How to Look Clever and Have Envious Neighbors: Average Volatility Managed Investment Timing

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ABSTRACT

A portfolio that manages investment by the average variance (AV) of the individual asset returns produces significantly higher average returns, performance ratios, and utility gains for investors above a portfolio that manages investment in the market by the variance of daily market returns. The AV managed portfolio is also cheaper, more practically implementable and displays better drawdown statistics. This performance gain arises because AV is uncompensated, non-systematic risk and making investment leverage a function of the inverse of AV reduces investor exposure when the risk does not accompany reward and increases investment when it does. The AV managed strategy works across countries and asset classes supporting the argument that reducing investment exposed to high levels of AV better aligns investment exposure with compensated systematic risk over time.

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While Warren Buffet may not wholly support taking leverage to invest[1], even at fixed levels of portfolio volatility timing investing can increase returns. Moreira and Muir (2017) show across investment strategies merely managing leverage in a portfolio by that portfolio's volatility produces greater expected returns and performance ratios. As Rutterford and Sotiropoulos (2016) document, the discussion of the risk-return trade-off is at least a century old and central to modern financial theory. Investment management that generates higher overall returns with no increase in total risk is of interest not only to investors but anyone seeking an understanding of the risk-return dynamic. Portfolio variance is the standard measure of investment risk at the foundation of modern portfolio theory and optimization. (Markowitz, 1952) Portfolio variance is a function of asset variance and the pairwise covariance between the assets in the portfolio. The signal from the average variance of the individual assets is a better investment timing signal and its better manages exposure to un-systematic, uncompensated risk.

Since the early identification of a low-risk anomaly, the absence of a need to take on more risk for higher returns as in Haugen and Heins (1972), a large number of researchers have sought to identify a positive relationship between return variance and expected returns. Campbell (1987); French, Schwert, and Stambaugh (1987); Glosten, Jagannathan, and Runkle (1993); Ang, Hodrick, Xing, and Zhang (2006) and many, many others have found insufficient evidence supporting a positive relationship between return variance and future returns. Some investors seek to exploit aspects of this disconnection through risk management and risk-parity funds. As of 2016, at least $150 and as much as $400 billion sat in these funds. (Steward, 2010; Cao, 2016) The Moreira and Muir (2017) result even holds for positions which already seek to exploit the low-risk anomaly like the "betting-against-beta" strategy of Frazzini and Pedersen (2014). Additionally, other researchers have applied similar variance or volatility management to specific assets or trading styles.[2] In short, it seems risk does not equal reward. However, the central premise of modern portfolio theory is that systematic risk is rewarded as investors are not able to avoid it and not all portfolio variance is systematic.

While the total variance of stock market returns (SV) is a function of both the variance of individual asset returns and the covariance between assets in the market, Pollet and Wilson (2010) show that the average pair-wise correlation (AC) of portfolio assets is the component of portfolio volatility most related to systematic changes in the economy and is the risk component compensated with higher returns. By managing monthly investment in the market portfolio by the other component of total volatility, the average variance of the individual asset returns (AV), I generate higher average annualized returns and significantly better Sharpe, Sortino, Kappa ratios, and factor alphas. In addition to identifying a better strategy for investors, decoupling AV from the SV sheds light on the risk-return trade-off dynamics of the market and allows timing investment across markets and asset classes when higher volatility will be compensated and pulling out when it will not.

AV is substantially better than SV as a investment management signal. Keeping the volatility of the AV managed portfolio the same as the buy and hold market portfolio, as in Moreira and

Muir (2017), an investor without borrowing constraints earns an annualized average return of 9.7%. This return is a statistically significant increase of more than one percentage point over the SV managed portfolio; the difference in annualized average monthly returns grows to more than two percentage points when practical leverage constraints are applied. With unconstrained borrowing, the AV managed portfolio has significantly better performance ratios like the symmetric Sharpe ratio, .52, and more asymmetric risk-return measures, e.g., Kappa$_3$ and Kappa$_4$ at .15 and .11 respectively. The advantage of managing with AV grows with risk aversion. The most risk-averse, $\gamma = 5$, constrained mean-variance investor sees a certainty equivalent return (CER) gain of more than two percentage points annualized; this return represents a 27.4% increase in utility nearly as substantial as the utility gains seen in return forecasting strategies. (Campbell, Lo, MacKinlay, et al., 1997) Stochastic dominance tests show the preference for AV management is not restricted to mean-variance investors. Investors with a broad class of utility functions, including those with higher order risk and prospect theory preferences, prefer the AV management strategy to SV. The significantly better performance of AV is also robust to other choices of target volatility; AV outperforms SV management when both portfolios seek to have 10% or 12% annual return volatility.

The AV and SV portfolio management strategies each generate a weight in the market portfolio as a function of the daily returns of the market the prior month. Some of the investments demanded by the SV management strategy are just unrealistic. The SV managed portfolio takes far more extreme leverage than AV and far more often. In addition to higher average monthly borrowing costs, 15.107 versus 11.411 basis points, the SV managed portfolio has higher turnover. The AV managed portfolio generates lower transactions costs with a break-even transaction cost more than twice the SV managed portfolio. This makes the AV managed portfolio cheaper for investors in both borrowing and transaction costs while generating higher returns. The AV managed portfolio is also cheaper to insure. Drawdowns are shallower and shorter for the AV managed portfolio at 9% versus 11.2% and 10.5 versus 15 months on average. The drawdown profiles for the AV and SV managed portfolios also reveal that the AV managed portfolio benefits fund managers in addition to fund investors. The SV managed portfolio exposes a fund manager to twice the risk of a drawdown severe enough to threaten their job, or possibly shutter the fund, and would be nearly 91.7% more expensive to insure against such a loss.

AV equity management is also better globally. The AV managed portfolio is a better performer in return, ratios, drawdowns, and costs in 8 of 9 non-US markets tested. AV management generates significantly better performance in both developed and emerging markets including Brazil, China, Germany, France, India, Italy, Japan, and Great Britain. For example, returns in China are more than quintupled over the buy and hold strategy by AV management. AV equity portfolio management works not only in the countries individually but for an equity portfolio diversified across the globe. AV management of the MSCI All Countries World Index (ACWI) produces higher annualized returns, 8.6%, with a better Sharpe ratio, .551, than either SV management or the buy and hold strategy. AV managed MSCI ACWI also has shallower drawdowns and is robust to much higher trading costs.

The identification of a better portfolio management signal is valuable, certainly for investors and fund managers. However, discovering the reason that AV outperforms SV makes a more significant contribution to our understanding of the market and the risk represented by portfolio variance. This requires examining the relationship of AV to future SV, AV, AC, and returns. The examination of regression results also allows for the correction for possible "volatility-feedback" effects, proposed by Campbell and Hentschel (1992), and makes it possible to judge the stability of the dynamic risk-return relationship mix revealed by AV and AC.

Using daily market returns from the Center for Research in Securities Prices (CRSP), I extend Pollet and Wilson (2010) generating monthly time series of stock market variance, SV, average correlation, AC, and average variance, AV. In results moved to the appendix for space, AV is a significant in-sample predictor of higher SV, AV, and AC. A one standard deviation increase in annualized AV, from .77 to 1.62, is related to an increase in next months annualized SV of .627 of a standard deviation or a .25 increase. The increase in SV makes it more than double its mean. AV remains a significant predictor of next months SV even when controlling for this month's SV. AV has a slightly higher R^2 value when predicting next month's SV and is better at predicting next month's AV. Controlling for this month's AV, SV is not a significant predictor of next month's AV. Neither SV nor AV is significantly related to future returns. However, as in the Pollet and Wilson (2010) results, AC is positively and significantly related to future returns. This evidence explains the performance of AV as a leverage management signal. Scaling investment in the market with the inverse of AV in the current month will pull funds out when the following month will have high SV without sacrificing higher expected returns when AC is high. These results support the existence of a long-run relationship between AC and future returns and support the conclusion that AV is not a compensated risk. When forecasting next month's excess log return over the full sample, the coefficient on AC is smaller and losses some significance compared to the post 1962 sample.

It is, of course, a difficult to test whether AV management better times investment to systematic risk. However, I find several results that suggest that AV is better than SV at signaling investors to times of better conditions across the economy. As always, asset pricing and portfolio tests are limited by the indices and proxies used. Pollet and Wilson (2010) show that AC is related to higher future returns and higher return correlation across the economy, but the relationship depends on the proportion of the market observed through the index proxy and the relationship of the stock market return and the aggregate wealth return. Thus, in a sub-sample where it is unlikely the daily returns used represent the whole market and the stock market itself is known not to be an insignificant part of aggregate wealth, AC should not predict returns. This is the case for the CRSP data before 1962 which covers only the New York Stock Exchange (NYSE) and covers a period when the average investment was meager. (Taylor, 2014) Indeed, in a placebo-like result, before 1962 AC does not predict next month's return even when controlling for AV.

The pre-1962 result supports the Pollet and Wilson (2010) arguement that the information AC, and thus AV, provide on the mix of compensated and uncompensated risk depends on the relationships of the index returns being used the stock market and aggregate wealth. This should

also be observable in portfolio performance. Taking market capitalization to GDP as a proxy for the proportion of the stock market in aggregate wealth, AV works better in the cross-section of countries when the market is more related to aggregate wealth. Investors capture average annualized returns over 11% with a Sharpe ratio above .77 from a portfolio using AV managed weights long in countries with high market capitalization to GDP ratio and short those with a low ratio. Moreover, the portfolio has a positive and significant Fama-French five-factor and five-factor plus Momentum alphas meaning the relationship of the stock market to aggregate wealth is systematically important to AV management as seen in the pre-1962 results above. This does not hold for SV management which has insignificant alphas, lower returns and a smaller Sharpe ratio.

The full in-sample results demonstrate AC forecasts next month's return and AV forecasts next month's risk without a relationship to returns. However, as Welch and Goyal (2008) argue, forecasting relationships maybe unstable and quite sensitive to sample period choice. They may not respond dynamically with the limited information available to investors in real-time and may not explain or support a trading strategy. Furthermore, robust out-of-sample results would indicate that there is more information available on real-time risk-return dynamics in AV than SV which would support the notion that AC is systematic and compensated; AV is un-systematic and uncompensated explaining the portfolio performance results.

Investors can only make decisions using the limited information available to them at a given time. For example in June of 2007 investors and investment models could only use historical information up to that month; the effects of November 2008 on the regression coefficients cannot affect the predictions for July 2007. To explain why AV is a better real-time market portfolio leverage signal, I run expanding window out-of-sample regressions using AV on SV, AV, AC, and log excess returns. From June 1926 to December 2016 and using the predictions from SV as a benchmark, AV is a significantly better forecaster of next months AV, AC, and SV. It generates better Diebold and Mariano (1995) test statistics, significantly lower mean squared forecast errors judging by the MSE-F statistic from McCracken (2007) and the encompassing test of Harvey, Leybourne, and Newbold (1998) show that AV contains all of the forecasting information in SV. As with the in-sample results, AV serves investors at least as well as SV and likely better in avoiding risk without giving up return. Out-of-sample testing always raises questions about model specification, recursive expansion versus rolling window parameter estimation, choices of the training period and prediction window and the influence of specific periods. Using the techniques in Rossi and Inoue (2012), the Diebold and Mariano (1995) and Harvey et al. (1998) measures can be calculated robust to concerns on window selection for either an expanding or rolling specification. The Rossi and Inoue (2012) robust statistics show that AV is a significantly better predictor than SV robust to the choice of window or regression specification. AV is a better investment timing signal than SV because it scales investment by future risk without giving up the future return.

A sharp test of AV management's ability to key investors to changes in systemic risk would be its ability to manage returns from other asset classes. Moreira and Muir (2017) find that equity SV is not a useful signal for currency investment. However, equity AV works as a management signal

not only for currency investments but real estate investment as well. Returns, ratios, drawdowns, and costs are all the better when managing investment in multiple currency and real estate indices by the inverse of the average variance of equities compared to both a buy and hold, and equity SV management. I test this using four currency indices and the S&P US Real Estate Investment Trust (REIT) index. From 2005 to 2015, the buy and hold and SV management strategies have annualized returns of -2.07% and -.36% for the Deutsche Bank Currency Carry Index. This carry trade index uses one of the oldest currency trade strategies based on the work on uncovered interest rate parity from Bilson (1981); Fama (1984) and studied by Lustig and Verdelhan (2007); Brunnermeier, Nagel, and Pedersen (2009); Burnside, Eichenbaum, and Rebelo (2011) among others. The average variance component of equity market variance is better globally and across asset classes supporting the argument that compensated systematic risk is higher when average variance is lower.

Using the decomposition of market variance, I identify a better portfolio leverage management signal. Weighting investment leverage by the inverse of the average of individual asset variance, AV, rather than SV, is a new addition to the portfolio management literature letting investors capture better performance as measured by expected annualized returns, performance ratios, costs and utility gains. My results complement recent work including the Moreira and Muir (2017) and Hocquard, Ng, and Papageorgiou (2013) who use volatility timing and a constant target to manage portfolio tail risk. The returns to the AV managed portfolio serve as another example in the risk-return dynamics literature exposing the returns to correlation risk obscured when SV measures risk. Recent work in this area has also attempted to generate more systematic signals from components of portfolio volatility or across many portfolios. Gonzlez-Urteaga and Rubio (2016) examine the average volatility across many portfolios and the risk premium compensation. Bollerslev, Hood, Huss, and Pedersen (2017) generate a better-realized volatility risk signals from high-frequency data of many assets demonstrating better forecasting performance and utility gains for investors. The formation and analysis of the AV signal come from the fundamental understanding of risk, portfolio variance and returns. This work requires a few publicly available data sets and a few considerations for the calculations at the monthly frequency. Most of the work is in the calculations required to show significance in portfolio performance and in the regressions which establish the relationship of AV, risk, and future returns.

I. Data

To calculate stock market variance (SV), average asset variance (AV), and average asset correlation (AC) for the US equities, I use daily return data from CRSP and calculate the variance of daily returns monthly. To simplify the analysis of individual assets, I require that the asset be traded on each in the month which mitigates any liquidity effects and ensures consistent variance, covariance and correlation calculations. These conditions make the calculation of asset variance:

$$\sigma_{m,t}^2 = \frac{1}{T-1} \sum_{\tau=1}^{T} \left(R_{m,\tau} - \frac{\sum_{\tau=1}^{T} R_{m,\tau}}{T} \right)^2. \tag{1}$$

where $R_{m,\tau}$ is the daily return, including dividends, on an asset for day τ in month t. When the asset is the market portfolio, so $R_{m,\tau} = R_{s,\tau}$, the result is the variation of market returns, SV. The standard Pearsons correlation where the correlation of assets m and n for month t is:

$$\rho_{m,n,t} = \frac{\sum_{\tau=1}^{T} \left(R_{m,\tau} - \frac{\sum_{\tau=1}^{T} R_{m,\tau}}{t} \right) \left(R_{n,\tau} - \frac{\sum_{\tau=1}^{T} R_{n,\tau}}{t} \right)}{\sqrt{\left(R_{m,\tau} - \frac{\sum_{\tau=1}^{T} R_{m,\tau}}{t} \right)^2 \sum_{\tau=1}^{T} \left(R_{n,\tau} - \frac{\sum_{\tau=1}^{T} R_{n,\tau}}{t} \right)^2}}. \tag{2}$$

Unfortunately, for samples as small as the monthly series of daily returns Pearsons correlation is not an unbiased estimator of the true correlation, even if the returns are normal. (Hotelling, 1953) The average month in my sample has 22 trading days however the number commonly drops into the teens.[3] For samples of these sizes, the bias causes an underestimation of the correlation which is worse the lower the true correlation. I employ an approximate correction from Olkin and Pratt (1958) such that the monthly correlation between two assets m and n is:

$$\rho_{m,n,t} = \widehat{\rho_{m,n,t}} \left(1 + \frac{1 + \widehat{\rho_{m,n}}^2}{2(t-3)} \right) \tag{3}$$

where $\widehat{\rho_{m,n,t}}$ is the Pearson correlation between a and b.[4] AV and AC are value-weighted so each month I calculate market capitalization for all of the assets available in CRSP. The capitalization used in month t for asset m is the product of the end of month price (PRC) and common shares outstanding (SHROUT) values for asset m in month t-1.

