How to Look Clever and Have Envious Neighbors: Average Volatility Managed Leverage Timing

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ABSTRACT

There's nothing very interesting here, but the format (achieved using the file `jf.sty`) makes it suitable for publication in the *Journal of Finance* even if the content doesn't. Here's a nice, informative, double-spaced abstract.

JEL classification: XXX, YYY.

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Discussions on when to risk investment capital in the pursuit of maximum return took place over telegraph lines nearly a century before Markowitz brought formality to the notion of risk, portfolio construction and optimization in the 1950s.[1] Modern portfolio theory states that portfolios with higher variance need to generate higher mean returns to attract rational investors.(Markowitz, 1952) However, several challenges have appeared to this foundational mean-variance claim. One of the most basic is commonly referred to as the low-risk or low-volatility anomaly. Haugen and Heins (1972) found there was little to no evidence for a "risk-premium" for increased portfolio volatility. While Warren Buffet may not be an enthusiastic supporter, low volatility portfolios with unusually high expected returns present an opportunity for leveraged investing.[2] Moreira and Muir (2017) show across investment strategies and asset classes that merely managing leverage in a portfolio by that portfolio's volatility produces greater expected returns and performance ratios. These results seem to challenge the mean-variance notion of investment risk premium fundamentally. This risk-return tradeoff is central to modern financial theory, so naturally, I address this problem by making it worse. Pollet and Wilson (2010) show that the average correlation of portfolio assets is the component of portfolio volatility most related to systemic changes in the economy and is the risk component compensated with higher returns. By managing the market portfolio using the average variance of the individual asset returns in the prior period rather than the variance of the market portfolio, I generate higher expected returns and significantly better performance ratios. In addition to identifying a better strategy for investors, decoupling the idiosyncratic variance of individual returns from the that of the market portfolio returns sheds light on the risk-return trade-off dynamics of the market and allows leveraging investment into times when higher risk will be compensated and pulling out when it will not. Thankfully, this worse problem exposes evidence consistent with the explanation of low-risk anomalies which claims that investors are constrained from taking the leverage necessary in the low-risk portfolios and inconsistent with the story that investors prefer the high-risk portfolios because they enjoy lottery-like investments. As such both average variance and volatility managed portfolios represent a realization of a practical limitation of one of the assumptions of modern portfolio theory and not a fundamental problem with the concept of the risk-premium.

Since the identification of a low-risk anomaly, or the absence of a risk premium, a large number of researchers have sought to identify a positive relationship between return variance and expected returns. Campbell (1987), French, Schwert, and Stambaugh (1987), Glosten, Jagannathan, and Runkle (1993) and many, many others have found very insufficient evidence identifying a positive relationship between return variance and future returns. On the other hand, Moreira and Muir (2017) demonstrate that volatility managed portfolios which decrease leverage when volatility is high produce large alphas, increase Sharpe ratios, and produce substantial utility gains for mean-variance investors. These results hold across investment styles, e.g., value or momentum, and in different asset classes, e.g., equities and currency portfolios. The results even hold for positions which already seek to exploit the low-risk anomaly like the "betting-against-beta" strategy of Frazzini and Pedersen (2014). Additionally, other researchers have applied similar variance or volatility management to specific assets or trading styles.[3] Practitioners call approaches like this risk-parity, and as of 2016, at least \$150 and as much as \$400 billion sits in these funds.(Steward, 2010; Cao, 2016) The generation of higher expected returns without a commensurate increase in portfolio volatility implies that a representative investor should time asset volatility, demanding less return when its higher meaning the investors risk appetite must be higher in periods like recession and market downturns when its expected to be lower. In short, it seems risk does not equal reward. However, as with pigs, some volatility is "more equal" than other volatility.[4]

Pollet and Wilson (2010) decompose market variance into the average correlation between pairs of assets and the average variance of the individual assets. This decomposition yields a series strongly related to both future volatility and higher returns, the average correlation of market assets series, and a one strongly related to future volatility but unrelated to returns, market asset average variance. Scaling investment in the market portfolio by the inverse of the previous period average variance should then improve return performance over and above managing investment the previous market variance since it avoids, somewhat, decreasing investment when the average correlation is high and its expected compensation. Average variance is also a better candidate for portfolio management because it has a better chance to pick up economic information sooner as individual assets respond to information about their own risk and expected returns with public disclosure while changes in aggregate materialize as data across assets are combined.(Campbell, Lo, MacKinlay, et al., 1997; Campbell, Lettau, Malkiel, and Xu, 2001)

Using daily market returns from the Center for Research in Securities Prices, I follow Pollet and Wilson (2010) generating quarterly time series of stock market variance, SV, average correlation, AC, and average variance, AV. I then extend this by calculating the time series monthly. As expected, AV is strongly related to next period market variance and unrelated, at best insignificantly negatively, related to next period excess log returns. From June 1962 to the end of 2016, encompassing the sample of Pollet and Wilson (2010), asset average variance is a significant in-sample predictor of higher daily market return variance, average asset variance, pairwise correlation, and lower log excess market returns at the monthly frequency. A one standard deviation increase in annualized average variance, from .77 to 1.62, is related to an increase in next months annualized market return variance of .545 of a standard deviation or a .22 increase. This makes next months expected market variance more than double the mean. AV remains a significant predictor or next month's SV even when this month's SV is included. A one standard deviation increase in AV also anticipates a .13 standard deviation, or .58 percentage point, lower log excess return. This the following month's expected return negative. When both AV and SV are used to predict next month's return, AV is significant but SV is not. These support results at the quarterly frequency in Pollet and Wilson (2010). However, over the full, 1926 to 2016, sample average variance is a signficant predictor of higher daily market return variance, average asset variance, pairwise correlation, but not log excess market returns, as shown in table IV. This evidence supports the use of average variance as a leverage management signal. Scaling investment in the market by the inverse of average asset variance in the current month will pull funds out when the following month will have high market variance without sacrificing higher expected returns. It may, in fact, avoid negative returns. These results support the intuition from the work on volatility management in Moreira and Muir (2017) and portfolio average variance and correlation in Pollet and Wilson (2010) but in-sample regression use all available information and do not necessarily identify tradeable strategies.(Welch and Goyal, 2008)

Investors can only make decisions using the limited information available to them at a given time. For example in June of 2007 investors and investment models could only use historical information up to that month; the effects of November 2008 on the variable coefficients do not affect the predictions for July 2007. Moreira and Muir (2017) demonstrate that market volatility is an effective market portfolio management technique across the CRSP data set from 1926 to 2015. To motivate average variance as a better market portfolio leverage signal, I run expanding window out-of-sample regressions using AV on market volatility, average variance, average correlation, and log excess returns. From June 1926 to December 2016 and using the predictions from SV as a benchmark, AV is a significantly better predictor or next months AV, AC and SV. It generates better Diebold and Mariano (1995) test statistics, significantly lower mean squared forecast errors judging by the MSE-F statistic from McCracken (2007) and the emcompassing test of Harvey, Leybourne, and Newbold (1998) show that average variance contains all of the predictive information in market variance. As with the in-sample results average variance serves investors at least as well as market variance and likely better in avoiding risk without giving up return. Out-of-sample testing always raises questions about model specification, recursive expansion versus rolling window parameter estimation, and choices of the training period and prediction window. Using the techniques in Rossi and Inoue (2012), the Diebold and Mariano (1995), McCracken (2007), and Harvey et al. (1998) measures can be calculated robust to concerns on window selection for either an expanding or rolling specification. The Rossi and Inoue (2012) robust statistics show that AV is a significantly better predictor than SV robust to the choice of window or regression specification. Thus, I expect managing leverage in the market portfolio by AV will produce substantially better return performance as compared to management by SV.

As promised by the out-of-sample results, AV is a substantially better than SV as a leverage management signal. Targeting the volatility of the buy and hold market portfolio return, as in Moreira and Muir (2017), an investor without borrowing constraints earns an annualized average monthly return of 9.7% from the average variance managed portfolio. This return is a statistically significant increase of more than 1% over the SV managed portfolio; the difference in annualized average monthly returns grows to more than 2% when practical leverage constraints are applied. With unconstrained borrowing, the AV managed portfolio has significantly better performance ratios like the symmetric Sharpe ratio, .52, and more asymmetric risk-return measures, e.g., Kappa 3 and Kappa 4 at .15 and .11 respectively. The advantage of managing with AV grows with risk aversion. The most risk-averse, $\gamma = 5$, constrained investor sees a certainty equivalent return (CER) gain of more than 2% annualized; this return represents a 26.4% increase in utility nearly as substantial as the utility gains seen in return timing strategies. (Campbell et al., 1997) Targeting the volatility of the buy and hold return requires seeing into the future and knowing the buy and hold return volatility. However, this look-ahead does not affect performance ratios like the Sharpe ratio moreover, the significantly better performance of AV is robust to other choices of target volatility. The asset allocation gains are not all perfect, however, and the lower Rachev ratio performance of AV hints at a possible explanation for the low-risk anomaly seen in volatility and average variance management.

There is no such thing as a free lunch, is not only a wonderfully pervasive adage, particularly loved by economists but a provable restriction on optimization problems. (Wolpert and Macready, 1997) Here, AV provides no free lunch. The improved performance measured by expected log excess returns, Sharpe, Sortino, and Kappa ratios is betrayed by worse performance in Rachev ratio. The Rachev ratio measures the right tail reward value at play relative to the left tail value at risk. In short, it measures the ratio of expected lottery rewards and losses. The volatility managed market portfolio has higher lottery winning potential for each dollar of potential lottery loss compared to the average variance managed portfolio. Both SV and AV generate better Rachev ratios than the buy and hold return. Prior literature on the low-risk strategies proposes two explanations. Either investors are leverage constrained and unable to form the positions which generate the abnormal returns or investors have a preference for the extreme right tail, lottery, returns which are not possible when employing risk-managed strategies. As AV and SV managed portfolios have better Rachev ratios than the buy and hold strategy, it would seem they are better lotteries to play. Asness, Frazzini, Gormsen, and Pedersen (2018) take a related approach to decomposing the betting-against-beta strategy of Frazzini and Pedersen (2014) into betting-against-correlation, BAC, and betting-against-variance factors, BAV, finding the BAC factor has a significant Fama-French five-factor alpha but unrelated to behavior explanations of the low-risk anomaly. (Fama and French, 2016) From the decomposition of daily market returns, I find similar results about the more general low-risk strategy of volatility management. Management by either market variance or average asset variance increases the lottery-like returns of the market portfolio. Both strategies increase the Rachev ratios of the returns. This change is inconsistent with the notion that the low-risk anomaly is the result of a behavioral preference of investors for lottery-like returns. (Barberis and Huang, 2008; Brunnermeier, Gollier, and Parker, 2007) Instead, the generation of higher log excess returns by the AV, SV, and betting-against-beta strategies support the notion that the low-risk anomaly arises from leverage constraints first suggested in Black (1972). Boguth and Simutin (2018) link capital constraints and the betting-against-beta strategy through mutual fund betas, and more generally Malkhozov, Mueller, Vedolin, and Venter (2017) show that international illiquidity predicts betting-against-beta returns. Testing directly for changes in the capital market line, first shown to flatten when leverage is costly by Jylh (2018), I find a significant effect of many proxies for credit constraints on the returns to AV management but no significant effect for proxies of lottery preference. Financial intermediary leverage, bank credit growth, growth in margin investing and lending rates all affect the return on the AV managed portfolio and flatten the capital market line while proxies for high investor lottery preference like market capitalization of gaming industry stocks and extreme values of market daily returns are not. The evidence is consistent with leverage constraints affecting the returns to low-risk strategies, not market-wide lottery preferences.

