that scales the variance of the volatility managed portfolio equal to the buy and hold market portfolio. In the basic portfolio weighting specification, I use the same approach so that the returns of both the SV and AV managed portfolios have the same variance as the buy and hold strategy. This constant is denoted  $c_{BH}$  and it takes different values for SV and AV. This scaling requires knowing the full sample buy and hold return variance. While this does not distortions in the performance ratios, to insure robustness two other specifications for the scaling targets are used. Annual volatilities of 12% and 10% are common in academic literature and fund management so  $c_{12}$  and  $c_{10}$  approximately target those levels.



Barroso and Santa-Clara (2015); Morrison and Tadrowski (2013); Verma (2018); Fleming, Kirby, and Ostdiek (2002); Hocquard, Ng, and Papageorgiou (2013) Using each of these constant an investor rebalances at the end of month  $t$  investing in the market portfolio with weight:

$$w_{x,t} = \frac{c}{x(t)} \quad (12)$$

where  $x(t)$  is either  $SV_t$  or  $AV_t$  and hold for month  $t + 1$ .

**[Place Table III about here]**

Table III shows summary statistics for the resulting investment weights for AV and SV for the three volatility targets. When targeting the volatility of the market portfolio, both portfolios are leveraged into the market on average with investment weights of 1.3 indicating 30% leverage. Regardless of the volatility target the SV managed portfolio calls for extreme levels of leverage. Figure 2 shows that the SV strategy targeting the buy and hold volatility calls for investment weights above the maximum AV weight in several periods. More than 500% leverage is needed at the end of the 1920s, throughout the 1960s, and in the 1990s. Given that these levels of leverage are unrealistic for most investors, it will be important to see if there is a difference in performance for the AV and SV strategies under real-world investment constraints and to investigate the associated costs generated by the trading needed for the SV managed portfolio.

**[Place Figure 2 about here]**

#### A. *Portfolio Performance*

Before examining the constrained portfolio performance, I present the results for SV and AV strategies targeting the buy and hold volatility without investment constraints in table IV. As in Moreira and Muir (2017) the portfolio performance is measured across the whole CRSP data set, however the relative performance is the same or better across the basis data set.

**[Place Table IV about here]**

Table IV presents the performance ratios for the SV and AV managed portfolios targeting the buy and hold volatility without investment constraints. The buy and hold market strategy is included for reference. However since Moreira and Muir (2017) establish that the SV managed portfolio out performs the buy and hold, statistical significance results are only presented for the comparison of the SV and AV managed portfolios. The AV managed portfolio generates a statistically significant 1.08 percentage points higher average annualized log excess return. As shown in the bottom panel of figure 3 the AV strategy builds its performance advantage slowly but consistently starting from the early 1950s and from that time the SV managed portfolio is never

a better investment. As both strategies are targeting the same volatility, the significant difference in return translates into a significant difference in Sharpe ratio. At .520 versus .462, for the SV managed portfolio, the AV managed Sharpe ratio is 12.6% higher. AV also generates significantly higher  $Kappa_3$  and  $Kappa_4$  ratios. So while the overall payment for downside risk, measured by Sortino ratio, may not be significantly higher, the payment for downside skewness and extreme downside return is.

[Place Figure 3 about here]

Like out-of-sample statistics, portfolio performance numbers always bring forward questions on performance in subsamples. While divisions of the sample by date are largely arbitrary and do not automatically convey the importance of the subsample, divisions along business cycles call out specific periods of investor sensitivity. Panels (b) and (c) in table IV present the performance of the buy and hold, SV and AV managed portfolios. The AV managed portfolio is a significantly better performer across all measures. In business cycle expansion, the AV managed portfolio provides significantly more compensation and significantly more compensation for every measure of risk. The results for NBER contractions are as stark in the other direction. The SV managed portfolio appears to be so much better that it might be a more desirable option given that we cannot know periods of extended contractions before they begin and investors may desire a portfolio that protects value through downturns more than one that maximizes returns during market upswings. However, as panel (d) shows, the better performance of the SV managed portfolio is due to one, albeit a rather important, data point. The significantly better performance of the SV managed portfolio through NBER contractions depends entirely on the 1929 to 1933 Great Depression. Excluding that time period, The SV managed portfolio still provides higher returns overall but there is no significant difference in performance by any portfolio risk measure. Additionally, investors that are concerned about the loss of portfolio value, regardless of when it occurs, will value the AV managed portfolio more than the SV managed portfolio.

Drawdowns, the peak-to-trough decline in the value of a portfolio, may be the most natural measure of real market risk. (Magdon-Ismail and Atiya, 2006) Maximum drawdown, the largest peak-to-trough decline in portfolio value, in particular is often used in place of return variance as a portfolio risk measure. (Johansen and Sornette, 2000; Vecer, 2006, 2007; Sornette, 2003) Drawdowns play a significant role in the lives of fund managers as deep losses not only rob the fund of capital but motivate investors to withdraw funds making drawdowns a significant determinant of fund survival. (Baba and Goko, 2009; Papaioannou, Park, Pihlman, and Hoorn, 2013; Lang and Prestbo, 2006) To compare SV and AV managed portfolios against the market buy and hold, I consider drawdowns longer than one month so two consecutive months of negative returns will start a drawdown. The drawdown continues until the portfolio regains the value it had at the beginning of the first month of the drawdown.

[Place Table VI about here]

Table VI presents the drawdown statistics for the buy and hold, SV managed, and AV managed portfolios. The AV managed portfolio has more discrete drawdown events, 87, than either the buy and hold or SV managed portfolio. However, the drawdowns are much less severe. The buy and hold strategy losses a maximum of 84.8% of its value at the deepest point of its maximum drawdown, Max DD. AV and SV lose only 60.3% and 63.6% at the bottom of their worst drawdowns. The SV managed portfolio has the worst average loss during a drawdown, Avg DD, at 11.2% of the portfolio value while AV's average loss is only 9%. SV also stays "underwater" the longest both on average, 15 months, and during its longest drawdown, 246 months. From figure 4, the deepest losses for AV and the buy and hold occur during and after the Great Depression. However, the deepest and longest sustained losses of value for the SV managed portfolio start in the 1960s and SV does not recover until 1989. The notion that there is a drawdown so severe that it causes the collapse of the fund, or at least a management turnover, is known as the "knockout" drawdown. (Pav, 2016) Given a knockout drawdown value, it is possible to estimate the likelihood of the knockout occurring, the fund or manager not surviving, by fitting a binomial distribution to the drawdown observations using the knockout drawdown level as a cutoff to create binary values indicating a drawdown exceeding that level, 1, or not, 0. (Pav, 2016) Setting the knockout drawdown at 45%, a loss of nearly half the current value, in any given month the SV managed and AV managed portfolios have probabilities of 1.06% and .55% of incurring a knockout drawdown in the next 12 months. The AV managed portfolio is far less risky in these terms as the SV managed portfolio would be nearly twice as likely to fail and 91.7% more expensive, in theory, to insure using the max drawdown insurance of Carr, Zhang, and Hadjiladis (2011).<sup>8</sup>

[Place Figure 4 about here]

### B. Leverage

More practical analysis of portfolio performance requires the incorporation of limits on the level of investment taken in the market portfolio. Leverage of 50%, a coefficient of 1.5 on the market, is a common constraint meant to mimic real market leverage constraints for the average investor based in part on the Reg. T margin requirement<sup>9</sup>. (Campbell and Thompson, 2008; Rapach, Strauss, and Zhou, 2010; Rapach and Zhou, 2013; Huang, Jiang, Tu, and Zhou, 2015; Rapach, Ringgenberg, and Zhou, 2016; Moreira and Muir, 2017; Deuskar, Kumar, and Poland, 2017) There are at least two exchange traded funds, ETFs, which three times the return of the SP500.<sup>10</sup> So, I take a market coefficient of three as the maximum feasible investment a typical investor can make in the market portfolio.

[Place Table V about here]

Table V panel (c) presents the results from applying investment constraints after calculating the weights for AV and SV targeting the buy and hold volatility. While both portfolios still outperform the buy and hold, The separations in average annualized excess return, 1.71% and 2.07%, are even

greater when investment constraints are applied. Panels (a) and (b) in figure 3 show the effects of the growing separation. While SV is barely able to clear the buy and hold strategy under typical brokerage constraints, returns to the AV managed portfolio remain clearly above. Investors that use a leveraged SP500 ETF to impliment the AV managed portfolio strategy are rewarded with returns significantly higher than the SV managed portfolio suggested weights. The brokerage investment restrictions pull the volatility of the AV managed portfolio too far from the SV returns to generate significant differences in performance ratios. However, investors using the leveraged ETFs are rewarded not only with higher returns but significantly better performance ratios across the board with the exception of the Rachev ratio which is at least no longer significantly better for the SV managed portfolio. The ETF leverage constrained AV strategy even generates better Sharpe and Sortino ratios than the unconstrained strategy. The results in Panels (a) and (b) demonstrate that better performance of AV is not a result of or contingent on looking to the volatility of the buy and hold strategy. As the targeted volatility is lowered the difference in performance between AV and SV becomes more significant.

The differences in investment weight profiles show in table III not only generate differences in returns but also in costs. As show in table VII the AV managed portfolio generates less than half the turnover of the SV managed portfolio. The average monthly absolute change in investment weight is .752 for the SV managed portfolio and only .317 for AV. Table VII also shows the Fama-French five factor and Fama-French five factor with momentum annualized alphas for the SV and AV managed portfolios. AV management results in higher annualized alphas, 3.036% and 3.152% vs 2.833% and 2.769%, than SV. To capture the effect of these difference in annualized alphas and turnover, I calculate the transaction costs needed to zero the strategy alphas so the portfolio only breaks even. (Frazzini, Israel, and Moskowitz, 2015; Moreira and Muir, 2017) Seen in table VII, the break even transaction costs are more than 2.5 time higher for the AV managed portfolio. The SV managed portfolio breaks even at 31.436 and 30.725 basis points while it takes costs of 79.992 and 83.061 basis points to zero out the AV managed annualized alphas. However, transaction costs are only the only expense incurred by the leveraged portfolios. To estimate the borrowing costs for each strategy, I assume that any month a strategy requires a position greater than one in the market the difference between the investment weight and one is borrowed. The average monthly cost of borrowing to invest for the AV managed strategy is nearly 25% lower than for the SV managed portfolio. Using the broker call money lending rates available in Bloomberg from September 1988 to October 2016, SV incurs an average monthly cost of 15.107 basis points while AV costs only 11.411 basis points. This ignores the possibility of the investor using saved gains rather than borrowing and the necessity of borrowing when the investor has lost money, however, as the AV managed portfolio has greater returns and better drawdown statistics included gains and losses would only further the seperation.

