Average Variance

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How to Look Clever and Have Envious Neighbors: Average Variance Managed Investment

Jeramia Poland



Indian School of Business

December 14, 2018

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Background

How should risk (portfolio volatility) be managed?

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- How should risk (portfolio volatility) be managed?
- Hocquard et al (2013) scaling investment to manage tail risk

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Central Idea I

 \bullet Use AV to avoid divesting high AC or investing high AV

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- Me : weight $= \frac{c_{AV}}{AV_t}$, c_{AV} to match buy and hold volatility

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- Use AV to avoid divesting high AC or investing high AV
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- Should find AV management outperforms SV management

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More Pollet and Wilson (2010)

• Variance of observed returns \neq variance of "market" returns \neq variance of aggregate wealth

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Central Idea II

• If AC = systematic, when AC is high \Rightarrow higher $r_{u,t+1}$ is expected

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- If AC = systematic, when AC is high \Rightarrow higher $r_{u,t+1}$ is expected
- $r_{u,t}$ is not observable through index returns

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 - Equity AV management has higher investment in times of higher $cov(r_{s,t+1}, r_{u,t+1})$, so it should work across asset classes

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Preview Results

• AV management outperforms SV management - returns, ratios, turnover, drawdowns, and utility

- AV management outperforms SV management returns, ratios, turnover, drawdowns, and utility
- AV management works better under practical investment constraints

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 - AV managed returns depend on the relationship of the proxy daily returns and aggregate wealth in time-series and cross-sectionally
 - The AV of equity returns can be used to manage investment in other assets across the economy

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Contribution

Variance Management

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Variance Management

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Risk Dynamics

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Risk Dynamics

 Gonzlez-Urteaga and Rubio (2016) and Bollerslev, Hood, Huss, and Pedersen (2017)

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- AV management informs about the risk mix across the economy

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Equity Data

Equity Data

Country	Start	Months	Index	Assets
USA	1926 - 8	1085	CRSP	500
AUS	2000 - 5	212	ASX	200
BRA	1995 - 2	275	iShares MSCI Brazil ETF	60
CHN	2005 - 5	152	CSI 300	300
DEU	1993 - 11	290	HDAX	110
FRA	1993 - 9	292	SBF 120	120
IND	2000 - 5	212	Nifty 50	50
ITA	2003 - 8	173	FTSE MIB	40
JPN	1993 - 6	295	Nikkei	255
UK	1993 - 6	295	FTSE	100
World	1995 - 3	274	MSCI ACWI	1735

Data

Non-Equity Data

Other Asset Data

Index	Start	Months	Asset Class
DI 110.0	2225	4-0	6
Bloomberg US Spot	2005 - 6	158	Currency
Deutsche Bank Currency	2005 - 6	158	Currency
Deutsche Bank Carry	2005 - 6	158	Currency
Deutsche Bank Momentum	2005 - 6	158	Currency
S&P REIT Index	2005 - 6	158	Real Estate
Bloomberg Commodity	2005 - 6	158	Commodities

Conclusio

AV Managed Investment

- $SV_t = \sigma_{S,t}^2$
- With m assets in the market, AV $_t = \sum_{m=1}^{M} w_{m,t} \sigma_{m,t}^2$
- $W_t = \frac{c_{target}}{X}$ is the investment weight in the portfolio, where $X \in \{AV_{t-1},SV_{t-1}\}$
- The constant c_{target} is used to control the volatility of the strategy
- c_{BH} matching the buy and hold
- For robustness, c_{10} and c_{12} targeting 10% or 12% annual return volatility

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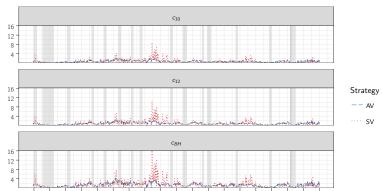
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Investment Weights

Strategy Investment Weight



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Investment Weight Again Again