By using the decomposition of market variance, I identify a better portfolio leverage management signal. Weighting investment leverage by the inverse of the average of individual asset variance, AV, rather than overall return variance, SV, is a new addition to the portfolio management literature letting investors capture better performance as measured by expected annualized returns. Investors also capture better investment ratios except for the Rachev ratio. The Rachev exposes the trade-off which investors must accept when managing risk by manipulating leverage conditional on average variance. The change in ratio contributes evidence against the lottery explanation of the low-risk anomaly in mean-variance analysis and supports the leverage constraints explanation. Furthur evidence shows that the returns and capital market line responds to proxies for tight lending conditions but not high lottery preference. This finding contrasts with conclusions reached in the study of cross-sectional low-risk anomalies which have been explained through the behavioral, lottery, channel.

The formation and analysis of the AV signal are relatively straightforward. It requires a few publically available datasets and a few considerations for the calculations at the monthly frequency. Most of the work is in the calculations required to show significant regression and portfolio performance.

I. Data

To calculate stock market variance, average asset variance, and average asset correlation, I use daily return data from CRSP and calculate the variance of daily returns monthly. To simplify the analysis of individual assets, I require that the asset be traded on each in the month which mitigates any liquidity effects and ensures consistent variance, covariance and correlation calculations. These conditions make the calculation of asset variance:

$$\sigma^2_{m,t} = \frac{1}{T-1}\sum_{\tau=1}^{T}\left(R_{m,\tau} - \frac{\sum_{\tau=1}^{T} R_{m,\tau}}{T}\right)^2. \tag{1}$$

where $R_{m,\tau}$ is the daily return, including dividends, on an asset for day τ in month t. When the asset is the market portfolio, so $R_{m,\tau} = R_{s,\tau}$, the result is the variation of market returns, SV. The standard Pearsons correlation where the correlation of assets m and n for month t is:

$$\rho_{m,n,t} = \frac{\sum_{\tau=1}^{T}\left(R_{m,\tau} - \frac{\sum_{\tau=1}^{T} R_{m,\tau}}{t}\right)\left(R_{n,\tau} - \frac{\sum_{\tau=1}^{T} R_{n,\tau}}{t}\right)}{\sqrt{\left(R_{m,\tau} - \frac{\sum_{\tau=1}^{T} R_{m,\tau}}{t}\right)^2 \sum_{\tau=1}^{T}\left(R_{n,\tau} - \frac{\sum_{\tau=1}^{T} R_{n,\tau}}{t}\right)^2}}. \tag{2}$$

Unfortunately, for samples as small as the monthly series of daily returns Pearsons correlation is not an unbiased estimator of the true correlation, even if the returns are normal. Hotelling (1953)

The average month in my sample has 22 trading days however the number commonly drops into the teens.[5] For samples of these sizes, the bias causes an underestimation of the correlation which is worse the lower the true correlation. I employ an approximate correction from Olkin and Pratt (1958) such that the monthly correlation between two assets m and n is:

$$\rho_{m,n,t} = \widehat{\rho_{m,n,t}} \left(1 + \frac{1 + \widehat{\rho^{m,n}}^2}{2(t-3)} \right) \tag{3}$$

where $\widehat{\rho_{m,n,t}}$ is the Pearson correlation between a and b.[6] Average variance and average correlation are value-weighted so each month I calculate market capitalization for all of the assets available in CRSP. The capitalization used in month t for asset m is the product of the end of month price (PRC) and common shares outstanding (SHROUT) values for asset m in month t-1.

$$MCAP_{m,t} = PRC_{m,t-1} \times SHROUT_{m,t-1} \tag{4}$$

To make the analysis more computationally trackable I use only, at most, the top 500 assets in CRSP by market capitalization for a given month.[7] Given this restriction, an assets market capitalization weight is defined by:

$$w_{m,t} = \frac{MCAP_{m,t}}{\sum_{j=1}^{J} MCAP_{n,t}} \tag{5}$$

with j $\leq$ 500. Thus, the other series of interest, market variance, SV, average variance, AV, and average correlation, AC, are defined by:

$$SV_t = \frac{1}{t-1} \sum_{\tau=1}^{T} \left(R_{s,\tau} - \frac{\sum_{\tau=1}^{T} R_{s,\tau}}{t} \right)^2 \tag{6}$$

$$AV_t = \sum_{m=1}^{M} w_{m,t} \sigma_{m,t}^2 \tag{7}$$

$$AC_t = \sum_{m=1}^{M} \sum_{m \neq n}^{N} w_{m,t} w_{n,t} \rho_{m,n,t} \tag{8}$$

Figure **??** shows the time series behavior of market and average variance, in percent, as well as average correlation. With the easily noticeable exception of October 1987, spikes in both market and average variance concentrate around NBER defined recessions.

[**Place Figure ?? about here**]

Table I shows the summary statistics for the calculated variables. Despite the use of the actual

number of trading days, the restriction to assets that trade every trading day, and the adjustment to the calculation of correlation, the quarterly calculated values are almost identical to those in Pollet and Wilson (2010) over the same sample. Over the expanded the period, the annualized monthly average variance has a mean value of .88%. The annualized stock market variance mean is much lower at .25% monthly. Average correlation is relatively consistent at .23 quarterly in the Pollet and Wilson (2010) sample, .261 monthly in the same sample and .276 over the full time period. Average variance is more volatile than the stock market variance, more than twice as much. In each sample average variance has the highest autocorrelation. While average correlation is also persistent, the stock market variance is only strongly persistent at the monthly frequency with autocorrelation of .61. All three time series are stationary rejecting the unit root null in the tests of Dickey and Fuller (1979), Ng and Perron (2001), and Elliott, Rothenberg, and Stock (1996).

[**Place Table I about here**]

As my primary interest is in the use of average variance versus market variance in the management of leverage in the CRSP market portfolio, I test AV and SV against CRSP log excess returns. Specifically, I take the difference between the natural log of one plus the CRSP market return and the natural log of one plus the risk-free rate using:

$$r_t = \log R_{m,t} - \log R_{f,t} \tag{9}$$

where $R_{f,t}$ the risk-free return for which I use the 1-month treasury bill rate from Ken French's website[8]. As shown in table II, over the full data period, each variance and correlation time series are contemporaneously correlated to lower log excess returns. Average variance, AV, is significantly correlated with next month's market variance AV, SV, and AC. Surprisingly, over the full data set, this month's AV is even nominally more correlated with next month's SV than this month's SV is, 0.625 versus .612. Over the basis period, AV is time series most negatively correlated with next months log excess return at -.129, while it is entirely unrelated to next month's return over the whole data set.

As in the prior literature, July 1963 serves as the start of the basis data period for the regression analysis.[9] For in and out-of-sample tests, I regress market and average variance, and average correlation against these excess log return values. Out-of-sample regressions require an in-sample training period which, I set this at 15% of the available time series for consistent calculation of robust out of sample statistics later in the analysis. This training window means that out of sample regressions, analysis begins at the end of July 1970 in the basis sample and December 1939 in the full sample.

Proxies for lottery preference and leverage constraints are needed to test which has a measured influence on the relationship of buy and hold market returns and volatility, or average variance managed portfolio returns. Bali, Cakici, and Whitelaw (2011) define MAX as the maximum daily return for an asset in a given month. High values for MAX are shown to be good indicators for future conditions associated with higher lottery preference behavior. High values of MAX are a significant indicator of lower future returns. I calculate $MAX1$ and $MAX5$ as the highest daily return and the average of the highest five daily returns for each portfolio in a given month. As another indicator of higher lottery preferences in the economy, I capture the market capitalization for all gaming stocks in CRSP. $GMCAP$ is the total market capitalization of all assets with SIC code 7999 in the CRSP dataset. Gaming industry firms should do better in times of high lottery preference driving higher levels of $GMCAP$. However, the total valuation may move with the overall market capitalization. To generate a measure independent of the size of the market, I calculate $GMCAP_{scaled}$ as the ratio of gaming market capitalization to total market capitalization.

To generate proxies for the lending constraints operating in the market, I look to measures of financial intermediary leverage, investors leverage, borrowing levels and interest rates. I use data on broker call money[10] rates from 1988 to 2014 from Bloomberg, $Broker_{Call}$, average bank call money rates for 1984-2005 from Datastream, $Bank_{Call}$, and bank prime lending rates for 1984-2014 from Datastream, $Bank_{Prime}$. I use a seasonally adjusted version year-on-year nominal bank credit growth, $Credit_{CHG}$, as another proxy of borrowing conditions from 1984 to 2014, similar to the variable of Gandhi (2016).[11] Typically investors use margin, borrowed from brokers, to take leverage investment positions. As a proxy for the capital available to financial intermediaries, like brokers, I use the intermediary leverage factor, LF_{AEM}, of Adrian, Etula, and Muir (2014). LF_{AEM} measures shocks to a seasonally-adjusted broker-dealer leverage ratio, the ratio of total broker-dealer financial assets and that value minus debt.

$$LF_{AEM} = \Delta Ln(\frac{FinAsst}{FinAsst - BankDbt}) \tag{10}$$

This variable is available from Asaf Manela's website[12]. The total amount of margin borrowing is available from the NYSE website.[13] I calculate MD_{1984} as the change in the level of margin debt, month to month, in real 1984 dollars.

I use the proxies for lottery preferences and lending constraints in in-sample regressions along with the Fama-French five-factors, which are taken from Ken French's website. In contrast, the fundamental relationship between AV and future returns is established through in-sample regressions needing only the time series of average variance and log excess monthly returns.

II. Regression Analysis

A. In Sample

The effectiveness of either market variance or average variance as an investment management signal will be driven primarily by their relationship with future risk and return. It is the trade-off which is key to the leverage management strategy. Assuming that investors hold a portfolio with whose risk-return ratio they are indifferent. When risk increases but expected returns do not, the risk-return ratio become more unattractive, and any risk-averse investor would like to decrease there position. Conversely, when the risk of the portfolio goes down without a decrease in return there is an opportunity to leverage into the position returning to the previous level of risk but now with a magnified return.