**[Place Table VII about here]**

### C. Investor Utility

One part still missing from the analysis of the difference in performance between AV and SV is a measure of the impact on different investors. Investors with different risk aversion will experience different utility effects to the constrained returns of AV and SV<sup>11</sup>. I consider a mean-variance investor as in Kandel and Stambaugh (1996), Campbell and Thompson (2008), Ferreira and Santa-Clara (2011), Rapach et al. (2016), and others. To measure changes to investor utility from switching between the SV and AV portfolios and due to leverage constraints, I use the difference in certainty equivalent return, CER. The CER change will measure the change in riskless return an investor demands, given their risk aversion, to use a given portfolio versus another given investment conditions. This can equivalently be thought of as a measure of the management fee an investor, given a specific risk aversion, would be willing to pay to have access to an investment fund. CER change is calculated as the difference in risk adjusted return using:

$$\Delta \text{ CER} = \left( \hat{\mu}_{r_x} - \frac{1}{2}\gamma\hat{\sigma}_{r_x}^2 \right) - \left( \hat{\mu}_{r_y} - \frac{1}{2}\gamma\hat{\sigma}_{r_y}^2 \right) \quad (13)$$

where  $\hat{\mu}_{r_x}$ ,  $\hat{\mu}_{r_y}$ ,  $\hat{\sigma}_{r_x}^2$ , and  $\hat{\sigma}_{r_y}^2$  are the means and variances of the returns to the x and y portfolios and  $\gamma$  is the investor risk aversion coefficient. I multiply the gains by 12 to annualize them. All investor risk aversion coefficients from 1 to 5 are tested for investors subject to investment constraints from a limit of 1 to 3 on the market portfolio, no leverage to 200% leverage.

**[Place Figure 5 about here]**

As shown in figure 5, CER losses due to leverage constraints accumulate for the SV managed portfolio sooner than for the AV managed portfolio. This is due to the extreme leverage positions needed for the SV managed portfolio. Across risk aversion coefficients, leverage constraints bite sooner and cut deeper into the SV managed portfolio until both are driven together when no leverage is allowed. The CER of the AV starts higher at 7.95%, 8.98%, and 9.33% vs 6.87%, 7.90%, and 8.25% for investors with risk aversion coefficients of 1, 3, and 5. When those investors are subject to 200% leverage constraint the CERs are 7.96%, 8.99%, 9.33%, versus 6.22%, 7.05%, and 7.33%. The AV managed portfolio provides the same investor utility while the benefit from the SV managed portfolio decreases by 9.46%, 10.76%, and 11.15%. To incorporate the starting difference in utility I look at the gains to moving from the SV to AV managed portfolio.

**[Place Figure 6 about here]**

As shown in figure 6, CER gains for the market variance targeting AV managed portfolio are increasing in both risk aversion and leverage use for constrained risk averse mean-variance investors. An investor with a risk aversion coefficient of 2 would capture an annualized CER gain of 1.49% using 50% leverage and 1.91% implementing the AV strategy through the 200% leveraged ETFs. The most risk averse investors subject to a 20% leverage limit see a CER gain of 1.35% while the

most risk tolerant see only 1.08%. The most risk averse investor, using the highest feasible leverage, realizes a CER gain of over 2% which translates to a utility increase of 27.4%. This increase is in the neighborhood of those typically seen from return timing strategies. (Campbell et al., 1997; Moreira and Muir, 2017) Risk averse, mean-variance investors see substantial utility gains switching from the SV to AV managed portfolio and these gains increase with leverage usage.

### III. Regression Analysis

#### A. In Sample

The effectiveness of either SV or AV as an investment management signal will be driven primarily by their relationship with future risk and return. It is the trade-off which is key to the leverage management strategy. Assuming that investors hold a portfolio with whose risk-return ratio they are indifferent. When risk increases but expected returns do not, the risk-return ratio become more unattractive, and any risk-averse investor would like to decrease there position. Conversely, when the risk of the portfolio goes down without a decrease in return there is an opportunity to leverage into the position returning to the previous level of risk but now with a magnified return.

To get an understanding of the relationship between stock market or average variance and returns, I begin with in-sample regressions. In each of these regressions, all of the information available in the sample is used to estimate the parameters. In general, the regressions take this form:

$$y_{t+1} = \alpha + \beta x_t + \epsilon_t. \quad (14)$$

The cotemporaneous regressions decomposing market variance are left unreported. The results show the same relationships found in Pollet and Wilson (2010) table 2. The only difference of note is that the relationship between average correlation, AC, and next months log excess return is weaker monthly than quarterly if average variance is not also included. For all in-sample regressions, the series are standardized to a mean of zero and standard deviation of one.

**[Place Table VIII about here]**

Table VIII contains the results of regressions run on the basis data set which spans 1962 to 2016. Panel A shows that AV is a significant predictor of next months SV in all specifications. A one standard deviation increase in AV means a .545 standard deviation increase in next months market variance. This change represents an increase from the mean of .2 to .42 or an increase from an annualized standard deviation of .45% to .65%. There does not appear to be any real advantage to using this months SV to predict next months market variance over AV. The coefficient values are nearly identical, .551 versus .545, as are the adjusted  $R^2$  values, 30.3% versus 29.6%. This months AV even remains significant in the specification including this months SV. Holding this months SV constant, a one standard deviation increase in AV still signals a .257 standard deviation increase

in next months SV. Again, the inclusion of SV appears to be of little to no help as the adjusted  $R^2$  only increases from 29.6% to 31.5%. Panels B shows predictive regressions of next months average variance. Here, unlike the prior relationship, there is a definite advantage to using this months AV in the prediction of next months average variance. The adjusted  $R^2$  of this month's AV, 44.5%, is nearly twice this month's SV, 27.2%, and the inclusion of SV with AV does not appear to be a significant improvement.

Pollet and Wilson (2010) argue that average correlation is a better measure of systemic market risk. Increases in average correlation are related to a higher covariance between labor income growth and the stock market, and the stock market and bond returns. Also, they show that in the context of Rolls Critique, average correlation should be a better indicator of risk for the true market portfolio rather than stock market variance. (Roll, 1977) Thus, the results in panel C of table VIII make the argument that leverage management using AV is a better idea. While both AV and SV are positively related to next months AC, SV is more highly related. The adjusted  $R^2$  value for the regression of next months AC on current SV is more than twice that of current AV. Additionally, each signal is related to higher levels of both AV and SV next month. Hence, the use of SV as a leverage signal means taking on less weight in the market portfolio when next months systemic risk is higher relative to idiosyncratic risk. Theoretically, this is avoiding times of higher compensated versus uncompensated risk. Panel D presents the results of directly regressing next months log excess return on the market variance, average variance, and average correlation series. Indeed, AV is a significant predictor of lower returns in the next month and a better predictor than SV. A one standard deviation increase in this months AV means a .13 standard deviation, .58 percentage points, lower log excess return next month. So, a one standard deviation increase in AV means an expected negative return next month. Average variance remains a significant predictor of lower returns next month even when SV is included while the coefficient on SV is insignificant.

**[Place Table IX about here]**

Table IX presents the results for the in-sample regressions across the whole CRSP data set, from 1926 to 2016. Across the whole data set, AV is an even better predictor of next months SV than current SV. The panel A results here are even better than in table VIII. The results in panel B show the same. The prior results are not only supported but appear better with AV being the best predictor of next months AV and SV providing no help and becoming insignificant with both variables are included. As before, across the whole data set stock market variance is more highly related to next months average correlation than is average variance. Panel C in table IX supports panel C in table VIII. Panel D is the first place the results appear different in a meaningful way. There is no relationship between AV and next months return in panel D of table IX. The coefficient on AV is insignificantly positive with an adjusted  $R^2$  of -0.1%. Over the full sample, SV is a better predictor of next month's log excess return. A one standard deviation increase in this month's stock market variance indicates a .056 standard deviation, .3 percentage points, lower return. However, with an adjusted  $R^2$  of .2%, there are better return predictors. While it is no longer obvious AV

is better than SV at both risk and return anticipation across the whole data set, the results still suggest that investors are likely to be better off using AV than SV as it is a better risk anticipation measure and at least unrelated to future returns. AV should capture more returns for the same level of risk, if not avoid negative returns.

### A.1. Robustness

Average variance is an autocorrelated time series; this opens the possibility that predictive regressions using AV have estimation bias as highlighted in Stambaugh (1999). Campbell and Hentschel (1992) show that the Stambaugh bias in predictive regressions involving volatility measures and future returns can be particularly severe because of a volatility feedback effect. To eliminate the Stambaugh bias in the estimated coefficients on AV in the regressions above, I follow the methodology in Amihud and Hurvich (2004) and further make the p-values used for coefficient significance robust through wild-bootstrapping as detailed in MacKinnon (2002). Table X shows that the relationships demonstrated above are unaffected by robust bias correction. Average variance is a predictor of higher average correlation and higher stock market variance across data sets. While AV is a significant predictor of lower returns in the 1962 forward period, it is unrelated to the next months log excess returns in the whole data set. So long as the relationships hold with the limited information that investors have available at the time they make investment decisions, AV is likely to be a better leverage management signal than SV.

[Place Table X about here]

### B. Out of Sample

Average variance is a good in-sample signal, however the out-of-sample performance remains in doubt. As Welch and Goyal (2008) definitively show, out-of-sample performance is not guaranteed by in-sample performance and is essential to any investment strategy which hopes to generate positive returns. To determine the out-of-sample relationships between market and average variance, average correlation and returns, I run regressions of the standard form

$$y_{t+1} = \alpha_t + \beta_t x_t + \epsilon_t \quad (15)$$

where  $\alpha_t$  and  $\beta_t$  are estimated with from the data available only until time  $t$ . That is, I estimate  $\alpha_t$  and  $\beta_t$  by regressing  $\{y_{s+1}\}_{s=1}^{t-1}$  on a constant and  $\{x_s\}_{s=1}^{t-1}$ . In all the reported results, I follow an expanding window approach so that for the next period  $t+2$ ,  $y_{t+2}$  is estimated as  $\alpha_{t+1} + \beta_{t+1}x_{t+1}$ , where  $\alpha_{t+1}$  and  $\beta_{t+1}$  by regressing  $\{y_{s+1}\}_{s=1}^t$  on a constant and  $\{x_s\}_{s=1}^t$ . I follow this process for all subsequent months. However, as part of a test on the robustness of the out-of-sample results, I demonstrate that the results do not depend on the use of an expanding window. Most critically, equation (15) prevents any look-ahead bias. The out-of-sample prediction tests use the



same set of variables as the in sample tests. Each out-of-sample test requires an in-sample training period in which parameters are estimated using all the data up to the time period before the first out-of-sample quarter or month.

