How to Look Clever and Have Envious Neighbors: Average Variance Managed Investment

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  - $r_{t+1} = \alpha_2 + \beta_2 AV_t + \epsilon_{2,t}$, β_2 insignificant

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- **AV management should work globally**

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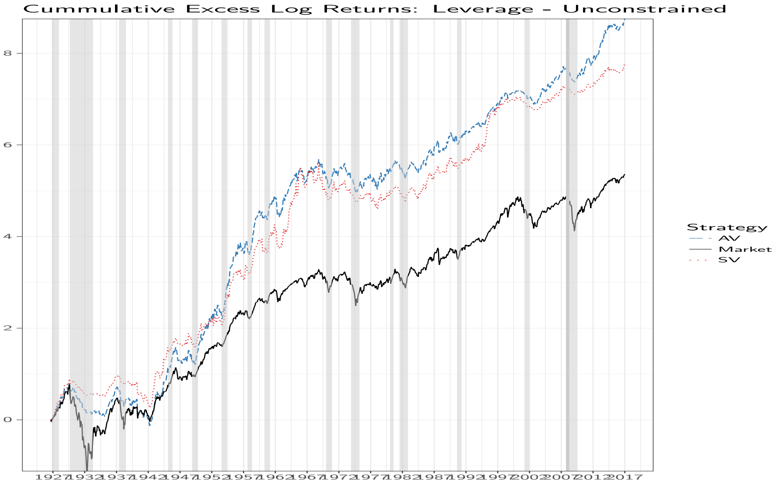
- **Equity AV management has higher investment in times of higher $cov(r_{s,t+1}, r_{u,t+1})$, so it should work across asset classes**

US Equity Performance



AV : 9.68% SV : 8.60% BH : 5.93%

Performance Measures

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- Certainty Equivalent Return gain (CER) $=$ Utility from AV - Utility from SV for mean-variance investor with risk aversion γ

Performance

1926:07-2016:12

c_{BH}: Unconstrained

	Return	Sharpe	Sortino	Kappa_3	Kappa_4	α_{FF3}	$\alpha_{FF3+Mom}$
BH	5.934	0.319	0.129	0.082	0.061		
SV	8.589	0.462	0.208	0.132	0.097	5.477	3.201
AV	9.676***	0.520*	0.225	0.150*	0.112*	5.594***	3.164

Constraint - 3

	Portfolio	Return	Sharpe	Sortino	Kappa_3	Kappa_4
c_{10}	SV	4.396	0.454	0.200	0.127	0.094
c_{10}	AV	5.225***	0.520*	0.225*	0.150**	0.112**
c_{12}	SV	5.219	0.452	0.198	0.127	0.094
c_{12}	AV	6.306***	0.520**	0.225**	0.150**	0.112**
c_{BH}	SV	7.606	0.456	0.199	0.129	0.096
c_{BH}	AV	9.677***	0.522**	0.226**	0.150**	0.112**

Notes: ***,**, and * Significant at the 1, 5, and 10 percent levels.

Investor Utility

- Δ CER $= \left(\hat{\mu}_{r_{AV}} - \frac{1}{2}\gamma\hat{\sigma}^2_{r_{AV}}\right) - \left(\hat{\mu}_{r_{SV}} - \frac{1}{2}\gamma\hat{\sigma}^2_{r_{SV}}\right)$

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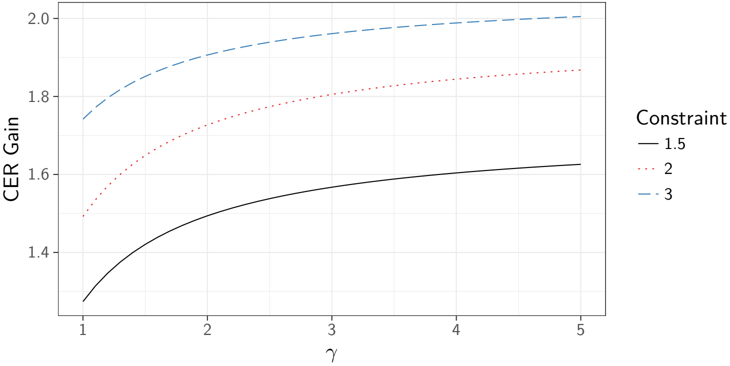