To get an understanding of the relationship between stock market or average variance and returns, I begin with in-sample regressions. In each of these regressions, all of the information available in the sample is used to estimate the parameters. In general, the regressions take this form:

$$y_{t+1} = \alpha + \beta x_t + \epsilon_t. \tag{11}$$

The cotemporaneous regressions decomposing market variance are left unreported. The results show the same relationships found in Pollet and Wilson (2010) table 2. The only difference of note is that the relationship between average correlation, AC, and next months log excess return is weaker monthly than quarterly if average variance is not also included. For all in-sample regressions, the series are standardized to a mean of zero and standard deviation of one.

Table III contains the results of regressions run on the basis data set which spans 1962 to 2016. Panel A shows that AV is a significant predictor of next months SV in all specifications. A one standard deviation increase in AV means a .545 standard deviation increase in next months market variance. This change represents an increase from the mean of .2 to .42 or an increase from an annualized standard deviation of .45% to .65%. There does not appear to be any real advantage to using this months SV to predict next months market variance over AV. The coefficient values are nearly identical, .551 versus .545, as are the adjusted R^2 values, 30.3% versus 29.6%. This months AV even remains significant in the specification including this months SV. Holding this months SV constant, a one standard deviation increase in AV still signals a .257 standard deviation increase in next months SV. Again, the inclusion of SV appears to be of little to no help as the adjusted R^2 only increases from 29.6% to 31.5%. Panels B shows predictive regressions of next months average variance. Here, unlike the prior relationship, there is a definite advantage to using this months AV in the prediction of next months average variance. The adjusted R^2 of this month's AV, 44.5%, is nearly twice this month's SV, 27.2%, and the inclusion of SV with AV does not appear to be a significant improvement.

Pollet and Wilson (2010) argue that average correlation is a better measure of systemic market risk. Increases in average correlation are related to a higher covariance between labor income growth and the stock market, and the stock market and bond returns. Also, they show that in the context of Rolls Critique, average correlation should be a better indicator of risk for the true market portfolio rather than stock market variance. (Roll, 1977) Thus, the results in panel C of table III make the argument that leverage management using AV is a better idea. While both AV and SV are positively related to next months AC, SV is more highly related. The adjusted R^2 value for the regression of next months AC on current SV is more than twice that of current AV. Additionally, each signal is related to higher levels of both AV and SV next month. Hence, the use of SV as a leverage signal means taking on less weight in the market portfolio when next months systemic risk is higher relative to idiosyncratic risk. Theoretically, this is avoiding times of higher compensated versus uncompensated risk. Panel D presents the results of directly regressing next months log excess return on the market variance, average variance, and average correlation series. Indeed, AV is a significant predictor or lower returns in the next month and a better predictor than SV. A one standard deviation increase in this months AV means a .13 standard deviation, .58 percentage points, lower log excess return next month. So, a one standard deviation increase in AV means an expected negative return next month. Average variance remains a significant predictor of lower returns next month even when SV is included while the coefficient on SV is insignificant.

Table IV presents the results for the in-sample regressions across the whole CRSP data set, from 1926 to 2016. Across the whole data set, AV is an even better predictor of next months SV than current SV. The panel A results here are even better than in table III. The results in panel B show the same. The prior results are not only supported but appear better with AV being the best predictor of next months AV and SV providing no help and becoming insignificant with both variables are included. As before, across the whole data set stock market variance is more highly related to next months average correlation than is average variance. Panel C in table IV supports panel C in table III. Panel D is the first place the results appear different in a meaningful way. There is no relationship between AV and next months return in panel D of table IV. The coefficient on AV is insignificantly positive with an adjusted R^2 of -0.1%. Over the full sample, SV is a better predictor of next month's log excess return. A one standard deviation increase in this month's stock market variance indicates a .056 standard deviation, .3 percentage points, lower return. However, with an adjusted R^2 of .2%, there are better return predictors. While it is no longer obvious AV is better than SV at both risk and return anticipation across the whole data set, the results still suggest that investors are likely to be better off using AV than SV as it is a better risk anticipation measure and at least unrelated to future returns. AV should capture more returns for the same level of risk, if not avoid negative returns.

A.1. Robustness

Average variance is an autocorrelated time series; this opens the possibility that predictive regressions using AV have estimation bias as highlighted in Stambaugh (1999). Campbell and Hentschel (1992) show that the Stambaugh bias in predictive regressions involving volatility measures and future returns can be particularly severe because of a volatility feedback effect. To eliminate the Stambaugh bias in the estimated coefficients on AV in the regressions above, I follow the methodology in Amihud and Hurvich (2004) and furthur make the p-values used for coefficient significance robust through wild-bootstraping as detailed in MacKinnon (2002). Table V shows that the relationships demonstrated above are unaffected by robust bias correction. Average variance is a predictor of higher average correlation and higher stock market variance across data sets. While AV is a significant predictor of lower returns in the 1962 forward period, it is unrelated to the next months log excess returns in the whole data set. So long as the relationships hold with the limited information that investors have available at the time they make investment decisions, AV is likely to be a better leverage management signal than SV.

B. Out of Sample

Average variance is a good in-sample signal, however the out-of-sample performance remains in doubt. As Welch and Goyal (2008) definitively show, out-of-sample performance is not guaranteed by in-sample performance and is essential to any investment strategy which hopes to generate positive returns. To determine the out-of-sample relationships between market and average variance, average correlation and returns, I run regressions of the standard form

$$y_{t+1} = \alpha_t + \beta_t x_t + \epsilon_t \tag{12}$$

where α_t and β_t are estimated with from the data available only until time t. That is, I estimate α_t and β_t by regressing $\{y_{s+1}\}_{s=1}^{t-1}$ on a constant and $\{x_s\}_{s=1}^{t-1}$. In all the reported results, I follow an expanding window approach so that for the next period $t+2$, y_{t+2} is estimated as $\alpha_{t+1} + \beta_{t+1} x_{t+1}$, where α_{t+1} and β_{t+1} by regressing $\{y_{s+1}\}_{s=1}^{t}$ on a constant and $\{x_s\}_{s=1}^{t}$. I follow this process for all subsequent months. However, as part of a test on the robustness of the out-of-sample results, I demonstrate that the results do not depend on the use of an expanding window. Most critically, equation (12) prevents any look-ahead bias. The out-of-sample prediction tests use the same set of variables as the in sample tests. Each out-of-sample test requires an in-sample training period in which parameters are estimated using all the data up to the time period before the first out-of-sample quarter or month.

For consistency, the first one-fourth of the data is used as the initial parameter estimation period with the remaining three-fourths of observations moved through recursively generating out-of-sample predictions. Three measures of out-of-sample performance are estimated. I use the Diebold and Mariano (1995) statistic and McCracken (2007) MSE-F as measures of the increased accuracy of AV based forecasts compared to forecasts from SV as a benchmark. The DM statistic is defined as:

$$DM = \frac{\bar{d}}{\sqrt{\frac{2\pi f_d(0)}{T}}} \tag{13}$$

where $\bar{d}$ is the mean difference in the loss differential. The loss differential is the function used to measure the difference between the forecasted and actual values. I use the squared forecast error, $(y_t - \hat{y}_t)^2$. So, $\bar{d}$ is the mean value of the difference between the squared error using AV and the squared error using the benchmark forecast from SV.

$$\bar{d} = \frac{1}{T} \sum_{T}^{\tau=1} ((y_t - \hat{y}_{AV,t})^2 - (y_t - \hat{y}_{SV,t})^2) \tag{14}$$

I use the same consistent estimator for the mean loss differential, $f_d(0)$ as in Diebold and Mariano (1995). The statistic is normally distributed under the null hypothesis of no difference in accuracy between the benchmark and proposed model. The standard postive critial values from the normal Z-table serve as cutoffs to establish a significant improvement in forecast accuracy. $MSFE_{SV}$ is mean squared forecast error when a benchmark model is used to generate out-of-sample predictions. Mean squared forecast error is defined as

$$MSFE_x = \frac{1}{T} \sum_{\tau=t}^{T} (y_\tau - \hat{y}_\tau^x)^2 \tag{15}$$

where $\hat{y}_t^x$ is the out-of-sample prediction of y_t generated from the a model using variable x, t is the first out-of-sample prediction time period, and T is the total number of out-of-sample time periods. The F-statistic in McCracken (2007) is calculated by

$$MSE - F = T\frac{MSFE_x - MSFE_b}{MSFE_b}. \tag{16}$$

The significance of the F-statistic is determined from bootstrapped values provided in McCracken (2007). Each of these two tests depends on the reduction of average squared error by the predictor x relative to a benchmark model. The final measure is a forecast encompassing statistic.

Encompassing tests the more stringent requirement that the benchmark forecasts contain no useful information absent in the forecasts of variable x. Forecast encompassing tests come from the literature on optimal forecast combination. (Chong and Hendry, 1986; Fair and Shiller, 1990) An optimal forecast as a convex combination of two forecasts for time period t +1 defined as

$$\hat{y}_t^* = (1-\lambda)\hat{y}_t^b + \lambda\hat{y}_t^x \tag{17}$$

where $\hat{y}_t^x$ are predicted values generated from the model using variable x and $\hat{y}_t^b$ are forecasts from the benchmark model. I use the forecast encompassing test of Harvey et al. (1998), ENC-HLN. The encompassing test of Harvey et al. (1998) directly tests the value and significance of the forecast combination λ. The test procedure rests on the calculation of a modification to the Diebold and Mariano (1995) test statistic and the consistent estimation of the long-run covariance between the difference in forecast error between the benchmark model and a model based on a competing variable, x. As such there is no one line equation that sums up the statistic used to judge the significance of λ. However, intuitively λ must be significantly different from zero for AV to have information above and beyond the forecasting information in SV and values close to one indicate that AV has all of the relevant information in SV and is optimal by itself.

The results in table VI show that AV is a significantly better out-of-sample predictor of AV and SV. For both the variables, all three measures of out-of-sample performance show significant imporvement in both the out-of-sample period starting in 1970 and starting in 1939. The forecast encompassing tests also show that AV contains all the forecasting information in SV and is optimal on its own. This means that investors concerned about the variance in the returns on their investment in the market are better off using this month's level of average asset variance to hedge next month's stock market variance than using this month's stock market variance. AV is also a significantly better predictor of next month's log excess return for the out-of-sample period starting in 1970. The DM statistic, 1.278, is nearly significant at the 10% level and both the MSE-F test and encompassing tests show significant imporvement for AV. Again the encompassing test shows that AV is optimal alone and no weight needs to be given to SV in the prediction of log excess returns. Over the period starting July 1970, investors attempting to predict next month's returns would have been better off using this month's average asset variance rather than total market variance and investors deleveraging based on high values of AV would have done better avoiding negative returns rather than deleveraging based on high values of SV. Supporting the results seen in the in-smaple tests, across the longer out-of-sample period, starting in 1939, AV is not a significant improvement over SV in the prediction of next month's log excess return. It will be important to look at the asset allocation performance of AV versus SV over the entire CRSP sample to see if portfolio management by AV does generate significantly higher returns than management by SV.