For consistency, the first one-fourth of the data is used as the initial parameter estimation period with the remaining three-fourths of observations moved through recursively generating out-of-sample predictions. Three measures of out-of-sample performance are estimated. I use the Diebold and Mariano (1995) statistic and McCracken (2007) MSE-F as measures of the increased accuracy of AV based forecasts compared to forecasts from SV as a benchmark. The DM statistic is defined as:

$$DM = \frac{\bar{d}}{\sqrt{\frac{2\pi f_d(0)}{T}}} \quad (16)$$

where  $\bar{d}$  is the mean difference in the loss differential. The loss differential is the function used to measure the difference between the forecasted and actual values. I use the squared forecast error,  $(y_t - \hat{y}_t)^2$ . So,  $\bar{d}$  is the mean value of the difference between the squared error using AV and the squared error using the benchmark forecast from SV.

$$\bar{d} = \frac{1}{T} \sum_{t=1}^T ((y_t - \hat{y}_{AV,t})^2 - (y_t - \hat{y}_{SV,t})^2) \quad (17)$$

I use the same consistent estimator for the mean loss differential,  $f_d(0)$  as in Diebold and Mariano (1995). The statistic is normally distributed under the null hypothesis of no difference in accuracy between the benchmark and proposed model. The standard positive critical values from the normal Z-table serve as cutoffs to establish a significant improvement in forecast accuracy.  $MSFE_{SV}$  is mean squared forecast error when a benchmark model is used to generate out-of-sample predictions. Mean squared forecast error is defined as

$$MSFE_x = \frac{1}{T} \sum_{\tau=t}^T (y_\tau - \hat{y}_\tau^x)^2 \quad (18)$$

where  $\hat{y}_t^x$  is the out-of-sample prediction of  $y_t$  generated from the a model using variable x, t is the first out-of-sample prediction time period, and T is the total number of out-of-sample time periods. The F-statistic in McCracken (2007) is calculated by

$$MSE - F = T \frac{MSFE_x - MSFE_b}{MSFE_b}. \quad (19)$$

The significance of the F-statistic is determined from bootstrapped values provided in McCracken (2007). Each of these two tests depends on the reduction of average squared error by the predictor x relative to a benchmark model. The final measure is a forecast encompassing statistic.

Encompassing tests the more stringent requirement that the benchmark forecasts contain no useful information absent in the forecasts of variable x. Forecast encompassing tests come from the

literature on optimal forecast combination. (Chong and Hendry, 1986; Fair and Shiller, 1990) An optimal forecast as a convex combination of two forecasts for time period  $t + 1$  defined as

$$\hat{y}_t^* = (1 - \lambda)\hat{y}_t^b + \lambda\hat{y}_t^x \quad (20)$$

where  $\hat{y}_t^x$  are predicted values generated from the model using variable  $x$  and  $\hat{y}_t^b$  are forecasts from the benchmark model. I use the forecast encompassing test of Harvey et al. (1998), ENC-HLN. The encompassing test of Harvey et al. (1998) directly tests the value and significance of the forecast combination  $\lambda$ . The test procedure rests on the calculation of a modification to the Diebold and Mariano (1995) test statistic and the consistent estimation of the long-run covariance between the difference in forecast error between the benchmark model and a model based on a competing variable,  $x$ . As such there is no one line equation that sums up the statistic used to judge the significance of  $\lambda$ . However, intuitively  $\lambda$  must be significantly different from zero for AV to have information above and beyond the forecasting information in SV and values close to one indicate that AV has all of the relevant information in SV and is optimal by itself.

**[Place Table XI about here]**

The results in table XI show that AV is a significantly better out-of-sample predictor of AV and SV. For both the variables, all three measures of out-of-sample performance show significant improvement in both the out-of-sample period starting in 1970 and starting in 1939. The forecast encompassing tests also show that AV contains all the forecasting information in SV and is optimal on its own. This means that investors concerned about the variance in the returns on their investment in the market are better off using this month's level of average asset variance to hedge next month's stock market variance than using this month's stock market variance. AV is also a significantly better predictor of next month's log excess return for the out-of-sample period starting in 1970. The DM statistic, 1.278, is nearly significant at the 10% level and both the MSE-F test and encompassing tests show significant improvement for AV. Again the encompassing test shows that AV is optimal alone and no weight needs to be given to SV in the prediction of log excess returns. Over the period starting July 1970, investors attempting to predict next month's returns would have been better off using this month's average asset variance rather than total market variance and investors deleveraging based on high values of AV would have done better avoiding negative returns rather than deleveraging based on high values of SV. Supporting the results seen in the in-sample tests, across the longer out-of-sample period, starting in 1939, AV is not a significant improvement over SV in the prediction of next month's log excess return. It will be important to look at the asset allocation performance of AV versus SV over the entire CRSP sample to see if portfolio management by AV does generate significantly higher returns than management by SV.

### B.1. Robustness

Out of sample estimation always raises issues with the choices made in the specification of the model and how to split the data into in and out of sample windows. Bluntly speaking, there are no good answers. The standard practice as in Rapach and Zhou (2013), Rapach et al. (2010), Rapach et al. (2016), and Huang et al. (2015), and many others, is to show performance in a few subsamples split by dates that the authors choose for unknown reasons. One of the concerns with subsample selection is that the window may be "ad-hoc" and the selection may mask significant results that would appear if the subsamples had been constructed differently. A second, more cynical, concern is that the presented subsample may represent significant performance that has been found either by chance or as the result of analyzing many subsample and only presenting the significant results. In any case, evaluation of the differences in performance across subsamples is often left to the imagination of the reader and whatever importance they place on the first half of the sample versus the second, the middle third versus the first and last thirds or however the data has been separated. While the selection of 1962 is not arbitrary as the daily return data is of much higher quality after, we have seen already a difference in return prediction performance for AV between the period after 1962 and the whole data set starting in 1926 which raises the question of the robustness of the out-of-sample results.

To address the robustness of the out of sample results and avoid the use of subsampling completely, I present out-of-sample statistics robust to both the specification of the prediction model, either expanding or rolling, and the choice of prediction window. Rossi and Inoue (2012) presents out of sample statistics robust to the choice of split between in and out of sample periods. The paper presents the calculation of the Diebold and Mariano (1995) statistic and the Harvey et al. (1998) encompassing test such that the choice of out-of-sample starting period is eliminated as a nuisance parameter and the asymptotic behavior of the statistics can be used to measure their observed significance. Fundamentally, this involves the calculation of each of the statistics for all feasible out-of-sample windows and in the case of rolling regression specification all feasible window sizes. The modifications are different for each of the statistics and the calculation of the robust statistic is different depending both on which statistic and which concern is being addressed. When the concern is that the chosen window could be unrepresentatively optimal, perhaps the best results of many tests, then it is possible that the null of no improvement is rejected based on the calculated statistics when in general it is true. To eliminate this possibility, Rossi and Inoue (2012) provide the  $R_T$  measure which essentially insures that the highest calculated statistics are so extreme that they could not occur without a underlying significant improvement in forecast accuracy from the benchmark to the proposed model. The  $A_T$  measure insures that the average calculated statistics are large enough that an arbitrarily selected out-of-sample starting period would not lead to the failure to reject the null of no accuracy improvement when it was indeed false. This two measures tackle the type I and II error questions. Given the results thus far, we will be looking for significant  $R_T$  values to support the significant ability of AV to predict stock market and average variance across the data set, and significant  $A_T$  values to tell us that the significant

ability of AV to predict log excess returns, seen from 1970 forward, is indicative of a real accuracy improvement while the lack of performance when including predictions from 1939 forward is simply a noisy period of poor performance obfuscating the superiority of the AV based model.

**[Place Table XII about here]**

Table XII shows the robust out-of-sample statistics. Panel A has the  $R_T$  and  $A_T$  statistics for the comparison of all possible expanding window forecasting models using AV with in-sample training windows of at least 15% of the data and out-of-sample forecasting periods of at least 15% of the data against a benchmark model using SV with the same specifications. The proportional data cut offs are necessary to use the critical values provided in the Rossi and Inoue (2012) paper, 15% is the smallest, and mean that the first feasible specification starts forecasting in December 1939 as in the out-of-sample results shown above and forecasts for at least April 2003 to December 2016 are made. Every DM statistic is significant indicating that the AV model is a significant improvement in forecasting accuracy for all variable of interest. Every encompassing test statistic is significant, however these are not  $\lambda$  values directly so while we know that AV contains information over and above SV they do not directly indicate it is optimal alone. Panel B has the results for comparisons of rolling window specification tests of AV and SV models, again using the 15% proportional cut-offs. While all DM statistics are significant, not all encompassing tests are. There are too many rolling window specifications in which SV contains significant forecasting information which is not captured by AV. A forecast combining the values from rolling window models using AV and SV will almost surely be better than using either rolling window forecast alone while using the expanding window AV model forecast alone may very well be better compared to any combination of that prediction with an expanding window SV forecast.

## IV. Additional Investment

Certainly, US equity investors and fund managers are better served by the AV managed portfolio than the SV. However clear this conclusion, it raises the questions of generalizability. Does the AV managed strategy work in equity markets outside of the US and does it still outperform the SV managed strategy? Testing the performance of AV management outside of US equities is even more important as confirmation of the argument that AV management works because returns to the AV managed portfolio depend more on AC and thus are more aligned with systemic risk. If AV management aligns better with systemic risk it should perform across assets.

### A. *Global Equity*

To test the performance of AV management across international equity markets, I collect daily returns for the Australian (AUS), Brazilian (BRA), Chinese (CHN), German (DEU), French (FRA), Indian (IND), Italian (ITA), Japanese (JPN), and English (UK) markets from Compustat - Capital

IQ. Table XIII shows the names of the indices used, the data time frames and the number of assets used for the calculation of market capitalization weights, AV and AC. In all cases the data series are much shorter. The primary limitations are the availability of daily returns and dividend data.

**[Place Table XIII about here]**

The AV management strategy is a better performer than SV across the world. Table XIV shows the differences in performance and the AV managed portfolio generates higher annualized average returns and better Sharpe ratios in all countries except Italy. In the fastest growing markets, China and India, the AV managed portfolio increases annualized average return by 2.455% and 2.637%. Both AV and SV management are improvements over the buy and hold in all markets. Its possible that some differences, e.g., the UK, are insignificant.<sup>12</sup> Australia shows the best buy and hold Sharpe ratio at .614, but the AV management strategy is able to increase it to .981, a very attractive result for any investor. The results for the AV management strategy for the Chinese market are the most alluring, a 27.381% annualized return with a .868 Sharpe ratio, but this also the shortest data series available only from 2005.