[Place Figure 1 about here]

To make the analysis more computationally tractable I use only, at most, the top 500 assets in CRSP by market capitalization for a given month.[5] To test the performance of AV management across international equity markets, I collect daily returns for the Australian (AUS), Brazilian (BRA), Chinese (CHN), German (DEU), French (FRA), Indian (IND), Italian (ITA), Japanese (JPN), and English (UK) markets from Compustat - Capital IQ. For international markets the number of assets is chosen to mimic the number of assets in the broad market index used to calculate SV. Table ?? shows the names of the indices used, the data time frames and the number of assets used for the calculation of market capitalization weights, AV and AC. In all cases the data series are much shorter. The primary limitations are the availability of daily returns and dividend data. To test AV and SV management against a globally diversified portfolio, I use the MSCI All Countries World Index (ACWI). To construct AV and SV leverage timing signals for the world index, I use a market capitalization weighted average of the country values. For each month the weight of the country is the index market capitalization in US dollars divided by the total market capitalization of the 10 country market indices, including the US. Globally, ten markets and more than 1700 assets are

covered. Given these asset restriction, an assets market capitalization weight is defined by:

$$w_{m,t} = \frac{MCAP_{m,t}}{\sum_{j=1}^{J} MCAP_{n,t}} \tag{4}$$

with $j \leq 500$ for the US market index. Thus, the series of interest, SV, AV, and AC are defined by:

$$SV_t = \sigma_{s,t}^2 = \frac{1}{T-1} \sum_{\tau=1}^{T} \left(R_{s,\tau} - \frac{\sum_{\tau=1}^{T} R_{s,\tau}}{T} \right)^2 \tag{5}$$

$$AV_t = \sum_{m=1}^{M} w_{m,t} \sigma_{m,t}^2 \tag{6}$$

$$AC_t = \sum_{m=1}^{M} \sum_{m \neq n}^{N} w_{m,t} w_{n,t} \rho_{m,n,t} \tag{7}$$

Figure 1 shows the time series behavior of SV and AV, in percent, as well as AC for the US market. With the easily noticeable exception of October 1987, spikes in both SV and AV concentrate around NBER defined recessions. Panel (b) of figure 1 shows that AV takes a larger weight in the market more often than SV but that SV takes far larger weights in the market in certain times. Figure 2 makes this clearer and shows that changing the target volatility only affects the most extreme investments made by the SV management strategy. **[Place Figure 2 about here]**

Table I panels (a) - (c) show the summary statistics for the calculated variables. Despite the use

of the actual number of trading days, the restriction to assets that trade every day, and the adjustment to the calculation of correlation, the quarterly calculated values are almost identical to those in Pollet and Wilson (2010) over the same sample. Over the expanded the period, the annualized monthly AV has a mean value of .88%. The annualized SV mean is much lower at .25% monthly. AC is relatively consistent at .23 quarterly in the Pollet and Wilson (2010) sample, .261 monthly in the same sample and .276 over the full time period. AV is more volatile than SV, more than twice as much. In each sample AV has the highest auto-correlation. While AC is also persistent, the stock market variance is only strongly persistent at the monthly frequency with autocorrelation of .61. All three time series are stationary rejecting the unit root null in the tests of Dickey and Fuller (1979), Ng and Perron (2001), and Elliott, Rothenberg, and Stock (1996). **[Place Table I about here]**

As my primary interest is in the use of AV versus SV in the management of leverage in the CRSP

market portfolio, I test AV and SV against log excess returns. Specifically, I take the difference between the natural log of one plus the CRSP market return and the natural log of one plus the risk-free rate using:

$$r_t = \log R_{m,t} - \log R_{f,t} \tag{8}$$

where $R_{f,t}$ the risk-free return for which I use the 1-month treasury bill, T-bill, rate from Ken French's website[6]. As shown in table II panel (a), over the full data period, each variance and correlation time series are contemporaneously correlated to lower log excess returns. AV is significantly correlated with next month's SV, AV, and AC. Surprisingly, over the full data set, this month's AV is even nominally more correlated with next month's SV than this month's SV is, 0.625 versus .612. Over the basis period, AV is the time series most negatively correlated with next months log excess return at -.129, while it is entirely unrelated to next month's return over the whole data set.

[**Place Table II about here**]

Panels (b) and (c) show international and cross asset class excess log return correlations. China is

the market least correlated with the rest of the countries and the global market index. Importantly, as panel (c) shows, the other asset class returns are not all highly correlated with the MSCI ACWI returns. The currency return indices are generally weakly negatively correlated while both real estate and commodity returns are positively correlated.

For in and out-of-sample tests, I regress market and average variance, and average correlation against these excess log return values next month. Out-of-sample regressions are run with expanding windows so coefficients estimated from a training window are used to forecast values for the first out of sample month. I set the first in-sample training period for the out-of-sample tests at 15% of the available time series for consistent calculation of robust out of sample statistics later in the analysis. This training window means that out of sample regressions, analysis begins in December 1939 and moves forward one month at a time as the training window expands.

II.   Investment

The most direct and practically relevant measure of AV as a portfolio leverage management tool is whether or not it generates portfolio gains. It is also important to make adjustments for the riskiness of the managed portfolio and compare performance across dimensions for a full investment picture. The AV managed portfolio may well generate higher annualized returns, but will it have a better Sharpe ratio than the SV managed portfolio?

To measure portfolio performance, in addition to annualized monthly log excess return, I will calculate each portfolio's Sharpe ratio, Sortino ratio, and two Kappa ratios and factor αs. The classic Sharpe ratio is a symmetric measure of risk and is defined as the ratio of the expected excess portfolio return over the standard deviation of portfolio returns.

$$\frac{\mathbb{E}[r_x]}{\sigma(r_x)} \tag{9}$$

While Sharpe measures each dollar of expected return for dollar of risk, the Sortino ratio attempts to more directly measure the risk most investors worry about. By using only downside deviation in the denominator, the Sortino quantifies each dollar of expected return for each dollar of loss. This

downside is measured relative to a target return. (Sortino and Price, 1994) As the log returns are already excess of the risk free rate, I set the Sortino target to 0 which makes the Sortino formula:

$$\frac{\mathbb{E}[r_x - 0]}{\sqrt{\int_{-\infty}^{0}(0 - r_x)^2 f(r_x)\, dr}} \tag{10}$$

The Sortino is just a specific instance of a more general risk measurement ratio formula. The Kappa ratio keeps the expected return relative to a target in the numerator but allows any lower partial moment in the denominator. (Kaplan and Knowles, 2004) With the target again set to 0 the general formula is of the form:

$$\frac{\mathbb{E}[r_x - 0]}{\sqrt[n]{LPM_n}} \tag{11}$$

The Sortino ratio is the $Kappa_2$ ratio. I calculate $Kappa_3$ and $Kappa_4$ also to see relative performance of AV and SV management adjusted for negative return skew and kurtosis. All of the measures are annualized, e.g., log returns are multiplied by 12 and ratios, like Sharpe, are multiplied by the square root of 12. Fama-French three and three-factor with momentum αs are also calculated and annualized. The αs come from the standard regression:

$$r_{x,t} = \alpha + \beta\chi_t + \epsilon_t \tag{12}$$

where χ contains the combinations of the small minus big, SMB, high minus low, HML, and market factors with and without the winners minus loser, WML or Mom, momentum factor for x equals AV or SV.

Measuring a difference in each of these measures for a pair of portfolios is not difficult; measuring a significant difference is. When evaluating the difference in Sharpe ratios, the method in Memmel (2003) seems to be popular. However, when returns are not normally distributed or auto-correlated this method is not valid. AV and SV managed returns, like market returns, are weakly auto-correlated, slightly skewed, and have much fatter tails when compared to normally distributed returns. Moreover, current period returns to either the AV or SV managed portfolio depend on the prior period variance of the market return strongly questioning the i.i.d assumption made in most hypothesis testing methodologies. Studentized time series bootstrap sampling preserves the time series properties of the AV and SV managed returns which is critical; for example, Scherer (2004) demonstrates that methods which loose the time dependence in the calculation of differences in Sortino ratios fail to properly estimate the sampling distribution and critical values. Time series bootstrap method preserves the original data structure and allows for efficient robust hypothesis testing. (Politis and Romano, 1994; Davison and Hinkley, 1997) Ledoit and Wolf (2008) show that this method is even more efficient than using Newey and West (1987) or Andrews and Monahan (1992) heteroskedasticity and auto-correlation corrected standard errors for testing the significance of differences between portfolio Sharpe ratios. I follow the p-value estimation method in Ledoit and Wolf (2008) to determine the significance of the difference between the ratios of the AV and

SV management strategies. This uses circular block bootstrapping of the return time series, robust centered studentized statistics computed from the bootstrap samples and is proven to be the most efficient hypothesis testing method.[7] (Politis and Romano, 1992; Ledoit and Wolf, 2008)

As in Moreira and Muir (2017), investment weight in the market portfolio is a function of the inverse of the variance of daily market returns, SV, or the average daily return asset variance, AV, scaled by a constant, c. Moreira and Muir (2017) use a constant that scales the variance of the volatility managed portfolio equal to the buy and hold market portfolio. In the basic portfolio weighting specification, I use the same approach so that the returns of both the SV and AV managed portfolios have the same variance as the buy and hold strategy. This constant is denoted c_{BH} and it takes different values for SV and AV. This scaling requires knowing the full sample buy and hold return variance. While this does not distort the performance ratios, to insure robustness two other specifications for the scaling targets are used. Annual volatilities of 12% and 10% are common in academic literature and fund management so c_{12} and c_{10} approximately target those levels. (Barroso and Santa-Clara, 2015; Morrison and Tadrowski, 2013; Verma, 2018; Fleming, Kirby, and Ostdiek, 2002; Hocquard et al., 2013) Using each of these constant an investor re-balances at the end of month t investing in the market portfolio with weight:

$$w_{x,t} = \frac{c_{target}}{x(t)} \tag{13}$$

where $x(t)$ is either SV_t or AV_t and hold for month t + 1.

[Place Table III about here]

Table III shows summary statistics for the resulting investment weights for AV and SV for the three volatility targets. The investment weight turnover, show later in VIII, mean, and standard deviation for the SV management strategy targeting the buy and hold are nearly identical to the values in Moreira and Muir (2017). When targeting the volatility of the market portfolio, both portfolios are leveraged into the market on average with investment weights of 1.3, indicating 30% leverage. Regardless of the volatility target the SV managed portfolio calls for extreme levels of leverage. Figure 2 shows that the SV strategy, regardless of target, calls for investment weights above the maximum AV weight in several periods. More than 500% leverage is needed at the end of the 1920s, throughout the 1960s, and in the 1990s. Given that these levels of leverage are unrealistic for most investors, it will be important to see if there is a difference in performance for the AV and SV strategies under real-world investment constraints and to investigate the associated costs generated by the trading needed for the SV managed portfolio.

[Place Figure 2 about here]

A.   Portfolio Performance

Before examining the constrained portfolio performance, I present the results for the SV and AV strategies targeting the buy and hold volatility without constraints in table IV. As in Moreira and Muir (2017) the portfolio performance is measured across the whole CRSP data set, however the relative performance is the same or better across the basis data set.

[**Place Table IV about here**]

Table IV presents the performance ratios for the SV and AV managed portfolios targeting the buy and hold volatility without investment constraints. The buy and hold market strategy is included for reference. However since Moreira and Muir (2017) establish that the SV managed portfolio out performs the buy and hold, statistical significance results are only presented for the comparison of the SV and AV managed portfolios. The AV managed portfolio generates a statistically significant 1.09 percentage points higher average annualized log excess return. As shown in the bottom panel of figure 3 the AV strategy builds its performance advantage slowly but consistently starting from the early 1950s and from that time the SV managed portfolio is never a better investment. As both strategies are targeting the same volatility, the significant difference in return translates into a significant difference in Sharpe ratio. At .520 versus .462, for the SV managed portfolio, the AV managed Sharpe ratio is 12.6% higher. There maybe some concern with the use of mean-variance symmetric performance measures like the Sharpe ratio, however AV management outperforms SV on asymmetric performance ratios as well. AV has a higher Sortino ratio. AV also generates significantly higher $Kappa_3$ and $Kappa_4$ ratios. The Fama-French three factor alpha is significantly higher for the AV versus SV managed portfolio. There is no difference in significance either way when the momentum factor is also included, but this is due to one data point as detailed below. So while the overall payment for downside risk, measured by Sortino ratio, may not be significantly higher, the payment for downside skewness and extreme downside return is. Measuring risk-adjusted returns using either the Fama-French five factor or five-factor with momentum model, AV management significantly outperforms AV. The differences are slightly higher than the difference in annualized average return.

[**Place Figure 3 about here**]

Portfolio performance numbers always bring forward questions on performance in sub-samples. While divisions of the sample by date are largely arbitrary and do not automatically convey the importance of the sub-sample, divisions along business cycles call out specific periods of investor sensitivity. Panels (b) and (c) in table IV present the performance of the buy and hold, SV and AV managed portfolios. The AV managed portfolio is a significantly better performer across all measures. In business cycle expansion, the AV managed portfolio provides significantly more compensation and significantly more compensation for every measure of risk. The results for NBER contractions go in the other direction. The SV managed portfolio appears to be so much better that

it might be a more desirable option given that we cannot know periods of extended contractions before they begin and investors may desire a portfolio that protects value through downturns more than one that maximizes returns during market upswings. However, as panel (d) shows, the better performance of the SV managed portfolio is due to one, albeit a rather important, data point. The significantly better performance of the SV managed portfolio through NBER contractions depends entirely on the 1929 to 1933 Great Depression. Excluding that time period, The AV managed portfolio, again, provides higher average and risk-adjusted returns but the difference in performance by any portfolio risk measure, while better for AV, is insignificant. As a result, its clear that without the 1929 to 1933 depression, the SV managed portfolio is unable to compete with AV management in measures of risk-return performance. Additionally, investors that are concerned about the loss of portfolio value, regardless of when it occurs, will value the AV managed portfolio more than the SV managed portfolio in drawdown and utility terms. Certainly, US equity investors are better served by the AV managed portfolio than the SV. However clear this conclusion, it raises the questions of generalization.

The AV management strategy is a better performer than SV across the world. Table V shows the differences in performance and the AV managed portfolio generates higher annualized average returns and better Sharpe ratios in all countries except Italy. In the fastest growing markets, China and India, the AV managed portfolio increases annualized average return by 2.455% and 2.637%. Both AV and SV management are improvements over the buy and hold in all markets. Australia shows the best buy and hold Sharpe ratio at .614, but the AV management strategy is able to increase it to .981, a very attractive result for any investor. The results for the AV management strategy for the Chinese market are the most alluring, a 27.381% annualized return with a .868 Sharpe ratio, but this also the shortest and the most volatile of the international return series. Both AV and SV management generate better returns and Sharpe ratios for the globally diversified world index, but again AV management is an improvement over SV.