B.1. Robustness

Out of sample estimation always raises issues with the choices made in the specification of the model and how to split the data into in and out of sample windows. Bluntly speaking, there are no good answers. The standard practice as in Rapach and Zhou (2013), Rapach, Strauss, and Zhou (2010), Rapach, Ringgenberg, and Zhou (2016), and Huang, Jiang, Tu, and Zhou (2015), and many others, is to show performance in a few subsamples split by dates that the authors choose for unknown reasons. The concerns with subsample selection are that the window may either be "ad-hoc" and the selection may mask significant results that would appear if the subsamples had been constructed differently. A second, more cynical, concern is that the presented subsample represent significant performance that has been found either by chance or as the result of analyzing many subsample and only presenting the significant results. In any case, evaluation of the differences in performance across subsamples is often left to the imagination of the reader and whatever importance they place on the first half of the sample versus the second, the middle third versus the first and last thirds or however the data has been separated. While the selection of 1962 is not arbitary as the daily return data is of much higher quality after, we have seen already a difference in return prediction performance for AV between the period after 1962 and the whole data set starting in 1926 which raises the question of the robustness of the out-of-sample results.

To address the robustness of the out of sample results and avoid the use of subsampling completely, I present out-of-sample statistics robust to both the specification of the prediction model, either expanding or rolling, and the choice of prediction window. Rossi and Inoue (2012) presents out of sample statistics robust to the choice of split between in and out of sample periods. The paper presents the calculation of the Diebold and Mariano (1995) statistic and the Harvey et al. (1998) encompassing test such that the choice of out-of-sample starting period is eliminated as a nuisance parameter and the asymptotic behavior of the statistics can be used to measure their observed significance. Fundamentally, this involves the calculation of each of the statistics for all feasible out-of-sample windows and in the case of rolling regression specification all feasible window sizes. The modifications are different for each of the statistics and the calculation of the robust statistic is different depending both on which statistic and which concern is being addressed. When the conern is that the choosen window could be unrepresentatively optimal, perhaps the best results of many tests, then it is possible that the null of no imporovement is rejected based on the calculated statistics when in general it is true. To eliminate this possibility, Rossi and Inoue (2012) provide the R_T measure which essentially insures that the highest calculated statistics are so extreme that they could not occur without a underlying significant improvement in forecast accuracy from the benchmark to the proposed model. The A_T measure insures that the average calculated statistics are large enough that an arbitrarily selected out-of-sample starting period would not lead to the failure to reject the null of no accuracy improvement when it was indeed false. This two measures tackle the type I and II error questions. Given the results thus far, we will be looking for significant R_T values to support the significant ability of AV to predict stock market and average variance across the data set, and significant A_T values to tell us that the significant ability of AV to predict log excess returns, seen from 1970 forward, is indicative of a real accuracy improvement while the lack of performance when including predictions from 1939 forward is simply a noisy period of poor performance obfuscating the superiority of the AV based model.

Table VII shows the robust out-of-sample statistics. Panel A has the R_T and A_T statistics for the comparision of all possible expanding window forecasting models using AV with in-sample training windows of at least 15% of the data and out-of-sample forecasting periods of at least 15% of the data against a benchmark model using SV with the same specifictions. The porportional data cut offs are necessary to use the critical values provided in the Rossi and Inoue (2012) paper, 15% is the smallest, and mean that the first feasible specification starts forecasting in December 1939 as in the out-of-sample results shown above and forecasts for at least April 2003 to December 2016 are made. Every DM statistic is significant indicating that the AV model is a significant improvment in forecasting accuracy for all variable of interest. Every encompassing test statistic is significant, however these are not λ values directly so while we know that AV contains information over and above SV they do not directly indicate it is optimal alone. Panel B has the results for comparisons of rolling window specification tests of AV and SV models, again using the 15% proportional cut-offs. While all DM statistics are significant, not all encompassing tests are. There are too many rolling window specifications in which SV contains significant forecasting information which is not captured by AV. A forecast combining the values from rolling window models using AV and SV will almost surely be better than using either rolling window forecast alone while using the expanding window AV model forecast alone may very well be better compared to any combination of that prediction with an expanding window SV forecast.

III. Asset Allocation

Of course, the most direct and practically relevant measure of AV as a portfolio leverage management tool is whether or not it generates portfolio gains. And while superior out-of-sample predictability usually translates into better portfolio performance, here timing leverage to risk, it is important to make adjustments for the riskiness of the managed portfolio and compare performance across dimensions for a full investment picture. The AV managed portfolio may well generate higher annualized returns, but will it have a better Sharpe ratio than the SV managed portfolio?

To measure portfolio performance, in addition to annualized monthly log excess return, I will calculate each portfolios Sharpe ratio, Sortino ratio, two Kappa ratios and Rachev ratio. The classic Sharpe ratio is a symmetric measure of risk and is defined as the ratio of the expected excess portfolio return over the standard deviation of portfolio returns.

$$\frac{\mathbb{E}[r_x]}{\sigma(r_x)} \tag{18}$$

While Sharpe measures each dollar of expected return for dollar of risk, the Sortino ratio attempts to more directly measure the risk most investors worry about. By using only downside deviation in the denominator, the Sortino quantifies each dollar of expected return for each dollar of loss. This downside is measured relative to a target return. (Sortino and Price, 1994) As the log returns are already excess of the risk free rate, I set the Sortino target to 0 which makes the Sortino formula:

$$\frac{\mathbb{E}[r_x - 0]}{\sqrt{\int_{-\infty}^{0} (0 - r_x)^2 f(r_x)\, dr}} \tag{19}$$

The Sortino is just a specific instance of a more general risk measurement ratio formula. The Kappa ratio keeps the expected return relative to a target in the numerator but allows any lower partial moment in the denominator. (Kaplan and Knowles, 2004) With the target again set to 0 the general formula is of the form:

$$\frac{\mathbb{E}[r_x - 0]}{\sqrt[n]{LPM_n}} \tag{20}$$

The Sortino ratio is the Kappa$_2$ ratio. I calculate Kappa$_3$ and Kappa$_4$ also to see relative performance of AV and SV management adjusted for negative return skew and kurtosis. The final performance ratio investigated is the Rachev. Developed in Biglova, Ortobelli, Rachev, and Stoyanov (2004), the Rachev ratio is the expected tail return in the best n% of the return distribution over the expected tail loss in the worst n%. I set n = 5 and measure the top 5% expected return over the expected loss in the worst 5%. The general Rachev ratio formula is, again against a target return level of 0:

$$\frac{ETL_{\alpha}\left(r_f - x'r\right)}{ETL_{\beta}\left(x'r - r_f\right)} \tag{21}$$

where $ETL_{\alpha} = \frac{1}{\alpha} \int_0^{\alpha} VaR_q\left(X\right) dq$.

Measuring a difference in each of these measures for a pair of portfolios is not difficult; measuring a significant difference is. When evaluating the difference in Sharpe ratios, the method in Memmel (2003) seems to be popular. However, when returns are not normally distributed or autocorrelated this method is not valid. AV and SV managed returns, like market returns, are weakly autocorrelated, slightly skewed, and have much fatter tails when compared to normally distributed returns. Moreover, current period returns to either the AV or SV managed portfolio depend on the prior period variance of the market return strongly questioning the i.i.d assumption made in most hyptothesis testing methodologies. Studentized time series bootstrap sampling preserves the time series properties of the AV and SV managed returns which is critical; for example, Scherer (2004) demonstrates that methods which loose the time dependence in the calculation of differences in Sortino ratios fail to properly estimate the sampling distribution and critical values.

Time seires bootstrap methods preserve the structure from the original data and allow for efficient robust hypothesis testing. (Politis and Romano, 1994; Davison and Hinkley, 1997) Ledoit and Wolf (2008) show that this method is even more efficient than using Newey and West (1987) or Andrews and Monahan (1992) heteroskedasticity and autocorrelation corrected standard errors for testing the significance of differences between two portfolio Sharpe ratios. I follow the p-value estimation method in Ledoit and Wolf (2008) to determine the significance of the difference between the portfolio performance ratio measures of the AV and SV mangement strategies. This uses circular block bootstrapping of the return time series, robust centered studentized statistics computed from the bootstrap samples and is proven to be the most efficient hypothesis testing method. Politis and Romano (1992); Ledoit and Wolf (2008)

As in Moreira and Muir (2017), investment weight in the market portfolio is a function of the variance of daily market returns, SV, or the average daily return asset variance, AV, scaled by a constant, c. Moreira and Muir (2017) use a constant that scales the variance of the volatility managed portfolio equal to the buy and hold market portfolio. In the basic portfolio weighting specification, I use the same approach so that the returns of both the SV and AV managed portfolios have the same variance as the buy and hold strategy. This constant is denoted c_{053} and it takes different values for SV and AV. This scaling requires knowing the full sample buy and hold return variance. While this induces distortions in the performance ratios, to insure robustness two other specifications for the scaling targets are used. Annual volatilities of 12% and 10% are common in academic literature and fund management so c_{035} and c_{029} approximately target those levels. Barroso and Santa-Clara (2015); Morrison and Tadrowski (2013); Verma (2018); Fleming, Kirby, and Ostdiek (2002); Hocquard, Ng, and Papageorgiou (2013) Using each of these constant an investor reblances at the end of month t investing in the market portfolio with weight:

$$w_{x,t} = \frac{c}{x(t)} \tag{22}$$

where $x(t)$ is either SV_t or AV_t and hold for month t + 1. Table VIII shows summary statistics for the resulting investment weights for AV and SV for the three volatility targets. When targeting the volatility of the market portfolio, both portfolios are leveraged into the market on averge with investment weights of 1.3 indicating 30% leverage. Regardless of the volatility target the SV managed portfolio calls for extreme levels of leverage. Figure 1 shows that the SV strategy targeting the buy and hold volatility calls for investment weights above the maximum AV weight in several peiords. More than 500% leverage is needed at the end of the 1920s, throughout the 1960s, and in the 1990s. Given that these levels of leverage are unrealistic for most investors, it will be important to see if there is a difference in performance for the AV and SV strategies under real-world investment constraints.

Before exaiming the constrained portfolio performance, I present the results for SV and AV strategies targeting the buy and hold volatility without investment constraints in table IX. As in Moreira and Muir (2017) the portfolio perforamnce is measured across the whole CRSP data set, however the relative performance is the same or better across the basis data set. Table **??** presents the performance ratios for the SV and AV managed portfolios targeting the buy and hold volatility without investment constraints. The buy and hold market strategy is included for reference. However since Moreira and Muir (2017) establish that the SV managed portfolio out performs the buy and hold, statistical significance results are only presented for the comparison of the SV and AV managed portfolios. The AV managed portfolio generates a statistically significant 1.08 percentage points higher average annualized log excess return. As shown in the bottom panel of figure 2 the AV strategy builds its performance advantage slowly but consistently starting from the early 1950s and from that time the SV managed portfolio is never a better investment. As both strategies are targeting the same volatility, the significant difference in return translates into a significant difference in Sharpe ratio. At .520 versus .462, for the SV managed portfolio, the AV managed Sharpe ratio is 12.6% higher. AV also generates significantly higher Kappa_3 and Kappa_4 ratios. So while the overall payment for downside risk, measured by Sortino ratio, may not be significantly higher, the payment for downside skewness and extreme downside return is higher. Unfortunately the compensation for downside tail risk is not perfect. SV has a significantly higher Rachev ratio. The volatility managed market portfolio has higher expected tail return potential for each dollar of potential tail loss to the average variance managed portfolio. The maximum annualized return to the SV managed portfolio is 35.3%, in August 1965. That month, the AV managed portfolio only returns 8.9% and its maximum return is an annualized 22.11%. The AV managed portfolio is a ticket with better winning chances and a higher expected return while the SV managed portfolio has a larger jackpot.