**[Place Table XIV about here]**

Table XV shows the drawdown statistics for each portfolio across countries. Unlike for the prior return measures, it is not always the case that SV management is an improvement over the buy and hold strategy. In Japan, India, and the UK, SV managed drawdowns are deeper, longer, and take longer to recover from on average. In every country but Australia, the AV managed portfolio has a shallower average drawdown. The AV managed portfolio has shorter average drawdowns in every country tested. In every country but China, the AV managed portfolio has a shorter average recovery time. Indeed, in every country but Japan, the AV management strategy is able to recover from a drawdown in less than 10 months on average.

**[Place Table XV about here]**

Table XVI highlights the differences in the trading costs of the strategies across country. This table differs from VII in that the break even trading costs are calculated in reference to reducing the annualized average returns of the AV and SV managed portfolios to the buy and hold and not reducing portfolio alphas to zero. The AV managed portfolio is able to tolerate higher trading costs in every country analyzed. In a majority of countries, 6 out of 9, the AV managed portfolio is able to tolerate trading costs twice as high as the SV managed portfolio.

**[Place Table XVI about here]**

Across the globe, the AV managed portfolio is a better leverage management signal. It results in higher annualized average returns. It captures in better Sharpe ratios. It generates better

drawdown statistics and is cheaper, in trading costs, to execute. Thus, AV management is the better equity investment strategy around the globe. However, if AV management better aligns returns with systemic risk it may be a better management strategy in more than just equities.

## V. Conclusion

I analyze the relationship between AV, the average variance of individual asset returns, and future risk and future returns. AV is a significant predictor of future portfolio variance, controlling for current variance, both in and out of sample. In contrast, AV is not significantly related to future returns and thus serves as a leverage timing signal. By using the decomposition of market variance, I add a better portfolio leverage management signal to the literature. Weighting investment by the inverse of the average asset variance, AV, rather than SV, increases market investment when total variance is expected to be low and decreases when higher total variance and lower returns are expected. The results are better Sharpe, Sortino and Kappa ratios with better Fama-French five factor and five factor plus momentum alphas. With better access to leverage timing on compensated risk, investors capture more utility with lower costs. AV portfolio investors, as well as fund managers, are more protected against drawdowns. These investment benefits manifest because the AV managed portfolio takes advantage of the correlation risk and return dynamics rather than the portfolio variance and return dynamics. Hence, the AV managed portfolio adds another dimension to the risk-return literature.

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## Notes

<sup>1</sup>”When leverage works, it magnifies your gains. Your spouse thinks you’re clever, and your neighbors get envious,...but leverage is addictive. Once having profited from its wonders, very few people retreat to more conservative practices. And as we all learned in third grade and some relearned in 2008 any series of positive numbers, however impressive the numbers may be, evaporates when multiplied by a single zero. History tells us that leverage all too often produces zeroes, even when it is employed by very smart people.” (McWhinnie, 2014)

<sup>2</sup>See, for example, Barroso and Santa-Clara (2015) and Kim, Tse, and Wald (2016) for discussions of volatility management of the momentum portfolio.

<sup>3</sup>The shortest trading month in the sample is September 2001 with 15 trading days while 17 is a common number in the months with holidays.

<sup>4</sup>The exact correction suggested in Olkin and Pratt (1958) is too computationally taxing for the equipment to which I have access.

<sup>5</sup>The least number of assets which trade every day in a given month is 392 in August of 1932. There are regularly 500 qualifying assets by the end of the 1930s.

<sup>6</sup>[http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data\\_library.html#Research](http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html#Research)

<sup>7</sup>CRSP has, as of 2005, backfilled NYSE daily returns to 1926, however the pre-1962 data is very different from the post-1962 data. The earlier data is much shallower having months with fewer than 400 assets total that meet the data requirements. Twice as many firms covering twice as many industries are available at the end of 1962 as compared to the end of 1961. As shown in Taylor (2014) the NYSE market was not a significant part of marginal wealth in the US following the Great Depression before the late 1950s. And, as documented in Jones (2002) the pre-1962 period is significantly and persistently more illiquid.

<sup>8</sup>Calculation of actual insurance costs require prices on the zero coupon bond, however given this common price the digital call option on the knockout value of the SV managed portfolio is 1.9166 times the price of the AV managed portfolio.

<sup>9</sup>Federal Reserve Board Regulation T (Reg T) establishes a baseline requirement that investors deposit 50% of an investment position in their margin trading accounts, however a brokerage house may set a higher equity requirement.

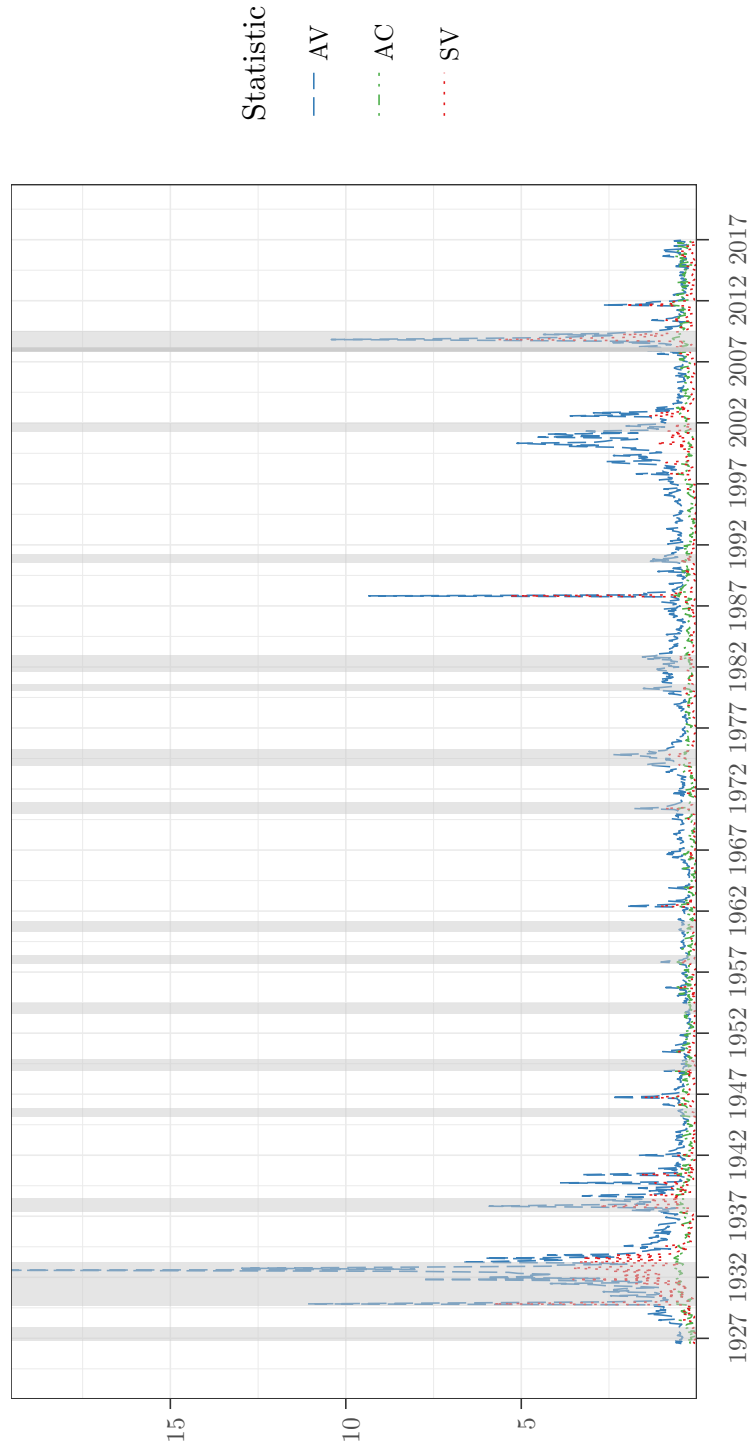
<sup>10</sup>The Direxion Daily S&P 500 Bull 3x Shares ETF, symbol SPXL, and ProShares Ultra Pro S&P 500 ETF, symbol UPRO, are two such funds.

<sup>11</sup>As Moreira and Muir (2017) note, "With no leverage limit, percentage utility gains are the same regardless of risk-aversion because investors can freely adjust their average risk exposure."

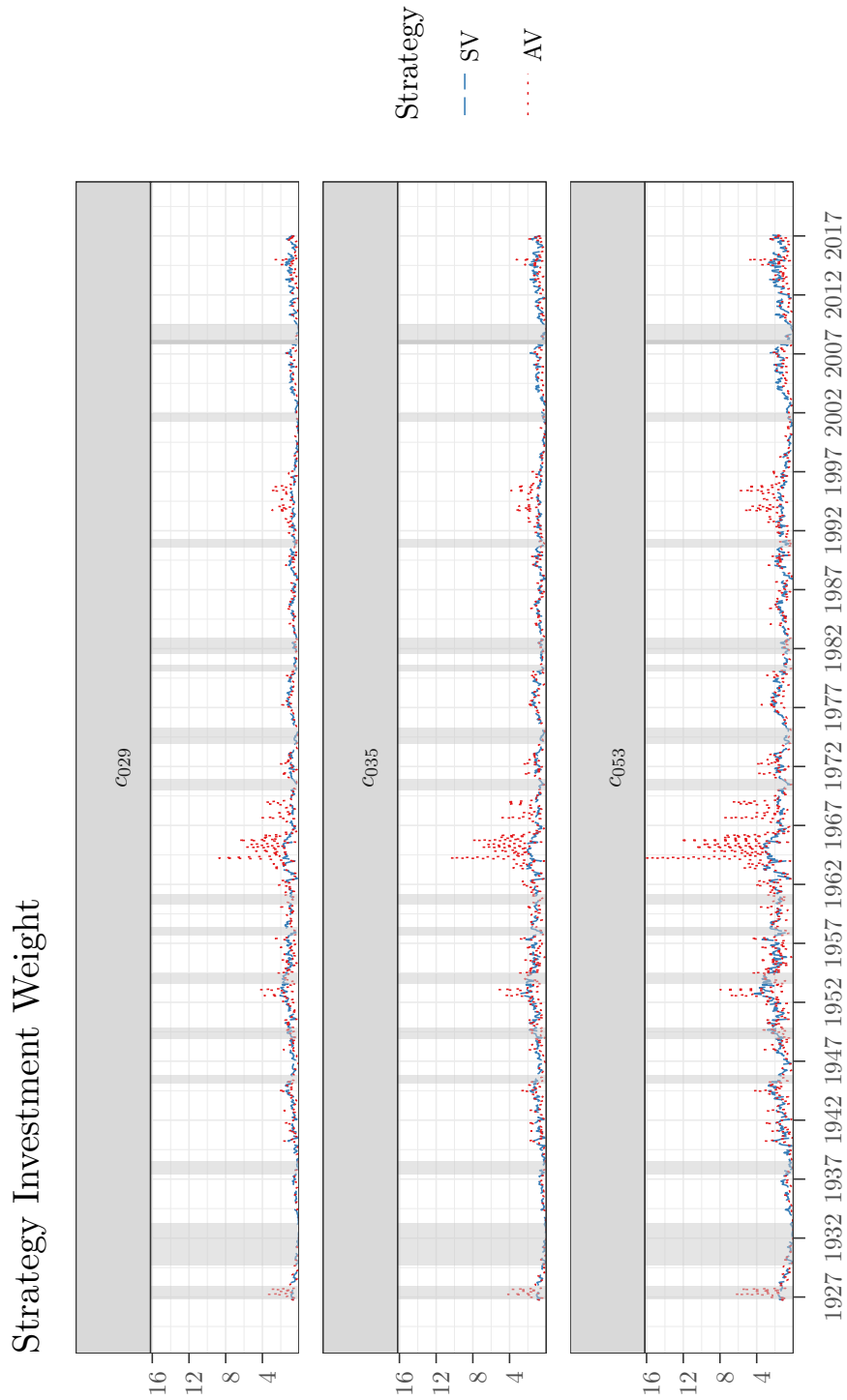
<sup>12</sup>International differences are not run through the bootstrapping significance tests due to computational tracability.