Table VIII panel (b) highlights the differences in the trading costs of the strategies across country. This table differs from VIII panel (a) in that the break even trading costs are calculated in reference to reducing the annualized average returns of the AV and SV managed portfolios to the buy and hold and not reducing portfolio alphas to zero. The AV managed portfolio is able to tolerate higher trading costs in every country analyzed. In a majority of countries, 6 out of 9, the AV managed portfolio is able to tolerate trading costs twice as high as the SV managed portfolio. As for the individual country indices, the AV managed world index is far cheaper than the SV managed portfolio allowing an investor to keep more of the higher returns generated.

Across the globe, the AV managed portfolio is a better leverage management signal. It results in higher annualized average returns. It captures in better Sharpe ratios. It generates better drawdown statistics and is cheaper, in trading costs, to execute. Thus, AV management is the better equity investment strategy around the globe. However, if AV management better aligns returns with systematic risk it may be a better management strategy in more than just equities.

B. Leverage

More practical analysis of portfolio performance requires recognition of the borrowing conditions and leverage constraints faced by real-world investors. Table VI shows the portfolio performance results in months with the highest and lowest 25% borrowing costs measured by the risk-free rate, one month T-bill. There is a clear difference. The SV managed portfolio has better returns and performance rations in the period with the highest risk-free rate. AV is the better performer in the months in the bottom 25% of the risk-free rate distribution. It is clear that lending constraints and borrowing conditions affect portfolio performance particularly for the SV managed portfolio which takes extremely large positions in the market in some months. One method of measuring the impact of investment constraints on the AV and SV strategies is to externally impose limits on the weights generated by each.

Leverage of 50%, a coefficient of 1.5 on the market, is a common constraint meant to mimic real market leverage constraints for the average investor based in part on the Reg. T margin requirement[8]. (Campbell and Thompson, 2008; Rapach, Strauss, and Zhou, 2010; Rapach and Zhou, 2013; Huang, Jiang, Tu, and Zhou, 2015; Rapach, Ringgenberg, and Zhou, 2016; Moreira and Muir, 2017; Deuskar, Kumar, and Poland, 2017) There are at least two exchange traded funds, ETFs, which three times the return of the SP500.[9] So, I take a market coefficient of three as the maximum feasible investment a typical investor can make in the market portfolio.

[**Place Table VII about here**]

Table VII presents the results from applying investment constraints after calculating the weights

for AV and SV targeting the buy and hold volatility. Panel (a) shows the results of applying the 50% leverage limit. The brokerage investment restrictions pull the volatility of the AV managed portfolio too far from SV to generate significant differences in performance ratios, but the difference in AV and SV managed returns is always greater than one percentage point and close to two when targeting the buy and hold volatility. Panel (b) shows the ETF, 200%, leverage constraint. The separations in buy and hold targeted average annualized excess return, 1.71% and 2.07%, are even greater. Investors using the leveraged ETFs are rewarded not only with higher returns but significantly better performance ratios across the board. The ETF leverage constrained AV strategy even generates better Sharpe and Sortino ratios than the unconstrained strategy. Panels (a) and (b) in figure 3 show the effects of the growing separation. While SV is barely able to clear the buy and hold strategy under typical brokerage constraints, returns to the AV managed portfolio remain clearly above. The results in Panels (a) and (b) of table VII demonstrate that better performance of AV is not a result of or contingent on looking to the volatility of the buy and hold strategy. As the targeted volatility is lowered the difference in performance between AV and SV becomes more significant.

The differences in investment weight profiles show in table III not only generate differences in returns but also in costs. As show in table VIII panel (a), for the US market, the AV managed portfolio generates less than half the turnover of the SV managed portfolio. The average

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monthly absolute change in investment weight is .752 for the SV managed portfolio and only .317 for AV. Table VIII shows the trading costs needed to reduce the annualized average return of the SV and AV managed portfolios to the buy and hold or to reduce the factor alphas to zero. (Frazzini, Israel, and Moskowitz, 2015; Moreira and Muir, 2017) Seen in table VIII, the break even transaction costs are more than twice that for the AV managed portfolio. The SV managed portfolio breaks even at 29.422, 60.694 and 35.472 basis points while it takes costs of 254.364, 176.467 and 83.176 basis points to reduce AV returns to the buy and hold or zero out the annualized alphas. However, transaction costs are not the only expense incurred by the leveraged portfolios. To estimate the borrowing costs for each strategy, I assume that any month a strategy requires a position greater than one in the market the difference between the investment weight and one is borrowed. The average monthly cost of borrowing to invest for the AV managed strategy is nearly 25% lower than for the SV managed portfolio. Using the broker call money lending rates available in Bloomberg from September 1988 to October 2016, SV incurs an average monthly cost of 15.107 basis points while AV costs only 11.411 basis points. This ignores the possibility of the investor using saved gains rather than borrowing and the necessity of borrowing when the investor has lost money, however, as the AV managed portfolio has greater returns and better drawdown statistics included gains and losses would only further the separation.

[**Place Table VIII about here**]

Table VIII panel (b) highlights the differences in the trading costs of the strategies across countries. The break even trading costs are calculated in reference to reducing the annualized average returns of the AV and SV managed portfolios to the buy and hold only. The AV managed portfolio is able to tolerate higher trading costs in every country analyzed. In all countries, as in the US market, the AV managed portfolio is able to tolerate higher transaction costs. In a majority of countries, 6 out of 9, the AV managed portfolio is able to tolerate trading costs twice as high as the SV managed portfolio. As for the individual country indices, the AV managed world index is far cheaper than the SV managed portfolio allowing an investor to keep more of the higher returns generated.

C. Drawdowns

Drawdowns, the peak-to-trough decline in the value of a portfolio, may be the most natural measure of real market risk. (Magdon-Ismail and Atiya, 2006) Maximum drawdown, the largest peak-to-trough decline in portfolio value, in particular is often used in place of return variance as a portfolio risk measure. (Johansen and Sornette, 2000; Vecer, 2006, 2007; Sornette, 2003) Drawdowns play a significant role in the lives of fund managers as deep losses not only rob the fund of capital but motivate investors to withdraw funds making drawdowns a significant determinant of fund survival. (Baba and Goko, 2009; Papaioannou, Park, Pihlman, and Hoorn, 2013; Lang and Prestbo, 2006) To compare SV and AV managed portfolios against the market buy and hold, I

consider drawdowns longer than one month so two consecutive months of negative returns will start a drawdown. The drawdown continues until the portfolio regains the value it had at the beginning of the first month of the drawdown.

[**Place Table IX about here**]

Table IX panel (a) presents the drawdown statistics for the buy and hold, SV managed, and AV managed portfolios. The AV managed portfolio has more discrete drawdown events, 87, than either the buy and hold or SV managed portfolio. However, the drawdowns are much less severe. The buy and hold strategy losses a maximum of 84.8% of its value at the deepest point of its maximum drawdown, Max DD. AV and SV lose only 60.3% and 63.6% at the bottom of their worst drawdowns. The SV managed portfolio has the worst average loss during a drawdown, Avg DD, at 11.2% of the portfolio value while AV's average loss is only 9%. SV also stays "underwater" the longest both on average, 15 months, and during its longest drawdown, 246 months. From figure 4, the deepest losses for AV and the buy and hold occur during and after the Great Depression. However, the deepest and longest sustained losses of value for the SV managed portfolio start in the 1960s and SV does not recover until 1989. The notion that there is a drawdown so severe that it causes the collapse of the fund, or at least a management turnover, is known as the "knockout" drawdown. (Pav, 2016) Given a knockout drawdown value, it is possible to estimate the likelihood of the knockout occurring, the fund or manager not surviving, by fitting a binomial distribution to the drawdown observations using the knockout drawdown level as a cutoff to create binary values indicating a drawdown exceeding that level, 1, or not, 0. (Pav, 2016) Setting the knockout drawdown at 45%, a loss of nearly half the current value, in any given month the SV managed and AV managed portfolios have probabilities of 1.06% and .55% of incurring a knockout drawdown in the next 12 months. The AV managed portfolio is far less risky in these terms as the SV managed portfolio would be nearly twice as likely to fail and 91.7% more expensive, in theory, to insure using the max drawdown insurance of Carr, Zhang, and Hadjiliadis (2011).[10]

[**Place Figure 4 about here**]

Table IX panel (b) shows the drawdown statistics for each portfolio across countries. Unlike for the prior return measures, it is not always the case that SV management is an improvement over the buy and hold strategy. In Japan, India, and the UK, SV managed drawdowns are deeper, longer, and take longer to recover from on average. In every country but Australia, the AV managed portfolio has a shallower average drawdown. The AV managed portfolio has shorter average drawdowns in every country tested. In every country but China, the AV managed portfolio has a shorter average recovery time. Indeed, in every country but Japan, the AV management strategy is able to recover from a drawdown in less than 10 months on average. SV management of the world index makes the average drawdown statistics worse across the board allowing AV management to draw some sharp distinctions. The AV management strategy has an average drawdown depth nearly 30% shallower than the SV strategy, a 6.982% versus 9.776% loss and the average drawdown

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length for the AV management strategy is more than 20% shorter than for SV, 9.9 versus 12.5 months.

D. Investor Utility

One part still missing from the analysis of the difference in performance between AV and SV is a measure of the impact on different investors. Investors with different risk aversion will experience different utility effects to the constrained returns of AV and SV[11]. I consider a mean-variance investor as in Kandel and Stambaugh (1996), Campbell and Thompson (2008), Ferreira and Santa-Clara (2011), Rapach et al. (2016), and others. To measure changes to investor utility from switching between the SV and AV portfolios and due to leverage constraints, I use the difference in certainty equivalent return, CER. The CER change will measure the change in risk-less return an investor demands, given their risk aversion, to use a given portfolio versus another given investment conditions. This can equivalently be though of as a measure of the management fee an investor, given a specific risk aversion, would be willing to pay to have access to an investment fund. CER change is calculated as the difference in risk adjusted return using:

$$\Delta \text{ CER} = \left(\hat{\mu}_{r_x} - \frac{1}{2}\gamma\hat{\sigma}^2_{r_x} \right) - \left(\hat{\mu}_{r_y} - \frac{1}{2}\gamma\hat{\sigma}^2_{r_y} \right) \tag{14}$$

where $\hat{\mu}_{r_x}$, $\hat{\mu}_{r_y}$, $\hat{\sigma}^2_{r_x}$, and $\hat{\sigma}^2_{r_y}$ are the means and variances of the returns to the x and y portfolios and γ is the investor risk aversion coefficient. I multiply the gains by 12 to annualize them. All investor risk aversion coefficients from 1 to 5 are tested for investors subject to investment constraints from a limit of 1 to 3 on the market portfolio, no leverage to 200% leverage.

[**Place Figure 5 about here**]

As shown in figure 5, CER losses due to leverage constraints accumulate for the SV managed portfolio sooner than for the AV managed portfolio. This is due to the extreme leverage positions needed for the SV managed portfolio. Across risk aversion coefficients, leverage constraints bite sooner and cut deeper into the SV managed portfolio until both are driven together when no leverage is allowed. The CER of the AV starts higher at 7.95%, 8.98%, and 9.33% vs 6.87%, 7.90%, and 8.25% for investors with risk aversion coefficients of 1, 3, and 5. When those investors are subject to 200% leverage constraint the CERs are 7.96%, 8.99%, 9.33%, versus 6.22%, 7.05%, and 7.33%. The AV manged portfolio provides the same investor utility while the benefit from the SV managed portfolio decreases by 9.46%, 10.76%, and 11.15%. To incorporate the starting difference in utility I look at the gains to moving from the SV to AV managed portfolio.

[**Place Figure 6 about here**]

As shown in figure 6, CER gains for the market variance targeting AV managed portfolio are increasing in both risk aversion and leverage use for constrained risk averse mean-variance investors.

An investor with a risk aversion coefficient of 2 would capture an annualized CER gain of 1.49% using 50% leverage and 1.91% implementing the AV strategy through the 200% leveraged ETFs. The most risk averse investors subject to a 20% leverage limit see a CER gain of 1.35% while the most risk tolerant see only 1.08%. The most risk averse investor, using the highest feasible leverage, realizes a CER gain of over 2% which translates to a utility increase of 27.4%. This increase is in the neighborhood of those typically seen from return timing strategies. (Campbell et al., 1997; Moreira and Muir, 2017) Risk averse, mean-variance investors see substantial utility gains switching from the SV to AV managed portfolio and these gains increase with leverage usage.

Again, there may be some concern about using mean-variance investors. However, it seems reasonable that investors still care about the variance of their portfolio return as the AV management strategy relies on the superiority of the AV signal not on investors taking AV as the measure of risk. Additionally, I test AV management against SV for a larger class of investors by moving away from the assumption of mean-variance utility using stochastic dominance. As Hadar and Russell (1969); Hanoch and Levy (1969); Rothschild and Stiglitz (1970); Levy and Levy (2002) detail, the use of stochastic dominance tests allow us to make conclusions about the preference for AV or SV management for investors whose expected utility functions differ from simple mean-variance optimization. In results reported in the appendix, I use the methods detailed in Vinod (2004, 2008) to test the stochastic dominance of AV over SV to the forth order. Second order stochastic dominance, AV over SV, means that any risk averse investor would prefer AV. (McFadden, 1989; Valle, Roman, and Mitra, 2017) Third order stochastic dominance means that any expected utility investor regardless of the form of the utility function would prefer AV to SV. (Whitmore, 1970; Chan, Clark, and Wong, 2016) At forth order stochastic dominance, in addition to the concept of risk aversion both prudence and temperance are incorporated and all all expected-utility theory (EUT) investors prefer AV management to SV. (Kimball, 1993; Eeckhoudt, Gollier, and Schneider, 1995) Moreover, investors with non-EUT, e.g. prospect theory, preferences still prefer AV management to SV. (Kahneman and Tversky, 1979) As, Prelec (1998) shows a probability weighting function can be used to incorporate the deviations in preference of non-EUT investors, for example loss aversion, into stochastic dominance analysis. I use the extensions in Vinod (2016) to show that the preference for AV is robust to deviations from expected utility theory. AV is the preferred management strategy for investors who can about utility whether they adhere to strict expected utility or behaviorally stray.

III. Systemic Risk

The effectiveness of either SV or AV as an investment management signal will be driven by their relationship with future risk and return. It is the trade-off which is key to the leverage management strategy. Assuming that investors hold a portfolio with whose risk-return ratio they are indifferent, when risk increases but expected returns do not, the risk-return ratio become more unattractive and any risk-averse investor would like to decrease there position. Conversely, when the risk goes

down without a decrease in return there is an opportunity to leverage into the position and return to the previous level of risk but now with a magnified return.

Pollet and Wilson (2010) argue that average correlation is a better measure of systematic market risk. Increases in average correlation are related to a higher covariance between labor income growth and the stock market, and the stock market and bond returns. Also, they show that in the context of Rolls Critique, average correlation should be a better indicator of risk for the true market portfolio rather than stock market variance. (Roll, 1977) The quarterly regressions in Pollet and Wilson (2010) show AC is related to higher returns in the following quarter.