More practical analysis of portfolio performance requires the incorporation of limits on the level of investment taken in the market portfolio. Leverage of 50%, a coefficient of 1.5 on the market, is a common constraint meant to mimic real market leverage constraints for the average investor based in part on the Reg. T margin requirement[14]. (Rapach et al., 2016; Moreira and Muir, 2017; Deuskar, Kumar, and Poland, 2017) There are at least two exchange traded funds, ETFs, which three times the return of the SP500.[15] So, I take a market coefficient of three as the maximum feasible investment a typical investor can make in the market portfolio. Table **??** panel c presents the results from applying investment constraints after calculating the weights for AV and SV targeting the buy and hold volatility. While both portfolios still outperform the buy and hold, The seperations in average annualized excess return, 1.71% and 2.07%, are even greater when investment constraints are applied. Panels a and b in figure 2 show the effects of the growing separation. While SV is barely able to clear the buy and hold strategy under typical brokerage constraints, returns to the AV managed portfolio remain clearly above. Investors that use a leveraged SP500 ETF to impliment the AV managed portfolio strategy are rewarded with returns significantly higher than the SV managed portfolio suggested weights. The investment restrictions pull the volatility of the AV managed portfolio too far from the SV returns to generate significant differences in performance ratios. However, investors using the leveraged ETFs are rewarded not only with higher returns but significantly better performance ratios across the board with the exception of the Rachev ratio which is at least no longer signficantly better for the SV managed portfolio. The ETF leverage constrained AV strategy even generates better Sharpe and Sortino ratios than the unconstrained strategy. The results in panels a and b demonstrate that better performance of AV is not a result of or contingent on looking to the volatility of the buy and hold strategy. As the targeted volatility is lowered the difference in performance between AV and SV becomes more significant. With the exception of the Rachev ratio, every performance measure is significantly better for the AV managed portfolio given lower volatility targets and investment constraints.

IV. Preferences

A. Changes to the Capital Market Line

B. Lottery Preference

We have some reason to suspect leverage already, the constraints drive returns toward BH and some reason to doubt lottery Rachev.

However, the fundamental argument that investors prefer the buy and hold market over the average variance or volatility managed investment because the market is more lottery-like. This itself is unclear.

Bali et al. (2011) show that the maximum daily return over the past one month, MAX, is a good measure of the lottery-like payoffs of a stock and a significant indicator of lower future returns robust to size, book-to-market, momentum, short-term reversals, liquidity, and skewness. This means that for lottery seeking investors to prefer the buy and hold market its MAX measures must be significantly different from the average variance and volatility managed portfolios. Yet, as seen in table **??** the mean and median values of the highest one day returns, MAX1, and the average of the five highest daily returns within the month, MAX5, are higher for the average variance managed portfolio than either volatility or the buy and hold market portfolio. Using daily return values scaled by the prior months portfolio volatility, as in Asness et al. (2018), the volatility managed portfolio is the most lottery like with the highest mean and median values of scaled MAX1, SMAX1, and scaled MAX5, SMAX5. Notably, the average variance managed portfolio still has higher mean SMAX1 and SMAX5 values than the buy and hold portfolio. This does not mean that the buy and hold portfolio is not viewed as a lottery and investors do not take some additional utility from holding it, however it seems very unlikely that this is even greater than the lottery utility provided by the average variance or volatility managed portfolio let alone large enough to compensate for the difference in return or CER gain.

It remains possible that on some other yet unknown measure the buy and hold strategy is more lottery like. To form a more direct test of the lottery preference explaination

C. Lending Constraints

High levels of bank credit growth are associated with the overextension of credit in the past, tighter current and future lending conditions, and lower future market returns.

Large positive shock, high value of LF_{AEM}, are associated with time of high intermediary funding illiquidity. Large bank debt constrains brokers abilities to aquire more funds to lend to investors and limits there willingness to lend to "risky" investors.

To justify adding a subsection here, from now on, we'll assume

CONDITION 1: $0 < \hat{\mu} < \gamma\sigma^2$.

This condition might be useful if there was a model.

C.1. A Subsubsection with a Proposition

Let's put a proposition here.

PROPOSITION 1: *If Condition 1 is satisfied, a solution to the central planner's problem,* $V(B, D, t) \in C^2\left(\mathbb{R}_+^2 \times [0, T]\right)$*, with control* $a : [0, 1] \times [0, T] \to [-\lambda, \lambda]$ *if* $\gamma > 1$ *is*

$$V(B, D, t) = -\frac{(B + D)^{1-\gamma}}{1 - \gamma} w\left(\frac{B}{B + D}, t\right). \tag{23}$$

Appendix A. An Appendix

Here's an appendix with an equation. Note that equation numbering is quite different in appendices and that the JF wants the word "Appendix" to appear before the letter in the appendix title. This is all handled in `jf.sty`.

$$E = mc^2. \tag{A1}$$

Appendix B. Another Appendix

Here's another appendix with an equation.

$$E = mc^2. \tag{B1}$$

Note that this is quite similar to Equation (A1) in Appendix A.

REFERENCES

Adrian, Tobias, Erkko Etula, and Tyler Muir, 2014, Financial intermediaries and the cross-section of asset returns, *Journal of Finance* 69, 2557–2596.

Amihud, Yakov, and Clifford M. Hurvich, 2004, Predictive regressions: A reduced-bias estimation method, *Journal of Financial and Quantitative Analysis* 39, 813–841.

Andrews, Donald W. K., and J. Christopher Monahan, 1992, An Improved Heteroskedasticity and Autocorrelation Consistent Covariance Matrix Estimator, *Econometrica* 60, 953–966.

Asness, Clifford S., Andrea Frazzini, Niels Joachim Gormsen, and Lasse Heje Pedersen, 2018, Betting against correlation: testing theories of the low-risk effect .

Bali, Turan G., Nusret Cakici, and Robert F. Whitelaw, 2011, Maxing out: Stocks as lotteries and the cross-section of expected returns, *Journal of Financial Economics* 99, 427–446.

Barberis, Nicholas, and Ming Huang, 2008, Stocks as Lotteries: The Implications of Probability Weighting for Security Prices, *American Economic Review* 98, 2066–2100.

Barroso, Pedro, and Pedro Santa-Clara, 2015, Momentum has its moments, *Journal of Financial Economics* 116, 111–120.

Biglova, Almira, Sergio Ortobelli, Svetlozar T. Rachev, and Stoyan Stoyanov, 2004, Different Approaches to Risk Estimation in Portfolio Theory, *The Journal of Portfolio Management* 31, 103–112.

Black, Fischer, 1972, Capital market equilibrium with restricted borrowing, *The Journal of business* 45, 444–455.

Boguth, Oliver, and Mikhail Simutin, 2018, Leverage constraints and asset prices: Insights from mutual fund risk taking, *Journal of Financial Economics* 127, 325–341.

Brunnermeier, Markus K., Christian Gollier, and Jonathan A. Parker, 2007, Optimal beliefs, asset prices, and the preference for skewed returns, *American Economic Review* 97, 159–165.

Campbell, John Y., 1987, Stock returns and the term structure, *Journal of Financial Economics* 18, 373 – 399.

Campbell, John Y., and Ludger Hentschel, 1992, No news is good news: An asymmetric model of changing volatility in stock returns, *Journal of Financial Economics* 31, 281–318.

Campbell, John Y., Martin Lettau, Burton G. Malkiel, and Yexiao Xu, 2001, Have individual stocks become more volatile? An empirical exploration of idiosyncratic risk, *The Journal of Finance* 56, 1–43.

Campbell, John Y, Andrew Wen-Chuan Lo, Archie Craig MacKinlay, et al., 1997, *The econometrics of financial markets*, volume 2 (princeton University press Princeton, NJ).

Cao, Larry, 2016, Risk Parity Made Easy: Cliffs Notes and Other Key Readings.

Chong, Yock Y, and David F Hendry, 1986, Econometric evaluation of linear macro-economic models, *Review of Economic Studies* 53, 671–690.

Davison, Anthony Christopher, and David Victor Hinkley, 1997, *Bootstrap methods and their application*, volume 1 (Cambridge university press).

Deuskar, Prachi, Nitin Kumar, and Jeramia Poland, 2017, Margin Credit and Stock Return Predictability, *SSRN Electronic Journal* .

Dickey, David A., and Wayne A. Fuller, 1979, Distribution of the Estimators for Autoregressive Time Series with a Unit Root, *Journal of the American Statistical Association* 74, 427–431.

Diebold, Francis X, and Robert S Mariano, 1995, Comparing predictive accuracy, *Journal of Business & Economic Statistics* 13, 253–263.

Elliott, Graham, Thomas J. Rothenberg, and James H. Stock, 1996, Efficient tests for an autoregressive unit root, *Econometrica* 64, 813–836.

Fair, Ray C, and Robert J Shiller, 1990, Comparing information in forecasts from econometric models, *American Economic Review* 3, 375–389.

Fama, Eugene F., and Kenneth R. French, 2016, Dissecting Anomalies with a Five-Factor Model, *Review of Financial Studies* 29, 69–103.

Fleming, Jeff, Chris Kirby, and Barbara Ostdiek, 2002, The Economic Value of Volatility Timing, *The Journal of Finance* 56, 329–352.

Frazzini, Andrea, and Lasse Heje Pedersen, 2014, Betting against beta, *Journal of Financial Economics* 111, 1–25.

French, Kenneth R. F, G.William Schwert, and Robert F. Stambaugh, 1987, Expected stock returns and volatility, *Journal of Financial Economics* 19, 3 – 29.

Gandhi, Priyank, 2016, From the 'Long Depression' to the 'Great Recession': Bank Credit, Macroeconomic Risk, and Equity Returns, University of Notre Dame Working Paper.

Glosten, Lawrence, Ravi Jagannathan, and David Runkle, 1993, On the relation between the expected value and the volatility of the nominal excess return on stocks, *The Journal of Finance* 48, 1779–1801.

Harvey, David S., Stephen J. Leybourne, and Paul Newbold, 1998, Tests for Forecast Encompassing, *Journal of Business & Economic Statistics* 16, 254–259.