**Figure 1. Time Series of Market Statistics:** The time series of the total variance of market returns,  $AV$  is the average variance of the daily returns of individual assets in percentage;  $SV$  is the total variance of the daily market return in percentage, and  $AC$  is the average pairwise correlation of daily asset returns in the market.

### Monthly Measures of Daily Return Statistics

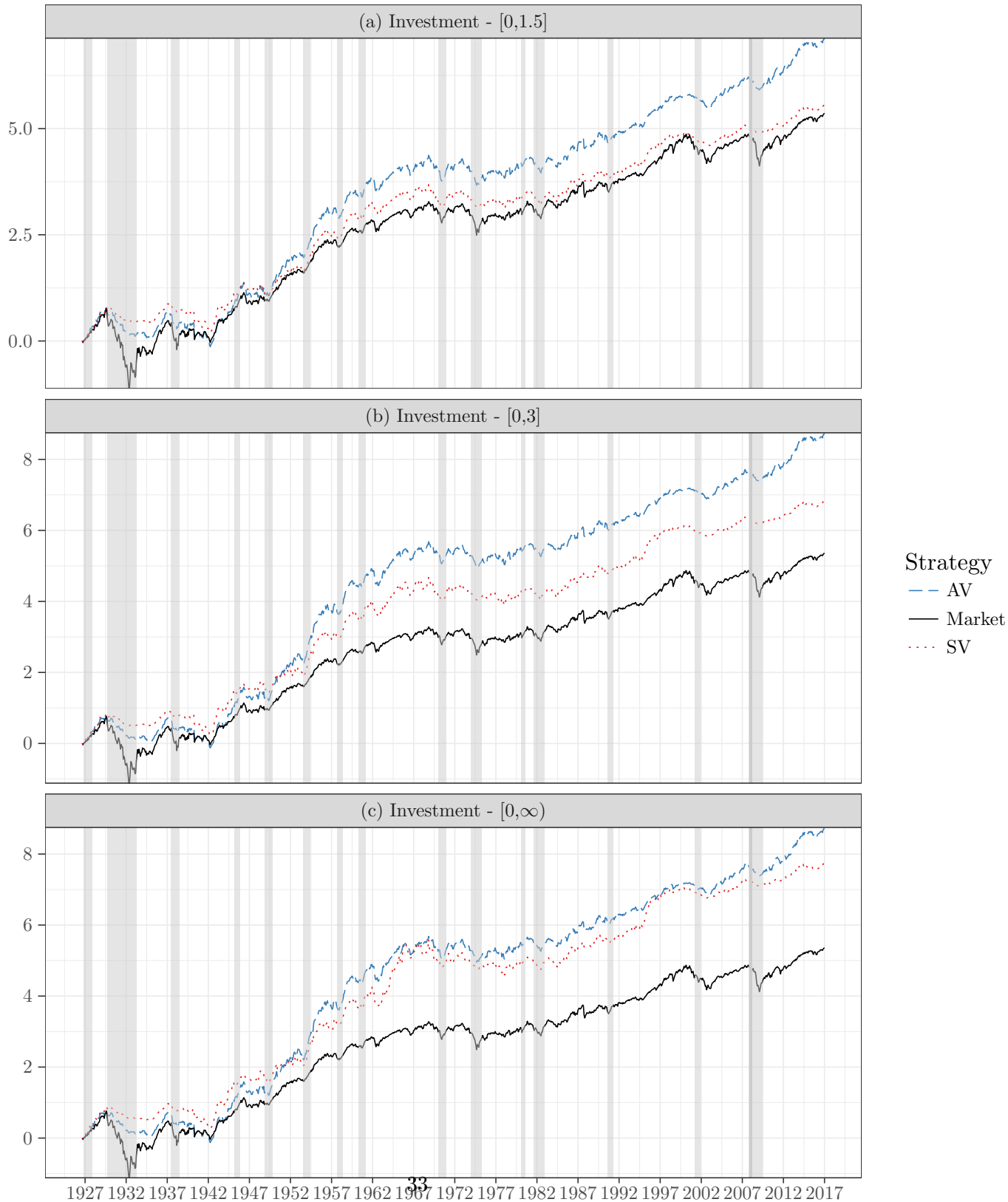


**Figure 2. Time Series of Investment Weights:** The time series of the investment weight into the market portfolio for SV and AV managed portfolios.  $c_{029}$  and  $c_{035}$  represents the weights when the SV and AV strategies target 10% and 12% annual volatilities while  $c_{053}$  represent targeting the buy and hold annual volatility over the 1926-2016 holding period.

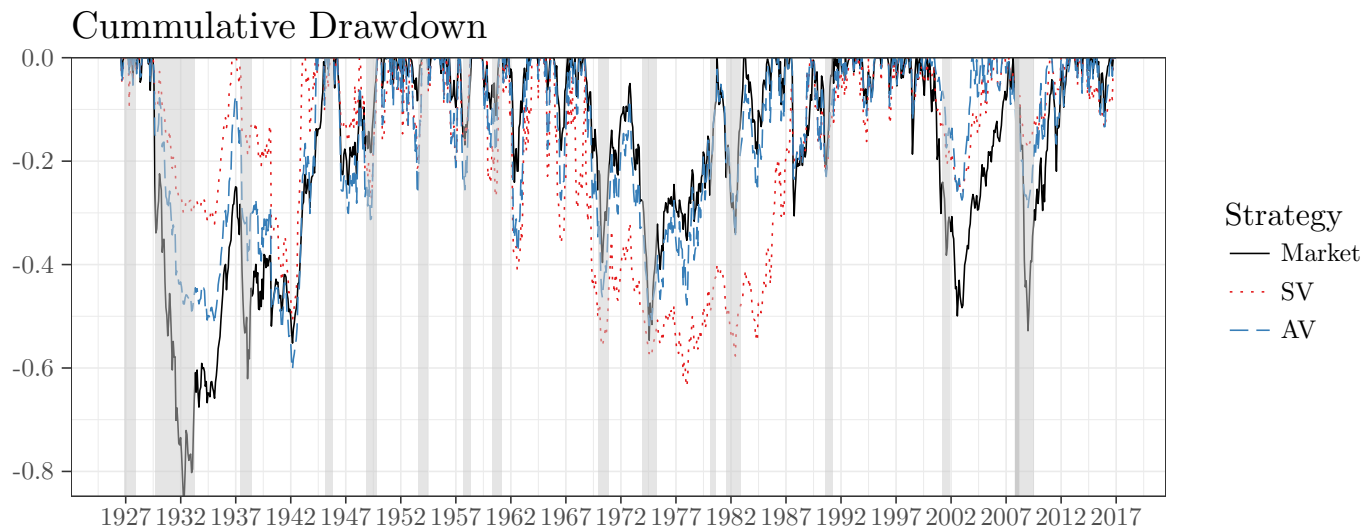




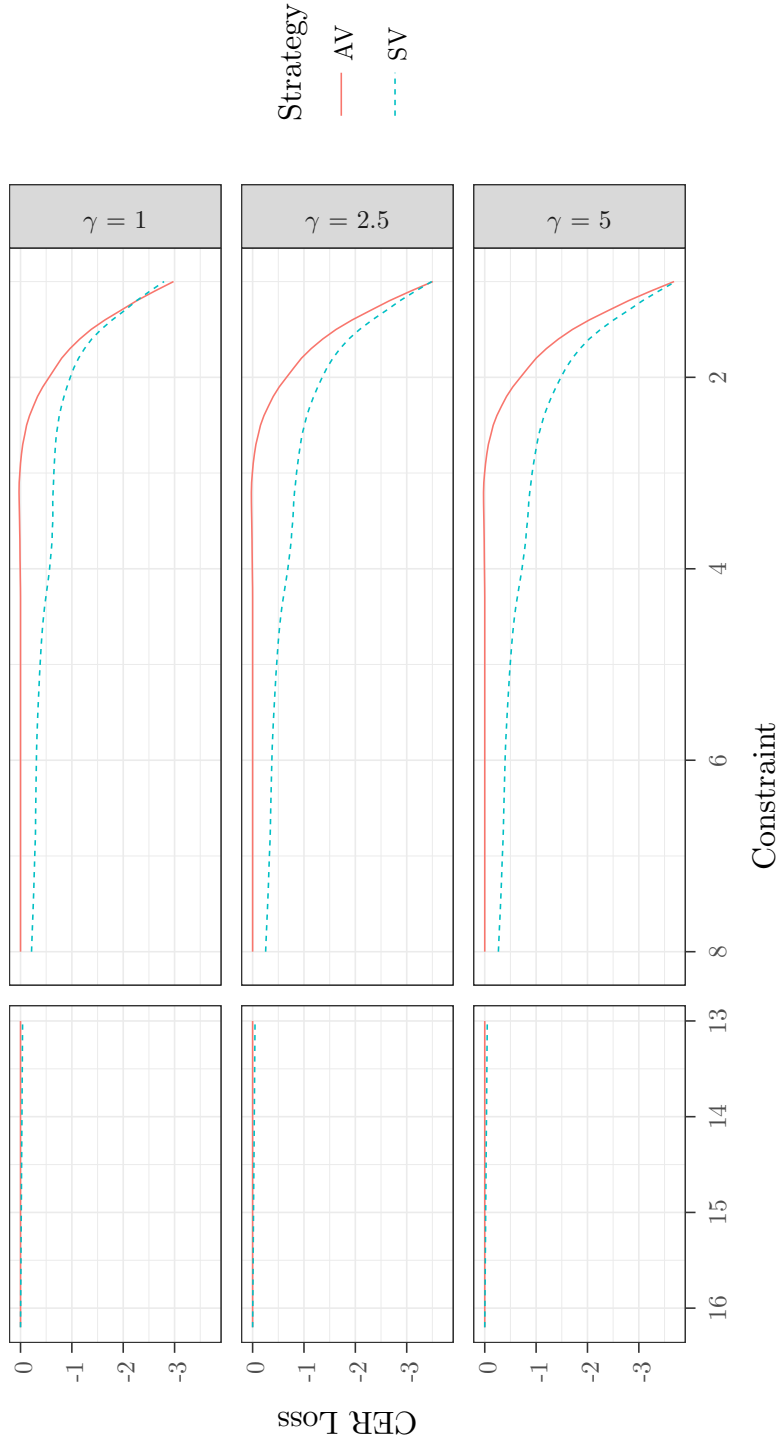
**Figure 3. Cumulative Log Excess Returns:** The time series of cumulative log excess returns for the buy and hold market investment as well as the AV and SV managed portfolios. Panel a limits the coefficient on the market portfolio between 0 and 1.5 for the AV and SV strategies; panel b limit them to weights from 0 to 3 and they are unconstrained in panel c.