A. In-Sample

Average variance is an auto-correlated time series; this opens the possibility that predictive regressions using AV have estimation bias as highlighted in Stambaugh (1999). Campbell and Hentschel (1992) show that the Stambaugh bias in predictive regressions involving volatility measures and future returns can be particularly severe because of a volatility feedback effect. This may also be distorting portfolio returns. To eliminate the Stambaugh bias in the estimated coefficients on AV, I follow the methodology in Amihud and Hurvich (2004) and further make the p-values used for coefficient significance robust through wild-bootstrapping as detailed in MacKinnon (2002). Robust coefficients and p-values are presented in all in-sample regression tables.

To get an understanding of the relationship between stock market or average variance and returns, I begin with in-sample regressions. In each of these regressions, all of the information available in the sample is used to estimate the parameters. In general, the regressions take this form:

$$y_{t+1} = \alpha + \beta x_t + \epsilon_t. \tag{15}$$

The contemporaneous regressions decomposing market variance are left unreported. The results show the same relationships found in Pollet and Wilson (2010) table 2. For all in-sample regressions, the series are standardized to a mean of zero and standard deviation of one.

Appendix table ?? contains the results of regressions run on the basis data set which spans 1926 to 2016. The results largely support the quarterly regressions in Pollet and Wilson (2010). AV is a significant predictor of next months SV in all specifications and slightly better in terms of R^2. This months AV even remains significant in the specification including this months SV and the inclusion of SV appears to be of little to no help as the adjusted R^2 increases only slightly. there is a definite advantage to using this months AV in the prediction of next months average variance. The adjusted R^2 of this month's AV is 51.5% versus 36.7%. When both AV and SV are included in the predictive regression, AV retains significance and SV does not. Investors are certainly no worse off using AV in the prediction of next month's SV and are better when predicting next months AV. Hence, investors have a signal at least as good for overall risk and better for, what will be shown to be, uncompensated risk. AC is a significant predictor of higher returns in the next month. So, investors lose out on slightly higher returns by divesting when SV is high because AC is high.

AV management avoids this. AV is not significantly related to returns in any specification. When included with AC, AV is insignificant. While no return specification is promising from purely a return timing strategy, the ability to manage risk without giving up return makes AV a better strategy than SV.

IV.   Systematic Risk

AV management works by limiting the risk investors to which investors are exposed to only compensated risk, as much as possible. AV is non-systematic, uncompensated risk. AC is compensated systematic risk. Equation (6) from Pollet and Wilson (2010) gives the relationship of AC, AV and the risk premium.

$$E_t[r_{s,t}] - r_{f,t} + \frac{\sigma_{s,t}^2}{2} = \frac{\gamma}{\beta_t(1-\theta_t)}\bar{\rho}\bar{\sigma}_t^2 - \frac{\gamma}{\beta_t(1-\theta_t)}\theta_t\bar{\sigma}_t^2$$

As they explain, ceteris paribus a change in AV has both positive and negative effects on returns with similar magnitudes. An increase in AC, all else equal, only increases future returns. Equation (8), in Pollet and Wilson (2010), relates AC and AV to the correlation of stock returns and the unobserved component of aggregate wealth.

$$cov(r_{s,t}, r_{u,t+1}) = \pi_0 + \frac{(1-w_{s,t}\beta(1-\theta))E[\bar{\sigma}_t^2]}{(1-w_{s,t})\beta(1-\theta)}\bar{\rho}_t - \frac{(1-w_{s,t}\beta(1-\theta))E[\bar{\rho}_t]-\theta}{(1-w_{s,t})\beta(1-\theta)}\bar{\sigma}_t^2$$

Again, they explain, the denominators for both coefficients of interest are positive if the β of the stock market on aggregate wealth is positive and the proportion of the market which is observed, w_s is greater than zero but less than one. Hence, the coefficient for AC is positive if 1-$w_s\beta \geq 0$ or equivalently if the covariance between the unobservable return and aggregate wealth is positive. For plausible parameter values 1-$w_s\beta(1-\theta)$ is small and $E[\bar{\rho}_t] - \theta$ is close to zero, as θ represents the portion of the shock to stock returns which is common among returns. Thus, the effect of AV will be negative but small. So, the relationship of AC to future stock returns and the ability of AC to signal increase in the correlation of returns across the economy depends on the portion that the observed proxy returns make of the market and the significance of the market in aggregate wealth.

A.   Regression Sub-sampling

The ability of AC to signal changes in systemic risk depends on both the representativeness of the daily returns used to calculate AC for the daily returns of the unobserved market and the correlation of the unobserved market with aggregate wealth. In short, if the daily returns are not a good proxy for market returns and the market is not a significant component of aggregate wealth it is unlikely AC will serve as a signal of systematic risk or changes in the economy. This is related to difference in results shown by Goyal and Santa-Clara (2003) and Bali, Cakici, and Zhang (2005) when the latter removes a significant number of daily returns and the forecasting ability of

idiosyncratic volatility disappears.

The CRSP daily return data contains only returns for assets traded on the New York Stock Exchange (NYSE) prior to 1962. This difference in coverage makes the pre-1962 data very different from the post-1962 data. The earlier data is much shallower having months with fewer than 400 assets total that meet the data requirements. Given the value of the assets traded outside the NYSE, as much as 13% of traded securities, by market capitalization, are missing from the CRSP data as of the 1950s. (NYSE, 2016; Investopedia, 2003) Twice as many firms covering twice as many industries are available in July 1962 versus June. As shown in Taylor (2014) the NYSE market was not a significant part of marginal wealth in the US following the Great Depression before the late 1950s. And, as documented in Jones (2002) the pre-1962 period is significantly and persistently more illiquid. Thus, AC calculated from daily returns prior to 1962 is not likely to be a good proxy for systemic risk. SV and AV will either be measures of the same risk or SV may outperform AV. This means regressions on the relationship of AC and future returns in the 1926 to 1962 sub-sample can provide us evidence suggesting that AV management works when systemic risk is better proxied.

Table XIV presents the results for the 1926 to 1962 CRSP sample while table XV has the results for the sample after 1962 which encompasses the Pollet and Wilson (2010) sample. Confirming the prior quarterly results, in the post 1962 sample AC is positively and significantly related to next month's return. Both AV and SV are both significantly related to future SV with nearly the same coefficient, .550 and .556, and similar R^2 values, 29.6% and 30.3%. As in the full sample AV is a better predictor than SV for next month's AV. As in the full sample, investors are served as well by AV as SV in timing risk and as AC is related to higher future returns, investor have the opportunity to stay invested in good times using AV management. In contrast, in the prior 1962 period, AC is not significantly related to future returns. The coefficient on AC is actually negative but insignificant. The relationships between AV, SV and future risk remain the same but as suspected AC no longer signals compensated risk. This supports the understanding that AC is a signal of systemic risk when calculated from a proxy that is a large portion of the market and sufficiently related to aggregate wealth.

B. Out-of-Sample

Average variance is a good in-sample signal, however the out-of-sample performance remains in doubt. As Welch and Goyal (2008) definitively show, out-of-sample performance is not guaranteed by in-sample performance and is essential to any investment strategy which hopes to generate positive returns. To determine the out-of-sample relationships between market and average variance, average correlation and returns, I run regressions of the standard form

$$y_{t+1} = \alpha_t + \beta_t x_t + \epsilon_t \tag{16}$$

where α_t and β_t are estimated with from the data available only until time t. That is, I estimate α_t and β_t by regressing $\{y_{s+1}\}_{s=1}^{t-1}$ on a constant and $\{x_s\}_{s=1}^{t-1}$. In all the reported results, I follow an expanding window approach so that for the next period $t+2$, y_{t+2} is estimated as $\alpha_{t+1} + \beta_{t+1}x_{t+1}$, where α_{t+1} and β_{t+1} by regressing $\{y_{s+1}\}_{s=1}^{t}$ on a constant and $\{x_s\}_{s=1}^{t}$. I follow this process for all subsequent months. However, as part of a test on the robustness of the out-of-sample results, I demonstrate that the results do not depend on the use of an expanding window. Most critically, equation (16) prevents any look-ahead bias. The out-of-sample prediction tests use the same set of variables as the in sample tests. Each out-of-sample test requires an in-sample training period in which parameters are estimated using all the data up to the time period before the first out-of-sample quarter or month.

For consistency, the first one-fourth of the data is used as the initial parameter estimation period with the remaining three-fourths of observations moved through recursively generating out-of-sample predictions. Three measures of out-of-sample performance are estimated. I use the Diebold and Mariano (1995) statistic and McCracken (2007) MSE-F as measures of the increased accuracy of AV based forecasts compared to forecasts from SV as a benchmark. The DM statistic is defined as:

$$DM = \frac{\bar{d}}{\sqrt{\frac{2\pi f_d(0)}{T}}} \tag{17}$$

where $\bar{d}$ is the mean difference in the loss differential. The loss differential is the function used to measure the difference between the forecasted and actual values. I use the squared forecast error, $(y_t - \hat{y}_t)^2$. So, $\bar{d}$ is the mean value of the difference between the squared error using AV and the squared error using the benchmark forecast from SV.

$$\bar{d} = \frac{1}{T} \sum_{T}^{\tau=1} ((y_t - \hat{y}_{AV,t})^2 - (y_t - \hat{y}_{SV,t})^2) \tag{18}$$

I use the same consistent estimator for the mean loss differential, $f_d(0)$ as in Diebold and Mariano (1995). The statistic is normally distributed under the null hypothesis of no difference in accuracy between the benchmark and proposed model. The standard positive critical values from the normal Z-table serve as cutoffs to establish a significant improvement in forecast accuracy. $MSFE_{SV}$ is mean squared forecast error when a benchmark model is used to generate out-of-sample predictions. Mean squared forecast error is defined as

$$MSFE_x = \frac{1}{T} \sum_{\tau=t}^{T} (y_\tau - \hat{y}_\tau^x)^2 \tag{19}$$

where $\hat{y}_t^x$ is the out-of-sample prediction of y_t generated from the a model using variable x, t is the first out-of-sample prediction time period, and T is the total number of out-of-sample time periods.

The F-statistic in McCracken (2007) is calculated by

$$MSE - F = T\frac{MSFE_x - MSFE_b}{MSFE_b}. \tag{20}$$

The significance of the F-statistic is determined from bootstrapped values provided in McCracken (2007). Each of these two tests depends on the reduction of average squared error by the predictor x relative to a benchmark model. The final measure is a forecast encompassing statistic.

Encompassing tests the more stringent requirement that the benchmark forecasts contain no useful information absent in the forecasts of variable x. Forecast encompassing tests come from the literature on optimal forecast combination. (Chong and Hendry, 1986; Fair and Shiller, 1990) An optimal forecast as a convex combination of two forecasts for time period t +1 defined as

$$\hat{y}_t^* = (1 - \lambda)\hat{y}_t^b + \lambda\hat{y}_t^x \tag{21}$$

where $\hat{y}_t^x$ are predicted values generated from the model using variable x and $\hat{y}_t^b$ are forecasts from the benchmark model. I use the forecast encompassing test of Harvey et al. (1998), ENC-HLN. The encompassing test of Harvey et al. (1998) directly tests the value and significance of the forecast combination λ. The test procedure rests on the calculation of a modification to the Diebold and Mariano (1995) test statistic and the consistent estimation of the long-run covariance between the difference in forecast error between the benchmark model and a model based on a competing variable, x. As such there is no one line equation that sums up the statistic used to judge the significance of λ. However, intuitively λ must be significantly different from zero for AV to have information above and beyond the forecasting information in SV and values close to one indicate that AV has all of the relevant information in SV and is optimal by itself.

[Place Table XI about here]

The results in table XI show that AV is a significantly better out-of-sample predictor of AV and SV. For both the variables, all three measures of out-of-sample performance show significant improvement in both the out-of-sample period starting in 1970 and starting in 1939. The forecast encompassing tests also show that AV contains all the forecasting information in SV and is optimal on its own. This means that investors concerned about the variance in the returns on their investment in the market are better off using this month's level of average asset variance to hedge next month's stock market variance than using this month's stock market variance. AV is also a significantly better predictor of next month's log excess return for the out-of-sample period starting in 1970. The DM statistic, 1.278, is nearly significant at the 10% level and both the MSE-F test and encompassing tests show significant improvement for AV. Again the encompassing test shows that AV is optimal alone and no weight needs to be given to SV in the prediction of log excess returns. Over the period starting July 1970, investors attempting to predict next month's returns would have been better off using this month's average asset variance rather than total market variance

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and investors deleveraging based on high values of AV would have done better avoiding negative returns rather than deleveraging based on high values of SV. Supporting the results seen in the in-sample tests, across the longer out-of-sample period, starting in 1939, AV is not a significant improvement over SV in the prediction of next month's log excess return. It will be important to look at the asset allocation performance of AV versus SV over the entire CRSP sample to see if portfolio management by AV does generate significantly higher returns than management by SV.

B.1. Robustness

Out of sample estimation always raises issues with the choices made in the specification of the model and how to split the data into in and out of sample windows. Bluntly speaking, there are no good answers. The standard practice as in Rapach and Zhou (2013), Rapach et al. (2010), Rapach et al. (2016), and Huang et al. (2015), and many others, is to show performance in a few subsamples split by dates that the authors choose for unknown reasons. One of the concerns with subsample selection is that the window may be "ad-hoc" and the selection may mask significant results that would appear if the subsamples had been constructed differently. A second, more cynical, concern is that the presented subsample may represent significant performance that has been found either by chance or as the result of analyzing many subsample and only presenting the significant results. In any case, evaluation of the differences in performance across subsamples is often left to the imagination of the reader and whatever importance they place on the first half of the sample versus the second, the middle third versus the first and last thirds or however the data has been separated. While the selection of 1962 is not arbitary as the daily return data is of much higher quality after, we have seen already a difference in return prediction performance for AV between the period after 1962 and the whole data set starting in 1926 which raises the question of the robustness of the out-of-sample results.

To address the robustness of the out of sample results and avoid the use of sub-sampling completely, I present out-of-sample statistics robust to both the specification of the prediction model, either expanding or rolling, and the choice of prediction window. Rossi and Inoue (2012) presents out of sample statistics robust to the choice of split between in and out of sample periods. The paper presents the calculation of the Diebold and Mariano (1995) statistic and the Harvey et al. (1998) encompassing test such that the choice of out-of-sample starting period is eliminated as a nuisance parameter and the asymptotic behavior of the statistics can be used to measure their observed significance. Fundamentally, this involves the calculation of each of the statistics for all feasible out-of-sample windows and in the case of rolling regression specification all feasible window sizes. The modifications are different for each of the statistics and the calculation of the robust statistic is different depending both on which statistic and which concern is being addressed. When the concern is that the chosen window could be overly optimal, perhaps the best results of many tests, then it is possible that the null of no improvement is rejected based on the calculated statistics when in general it is true. To eliminate this possibility, Rossi and Inoue (2012) provide the R_T measure which essentially insures that the highest calculated statistics are

so extreme that they could not occur without a underlying significant improvement in forecast accuracy from the benchmark to the proposed model. The A_T measure insures that the average calculated statistics are large enough that an arbitrarily selected out-of-sample starting period would not lead to the failure to reject the null of no accuracy improvement when it was indeed false. This two measures tackle the type I and II error questions. Given the results thus far, we will be looking for significant R_T values to support the significant ability of AV to predict stock market and average variance across the data set, and significant A_T values to tell us that the significant ability of AV to predict log excess returns, seen from 1970 forward, is indicative of a real accuracy improvement while the lack of performance when including predictions from 1939 forward is simply a noisy period of poor performance obfuscating the superiority of the AV based model.