CRSP Market Portfolio Investment Weights

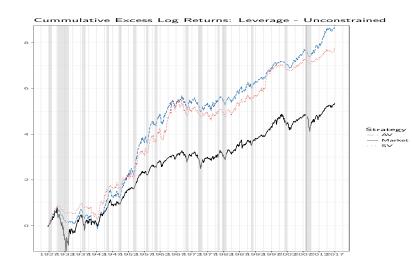
Portfolio	Target	Mean	St. Dev.	Min	Pctl(25)	Median	Pctl(75)	Max
SV	C ₁₀	0.697	0.762	0.009	0.246	0.512	0.874	8.743
AV	c ₁₀	0.702	0.383	0.018	0.425	0.667	0.915	2.296
SV	c ₁₂	0.841	0.920	0.011	0.297	0.618	1.055	10.552
AV	C ₁₂	0.848	0.463	0.022	0.513	0.805	1.104	2.772
SV	CBH	1.290	1.412	0.017	0.455	0.948	1.619	16.193
AV	CBH	1.301	0.710	0.033	0.787	1.235	1.694	4.253

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US Equity Performance



AV: 9.68% SV: 8.60% BH: 5.93%

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Performance Measures

ullet RET = annualized average log excess return

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- Kappa $_n = \frac{\mathbb{E}[R_{\times} 0]}{\sqrt[n]{LPM_n}}$, where LPM is lower partial moment Kappa $_2 =$ Sortino

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- Certainty Equivalent Return gain (CER) = Average utility from AV Average utility from SV for mean-variance investor with risk aversion γ

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Unconstrained

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 $c_{\emph{BH}}$: Unconstrained

	Return	Sharpe	Sortino	Kappa ₃	$Kappa_4$	α_{FF3}	$lpha_{\it FF3+Mom}$
ВН	5.934	0.319	0.129	0.082	0.061		
SV	8.589	0.462	0.208	0.132	0.097	5.477	3.201
AV	9.676***	0.520*	0.225	0.150*	0.112*	5.594***	3.164

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Outperforms with higher returns for the same variance

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Results Performance

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Unconstrained

- Outperforms with higher returns for the same variance
- Outperforms with other risk specifications downside, skewness, kurtosis, α

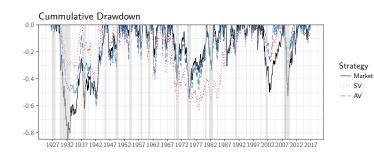
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Drawdowns: c_{BH}



Strategy	N	Max DD	Avg DD	Max Length	Avg Length	Max Recovery	Avg Recovery
ВН	82	-84.803	-8.069	188	11.549	154	7.207
SV	65	-63.637	-11.196	246	14.954	135	7.446
AV	87	-60.264	-9.026	205	10.851	135	5.034

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Drawdown Insurance: Unconstrained

Knockout

- Drawndown large enough to shutter fund (investor pull-out), cost manager job
- Assuming 45% loss in a 12-month period as knockout
- SV 1.06% and AV .55% using Pav (2016)
- AV \approx half the cost to insure, Carr, Zhang, and Hadjiliadis (2011)

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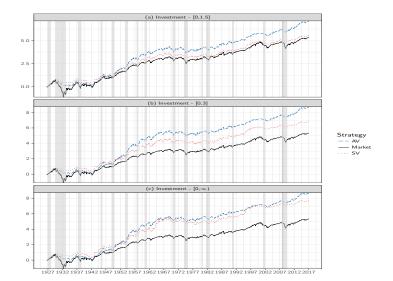
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Leverage and Returns



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Leverage

	c _{BH} : Constraint - 1.5							
Portfolio	Return	Sharpe	Sortino	$Kappa_3$	$Kappa_4$			
ВН	5.932	0.319	0.129	0.082	0.061			
SV	6.171	0.467	0.200	0.128	0.091			
AV	7.885***	0.486	0.204	0.133	0.097			

c_{BH}: Constraint - 3

		Di	,		
Portfolio	Return	Sharpe	Sortino	$Kappa_3$	$Kappa_4$
BH	5.932	0.319	0.129	0.082	0.061
SV	7.606	0.456	0.199	0.129	0.096
AV	9.677***	0.522**	0.226**	0.150**	0.112**

Notes: ***,**, and * Significant at the 1, 5, and 10 percent levels.