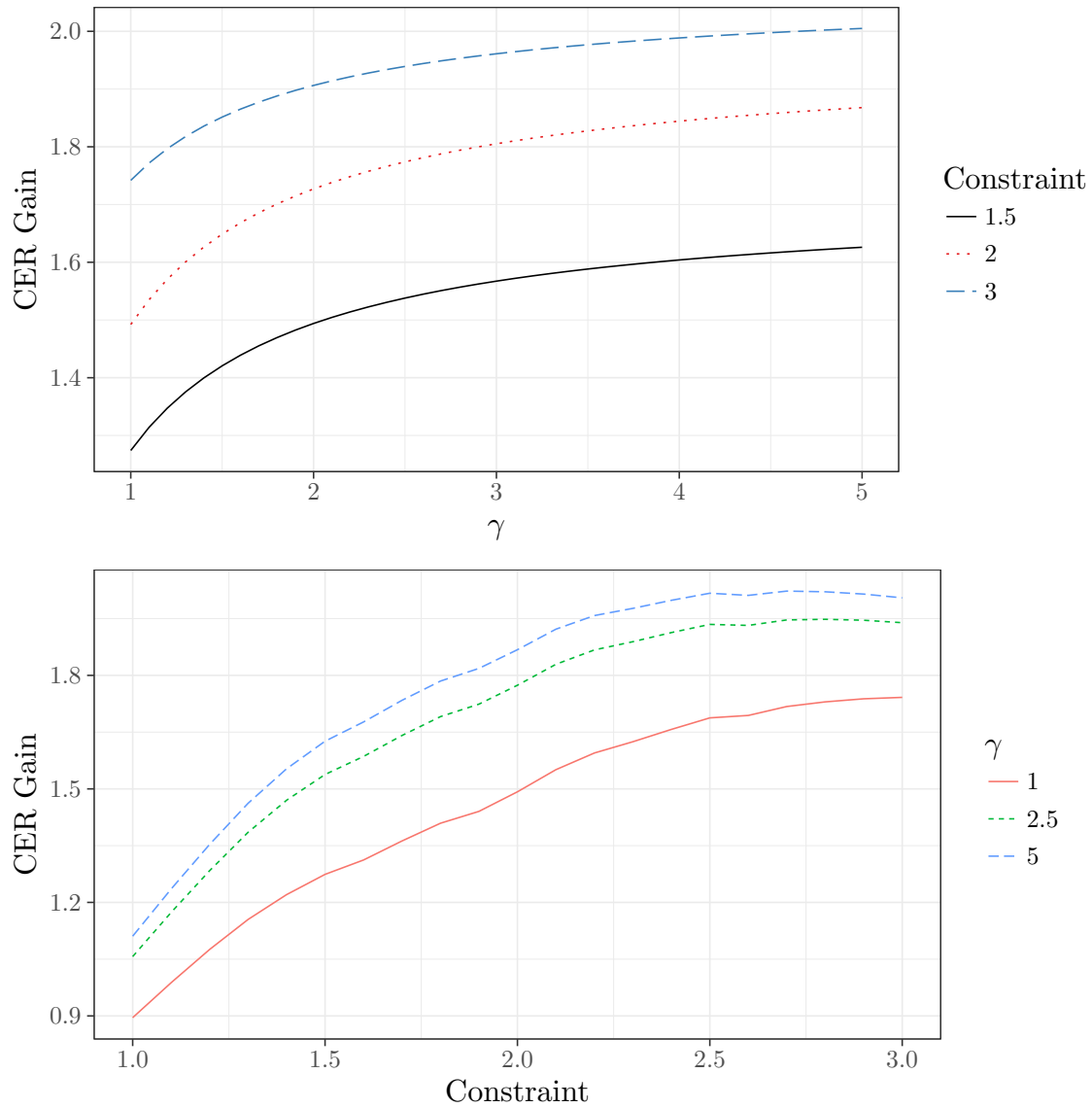
**Figure 4. Portfolio Drawdowns:** The time series of cummulative drawdowns for the buy and hold market investment as well as the AV and SV managed portfolios. SV and AV managed portfolios are targeting the buy and hold market volatility and have no investment constraints. See section II for details.



**Figure 5. CER Losses:** This figure displays the loss of certainty equivalent return as leverage constraints are applied to the AV and SV managed portfolios. The losses are expressed in percentage point differences of the constrained returns from the unconstrained returns for investors with  $\gamma$  risk aversion coefficients of 1, 2.5, and 5.



**Figure 6. CER Gains:** Certainty Equivalent Return gains for mean variance investors with risk aversion coefficients ranging from 1 to 5 and subject to investment constraints ranging from 1 to 3. See section II for details.



**Table I: Summary statistics**

The table displays summary statistics for the market variance and correlation statistics, and returns. RET is the log excess return of the CRSP market portfolio. AC is the average pairwise correlation of the daily returns of the 500 largest firms in the CRSP data set over the month or quarter. AV is the average of the individual variances of daily returns for the 500 largest firms in the CRSP data set. SV is the variance of daily CRSP market returns. See section I for details on construction.

**(a)** Pollet and Wilson Sample 1963Q1:2006Q4

Statistic	N	Mean	St. Dev.	Min	Max	Autocorrelation
RET	176	1.169	8.372	−30.072	19.956	0.000
AC	176	0.231	0.091	0.034	0.648	0.556
AV	176	2.221	1.827	0.634	12.044	0.695
SV	176	0.484	0.615	0.029	6.397	0.310

**(b)** Sample 1962M6:2016M12

Statistic	N	Mean	St. Dev.	Min	Max	Autocorrelation
RET	655	0.409	4.453	−25.985	14.515	0.081
AC	655	0.261	0.129	0.019	0.762	0.620
AV	655	0.770	0.849	0.198	10.416	0.667
SV	655	0.200	0.406	0.006	5.664	0.551

**(c)** Full Sample 1926M7:2016M12

Statistic	N	Mean	St. Dev.	Min	Max	Autocorrelation
RET	1,085	0.495	5.371	−34.523	33.188	0.106
AC	1,085	0.276	0.134	0.019	0.762	0.610
AV	1,085	0.881	1.281	0.154	19.540	0.718
SV	1,085	0.248	0.502	0.006	5.808	0.612

**Table II:Correlations**

The table displays Pearson correlation statistics for the market variance and correlation statistics, and returns. RET is the log excess return of the CRSP market portfolio. AC is the average pairwise correlation of the daily returns of the 500 largest firms in the CRSP data set over the month or quarter. AV is the average of the individual variances of daily returns for the 500 largest firms in the CRSP data set. SV is the variance of daily CRSP market returns. See section I for details on construction.

**(a)** Sample 1962M6:2016M12

	RET	AC	AV	SV	RET <sub>t+1</sub>	AC <sub>t+1</sub>	AV <sub>t+1</sub>	SV <sub>t+1</sub>
RET	1	0	0	0	0	0	0	0
AC	-0.244	1	0	0	0	0	0	0
AV	-0.284	0.352	1	0	0	0	0	0
SV	-0.353	0.529	0.899	1	0	0	0	0
RET <sub>t+1</sub>	0.081	0.049	-0.129	-0.107	1	0	0	0
AC <sub>t+1</sub>	-0.223	0.622	0.240	0.360	-0.239	1	0	0
AV <sub>t+1</sub>	-0.268	0.218	0.667	0.522	-0.283	0.351	1	0
SV <sub>t+1</sub>	-0.289	0.332	0.545	0.552	-0.351	0.528	0.899	1

**(b)** Full Sample 1926M7:2016M12

	RET	AC	AV	SV	RET <sub>t+1</sub>	AC <sub>t+1</sub>	AV <sub>t+1</sub>	SV <sub>t+1</sub>
RET	1	0	0	0	0	0	0	0
AC	-0.295	1	0	0	0	0	0	0
AV	-0.136	0.467	1	0	0	0	0	0
SV	-0.279	0.619	0.857	1	0	0	0	0
RET <sub>t+1</sub>	0.106	0.011	0	-0.057	1	0	0	0
AC <sub>t+1</sub>	-0.229	0.610	0.383	0.453	-0.295	1	0	0
AV <sub>t+1</sub>	-0.191	0.358	0.718	0.607	-0.136	0.467	1	0
SV <sub>t+1</sub>	-0.259	0.416	0.625	0.612	-0.279	0.619	0.857	1

**Table III:Investment Weights**

This table displays summary statistics for the time series of investment weights used by both the AV and SV managed portfolio strategies with different volatility targets.  $c_{053}$  represents targeting the annual volatility of the buy and hold market portfolio over the whole data set, 1926 to 2016.  $c_{029}$  and  $c_{035}$  target, approximately, 10% and 12% annual return volatility for the AV and SV managed portfolios.

