Don't Throw out the Return with the Risk: Average Variance Portfolio Management

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- Time leverage on a component which predicts higher risk you can decrease exposure ahead of risky times
- Are you giving up potential returns?

Equity Premium

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- Markowitz (1952) - formal portfolio variance, return optimization
- Haugen (1972) - low risk portfolios out perform
- Moreira and Muir (2017) - portfolios scaled by last months realized volatilty outperform the underlying

Volatiltiy Managed Market Investment

- $W_t R_{st}$ where R_{st} is the monthly return to the CRSP market portfolio in month t.
- $\sigma^2(r_{s,t-1})$ is the variance, where $r_{s,t-1}$ is the series of daily returns of the CRSP market portfolio for month t-1
- $W_t = \frac{1}{\sigma^2(r_{s,t-1})}$ is the investment weight on the CRSP market portfolio for month t

Moreira and Muir 2017

1620 *The Journal of Finance®*

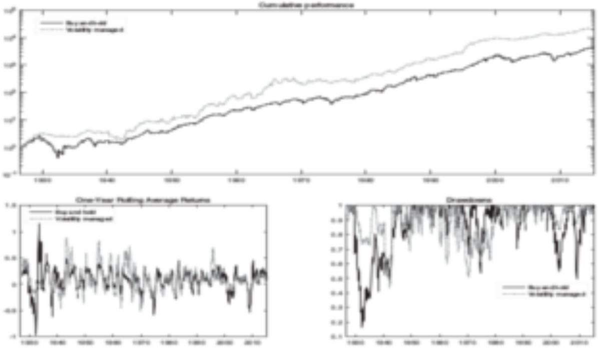


Figure 3. Cumulative returns to the volatility-managed market return. The top panel plots the cumulative returns to a buy-and-hold strategy versus a volatility-managed strategy for the market portfolio from 1926 to 2015. The y-axis is on a log scale and both strategies have the same unconditional monthly standard deviation. The lower left panel plots rolling one-year returns from each strategy and the lower right panel shows the drawdown of each strategy.

Market Variance

- Campbell, Lettau, and Xu (2001) - variance of individual assets vs market variance and CAPM
- Pollet and Wilson (2010) - decompose quarterly variance of market portfolio - Avg cor and Avg var

Avg Var and Avg Cor

$$R_{s,t} = \sum_{1}^{N} w_{n,t} R_{n,t}$$

$$\sigma^2(r_{s,t}) = \sum_{n=1}^{N} \sum_{m=1}^{N} w_{n,t} w_{m,t} \sigma_{n,t}^2 \sigma_{m,t}^2 \rho_{n,m,t}$$

$$\sigma_{s,t}^2 = \sum_{n=1}^{N} w_{n,t} \sigma_{n,t}^2 \times \sum_{n=1}^{N} \sum_{m \neq n}^{N} w_{n,t} w_{m,t} \rho_{n,m,t}$$

$$AV_t = \sum_{n=1}^{N} w_{n,t} \sigma_{n,t}^2 \quad \text{and} \quad AC_t = \sum_{n=1}^{N} \sum_{m \neq n}^{N} w_{n,t} w_{m,t} \rho_{n,m,t}$$

Pollet and Wilson 2010 - Risk

Table: 1963Q2:2007Q1

	SV_{t+1}				
AC_t	0.014***		0.005		
	(0.005)		(0.005)		
AV_t		0.144***	0.136***		0.188***
		(0.023)	(0.024)		(0.042)
SV_t				0.310***	−0.156
				(0.072)	(0.124)
Constant	0.002	0.002**	0.001	0.003***	0.001**
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Observations	176	176	176	176	176
R^2	0.042	0.184	0.096	0.096	0.191
Adjusted R^2	0.037	0.179	0.091	0.091	0.182

