Does Idiosyncratic Volatility Proxy for Risk Exposure?

Zhanhui Chen
Nanyang Technological University

Ralitsa Petkova
Purdue University

We decompose aggregate market variance into an average correlation component and an average variance component. Only the latter commands a negative price of risk in the cross section of portfolios sorted by idiosyncratic volatility. Portfolios with high (low) idiosyncratic volatility relative to the Fama-French (1993) model have positive (negative) exposures to innovations in average stock variance and therefore lower (higher) expected returns. These two findings explain the idiosyncratic volatility puzzle of Ang et al. (2006, 2009). The factor related to innovations in average variance also reduces the pricing errors of book-to-market and momentum portfolios relative to the Fama-French (1993) model. (*JEL* G12)

In an influential study, Ang, Hodrick, Xing, and Zhang (2006, 2009; AHXZ hereafter) show that stocks with high idiosyncratic risk, defined as the standard deviation of the residuals from the Fama-French (1993) model, have anomalously low future returns.[1] This finding is puzzling in light of theories that suggest that idiosyncratic volatility (denoted as IV) should be irrelevant or positively related to expected returns.[2]

If a factor is missing from the Fama-French model, the sensitivity of stocks to the missing factor times the movement in the missing factor will show up in the residuals of the model. Firms with greater sensitivities to the missing factor

---

We thank Geert Bekaert (editor), two anonymous referees, and seminar participants at the Norwegian Business School (BI) and Texas A&M University for many valuable comments and suggestions. Chen acknowledges financial support from a Nanyang Technological University Start-up Grant. Send correspondence to Ralitsa Petkova, Finance Department, Krannert School of Management, Purdue University, 403 W. State Street, West Lafayette, IN 47907; telephone: (216) 235-0558. E-mail: rpetkova@purdue.edu.

[1] Some articles challenge this result from an empirical methodology perspective. For example, Fu (2009) argues that the estimate in AHXZ (2006) is not a good proxy for expected idiosyncratic risk and shows that conditional idiosyncratic volatility computed from an EGARCH model is positively related to expected returns. Both Fu (2009) and Huang et al. (2010) demonstrate that return reversals from stocks with high idiosyncratic risk in the last month lead to AHXZ's results. Bali and Cakici (2008) show that the idiosyncratic risk puzzle is not robust for different portfolio weighting schemes and sample data choices. However, Barinov (2010) points out a sample selection bias in Bali and Cakici (2008).

[2] The CAPM suggests that idiosyncratic risk should not be priced, but Merton (1987) argues that if investors cannot diversify properly, then idiosyncratic risk should be rewarded with higher expected returns.

doi:10.1093/rfs/hhs084

should therefore have larger idiosyncratic volatilities relative to the Fama-French model, everything else being equal. AHXZ follow this argument and, motivated by the Intertemporal Capital Asset Pricing Model (ICAPM), include aggregate market variance as a potential missing factor in the Fama-French model. They find that market variance is a significant cross-sectional asset pricing factor but the spread in the market variance loadings between high and low IV stocks cannot fully explain the IV puzzle. In this article, we address an important but still unanswered question: Is there a risk-based explanation behind the low average returns of stocks with high idiosyncratic volatility?

A risk-based explanation behind the IV puzzle needs to: 1) identify a risk factor missing from the Fama-French model and show that exposure to this risk factor is priced; and 2) show that the loadings of high IV stocks relative to the missing factor differ from those of low IV stocks, and the spread in loadings is large enough to explain the difference in average returns between high and low IV stocks. We provide evidence consistent with both of these objectives.

First, motivated by the intertemporal models of Campbell (1993, 1996) and Chen (2003), we focus on state variables that govern market variance.[3] To do that, we decompose aggregate market variance as market variance $\approx$ average stock variance $*$ average stock correlation. Therefore, exposure to aggregate market variance has two components as well: exposure to average variance risk and exposure to correlation risk. We estimate separately the loadings to average variance and average correlation of portfolios sorted by size and IV. For the period from July 1966 to December 2009, only exposure to average variance (and *not* correlation) is priced, in addition to the Fama-French factors, and its price of risk is negative.

Second, we show that portfolios with high (low) IV have positive (negative) loadings with respect to innovations in average stock variance and thus lower (higher) expected returns. This difference in the loadings between high and low IV stocks, combined with the negative premium for average stock variance, completely explains the average return difference between high and low IV assets. For example, among small stocks, the realized Fama-French alpha of the high-minus-low IV portfolio is -1.79% per month. This alpha is completely explained by the combined effect of a negative average variance premium of 7.7% per month and a difference in the average variance loadings of high (low) IV stocks of 0.24 ($-7.7\%*0.24=-1.85\%$). Similar results hold for medium and large stocks.

Finally, we show that in the presence of loadings with respect to innovations in average variance, individual idiosyncratic risk does not affect expected returns. This result holds for a set of portfolios sorted by IV and the

[3] These models differ from asset pricing models that use important macroeconomic variables as sources of risk. Instead, Campbell (1993) and Chen (2003) propose that state variables that predict investment opportunities in the time series should be used as risk factors in the cross section. The advantage of this approach is that it provides a link between time-series and cross-sectional return predictability. Variables that indicate that investment opportunities deteriorate should command negative prices of risk.

cross section of individual stock returns. It is robust to the inclusion of other stock characteristics such as size, book-to-market, and past returns.

The main message of this article is that although aggregate market variance is priced cross-sectionally (as AHXZ find), only one component of it (average variance) is priced in the cross section of portfolios sorted by IV. Exposure to average correlation is not an important determinant of the average returns of these portfolios. Because of the confounding effect of correlations in aggregate market variance, AHXZ find that loadings with respect to aggregate market variance cannot explain the IV puzzle. The novel result in our article is that once the effects of average variance and average correlation on stock returns are disentangled, the role of average variance in explaining the IV puzzle clearly stands out. To the best of our knowledge, this has not been documented before.

Why is the correlation component of total market variance not priced in the cross section of returns, while the variance component is priced? We offer two explanations. First, Campbell (1993) shows that any variable that forecasts future market returns or volatility is a good candidate state variable for cross-sectional pricing. We find that average variance predicts lower future market returns and higher future market variance. Therefore, high average variance worsens the investor's risk-return trade-off and commands a risk premium. Average correlation, on the other hand, predicts higher future market returns and higher future market variance. Therefore, the overall effect of average correlation on the risk-return trade-off is ambiguous.

Second, we find that high (low) IV stocks have high (low) research and development expenditure (R&D), which is considered to be an indicator for the presence of real options. Therefore, a large portion of the value of high IV stocks may come from their individual real options. Recent evidence suggests that individual options are not significantly exposed to correlation risk. Namely, Driessen et al. (2009) find that individual option returns are much less dependent on correlation shocks compared to index option returns. Intuitively, index options are expensive and earn low returns because they offer a valuable hedge against correlation increases and insure against the risk of a loss in diversification benefits. The same does not hold for individual options. Therefore, our finding that average correlation risk is not priced in the cross section of assets sorted by IV is consistent with Driessen et al. (2009).

We also examine why the loadings of high IV stocks with respect to average variance are positive, conditional on their Fama-French betas. This indicates that in times of high volatility, high IV stocks perform better than predicted by the Fama-French model. Given that these stocks have high R&D expenditures, our results are consistent with predictions from the real options literature. Theoretical models from this literature predict that the value of a real option should be increasing in the volatility of the underlying asset. Therefore, the value of a firm with a lot of real options should be less negatively affected by increasing volatility, both idiosyncratic and systematic. This makes high IV stocks good hedges for times of increasing market-wide variance.

To provide an economic interpretation of average variance as a pricing factor, we relate it to aggregate liquidity, the variance of consumption growth, and the aggregate market-to-book ratio, which is a measure of aggregate growth options. We show that the component of average variance projected on these three variables has the same pricing implications as total average variance.

In summary, our results contribute to the understanding of the IV puzzle documented by AHXZ (2006). AHXZ (2009) show that their earlier findings are robust, and they provide supporting out-of-sample evidence from 23 different countries. After documenting that high-minus-low IV portfolios comove across countries, AHXZ (2009) conclude that a missing risk factor is the most likely explanation for the IV puzzle. Our article contributes to the literature by directly examining the hypothesis that exposure to a risk factor, which is missing from the Fama-French model, explains the IV effect. We provide empirical support for this hypothesis. We find that high IV assets have low expected returns since they provide hedging opportunities relative to increases in average stock variance. When average stock variance goes up, investment opportunities deteriorate. Therefore, investors are willing to pay an insurance premium for high IV stocks since their payoff is less negative when average return variance is large.

The rest of this article is organized as follows. Section 1 discusses the relation between idiosyncratic risk defined relative to the Fama-French model and exposure to a missing risk factor. It argues that the factors missing from the Fama-French model are the two components of market variance. In Section 2, we compute the two separate components of aggregate market variance, average variance and average correlation, and examine their time-series properties. Section 3 is the main section of the article. It contains cross-sectional regressions that estimate factor prices of risk for average variance and correlation using portfolios sorted by size and IV. Section 4 examines the performance of the average variance factor in the cross section of alternative test assets. Section 5 explores the characteristics of stocks that have different loadings with respect to average variance and provides an economic interpretation of the average variance factor. Section 6 provides a comparison between several alternative explanations of the IV puzzle and ours, and Section 7 concludes. The Appendix contains some further extensions and robustness checks.[4]

1. The Fama-French Model Augmented with Average Variance and Average Correlation

1.1 Idiosyncratic volatility as a proxy for an exposure to a missing factor

The following analysis summarizes the relation between idiosyncratic volatility relative to the Fama-French model and loadings with respect to a missing factor.

---

[4] All appendices are available online at http://www.sfsrfs.org.

The analysis follows MacKinlay and Pastor (2000). Let R_{it} denote the excess return on asset i in period t. The linear relation between the asset returns and the risk factors is

$$R_{it} = \alpha_i + \beta_i R_{Mt} + h_i HML_t + s_i SMB_t + \varepsilon_{it}, \tag{1}$$

where R_M, HML, and SMB are the excess return on the market portfolio, the value factor, and the size factor, respectively, and α_i is the mispricing of asset i.

If exact pricing does not hold due to a missing factor, then α_i is not zero. In that case, α_i can be shown to be related to the variance of ε_{it}, using the optimal orthogonal portfolio op.[5] It is optimal since it can be combined with the factor portfolios to form the tangency portfolio. It is also orthogonal to the factor portfolios.