Portfolio	Target	Mean	St. Dev.	Min	Pctl(25)	Median	Pctl(75)	Max
SV	$c_{029}$	0.697	0.762	0.009	0.246	0.512	0.874	8.743
AV	$c_{029}$	0.702	0.383	0.018	0.425	0.667	0.915	2.296
SV	$c_{035}$	0.841	0.920	0.011	0.297	0.618	1.055	10.552
AV	$c_{035}$	0.848	0.463	0.022	0.513	0.805	1.104	2.772
SV	$c_{053}$	1.290	1.412	0.017	0.455	0.948	1.619	16.193
AV	$c_{053}$	1.301	0.710	0.033	0.787	1.235	1.694	4.253

**Table IV: Portfolio Performance - Unconstrained**

This table displays portfolio performance measures for the AV and SV managed portfolio strategies using  $c_{053}$  to target the annual volatility of the buy and hold market portfolio over the whole data set, 1926 to 2016, and over NBER business cycle expansions, contractions and contractions excluding the Great Depression. RET is the average annualized monthly log excess return. Sharpe and Sortino are the Sharpe and Sortino ratios respectively; Kappa<sub>3</sub> and Kappa<sub>4</sub> are the lower partial skewness and lower partial kurtosis Kappa measures. See section II for details. No constraints are placed on the level of investment in the market portfolio for the AV and SV managed portfolio; the buy and hold strategy always has an investment weight of one in the market. Stars on the lines for the AV and SV managed portfolios indicate a significant positive performance difference between those two portfolios.

**(a) Full Sample**

	Return	Sharpe	Sortino	Kappa <sub>3</sub>	Kappa <sub>4</sub>
BH	5.932	0.319	0.129	0.082	0.061
SV	8.598	0.462	0.208	0.132	0.097
AV	9.677***	0.520*	0.225	0.150*	0.112**

**(b) NBER Expansions**

	Return	Sharpe	Sortino	Kappa <sub>3</sub>	Kappa <sub>4</sub>
BH	7.947	0.524	0.221	0.144	0.105
SV	10.046	0.52	0.236	0.149	0.11
AV	11.869***	0.639**	0.282**	0.184**	0.136**

**(c) NBER Contractions**

	Return	Sharpe	Sortino	Kappa <sub>3</sub>	Kappa <sub>4</sub>
BH	-4.065	-0.134	-0.0520	-0.036	-0.029
SV	1.216***	0.084**	0.035***	0.025***	0.021**
AV	-1.467	-0.080	-0.031	-0.022	-0.018

**(d) NBER Contractions x1929:09-1933:03**

	Return	Sharpe	Sortino	Kappa <sub>3</sub>	Kappa <sub>4</sub>
BH	5.031	0.225	0.088	0.060	0.047
SV	4.65***	0.298	0.130	0.093	0.077
AV	3.978	0.207	0.084	0.061	0.050

*Notes:* \*\*\*, \*\*, and \* Significant at the 1, 5, and 10 percent levels.



**Table V: Portfolio Performance - Constrained**

This table displays portfolio performance measures for the AV and SV managed portfolio strategies using  $c_{10}$ ,  $c_{12}$ , and  $c_{BH}$  to target the annual volatilities of 10%, 12% and equal to the buy hold market portfolio over the whole data set, 1926 to 2016. Performance ratios are calculated for investment constraints of a maximum of 1.5 and 3, 50% and 200% leverage. RET is the average annualized monthly log excess return. Sharpe and Sortino are the Sharpe and Sortino ratios respectively; Kappa<sub>3</sub> and Kappa<sub>4</sub> are the lower partial skewness and lower partial kurtosis Kappa measures. See section II for details. No constraints are placed on the level of investment in the market portfolio for the AV and SV managed portfolio; the buy and hold strategy always has an investment weight of one in the market. Stars on the lines for the AV and SV managed portfolios indicate a significant positive performance difference between those two portfolios.

**(a) 10% Volatility Target**

Portfolio	Constraint - 1.5					Constraint - 3				
	Return	Sharpe	Sortino	Kappa3	Kappa4	Return	Sharpe	Sortino	Kappa3	Kappa4
SV	4.065	0.461	0.201	0.130	0.097	4.396	0.454	0.200	0.127	0.094
AV	5.196***	0.522**	0.225**	0.150**	0.966	5.225***	0.520*	0.225*	0.150**	0.112**

**(b) 12% Volatility Target**

Portfolio	Constraint - 1.5					Constraint - 3				
	Return	Sharpe	Sortino	Kappa3	Kappa4	Return	Sharpe	Sortino	Kappa3	Kappa4
SV	4.735	0.470	0.205	0.133	0.098	5.219	0.452	0.198	0.127	0.094
AV	6.081***	0.516*	0.221*	0.147*	0.945	6.306***	0.520**	0.225**	0.150**	0.112**

**(c) Buy and Hold Volatility Target**

Portfolio	Constraint - 1.5					Constraint - 3				
	Return	Sharpe	Sortino	Kappa3	Kappa4	Return	Sharpe	Sortino	Kappa3	Kappa4
BH	5.932	0.319	0.129	0.082	0.061	5.932	0.319	0.129	0.082	0.061
SV	6.171	0.467	0.200	0.128	0.091	7.606	0.456	0.199	0.129	0.096
AV	7.885***	0.486	0.204	0.133	0.097	9.677***	0.522**	0.226***	0.150**	0.112**

Notes:

\*\*\* \*\*, and \* Significant at the 1, 5, and 10 percent levels.

**Table VI: Portfolio Drawdowns**

This table displays drawdown statistics for the buy and hold, AV, and SV managed portfolio strategies targeting the buy and hold strategy volatility without investment constraints. N represents the number of drawdowns longer than one month. Max DD and Avg DD are the maximum and mean drawdown in return percentage terms. Max Length and Avg Length are the lengths in months of the maximum drawdown and mean drawdown. Max Recovery and Avg Recovery are the maximum and mean times to recover back to the peak portfolio value at the start of the drawdown. See section II for details.

Strategy	N	Max DD	Avg DD	Max Length	Avg Length	Max Recovery	Avg Recovery
BH	82	-84.803	-8.069	188	11.549	154	7.207
SV	65	-63.637	-11.196	246	14.954	135	7.446
AV	87	-60.264	-9.026	205	10.851	135	5.034

**Table VII:Costs**

This table displays the average change in absolute market investment weight for the buy and hold volatility targeting, unconstrained, SV and AV managed portfolio strategies and the costs associated with trading and borrowing. Fama-French five factor and five factor with Momentum alphas are calculated using the factor portfolio return data from Ken French's website. Strategy break-even points are calculated in basis points as the cost to investment weight turnover which drives the alphas to zero. Borrowing costs are calculated in basis points as the average monthly cost incurred borrowing to take a position in the market greater than one for the SV and AV managed portfolios at the Bloomberg broker call money rate, 1984-2014. See section II.B for details.

Strategy	$ \Delta\omega $	FF-5		FF-5 + Mom		Borrowing
		$\alpha$	Break Even	$\alpha$	Break Even	
SV	0.752	2.833	31.436	2.769	30.725	15.107
AV	0.317	3.036	79.992	3.152	83.061	11.411

**Table VIII: In Sample Results - Post 1962**

The table displays in sample regression results for monthly market variance, correlation and return statistics. SV is the annualized monthly variance of daily CRSP market returns. AV and AC are the monthly average variance and average pairwise correlation of daily returns for the top 500 assets in the CRSP market, as in Pollet and Wilson (2010). RET is the log return of the CRSP market portfolio minus the log return on the 1 month treasury bill. The sample period is from 1962:06 to 2016:12. The series are standardized to a mean of zero and standard deviation of one.

(a) Market Return Variance - $SV_{t+1}$					
AV	0.545*** p = 0.000			0.489*** p = 0.000	0.257*** p = 0.001
AC		0.332*** p = 0.000		0.160*** p = 0.00001	
SV			0.551*** p = 0.000		0.320*** p = 0.00002
Constant	-0.0005 p = 0.989	-0.0001 p = 0.999	-0.0003 p = 0.993	-0.0005 p = 0.989	-0.0004 p = 0.991
R <sup>2</sup>	0.297	0.110	0.304	0.320	0.317
Adjusted R <sup>2</sup>	0.296	0.109	0.303	0.318	0.315
(b) Average Asset Return Variance - $AV_{t+1}$					
AV	0.667*** p = 0.000			0.674*** p = 0.000	1.030*** p = 0.000
AC		0.218*** p = 0.00000		-0.019 p = 0.544	
SV			0.522*** p = 0.000		-0.403*** p = 0.000
Constant	-0.001 p = 0.985	-0.00004 p = 1.000	-0.0003 p = 0.994	-0.001 p = 0.984	-0.001 p = 0.981
R <sup>2</sup>	0.445	0.048	0.273	0.446	0.477
Adjusted R <sup>2</sup>	0.445	0.046	0.272	0.444	0.475
(c) Average Asset Return Correlation - $AC_{t+1}$					
AV	0.239*** p = 0.000			0.024 p = 0.470	-0.438*** p = 0.00000
AC		0.621*** p = 0.000		0.613*** p = 0.000	
SV			0.360*** p = 0.000		0.753*** p = 0.000
Constant	-0.0002 p = 0.996	-0.0001 p = 0.998	-0.0002 p = 0.996	-0.0001 p = 0.997	-0.00003 p = 1.000
R <sup>2</sup>	0.057	0.387	0.130	0.387	0.167
Adjusted R <sup>2</sup>	0.056	0.386	0.128	0.385	0.164
(d) Log Excess Market Return - $RET_{t+1}$					
AV	-0.130*** p = 0.001			-0.168*** p = 0.0001	-0.173* p = 0.052
AC		0.049 p = 0.212		0.108*** p = 0.010	
SV			-0.107*** p = 0.006		0.048 p = 0.588
Constant	-0.000 p = 1.000	-0.000 p = 1.000	-0.000 p = 1.000	-0.000 p = 1.000	-0.000 p = 1.000
N	655	655	655	655	655
R <sup>2</sup>	0.017	0.002	0.012	0.027	0.017
Adjusted R <sup>2</sup>	0.015	0.001	0.010	0.024	0.014

Notes:

\*\*\*, \*\*, and \* Significant at the 1, 5, and 10 percent levels.

**Table IX: Full In Sample Results**

The table displays in sample regression results for monthly market variance, correlation and return statistics. SV is the annualized monthly variance of daily CRSP market returns. AV and AC are the monthly average variance and average pairwise correlation of daily returns for the top 500 assets in the CRSP market, as in Pollet and Wilson (2010). RET is the log return of the CRSP market portfolio minus the log return on the 1 month treasury bill. The sample period is from 1926:07 to 2016:12.