Note: $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

Pollet and Wilson 2010 - Returns

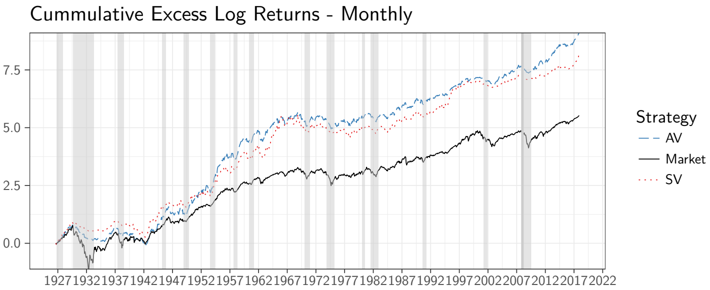
Table: 1963Q2:2007Q1

	RET_{t+1}				
AC_t	0.215***		0.248***		
	(0.068)		(0.072)		
AV_t		−0.116	−0.512		−1.746***
		(0.347)	(0.356)		(0.615)
SV_t				1.466	5.795***
				(1.026)	(1.828)
Constant	−0.038**	0.014	−0.034**	0.005	0.022**
	(0.017)	(0.010)	(0.017)	(0.008)	(0.010)
Observations	176	176	176	176	176
R^2	0.054	0.001	0.065	0.012	0.056
Adjusted R^2	0.049	−0.005	0.054	0.006	0.045

Note: $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

Average Variance

- Timing leverage by variance generates higher returns
- Market variance contains average correlation
- Average variance is at least unrelated to future returns
- $W_t = \frac{1}{AV_{t-1}}$ is the investment weight on the CRSP market portfolio



Data

CRSP daily returns

- NYSE daily return (1926-2017)
- NYSE-AMEX daily returns (1962-2017)
- NASDAQ daily returns (1974-2017)

Summary Stats

Monthly 1962M6:2016M12

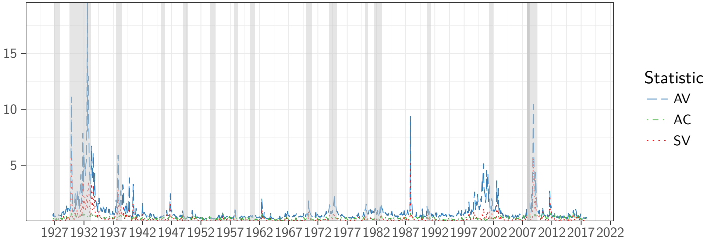
Statistic	N	Mean	St. Dev.	Min	Max	Autocorrelation
RET	655	0.410	4.460	−26.134	14.814	0.081
AC	655	0.261	0.129	0.019	0.762	0.620
AV	655	0.770	0.849	0.198	10.416	0.667
SV	655	0.200	0.406	0.006	5.664	0.551

Monthly 1926M7:2016M12

Statistic	N	Mean	St. Dev.	Min	Max	Autocorrelation
RET	1,085	0.495	5.371	−34.523	33.188	0.106
AC	1,085	0.276	0.134	0.019	0.762	0.610
AV	1,085	0.881	1.281	0.154	19.540	0.718
SV	1,085	0.248	0.502	0.006	5.808	0.612

Time Series

Monthly Measures of Daily Return Statistics



Variance Prediction

Sample 1962M6:2016M12

	SV_{t+1}				
AC_t	0.010***			0.005***	
	(0.001)			(0.001)	
AV_t		0.261***		0.234***	0.123***
		(0.016)		(0.017)	(0.035)
SV_t			0.551***		0.320***
			(0.033)		(0.074)
Constant	−0.001**	−0.00001	0.001***	−0.001***	0.0004**
	(0.0003)	(0.0002)	(0.0001)	(0.0003)	(0.0002)
Observations	654	654	654	654	654
R^2	0.110	0.297	0.304	0.320	0.317
Adjusted R^2	0.109	0.296	0.303	0.318	0.315

Note: $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

AV Prediction

Sample 1962M6:2016M12

	AV_{t+1}				
AC_t	0.014***			−0.001	
	(0.003)			(0.002)	
AV_t		0.667***		0.674***	1.030***
		(0.029)		(0.031)	(0.065)
SV_t			1.092***		−0.844***
			(0.070)		(0.135)
Constant	0.004***	0.003***	0.006***	0.003***	0.001***
	(0.001)	(0.0003)	(0.0003)	(0.001)	(0.0004)
Observations	654	654	654	654	654
R^2	0.048	0.445	0.273	0.446	0.477
Adjusted R^2	0.046	0.445	0.272	0.444	0.475

Note: $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

Return Prediction

Sample 1962M6:2016M12

	RET_{t+1}				
AC_t	0.017			0.037***	
	(0.013)			(0.014)	
AV_t		−0.678***		−0.877***	−0.905*
		(0.203)		(0.216)	(0.463)
SV_t			−1.174***		0.526
			(0.426)		(0.969)
Constant	−0.0001	0.009***	0.007***	0.001	0.010***
	(0.004)	(0.002)	(0.002)	(0.004)	(0.003)
Observations	655	655	655	655	655
R^2	0.002	0.017	0.012	0.027	0.017
Adjusted R^2	0.001	0.015	0.010	0.024	0.014