Since op is optimal, when it is included in the Fama-French model, the intercept α_i disappears. In addition, the orthogonality property of op preserves the coefficient β, h, and s unchanged. Due to these properties, op can be thought of as an omitted factor in a linear factor model. When the omitted factor is added to the model, the following relation holds:

$$R_{it} = \beta_{opi} R_{opt} + \beta_i R_{Mt} + h_i HML_t + s_i SMB_t + u_{it}, \tag{2}$$

where β_{opi} is the sensitivity to the omitted factor op, and R_{opt} is the return on portfolio op. The link between β_{opi} and the variance of ε_{it} results from comparing Equations (1) and (2). If we equate the variance of ε_{it} with the variance of $\beta_{opi} R_{opt} + u_{it}$, we have

$$Var(\varepsilon_{it}) = \beta_{opi}^2 Var(R_{opt}) + Var(u_{it}). \tag{3}$$

Equation (3) reveals that if an asset has a significant mispricing relative to the Fama-French model, then there is a positive relation between the idiosyncratic volatility relative to the model, $Var(\varepsilon_{it})$, and the asset's exposure to the missing factor, β_{opi}^2. Therefore, the measure of idiosyncratic volatility from the misspecified model in Equation (1) depends on the asset's beta with respect to the missing factor and the true idiosyncratic volatility, $Var(u_{it})$, relative to the correct model in Equation (2).

MacKinlay and Pastor (2000) point out that if α_i is related to a missing factor, then there should be a positive relation between this mispricing and the residual variance. They state that in the absence of such a relation, mispriced securities could be collected to form asymptotic arbitrage opportunities. Using the fact that $\alpha_i = \beta_{opi} E(R_{opt})$, we can further expand Equation (3):

$$Var(\varepsilon_{it}) = \frac{\alpha_i^2}{S^2(R_{opt})} + Var(u_{it}), \tag{4}$$

where $S^2(R_{opt})$ is the squared Sharpe ratio of the missing factor. Equation (4) reveals that when a factor is missing from the Fama-French model,

[5] See MacKinlay (1995) for a more detailed discussion on the optimal orthogonal portfolio.

the resulting mispricing α_i^2 should be positively correlated with the residual variance $Var(\varepsilon_{it})$.

Therefore, if an asset has a significant alpha relative to the Fama-French model, then AHXZ's measure of IV may proxy for the asset's exposure to a missing risk factor. We find that for every month in our sample, a large percentage of stocks have significant alphas relative to the Fama-French model during the period when it is used to compute their idiosyncratic volatilities.

The sensitivity with respect to the omitted factor is squared in Equation (3). This might suggest that only the magnitude of the loading is important, but that is misleading. The sign of the loading is crucial. AHXZ show that high IV portfolios have negative alphas with respect to the Fama-French model after portfolio formation, while the alphas of low IV portfolios are positive. This suggests that the model overestimates the expected returns of high IV stocks, and underestimates them for their low IV counterparts. If a missing factor is to account for the IV puzzle, then the product of the price of risk of the missing factor and the exposure to this factor should account for the mispricing for both high and low IV stocks. Therefore, their betas with respect to the missing factor must have opposite signs.

1.2 What is the factor missing from the Fama-French model?

In the discrete-time version of the ICAPM, expected returns are linear functions of covariances with state variables that describe investment opportunities. Campbell (1993) and Chen (2003) develop asset pricing models that specify the identity of the ICAPM state variables. Namely, they show that expected returns depend on covariances with variables that predict the market return and variance. The literature on the time series of market variance shows that aggregate variance has two separate components, one related to stock variances and the other related to stock correlations. We combine these insights from the market variance and the asset pricing literature and conjecture that the factors missing from the Fama-French model are the two components of market variance.

The two components of market variance behave differently. Driessen et al. (2009) point out that there is a priced risk factor in index-based variance, like VIX, that is not present in individual stock variance. This factor is the stochastic correlation between stocks. Therefore, the VIX index, and more generally, total market variance, can be decomposed into average variance and average correlation. Driessen et al. (2009) show that individual options are not exposed to correlation risk, while index options are. Pollet and Wilson (2010) show that average correlation predicts the market return, while average variance does not.

Motivated by the findings of Driessen et al. (2009) and Pollet and Wilson (2010), we decompose market variance into an average variance and an average correlation component. It is interesting to analyze the pricing abilities of both components not only in options but also in the cross section of equity returns. We examine to what extent cross-sectional differences in expected returns for portfolios sorted by IV are driven by differences in exposure to average variance or by differences in exposure to average correlation.

Let M denote the value-weighted market portfolio of all stocks where w_{it} is the weight of asset i at time t in the market. Then, the variance of the market return is

$$V_t = \sum_{i=1}^{N}\sum_{j=1}^{N} w_{it} w_{jt} Corr(R_{it}, R_{jt}) SD(R_{it}) SD(R_{jt}), \tag{5}$$

where N stands for the number of stocks in the market portfolio. We employ a useful approximation to decompose total market variance into an average variance and an average correlation component. The approximation states that market variance is the product of the average variance of all individual stocks and the average correlation between all pairs of stocks. We define AV_t to be the cross-sectional average variance for the N stocks in the market portfolio at time t:

$$AV_t = \sum_{i=1}^{N} w_{it} V(R_{it}), \tag{6}$$

and AC_t to be the cross-sectional average correlation between all pairs of stocks at time t:

$$AC_t = \sum_{i=1}^{N}\sum_{j=1}^{N} w_{it} w_{jt} Corr(R_{it}, R_{jt}). \tag{7}$$

Assuming that all stocks have the same individual variances, expression (5) simplifies to

$$V_t = AV_t AC_t. \tag{8}$$

The intuition from Campbell (1993) and Chen (2003) suggests that investors would want to hedge against changes in average variance and average correlation because they affect market variance. To capture that intuition, we adopt the linear multifactor framework of the discrete-time ICAPM. Given the linearity of the ICAPM framework, to examine the asset pricing implications of Equation (8) we consider a linear approximation around the expectations of average variance, $E(AV_t)$, and average correlation, $E(AC_t)$. We obtain the following expression for total market variance:

$$V_t = -c_0 + c_1 AV_t + c_2 AC_t, \tag{9}$$

where $c_0 = E(AV_t)E(AC_t)$, $c_1 = E(AC_t)$, and $c_2 = E(AV_t)$. According to (9), market variance changes are driven by shocks to individual variances and shocks to correlations. Therefore, the equilibrium unconditional expected excess return on asset i is

$$E(R_{it}) = \gamma_M \beta_{Mi} + \gamma_{HML} \beta_{HMLi} + \gamma_{SMB} \beta_{SMBi} + \gamma_{\triangle AV} \beta_{\triangle AVi} + \gamma_{\triangle AC} \beta_{\triangle ACi}, \tag{10}$$

where the γ terms are the prices of risk related to the market, HML, SMB, changes in AV, and changes in AC, respectively, and the βs are factor loadings.

The implication of the model in Equation (10) is that assets with different loadings with respect to the risk factors have different average returns. Our goal is to examine whether portfolios with high and low IV have loadings with opposite signs relative to the two separate components of market variance. In addition, we are interested in the extent to which exposure to these two types of shocks is priced in the cross section of portfolios sorted by IV.

It is important to emphasize the difference between IV and AV. The former, IV, is a stock-specific volatility characteristic that is negatively related to average returns. The latter, AV, is a market-wide volatility variable that contains both systematic and idiosyncratic components. Even though both IV and AV are measures of volatility, it does not automatically follow that stocks with high IV necessarily have high $\triangle AV$ loadings. This is the case since AV also contains systematic volatility components.

2. Estimation of Average Variance and Average Correlation

2.1 Data and descriptive statistics

We use monthly and daily stock returns from CRSP for the period from July 1963 to December 2009. We include all ordinary common equities (share codes 10 or 11) on the NYSE, AMEX, and NASDAQ. The market portfolio is the value-weighted NYSE/AMEX/NASDAQ index return. Excess returns are computed relative to the 30-day T-bill rate.

Each month, we compute the variance of the market portfolio using within-month daily returns:

$$V_{Mt} = \sum_{d=1}^{D_t} R_{Md}^2 + 2\sum_{d=2}^{D_t} R_{Md} R_{Md-1}, \tag{11}$$

where D_t is the number of days in month t and R_{Md} is the portfolio's return on day d. The second term on the right-hand side adjusts for the autocorrelation in daily returns, following French, Schwert, and Stambaugh (1987).

Next, we derive the two separate parts of market variance. Average stock variance, AV_t, is the value-weighted average of monthly stock variances using daily data:

$$AV_t = \sum_{i=1}^{N_t} w_{it} \left[\sum_{d=1}^{D_t} R_{id}^2 + 2\sum_{d=2}^{D_t} R_{id} R_{id-1} \right], \tag{12}$$

where R_{id} is the return of stock i in day d and N_t is the number of stocks that exist in month t.[6] This measure is based on total stock variance, and therefore, it includes both systematic and idiosyncratic components.

[6] We also compute the arithmetic average of monthly stock variances, $AV_t = \frac{1}{N_t}\sum_{i=1}^{N_t}\left[\sum_{d=1}^{D_t} R_{id}^2 + 2\sum_{d=2}^{D_t} R_{id} R_{id-1}\right]$, and obtain similar results. It is possible that the second term in Equation (12) may dominate the first term when there are negative autocorrelations, which produces negative estimates of variance. For these stocks, we follow Goyal and Santa-Clara (2003) and use only the first term in the calculation.