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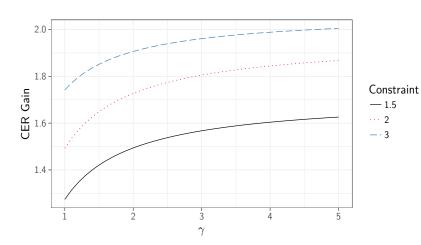
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Investor Utility



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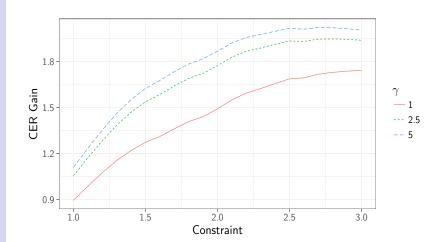
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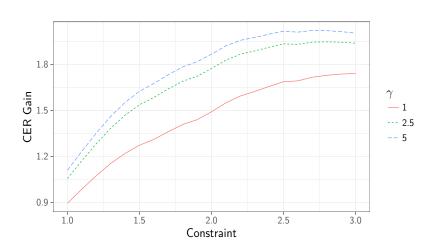


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Investor Utility



 Stochastic dominance tests show that non-EUT investors prefer AV management. ◆ロ > ← (回) ← (重) を (重) を (で)

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Suggestively Systematic

• 1926-1962 CRSP dialy returns - low $\textit{w}_{\textit{s},\textit{t}}$ and $\beta_{\textit{t}}$

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 - persistently illiquid

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Results

Systematic Risk

- 1926-1962 CRSP dialy returns low $w_{s,t}$ and β_t
 - shallower with only NYSE firms, est below 87% market cap
 - persistently illiquid
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Risk and Reward

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Performance Systematic Risk

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 - AC should not predict returns 1926 1962
- AV managed returns should depend systematically on the representativeness of daily
- AV management should work globally
- AV management using the AV of equity returns should generate better performance across asset classes

Performance Systematic Risk

Conclusio

Global Performance

	AV		SV		ВН	
Country	RET	Sharpe	RET	Sharpe	RET	Sharpe
AUS	12.477***	0.981	11.993	0.943	7.805	0.614
BRA	11.000***	0.291	9.037	0.240	6.163	0.164
CHN	27.381	0.868	24.926	0.790	12.286	0.390
DEU	11.064***	0.537*	7.633	0.371	5.399	0.262
FRA	7.243***	0.404	6.128	0.341	4.904	0.273
IND	14.893***	0.633	12.256	0.521	11.460	0.487
ITA	3.838	0.194	3.912	0.198	1.451	0.073
JPN	1.375***	0.068	0.129	0.006	-0.775	-0.038
UK	6.591***	0.485	5.984	0.441	5.111	0.376
World	8.603***	0.551	8.306	0.536	4.484	0.290

Average Variance

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Regressions

 Expect AC not to predict returns in the pre-1962 data but it should post-1962 Methodolog

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Regressions

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Systematic Risk

Regressions

- Expect AC not to predict returns in the pre-1962 data but it should post-1962
- In-sample regression coefficients can be corrected for possible "volatility feedback" - Campbell and Hentschel (1992)
- Amihud and Hurvich (2004) bias correction
- forecasts unstable, sensitive to period; may not be dynamic or informative of anything - Welch and Goyal (2008)

Systematic Risk

In-sample Sub-samples

RET_{t+1} -	1926M7:1962M6
---------------	---------------

AV	0.061			0.121	0.315
AC		-0.032		-0.099	
SV			-0.028		-0.264
R^2	0.004	0.001	0.001	0.010	0.026
Adjusted R^2	0.002	-0.002	-0.002	0.005	0.021