Haugen, Robert, and A. Heins, 1972, On the evidence supporting the existence of risk premiums in the capital market, Technical report, Working Paper, University of Wisconsin-Madison.

Hocquard, Alexandre, Sunny Ng, and Nicolas Papageorgiou, 2013, A Constant-Volatility Framework for Managing Tail Risk, *Journal of Portfolio Management; New York* 39, 28–40,6,8.

Hotelling, Harold, 1953, New light on the correlation coefficient and its transforms, *Journal of the Royal Statistical Society. Series B (Methodological)* 15, 193–232.

Huang, Dashan, Fuwei Jiang, Jun Tu, and Guofu Zhou, 2015, Investor sentiment aligned: A powerful predictor of stock returns, *Review of Financial Studies* 28, 791–837.

Jones, Charles, 2002, A century of stock market liquidity and trading costs .

Jylh, Petri, 2018, Margin Requirements and the Security Market Line: Margin Requirements and the Security Market Line, *The Journal of Finance* .

Kandel, Shmuel, and Robert F Stambaugh, 1996, On the predictability of stock returns: An asset-allocation perspective, *Journal of Finance* 51, 385–424.

Kaplan, Paul D., and James A. Knowles, 2004, Kappa: A generalized downside risk-adjusted performance measure, *Journal of Performance Measurement.* 8, 42–54.

Kim, Abby Y., Yiuman Tse, and John K. Wald, 2016, Time series momentum and volatility scaling, *Journal of Financial Markets* 30, 103–124.

Ledoit, Oliver, and Michael Wolf, 2008, Robust performance hypothesis testing with the Sharpe ratio, *Journal of Empirical Finance* 15, 850–859.

MacKinnon, James G., 2002, Bootstrap inference in econometrics, *Canadian Journal of Economics/Revue Canadienne d'Economique* 35, 615–645.

Malkhozov, Aytek, Philippe Mueller, Andrea Vedolin, and Gyuri Venter, 2017, International illiquidity .

Markowitz, Harry, 1952, Portfolio Selection, *The Journal of Finance* 7, 77.

McCracken, Michael W., 2007, Asymptotics for out of sample tests of Granger causality, *Journal of Econometrics* 140, 719–752.

McWhinnie, Eric, 2014, Warren Buffett on using leverage to invest.

Memmel, Christoph, 2003, Performance Hypothesis Testing with the Sharpe Ratio, SSRN Scholarly Paper ID 412588, Social Science Research Network, Rochester, NY.

Moreira, Alan, and Tyler Muir, 2017, Volatility-Managed Portfolios: Volatility-Managed Portfolios, *The Journal of Finance* 72, 1611–1644.

Morrison, Steven, and Laura Tadrowski, 2013, Guarantees and Target Volatility Funds 12.

Newey, Whitney K., and Kenneth D. West, 1987, A Simple, Positive Semi-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix, *Econometrica* 55, 703–708.

Ng, Serena, and Pierre Perron, 2001, Lag Length Selection and the Construction of Unit Root Tests with Good Size and Power, *Econometrica* 69, 1519–1554.

Olkin, Ingram, and John W. Pratt, 1958, Unbiased estimation of certain correlation coefficients, *The Annals of Mathematical Statistics* 29, 201–211.

Orwell, George, 1946, *Animal farm* (Harcourt, Brace and Company, New York, NY).

Politis, Dimitris N., and Joseph P. Romano, 1992, A General Resampling Scheme for Triangular Arrays of α-Mixing Random Variables with Application to the Problem of Spectral Density Estimation, *The Annals of Statistics* 20, 1985–2007.

Politis, Dimitris N., and Joseph P. Romano, 1994, The Stationary Bootstrap, *Journal of the American Statistical Association* 89, 1303–1313.

Pollet, Joshua M., and Mungo Wilson, 2010, Average correlation and stock market returns, *Journal of Financial Economics* 96, 364–380.

Rapach, David, and Guofu Zhou, 2013, Forecasting stock returns, in Graham Elliott, and Allan Timmermann, eds., *Handbook of Economic Forecasting*, volume 2A, 327–383 (Elsevier B.V.).

Rapach, David E., Matthew C. Ringgenberg, and Guofu Zhou, 2016, Short interest and aggregate stock returns, *Journal of Financial Economics* 121, 46–65.

Rapach, David E., Jack K. Strauss, and Guofu Zhou, 2010, Out-of-sample equity premium prediction: Combination forecasts and links to the real economy, *Review of Financial Studies* 23, 821–862.

Roll, Richard, 1977, A critique of the asset pricing theory's tests Part I: On past and potential testability of the theory, *Journal of Financial Economics* 4, 129 – 176.

Rossi, Barbara, and Atsushi Inoue, 2012, Out-of-Sample Forecast Tests Robust to the Choice of Window Size, *Journal of Business & Economic Statistics* 30, 432–453.

Rutterford, Janette, and Dimitris P. Sotiropoulos, 2016, Financial diversification before modern portfolio theory: UK financial advice documents in the late nineteenth and the beginning of the twentieth century, *The European Journal of the History of Economic Thought* 23, 919–945.

Scherer, Bernd, 2004, An alternative route to performance hypothesis testing, *Journal of Asset Management* 5, 5–12.

Sortino, Frank A., and Lee N. Price, 1994, Performance Measurement in a Downside Risk Framework, *The Journal of Investing* 3, 59–64.

Stambaugh, Robert F., 1999, Predictive regressions, *Journal of Financial Economics* 54, 375–421.

Steward, Martin, 2010, The truly balanced portfolio.

Verma, Sid, 2018, Volatility-Targeting Funds Could Sell $225 Billion of Stocks, *Bloomberg.com* .

Welch, Ivo, and Amit Goyal, 2008, A comprehensive look at the empirical performance of equity premium prediction, *Review of Financial Studies* 21, 1455–1508.

Wolpert, David H., and William G. Macready, 1997, No free lunch theorems for optimization, *IEEE transactions on evolutionary computation* 1, 67–82.

Notes

[1]Invented by Edward A. Calahan, an employee of the American Telegraph Company in 1867 and in wide spread use starting in the 1870s, the stock ticker provided the first reliable means of conveying up to the minute stock prices over a long distances and market participants have been discusing the relationship between returns, risk, and portfolios for at least as long.(Rutterford and Sotiropoulos, 2016)

[2](McWhinnie, 2014)

[3]See, for example, Barroso and Santa-Clara (2015) and Kim, Tse, and Wald (2016) for discussions of volatility managment of the momentum portfolio.

[4](Orwell, 1946)

[5]The shortest trading month in the sample is September 2001 with 15 trading days while 17 is a common number in the months with holidays.

[6]The exact correction suggested in Olkin and Pratt (1958) is too computationally taxing for the equiptment to which I have access.

[7]The least number of assets which trade every day in a given month is 392 in August of 1932. There are regularly 500 qualifying assets by the end of the 1930s.

[8]http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html#Research

[9]CRSP has, as of 2005, backfilled NYSE daily returns to 1926, however the pre-1962 data is very different from the post-1962 data. The earlier data is much shallower having months with fewer than 400 assets total that meet the data requirements. Twice as many firms covering twice as many industries are available at the end of 1962 as compared to the end of 1961. And, as documented in Jones (2002) the pre-1962 period is significantly and persistantly more illiquid.

[10]Call money is the money loaned by a bank or other institution which is repayable on demand.

[11]Seaonally adjusted nominal monthly bank credit is available in statistical release H.8 (Assets and Liabilities of Commercial Banks in the U.S.) of the Board of Governors of the Federal Reserve System.

[12]http://apps.olin.wustl.edu/faculty/manela/

[13]http://www.nyxdata.com/nysedata/asp/factbook/viewer_edition.asp?mode=table&key=3153&category=8

[14]Federal Reserve Board Regulation T (Reg T) establishes a baseline requirement that investors

deposit 50% of an investment position in their margin trading accounts, however a brokerage house may set a higher equity requirement.

[15]The Direxion Daily S&P 500 Bull 3x Shares ETF, symbol SPXL, and ProShares Ultra Pro S&P 500 ETF, symbol UPRO, are two such funds.

Figure 1. Time Series of Investment Weights: The time series of the investment weight into the market portfolio for SV and AV managed portfolios targeting the buy and hold volatility.

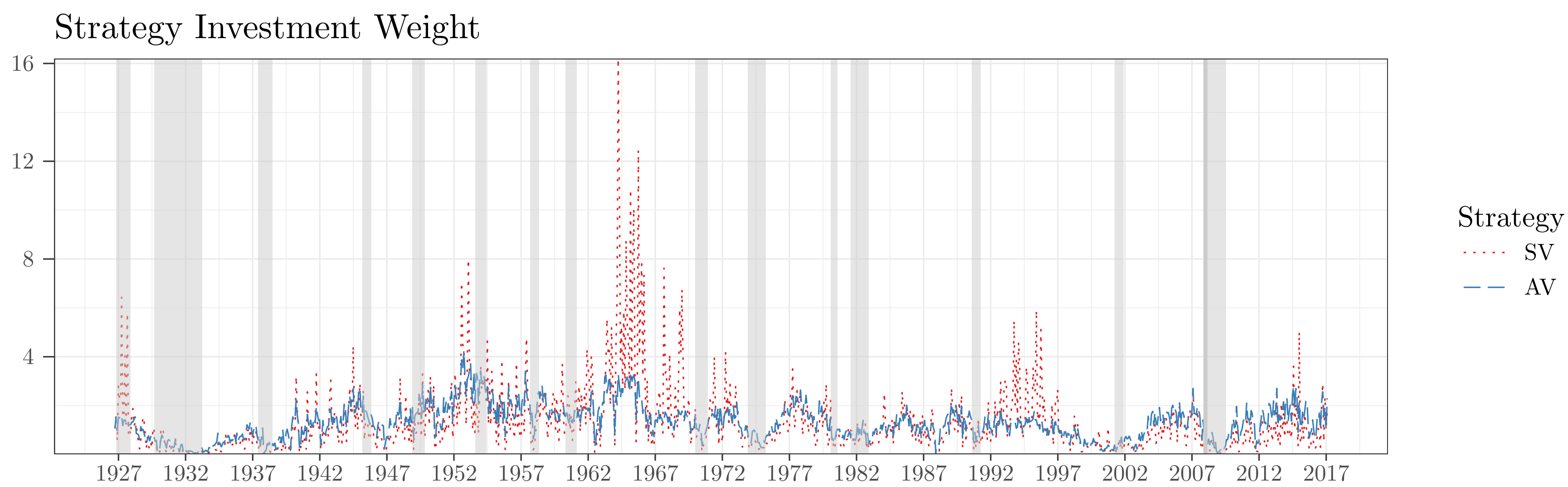

Figure 2. Cummulative Log Excess Returns: The time series of cummulative log excess returns for the buy and hold market investment as well as the AV and SV managed portfolios. Panel a limits the coefficient on the market portfolio between 0 and 1.5 for the AV and SV strategies; panel b limit them to weights from 0 to 3 and they are unconstrained in panel c.