Table 1
Market variance and its components: Descriptive statistics and time-series regressions

Panel A: Descriptive statistics

Variable	Mean	Median	Std. Dev.	Min	Max	AR(1)	AR(2)	AR(3)
V	0.0023	0.0013	0.0045	0.0001	0.0671	0.3653	0.1733	0.1292
AV	0.0104	0.0079	0.0097	0.0024	0.1030	0.6478	0.5153	0.4775
AC	0.2086	0.1955	0.1054	0.0174	0.6530	0.6358	0.5269	0.4865

Panel B: Time-series regressions

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$Constant$	0.00	−0.24	−0.18	−0.33	−0.09	−0.11	0.32	−0.42
	(0.22)	(−2.25)	(−2.67)	(−5.52)	(1.25)	(−1.28)	(0.68)	(−0.59)
$AV_t * AC_t$	87.52							
	(13.32)							
AC_t		2.26		0.93				
		(3.84)		(3.64)				
AV_t			39.32	35.14				
			(5.16)	(4.66)				
AV_{t-1}					17.01	17.29	−65.31	−59.42
					(3.23)	(3.00)	(−3.43)	(−2.81)
AC_{t-1}					0.71	0.58	3.75	2.12
					(3.02)	(2.73)	(1.68)	(1.04)
DIV_{t-1}						2.58		34.88
						(0.89)		(1.25)
$TERM_{t-1}$						−2.36		19.89
						(−1.75)		(0.87)
DEF_{t-1}						5.42		42.43
						(1.17)		(0.63)
RF_{t-1}						−18.64		−138.30
						(−1.56)		(−0.82)
R^2	0.93	0.29	0.73	0.77	0.22	0.22	0.02	0.03

Panel C: Factor means, volatilities, and correlations

Factor	Mean	Std. Dev.	HML	SMB	$\triangle AV$	$\triangle AC$	PAV	PAC
R_M	0.42	4.51	−0.33	0.30	−0.16	0.12	−0.32	0.37
HML	0.42	2.94		−0.25	0.02	−0.08	0.01	−0.26
SMB	0.25	3.19			−0.16	−0.01	−0.35	−0.02
$\triangle AV$	0.00	4.50				−0.24	0.35	−0.09
$\triangle AC$	0.00	4.50					−0.05	0.20
PAV	−0.63	1.59						−0.26
PAC	0.15	0.88						

Panel A shows descriptive statistics for market variance, V, average stock variance, AV, and average stock correlation, AC. V is calculated as in Equation (11), AV is computed as in Equation (12), and AC is computed as the cross-sectional average of the pairwise correlations of daily returns during each month for all stocks trading on the NYSE/NASDAQ/AMEX. AR(i) denotes the i^{th}-order autocorrelation of each series. Panel B shows contemporaneous (Columns (1)–(4)) or predictive (Columns (5)–(6)) time-series regressions for V, and predictive regressions (Columns (7)–(8)) for the excess market return, R_M. The explanatory variables are $AV * AC$, AV, AC, DIV (dividend yield), $TERM$ (term spread), DEF (default spread), and RF (30-day T-bill rate). All coefficients are multiplied by 100. Newey-West t-statistics with six lags are in parentheses. Panel C reports the sample means (in %), volatilities (in %), and correlations for the Fama-French factors, innovations in AV and AC, and the mimicking portfolios for innovations in average variance and average correlation, PAV and PAC, computed as described in Section 3.4. The sample period is from July 1963 to December 2009.

Average stock correlation, AC_t, is the value-weighted average of pairwise correlations of daily returns during each month for all stocks. Summary statistics for value-weighted market variance, average stock variance, and average stock correlation are provided in Panel A of Table 1.

Panel A of Figure 1 plots the time series of monthly market variance (solid line) and the product of average variance and average correlation (dotted line) for the period July 1963 to December 2009. The figure shows that the two series track each other very closely. The correlation between the two is 97%. Panel B plots the time series of average variance, while Panel C plots average correlation. The sample correlation between AV and AC is 41%. The series do not exhibit a significant trend over time.

In Table 1, Column (1) of Panel B reports a contemporaneous OLS regression of market variance from Equation (11) on the product of average variance from Equation (12) and average correlation. We use Newey-West t-statistics with six lags. The R^2 of the regression is 93%, which indicates that the variation in market variance is almost entirely captured by the product of contemporaneous average variance and average correlation.

Columns (2) and (3) in Table 1 present estimates of the relative importance of average variance and average correlation for changes in market variance. Column (2) shows that average correlation accounts for 29% of the variation in market variance, while Column (3) shows that average variance accounts for 73%. When both AV and AC are included in the regression in Column (4), they explain 77% of the contemporaneous movements in market variance. The results in Column (4) indicate that the linearization in Equation (9) is reasonable because we are able to explain most of the variation in total market variance. Furthermore, they reveal that the major component of total market variance is average stock variance.

Next, we analyze the ability of AV and AC to predict future market variance. Column (5) of Panel B in Table 1 reports a predictive OLS regression of market variance on average variance and average correlation. Both variables predict higher market variance in the next period. The R^2 of the regression is 22%, and the two variables are jointly significant. If the only variable in the regression is AV, the explanatory power of the model is 19%. In Column (6), we control for the aggregate dividend yield (DIV), term spread ($TERM$), default spread (DEF), and the short-term T-bill rate (RF). DIV is computed as the sum of aggregate dividends over the last 12 months, divided by the level of the market index, $TERM$ is the difference between the yields of a ten-year and a one-year government bond, and DEF is the difference between the yields of long-term corporate Baa and Aaa bonds. Bond yields are from the FRED database of the Federal Reserve Bank of St. Louis. Average variance and average correlation remain significant predictors of aggregate market variance. Average stock variance appears to be the dominant predictor of realized market variance.[7]

[7] We also augment the predictive regression from above with past values of realized market variance, V, orthogonalized to AV and AC. The goal is to test whether there are some remaining components of market variance, which are not captured previously. The results show that past values of realized market variance do not contribute any explanatory power over and above the six predictive variables from Column (6). The results

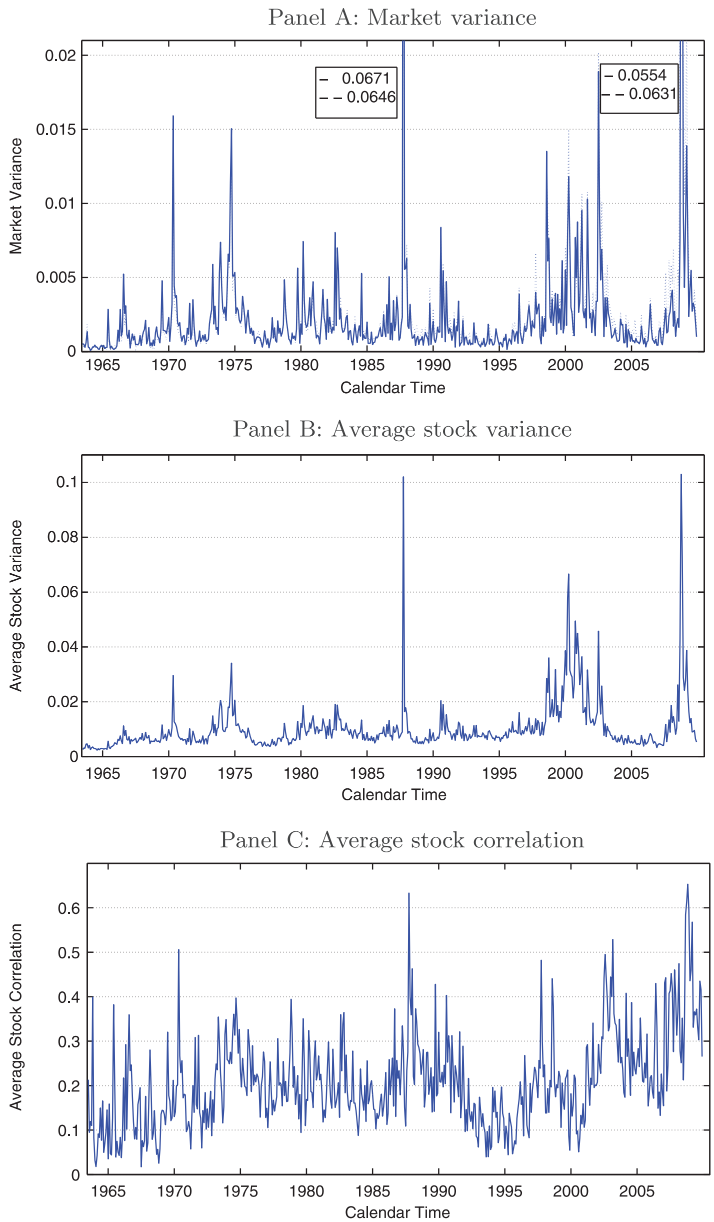

Figure 1
Market variance and its components
Panel A plots the monthly variance of the market portfolio (solid line) and the product of average stock variance and average stock correlation (dotted line). Market variance is calculated using Equation (11), and average stock variance is computed using Equation (12). Average stock correlation is computed as the value-weighted cross-sectional average of the pairwise correlations of daily returns during each month for all stocks trading on the NYSE/NASDAQ/AMEX. In Panel A, the y-axis scale is cut off at 0.02. The values outside the scale are presented in text boxes. Panels B and C plot separately average stock variance and average stock correlation, respectively. The sample period is from July 1963 to December 2009.

Columns (7) and (8) of Panel B in Table 1 examine the ability of average variance and average correlation to predict future market returns. Column (7) shows that AV is significantly negatively related to the one-month-ahead market return. In contrast, AC is positively related to future market returns, but the relation is not significant. Similar results hold in Column (8) when we control for other commonly used predictive variables. The R^2 of the predictive regression is comparable to other studies that analyze the predictability of the monthly market return.

Pollet and Wilson (2010) also document that AV is negatively related to future market returns, while AC is positively related. However, they find that only the latter relationship is significant. This is in contrast to our finding that AV is the only significant predictor of the excess market return.[8] The difference in significance between our results and theirs could stem from using different sample periods, different data frequency, and different sets of stocks to compute AV and AC. Namely, Pollet and Wilson (2010) use quarterly data and the 500 largest stocks. We use all stocks to compute AV and AC since our main focus is on explaining the cross section of stock returns that contains stocks with various market capitalizations.

The negative relation between AV and future market returns may be a result of the positive correlation between AV and the aggregate market-to-book ratio (51% in our sample). The market-to-book ratio is closely related to firms' growth opportunities, and it is also a negative predictor of future market returns. We explore the relation between AV and aggregate market-to-book in more detail in Section 5.3.

The predictive regressions in Panel B of Table 1 have implications for the cross-sectional pricing of AV. Given that AV is a negative predictor of future market returns and a positive predictor of future market variance, its role as a pricing factor can be interpreted in the context of Campbell (1993). Campbell suggests that a positive shock to any variable that predicts a decrease in the expected market return would signal that investors face deteriorating investment opportunities. Chen (2003) extends Campbell's (1993) results and shows that investment opportunities also depend on movements in market variance. Since AV predicts higher future market variance, positive shocks to AV represent deterioration in investment opportunities along the risk dimension as well. This in turn causes risk-averse investors to increase precautionary savings and reduce current consumption. Therefore, positive shocks to AV indicate that investors will face lower expected returns and higher risk in the future. Such a variable should command a negative price of risk in

---

are not surprising since average stock variance and correlation are more persistent than market variance, with first-order autocorrelations of 0.65 and 0.64, respectively (Panel A of Table 1).

[8] Guo and Savickas (2008) examine a predictive regression that contains AV and aggregate market variance. They find that AV is a significant and negative predictor of the excess market return in the United States and G7 countries.

the cross section of expected returns. Assets that pay off well when shocks to AV are positive provide a hedge against worsening investment opportunities and should earn lower expected returns.

Similarly, the cross-sectional pricing of AC should be related to its ability to predict investment opportunities. Given that AC is a positive predictor of future market returns and a positive predictor of future market variance, its role as a pricing factor is ambiguous.