(a) Market Return Variance - $SV_{t+1}$					
AV	0.625*** p = 0.000			0.551*** p = 0.000	0.379*** p = 0.000
AC		0.416*** p = 0.000		0.159*** p = 0.000	
SV			0.612*** p = 0.000		0.288*** p = 0.000
Constant	-0.0003 p = 0.991	-0.0001 p = 0.998	-0.0002 p = 0.993	-0.0003 p = 0.991	-0.0003 p = 0.991
R <sup>2</sup>	0.391	0.173	0.375	0.410	0.413
Adjusted R <sup>2</sup>	0.390	0.173	0.374	0.409	0.412
(b) Average Asset Return Variance - $AV_{t+1}$					
AV	0.718*** p = 0.000			0.704*** p = 0.000	0.745*** p = 0.000
AC		0.358*** p = 0.000		0.029 p = 0.232	
SV			0.606*** p = 0.000		-0.031 p = 0.445
Constant	-0.0003 p = 0.989	-0.0001 p = 0.998	-0.0002 p = 0.993	-0.0003 p = 0.989	-0.0003 p = 0.989
R <sup>2</sup>	0.515	0.128	0.368	0.516	0.516
Adjusted R <sup>2</sup>	0.515	0.127	0.367	0.515	0.515
(c) Average Asset Return Correlation - $AC_{t+1}$					
AV	0.383*** p = 0.000			0.125*** p = 0.00001	-0.018 p = 0.738
AC		0.610*** p = 0.000		0.551*** p = 0.000	
SV			0.453*** p = 0.000		0.468*** p = 0.000
Constant	-0.0002 p = 0.996	-0.0001 p = 0.996	-0.0002 p = 0.996	-0.0002 p = 0.995	-0.0002 p = 0.996
R <sup>2</sup>	0.147	0.372	0.205	0.385	0.205
Adjusted R <sup>2</sup>	0.146	0.372	0.204	0.384	0.204
(d) Log Excess Market Return - $RET_{t+1}$					
AV	0.0002 p = 0.996			-0.006 p = 0.857	0.182*** p = 0.002
AC		0.011 p = 0.724		0.014 p = 0.692	
SV			-0.056* p = 0.064		-0.213*** p = 0.0003
Constant	-0.00000 p = 1.000	-0.00001 p = 1.000	0.00002 p = 1.000	-0.00001 p = 1.000	0.00002 p = 1.000
N	1,085	1,085	1,085	1,085	1,085
R <sup>2</sup>	0.00000	0.0001	0.003	0.0001	0.012
Adjusted R <sup>2</sup>	-0.001	-0.001	0.002	-0.002	0.010

Notes: \*\*\*, \*\*, and \* Significant at the 1, 5, and 10 percent levels.

**Table X: In Sample Robust Results**

The table displays in-sample regression results of  $AC_{t+1}$ ,  $SV_{t+1}$  and  $RET_{t+1}$  on  $AV_t$ . The coefficients and standard errors calculated robust to Kandel and Stambaugh (1996) bias using the correction in Amihud and Hurvich (2004). Robust p-values are calculated through t-statistic wild-bootstrap simulation, as in MacKinnon (2002).

**(a)**  $AV_t$ : Sample 1962:06 to 2016:12

	$\beta$	t.stat	p
$AC_{t+1}$	0.241	6.567	0.000
$SV_{t+1}$	0.550	32.442	0.000
$RET_{t+1}$	-0.131	-3.494	0.166

**(b)**  $AV_t$ : Sample 1926:07 to 2016:12

	$\beta$	t.stat	p
$AC_{t+1}$	0.384	14.342	0.000
$SV_{t+1}$	0.627	40.002	0.000
$RET_{t+1}$	0.000	-0.016	0.562

**Table XI: Full Out-of-Sample Results**

The table displays out-of-sample expanding window regression results for monthly market variance, correlation and return statistics.  $SV$  is the annualized monthly variance of daily CRSP market returns.  $AV$  and  $AC$  are the monthly average variance and average pairwise correlation of daily returns for the top 500 assets in the CRSP market, as in Pollet and Wilson (2010).  $RET$  is the log return of the CRSP market portfolio minus the log return on the 1 month treasury bill.  $DM$  is the Diebold and Mariano (1995) statistic measuring for cast accuracy.  $MSE-F$  is the mean squared error improvement F-test of in McCracken (2007) and  $ENC-HLN$  is the forecast encompassing test of Harvey et al. (1998). In each panel the benchmark forecasts come from a model which uses  $SV_t$  to predict the independent variable.

**(a)** Sample 1970:07 to 2016:12

	DM	MSE-F	ENC-HLN
$AC_{t+1}$	1.074	109.736***	1
$SV_{t+1}$	1.53*	29.252***	1**
$AV_{t+1}$	2.286**	109.333***	1***
$RET_{t+1}$	1.278	11.801***	1*

**(b)** Sample 1939:12 to 2016:12

	DM	MSE-F	ENC-HLN
$AC_{t+1}$	1.604*	46.251***	1**
$SV_{t+1}$	1.041	21.57***	0.956**
$AV_{t+1}$	3.104***	198.267***	1***
$RET_{t+1}$	-2.027	-8.702	0

*Notes:*           \*\*\*, \*\*, and \* Significant at the 1, 5, and 10 percent levels.

**Table XII: Out of Sample Robust Results**

The table displays out-of-sample regression results of forecasts using  $AV_{t+1}$  as a predictor. Rossi and Inoue (2012) provides the methodology to make the calculations of the out-of-sample accuracy improvements of Diebold and Mariano (1995) and McCracken (2007) and the encompassing test of Harvey et al. (1998). In each panel the benchmark forecasts come from a model which uses  $SV_t$  to predict the independent variable.

**(a) Robust Expanding Window Results**

Stat	Variable	DM	ENC-HLN
$R_T$	$AC_{t+1}$	28.532***	6.769***
$R_T$	$SV_{t+1}$	8.874***	1.838***
$R_T$	$AV_{t+1}$	34.347***	18.197***
$R_T$	$RET_{t+1}$	29.124***	4.871***
$A_T$	$AC_{t+1}$	19.867***	1.828***
$A_T$	$SV_{t+1}$	2.647***	0.949***
$A_T$	$AV_{t+1}$	21.751***	10.7***
$A_T$	$RET_{t+1}$	13.347***	1.68***

**(b) Robust Rolling Window Results**

Stat	Variable	DM	ENC-HLN
$R_T$	$AC_{t+1}$	27.398***	8.706**
$R_T$	$SV_{t+1}$	21.92***	3.973
$R_T$	$AV_{t+1}$	34.292***	29.804***
$R_T$	$RET_{t+1}$	15.964***	3.884
$A_T$	$AC_{t+1}$	8.08***	1.542
$A_T$	$SV_{t+1}$	8.218***	2.062
$A_T$	$AV_{t+1}$	21.631***	19.449***
$A_T$	$RET_{t+1}$	9.209***	1.78

Notes: \*\*\*, \*\*, and \* Significant at the 1, 5, and 10 percent levels.



**Table XIII:Country Indices**

The table displays summary statistics for international data used. The start year and month, number of months, name of the market proxy index, and the average number of assets meeting the trading and liquidity requirements for each country where the performance of AV and SV managed equity portfolios.

Country	Start	Months	Index	Avg_Assests
AUS	2000 - 5	212	ASX	200
BRA	1995 - 2	275	iShares MSCI Brazil ETF	60
CHN	2005 - 5	152	CSI 300	298.400
DEU	1993 - 11	290	HDAX	110
FRA	1993 - 9	292	SBF 120	120
IND	2000 - 5	212	Nifty 50	50
ITA	2003 - 8	173	FTSE MIB	40
JPN	1993 - 6	295	Nikkei	255
UK	1993 - 6	295	FTSE	100
World	1995 - 3	274	MSCI ACWI	2400

**Table XIV:**International Portfolio Performance - this table shows annualized average log excess returns and Sharpe ratios for the AV and SV managed portfolios compared to the market buy and hold for various international markets. See table XIII for details.

	AV		SV		BH	
Country	RET	Sharpe	RET	Sharpe	RET	Sharpe
AUS	12.477	0.981	11.993	0.943	7.805	0.614
BRA	11.000	0.291	9.037	0.240	6.163	0.164
CHN	27.381	0.868	24.926	0.790	12.286	0.390
DEU	11.064	0.537	7.633	0.371	5.399	0.262
FRA	7.243	0.404	6.128	0.341	4.904	0.273
IND	14.893	0.633	12.256	0.521	11.460	0.487
ITA	3.838	0.194	3.912	0.198	1.451	0.073
JPN	1.375	0.068	0.129	0.006	-0.775	-0.038
UK	6.591	0.485	5.984	0.441	5.111	0.376
World	8.604	0.551	8.306	0.536	4.484	0.290

**Table XV:**International Portfolio Drawdowns - this table shows drawdown statistics for the AV and SV managed portfolios compared to the market buy and hold for various international markets.

Country	AV			SV			BH		
	Avg DD	Avg Length	Avg Recovery	Avg DD	Avg Length	Avg Recovery	Avg DD	Avg Length	Avg Recovery
AUS	-6.302	7.174	3.348	-5.322	9.263	5.421	-6.318	8.600	4.550
BRA	-8.059	9.560	4.208	-17.469	15.235	5.500	-15.064	17.067	4.286
CHN	-9.511	10.333	5.917	-10.074	10.583	3.727	-19.374	27.400	2.000
DEU	-11.051	10.625	5.783	-12.587	16.812	9.933	-10.706	17.125	12.333
FRA	-10.263	14.111	5.941	-15.260	18.267	10.214	-11.590	19.071	15.077
IND	-8.170	6.500	2.885	-12.545	12.467	5.733	-10.862	8.318	4.500
ITA	-14.625	19.500	2.143	-18.174	22.571	2.333	-8.919	15.400	1.667
JPN	-30.655	72.750	41.750	-78.514	294.000	175.000	-40.792	148.00	2.000
UK	-6.060	11.609	4.652	-7.872	14.158	8.158	-6.018	10.560	7.240
World	-6.982	9.909	7.333	-9.776	12.5	7.059	-8.209	10.091	6.429

**Table XVI:**International Portfolio Costs - this table shows the break even transaction costs for the AV and SV managed portfolios. These are the trading costs which reduced the average return to the AV and SV managed portfolios to the buy and hold market return in each country.

Country	AV			SV			RET <sub>BH</sub>
	RET	$ \Delta\omega $	Break Even	RET	$ \Delta\omega $	Break Even	
AUS	12.477	0.486	80.139	11.993	0.466	74.914	7.805
BRA	11.000	0.253	159.118	9.037	0.623	38.462	6.163
CHN	27.381	0.305	412.715	24.926	0.538	195.972	12.286
DEU	11.064	0.499	94.545	7.633	0.581	32.052	5.399
FRA	7.243	0.468	41.656	6.128	0.536	19.041	4.904
IND	14.893	0.710	40.316	12.256	0.507	13.097	11.460
ITA	3.838	0.448	44.366	3.912	0.603	33.991	1.451
JPN	1.375	0.442	40.518	0.129	0.551	13.675	-0.775
UK	6.591	0.473	26.113	5.984	0.509	14.287	5.111
World	8.604	0.439	78.113	8.306	0.642	49.586	4.484