RET_{t+1} - 1962M6:2016M12

AV	-0.131			-0.168**	0.016
AC		0.047***		0.106***	
SV			-0.109		0.254
R^2	0.017	0.002	0.012	0.027	0.017
Adjusted R ²	0.015	0.001	0.010	0.024	0.014

Out of Sample Stats

Diebold-Marino Statistic (1995)

• DM =
$$\frac{\bar{d}}{\sqrt{\frac{2\pi f_d(0)}{T}}}$$

 Asymptotically normally statistic comparing significance of accuracy ratio

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• MSE-F =
$$T \times \frac{\frac{1}{T} \sum_{1}^{T} (e_{b,t}^2 - e_{x,t}^2)}{MSFE_x}$$

• MSE-F = F-type test for significance in squared residual

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ENC-HLN Harvey, Lebourne and Newbold (1998)

- Optimal forecast $= \hat{y}_t^* = (1 \lambda)\hat{y}_{b,t} + \lambda\hat{y}_{x,t}$
- ullet $\lambda=$ measure of the optimal combination of forecasts

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Rossi and Inoue (2012)

- Calculate OOS stats on all feasible window specifications
- Use asymptotic distribution \rightarrow stat critical values
- Ose asymptotic distribution stat critical values

• Different critical values for Type I (R_T) and Type II ($A_{T_{25/30}}$)

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Risk and Reward

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Out of Sample Results

Table: Sample 1939:12 to 2016:12

	DM	MSE-F	ENC-HLN
AC_{t+1}	1.604*	46.251***	1**
SV_{t+1}	1.041	21.57***	0.956**
AV_{t+1}	3.104***	198.267***	1***
RET_{t+1}	-2.027	-8.702	0

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Robust Out of Sample Results

Table: Sample 1939:12 to 2016:12

Stat	Variable	DM	ENC-HLN
R_T	SV_{t+1}	8.874***	1.838***
R_T	RET_{t+1}	29.124***	4.871***
A_T	SV_{t+1}	2.647***	0.949***
A_T	RET_{t+1}	13.347***	1.68***

Notes: ***,**, and * Significant at the 1, 5, and 10 percent levels.

• AC fails when it "should"

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Notes: ***,**, and * Significant at the 1, 5, and 10 percent levels.

- AC fails when it "should"
- AV forecasts better than SV with robust in and out-of-sample performance
- AV is a more informative signal

Performance Systematic Risk

Conclusio

Systematic Performance

• Use a proxy for the weight of stock returns in the return on aggregate wealth, $w_{s,t}$

Global Long - Short Ratio of Market RET to Wealth† RET $(w_{s,t})$

	RET	Sharpe	$lpha_{\it FF3}$	$lpha_{\it FF5}$	$lpha_{\it FF5+Mom}$
Long	12.601	0.747	9.484**	7.909*	7.725*
Short	7.537	0.562	5.038*	5.422*	5.318*
Long - Short	5.065	0.405	4.446***	2.488***	2.407**

- † Credit Suisse annual reports on global wealth (2000-2017)
 - Market Capt to GDP is also used to form long/short portfolios for robustness

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Systematic Performance

- Use a proxy for the weight of stock returns in the return on aggregate wealth, $w_{s,t}$
- Form long/short portfolios for above and below median countries

Global Long - Short Ratio of Market RET to Wealth \dagger RET $(w_{s,t})$

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Economy-Wide Performance

 Equity SV does not manage Currency investments -Moreira and Muir (2017)