If portfolios with high (low) IV relative to the Fama-French model have positive (negative) loadings with respect to changes in AV, then they should have lower (higher) expected returns. If IV proxies for exposure to average variance, then IV should have no additional explanatory power for average returns over and above loadings to average variance. As we show later, these predictions are supported for the case of average variance. In the sample that we examine, average correlation does not appear to be priced. This is consistent with the previous results, which show that AC predicts both higher future returns and higher future aggregate risk.

2.2 Extracting the innovations in average variance and average correlation

To test the model in Equation (10), we need to estimate the innovations in average variance and average correlation. We adopt the vector autoregressive (VAR) approach of Campbell (1996) and specify a state vector z_t that contains the excess market return, HML, SMB, AV, and AC. The demeaned vector z_t follows a first-order VAR:

$$z_t = Az_{t-1} + u_t. \tag{13}$$

The residuals in the vector u_t are the innovation terms that will be used as risk factors.

The innovations at each time t are computed by estimating the VAR using data available up to time t. This eliminates a potential look-ahead bias if the full sample is used to estimate the VAR. The first VAR in the series contains 36 months, and the first observation for the innovation factors is for July 1966.

Campbell (1996) emphasizes that it is hard to interpret estimation results for a VAR factor model unless the factors are orthogonalized and scaled in some way. Following Campbell (1996), we triangularize the VAR system in Equation (13) so that the innovation in the excess market return is unaffected and the orthogonalized innovation in AV is the component of the original AV innovation orthogonal to the excess market return, HML, and SMB. The orthogonalized innovation in AC is the component of the original AC innovation orthogonal to the excess market return, HML, SMB, and AV, and so on. We also scale all innovations to have the same variance as the innovation in the excess market return. The variables in the VAR system are ordered so that the resulting factors are easy to interpret. The orthogonalized innovation to AV is a change in average stock variance with no change in the stock return, HML and SMB. Thus, it can be interpreted as a shock to average stock variance. Similarly, the orthogonal innovation to AC measures shocks to average correlation that are orthogonal to stock returns, stock variance, HML, and SMB.[9] Panel C of Table 1 reports the mean values, the volatilities, and the correlations between the Fama-French factors and innovations in AV and AC.

Da and Schaumburg (2011) construct a factor similar to innovations in average variance. Their factor performs well in explaining the cross section of returns across equity portfolios, options, and corporate bonds. However, they do not study the idiosyncratic volatility puzzle and the relation between their volatility factor and other macroeconomic variables.

3. The Cross Section of Portfolios Sorted by Size and Idiosyncratic Volatility

3.1 Revisiting the idiosyncratic volatility puzzle

We begin by documenting that the IV effect exists in our sample and that it cannot be explained by exposure to total market variance.

Every month, we sort stocks into five size quintiles and then we further sort them by IV relative to the Fama-French model. We use NYSE size breakpoints to avoid the small size issues noted in Bali and Cakici (2008). Monthly IV is computed as the standard deviation of the residuals from a Fama-French (1993) regression based on daily returns within the month. At least 15 daily observations are required in estimating IV, except on 9/2001 when only 10 observations are required. We form 25 value-weighted portfolios and record their monthly returns for the period from July 1963 to December 2009. These portfolios represent our basic set of test assets.[10]

Panel A of Table 2 reports the Fama-French alphas of the 25 portfolios. High (low) IV portfolios have negative (positive) Fama-French alphas. The difference in alphas between high and low IV stocks is statistically significant in size quintiles 1, 2, and 3. The average difference in alphas between high and low IV portfolios across all size quintiles is -0.75%, with a t-statistic of -4.54.

Next, we augment the Fama-French model with total market variance to test whether this model captures the negative IV premium in the cross section of 25 size-IV portfolios. We estimate a VAR system, as described in Section 2.2, with the excess market return, HML, SMB, and total variance, V. The innovations in market variance from the VAR system are used as risk factors in the cross section of returns. We estimate prices of risk using the Fama-MacBeth (1973) two-stage method. In the first stage, betas are estimated over the full sample as the slope coefficients from the following return-generating process:

$$R_{it}=\alpha_i+\beta_{Mi}R_{Mt}+\beta_{HMLi}HML_t+\beta_{SMBi}SMB_t+\beta_{\triangle Vi}\triangle V_t+\varepsilon_{it}, \qquad (14)$$

9 We find similar results using a different ordering of the VAR system. In addition, including the predictive variables from Panel B of Table 1 in the VAR produces similar results.

10 Results for equally weighted portfolios are similar and are available upon request.

Table 2
Portfolios sorted by size and idiosyncratic volatility: Revisiting the idiosyncratic volatility puzzle

Panel A: Fama-French alpha

	Low IV	2	3	4	High IV	
			α_{FF}			$diff$
Small	0.21*	0.22*	−0.06	−0.54*	−1.58*	−1.79*
2	0.17	0.22*	0.13	0.04	−0.74*	−0.91*
3	0.04	0.18*	0.11	0.03	−0.46*	−0.50*
4	0.01	0.04	0.14	0.10	−0.27*	−0.28
Large	0.11	0.09	0.09	−0.08	−0.17	−0.28

Panel B: Fama-French model augmented with $\triangle V$

γ_0	γ_M	γ_{HML}	γ_{SMB}	$\gamma_{\triangle V}$	γ_{ivol}	R^2
−0.14	0.50	0.19	0.20	−7.27		0.71
(−1.95)	(2.26)	(0.80)	(1.22)	(−4.21)		
−0.16	0.76	0.48	0.39	−4.00	−4.56	0.89
(−1.85)	(3.48)	(2.45)	(2.83)	(−3.78)	(−3.71)	

Panel C: Loadings on $\triangle V$

	Low IV	2	3	4	High IV	
			$\beta_{\triangle V}$			$diff$
Small	−0.09*	−0.08*	−0.07	−0.05	−0.02	0.07
2	−0.10*	−0.07*	−0.08*	−0.05	−0.07	0.03
3	−0.05*	−0.05*	−0.07*	−0.06*	−0.03	0.02
4	−0.03	−0.03	−0.03*	−0.03	−0.04	0.01
Large	0.03*	0.04*	0.04*	0.03*	0.01	−0.02

Panel A presents the Fama-French alphas (in % per month) of 25 portfolios sorted by size and idiosyncratic volatility (IV). The last column of Panel A reports the difference in alpha between high and low IV stocks within each size quintile. Panels B presents Fama-MacBeth cross-sectional regressions using the excess returns of 25 size-IV portfolios. The factor betas, which are the independent variables in the regressions, are computed over the full sample. The model is the Fama-French model augmented with innovations in total market variance ($\triangle V$). The panel examines whether portfolio-level idiosyncratic volatility, $ivol$, has incremental explanatory power in the Fama-French model augmented with $\triangle V$. The variable $ivol$ is lagged one month relative to excess returns. The adjusted R^2 follows Jagannathan and Wang (1996). The t-statistics are in parentheses and adjusted for errors-in-variables, following Shanken (1992). All coefficients in Panel B are multiplied by 100, and the market portfolio, HML, and SMB are included among the test assets. Panel C reports the $\triangle V$ loadings of the 25 size-IV portfolios from the time-series regressions. The last column of Panel C reports the difference in loadings between high and low IV stocks within each size quintile. The asterisks indicate significance at the 5% level or higher, based on Newey-West t-statistics with six lags. The sample period is from July 1966 to December 2009.

where $\triangle V$ stands for innovations in aggregate market variance.

The slope coefficients from (14) are used as independent variables in

$$R_{it} = \gamma_0 + \gamma_M \hat{\beta}_{Mi} + \gamma_{HML} \hat{\beta}_{HMLi} + \gamma_{SMB} \hat{\beta}_{SMBi} + \gamma_{\triangle V} \hat{\beta}_{\triangle Vi} + \epsilon_{it}. \qquad (15)$$

We also compute the adjusted cross-sectional R^2, which follows Jagannathan and Wang (1996). Since the betas are generated regressors in (15), the t-statistics associated with the γ terms are adjusted for errors-in-variables, following Shanken (1992).

Panel B of Table 2 present results from estimating Equation (15) for 25 size-IV portfolios. We also include the market return, HML, and SMB among the test assets. This is motivated by Lewellen, Nagel, and Shanken (2010), who suggest that when some of the asset pricing factors are traded portfolios, they should be included in the set of test assets. The price of risk for $\triangle V$ is negative and significant, which is consistent with AHXZ. The intercept γ_0 is significant at the 10% level, which suggests that some of the 25 portfolios might be mispriced relative to this model.

Panel B in Table 2 also examines whether portfolio-level IV has incremental explanatory power over and above portfolio loadings with respect to $\triangle V$. Portfolio IV is computed as the value-weighted average of the IVs of the stocks in the portfolio and is denoted as $ivol$. The panel shows that the model from (15) does not capture the IV effect since the coefficient in front of $ivol$ is negative and significant. Individual IV adds 18% of explanatory power over and above the factor loadings. Therefore, loadings to innovations in market variance cannot completely capture the IV effect.

Panel C of Table 2 reports the full-sample loadings of the 25 portfolios with respect to $\triangle V$, estimated from Equation (14). With the exception of the largest quintile, all portfolios have negative $\triangle V$ betas. Combined with the negative price of variance risk, this indicates that exposure to aggregate variance predicts higher expected returns for these portfolios than predicted by the Fama-French model. This is not consistent with the fact that high IV stocks have negative Fama-French alphas.

The $\triangle V$ loadings of high IV stocks in the three smallest quintiles are lower in magnitude than those of low IV stocks. This is not consistent with Equation (3), which shows that IV relative to the Fama-French model is an increasing function of the magnitude of beta with respect to the missing factor. Finally, the spread in $\triangle V$ betas between high and low IV stocks is not significant in any size quintile. Therefore, changes in total variance do not seem to capture the factor missing from the Fama-French model.

Our findings in Table 2 are consistent with AHXZ, who find that innovations in the VIX index are not able to explain the IV puzzle. They show that the $\triangle VIX$ loadings of high and low IV portfolios have the same sign, while opposite signs are necessary to explain the puzzle.

Other studies that examine the pricing of total market variance include Adrian and Rosenberg (2008), Moise (2010), and Da and Schaumburg (2011). They also show that changes in aggregate market variance command a negative price of risk in the cross section of various portfolios. However, they do not examine the IV puzzle. Our results suggest that a different factor is needed to address the puzzle.