[**Place Table ?? about here**]

Table **??** shows the robust out-of-sample statistics. Panel A has the R_T and A_T statistics for the comparison of all possible expanding window forecasting models using AV with in-sample training windows of at least 15% of the data and out-of-sample forecasting periods of at least 15% of the data against a benchmark model using SV with the same specifications. The proportional data cut offs are necessary to use the critical values provided in the Rossi and Inoue (2012) paper, 15% is the smallest, and mean that the first feasible specification starts forecasting in December 1939 as in the out-of-sample results shown above and forecasts for at least April 2003 to December 2016 are made. Every DM statistic is significant indicating that the AV model is a significant improvement in forecasting accuracy for all variable of interest. Every encompassing test statistic is significant, however these are not λ values directly so while we know that AV contains information over and above SV they do not directly indicate it is optimal alone. Panel B has the results for comparisons of rolling window specification tests of AV and SV models, again using the 15% proportional cut-offs. While all DM statistics are significant, not all encompassing tests are. There are too many rolling window specifications in which SV contains significant forecasting information which is not captured by AV. A forecast combining the values from rolling window models using AV and SV will almost surely be better than using either rolling window forecast alone while using the expanding window AV model forecast alone may very well be better compared to any combination of that prediction with an expanding window SV forecast.

C.   Additional Investment

C.1.   Other Asset Classes

If investment management by the average variance of equity market returns better aligns leverage with changes in systematic risk then it should be able to generate better results for investors across asset classes. This will demonstrate a stark contrast to Moreira and Muir (2017) whose results show that equity variance is not a leverage timing signal for currency returns. To test AV

and SV in other asset classes, I use the world market capitalization weighted AV and SV values constructed before and several asset class indices. These indices include the Bloomberg US Dollar Spot Index, the Deutsche Bank Currency Return Index, The Deutsche Bank Currency Carry Index, the Deutsche Bank Currency Momentum Index, the S&P U.S. Real Estate Investment Trust (REIT), and the Bloomberg Commodity Index. All series start in July of 2005 and end in December 2015.

[Place Table XII about here]

Table XII shows the average annualized average returns and Sharpe ratios for each of the different asset class indices. The results for the currency indices support the conclusion in Moreira and Muir (2017). Management of currency investments using the total volatility of equity returns does not work. In many cases returns and Sharpe ratios are worse, e.g. the Deutsche Bank Currency and Currency Momentum indices. In contrast, management by equity AV is better than both the buy and hold and SV management strategies for currencies and real estate investments. Negative returns on the Bloomberg Dollar Spot, Deutsche Bank Currency, and Deutsche Bank Currency Carry indices become positive when managed by AV. The AV managed real estate investment nearly quintuples returns to the S&P REIT index, 26.7% vs 5.3% and the Sharpe ratio is more than four times as high, .995 vs .198. As shown in table **??** this is not simply due to the correlation of world equity returns and the other asset classes. Many of the currency indices are negatively correlated with the world index.

Unfortunately, equity AV management does not appear to improve the return or Sharpe ratio to investment in commodities. While AV does outperform SV management for the Bloomberg Commodity index, it generates a lower return and worse Sharpe ratio than the buy and hold strategy. This is certainly disappointing, however it may be expected. Gorton and Rouwenhorst (2006); Buyuksahin, Haigh, and Robe (2008); Bhardwaj, Gorton, and Rouwenhorst (2015) document the lack of relationship between equity and commodity returns. This is likely the genesis for the advice suggesting commodities as a portfolio hedge. Additionally, as Erb and Harvey (Forthcoming) show commodity returns are linked to income returns and not prices and link to systematic wealth in a different manner than equities. This, it is possible that the systematic risk to investor wealth to which AV management aligns is different from the wealth related to the performance of commodity investments.

[Place Table ?? about here]

Table **??** shows the drawdown statistics for the equity AV and SV management strategies applied to the alternative asset class indices. In all cases, even for commodities, AV management results in better drawdown statistics. The average depths are shallower, lengths and recoveries shorter. Equity SV management is not as successful. For currencies, SV management is only successful in making the average drawdown depth less for the Bloomberg Dollar Spot Index, a 10.1% versus 13.6% loss, but still the AV managed average loss is less at 9.7%. AV management cuts the average commodity loss by more than 50%, reducing a 26.6% loss to 10.1%; the average length by nearly

26

70%, 39.3 versus 12.2 months; and the average recovery time by more than 50%, 4.3 versus 2.1 months. For all statistics SV management makes the drawdown statistics worse. So, while it may not generate higher returns or better Sharpe ratios, AV management of a commodity investment may allow better sleep at night.

[**Place Table ?? about here**]

Finally, equity AV management of the other asset classes is cheaper than SV management. The equity AV and SV management signals carryover the same relationship from prior tables, the AV managed investments have less turnover and thus incur lower transaction costs than SV. This means that they are able to tolerate higher trading costs and have higher break even costs. Neither strategy improves commodity investment returns so neither strategy can tolerate even zero trading costs when managing the Bloomberg commodity index. However, the SV managed investments in the Deutsche Bank Currency and Currency Momentum indices cannot even tolerate zero trading costs while the break even costs for all of the AV managed investments are reasonable.

Clear, the use of equity AV to manage investments in other asset classes is better than the use of equity SV. Returns, Sharpe ratios, drawdown statistics and trading costs are better in currency and real estate investments. There may be no benefit to AV management in commodities, but SV management appears to be clearly worse than the buy and hold. This is likely due to commodity returns stemming from a risk unrelated to the risk identified by equity returns. The evidence from global equity and across asset class investments suggests that AV management better aligns investment positions to compensated systematic risk.

V.   Conclusion

Weighting investment by the inverse of the average asset variance, AV, rather than SV, increases market investment when total variance is expected to be low and decreases when higher total variance and lower returns are expected. The results are better Sharpe, Sortino and Kappa ratios with better Fama-French five factor and five factor plus momentum alphas. With better access to leverage timing on compensated risk, investors capture more utility with lower costs. AV portfolio investors, as well as fund managers, are more protected agains drawdowns. I analyze the relationship between AV, the average variance of individual asset returns, and future risk and future returns to reveal the mechanism that makes AV a better timing signal. AV is a significant predictor of future portfolio variance, controlling for current variance, both in and out of sample. In contrast, AV is not significantly related to future returns and thus serves as a leverage timing signal. By using the decomposition of market variance, I add a better portfolio leverage management signal to the literature, because AV management better aligns investment to times of compensated risk. These investment benefits manifest because the AV managed portfolio takes advantage of the correlation risk and return dynamics rather than the portfolio variance and return dynamics.

AV management outperforms across the globe. Returns, Sharpe ratios, drawdown statistics and

costs are better in 9 of 10 countries studied. Performance is also better for a globally diversified equity investment. Not only is the average variance of equity returns better for the management of equity investments but it is better across asset classes. AV generates better performance in currency and real estate investment supporting the argument that AV management better aligns investment to times that compensated systematic risk is higher. SV management fails to perform, in many cases its worse than the buy and hold strategy, across asset classes. Hence, the AV managed portfolio adds another dimension to the risk-return literature.

VI. Acknowledgments

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Notes

[1]"When leverage works, it magnifies your gains. Your spouse thinks you're clever, and your neighbors get envious,...but leverage is addictive. Once having profited from its wonders, very few people retreat to more conservative practices. And as we all learned in third grade and some relearned in 2008 any series of positive numbers, however impressive the numbers may be, evaporates when multiplied by a single zero. History tells us that leverage all too often produces zeroes, even when it is employed by very smart people." (McWhinnie, 2014)

[2]See, for example, Barroso and Santa-Clara (2015) and Kim, Tse, and Wald (2016) for discussions of volatility management of the momentum portfolio.

[3]The shortest trading month in the sample is September 2001 with 15 trading days while 17 is a common number in the months with holidays.

[4]The exact correction suggested in Olkin and Pratt (1958) is too computationally taxing for the equipment to which I have access.

[5]The least number of assets which trade every day in a given month is 392 in August of 1932. There are regularly 500 qualifying assets by the end of the 1930s.

[6]http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html#Research

[7]I randomize the block sampling size rather than calculate the optimal size. This results in wider standard errors and a more conservative test. Lahiri (1999).

[8]Federal Reserve Board Regulation T (Reg T) establishes a baseline requirement that investors deposit 50% of an investment position in their margin trading accounts, however a brokerage house may set a higher equity requirement.

[9]The Direxion Daily S&P 500 Bull 3x Shares ETF, symbol SPXL, and ProShares Ultra Pro S&P 500 ETF, symbol UPRO, are two such funds.

[10]Calculation of actual insurance costs require prices on the zero coupon bond, however given this common price the digital call option on the knockout value of the SV managed portfolio is 1.9166 times the price of the AV managed portfolio.

[11]As Moreira and Muir (2017) note, "With no leverage limit, percentage utility gains are the same regardless of risk-aversion because investors can freely adjust theiraverage risk exposure."

Figure 1. Time Series of Market Statistics: Panel (a) the time series of the total variance of market returns, AV is the average variance of the daily returns of individual assets in percentage; SV is the total variance of the daily market return in percentage, and AC is the average pairwise correlation of daily asset returns in the market. Panel (b) shows the ratio of AV to SV.

(a) Monthly Measures of Daily Return Statistics
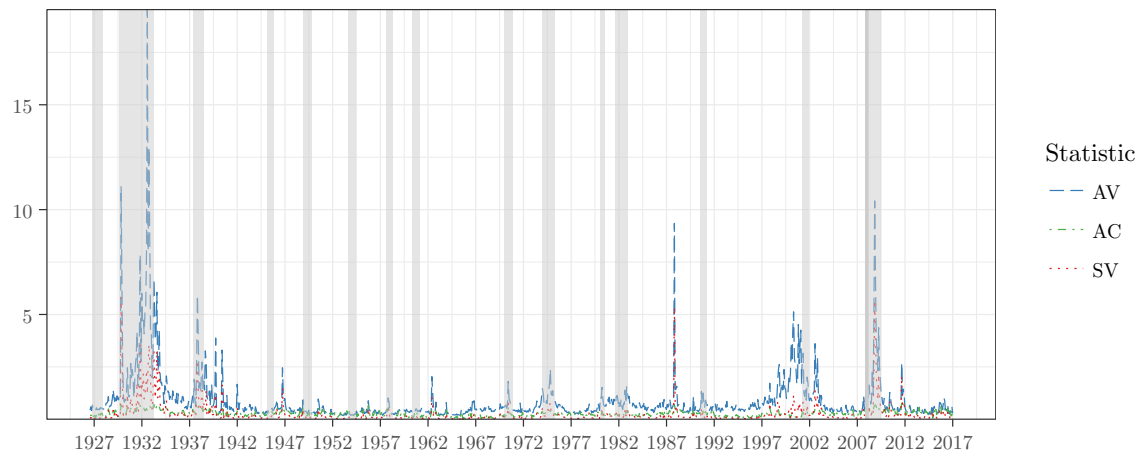
(b) AV Investment Weight Minus SV Investment Weight
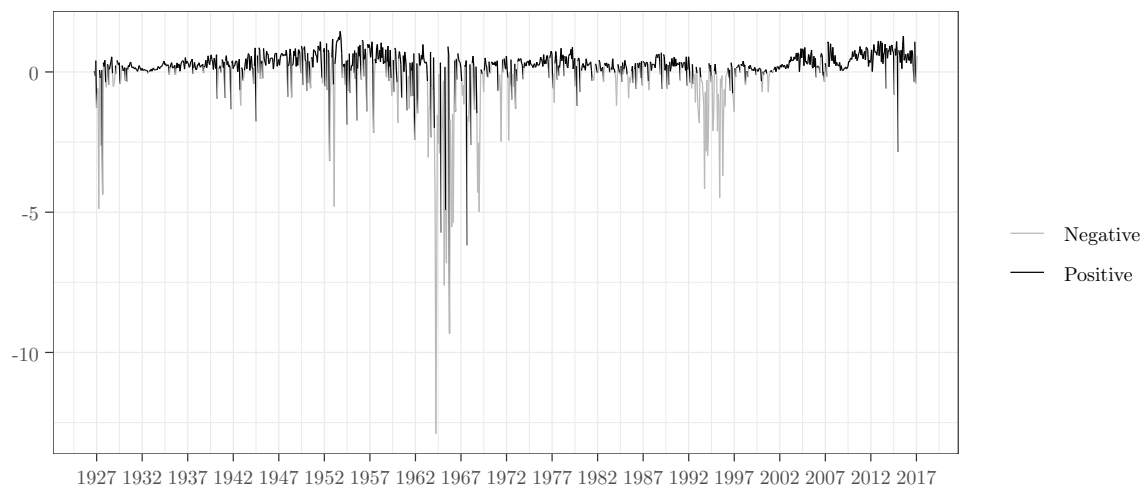
Figure 2. Time Series of Investment Weights: The time series of the investment weight into the market portfolio for SV and AV managed portfolios. c_{029} and c_{035} represents the weights when the SV and AV strategies target 10% and 12% annual volatilities while c_{053} represent targeting the buy and hold annual volatility over the 1926-2016 holding period.

Strategy Investment Weight

Figure 3. Cummulative Log Excess Returns: The time series of cummulative log excess returns for the buy and hold market investment as well as the AV and SV managed portfolios. Panel a limits the coefficient on the market portfolio between 0 and 1.5 for the AV and SV strategies; panel b limit them to weights from 0 to 3 and they are unconstrained in panel c.
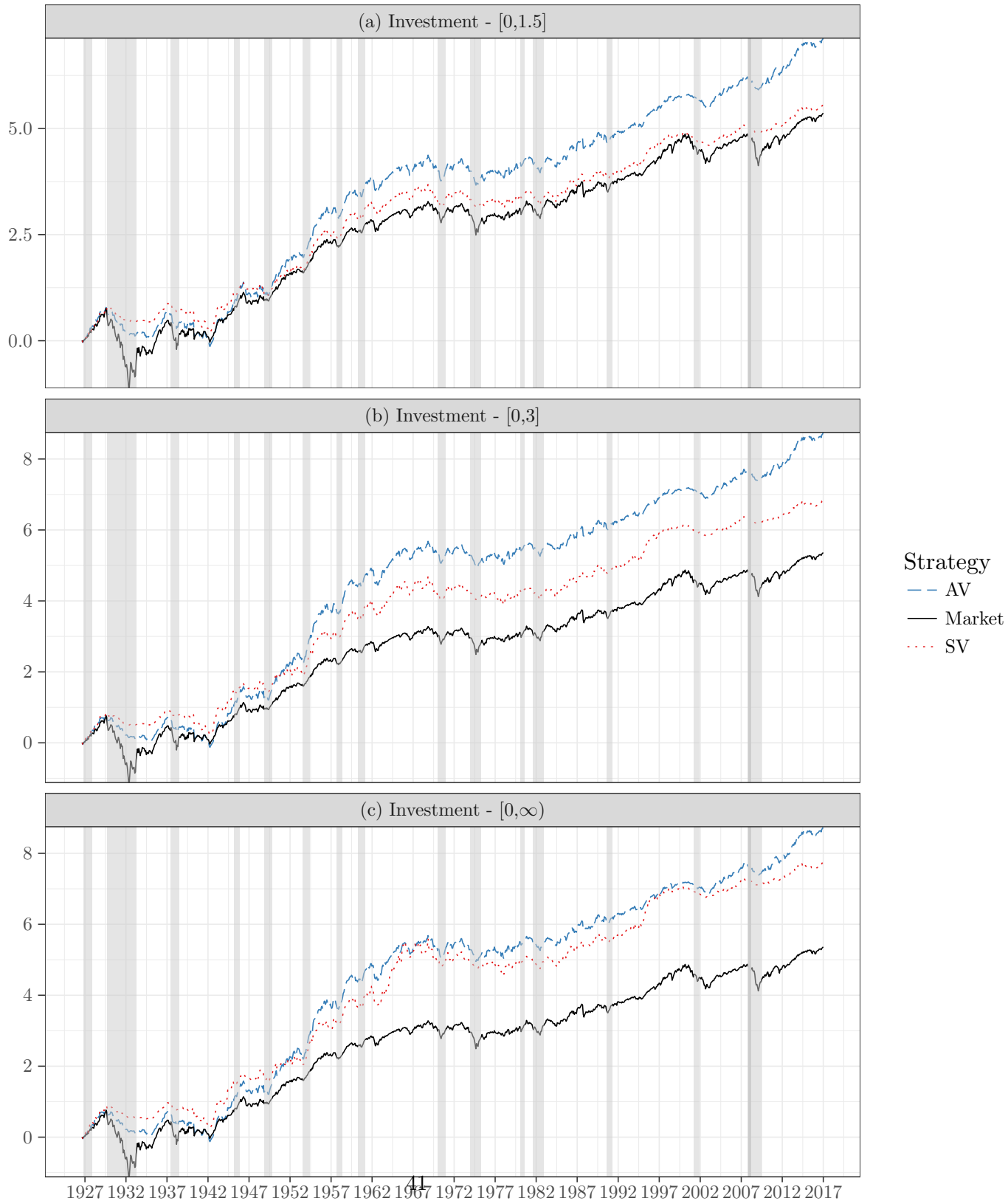
Figure 4. Portfolio Drawdowns: The time series of cummulative drawdowns for the buy and hold market investment as well as the AV and SV managed portfolios. SV and AV managed portfolios are targeting the buy and hold market volatility and have no investment constraints. See section II for details.