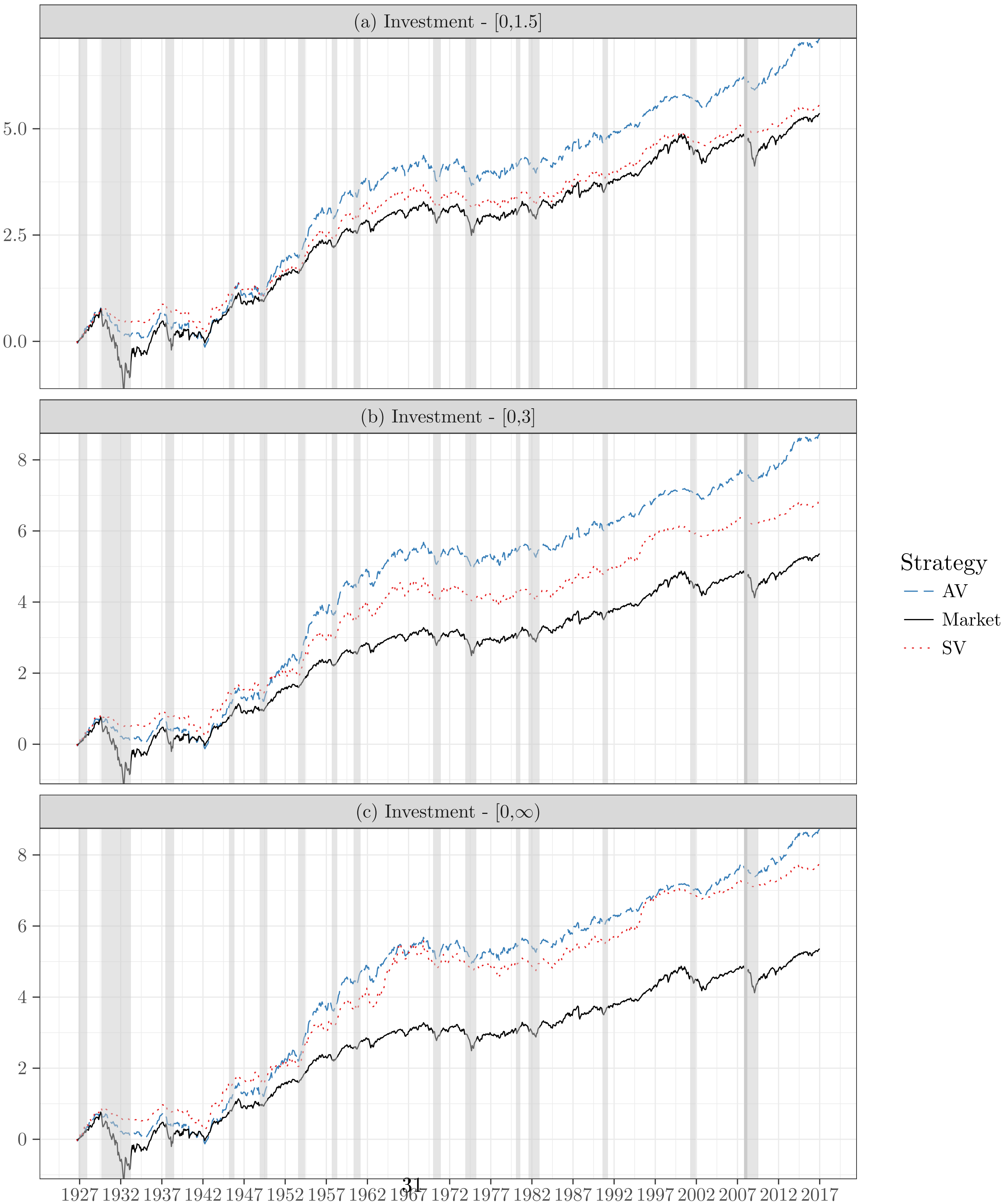

Table I:Summary statistics

The table displays summary statistics for the market variance and correlation statistics, and returns. RET is the log excess return of the CRSP market portfolio. AC is the average pairwise correlation of the daily returns of the 500 largest firms in the CRSP data set over the month or quarter. AV is the average of the individual variances of daily returns for the 500 largest firms in the CRSP data set. SV is the variance of daily CRSP market returns. See section I for details on construction.

(a) Pollet and Wilson Sample 1963Q1:2006Q4

Statistic	N	Mean	St. Dev.	Min	Max	Autocorrelation
RET	176	1.169	8.372	−30.072	19.956	0.000
AC	176	0.231	0.091	0.034	0.648	0.556
AV	176	2.221	1.827	0.634	12.044	0.695
SV	176	0.484	0.615	0.029	6.397	0.310

(b) Sample 1962M6:2016M12

Statistic	N	Mean	St. Dev.	Min	Max	Autocorrelation
RET	655	0.409	4.453	−25.985	14.515	0.081
AC	655	0.261	0.129	0.019	0.762	0.620
AV	655	0.770	0.849	0.198	10.416	0.667
SV	655	0.200	0.406	0.006	5.664	0.551

(c) Full Sample 1926M7:2016M12

Statistic	N	Mean	St. Dev.	Min	Max	Autocorrelation
RET	1,085	0.495	5.371	−34.523	33.188	0.106
AC	1,085	0.276	0.134	0.019	0.762	0.610
AV	1,085	0.881	1.281	0.154	19.540	0.718
SV	1,085	0.248	0.502	0.006	5.808	0.612

Table II:Correlations

The table displays Pearson correlation statistics for the market variance and correlation statistics, and returns. RET is the log excess return of the CRSP market portfolio. AC is the average pairwise correlation of the daily returns of the 500 largest firms in the CRSP data set over the month or quarter. AV is the average of the individual variances of daily returns for the 500 largest firms in the CRSP data set. SV is the variance of daily CRSP market returns. See section I for details on construction.

(a) Sample 1962M6:2016M12

	RET	AC	AV	SV	RET_{t+1}	AC_{t+1}	AV_{t+1}	SV_{t+1}
RET	1	0	0	0	0	0	0	0
AC	-0.244	1	0	0	0	0	0	0
AV	-0.284	0.352	1	0	0	0	0	0
SV	-0.353	0.529	0.899	1	0	0	0	0
RET_{t+1}	0.081	0.049	-0.129	-0.107	1	0	0	0
AC_{t+1}	-0.223	0.622	0.240	0.360	-0.239	1	0	0
AV_{t+1}	-0.268	0.218	0.667	0.522	-0.283	0.351	1	0
SV_{t+1}	-0.289	0.332	0.545	0.552	-0.351	0.528	0.899	1

(b) Full Sample 1926M7:2016M12

	RET	AC	AV	SV	RET_{t+1}	AC_{t+1}	AV_{t+1}	SV_{t+1}
RET	1	0	0	0	0	0	0	0
AC	-0.295	1	0	0	0	0	0	0
AV	-0.136	0.467	1	0	0	0	0	0
SV	-0.279	0.619	0.857	1	0	0	0	0
RET_{t+1}	0.106	0.011	0	-0.057	1	0	0	0
AC_{t+1}	-0.229	0.610	0.383	0.453	-0.295	1	0	0
AV_{t+1}	-0.191	0.358	0.718	0.607	-0.136	0.467	1	0
SV_{t+1}	-0.259	0.416	0.625	0.612	-0.279	0.619	0.857	1

Table III:In Sample Results - Post 1962

The table displays in sample regression results for monthly market variance, correlation and return statistics. SV is the annualized monthly variance of daily CRSP market returns. AV and AC are the monthly average variance and average pairwise correlation of daily returns for the top 500 assets in the CRSP market, as in Pollet and Wilson (2010). RET is the log return of the CRSP market portfolio minus the log return on the 1 month treasury bill. The sample period is from 1962:06 to 2016:12. The series are standardized to a mean of zero and standard deviation of one.

(a) Market Return Variance - SV_{t+1}

AV	0.545***			0.489***	0.257***
	p = 0.000			p = 0.000	p = 0.001
AC		0.332***		0.160***	
		p = 0.000		p = 0.00001	
SV			0.551***		0.320***
			p = 0.000		p = 0.00002
Constant	−0.0005	−0.0001	−0.0003	−0.0005	−0.0004
	p = 0.989	p = 0.999	p = 0.993	p = 0.989	p = 0.991
R^2	0.297	0.110	0.304	0.320	0.317
Adjusted R^2	0.296	0.109	0.303	0.318	0.315

(b) Average Asset Return Variance - AV_{t+1}

AV	0.667***			0.674***	1.030***
	p = 0.000			p = 0.000	p = 0.000
AC		0.218***		−0.019	
		p = 0.00000		p = 0.544	
SV			0.522***		−0.403***
			p = 0.000		p = 0.000
Constant	−0.001	−0.00004	−0.0003	−0.001	−0.001
	p = 0.985	p = 1.000	p = 0.994	p = 0.984	p = 0.981
R^2	0.445	0.048	0.273	0.446	0.477
Adjusted R^2	0.445	0.046	0.272	0.444	0.475

(c) Average Asset Return Correlation - AC_{t+1}

AV	0.239***			0.024	−0.438***
	p = 0.000			p = 0.470	p = 0.00000
AC		0.621***		0.613***	
		p = 0.000		p = 0.000	
SV			0.360***		0.753***
			p = 0.000		p = 0.000
Constant	−0.0002	−0.0001	−0.0002	−0.0001	−0.00003
	p = 0.996	p = 0.998	p = 0.996	p = 0.997	p = 1.000
R^2	0.057	0.387	0.130	0.387	0.167
Adjusted R^2	0.056	0.386	0.128	0.385	0.164

(d) Log Excess Market Return - RET_{t+1}

AV	−0.130***			−0.168***	−0.173*
	p = 0.001			p = 0.0001	p = 0.052
AC		0.049		0.108***	
		p = 0.212		p = 0.010	
SV			−0.107***		0.048
			p = 0.006		p = 0.588
Constant	−0.000	−0.000	−0.000	−0.000	−0.000
	p = 1.000	p = 1.000	p = 1.000	p = 1.000	p = 1.000
N	655	655	655	655	655
R^2	0.017	0.002	0.012	0.027	0.017
Adjusted R^2	0.015	0.001	0.010	0.024	0.014

Notes: ***,**, and * Significant at the 1, 5, and 10 percent levels.

Table IV:Full In Sample Results

The table displays in sample regression results for monthly market variance, correlation and return statistics. SV is the annualized monthly variance of daily CRSP market returns. AV and AC are the monthly average variance and average pairwise correlation of daily returns for the top 500 assets in the CRSP market, as in Pollet and Wilson (2010). RET is the log return of the CRSP market portfolio minus the log return on the 1 month treasury bill. The sample period is from 1926:07 to 2016:12.