3.2 Prices of risk for average variance and average correlation

The key to explaining the IV puzzle is in separating the two components of market variance, AV and AC. We estimate the factor prices of risk from model (10) using the excess returns of 25 size-IV portfolios and the Fama-MacBeth (1973) two-stage method. In the first stage, betas are estimated as the slope coefficients from the following process for excess returns:

$$R_{it}=\alpha_i+\beta_{Mi}R_{Mt}+\beta_{HMLi}HML_t+\beta_{SMBi}SMB_t+\beta_{\triangle AVi}\triangle AV_t+\beta_{\triangle ACi}\triangle AC_t+\varepsilon_{it}. \quad (16)$$

We use two different sets of betas. Following Black, Jensen, and Scholes (1972) and Lettau and Ludvigson (2001), we use the full sample from July 1966 to December 2009 to estimate regression (16). The asset pricing test starts in July 1966 since we use the first 36 months of the sample to compute the first observations for the innovation factors. If the true factor loadings are constant, the full-sample betas should be the most precise. Alternatively, following Ferson and Harvey (1999), we estimate regression (16) using 60-month rolling windows. The rolling windows start in July 1966 as well, and the corresponding betas are called rolling betas. In the second stage, we use cross-sectional regressions to estimate the factor prices of risk:

$$R_{it}=\gamma_0+\gamma_M\hat{\beta}_{Mi}+\gamma_{HML}\hat{\beta}_{HMLi}+\gamma_{SMB}\hat{\beta}_{SMBi}+\gamma_{\triangle AV}\hat{\beta}_{\triangle AVi}+\gamma_{\triangle AC}\hat{\beta}_{\triangle ACi}+\epsilon_{it}. \tag{17}$$

For the case of full-sample betas, we use the same betas every month, while for the case of rolling betas, portfolio excess returns at t are regressed on factor loadings estimated using information from $t-60$ to $t-1$. Following Lewellen, Nagel, and Shanken (2010), we include the market return, HML, and SMB in the set of test assets. Therefore, the asset pricing model is asked to price the traded factor portfolios as well.

Columns (1), (2), (6), and (7) of Table 3 report results for the benchmark Fama-French model. For both full-sample and rolling betas, the cross-sectional intercept is significant, indicating that the pricing error of the model is not zero. The explanatory power of the model is low, and individual portfolio IV is significantly priced in the presence of the Fama-French betas.

Columns (3) and (8) of Table 3 report the results for Equation (17). For the case of full-sample betas, $\triangle AV$ loadings represent a significant determinant of expected returns. The price of risk for $\triangle AV$ is negative at -7.7%. For the 25 size-IV portfolios, the 1st-percentile $\triangle AV$ beta is -0.06, while the 99th-percentile $\triangle AV$ beta is 0.17. Since the price of $\triangle AV$ risk is -7.7%, if $\triangle AV$ beta increases from the 1st to the 99th percentile, expected return will decrease by 1.8% per month.

The market betas of the 25 portfolios are also significant determinants of their average returns. The estimated market price of risk is positive at 0.48% and not statistically different from the average excess market return of 0.42%. All the factors in the model are jointly significant.

Since we use excess portfolio returns, the intercept γ_0 is the pricing error of the model and it should be zero if the model is correct. This hypothesis cannot be rejected. Overall, the model is able to explain 80% of the variation in average returns. In Appendix A, we present a Monte Carlo experiment that derives the finite-sample distribution of the cross-sectional t-statistics. The conclusions based on the small-sample distribution of the t-statistics are in line with the asymptotic results reported in Table 3.

For the case of rolling betas in Column (8) of Table 3, loadings with respect to $\triangle AV$ are again significant. The price of risk for $\triangle AV$ is still negative; however, its magnitude is smaller at -2.60%. For the 25 size-IV portfolios,

Table 3
Cross-sectional regressions: Main results

	Full-sample betas					Rolling betas				
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
γ_0	0.16	−0.23	0.02	0.01	−0.08	0.32	−0.16	0.12	0.17	0.13
	(2.71)	(−3.23)	(0.02)	(0.11)	(−0.69)	(4.88)	(−2.69)	(1.63)	(1.62)	(1.65)
γ_M	0.34	0.83	0.48	0.46	0.64	0.18	0.76	0.33	0.28	0.37
	(1.66)	(3.82)	(2.12)	(2.09)	(2.91)	(0.81)	(3.55)	(1.48)	(1.25)	(1.60)
γ_{HML}	0.67	0.80	0.34	0.26	0.45	0.39	0.69	0.40	0.36	0.37
	(3.63)	(4.49)	(1.27)	(1.29)	(1.86)	(2.37)	(4.29)	(2.18)	(2.07)	(2.04)
γ_{SMB}	−0.15	0.39	0.19	0.22	0.29	−0.07	0.43	0.10	0.09	0.09
	(−0.95)	(2.71)	(1.18)	(1.43)	(2.15)	(−0.43)	(3.04)	(0.58)	(0.56)	(0.57)
$\gamma_{\triangle AV}$			−7.70	−6.52	−5.23			−2.60	−2.61	−2.69
			(−4.09)	(−4.28)	(−3.35)			(−4.99)	(−5.13)	(−5.12)
$\gamma_{\triangle AC}$			5.21		5.48			−1.35		−1.56
			(1.39)		(1.80)			(−1.67)		(−1.81)
$\gamma_{\triangle V}$				1.91					1.40	
				(0.89)					(1.55)	
γ_{ivol}		−7.65			−2.46		−7.52			−0.63
		(−8.18)			(−1.52)		(−9.14)			(−1.14)
R^2	0.22	0.84	0.80	0.75	0.85	0.19	0.83	0.70	0.62	0.71

This table presents Fama-MacBeth regressions using the excess returns of 25 portfolios sorted by size and idiosyncratic volatility. The factor betas, which are the independent variables in the regressions, are computed either over the full sample (full-sample betas) or in 60-month rolling regressions (rolling betas). The variables $\triangle AV$ and $\triangle AC$ refer to innovations in average variance and average correlation, respectively, computed as described in Section 2.2. The variable $\triangle V$ refers to innovations in total market variance. The variable $ivol$ refers to individual portfolio idiosyncratic volatility and it is lagged one month relative to excess returns on the left-hand side. The adjusted R^2 follows Jagannathan and Wang (1996). The t-statistics are in parentheses and adjusted for errors-in-variables, following Shanken (1992). All coefficients are multiplied by 100, and the market portfolio, HML, and SMB are included among the test assets. The sample period is from July 1966 to December 2009.

the 1st-percentile $\triangle AV$ rolling beta is −0.09, while the 99th-percentile $\triangle AV$ rolling beta is 0.25. Therefore, if $\triangle AV$ rolling beta increases from the 1st to the 99th percentile, expected return will decrease by 0.9%. We find that the full-sample regressions in the first stage of the Fama-MacBeth method yield more precise $\triangle AV$ beta estimates than 60-month rolling regressions. Therefore, the attenuation bias seems to be less severe with full-sample $\triangle AV$ betas, and that is why they yield higher $\gamma_{\triangle AV}$ estimates.[11] The intercept γ_0 is not significantly different from zero at conventional significance levels.

The price of risk for $\triangle AC$ is not significant. It switches from positive in the case of full-sample betas to negative in the case of rolling betas.

It is also helpful to provide a visual comparison of the performance of the Fama-French model and the model augmented with $\triangle AV$ and $\triangle AC$. To do that, we plot the fitted expected return of each portfolio against its realized average return in Figure 2. The fitted expected return is computed using the estimated parameter values from a given model specification. The realized average return is the time-series average of the portfolio return. If the fitted expected return and the realized average return for each portfolio are the same, then they should lie on a 45-degree line through the origin. Each two-digit number in Figure 2

[11] A similar point is made in Liu and Zhang (2008), who use the Fama-MacBeth approach to study the cross section of momentum portfolios.

represents a separate portfolio. The first digit refers to the size quintile of the portfolio (1 being the smallest and 5 the biggest), while the second digit refers to the IV quintile (1 being the lowest and 5 the highest).

Panel A of Figure 2 shows the performance of the Fama-French model. The model produces significant pricing errors for the high IV portfolios within size quintiles 1 and 2. In contrast, Panel B shows that the Fama-French model augmented with $\triangle AV$ and $\triangle AC$ is more successful at pricing the portfolios that are challenging for the Fama-French model. The high IV portfolios in the small quintiles move closer to the 45-degree line in the presence of the $\triangle AV$ and $\triangle AC$ factors.

Next, we test whether aggregate market variance has incremental explanatory power over and above average variance. We first run a VAR that contains the market return, HML, SMB, AV, and V. The innovations from the VAR are the factors in the asset pricing model. Innovations in V are orthogonal to innovations in AV. Since average variance is a component of aggregate market variance, when both of them are included in the asset pricing equation it constitutes a direct test of the marginal explanatory power of V. The results are presented in Columns (4) and (9) of Table 3. The component of aggregate market variance that is orthogonal to average variance is not priced in the cross section of returns. The results are robust to including average correlation in the model.

Finally, we perform a direct test of whether individual portfolio IV has incremental explanatory power over and above portfolio loadings with respect to innovations in AV. We include portfolio-specific idiosyncratic volatility, denoted as $ivol$, in Equation (17). If loadings with respect to innovations in average variance explain the IV puzzle, then the coefficient in front of $ivol$ should be zero.

Columns (5) and (10) of Table 3 show that there is no residual IV effect in the model that contains innovations in average variance. With full-sample betas, the risk premium of $\triangle AV$ remains significant. The cross-sectional R^2 indicates that individual portfolio IV does not add much explanatory power over and above the factor loadings. The same conclusions hold for rolling betas.

In summary, the results are in line with the argument that changes in average variance represent the factor omitted from the Fama-French model. In the context of Equation (3), our results suggest that IV relative to the Fama-French model proxies for assets' loadings with respect to innovations in average variance. In the presence of these loadings, the IV puzzle of AHXZ disappears.