Investment weight =
$$\frac{c_{Asset}}{AV_{MSCI}}$$

	AV		S	SV		ВН	
Index	RET	Sharpe	RET	Sharpe	RET	Sharpe	
Dollar _{BB}	1.324***	0.170	0.606	0.078	-0.296	-0.038	
Curr _{DB}	1.195***	0.272*	-0.668	-0.152	-0.244	-0.056	
Carry _{DB}	1.440***	0.134	-0.361	-0.033	-2.071	-0.192	
Mom _{DB}	1.942***	0.214	0.413	0.045	1.095	0.120	
REIT _{S&P}	26.706***	0.995	14.980	0.558	5.302	0.198	
Comm _{BB}	-5.579***	-0.303	-6.431	-0.349	-5.279	-0.286	
$Bond_{\mathit{Univ}}$	3.951***	1.168***	1.436	0.425	3.276	0.969	

Conclusio

Economy-Wide Performance

- Equity SV does not manage Currency investments -Moreira and Muir (2017)
- Equity AV should still signal unsystematic risk and manage other assets

Investment weight
$$=\frac{c_{Asset}}{AV_{MSCI}}$$

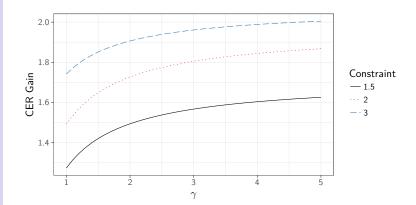
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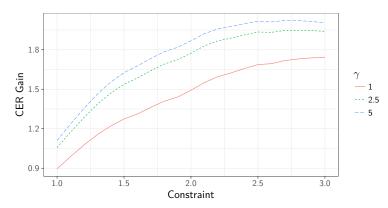
- AV management is better than SV: higher returns, better ratios, lower costs
- AV management is better because it times moving in and out of investments to changes in systematic risk which is compensated and non-systematic risk which is not
- As such, AV management is a useful signal both globally and across assets classes where SV management does not perform
- Thank you

J. Poland

Leverage



Leverage



Risk averse, mean-variance investors see substantial utility gains switching from the SV to AV managed portfolio and these gains increase with leverage usage and risk aversion J. Poland

Return Prediction

1962:07 - 2016:12

			RET_{t+1}		
AV	-0.131 p = 0.166			-0.168** $p = 0.020$	$\begin{array}{c} 0.016 \\ p = 0.739 \end{array}$
AC		0.047^{***} $p = 0.001$		0.106^{***} $p = 0.0001$	
SV			-0.109 p = 0.746		0.254 $p = 0.893$
Constant	-0.000 $p = 1.000$	-0.000 $p = 1.000$	-0.000 p = 1.000	-0.000 p = 1.000	-0.000 $p = 1.000$
<i>N</i> R ²	655 0.017	655 0.002	655 0.012	655 0.027	655 0.017
Adjusted R ²	0.015	0.001	0.010	0.024	0.014

Notes:

^{***}Significant at the 1 percent level.

^{**}Significant at the 5 percent level.

^{*}Significant at the 10 percent level.

J. Poland

Return Prediction

1926:08 - 1962:07

 RET_{t+1} - 1926M7:1962M6

AV	0.061			0.121	0.315
AC		-0.032		-0.099	
SV			-0.028		-0.264
R^2	0.004	0.001	0.001	0.010	0.026
Adjusted R^2	0.002	-0.002	-0.002	0.005	0.021

Global Equity

- If AV management times investment to compensated risk because it changes in response to changes in systematic vs non-systematic risk it should work outside the US
- World AV and SV are market cap weighted averages of country values, US included

	AV		S	V	ВН		
Country	RET	Sharpe	RET	Sharpe	RET	Sharpe	
AUS	12.477	0.981	11.993	0.943	7.805	0.614	
BRA	11.000	0.291	9.037	0.240	6.163	0.164	
CHN	27.381	0.868	24.926	0.790	12.286	0.390	
DEU	11.064	0.537	7.633	0.371	5.399	0.262	
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JPN	1.375	0.068	0.129	0.006	-0.775	-0.038	
UK	6.591	0.485	5.984	0.441	5.111	0.376	
World	8.604	0.551	8.306	0.536	4.484	0.290 5	