Global Performance

	AV		SV		BH	
Country	RET	Sharpe	RET	Sharpe	RET	Sharpe
AUS	12.477***	0.981	11.993	0.943	7.805	0.614
BRA	11.000***	0.291	9.037	0.240	6.163	0.164
CHN	27.381	0.868	24.926	0.790	12.286	0.390
DEU	11.064***	0.537*	7.633	0.371	5.399	0.262
FRA	7.243***	0.404	6.128	0.341	4.904	0.273
IND	14.893***	0.633	12.256	0.521	11.460	0.487
ITA	3.838	0.194	3.912	0.198	1.451	0.073
JPN	1.375***	0.068	0.129	0.006	-0.775	-0.038
UK	6.591***	0.485	5.984	0.441	5.111	0.376
World	8.603***	0.551	8.306	0.536	4.484	0.290

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AV Managed Returns

	RET	Sharpe	α_{FF3}	α_{FF5}	$\alpha_{FF5+Mom}$
Long	12.601	0.747	9.484**	7.909*	7.725*
Short	7.537	0.562	5.038*	5.422*	5.318*
Long - Short	5.065	0.405	4.446***	2.488***	2.407**

† Credit Suisse annual reports on global wealth (2000-2017)

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- Market Cap to GDP also used for robustness

Economy-Wide Performance

- AV management times return coordination across the economy
- AV calculated from equities, weights are then used on other indices

2005M7:2015M12

	AV		SV		BH	
Index	RET	Sharpe	RET	Sharpe	RET	Sharpe
Dollar_{BB}	1.324***	0.170	0.606	0.078	−0.296	−0.038
Curr_{DB}	1.195***	0.272*	−0.668	−0.152	−0.244	−0.056
Carry_{DB}	1.440***	0.134	−0.361	−0.033	−2.071	−0.192
Mom_{DB}	1.942***	0.214	0.413	0.045	1.095	0.120
$\text{REIT}_{S\&P}$	26.706***	0.995	14.980	0.558	5.302	0.198
Comm_{BB}	−5.579***	−0.303	−6.431	−0.349	−5.279	−0.286
Bond_{Univ}	3.951***	1.168***	1.436	0.425	3.276	0.969

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- AV comes from the foundations of investment risk
- AV management informs about the risk mix across the economy

Equity Data

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Country	Start	Obs	Index	Assets
USA	1926 - 8	1085	CRSP	500
AUS	2000 - 5	212	ASX	200
BRA	1995 - 2	275	iShares MSCI Brazil ETF	60
CHN	2005 - 5	152	CSI 300	300
DEU	1993 - 11	290	HDAX	110
FRA	1993 - 9	292	SBF 120	120
IND	2000 - 5	212	Nifty 50	50
ITA	2003 - 8	173	FTSE MIB	40
JPN	1993 - 6	295	Nikkei	255
UK	1993 - 6	295	FTSE	100
World	1995 - 3	274	MSCI ACWI	1735

Non-Equity Data

Other Asset Data

Index	Start	Obs	Asset Class
Bloomberg US Spot	2005 - 6	158	Currency
Deutsche Bank Currency	2005 - 6	158	Currency
Deutsche Bank Carry	2005 - 6	158	Currency
Deutsche Bank Momentum	2005 - 6	158	Currency
S&P REIT Index	2005 - 6	158	Real Estate
Bloomberg Commodity	2005 - 6	158	Commodities

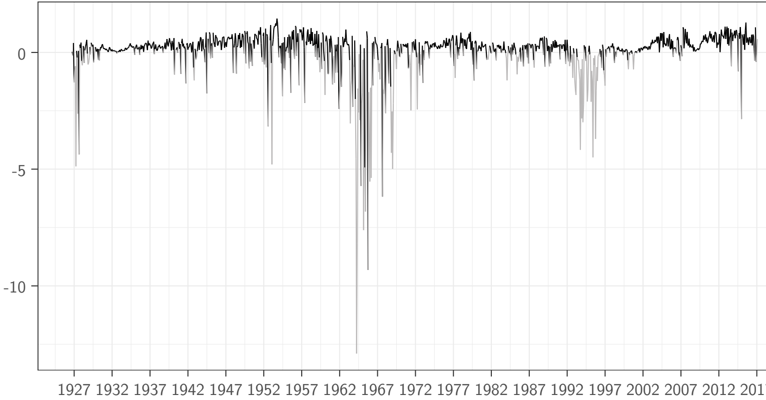
AV Construction

- $\mathrm{SV}_t = \sigma^2_{S,t}$
- With m assets in the market, $\mathrm{AV}_t = \sum_{m=1}^{M} w_{m,t}\sigma^2_{m,t}$
- $W_t = \frac{c}{X}$ is the investment weight in the portfolio, where X $\in \{\mathrm{AV}_{t-1},\mathrm{SV}_{t-1}\}$
- The constant c_{target} is used to control the volatility of the strategy
- c_{BH} matching the buy and hold
- For robustness, c_{10} and c_{12} targeting 10% or 12% annual return volatility

Investment Weights

Strategy Investment Weight

Investment Weights Again



Investment Weight Again Again

Portfolio	Target	Mean	St. Dev.	Min	Pctl(25)	Median	Pctl(75)	Max
SV	c_{10}	0.697	0.762	0.009	0.246	0.512	0.874	8.743
AV	c_{10}	0.702	0.383	0.018	0.425	0.667	0.915	2.296
SV	c_{12}	0.841	0.920	0.011	0.297	0.618	1.055	10.552
AV	c_{12}	0.848	0.463	0.022	0.513	0.805	1.104	2.772
SV	c_{BH}	1.290	1.412	0.017	0.455	0.948	1.619	16.193
AV	c_{BH}	1.301	0.710	0.033	0.787	1.235	1.694	4.253

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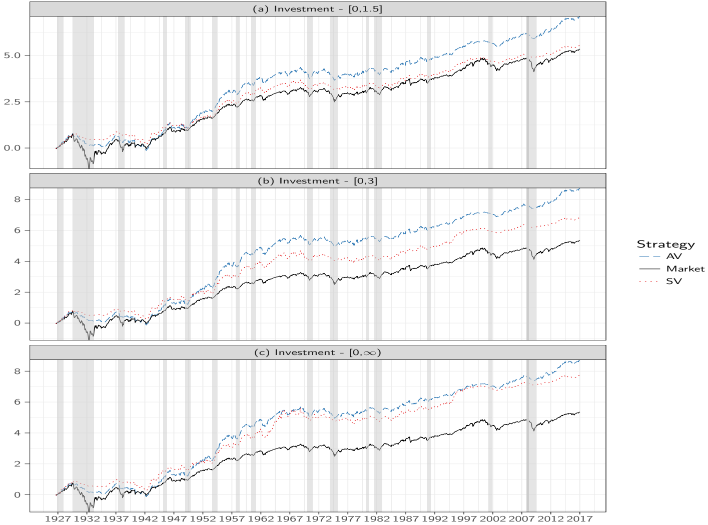
- Break Even - trading costs, basis points, which erase gains
- Certainty Equivalent Return gain (CER) = Average utility from AV - Average utility from SV for mean-variance investor with risk aversion γ

Returns

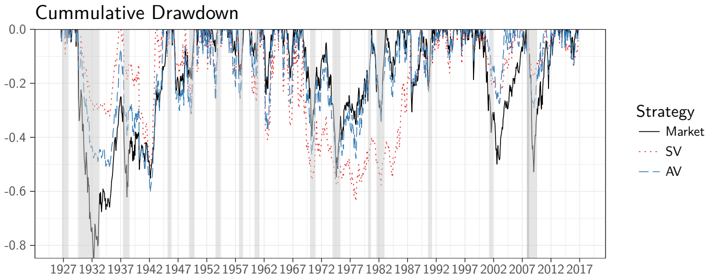

(a) Investment - [0,1.5]
5.0
2.5
0.0
(b) Investment - [0,3]
8
6
4
2
0
(c) Investment - [0,∞)
8
6
4
2
0
1927 1932 1937 1942 1947 1952 1957 1962 1967 1972 1977 1982 1987 1992 1997 2002 2007 2012 2017
Strategy
AV
Market
SV

Performance

c_{BH}: 1926:07-2016:12

	Return	Sharpe	Sortino	Kappa_3	Kappa_4	α_{FF5}	$\alpha_{FF5+Mom}$
BH	5.932	0.319	0.129	0.082	0.061		
SV	8.598	0.462	0.208	0.132	0.097	5.477	3.201
AV	9.677***	0.520*	0.225	0.150*	0.112*	5.594***	3.164***

Drawdowns: c_{BH}



Strategy	N	Max DD	Avg DD	Max Length	Avg Length	Max Recovery	Avg Recovery
BH	82	-84.803	-8.069	188	11.549	154	7.207
SV	65	-63.637	-11.196	246	14.954	135	7.446
AV	87	-60.264	-9.026	205	10.851	135	5.034

Drawdown Insurance: c_{BH}

Knockout

- Drawndown large enough to shutter fund (investor pull-out), cost manager job
- Assuming 45% loss in a 12-month period as knockout
- SV 1.06% and AV .55% using Pav (2016)
- AV $\approx$ half the cost to insure, Carr, Zhang, and Hadjiliadis (2011)

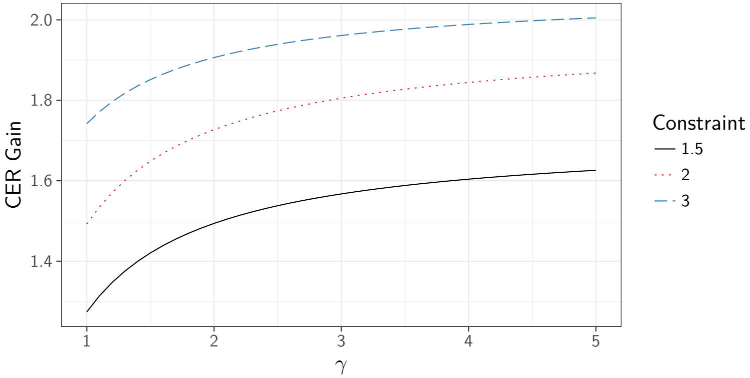
Leverage

Portfolio	c_{BH}: Constraint - 1.5				
	Return	Sharpe	Sortino	Kappa_3	Kappa_4
BH	5.932	0.319	0.129	0.082	0.061
SV	6.171	0.467	0.200	0.128	0.091
AV	7.885***	0.486	0.204	0.133	0.097

Portfolio	c_{BH}: Constraint - 3				
	Return	Sharpe	Sortino	Kappa_3	Kappa_4
BH	5.932	0.319	0.129	0.082	0.061
SV	7.606	0.456	0.199	0.129	0.096
AV	9.677***	0.522**	0.226**	0.150**	0.112**

Notes: ***,**, and * Significant at the 1, 5, and 10 percent levels.

Leverage



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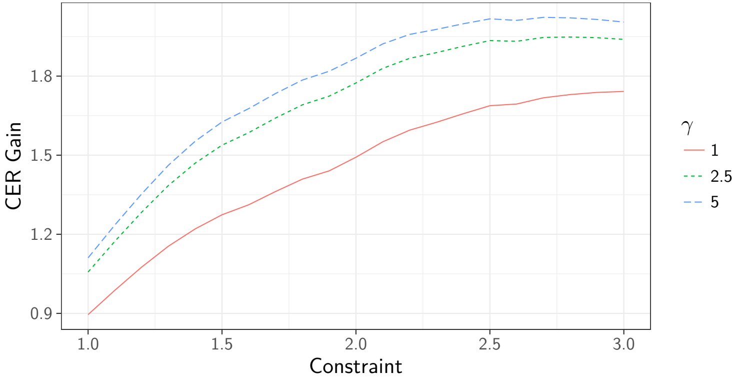

Risk averse, mean-variance investors see substantial utility gains switching from the SV to AV managed portfolio and these gains increase with leverage usage and risk aversion

AC/AV and Systematic Risk

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- This is similar to the difference in results between Goyal and Santa Clara (2003) and Bali et all (2005) when the latter removes a significant number of daily returns and the forecasting ability of idiosyncratic volatility disappears
- Thus we can run a placebo-like test on a sub-sample where the daily returns are not representative

Subsample Tests

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- As much as 13% of market capitalization is not captured by CRSP data as of the 1950s.
- Twice as many firms covering twice as many industries are available at the end of 1962 as compared to the end of 1961.
- As shown in Taylor (2014) the NYSE market was not a significant part of marginal wealth in the US following the Great Depression before the late 1950s.

Regressions

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- Omit variance (SV_{t+1}) prediction by AV as it works in both sub-samples
- Goyal and Welch (2008) forecasting relationships maybe unstable and quite sensitive to sample period choice; they may not respond dynamically with the limited information available to investors in real-time and may not explain or support a trading strategy

Return Prediction

1962:07 - 2016:12

	RET_{t+1}				
AV	-0.131			-0.168^{**}	0.016
	$p = 0.166$			$p = 0.020$	$p = 0.739$
AC		0.047^{***}		0.106^{***}	
		$p = 0.001$		$p = 0.0001$	
SV			-0.109		0.254
			$p = 0.746$		$p = 0.893$
Constant	-0.000	-0.000	-0.000	-0.000	-0.000
	$p = 1.000$	$p = 1.000$	$p = 1.000$	$p = 1.000$	$p = 1.000$
N	655	655	655	655	655
R^2	0.017	0.002	0.012	0.027	0.017
Adjusted R^2	0.015	0.001	0.010	0.024	0.014

Notes: ***Significant at the 1 percent level.
**Significant at the 5 percent level.
*Significant at the 10 percent level.

Return Prediction

1926:08 - 1962:07

	RET_{t+1} - 1926M7:1962M6				
AV	0.061			0.121	0.315
AC		-0.032		-0.099	
SV			-0.028		-0.264
R^2	0.004	0.001	0.001	0.010	0.026
Adjusted R^2	0.002	-0.002	-0.002	0.005	0.021

Out of Sample Stats

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- $\text{DM} = \frac{\bar{d}}{\sqrt{\frac{2\pi f_d(0)}{T}}}$
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ENC-HLN Harvey, Lebourne and Newbold (1998)

- Optimal forecast $= \hat{y}_t^* = (1-\lambda)\hat{y}_{b,t} + \lambda \hat{y}_{x,t}$
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Rossi and Inoue (2012)

- Calculate OOS stats on all feasible window specifications
- Use asymptotic distribution $\rightarrow$ stat critical values
- Different critical values for Type I (R_T) and Type II (A_T) error tests

Out of Sample Results

Table: Sample 1939:12 to 2016:12

	DM	MSE-F	ENC-HLN
AC_{t+1}	1.604*	46.251***	1**
SV_{t+1}	1.041	21.57***	0.956**
AV_{t+1}	3.104***	198.267***	1***
RET_{t+1}	-2.027	-8.702	0

Robust Out of Sample Results

Table: Sample 1939:12 to 2016:12

Stat	Variable	DM	ENC-HLN
R_T	SV_{t+1}	8.874***	1.838***
R_T	RET_{t+1}	29.124***	4.871***
A_T	SV_{t+1}	2.647***	0.949***
A_T	RET_{t+1}	13.347***	1.68***

Notes: ***,**, and * Significant at the 1, 5, and 10 percent levels.

- These results compare the use AV to SV in forecasting - not case either is good (RET) but AV is better

Global Equity

- If AV management times investment to compensated risk because it changes in response to changes in systematic vs non-systematic risk it should work outside the US
- World AV and SV are market cap weighted averages of country values, US included

	AV		SV		BH	
Country	RET	Sharpe	RET	Sharpe	RET	Sharpe
AUS	12.477	0.981	11.993	0.943	7.805	0.614
BRA	11.000	0.291	9.037	0.240	6.163	0.164
CHN	27.381	0.868	24.926	0.790	12.286	0.390
DEU	11.064	0.537	7.633	0.371	5.399	0.262
FRA	7.243	0.404	6.128	0.341	4.904	0.273
IND	14.893	0.633	12.256	0.521	11.460	0.487
ITA	3.838	0.194	3.912	0.198	1.451	0.073
JPN	1.375	0.068	0.129	0.006	-0.775	-0.038
UK	6.591	0.485	5.984	0.441	5.111	0.376
World	8.604	0.551	8.306	0.536	4.484	0.290

Global Equity Again

Drawdown Statistics

	AV			SV			BH		
Country	Avg DD	Avg Length	Avg Recovery	Avg DD	Avg Length	Avg Recovery	Avg DD	Avg Length	Avg Recovery
AUS	-6.302	7.174	3.348	-5.322	9.263	5.421	-6.318	8.600	4.550
BRA	-8.059	9.560	4.208	-17.469	15.235	5.500	-15.064	17.067	4.286
CHN	-9.511	10.333	5.917	-10.074	10.583	3.727	-19.374	27.400	2.000
DEU	-11.051	10.625	5.783	-12.587	16.812	9.933	-10.706	17.125	12.333
FRA	-10.263	14.111	5.941	-15.260	18.267	10.214	-11.590	19.071	15.077
IND	-8.170	6.500	2.885	-12.545	12.467	5.733	-10.862	8.318	4.500
ITA	-14.625	19.500	2.143	-18.174	22.571	2.333	-8.919	15.400	1.667
JPN	-30.655	72.750	41.750	-78.514	294.000	175.000	-40.792	148.00	2.000
UK	-6.060	11.609	4.652	-7.872	14.158	8.158	-6.018	10.560	7.240
World	-6.982	9.909	7.333	-9.776	12.500	7.059	-8.209	10.091	6.429

Global Equity Again Again

Trading Costs

Country	AV RET	AV $\lvert\Delta\omega\rvert$	AV Break Even	SV RET	SV $\lvert\Delta\omega\rvert$	SV Break Even	RET_{BH}
AUS	12.477	0.486	80.139	11.993	0.466	74.914	7.805
BRA	11.000	0.253	159.118	9.037	0.623	38.462	6.163
CHN	27.381	0.305	412.715	24.926	0.538	195.972	12.286
DEU	11.064	0.499	94.545	7.633	0.581	32.052	5.399
FRA	7.243	0.468	41.656	6.128	0.536	19.041	4.904
IND	14.893	0.710	40.316	12.256	0.507	13.097	11.460
ITA	3.838	0.448	44.366	3.912	0.603	33.991	1.451
JPN	1.375	0.442	40.518	0.129	0.551	13.675	-0.775
UK	6.591	0.473	26.113	5.984	0.509	14.287	5.111
World	8.604	0.439	78.113	8.306	0.642	49.586	4.484

Asset Classes

- If AV management times to changes systematic vs non-systematic risk, equity AV should provide a management signal for more than equities
- Moriera and Muir (2017) show that equity SV does not work as a signal for currency investment
- World AV and SV used with c calculated to match buy and hold for each index

	AV		SV		BH	
Index	RET	Sharpe	RET	Sharpe	RET	Sharpe
Bloomberg Dollar	1.324	0.170	0.606	0.078	-0.296	-0.038
DB Currency	1.195	0.272	-0.668	-0.152	-0.244	-0.056
DB Carry	1.440	0.134	-0.361	-0.033	-2.071	-0.192
DB Mom	1.942	0.214	0.413	0.045	1.095	0.120
S&P REIT	26.706	0.995	14.980	0.558	5.302	0.198
Bloomberg Commodity	-5.579	-0.303	-6.431	-0.349	-5.279	-0.286

Asset Classes Again

Drawdown Statistics

Index	AV Avg DD	AV Avg Length	AV Avg Recovery	SV Avg DD	SV Avg Length	SV Avg Recovery	BH Avg DD	BH Avg Length	BH Avg Recovery
Bloomberg Dollar	-8.393	29.000	12.750	-10.632	39.333	21.333	-13.565	60.000	27.000
DB Currency	-2.236	9.750	2.667	-10.471	59.500	20.500	-8.839	59.500	41.500
DB Carry	-7.336	14.250	7.375	-33.972	121.000	98.000	-30.332	60.000	21.000
DB Mom	-4.748	11.900	3.300	-14.679	59.000	17.000	-12.278	38.333	18.333
S&P REIT	-7.692	4.400	1.800	-15.016	9.455	5.000	-17.004	15.143	9.286
Bloomberg Commodity	-9.784	12.222	2.111	-31.116	39.000	12.333	-26.638	39.333	4.333

Asset Classes Again Again

Trading Costs

	AV			SV			
Index	RET	$\lvert\Delta\omega\rvert$	Break Even	RET	$\lvert\Delta\omega\rvert$	Break Even	RET_{BH}
Bloomberg Dollar	1.324	0.411	32.846	0.606	0.620	12.126	-0.296
DB Currency	1.195	0.430	27.851	-0.668	0.482	-7.339	-0.244
DB Carry	1.440	0.427	68.600	-0.361	0.510	27.947	-2.071
DB Mom	1.942	0.441	16.010	0.413	0.599	-9.501	1.095
S&P REIT	26.706	0.592	301.254	14.980	0.807	99.908	5.302
Bloomberg Commodity	-5.579	0.460	-5.430	-6.431	0.555	-17.285	-5.279

Conclusion

- AV management is better than SV: higher returns, better ratios, lower costs
- AV management is better because it times moving in and out of investments to changes in systematic risk which is compensated and non-systematic risk which is not
- As such, AV management is a useful signal both globally and across assets classes where SV management does not perform
- Thank you

More Pollet and Wilson (2010)

PW Details

- Start with Campbell and Viceira (2002) :
  $r_{i,t+1} \approx \gamma\sigma_{i,m,t} - \frac{\sigma^2_{i,t}}{2}$, m is true market
- holds for i = s, stock market portfolio
- $r_{s,t+1} \approx \gamma cov_t(r_{s,t+1}, r_{m,t+1}) - \frac{\sigma^2_{s,t}}{2}$
- $r_{s,t+1} \approx \gamma cov_t(r_{s,t+1}, w_{s,t}r_{s,t+1} + (1 - w_{s,t})r_{u,t+1}) - \frac{\sigma^2_{s,t}}{2}$,
  u is observable component
- $r_{s,t+1} \approx \gamma cov_t(r_{s,t+1}, w_{s,t}r_{s,t+1} + (1 - w_{s,t})r_{u,t+1}) - \frac{\sigma^2_{s,t}}{2}$
- $r_{s,t+1} \approx \gamma w_{s,t}var_t(r_{s,t+1}) + cov(r_{s,t}, (1 - w_{s,t})r_{u,t+1}) - \frac{\sigma^2_{s,t}}{2}$

More Pollet and Wilson (2010)

- assume shocks to stock returns : $\bar{\epsilon}_{z,t+1} + \epsilon_{i,t+1}$, z common i idiosyncratic
- $r_{s,t+1} = \beta_t r_{m,t} + \bar{\epsilon}_{z,t+1}$
- var$(\bar{\epsilon}_{z,t+1} + \epsilon_{i,t+1}) = \sigma^2_{z,t} = \theta_t \sigma^2_{z,t} + (1-\theta_t)\sigma^2_{i,t}$, θ common part
- $r_{u,t+1} = \frac{1-w_{s,t}\beta_t}{1-w_{s,t}} r_{m,t} - \frac{w_{s,t}\beta_t}{1-w_{s,t}}$
- substitute and simplify (many steps)
- $cov(r_{s,t}, r_{u,t+1}) = \frac{1-w_{s,t}\beta_t}{1-w_{s,t}} \frac{\bar{\sigma}^2_t}{\beta_t} \frac{\bar{\rho}_t - \theta_t}{1-\theta_t} \bar{\rho}_t - \frac{w_{s,t}\theta_t}{1-w_{s,t}} \frac{\bar{\sigma}^2_t}{\beta_t} \frac{1-\bar{\rho}_t}{1-\theta_t} \bar{\sigma}^2_t$
- more simplification
- $cov(r_{s,t}, r_{u,t+1}) = \pi_0 + \zeta_1 \bar{\rho}_t + \zeta_2 \bar{\sigma}^2_t$
- ζ_1 positive but small for plausible values of $w_{s,t}$ and β_t, ζ_2 negative but small for plausible values
- Return