3.3 Factor loadings

A negative price of risk for $\triangle AV$ means that assets that covary positively (negatively) with innovations in AV should have lower (higher) expected returns since they have higher (lower) payoffs when future investment opportunities turn for the worse. Thus, if exposure to changes in average variance is to explain the IV puzzle, stocks with high (low) IV must have

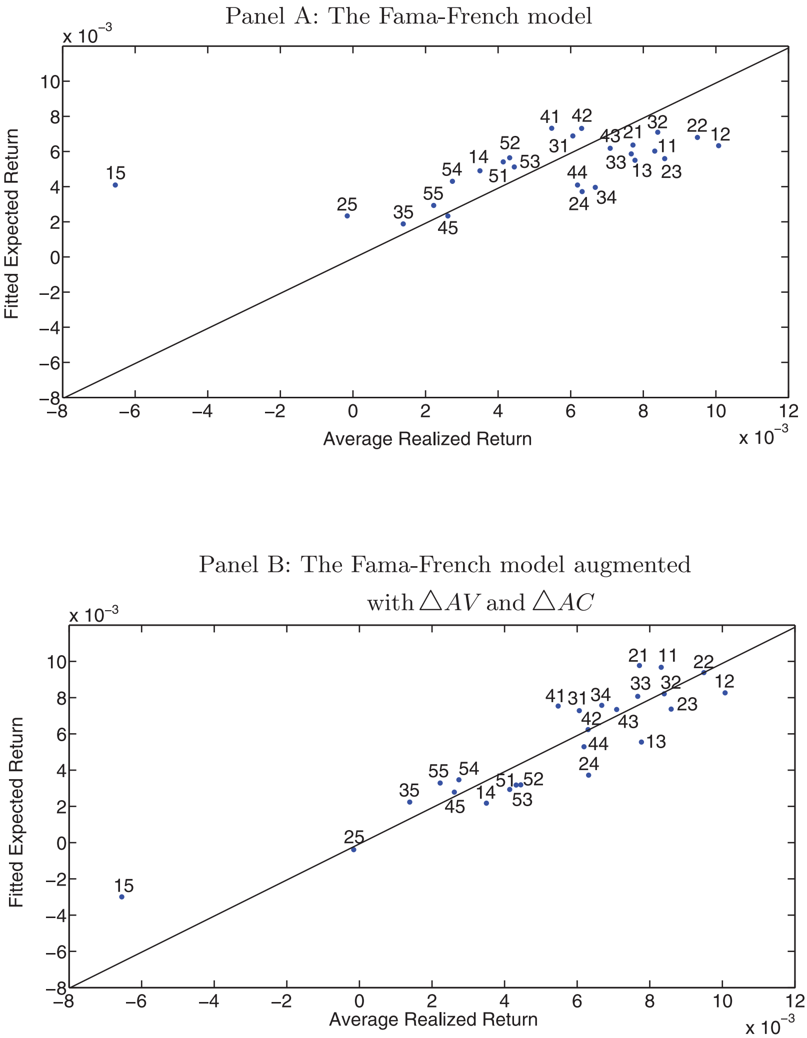

Figure 2
Fitted expected returns versus average realized returns
This figure shows realized average returns (in %) on the horizontal axis and fitted expected returns (in %) on the vertical axis for 25 portfolios sorted by size and idiosyncratic volatility. Each two-digit number represents a separate portfolio. The first digit refers to the size quintile (1 being the smallest and 5 the largest), while the second digit refers to the idiosyncratic risk quintile (1 being the lowest and 5 the highest). For each portfolio, the realized average return is the time-series average of the portfolio return and the fitted expected return is the fitted value for the expected return from the corresponding model. The straight line is the 45-degree line from the origin. The sample period is from July 1963 to December 2009.

positive (negative) $\triangle AV$ betas. Next, we report the full-sample factor loadings for the 25 portfolios estimated from regression (16).

Panel A of Table 4 shows that stocks with high IV tend to be small growth stocks with high market betas, while stocks with low IV tend to be large value stocks with low market betas. The differences in R_M, HML, and SMB loadings between high and low IV stocks are significant in each size group.

Panel A of Table 4 also reports that within each size quintile except quintile 5, high IV stocks have positive $\triangle AV$ betas while low IV stocks have negative $\triangle AV$ betas. In addition, as we move from larger to smaller quintiles, the magnitude of the betas of the two extreme idiosyncratic groups increases. The portfolios that have significant $\triangle AV$ betas tend to be concentrated in size quintiles 1 and 2. All 25 $\triangle AV$ betas are jointly significant. In judging the significance of the $\triangle AV$ factor loadings, it is also useful to look at the difference in $\beta_{\triangle AV}$ between high and low IV assets. Since the IV puzzle documented by AHXZ is a cross-sectional result, if the $\triangle AV$ factor is to explain the puzzle, then the $\triangle AV$ loadings of assets that differ in IV must differ from each other. As Table 4 shows, the difference in $\beta_{\triangle AV}$ between high and low IV stocks is significant in the first three size quintiles. These are the quintiles in which the IV puzzle is observed (Table 2, Panel A). Even though the IV effect and the significant spread in $\triangle AV$ betas are concentrated in size quintiles 1, 2, and 3, the results are not likely to be driven by the smallest stocks. This is the case since we use NYSE breakpoints to construct the 25 size-IV portfolios. When we use CRSP breakpoints to construct these portfolios, the IV effect is present in all CRSP quintiles, but it is weaker in the smallest quintile. These results are available upon request.

The $\triangle AV$ betas of high IV portfolios in all size groups (except quintile 4) are larger in magnitude than the $\triangle AV$ betas of low IV portfolios. This is consistent with Equation (3), which indicates that IV relative to the Fama-French model is an increasing function of the magnitude of beta with respect to the missing factor.

Since the $\triangle AV$ betas are derived in a multiple time-series regression, they are conditional on the other factor betas. So, the positive $\triangle AV$ betas of high IV stocks indicate that these stocks do better than predicted by the Fama-French model in times of high volatility. Therefore, while all stocks may be negatively affected by increasing market-wide volatility, high IV stocks are less so.

Do high IV stocks have positive $\triangle AV$ betas mechanically since AV contains idiosyncratic components? We address this question by noting that the $\triangle AV$ factor is not a traded portfolio. Therefore, it is not weighted by design toward stocks that are likely to exhibit a high IV characteristic. Among portfolios with similar IVs, there is a sizable spread in $\triangle AV$ betas. For example, in the highest IV quintile, the spread in $\triangle AV$ loadings goes from 0.02 to 0.19 and the difference is significant. In the third IV quintile, some portfolios have negative $\triangle AV$ betas, while others have positive ones. There are also instances in which a portfolio with high IV has a lower $\triangle AV$ beta than a portfolio with a lower

Table 4
Portfolio loadings and intercept restrictions from the Fama-French model augmented with $\triangle AV$ and $\triangle AC$

	Low IV	2	3	4	High IV		Low IV	2	3	4	High IV	
Panel A: Time-series coefficients												
			α			$diff$			β_{R_M}			$diff$
Small	0.24	0.21	−0.09	−0.60	−1.68	−1.92	0.70*	0.99*	1.15*	1.22*	1.24*	0.54*
2	0.21	0.23	0.13	−0.12	−0.80	−1.00	0.77*	1.00*	1.10*	1.21*	1.36*	0.59*
3	0.05	0.19	0.13	0.03	−0.49	−0.55	0.78*	1.00*	1.08*	1.17*	1.31*	0.52*
4	0.03	0.05	0.15	0.10	−0.28	−0.31	0.83*	1.01*	1.09*	1.13*	1.31*	0.48*
Large	0.11	0.09	0.08	−0.08	−0.17	−0.28	0.82*	0.93*	1.02*	1.08*	1.23*	0.41*
			β_{HML}			$diff$			β_{SMB}			$diff$
Small	0.44*	0.41*	0.26*	0.19	0.13	−0.31	0.63	0.93	1.16	1.38	1.66	1.03*
2	0.42*	0.41*	0.22*	−0.05	−0.28*	−0.70*	0.47	0.63	0.81	1.10	1.30	0.83*
3	0.45*	0.39*	0.21*	−0.07	−0.40*	−0.85*	0.25	0.37	0.55	0.78	0.99	0.74*
4	0.44*	0.36*	0.17*	−0.11*	−0.41*	−0.84*	0.03	0.10	0.21	0.42	0.68	0.65*
Large	0.09	0.07	−0.04	−0.17*	−0.39*	−0.47*	−0.31	−0.29	−0.21	−0.10	0.19	0.49*
			$\beta_{\triangle AV}$			$diff$			$\beta_{\triangle AC}$			$diff$
Small	−0.05*	0.00	0.05*	0.11*	0.19*	0.24*	−0.09*	−0.10*	−0.11*	−0.12*	−0.13*	−0.04
2	−0.06*	−0.02	0.00	0.05*	0.11*	0.17*	−0.09*	−0.08*	−0.10*	−0.11*	−0.12*	−0.03
3	−0.02	−0.02	−0.03	0.00	0.07	0.09*	−0.06*	−0.06*	−0.08*	−0.06*	−0.06*	−0.01
4	−0.03	−0.01	−0.02	0.00	0.03	0.07	−0.03	−0.05*	−0.03*	−0.04*	−0.06*	−0.03
Large	0.01	0.01	0.02	0.01	0.02	0.01	0.02	0.02	0.03*	0.02	0.00	−0.02
Panel B: Intercept restrictions												
			$\alpha_i-\beta_i'(\gamma-E(f))$			$diff$			$t_{\alpha_i-\beta_i'(\gamma-E(f))}$			$diff$
Small	−0.09	0.21	0.24	0.13	−0.39	−0.30	−0.40	0.72	0.69	0.32	−0.84	−0.79
							[−0.7, 0.1]	[0.1, 0.7]	[0.0, 0.6]	[−0.2, 0.5]	[−1.1, −0.1]	[−1.6, 0.2]
2	−0.15	0.04	0.15	0.28	0.02	0.17	−0.69	0.16	0.52	0.79	0.05	0.49
							[−0.9, −0.1]	[−0.3, 0.3]	[−0.0, 0.5]	[0.2, 0.6]	[−0.3, 0.3]	[−0.3, 1.0]
3	−0.09	0.05	0.00	−0.07	−0.09	0.00	−0.46	0.20	−0.01	−0.21	−0.22	0.02
							[−0.8, 0.1]	[−0.3, 0.4]	[−0.3, 0.2]	[−0.4, 0.1]	[−0.5, 0.2]	[−0.7, 0.7]
4	−0.18	0.03	0.00	0.11	0.00	0.18	−0.88	0.14	−0.01	0.40	−0.01	0.56
							[−1.1, −0.1]	[−0.3, 0.4]	[−0.4, 0.3]	[−0.1, 0.5]	[−0.4, 0.3]	[−0.4, 1.1]
Large	0.12	0.12	0.12	−0.07	−0.10	−0.22	0.71	0.60	0.57	−0.29	−0.33	−0.96
							[0.0, 0.8]	[−0.0, 0.7]	[0.0, 0.6]	[−0.5, 0.1]	[−0.6, 0.2]	[−1.6, 0.2]

Panel A reports intercepts (% per month) and factor loadings for 25 portfolios sorted by size and idiosyncratic volatility (IV). The difference in alpha and factor loadings between high and low IV stocks within each size quintile is also reported. The factor loadings are computed in a time-series regression over the full sample: $R_{it}=\alpha_i+\beta_{Mi}R_{Mt}+\beta_{HMLi}HML_t+\beta_{SMBi}SMB_t+\beta_{\triangle AVi}\triangle AV_t+\beta_{\triangle ACi}\triangle AC_t+\varepsilon_{it}$. Panel B presents the difference $\alpha_i-\beta_i'(\gamma-E(f))$, and its corresponding asymptotic t-statistic, for each of the 25 size-IV portfolios. The vector γ is from the second stage of the Fama-MacBeth regression in Column (3) of Table 3. $E(f)$ is the expected value of each factor. Below each t-statistic is its 95% confidence interval based on a small-sample distribution derived in a Monte Carlo experiment described in Appendix A. The panels report the difference in $\alpha_i-\beta_i'(\gamma-E(f))$ measures between high and low IV stocks in each size quintile, together with the corresponding significance. The asterisks indicate significance at the 5% level or higher, based on Newey-West t-statistics with six lags. The sample period is from July 1966 to December 2009.