Global Equity Again

Drawdown Statistics

		AV			SV		ВН		
Country	Avg DD	Avg Length	Avg Recovery	Avg DD	Avg Length	Avg Recovery	Avg DD	Avg Length	Avg Recovery
AUS	-6.302	7.174	3.348	-5.322	9.263	5.421	-6.318	8.600	4.550
BRA	-8.059	9.560	4.208	-17.469	15.235	5.500	-15.064	17.067	4.286
CHN	-9.511	10.333	5.917	-10.074	10.583	3.727	-19.374	27.400	2.000
DEU	-11.051	10.625	5.783	-12.587	16.812	9.933	-10.706	17.125	12.333
FRA	-10.263	14.111	5.941	-15.260	18.267	10.214	-11.590	19.071	15.077
IND	-8.170	6.500	2.885	-12.545	12.467	5.733	-10.862	8.318	4.500
ITA	-14.625	19.500	2.143	-18.174	22.571	2.333	-8.919	15.400	1.667
JPN	-30.655	72.750	41.750	-78.514	294.000	175.000	-40.792	148.00	2.000
UK	-6.060	11.609	4.652	-7.872	14.158	8.158	-6.018	10.560	7.240
World	-6.982	9.909	7.333	-9.776	12.500	7.059	-8.209	10.091	6.429

Global Equity Again Again

Trading Costs

		AV					
Country	RET	$ \Delta\omega $	Break Even	RET	$ \Delta\omega $	Break Even	RET _{BH}
AUS	12.477	0.486	80.139	11.993	0.466	74.914	7.805
BRA	11.000	0.253	159.118	9.037	0.623	38.462	6.163
CHN	27.381	0.305	412.715	24.926	0.538	195.972	12.286
DEU	11.064	0.499	94.545	7.633	0.581	32.052	5.399
FRA	7.243	0.468	41.656	6.128	0.536	19.041	4.904
IND	14.893	0.710	40.316	12.256	0.507	13.097	11.460
ITA	3.838	0.448	44.366	3.912	0.603	33.991	1.451
JPN	1.375	0.442	40.518	0.129	0.551	13.675	-0.775
UK	6.591	0.473	26.113	5.984	0.509	14.287	5.111
World	8.604	0.439	78.113	8.306	0.642	49.586	4.484

Asset Classes

- If AV management times to changes systematic vs non-systematic risk, equity AV should provide a management signal for more than equities
- Moriera and Muir (2017) show that equity SV does not work as a signal for currency investment
- World AV and SV used with c calculated to match buy and hold for each index

	AV		S	iV	ВН	
Index	RET	Sharpe	RET	Sharpe	RET	Sharpe
Bloomberg Dollar	1.324	0.170	0.606	0.078	-0.296	-0.038
DB Currency	1.195	0.272	-0.668	-0.152	-0.244	-0.056
DB Carry	1.440	0.134	-0.361	-0.033	-2.071	-0.192
DB Mom	1.942	0.214	0.413	0.045	1.095	0.120
S&P REIT	26.706	0.995	14.980	0.558	5.302	0.198
Bloomberg Commodity	-5.579	-0.303	-6.431	-0.349	-5.279	-0.286

Asset Classes Again

Drawdown Statistics

	AV				SV			ВН		
Index	Avg DD	Avg Length	Avg Recovery	Avg DD	Avg Length	Avg Recovery	Avg DD	Avg Length	Avg Recovery	
Bloomberg Dollar	-8.393	29.000	12.750	-10.632	39.333	21.333	-13.565	60.000	27.000	
DB Currency	-2.236	9.750	2.667	-10.471	59.500	20.500	-8.839	59.500	41.500	
DB Carry	-7.336	14.250	7.375	-33.972	121.000	98.000	-30.332	60.000	21.000	
DB Mom	-4.748	11.900	3.300	-14.679	59.000	17.000	-12.278	38.333	18.333	
S&P REIT	-7.692	4.400	1.800	-15.016	9.455	5.000	-17.004	15.143	9.286	
Bloomberg Commodity	-9.784	12.222	2.111	-31.116	39.000	12.333	-26.638	39.333	4.333	

J. Poland

Asset Classes Again Again

Trading Costs

	AV		SV				
Index	RET	$ \Delta\omega $	Break Even	RET	$ \Delta\omega $	Break Even	RET _{BH}
Bloomberg Dollar	1.324	0.411	32.846	0.606	0.620	12.126	-0.296
DB Currency	1.195	0.430	27.851	-0.668	0.482	-7.339	-0.244
DB Carry	1.440	0.427	68.600	-0.361	0.510	27.947	-2.071
DB Mom	1.942	0.441	16.010	0.413	0.599	-9.501	1.095
S&P REIT	26.706	0.592	301.254	14.980	0.807	99.908	5.302
Bloomberg Commodity	-5.579	0.460	-5.430	-6.431	0.555	-17.285	-5.279

Conclusion

- AV management is better than SV: higher returns, better ratios, lower costs
- AV management is better because it times moving in and out of investments to changes in systematic risk which is compensated and non-systematic risk which is not
- As such, AV management is a useful signal both globally and across assets classes where SV management does not perform
- Thank you

More Pollet and Wilson (2010)

PW Details

- Start with Campbell and Viceira (2002) : $r_{i,t+1} \approx \gamma \sigma_{i,m,t} \frac{\sigma_{i,t}^2}{2}$, m is true market
- holds for i = s, stock market portfolio
- $r_{s,t+1} \approx \gamma cov_t(r_{s,t+1},r_{m,t+1}) \frac{\sigma_{s,t}^2}{2}$
- $r_{s,t+1} \approx \gamma cov_t(r_{s,t+1}, w_{s,t}r_{s,t+1} + (1-w_{s,t})r_{u,t+1}) \frac{\sigma_{s,t}^2}{2}$, u is observable component
- $r_{s,t+1} \approx \gamma cov_t(r_{s,t+1}, w_{s,t}r_{s,t+1} + (1 w_{s,t})r_{u,t+1}) \frac{\sigma_{s,t}^2}{2}$
- $r_{s,t+1} \approx \gamma w_{s,t} var_t(r_{s,t+1}) + cov(r_{s,t}, (1-w_{s,t})r_{u,t+1}) \frac{\sigma_{s,t}^2}{2}$

More Pollet and Wilson (2010)

- assume shocks to stock returns : $\bar{\epsilon}_{z,t+1} + \epsilon_{i,t+1}$, z common i idiosyncratic
- $r_{s,t+1} = \beta_t r_{m,t} + \overline{\epsilon}_{z,t+1}$
- $\operatorname{var}(\overline{\epsilon}_{z,t+1} + \epsilon_{i,t+1}) = \sigma_{z,t}^2 = \theta_t \sigma_{z,t}^2 + (1 \theta_t) \sigma_{i,t}^2$, θ common part
- $r_{u,t+1} = \frac{1 w_{s,t}\beta_t}{1 w_{s,t}} r_{m,t} \frac{w_{s,t}\beta_t}{1 w_{s,t}}$
- substitute and simplify (many steps)
- $cov(r_{s,t}, r_{u,t+1}) = \frac{1 w_{s,t}\beta_t}{1 w_{s,t}} \frac{\bar{\sigma}_t^2}{\beta_t} \frac{\bar{\rho}_t \theta_t}{1 \theta_t} \bar{\rho}_t \frac{w_{s,t}\theta_t}{1 w_{s,t}} \frac{\bar{\sigma}_t^2}{\beta_t} \frac{1 \bar{\rho}_t}{1 \theta_t} \bar{\sigma}_t^2$
- more simplification
- $cov(r_{s,t}, r_{u,t+1}) = \pi_0 + \zeta_1 \bar{\rho_t} + \zeta_2 \bar{\sigma}_t^2$
- ζ_1 positive but small for plausible values of $w_{s,t}$ and β_t , ζ_2 negative but small for plausible values
- Return