Cummulative Drawdown
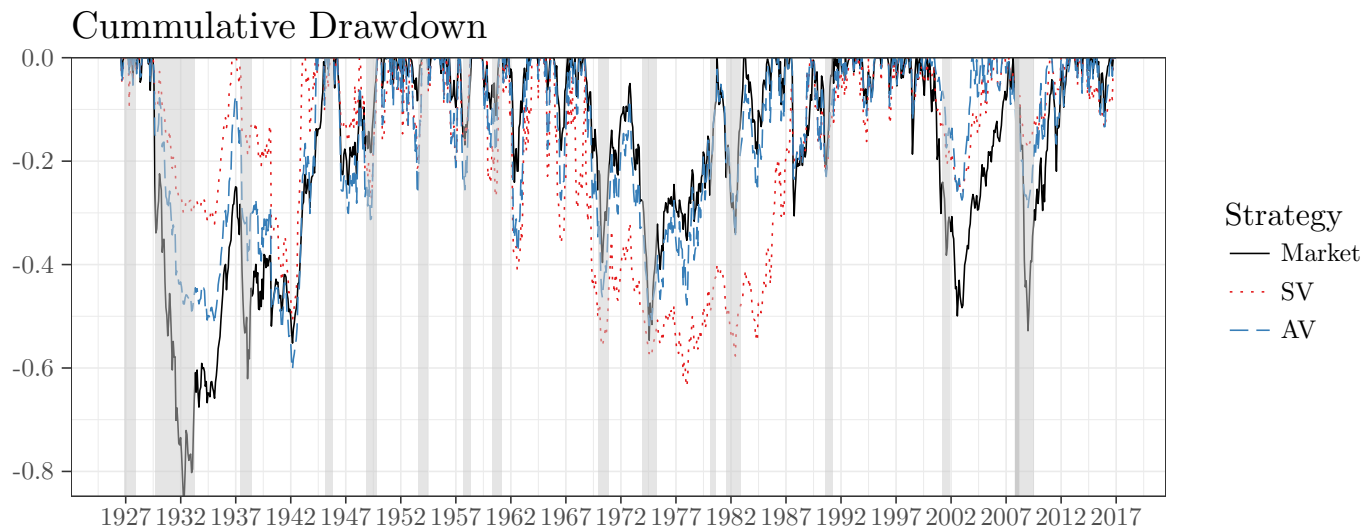
Figure 5. CER Losses: This figure displays the loss of certainty equivalent return as leverage constraints are applied to the AV and SV managed portfolios. The losses are expressed in percentage point differences of the constrained returns from the unconstrained returns for investors with γ risk aversion coefficients of 1, 2.5, and 5.
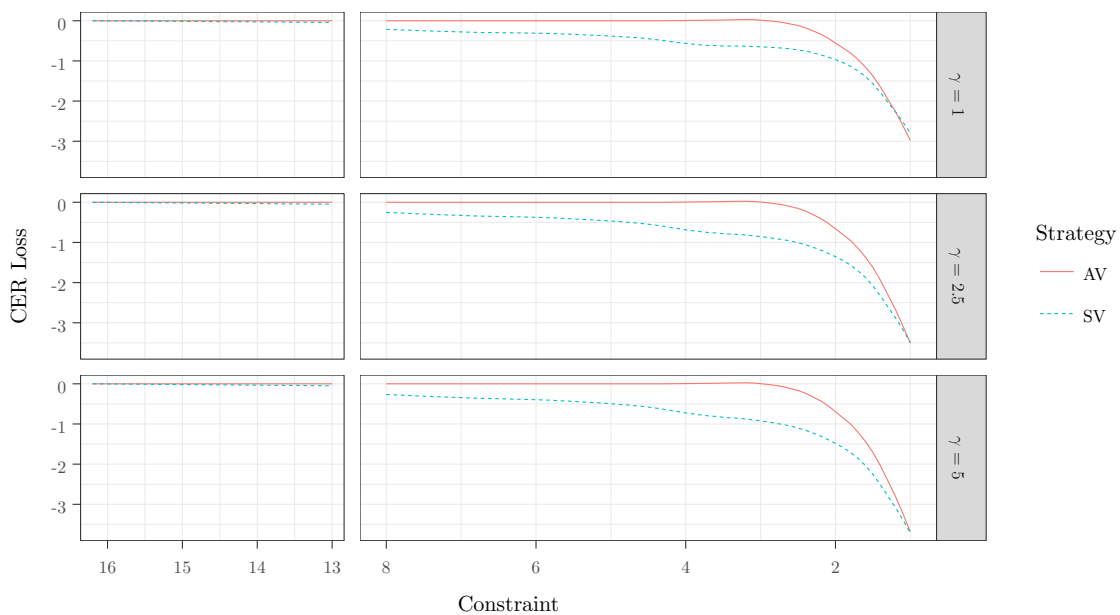
Figure 6. CER Gains: Certainity Equivalent Return gains for mean variance investors with risk aversion coefficients ranging from 1 to 5 and subject to investment constraints ranging from 1 to 3. See section II for details.
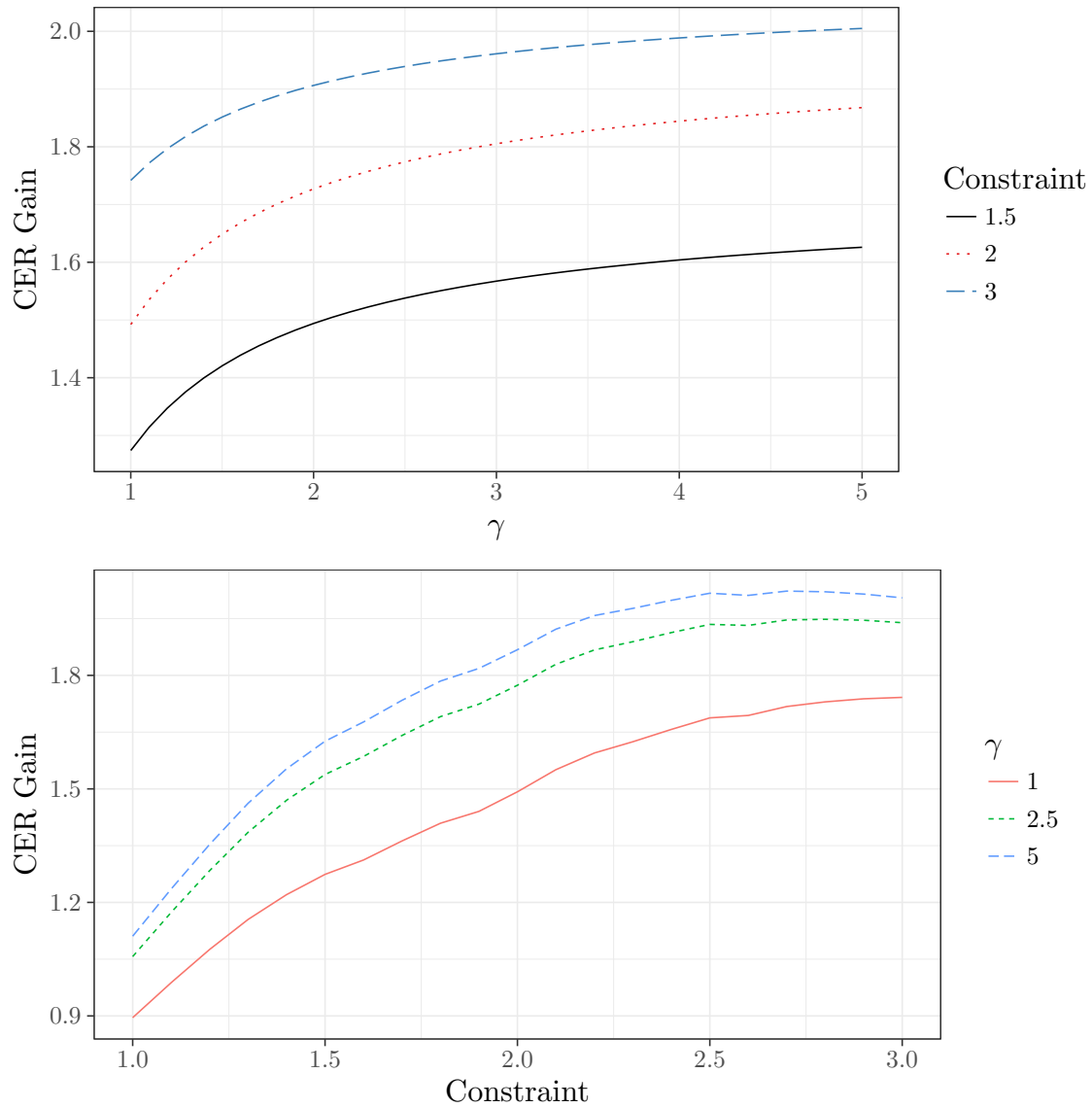
Table I:Summary statistics

Panels (a) - (c) display summary statistics for the market variance and correlation statistics, and returns. RET is the log excess return of the CRSP market portfolio. AC is the average pairwise correlation of the daily returns of the 500 largest firms in the CRSP data set over the month or quarter. AV is the average of the individual variances of daily returns for the 500 largest firms in the CRSP data set. SV is the variance of daily CRSP market returns. See section I for details on construction. Panel (d) displays summary statistics for international data used. The start year and month, number of months, name of the market proxy index, and the average number of assets meeting the trading and liquidity requirements for each country where the performance of AV and SV managed equity portfolios.

(a) Pollet and Wilson Sample 1963Q1:2006Q4

Statistic	N	Mean	St. Dev.	Min	Max	Autocorrelation
RET	176	1.169	8.372	−30.072	19.956	0.000
AC	176	0.231	0.091	0.034	0.648	0.556
AV	176	2.221	1.827	0.634	12.044	0.695
SV	176	0.484	0.615	0.029	6.397	0.310

(b) Sample 1962M6:2016M12

Statistic	N	Mean	St. Dev.	Min	Max	Autocorrelation
RET	655	0.409	4.453	−25.985	14.515	0.081
AC	655	0.261	0.129	0.019	0.762	0.620
AV	655	0.770	0.849	0.198	10.416	0.667
SV	655	0.200	0.406	0.006	5.664	0.551

(c) Full Sample 1926M7:2016M12

Statistic	N	Mean	St. Dev.	Min	Max	Autocorrelation
RET	1,085	0.495	5.371	−34.523	33.188	0.106
AC	1,085	0.276	0.134	0.019	0.762	0.610
AV	1,085	0.881	1.281	0.154	19.540	0.718
SV	1,085	0.248	0.502	0.006	5.808	0.612

(d) Country Indices - Summary

Country	Start	Months	Index	Avg_Assests
AUS	2000 - 5	212	ASX	200
BRA	1995 - 2	275	iShares MSCI Brazil ETF	60
CHN	2005 - 5	152	CSI 300	300
DEU	1993 - 11	290	HDAX	110
FRA	1993 - 9	292	SBF 120	120
IND	2000 - 5	212	Nifty 50	50
ITA	2003 - 8	173	FTSE MIB	40
JPN	1993 - 6	295	Nikkei	255
UK	1993 - 6	295	FTSE	100
USA	1926 - 7	1085	CRSP	500
World	1995 - 3	274	MSCI ACWI	1735

Table II:Correlations

Panel (a) displays Pearson correlation statistics for the market variance and correlation statistics, and returns. RET is the log excess return of the CRSP market portfolio. AC is the average pairwise correlation of the daily returns of the 500 largest firms in the CRSP data set over the month or quarter. AV is the average of the individual variances of daily returns for the 500 largest firms in the CRSP data set. SV is the variance of daily CRSP market returns. See section I for details on construction. Panel (b) shows the Pearson correlation statistics for the excess log returns for each country. Panel (c) shows the correlation statistics for the excess returns of the other asset classes with the US and World excess log returns. See section IV.B for details.

(a) US 1926M7:2016M12

	RET	AC	AV	SV	RET_{t+1}	AC_{t+1}	AV_{t+1}	SV_{t+1}
RET	1	0	0	0	0	0	0	0
AC	-0.295	1	0	0	0	0	0	0
AV	-0.136	0.467	1	0	0	0	0	0
SV	-0.279	0.619	0.857	1	0	0	0	0
RET_{t+1}	0.106	0.011	0	-0.057	1	0	0	0
AC_{t+1}	-0.229	0.610	0.383	0.453	-0.295	1	0	0
AV_{t+1}	-0.191	0.358	0.718	0.607	-0.136	0.467	1	0
SV_{t+1}	-0.259	0.416	0.625	0.612	-0.279	0.619	0.857	1

(b) Global Equities

	AUS	BRA	CHN	DEU	FRA	IND	ITA	JPN	UK	USA
AUS	1									
BRA	0.590	1								
CHN	0.381	0.390	1							
DEU	0.663	0.574	0.336	1						
FRA	0.714	0.539	0.292	0.899	1					
IND	0.575	0.576	0.363	0.538	0.538	1				
ITA	0.665	0.448	0.288	0.823	0.885	0.547	1			
JPN	0.616	0.427	0.339	0.543	0.547	0.555	0.608	1		
UK	0.713	0.614	0.279	0.769	0.812	0.552	0.735	0.489	1	
USA	0.744	0.604	0.365	0.771	0.774	0.573	0.719	0.568	0.802	1
World	0.781	0.686	0.381	0.801	0.831	0.635	0.786	0.657	0.862	0.956

(c) Global and Other Asset Classes

	USA	World	$Dollar_{BB}$	$Curr_{DB}$	$Carry_{DB}$	Mom_{DB}	$REIT_{SP}$	$Comm_{BB}$
USA	1							
World	0.956	1						
$Dollar_{BB}$	-0.557	-0.669	1					
$Curr_{DB}$	0.221	0.205	0.047	1				
$Carry_{DB}$	-0.044	-0.107	0.033	0.216	1			
Mom_{DB}	-0.188	-0.193	0.234	0.557	0.289	1		
$REIT_{SP}$	0.763	0.736	-0.448	0.032	0.004	-0.173	1	
$Comm_{BB}$	0.518	0.611	-0.712	0.017	-0.116	-0.177	0.345	1

Table III:Investment Weights

This table displays summary statistics for the time series of investment weights used by both the AV and SV managed portfolio strategies with differenct volatility targets. c_{BH} represents targeting the annual volatility of the buy and hold market portfolio over the whole data set, 1926 to 2016. c_{10} and c_{12} target, approximately, 10% and 12% annual return volatility for the AV and SV managed portfolios.