IV (e.g., the high IV portfolios in size quintiles 4 and 5 vs. the small portfolio in IV quintile 3).

In Appendix B, we decompose AV into a systematic component and an idiosyncratic component. The results suggest that high (low) IV portfolios have positive (negative) loadings to the systematic component of AV, and these loadings are significant determinants of expected returns. Therefore, it is unlikely that the previously documented relation between the IV of a portfolio and its exposure to $\triangle AV$ is purely mechanical.

Panel A of Table 4 also shows the loadings of the 25 portfolios with respect to $\triangle AC$. All of the loadings (except for quintile 5) are negative, and the spread in $\triangle AC$ betas between high and low IV stocks does not seem high enough to explain differences in average returns. The spread in $\triangle AC$ betas between high and low IV stocks is not significant, except for the largest quintile.

If we combine the patterns of $\triangle AV$ and $\triangle AC$ betas from Table 4, we will get a pattern that resembles the one for $\triangle V$ betas in Panel C of Table 2. Still, the pattern of $\triangle V$ betas is closer to the one of $\triangle AC$ betas. This finding suggests that because of the confounding effect of correlations, loadings with respect to changes in aggregate market variance are not able to price all portfolios sorted by IV.

Finally, Panel A of Table 4 shows the time-series intercepts α_i of the 25 portfolios. Since some of the factors in our model are not traded portfolios, the restriction on the time-series intercepts is

$$\alpha_i - \beta_i^{'}(\gamma - E(f)) = 0, \tag{18}$$

where $\beta_i^{'} = [\beta_{Mi}, \beta_{HMLi}, \beta_{SMBi}, \beta_{\triangle AVi}, \beta_{\triangle ACi}]$, $\gamma = [\gamma_M, \gamma_{HML}, \gamma_{SMB}, \gamma_{\triangle AV}, \gamma_{\triangle AC}]'$, and $E(f) = [E(R_M), E(HML), E(SMB), E(\triangle AV), E(\triangle AC)]'$. The pattern in the α_is from Panel A of Table 4 shows that high IV stocks have lower expected returns than low IV stocks in each size quintile. Note that we do not report the significance of the individual α_is in Panel A of Table 4 since the null hypothesis is not $H_0: \alpha_i = 0$.

Panel B of Table 4 reports the measure from Equation (18) for each portfolio, and the corresponding asymptotic t-statistics for the null hypothesis H_0: $\alpha_i - \beta_i^{'}(\gamma - E(f)) = 0$. The results indicate that the model-implied restriction on the time-series intercept of each portfolio cannot be rejected according to conventional asymptotic testing. Since the βs and γs are estimated parameters, we also derive the small-sample distribution of the t-statistic associated with the null hypothesis in (18). More details about the derivation are provided in Appendix A. The 2.5th- and 97.5th-percentile values of this distribution are reported below each t-statistic. In general, the pattern of statistical significance of $\alpha_i - \beta_i^{'}(\gamma - E(f))$ from the small-sample distributions matches that of the asymptotic distributions.

3.4 Mimicking portfolios for innovations in average variance and average correlation

The results so far suggest that the risk associated with increasing average variance is priced. Therefore, investors might be willing to hold a portfolio that hedges unexpected increases in average variance. In this section, we derive such a portfolio that tracks innovations in $\triangle AV$, and examine its ability to explain the time-series and cross-sectional variation in returns sorted by IV. We also derive a mimicking portfolio for $\triangle AC$. The advantage of using mimicking portfolios for innovations in AV and AC is that the excess returns of the mimicking portfolios measure the prices of risk associated with innovations in the state variables.

Following Breeden, Gibbons, and Litzenberger (1989), we form a mimicking portfolio for $\triangle AV$ by estimating the fitted value from the following regression:

$$\triangle AV_t = c + bX_t + u_t, \tag{19}$$

where X_t represents the excess returns on base assets. The return on the portfolio $\hat{b}X_t$ is the factor that mimics innovations in average variance. It is denoted as PAV. We use 25 portfolios sorted by size and $\triangle AV$ loadings as base assets.[12] Panel C of Table 1 reports summary statistics for the PAV factor. The correlation between PAV and $\triangle AV$ is 35%. The average return of portfolio PAV over the full sample period is -0.63% per month. This is the price of risk associated with innovations in average variance.

Similarly, we use 25 portfolios sorted by size and $\triangle AC$ loadings to form a mimicking portfolio for innovations in average correlation. That portfolio is denoted as PAC. Summary statistics for PAC are in Panel C of Table 1. The correlation between PAC and $\triangle AC$ is 20%.

Next we augment the Fama-French model with PAV and PAC to test whether it can capture the IV effect. In the first step, we regress the time series of excess returns of each portfolio on the market return, HML, SMB, PAV, and PAC. The regression and the two mimicking portfolios are estimated simultaneously through GMM. Since all factors are traded portfolios, the time-series intercept is a risk-adjusted return and it should be zero under the null hypothesis. Panel A of Table 5 reports the time-series intercepts from the Fama-French model augmented with PAV and PAC, together with the factor loadings. Compared to the Fama-French alphas in Panel A of Table 2, the alphas of the high IV stocks in Panel A of Table 5 are substantially smaller (except for the largest stocks). None of the alphas are statistically significant. The chi-square statistic for the joint significance of the alphas, estimated with GMM, is 30.15 (p-value = .22) with 25 d.f. Therefore, the alphas are not jointly

[12] For each stock, we estimate factor loadings based on Equation (16). To estimate loadings for month t, we use the previous 60 months of data. Each month, we form 25 value-weighted portfolios based on a double sort by size and $\triangle AV$ loadings estimated with returns from the previous 60 months.

Table 5
Mimicking portfolios for $\triangle AV$ and $\triangle AC$

Panel A: Time-series coefficients

	Low IV	2	3	4	High IV		Low IV	2	3	4	High IV	
			α			$diff$			β_{R_M}			$diff$
Small	0.01	0.18	0.09	−0.16	−0.86	−0.87	0.65*	0.97*	1.18*	1.32*	1.43*	0.78*
2	−0.02	0.08	0.12	0.11	−0.29	−0.27	0.77*	0.93*	1.11*	1.30*	1.53*	0.76*
3	−0.03	0.09	0.03	−0.10	−0.29	−0.26	0.79*	0.98*	1.09*	1.08*	1.32*	0.52*
4	−0.10	0.06	0.03	0.08	−0.14	−0.04	0.82*	1.07*	1.05*	1.14*	1.37*	0.55*
Large	0.12	0.11	0.09	−0.10	−0.17	−0.29	0.82*	0.94*	0.99*	1.06*	1.21*	0.39*
			β_{HML}			$diff$			β_{SMB}			$diff$
Small	0.78*	0.76*	0.52*	0.55*	0.39	−0.03	0.51*	0.93*	1.25*	1.59*	2.02*	1.50*
2	0.38*	0.41*	0.31	0.31	0.34	−0.51	0.28*	0.60*	0.78*	1.14*	1.45*	1.17*
3	0.32*	0.39*	0.20	−0.02	−0.20*	−0.72*	0.17	0.30*	0.44*	0.80*	1.12*	0.95*
4	0.40*	0.36*	0.15	−0.06*	−0.32*	−0.78*	−0.06	0.01	0.16*	0.39*	0.70*	0.76*
Large	0.38	0.31	0.15	−0.13*	−0.40*	−0.46*	−0.31*	−0.29*	−0.17*	−0.09	0.21	0.51*
			β_{PAV}			$diff$			β_{PAC}			$diff$
Small	−0.60*	−0.03	0.46*	1.08*	1.96*	2.56*	−0.21	0.15	0.17	0.26	0.28	0.48
2	−0.77*	−0.28	−0.11	0.40*	1.01*	1.78*	−0.81	0.26	−0.25	−0.39	−0.48	0.33
3	−0.33	−0.30*	−0.39*	−0.12	0.59*	0.92*	−0.44	−0.19	−0.55	0.68	0.50	0.94
4	−0.40*	−0.23*	−0.28	−0.11	0.21	0.61	−0.33	−0.80	0.02	−0.15	−0.35	−0.02
Large	0.01	0.02	0.13	0.01	0.06	0.05	−0.05	−0.08	0.41	0.20	0.21	0.25

Panel B: Cross-sectional prices of risk

	Full-sample betas		Rolling betas	
	(1)	(2)	(3)	(4)
γ_0	−0.05	−0.04	0.10	0.10
	(−0.78)	(−0.95)	(1.16)	(1.56)
γ_M	0.45	0.53	0.32	0.31
	(2.12)	(2.71)	(1.44)	(1.37)
γ_{HML}	0.46	0.51	0.37	0.36
	(2.41)	(2.97)	(2.26)	(2.21)
γ_{SMB}	0.21	0.29	0.10	0.13
	(1.42)	(2.08)	(0.68)	(0.85)
γ_{PAV}	−0.79	−0.58	−0.56	−0.59
	(−8.08)	(−7.08)	(−6.88)	(−7.08)
γ_{PAC}	0.19	0.18	0.07	0.06
	(1.15)	(1.19)	(1.51)	(1.29)
γ_{ivol}		−2.23		−0.25
		(−1.62)		(−0.58)
R^2	0.81	0.84	0.69	0.68

Panel A reports intercepts (% per month) and factor loadings for 25 portfolios sorted by size and idiosyncratic volatility (IV). The difference in alpha and factor loadings between high and low IV stocks within each size quintile is also reported. The factor loadings are computed in a time-series regression over the full sample: $R_{it}=\alpha_i+\beta_{Mi}R_{Mt}+\beta_{HMLi}HML_t+\beta_{SMBi}SMB_t+\beta_{PAVi}PAV_t+\beta_{PACi}PAC_t+\varepsilon_{it}$. PAV and PAC are mimicking portfolios for innovations in average variance and average correlation, respectively, computed as described in Section 3.4. The asterisks indicate significance at the 5% level or higher, based on Newey-West t-statistics with six lags. Panels B presents Fama-MacBeth regressions using the excess returns of 25 size-IV portfolios. The factor betas, which are the independent variables in the regressions, are computed either over the full sample (full-sample betas) or in 60-month rolling regressions (rolling betas). The model is the Fama-French model augmented with PAV and PAC. The variable $ivol$ is individual portfolio idiosyncratic volatility, and it is lagged one month relative to excess returns. The adjusted R^2 follows Jagannathan and Wang (1996). The t-statistics are in parentheses and adjusted for errors-in-variables, following Shanken (1992). All coefficients in Panel B are multiplied by 100, and the market portfolio, HML, SMB, PAV, and PAC are included among the test assets. The sample period is July 1966 to December 2009.

significant. The difference in risk-adjusted returns between high and low IV stocks is not significant in all size quintiles.