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Out-of-Sample Tests

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- Generate one period ahead prediction
  - $\hat{r}_{t+1} = \hat{\alpha}_t + \hat{\beta}_t x_t$
- Each following month the "training" window expands by one month

Out of Sample Stats

- y_t - $\hat{y}_{x,t} = e_{x,t}$: forecast error of preditor x
- $\frac{1}{T}\sum_1^T (e_{x,t})^2 = \text{MSFE}_x$: mean squared forecast error based on predictor x

R^2_{oos} Campbell and Thompson 2007

- $R^2_{os} = 1 - \frac{MSFE_x}{MSFE_b}$
- R^2_{os} = proportional reduction in MSFE

MSE-F Mcracken 2004

- $\text{MSE-F} = T \times \frac{\frac{1}{T}\sum_1^T (e^2_{b,t} - e^2_{x,t})}{MSFE_x}$
- MSE-F = F-type test for significance in squared residual (like in sample regression)

Out of Sample Stats

- R^2_{oos} and MSE-F test improvement in forecast accuracy relative to a benchmark
- Encompassing tests impose the greater requirement that the benchmark have no valuable forecasting information

ENC-NEW Mcracken and Clark 2009

- ENC-NEW $= T \times \frac{\frac{1}{T}\sum_1^T (e_{b,t}^2 - e_{b,t}e_{x,t})}{MSFE_x}$
- ENC-NEW = F-type statistic on the imporvement of including the benchmark

ENC-HLN Harvey, Lebourne and Newbold 1998

- Optimal forecast $= \hat{y}_t^* = (1-\lambda)\hat{y}_{b,t} + \lambda \hat{y}_{x,t}$
- $\lambda =$ measure of the optimal combination of forecasts from x and the benchmark

Out of Sample Results

Table: 1970M7:2017M12

Benchmark: Historical Average

	Sample	R^2_{oos}	MSE-F	ENC-NEW	ENC-HLN
SV_{t+1}	Monthly	25.414*	189.790***	160.994**	1***
AV_{t+1}	Monthly	38.11**	342.979***	355.228**	0.967***
RET_{t+1}	Monthly	-0.059	-0.328	3.493**	0.478

Benchmark: SV_t

	Sample	R^2_{oos}	MSE-F	ENC-NEW	ENC-HLN
SV_{t+1}	Monthly	4.041	23.454***	25.409**	0.929*
AV_{t+1}	Monthly	26.853	204.485***	135.494**	1***
RET_{t+1}	Monthly	2.116	12.043***	8.2**	1

Out of Sample Results

Table: 1926M7:1962M6

Benchmark: Historical Average

	Sample	R^2_{oos}	MSE-F	ENC-NEW	ENC-HLN
SV_{t+1}	Monthly	49.972***	367.592***	397.183**	0.931***
AV_{t+1}	Monthly	50.747**	379.160***	409.061**	0.932***
RET_{t+1}	Monthly	-8.708	-29.479	-9.96	0

Benchmark: SV_t

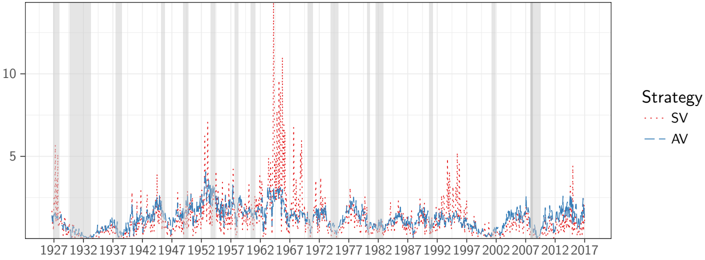
	Sample	R^2_{oos}	MSE-F	ENC-NEW	ENC-HLN
SV_{t+1}	Monthly	-1.289	-4.682	76.562**	0.485*
AV_{t+1}	Monthly	11.328	47.013***	121.513**	0.62**
RET_{t+1}	Monthly	-6.098	-21.152	-6.192	0

Investment Weight

$w_{AV,t} = \frac{c_{AV}}{AV_{t-1}}$ and $w_{SV,t} = \frac{c_{SV}}{SV_{t-1}}$

c is a constant used to equalize the standard deviation of strategies to the buy and hold

Strategy Investment Weight



Statistic	N	Mean	St. Dev.	Min	Max
$w_{SV,t}$	1,085	1.290	1.412	0.017	16.193
$w_{AV,t}$	1,085	1.301	0.710	0.033	4.253

Performance Measures

- RET = annualized average log excess return
- Sharpe $= \frac{\mathbb{E}[R_x]}{\sigma(R_x)}$, dollar of returns for dollar of variance
- Sortino $= \frac{\mathbb{E}[R_x - 0]}{\sqrt{\int_{-\infty}^{0} (0 - R_x)^2 f(R_x)\, dR}}$, return for downside
- Kappa(n) $= \frac{\mathbb{E}[R_x - 0]}{\sqrt[n]{LPM_n}}$, where LPM is lower partial moment
  Kappa[2] = Sortino
- UpsidePotential $= \frac{\mathbb{E}[(R_x - 0)_+]}{\sqrt{\mathbb{E}[(R_x - 0)_-^2]}}$, dollar of average gain for downside risk
- Rachev $= \frac{ETL_\alpha(r_f - x'r)}{ETL_\beta(x'r - r_f)}$ where $ETL_\alpha = \frac{1}{\alpha} \int_0^\alpha VaR_q(X)\, dq$, dollar of possible extreme gain for dollar of possible extreme loss

Performance

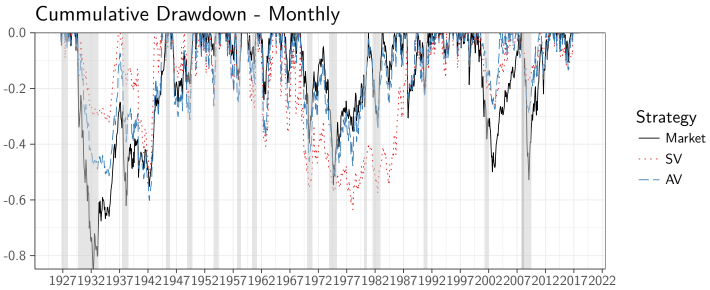
1926M7:2016M12

Strategy	RET	Sharpe	Sortino	Kappa	UpsidePotential	Rachev
BH	5.932	0.319	0.447	0.082	0.584	0.841
SV	8.598	0.462	0.722	0.132	0.650	1.151
AV	9.677	0.520	0.778	0.150	0.706	0.972

1962M6:2016M12

Strategy	RET	Sharpe	Sortino	Kappa	UpsidePotential	Rachev
BH	5.112	0.332	0.463	0.089	0.635	0.826
SV	7.311	0.406	0.647	0.122	0.663	1.212
AV	7.857	0.470	0.702	0.139	0.719	0.987

Drawdowns



Strategy	N	Max DD	Avg DD	Max Length	Avg Length	Max Recovery	Avg Recovery
BH	82	-84.803	-8.069	188	11.549	154	7.207
SV	65	-63.508	-11.162	246	14.954	135	7.446
AV	87	-60.208	-9.014	205	10.851	135	5.034

Risk over Reward

- The higher excess returns of low-risk strategies (assets) comes from a preference for the lottery like extreme returns possible from higher risk investments - Barberis and Huang (2008); Brunnermeier, Gollier, and Parker (2007)
- Leverage constraints prevent investors from taking the low-risk position - Black (1972)

Lottery

- For lotter preferences to explain the higher returns of either SV or AV, the Buy and Hold strategy must be more lottery-like than either
- It is not

	MAX1				SMAX1			
Strategy	Mean	Median	Sd	KS	Mean	Median	Sd	KS
BH	1.776	1.422	1.398		2.186	1.971	1.046	
SV	1.569	1.258	1.243	0.539	3.229	2.167	4.661	0
AV	1.796	1.650	0.960	0	2.884	1.691	4.992	0
	MAX5				**SMAX5**			
Strategy	Mean	Median	Sd	KS	Mean	Median	Sd	KS
BH	1.134	0.922	0.774		1.410	1.341	0.540	
SV	1.023	0.842	0.787	0.393	2.084	1.377	2.765	0
AV	1.164	1.088	0.534	0	1.827	1.121	2.833	0

Leverage

$$R_t^{AV} = \alpha_t + \beta_t^1 R_t^M + \boldsymbol{\beta}_t \boldsymbol{\chi}_t \tag{1}$$

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- Testing market proxies gives no insight into the falsity of the theory.
- The results of BJS, FM, BF and others are consistent with the S-L theory.