Portfolio	Target	Mean	St. Dev.	Min	Pctl(25)	Median	Pctl(75)	Max
SV	c_{10}	0.697	0.762	0.009	0.246	0.512	0.874	8.743
AV	c_{10}	0.702	0.383	0.018	0.425	0.667	0.915	2.296
SV	c_{12}	0.841	0.920	0.011	0.297	0.618	1.055	10.552
AV	c_{12}	0.848	0.463	0.022	0.513	0.805	1.104	2.772
SV	c_{BH}	1.290	1.412	0.017	0.455	0.948	1.619	16.193
AV	c_{BH}	1.301	0.710	0.033	0.787	1.235	1.694	4.253

Table IV:Portfolio Performance - Unconstrained

This table displays portfolio performance measures for the AV and SV managed portfolio strategies using c_{BH} to target the annual volatility of the buy and hold market portfolio over the whole data set, 1926 to 2016, and over NBER business cycle expansions, contractions and contractions excluding the Great Depression. RET is the average annualized monthly log excess return. Sharpe and Sortino are the Sharpe and Sortino ratios respectively; $Kappa_3$ and $Kappa_4$ are the lower parital skewness and lower partial kurtosis Kappa measures. See section II for details. No constraints are plased on the level of investment in the market portfolio for the AV and SV managed portfolio; the buy and hold strategy always has an investment weight of one in the market. Stars on the lines for the AV and SV managed portfolios indicate a significant postive performance difference between those two portfolios.

(a) Full Sample

	Return	Sharpe	Sortino	$Kappa_3$	$Kappa_4$	α_{FF3}	$\alpha_{FF3+Mom}$
BH	5.934	0.319	0.129	0.082	0.061		
SV	8.589	0.462	0.208	0.132	0.097	5.477	3.201
AV	9.676***	0.520*	0.225	0.150*	0.112*	5.594***	3.164

(b) NBER Expansions

	Return	Sharpe	Sortino	$Kappa_3$	$Kappa_4$	α_{FF3}	$\alpha_{FF3+Mom}$
BH	9.598	0.626	0.278	0.179	0.130		
SV	10.05	0.521	0.236	0.149	0.110	2.342	1.192
AV	11.882***	0.640***	0.283**	0.184**	0.136**	2.410**	1.318***

(c) NBER Contractions

	Return	Sharpe	Sortino	$Kappa_3$	$Kappa_4$	α_{FF3}	$\alpha_{FF3+Mom}$
BH	-12.246	-0.416	-0.150	-0.108	-0.087		
SV	1.344***	0.092*	0.039***	0.028***	0.023***	5.158***	5.595*
AV	-1.272	-0.069	-0.027	-0.019	-0.016	4.582	5.462

(d) NBER Contractions x1929:09-1933:03

	Return	Sharpe	Sortino	$Kappa_3$	$Kappa_4$	α_{FF3}	$\alpha_{FF3+Mom}$
BH	5.205	0.233	0.091	0.062	0.048		
SV	12.542	0.799	0.414	0.287	0.230	10.518	9.683
AV	13.951***	0.752	0.349	0.244	0.194	10.603	10.063***

Table V:International Portfolio Performance - this table shows annualized average log excess returns and Sharpe ratios for the AV and SV managed portfolios compared to the market buy and hold for various international markets. See table **??** for details.

	AV		SV		BH	
Country	RET	Sharpe	RET	Sharpe	RET	Sharpe
AUS	12.477***	0.981	11.993	0.943	7.805	0.614
BRA	11.000***	0.291	9.037	0.240	6.163	0.164
CHN	27.381	0.868	24.926	0.790	12.286	0.390
DEU	11.064***	0.537*	7.633	0.371	5.399	0.262
FRA	7.243***	0.404	6.128	0.341	4.904	0.273
IND	14.893***	0.633	12.256	0.521	11.460	0.487
ITA	3.838	0.194	3.912	0.198	1.451	0.073
JPN	1.375***	0.068	0.129	0.006	-0.775	-0.038
UK	6.591***	0.485	5.984	0.441	5.111	0.376
World	8.603***	0.551	8.306	0.536	4.484	0.290

Table VI:Portfolio Performance - Lending Conditions

This table displays portfolio performance measures for the AV and SV managed portfolio strategies using c_{BH} to target the annual buy hold market portfolio variance over sub-samples of the 1926 to 2016 data set. Panel (a) shows results for the months with one month T-bill rates in the top 25%; panel (b) shows results for the months in the bottom 25%.

(a) RF - Top 25%

	Return	Sharpe	Sortino	Kappa$_3$	Kappa$_4$
BH	1.134	0.065	0.025	0.017	0.013
SV	5.567***	0.318	0.145**	0.101**	0.081**
AV	4.187	0.226	0.092	0.063	0.049

(b) RF - Top 25%

	Return	Sharpe	Sortino	Kappa$_3$	Kappa$_4$
BH	11.333	0.529	0.237	0.153	0.115
SV	7.734	0.529	0.234	0.131	0.089
AV	10.703***	0.578	0.239	0.148*	0.105**

Table VII:Portfolio Performance - Constrained

This table displays portfolio performance measures for the AV and SV managed portfolio strategies using c_{10}, c_{12}, and c_{BH} to target the annual volatilities of 10%, 12% and equal to the buy hold market portfolio over the whole data set, 1926 to 2016. Performance ratios are calculated for investment constraints of a maximum of 1.5 and 3, 50% and 200% leverage. RET is the average annualized monthly log excess return. Sharpe and Sortino are the Sharpe and Sortino ratios respectively; Kappa$_3$ and Kappa$_4$ are the lower parital skewness and lower partial kurtosis Kappa measures. See section II for details. No constraints are plased on the level of investment in the market portfolio for the AV and SV managed portfolio; the buy and hold strategy always has an investment weight of one in the market. Stars on the lines for the AV and SV managed portfolios indicate a significant postive performance difference between those two portfolios.

(a) Constraint - 1.5

	Portfolio	Return	Sharpe	Sortino	Kappa$_3$	Kappa$_4$
c_{10}	SV	4.065	0.461	0.201	0.130	0.097
c_{10}	AV	5.196***	0.522**	0.225**	0.150**	0.966
c_{12}	SV	4.735	0.470	0.205	0.133	0.098
c_{12}	AV	6.081***	0.516*	0.221*	0.147*	0.945
c_{BH}	SV	6.171	0.467	0.200	0.128	0.091
c_{BH}	AV	7.885***	0.486	0.204	0.133	0.097

(b) Constraint - 3

	Portfolio	Return	Sharpe	Sortino	Kappa$_3$	Kappa$_4$
c_{10}	SV	4.396	0.454	0.200	0.127	0.094
c_{10}	AV	5.225***	0.520*	0.225*	0.150**	0.112**
c_{12}	SV	5.219	0.452	0.198	0.127	0.094
c_{12}	AV	6.306***	0.520**	0.225**	0.150**	0.112**
c_{BH}	SV	7.606	0.456	0.199	0.129	0.096
c_{BH}	AV	9.677***	0.522**	0.226**	0.150**	0.112**

Notes: ***,**, and * Significant at the 1, 5, and 10 percent levels.

Table VIII:Costs

Panel (a) displays the average change in absolute market investment weight for the buy and hold volatility targeting, unconstrained, SV and AV managed portfolio strategies and the costs associated with trading and borrowing. Fama-French three factor and three factor with Momentum alphas are calculated using the factor portfolio return data from Ken French's website. Strategy break-even points are calculated in basis points as the cost to investment weight turnover which drives the alphas to zero or average annualized return equal to the buy and hold. Borrowing costs are calculated in basis points as the average monthly cost incurred borrowing to take a position in the market greater than one for the SV and AV manged portfolios at the Bloomberg broker call money rate, 1984-2014. See section II.B for details. Panel (b) shows the break even transaction costs for the AV and SV managed portfolios. These are the trading costs which reduced the average return to the AV and SV managed portfolios to the buy and hold market return in each country.

(a) US Equity 1926M8:2016M12

Strategy	$|\Delta\omega|$	RET	Break Even	FF-3 α	FF-3 Break Even	FF-3 + Mom α	FF-3 + Mom Break Even	Borrowing
SV	0.752	8.589	29.422	5.477	60.694	3.201	35.472	15.107
AV	0.317	9.676	254.364	5.594	176.467	3.164	83.176	11.411

(b) Global Equities

Country	AV RET	AV $|\Delta\omega|$	AV Break Even	SV RET	SV $|\Delta\omega|$	SV Break Even	RET_{BH}
AUS	12.477	0.486	80.139	11.993	0.466	74.914	7.805
BRA	11.000	0.253	159.118	9.037	0.623	38.462	6.163
CHN	27.381	0.305	412.715	24.926	0.538	195.972	12.286
DEU	11.064	0.499	94.545	7.633	0.581	32.052	5.399
FRA	7.243	0.468	41.656	6.128	0.536	19.041	4.904
IND	14.893	0.710	40.316	12.256	0.507	13.097	11.460
ITA	3.838	0.448	44.366	3.912	0.603	33.991	1.451
JPN	1.375	0.442	40.518	0.129	0.551	13.675	-0.775
UK	6.591	0.473	26.113	5.984	0.509	14.287	5.111
World	8.603	0.439	78.113	8.306	0.642	49.586	4.484

Table IX:Portfolio Drawdowns

This table displays drawdown statistics for the buy and hold, AV, and SV managed portfolio strategies targeting the buy and hold strategy volatility without investment constraints. N represents the number of drawdowns longer than one month. Max DD and Avg DD are the maximum and mean drawdown in return percentage terms. Max Length and Avg Length are the lengths in months of the maximum drawdown and mean drawdown. Max Recovery and Avg Recovery are the maximum and mean times to recover back to the peak portfolio value at the start of the drawdown. See section II for details.

(a) US Equities 1926M8:2016M12

Strategy	N	Max DD	Avg DD	Max Length	Avg Length	Max Recovery	Avg Recovery
BH	82	-84.803	-8.069	188	11.549	154	7.207
SV	65	-63.637	-11.196	246	14.954	135	7.446
AV	87	-60.264	-9.026	205	10.851	135	5.034

(b) Global Equities

Country	AV Avg DD	AV Avg Length	AV Avg Recovery	SV Avg DD	SV Avg Length	SV Avg Recovery	BH Avg DD	BH Avg Length	BH Avg Recovery
AUS	-6.302	7.174	3.348	-5.322	9.263	5.421	-6.318	8.600	4.550
BRA	-8.059	9.560	4.208	-17.469	15.235	5.500	-15.064	17.067	4.286
CHN	-9.511	10.333	5.917	-10.074	10.583	3.727	-19.374	27.400	2.000
DEU	-11.051	10.625	5.783	-12.587	16.812	9.933	-10.706	17.125	12.333
FRA	-10.263	14.111	5.941	-15.260	18.267	10.214	-11.590	19.071	15.077
IND	-8.170	6.500	2.885	-12.545	12.467	5.733	-10.862	8.318	4.500
ITA	-14.625	19.500	2.143	-18.174	22.571	2.333	-8.919	15.400	1.667
JPN	-30.655	72.750	41.750	-78.514	294.000	175.000	-40.792	148.00	2.000
UK	-6.060	11.609	4.652	-7.872	14.158	8.158	-6.018	10.560	7.240
World	-6.982	9.909	7.333	-9.776	12.500	7.059	-8.209	10.091	6.429

Table X:Systemic Risk

Panel (a) displays in sample regression results for monthly market variance, correlation and return statistics. SV is the annualized monthly variance of daily CRSP market returns. AV and AC are the monthly average variance and average pairwise correlation of daily returns for the top 500 assets in the CRSP market, as in Pollet and Wilson (2010). RET is the log return of the CRSP market portfolio minus the log return on the 1 month treasury bill. The coefficients and p-values bootstrapped for robustness, see section III.A for details. Panel (b) displays the effect of sorting AV managed country returns on Market Capitalization-to-GDP ratio and forming a strategy long on countries with high ratios and shorting low ratio countries. See **??** for details.

(a) In Sample Results

			RET_{t+1} - 1926M7:2016M12		
AV	-0.0002			-0.006	0.192
	p = 0.562			p = 0.423	p = 0.833
AC		0.010**		0.012**	
		p = 0.0.052		p = 0.068	
SV			-0.057		-0.204
			p = 0.612		p = 0.865
R^2	0.00000	0.0001	0.003	0.0001	0.012
Adjusted R^2	-0.001	-0.001	0.002	-0.002	0.010
			RET_{t+1} - 1926M7:1962M6		
AV	0.061			0.121	0.315
	p = 0.609			p = 0.741	p = 0.954
AC		-0.032		-0.099	
		p = 0.520		p = 0.862	
SV			-0.028		-0.264
			p = 0.418		p = 0.948
R^2	0.004	0.001	0.001	0.010	0.026
Adjusted R^2	0.002	-0.002	-0.002	0.005	0.021
			RET_{t+1} - 1962M6:2016M12		
AV	-0.131			-0.168**	0.016
	p = 0.161			p = 0.020	p = 0.739
AC		0.047***		0.106***	
		p = 0.001		p = 0.000	
SV			-0.109		0.254
			p = 0.746		p = 0.107
R^2	0.017	0.002	0.012	0.027	0.017
Adjusted R^2	0.015	0.001	0.010	0.024	0.014

(b) Market-to-GDP AV Strategy

	RET	Sharpe	α_{FF5}	$\alpha_{FF5+Mom}$
SV	7.752	0.675	4.353	4.211
AV	11.156	0.774	7.445***	7.320***
Notes:		***,**, and * Significant at the 1, 5, and 10 percent levels.		

Table XI:Full Out-of-Sample Results

Panels (a) and (b) displays out-of-sample expanding window regression results for monthly market variance, correlation and return statistics. SV is the annualized monthly variance of daily CRSP market returns. AV and AC are the monthly average variance and average pairwise correlation of daily returns for the top 500 assets in the CRSP market, as in Pollet and Wilson (2010). RET is the log return of the CRSP market portfolio minus the log return on the 1 month treasury bill. DM is the Diebold and Mariano (1995) statistic measuring for cast accuracy. MSE-F is the mean squared error improvement F-test of in McCracken (2007) and ENC-HLN is the forecast encompassing test of Harvey et al. (1998). In each panel the benchmark forecasts come from a model which uses SV_t to predict the independent variable. Panel (c) displays out-of-sample regression results of forecasts using AV_{t+1} as a predictor. Rossi and Inoue (2012) provides the methodology to make the calculations of the out-of-sample accuracy improvements of Diebold and Mariano (1995) and McCracken (2007) and the encompassing test of Harvey et al. (1998). In each panel the benchmark forecasts come from a model which uses SV_t to predict the independent variable.