Panel A of Table 5 also shows that the PAV betas of high IV stocks are positive, while the PAV betas of low IV stocks are negative (except for the largest quintile). Almost half of the size-IV portfolios are significantly exposed to PAV. The difference in PAV betas between high and low IV portfolios is statistically significant in quintiles 1, 2, and 3.

In the second step, we test whether exposure to the PAV and PAC factors is significantly priced in the cross section of returns. Panel B of Table 5 reports the results. Using full-sample betas in Column (1), the price of risk for PAV is -0.79% and significant. For the 25 size-IV portfolios, the 1st-percentile PAV beta is -0.7, while the 99th-percentile PAV beta is 1.7. Therefore, if PAV beta increases from the 1st to the 99th percentile, expected return will decrease by 1.9%. This number is very close to the 1.8% reported in the case when innovations in AV are used rather than a mimicking portfolio. In addition, since PAV is a traded portfolio, its price of risk should be equal to the average return of PAV. The estimated price of risk for PAV (-0.79%) is not statistically different from the average monthly PAV return (-0.63%).

The cross-sectional intercept in Column (1) of Panel B of Table 5 is not significant. Column (2) shows that there is no residual IV effect in the presence of PAV loadings. Similar conclusions hold for rolling betas in Columns (3) and (4). Overall, when using mimicking portfolios, the results are very similar to the ones reported with AV innovations. The mimicking portfolio approach provides an alternative way of measuring the risk premium associated with exposure to average variance.

4. Alternative Test Assets

4.1 Alternative portfolio sorts

In this section, we use other portfolios to check the robustness of the model. If the AV factor is indeed an important state variable, it should be able to price other assets. Also, as Lewellen, Nagel, and Shanken (2010) point out, it is important to expand the set of test assets when a couple of factors seem to explain nearly all of the variation in returns.

We use three additional sets of portfolios. The first set consists of 25 portfolios sorted by size and book-to-market (BM). The second set includes 25 portfolios sorted by size and past returns. The third set includes 49 industry portfolios. The returns of the equity portfolios come from Ken French's website.

We estimate Fama-MacBeth regressions for each of the three sets of alternative test portfolios. The traded factors in each model are included among the test assets. Table 6 presents the results using full-sample betas.[13] Columns

[13] Results for rolling betas are similar and available upon request.

Table 6
Cross-sectional regressions: Alternative test assets

	Size-BM port.			Size-mom. port.			Industry port.			Stocks	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
γ_0	0.05	0.04	0.03	−0.07	−0.10	−0.09	0.53	0.40	0.69	0.71	0.71
	(2.73)	(1.77)	(1.44)	(−2.08)	(−1.62)	(−1.58)	(4.82)	(3.04)	(3.36)	(4.41)	(4.73)
γ_M	0.36	0.33	0.36	0.50	0.49	0.48	−0.22	0.08	−0.26	−0.02	0.12
	(1.74)	(1.52)	(1.73)	(2.40)	(2.30)	(2.30)	(−0.95)	(0.33)	(−0.86)	(−0.18)	(1.31)
γ_{HML}	0.45	0.44	0.41	0.51	0.57	0.51	0.07	−0.03	0.53	0.18	0.06
	(3.34)	(3.11)	(3.03)	(3.19)	(3.81)	(3.19)	(0.41)	(−0.15)	(1.88)	(2.21)	(0.95)
γ_{SMB}	0.20	0.15	0.26	0.39	0.36	0.38	0.25	−0.19	−0.11	0.01	−0.02
	(1.35)	(1.00)	(1.83)	(2.43)	(2.10)	(2.37)	(1.32)	(−0.97)	(−0.45)	(0.07)	(−0.37)
γ_{UMD}				0.73	0.74	0.72					
				(3.63)	(3.56)	(3.53)					
$\gamma_{\triangle AV}$		−3.75			−2.93			−3.00		−0.22	−0.14
		(−2.26)			(−2.35)			(−2.17)		(−3.15)	(−2.51)
$\gamma_{\triangle AC}$		−1.58			−0.56			−0.93		−0.12	−0.10
		(−1.11)			(−0.50)			(−1.11)		(−1.88)	(−1.89)
γ_{PAV}			−0.79			−0.29			0.07		
			(−4.86)			(−2.01)			(0.25)		
γ_{PAC}			0.37			0.16			1.03		
			(1.74)			(1.54)			(1.16)		
γ_{size}											−0.14
											(−3.97)
γ_{bm}											0.40
											(8.00)
γ_{mom}											0.37
											(2.08)
γ_{ivol}											−4.76
											(−1.61)
R^2	0.65	0.67	0.81	0.84	0.88	0.87	0.05	0.52	0.19	0.03	0.05

This table presents Fama-MacBeth regressions using the excess returns of different test assets. The betas, which are the independent variables in the regressions, are computed over the full sample. $\triangle AV$ and $\triangle AC$ refer to innovations in average variance and average correlation, respectively, computed as in Section 2.2. PAV and PAC refer to mimicking portfolios for innovations in average variance and average correlation, respectively, computed as in Section 3.4. UMD is the momentum factor. Columns (1)–(3) correspond to a set of 25 size-book-to-market portfolios. Columns (4)–(6) correspond to a set of 25 size-momentum portfolios. Columns (7)–(9) correspond to a set of 49 industry portfolios. In Columns (1)–(9), the traded factors are included in the set of test assets. Columns (10) and (11) correspond to the cross section of individual stocks. Stock betas are estimated in 60-month rolling window regressions. $ivol$ is measured as the standard deviation of the residuals from the Fama-French model estimated with daily data within a month, and it is lagged one month relative to excess returns; bm is the book-to-market ratio of the stock available six months prior; $size$ is the log market capitalization of the firm at the end of the previous month; mom is the stock return over the previous six months after skipping a month. The adjusted R^2 follows Jagannathan and Wang (1996). The t-statistics are in parentheses and adjusted for errors-in-variables, following Shanken (1992). All coefficients are multiplied by 100. The period is July 1966 to December 2009.

(1)–(3) correspond to 25 size and BM portfolios. The benchmark Fama-French model shows significant pricing errors. Untabulated results show that the significant pricing errors are due to the growth portfolios in the two smallest quintiles. When the Fama-French model is augmented with $\triangle AV$ and $\triangle AC$, the pricing errors of the model become insignificant. Exposure to average variance is significantly negative priced in the cross section of size and BM portfolios. Untabulated results show that the $\triangle AV$ betas of growth stocks are on average higher that the $\triangle AV$ betas of value stocks. Small growth portfolios appear to be good hedged for times when average variance is high. Seven of the 25 size and BM portfolios have significant $\triangle AV$ betas. The difference in exposure to $\triangle AV$ between value and growth stocks is significant in the smallest quintile. If $\triangle AV$ beta moves from the 1st to the 99th percentile, a price of risk estimate of -3.75% implies that expected return will decrease by 1.3%.

When the Fama-French model is augmented with the mimicking portfolios for $\triangle AV$ and $\triangle AC$, PAV and PAC, respectively, the pricing errors of the model are also insignificant. Untabulated results show that more than half of the 25 size and BM portfolios have significant PAV betas. The difference in exposure to PAV between value and growth stocks is significant in size quintiles 1 and 2.

Overall, the results suggest that innovations in average variance represent a significant factor in the cross section of assets sorted by size and BM. The $\triangle AV$ factor adds additional information over and above the Fama-French factors. This is in line with our argument that $\triangle AV$ is a factor missing from the Fama-French model.

Columns (4)–(6) of Table 6 examine the pricing of 25 size-momentum portfolios. The benchmark Fama-French model augmented with the momentum factor UMD shows significant pricing errors. Untabulated results show that the significant pricing errors are due to the portfolios in the smallest size quintile. Next, we augment the benchmark model with $\triangle AV$ and $\triangle AC$. These factors are derived from a VAR system that contains the market return, HML, SMB, UMD, AV, and AC. Column (5) shows that the pricing errors in the presence of $\triangle AV$ become insignificant. The price of risk for average variance is negative and significant. Untabulated results show that the $\triangle AV$ betas of losers (winners) are positive (negative). Five of the 25 size and BM portfolios have significant $\triangle AV$ betas. The difference in exposure to $\triangle AV$ between winners and losers is significant in size quintiles 1,2, 4, and 5. If $\triangle AV$ beta moves from the 1st to the 99th percentile, a price of risk estimate of -3.12% implies that expected return will decrease by 1%.

When $\triangle AV$ and $\triangle AV$ are replaced with their mimicking portfolios in Column (6) of Table 6, we obtain similar results. Untabulated results show that more than half of the 25 size-momentum portfolios have significant exposure to the mimicking portfolio for average variance. Overall, $\triangle AV$ and its mimicking portfolio add explanatory power to the Fama-French model augmented with the momentum factor.

Columns (7)–(9) of Table 6 examine the pricing of 49 industry portfolios. We use the Fama-French model as the benchmark. Column (7) shows that the benchmark model has a very low explanatory power (5%) and it generates significant pricing errors. The results in Column (8) show that $\triangle AV$ betas are significant determinants of the expected returns of industry portfolios and the explanatory power of the Fama-French model augmented with $\triangle AV$ and $\triangle AC$ is substantially higher (52%). Untabulated results show that ten industries have significant exposure to innovations in average variance. The $\triangle AV$ betas of Hardware, Software, Chips, and Lab Equipment are significantly positive. This is in line with our argument that growth firms tend to do well when average variance is high.

When the mimicking portfolios PAV and PAC are used in Column (9) of Table 6, the explanatory power of the model decreases and none of the risk factors are significant. Untabulated results show that almost half of the industry portfolios have significant β_{PAV} coefficients. