Average Variance

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How to Look Clever and Have Envious Neighbors: Average Variance Managed Investment

Jeramia Poland



Indian School of Business

December 17, 2018

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Risk and Reward

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Performance

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Background

 Hocquard et al (2013) - scaling investment to manage tail risk

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Systematic Risk

- Hocquard et al (2013) scaling investment to manage tail risk
- Moreira and Muir (2017) scaling month t+1 investment by month t daily return variance for alpha (Sharpe)

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 - $r_{t+1} = \alpha_1 + \beta_1 AC_t + \epsilon_{1,t}$, β_1 positive and significant
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Central Idea I

 \bullet Use AV to avoid divesting high AC or investing high AV

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- Should find AV management outperforms SV management

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Central Idea II

 Pollet and Wilson make a theoretical argument that AC is a signal of systematic risk across the economy - index returns and unobservable returns

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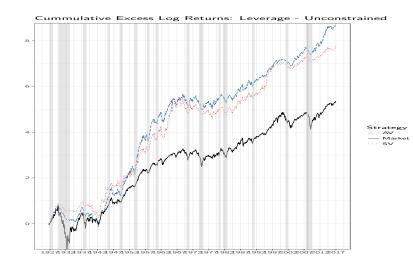
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- Equity AV management has higher investment in times of higher $cov(r_{s,t+1}, r_{u,t+1})$, so it should work across asset classes

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US Equity Performance



AV: 9.68% SV: 8.60% BH: 5.93%



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Performance Measures

ullet RET = annualized average log excess return

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- Certainty Equivalent Return gain (CER) = Average utility from AV Average utility from SV for mean-variance investor with risk aversion γ

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c_{BH}: Unconstrained

	Return	Sharpe	Sortino	Kappa ₃	Kappa ₄	α_{FF3}	$\alpha_{FF3+Mom}$
ВН	5.934	0.319	0.129	0.082	0.061		
SV	8.589	0.462	0.208	0.132	0.097	5.477	3.201
AV	9.676***	0.520*	0.225	0.150*	0.112*	5.594***	3.164

Constraint - 3

	Portfolio	Return	Sharpe	Sortino	$Kappa_3$	$Kappa_4$	
c ₁₀	SV	4.396	0.454	0.200	0.127	0.094	
c ₁₀	AV	5.225***	0.520*	0.225*	0.150**	0.112**	
c ₁₂	SV	5.219	0.452	0.198	0.127	0.094	
c ₁₂	AV	6.306***	0.520**	0.225**	0.150**	0.112**	
СВН	SV	7.606	0.456	0.199	0.129	0.096	
c _{BH}	AV	9.677***	0.522**	0.226**	0.150**	0.112**	

Notes:

***, **, and * Significant at the 1, 5, and 10 percent levels.

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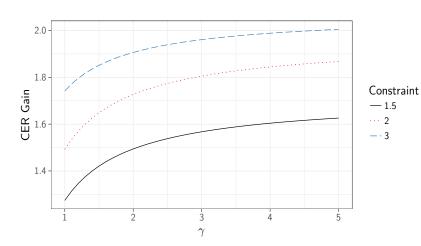
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Investor Utility



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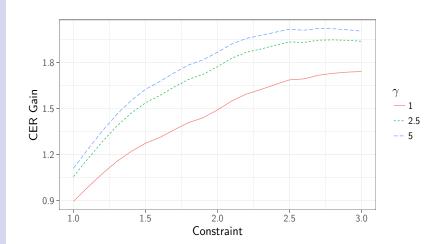
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Average Variance

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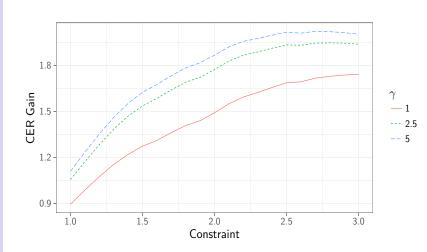
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Investor Utility



• Stochastic dominance tests show that non-EUT investors prefer AV management.

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Suggestively Systematic

• 1926-1962 CRSP dialy returns - low $\textit{w}_{\textit{s},\textit{t}}$ and $\beta_{\textit{t}}$

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- 1926-1962 CRSP dialy returns low $\textit{w}_{\textit{s},\textit{t}}$ and $\beta_{\textit{t}}$
 - \bullet shallower with only NYSE firms, est below 87% market cap

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- AV management using the AV of equity returns should generate better performance across asset classes

Results

Performance Systematic Risk

Regression Sub-samples

AV	0.061			0.121	0.315
AC		-0.032		-0.099	
SV			-0.028		-0.264
R^2	0.004	0.001	0.001	0.010	0.026
Adjusted R ²	0.002	-0.002	-0.002	0.005	0.021

RET_{t+1} - 1962M6:2016M12

AV	-0.131			-0.168**	0.016
AC		0.047***		0.106***	
SV			-0.109		0.254
R^2	0.017	0.002	0.012	0.027	0.017
Adjusted R ²	0.015	0.001	0.010	0.024	0.014

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Global Performance

	AV		S	V	ВН	
Country	RET	Sharpe	RET	Sharpe	RET	Sharpe
AUS	12.477***	0.981	11.993	0.943	7.805	0.614
BRA	11.000***	0.291	9.037	0.240	6.163	0.164
CHN	27.381	0.868	24.926	0.790	12.286	0.390
DEU	11.064***	0.537*	7.633	0.371	5.399	0.262
FRA	7.243***	0.404	6.128	0.341	4.904	0.273
IND	14.893***	0.633	12.256	0.521	11.460	0.487
ITA	3.838	0.194	3.912	0.198	1.451	0.073
JPN	1.375***	0.068	0.129	0.006	-0.775	-0.038
UK	6.591***	0.485	5.984	0.441	5.111	0.376
World	8.603***	0.551	8.306	0.536	4.484	0.290

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Systematic Performance

Global Long - Short Ratio of Market Index RET to Wealth† RET $(w_{s,t})$

AV Managed Returns								
	RET	Sharpe	$lpha_{\it FF3}$	$lpha_{\it FF5}$	$lpha_{\it FF5+Mom}$			
Long	12.601	0.747	9.484**	7.909*	7.725*			
Short	7.537	0.562	5.038*	5.422*	5.318*			
Long - Short	5.065	0.405	4.446***	2.488***	2.407**			

[†] Credit Suisse annual reports on global wealth (2000-2017)

Economy-Wide Performance

Investment weight in other asset classes managed by equity AV

	2005M7:2015M12								
	AV		S	V	ВН				
Index	RET	Sharpe	RET	Sharpe	RET	Sharpe			
Dollar _{BB}	1.324***	0.170	0.606	0.078	-0.296	-0.038			
Curr _{DB}	1.195***	0.272*	-0.668	-0.152	-0.244	-0.056			
Carry _{DB}	1.440***	0.134	-0.361	-0.033	-2.071	-0.192			
Mom_{DB}	1.942***	0.214	0.413	0.045	1.095	0.120			
REIT _{S&P}	26.706***	0.995	14.980	0.558	5.302	0.198			
Comm _{BB}	-5.579***	-0.303	-6.431	-0.349	-5.279	-0.286			
$Bond_{\mathit{Univ}}$	3.951***	1.168***	1.436	0.425	3.276	0.969			

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Variance Management

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- AV comes from the foundations of investment risk
- AV management informs about the risk mix across the economy

Equity Data

Equity Data

Country	Start	Obs	Index	Assets
USA	1926 - 8	1085	CRSP	500
AUS	2000 - 5	212	ASX	200
BRA	1995 - 2	275	iShares MSCI Brazil ETF	60
CHN	2005 - 5	152	CSI 300	300
DEU	1993 - 11	290	HDAX	110
FRA	1993 - 9	292	SBF 120	120
IND	2000 - 5	212	Nifty 50	50
ITA	2003 - 8	173	FTSE MIB	40
JPN	1993 - 6	295	Nikkei	255
UK	1993 - 6	295	FTSE	100
World	1995 - 3	274	MSCI ACWI	1735

Investment

Non-Equity Data

Other Asset Data

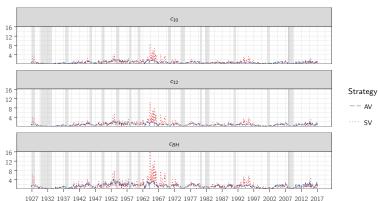
Index	Start	Obs	Asset Class
Dia soule sour LIC Coort	2005 6	150	C
Bloomberg US Spot	2005 - 6	158	Currency
Deutsche Bank Currency	2005 - 6	158	Currency
Deutsche Bank Carry	2005 - 6	158	Currency
Deutsche Bank Momentum	2005 - 6	158	Currency
S&P REIT Index	2005 - 6	158	Real Estate
Bloomberg Commodity	2005 - 6	158	Commodities

AV Construction

- $\mathsf{SV}_t = \sigma_{\mathsf{S},t}^2$
- With m assets in the market, AV $_t = \sum_{m=1}^{M} w_{m,t} \sigma_{m,t}^2$
- $W_t = \frac{c}{X}$ is the investment weight in the portfolio, where X $\in \{AV_{t-1}, SV_{t-1}\}$
- The constant c_{target} is used to control the volatility of the strategy
- c_{BH} matching the buy and hold
- For robustness, c_{10} and c_{12} targeting 10% or 12% annual return volatility

Investment Weights

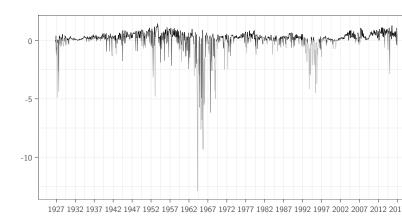
Strategy Investment Weight



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Investment

Investment Weights Again



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Investment Weight Again Again

Portfolio	Target	Mean	St. Dev.	Min	Pctl(25)	Median	Pctl(75)	Max
SV	C ₁₀	0.697	0.762	0.009	0.246	0.512	0.874	8.743
AV	c ₁₀	0.702	0.383	0.018	0.425	0.667	0.915	2.296
SV	c ₁₂	0.841	0.920	0.011	0.297	0.618	1.055	10.552
AV	C ₁₂	0.848	0.463	0.022	0.513	0.805	1.104	2.772
SV	c_{BH}	1.290	1.412	0.017	0.455	0.948	1.619	16.193
AV	c_{BH}	1.301	0.710	0.033	0.787	1.235	1.694	4.253

Investment

Performance Measures

• RET = annualized average log excess return

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- Kappa $_n = \frac{\mathbb{E}[R_{\times} 0]}{\sqrt[n]{LPM_n}}$, where LPM is lower partial moment Kappa $_2 = \mathsf{Sortino}$

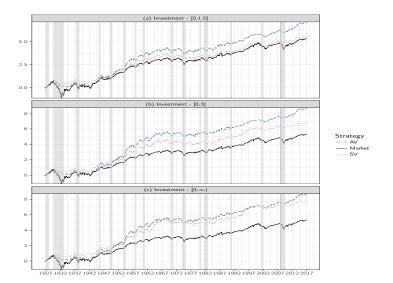
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- Certainty Equivalent Return gain (CER) = Average utility from AV Average utility from SV for mean-variance investor with risk aversion γ

Returns

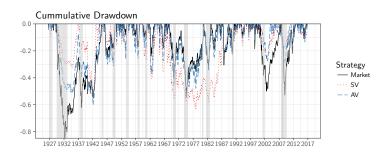


Performance

 c_{BH} : 1926:07-2016:12

	Return	Sharpe	Sortino	Kappa ₃	Kappa ₄	α_{FF5}	$lpha_{\it FF5+Mom}$
ВН	5.932	0.319	0.129	0.082	0.061		
SV	8.598	0.462	0.208	0.132	0.097	5.477	3.201
AV	9.677***	0.520*	0.225	0.150*	0.112*	5.594***	3.164***

Drawdowns: c_{BH}



Strategy	N	Max DD	Avg DD	Max Length	Avg Length	Max Recovery	Avg Recovery
ВН	82	-84.803	-8.069	188	11.549	154	7.207
SV	65	-63.637	-11.196	246	14.954	135	7.446
AV	87	-60.264	-9.026	205	10.851	135	5.034

Drawdown Insurance: c_{BH}

Knockout

- Drawndown large enough to shutter fund (investor pull-out), cost manager job
- Assuming 45% loss in a 12-month period as knockout
- SV 1.06% and AV .55% using Pav (2016)
- AV \approx half the cost to insure, Carr, Zhang, and Hadjiliadis (2011)

Leverage

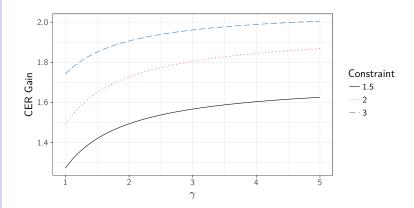
	c _{BH} : Constraint - 1.5								
Portfolio	Return	Sharpe	Sortino	$Kappa_3$	Kappa ₄				
ВН	5.932	0.319	0.129	0.082	0.061				
SV	6.171	0.467	0.200	0.128	0.091				
AV	7.885***	0.486	0.204	0.133	0.097				

c_{BH}: Constraint - 3

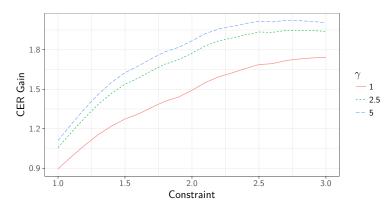
		Di	,		
Portfolio	Return	Sharpe	Sortino	$Kappa_3$	$Kappa_4$
BH	5.932	0.319	0.129	0.082	0.061
SV	7.606	0.456	0.199	0.129	0.096
AV	9.677***	0.522**	0.226**	0.150**	0.112**

Notes: ***,**, and * Significant at the 1, 5, and 10 percent levels.

Leverage



Leverage



Risk averse, mean-variance investors see substantial utility gains switching from the SV to AV managed portfolio and these gains increase with leverage usage and risk aversion

AC/AV and Systematic Risk

 Pollet and Wilson (2010) - AC is positively related to the correlation of market returns and aggregate wealth, including the unobserved component of the "true market"

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- This is similar to the difference in results between Goyal and Santa Clara (2003) and Bali et all (2005) when the latter removes a significant number of daily returns and the forecasting ability of idiosyncratic volatility disappears
- Thus we can run a placebo-like test on a sub-sample where the daily returns are not representative

Subsample Tests

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- As much as 13% of market capitalization is not captured by CRSP data as of the 1950s.
- Twice as many firms covering twice as many industries are available at the end of 1962 as compared to the end of 1961.
- As shown in Taylor (2014) the NYSE market was not a significant part of marginal wealth in the US following the Great Depression before the late 1950s.

Regressions

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- Amihud and Hurvich (2004) bias correction
- Omit variance (SV_{t+1}) prediction by AV as it works in both sub-samples
- Goyal and Welch (2008) forecasting relationships maybe unstable and quite sensitive to sample period choice; they may not respond dynamically with the limited information available to investors in real-time and may not explain or support a trading strategy

Return Prediction

1962:07 - 2016:12

			RET_{t+1}		
AV	-0.131 p = 0.166			-0.168** $p = 0.020$	$\begin{array}{c} 0.016 \\ p = 0.739 \end{array}$
AC		0.047^{***} $p = 0.001$		0.106^{***} $p = 0.0001$	
SV			-0.109 p = 0.746		0.254 $p = 0.893$
Constant	-0.000 $p = 1.000$	-0.000 $p = 1.000$	-0.000 $p = 1.000$	-0.000 p = 1.000	-0.000 $p = 1.000$
NR ²	655 0.017	655 0.002	655 0.012	655 0.027	655 0.017
Adjusted R ²	0.015	0.001	0.010	0.024	0.014

Notes:

^{***}Significant at the 1 percent level.

^{**}Significant at the 5 percent level.

^{*}Significant at the 10 percent level.

Return Prediction

1926:08 - 1962:07

 RET_{t+1} - 1926M7:1962M6

AV	0.061			0.121	0.315
AC		-0.032		-0.099	
SV			-0.028		-0.264
R^2	0.004	0.001	0.001	0.010	0.026
Adjusted R^2	0.002	-0.002	-0.002	0.005	0.021

Diebold-Marino Statistic (1995)

• DM =
$$\frac{\bar{d}}{\sqrt{\frac{2\pi f_d(0)}{T}}}$$

 Asymptotically normally statistic comparing significance of accuracy ratio

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ENC-HLN Harvey, Lebourne and Newbold (1998)

- Optimal forecast $= \hat{y}_t^* = (1 \lambda)\hat{y}_{b,t} + \lambda\hat{y}_{x,t}$
- ullet $\lambda=$ measure of the optimal combination of forecasts

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Rossi and Inoue (2012)

- Calculate OOS stats on all feasible window specifications
- Use asymptotic distribution \rightarrow stat critical values
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Out of Sample Results

Table: Sample 1939:12 to 2016:12

	DM	MSE-F	ENC-HLN
AC_{t+1}	1.604*	46.251***	1**
SV_{t+1}	1.041	21.57***	0.956**
AV_{t+1}	3.104***	198.267***	1***
RET_{t+1}	-2.027	-8.702	0

Robust Out of Sample Results

Table: Sample 1939:12 to 2016:12

Stat	Variable	DM	ENC-HLN
R_T	SV_{t+1}	8.874***	1.838***
R_T	RET_{t+1}	29.124***	4.871***
A_T	SV_{t+1}	2.647***	0.949***
A_T	RET_{t+1}	13.347***	1.68***

Notes: ***,**, and * Significant at the 1, 5, and 10 percent levels.

 These results compare the use AV to SV in forecasting not case either is good (RET) but AV is better

Global Equity

- If AV management times investment to compensated risk because it changes in response to changes in systematic vs non-systematic risk it should work outside the US
- World AV and SV are market cap weighted averages of country values, US included

	Д	V	S	V	В	Н
Country	RET	Sharpe	RET	Sharpe	RET	Sharpe
AUS	12.477	0.981	11.993	0.943	7.805	0.614
BRA	11.000	0.291	9.037	0.240	6.163	0.164
CHN	27.381	0.868	24.926	0.790	12.286	0.390
DEU	11.064	0.537	7.633	0.371	5.399	0.262
FRA	7.243	0.404	6.128	0.341	4.904	0.273
IND	14.893	0.633	12.256	0.521	11.460	0.487
ITA	3.838	0.194	3.912	0.198	1.451	0.073
JPN	1.375	0.068	0.129	0.006	-0.775	-0.038
UK	6.591	0.485	5.984	0.441	5.111	0.376
World	8.604	0.551	8.306	0.536	4.484	0.2902

Global Equity Again

Drawdown Statistics

		AV		SV			ВН		
Country	Avg DD	Avg Length	Avg Recovery	Avg DD	Avg Length	Avg Recovery	Avg DD	Avg Length	Avg Recovery
AUS	-6.302	7.174	3.348	-5.322	9.263	5.421	-6.318	8.600	4.550
BRA	-8.059	9.560	4.208	-17.469	15.235	5.500	-15.064	17.067	4.286
CHN	-9.511	10.333	5.917	-10.074	10.583	3.727	-19.374	27.400	2.000
DEU	-11.051	10.625	5.783	-12.587	16.812	9.933	-10.706	17.125	12.333
FRA	-10.263	14.111	5.941	-15.260	18.267	10.214	-11.590	19.071	15.077
IND	-8.170	6.500	2.885	-12.545	12.467	5.733	-10.862	8.318	4.500
ITA	-14.625	19.500	2.143	-18.174	22.571	2.333	-8.919	15.400	1.667
JPN	-30.655	72.750	41.750	-78.514	294.000	175.000	-40.792	148.00	2.000
UK	-6.060	11.609	4.652	-7.872	14.158	8.158	-6.018	10.560	7.240
World	-6.982	9.909	7.333	-9.776	12.500	7.059	-8.209	10.091	6.429

Global Equity Again Again

Trading Costs

	AV				SV			
Country	RET	$ \Delta\omega $	Break Even	RET	$ \Delta\omega $	Break Even	RET _{BH}	
AUS	12.477	0.486	80.139	11.993	0.466	74.914	7.805	
BRA	11.000	0.253	159.118	9.037	0.623	38.462	6.163	
CHN	27.381	0.305	412.715	24.926	0.538	195.972	12.286	
DEU	11.064	0.499	94.545	7.633	0.581	32.052	5.399	
FRA	7.243	0.468	41.656	6.128	0.536	19.041	4.904	
IND	14.893	0.710	40.316	12.256	0.507	13.097	11.460	
ITA	3.838	0.448	44.366	3.912	0.603	33.991	1.451	
JPN	1.375	0.442	40.518	0.129	0.551	13.675	-0.775	
UK	6.591	0.473	26.113	5.984	0.509	14.287	5.111	
World	8.604	0.439	78.113	8.306	0.642	49.586	4.484	

Asset Classes

- If AV management times to changes systematic vs non-systematic risk, equity AV should provide a management signal for more than equities
- Moriera and Muir (2017) show that equity SV does not work as a signal for currency investment
- World AV and SV used with c calculated to match buy and hold for each index

	AV		SV		ВН	
Index	RET	Sharpe	RET	Sharpe	RET	Sharpe
Bloomberg Dollar	1.324	0.170	0.606	0.078	-0.296	-0.038
DB Currency	1.195	0.272	-0.668	-0.152	-0.244	-0.056
DB Carry	1.440	0.134	-0.361	-0.033	-2.071	-0.192
DB Mom	1.942	0.214	0.413	0.045	1.095	0.120
S&P REIT	26.706	0.995	14.980	0.558	5.302	0.198
Bloomberg Commodity	-5.579	-0.303	-6.431	-0.349	-5.279	-0.286

J. Poland

Investment

Asset Classes Again

Drawdown Statistics

		AV			SV			ВН		
Index	Avg DD	Avg Length	Avg Recovery	Avg DD	Avg Length	Avg Recovery	Avg DD	Avg Length	Avg Recovery	
Bloomberg Dollar	-8.393	29.000	12.750	-10.632	39.333	21.333	-13.565	60.000	27.000	
DB Currency	-2.236	9.750	2.667	-10.471	59.500	20.500	-8.839	59.500	41.500	
DB Carry	-7.336	14.250	7.375	-33.972	121.000	98.000	-30.332	60.000	21.000	
DB Mom	-4.748	11.900	3.300	-14.679	59.000	17.000	-12.278	38.333	18.333	
S&P REIT	-7.692	4.400	1.800	-15.016	9.455	5.000	-17.004	15.143	9.286	
Bloomberg Commodity	-9.784	12.222	2.111	-31.116	39.000	12.333	-26.638	39.333	4.333	

Asset Classes Again Again

Trading Costs

	AV			SV			
Index	RET	$ \Delta\omega $	Break Even	RET	$ \Delta\omega $	Break Even	RET _{BH}
Bloomberg Dollar	1.324	0.411	32.846	0.606	0.620	12.126	-0.296
DB Currency	1.195	0.430	27.851	-0.668	0.482	-7.339	-0.244
DB Carry	1.440	0.427	68.600	-0.361	0.510	27.947	-2.071
DB Mom	1.942	0.441	16.010	0.413	0.599	-9.501	1.095
S&P REIT	26.706	0.592	301.254	14.980	0.807	99.908	5.302
Bloomberg Commodity	-5.579	0.460	-5.430	-6.431	0.555	-17.285	-5.279

Conclusion

- AV management is better than SV: higher returns, better ratios, lower costs
- AV management is better because it times moving in and out of investments to changes in systematic risk which is compensated and non-systematic risk which is not
- As such, AV management is a useful signal both globally and across assets classes where SV management does not perform
- Thank you

More Pollet and Wilson (2010)

PW Details

- Start with Campbell and Viceira (2002) : $r_{i,t+1} \approx \gamma \sigma_{i,m,t} \frac{\sigma_{i,t}^2}{2}$, m is true market
- holds for i = s, stock market portfolio
- $r_{s,t+1} \approx \gamma cov_t(r_{s,t+1},r_{m,t+1}) \frac{\sigma_{s,t}^2}{2}$
- $r_{s,t+1} \approx \gamma cov_t(r_{s,t+1}, w_{s,t}r_{s,t+1} + (1-w_{s,t})r_{u,t+1}) \frac{\sigma_{s,t}^2}{2}$, u is observable component
- $r_{s,t+1} \approx \gamma cov_t(r_{s,t+1}, w_{s,t}r_{s,t+1} + (1 w_{s,t})r_{u,t+1}) \frac{\sigma_{s,t}^2}{2}$
- $r_{s,t+1} \approx \gamma w_{s,t} var_t(r_{s,t+1}) + cov(r_{s,t}, (1-w_{s,t})r_{u,t+1}) \frac{\sigma_{s,t}^2}{2}$

More Pollet and Wilson (2010)

- assume shocks to stock returns : $\bar{\epsilon}_{z,t+1} + \epsilon_{i,t+1}$, z common i idiosyncratic
- $r_{s,t+1} = \beta_t r_{m,t} + \overline{\epsilon}_{z,t+1}$
- $\operatorname{var}(\overline{\epsilon}_{z,t+1} + \epsilon_{i,t+1}) = \sigma_{z,t}^2 = \theta_t \sigma_{z,t}^2 + (1 \theta_t) \sigma_{i,t}^2$, θ common part
- $r_{u,t+1} = \frac{1 w_{s,t}\beta_t}{1 w_{s,t}} r_{m,t} \frac{w_{s,t}\beta_t}{1 w_{s,t}}$
- substitute and simplify (many steps)
- $cov(r_{s,t}, r_{u,t+1}) = \frac{1 w_{s,t}\beta_t}{1 w_{s,t}} \frac{\bar{\sigma}_t^2}{\beta_t} \frac{\bar{\rho}_t \theta_t}{1 \theta_t} \bar{\rho}_t \frac{w_{s,t}\theta_t}{1 w_{s,t}} \frac{\bar{\sigma}_t^2}{\beta_t} \frac{1 \bar{\rho}_t}{1 \theta_t} \bar{\sigma}_t^2$
- more simplification
- $cov(r_{s,t}, r_{u,t+1}) = \pi_0 + \zeta_1 \bar{\rho_t} + \zeta_2 \bar{\sigma}_t^2$
- ζ_1 positive but small for plausible values of $w_{s,t}$ and β_t , ζ_2 negative but small for plausible values
- Return