(a) Sample 1970M7:2016M12

	DM	MSE-F	ENC-HLN
AC_{t+1}	1.074	109.736***	1
SV_{t+1}	1.53*	29.252***	1**
AV_{t+1}	2.286**	109.333***	1***
RET_{t+1}	1.278	11.801***	1*

(b) Sample 1939M12:2016M12

	DM	MSE-F	ENC-HLN
AC_{t+1}	1.604*	46.251***	1**
SV_{t+1}	1.041	21.57***	0.956**
AV_{t+1}	3.104***	198.267***	1***
RET_{t+1}	-2.027	-8.702	0

(c) Robust Expanding Window Results

Stat	Variable	DM	ENC-HLN
R_T	AC_{t+1}	28.532***	6.769***
R_T	SV_{t+1}	8.874***	1.838***
R_T	AV_{t+1}	34.347***	18.197***
R_T	RET_{t+1}	29.124***	4.871***
A_T	AC_{t+1}	19.867***	1.828***
A_T	SV_{t+1}	2.647***	0.949***
A_T	AV_{t+1}	21.751***	10.7***
A_T	RET_{t+1}	13.347***	1.68***

Notes: ***,**, and * Significant at the 1, 5, and 10 percent levels.

Table XII:Alternative Asset Class Performance - This table shows the annualized average monthly returns and Sharpe ratios to managing investment in the asset class index using the global market capitalization weighted values of equity AV and SV, see section **??** for details.

Index	AV		SV		BH	
	RET	Sharpe	RET	Sharpe	RET	Sharpe
Bloomberg Dollar	1.324***	0.170	0.606	0.078	−0.296	−0.038
DB Currency	1.195***	0.272*	−0.668	−0.152	−0.244	−0.056
DB Carry	1.440***	0.134	−0.361	−0.033	−2.071	−0.192
DB Mom	1.942***	0.214	0.413	0.045	1.095	0.120
S&P REIT	26.706***	0.995	14.980	0.558	5.302	0.198
Bloomberg Commodity	−5.579***	−0.303	−6.431	−0.349	−5.279	−0.286

VII.   Appendix

A.   In Sample Regressions

Table XIII:Full In Sample Results

The table displays in sample regression results for monthly market variance, correlation and return statistics. SV is the annualized monthly variance of daily CRSP market returns. AV and AC are the monthly average variance and average pairwise correlation of daily returns for the top 500 assets in the CRSP market, as in Pollet and Wilson (2010). RET is the log return of the CRSP market portfolio minus the log return on the 1 month treasury bill. The sample period is from 1926:07 to 2016:12. The coefficients and p-values are robust, see section III.A for details.

(a) Market Return Variance - SV_{t+1}

AV	0.627***			0.553***	0.368***
	p = 0.000			p = 0.000	p = 0.000
AC		0.418***		0.160***	
		p = 0.000		p = 0.000	
SV			0.615***		0.278***
			p = 0.000		p = 0.000
Constant	−0.0003	−0.0001	−0.0002	−0.0003	−0.0003
	p = 0.991	p = 0.998	p = 0.993	p = 0.991	p = 0.991
R^2	0.391	0.173	0.375	0.410	0.413
Adjusted R^2	0.390	0.173	0.374	0.409	0.412

(b) Average Asset Return Variance - AV_{t+1}

AV	0.721***			0.709***	0.759***
	p = 0.000			p = 0.000	p = 0.000
AC		0.359***		0.029***	
		p = 0.000		p = 0.000	
SV			0.609***		−0.019
			p = 0.000		p = 0.991
Constant	−0.0003	−0.0001	−0.0002	−0.0003	−0.0003
	p = 0.989	p = 0.998	p = 0.993	p = 0.989	p = 0.989
R^2	0.515	0.128	0.368	0.516	0.516
Adjusted R^2	0.515	0.127	0.367	0.515	0.515

(c) Average Asset Return Correlation - AC_{t+1}

AV	0.384***			0.125***	−0.029
	p = 0.000			p = 0.00001	p = 0.641
AC		0.613***		0.554***	
		p = 0.000		p = 0.000	
SV			0.454***		0.458***
			p = 0.000		p = 0.000
Constant	−0.0002	−0.0001	−0.0002	−0.0002	−0.0002
	p = 0.996	p = 0.996	p = 0.996	p = 0.995	p = 0.996
R^2	0.147	0.372	0.205	0.385	0.205
Adjusted R^2	0.146	0.372	0.204	0.384	0.204

(d) Log Excess Market Return - RET_{t+1}

AV	-0.0002			−0.006	0.192
	p = 0.562			p = 0.423	p = 0.833
AC		0.010**		0.012**	
		p = 0.0052		p = 0.068	
SV			−0.057		−0.204
			p = 0.612		p = 0.865
R^2	0.00000	0.0001	0.003	0.0001	0.012
Adjusted R^2	−0.001	−0.001	0.002	−0.002	0.010

Table XIV: In Sample Results - Pre 1962

The table displays in sample regression results for monthly market variance, correlation and return statistics. SV is the annualized monthly variance of daily CRSP market returns. AV and AC are the monthly average variance and average pairwise correlation of daily returns for the top 500 assets in the CRSP market, as in Pollet and Wilson (2010). RET is the log return of the CRSP market portfolio minus the log return on the 1 month treasury bill. The sample period is from 1926:07 to 2016:1962:06. The series are standardized to a mean of zero and standard deviation of one. The coefficients and p-values are robust, see section III.A for details.

(a) Market Return Variance - SV_{t+1}

AV	0.672***			0.583***	0.414***
	p = 0.000			p = 0.000	p = 0.000
AC		0.492***		0.160***	
		p = 0.000		p = 0.0004	
SV			0.650***		0.266***
			p = 0.000		p = 0.00004
Constant	−0.000	−0.000	0.000	−0.000	0.000
	p = 1.000	p = 1.000	p = 1.000	p = 1.000	p = 1.000
R^2	0.444	0.236	0.413	0.460	0.466
Adjusted R^2	0.443	0.235	0.412	0.458	0.463

(b) Average Asset Return Variance - AV_{t+1}

AV	0.739***			0.702***	0.665***
	p = 0.000			p = 0.000	p = 0.000
AC		0.473***		0.078***	
		p = 0.000		p = 0.000	
SV			0.655***		0.128***
			p = 0.000		p = 0.000
Constant	−0.000	−0.000	0.000	−0.000	−0.000
	p = 1.000	p = 1.000	p = 1.000	p = 1.000	p = 1.000
R^2	0.535	0.218	0.422	0.539	0.539
Adjusted R^2	0.534	0.216	0.420	0.537	0.537

(c) Average Asset Return Correlation - AC_{t+1}

AV	0.503***			0.250***	0.150**
	p = 0.000			p = 0.000	p = 0.030
AC		0.580***		0.432***	
		p = 0.000		p = 0.000	
SV			0.539***		0.384***
			p = 0.000		p = 0.000
Constant	0.000	0.000	0.000	0.000	0.000
	p = 1.000	p = 1.000	p = 1.000	p = 1.000	p = 1.000
R^2	0.249	0.331	0.286	0.374	0.294
Adjusted R^2	0.248	0.329	0.285	0.371	0.291

(d) Log Excess Market Return - RET_{t+1}

AV	0.061			0.121	0.315
	p = 0.609			p = 0.741	p = 0.954
AC		−0.032		−0.099	
		p = 0.520		p = 0.862	
SV			−0.028		−0.264
			p = 0.418		p = 0.948
R^2	0.004	0.001	0.001	0.010	0.026
Adjusted R^2	0.002	−0.002	−0.002	0.005	0.021

Table XV:In Sample Results - Post 1962

The table displays in sample regression results for monthly market variance, correlation and return statistics. SV is the annualized monthly variance of daily CRSP market returns. AV and AC are the monthly average variance and average pairwise correlation of daily returns for the top 500 assets in the CRSP market, as in Pollet and Wilson (2010). RET is the log return of the CRSP market portfolio minus the log return on the 1 month treasury bill. The sample period is from 1962:06 to 2016:12. The series are standardized to a mean of zero and standard deviation of one. The coefficients and p-values are robust, see section III.A for details.

(a) Market Return Variance - SV_{t+1}

AV	0.550***			0.494***	0.135***
	p = 0.000			p = 0.000	p = 0.001
AC		0.334***		0.162***	
		p = 0.000		p = 0.00001	
SV			0.556***		0.187***
			p = 0.000		p = 0.00002
Constant	−0.0005	−0.0001	−0.0003	−0.0005	−0.0004
	p = 0.989	p = 0.999	p = 0.993	p = 0.989	p = 0.991
R^2	0.297	0.110	0.304	0.320	0.317
Adjusted R^2	0.296	0.109	0.303	0.318	0.315

(b) Average Asset Return Variance - AV_{t+1}

AV	0.672***			0.680***	1.158***
	p = 0.000			p = 0.000	p = 0.000
AC		0.219***		−0.019	
		p = 0.003		p = 0.544	
SV			0.526***		−0.265
			p = 0.000		p = 0.999
Constant	−0.001	−0.00004	−0.0003	−0.001	−0.001
	p = 0.985	p = 1.000	p = 0.994	p = 0.984	p = 0.981
R^2	0.445	0.048	0.273	0.446	0.477
Adjusted R^2	0.445	0.046	0.272	0.444	0.475

(c) Average Asset Return Correlation - AC_{t+1}

AV	0.241***			0.023***	−0.634
	p = 0.000			p = 0.000	p = 0.999
AC		0.626***		0.619***	
		p = 0.000		p = 0.000	
SV			0.362***		0.539***
			p = 0.000		p = 0.008
Constant	−0.0002	−0.0001	−0.0002	−0.0001	−0.00003
	p = 0.996	p = 0.998	p = 0.996	p = 0.997	p = 1.000
R^2	0.057	0.387	0.130	0.387	0.167
Adjusted R^2	0.056	0.386	0.128	0.385	0.164

(d) Log Excess Market Return - RET_{t+1}

AV	−0.131			−0.168**	0.016
	p = 0.161			p = 0.020	p = 0.739
AC		0.047***		0.106***	
		p = 0.001		p = 0.000	
SV			−0.109		0.254
			p = 0.746		p = 0.107
R^2	0.017	0.002	0.012	0.027	0.017
Adjusted R^2	0.015	0.001	0.010	0.024	0.014

B. Investor Utility

Prelec (1998) defines a flexible function with parameter α controlling the "slope" of the "weight" an investor places on an observed return, ordered from smallest to largest. When α equals zero, investors are EUT-compliant and place equal weight on all observed utilities from returns regardless of size when evaluating two possible portfolios. When α is less than one but greater than zero, investors place greater weight on the lower, negative, realized returns than the higher a result consistent with loss aversion from prospect theory. Figure 7 shows the relationship between α, the ordered cumulative return probability and the weight the investor places on the probability of such a return.

Using α, I test the stochastic dominance of AV relative to SV. As Hadar and Russell (1969); Hanoch

Figure 7. Non-EUT Investor Decision Weights: This figure illustrates the incorporation of non-EUT investor preferences in the decision between two investment portfolios used in the stochastic dominance analysis. The cumulative probability, p, of the rank order statistics of the portfolio returns are on the x-axis. The perceived utility weight, W(p,α), or importance placed on the possibility of incurring a return with probability p. The parameter α measures EUT compliance, $\alpha = 1$ when investors care equally about all returns and α ¡ 1 when investors care about extreme loses more than they should".
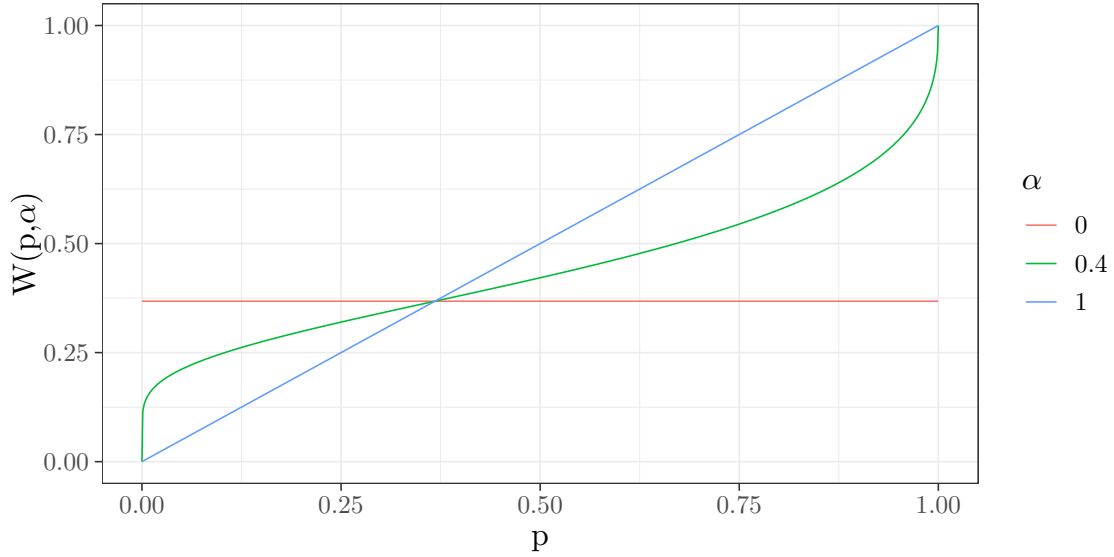
and Levy (1969); Rothschild and Stiglitz (1970); Levy and Levy (2002) detail, the use of stochastic dominance tests allow us to make conclusions about the preference for AV or SV management for investors whose expected utility functions differ from simple mean-variance optimization. In results reported in the appendix, I use the methods detailed in Vinod (2004, 2008) to test the stochastic dominance of AV over SV to the forth order. Second order stochastic dominance, AV over SV, means that any risk averse investor would prefer AV. (McFadden, 1989; Valle et al., 2017) Third order stochastic dominance means that any expected utility investor regardless of the form of the utility function would prefer AV to SV. (Whitmore, 1970; Chan et al., 2016) At forth order

stochastic dominance, in addition to the concept of risk aversion both prudence and temperance are incorporated and all all expected-utility theory (EUT) investors prefer AV management to SV. (Kimball, 1993; Eeckhoudt et al., 1995) As table XVI shows, expected utility investors whose α equals 1 prefer AV management to SV management. The positive numbers in across columns SD2, SD3, and SD4 indicate up to forth order stochastic dominance. Moreover, investors with non-EUT, e.g. prospect theory, preferences still prefer AV management to SV. (Kahneman and Tversky, 1979) For all values of α AV management stochastically dominants to the forth order over the SV portfolio.

Table XVI:Stocastic Dominance- This table presents the results for tests of stochastic dominance of the AV managed portfolio over the SV managed portfolio. Values of α indicate investor compliance, $\alpha = 1$, or deviation from expected utility theory (EUT). See sections II.D and VII for details.

α	SD1	SD2	SD3	SD4
0.010	238.626	55,366.640	9,760,554.000	1,385,264,265.000
0.100	191.504	44,487.770	7,849,769.000	1,114,596,326.000
0.200	141.808	33,021.230	5,836,320.000	829,429,477.000
0.300	97.646	22,844.570	4,050,563.000	576,605,891.000
0.400	61.450	14,523.020	2,592,250.000	370,297,258.000
0.500	34.620	8,381.752	1,518,794.000	218,664,964.000
0.600	17.052	4,396.376	825,963.300	121,119,656.000
0.700	7.118	2,188.643	447,116.200	68,211,642.000
0.800	2.234	1,158.376	276,458.800	44,930,567.000
0.900	-0.196	702.314	207,620.100	36,179,509.000
0.990	-1.799	430.734	170,608.100	31,907,367.000