(a) Market Return Variance - SV_{t+1}

AV	0.625***			0.551***	0.379***
	p = 0.000			p = 0.000	p = 0.000
AC		0.416***		0.159***	
		p = 0.000		p = 0.000	
SV			0.612***		0.288***
			p = 0.000		p = 0.000
Constant	−0.0003	−0.0001	−0.0002	−0.0003	−0.0003
	p = 0.991	p = 0.998	p = 0.993	p = 0.991	p = 0.991
R^2	0.391	0.173	0.375	0.410	0.413
Adjusted R^2	0.390	0.173	0.374	0.409	0.412

(b) Average Asset Return Variance - AV_{t+1}

AV	0.718***			0.704***	0.745***
	p = 0.000			p = 0.000	p = 0.000
AC		0.358***		0.029	
		p = 0.000		p = 0.232	
SV			0.606***		−0.031
			p = 0.000		p = 0.445
Constant	−0.0003	−0.0001	−0.0002	−0.0003	−0.0003
	p = 0.989	p = 0.998	p = 0.993	p = 0.989	p = 0.989
R^2	0.515	0.128	0.368	0.516	0.516
Adjusted R^2	0.515	0.127	0.367	0.515	0.515

(c) Average Asset Return Correlation - AC_{t+1}

AV	0.383***			0.125***	−0.018
	p = 0.000			p = 0.00001	p = 0.738
AC		0.610***		0.551***	
		p = 0.000		p = 0.000	
SV			0.453***		0.468***
			p = 0.000		p = 0.000
Constant	−0.0002	−0.0001	−0.0002	−0.0002	−0.0002
	p = 0.996	p = 0.996	p = 0.996	p = 0.995	p = 0.996
R^2	0.147	0.372	0.205	0.385	0.205
Adjusted R^2	0.146	0.372	0.204	0.384	0.204

(d) Log Excess Market Return - RET_{t+1}

AV	0.0002			−0.006	0.182***
	p = 0.996			p = 0.857	p = 0.002
AC		0.011		0.014	
		p = 0.724		p = 0.692	
SV			−0.056*		−0.213***
			p = 0.064		p = 0.0003
Constant	−0.00000	−0.00001	0.00002	−0.00001	0.00002
	p = 1.000	p = 1.000	p = 1.000	p = 1.000	p = 1.000
N	1,085	1,085	1,085	1,085	1,085
R^2	0.00000	0.0001	0.003	0.0001	0.012
Adjusted R^2	−0.001	−0.001	0.002	−0.002	0.010

Notes: ***,**, and * Significant at the 1, 5, and 10 percent levels.

Table V:In Sample Robust Results

The table displays in-sample regression results of AC_{t+1}, SV_{t+1} and RET_{t+1} on AV_t. The coeffiecients and standard errors calculated robust to Kandel and Stambaugh (1996) bias using the correction in Amihud and Hurvich (2004). Robust p-values are calculated through t-statistic wild-bootstrap simulation, as in MacKinnon (2002).

(a) AV_t: Sample 1962:06 to 2016:12

	β	t.stat	p
AC_{t+1}	0.241	6.567	0.000
SV_{t+1}	0.550	32.442	0.000
RET_{t+1}	-0.131	-3.494	0.166

(b) AV_t: Sample 1926:07 to 2016:12

	β	t.stat	p
AC_{t+1}	0.384	14.342	0.000
SV_{t+1}	0.627	40.002	0.000
RET_{t+1}	0.000	-0.016	0.562

Table VI:Full Out-of-Sample Results

The table displays out-of-sample expanding window regression results for monthly market variance, correlation and return statistics. SV is the annualized monthly variance of daily CRSP market returns. AV and AC are the monthly average variance and average pairwise correlation of daily returns for the top 500 assets in the CRSP market, as in Pollet and Wilson (2010). RET is the log return of the CRSP market portfolio minus the log return on the 1 month treasury bill. DM is the Diebold and Mariano (1995) statistic measuring for cast accuracy. MSE-F is the mean squared error improvement F-test of in McCracken (2007) and ENC-HLN is the forecast encompassing test of Harvey et al. (1998). In each panel the benchmark forecasts come from a model which uses SV_t to predict the independent variable.

(a) Sample 1970:07 to 2016:12

	DM	MSE-F	ENC-HLN
AC_{t+1}	1.074	109.736***	1
SV_{t+1}	1.53*	29.252***	1**
AV_{t+1}	2.286**	109.333***	1***
RET_{t+1}	1.278	11.801***	1*

(b) Sample 1939:12 to 2016:12

	DM	MSE-F	ENC-HLN
AC_{t+1}	1.604*	46.251***	1**
SV_{t+1}	1.041	21.57***	0.956**
AV_{t+1}	3.104***	198.267***	1***
RET_{t+1}	-2.027	-8.702	0

Notes: ***,**, and * Significant at the 1, 5, and 10 percent levels.

Table VII:Out of Sample Robust Results

The table displays out-of-sample regression results of forecasts using AV_{t+1} as a predictor. Rossi and Inoue (2012) provides the methodology to make the calculations of the out-of-sample accuracy improvements of Diebold and Mariano (1995) and McCracken (2007) and the encompassing test of Harvey et al. (1998). In each panel the benchmark forecasts come from a model which uses SV_t to predict the independent variable.

(a) Robust Expanding Window Results

Stat	Variable	DM	ENC-HLN
R_T	AC_{t+1}	28.532***	6.769***
R_T	SV_{t+1}	8.874***	1.838***
R_T	AV_{t+1}	34.347***	18.197***
R_T	RET_{t+1}	29.124***	4.871***
A_T	AC_{t+1}	19.867***	1.828***
A_T	SV_{t+1}	2.647***	0.949***
A_T	AV_{t+1}	21.751***	10.7***
A_T	RET_{t+1}	13.347***	1.68***

(b) Robust Rolling Window Results

Stat	Variable	DM	ENC-HLN
R_T	AC_{t+1}	27.398***	8.706**
R_T	SV_{t+1}	21.92***	3.973
R_T	AV_{t+1}	34.292***	29.804***
R_T	RET_{t+1}	15.964***	3.884
A_T	AC_{t+1}	8.08***	1.542
A_T	SV_{t+1}	8.218***	2.062
A_T	AV_{t+1}	21.631***	19.449***
A_T	RET_{t+1}	9.209***	1.78

Notes: ***,**, and * Significant at the 1, 5, and 10 percent levels.

Table VIII:Investment Weights

This table displays summary statistics for the time series of investment weights used by both the AV and SV managed portfolio strategies with differenct volatility targets. c_{053} represents targeting the annual volatility of the buy and hold market portfolio over the whole data set, 1926 to 2016. c_{029} and c_{035} target, approximately, 10% and 12% annual return volatility for the AV and SV managed portfolios.

Portfolio	Target	Mean	St. Dev.	Min	Pctl(25)	Median	Pctl(75)	Max
SV	c_{053}	1.290	1.412	0.017	0.455	0.948	1.619	16.193
AV	c_{053}	1.301	0.710	0.033	0.787	1.235	1.694	4.253
SV	c_{029}	0.697	0.762	0.009	0.246	0.512	0.874	8.743
AV	c_{029}	0.702	0.383	0.018	0.425	0.667	0.915	2.296
SV	c_{035}	0.841	0.920	0.011	0.297	0.618	1.055	10.552
AV	c_{035}	0.848	0.463	0.022	0.513	0.805	1.104	2.772

Table IX:Portfolio Performance - Unconstrained

This table displays portfolio performance measures for the AV and SV managed portfolio strategies using c_{053} to target the annual volatility of the buy and hold market portfolio over the whole data set, 1926 to 2016. RET is the average annualized monthly log excess return. Sharpe, Sortino and Rachev are the Sharpe, Sortion and Rachev ratios respectively; Kappa_3 and Kappa_4 are the lower parital skewness and lower partial kurtosis Kappa measures. See section III for details. No constraints are plased on the level of investment in the market portfolio for the AV and SV managed portfolio; the buy and hold strategy always has an investment weight of one in the market. Stars on the lines for the AV and SV managed portfolios indicate a significant postive performance difference between those two portfolios.

Return	Sharpe	Sortino	Kappa_3	Kappa_4	Rachev
5.932	0.319	0.129	0.082	0.061	0.841
8.598	0.462	0.208	0.132	0.097	1.151**
9.677***	0.520*	0.225	0.150*	0.112**	0.972

Notes: ***,**, and * Significant at the 1, 5, and 10 percent levels.

Table X:Portfolio Performance - Constrained

This table displays portfolio performance measures for the AV and SV managed portfolio strategies using c_{029}, c_{035}, and c_{053} to target the annual volatilities of 10%, 12% and equal to the buy hold market portfolio over the whole data set, 1926 to 2016. Performance ratios are calculated for investment constraints of a maximum of 1.5 and 3, 50% and 200% leverage. RET is the average annualized monthly log excess return. Sharpe, Sortino and Rachev are the Sharpe, Sortion and Rachev ratios respectively; Kappa_3 and Kappa_4 are the lower parital skewness and lower partial kurtosis Kappa measures. See section III for details. No constraints are plased on the level of investment in the market portfolio for the AV and SV managed portfolio; the buy and hold strategy always has an investment weight of one in the market. Stars on the lines for the AV and SV managed portfolios indicate a significant postive performance difference between those two portfolios.

(a) 10% Volatility Target

	Constraint - 1.5						Constraint - 3					
Portfolio	Return	Sharpe	Sortino	Kappa_3	Kappa_4	Rachev	Return	Sharpe	Sortino	Kappa_3	Kappa_4	Rachev
SV	4.065	0.461	0.201	0.130	0.097	1.038	4.396	0.454	0.200	0.127	0.094	1.096*
AV	5.196***	0.522**	0.225**	0.150**	0.112**	0.966	5.225***	0.520*	0.225*	0.150**	0.112**	0.972

(b) 12% Volatility Target

	Constraint - 1.5						Constraint - 3					
Portfolio	Return	Sharpe	Sortino	Kappa_3	Kappa_4	Rachev	Return	Sharpe	Sortino	Kappa_3	Kappa_4	Rachev
SV	4.735	0.470	0.205	0.133	0.098	1.027	5.219	0.452	0.198	0.127	0.094	1.083*
AV	6.081***	0.516*	0.221*	0.147*	0.110**	0.945	6.306***	0.520**	0.225**	0.150**	0.112**	0.972

(c) Buy and Hold Volatility Target

	Constraint - 1.5						Constraint - 3					
Portfolio	Return	Sharpe	Sortino	Kappa_3	Kappa_4	Rachev	Return	Sharpe	Sortino	Kappa_3	Kappa_4	Rachev
BH	5.932	0.319	0.129	0.082	0.061	0.841	5.932	0.319	0.129	0.082	0.061	0.841
SV	6.171	0.467	0.200	0.128	0.091	0.982	7.606	0.456	0.199	0.129	0.096	1.044
AV	7.885***	0.486	0.204	0.133	0.097	0.896	9.677***	0.522**	0.226**	0.150**	0.112**	0.969

Notes: ***,**, and * Significant at the 1, 5, and 10 percent levels.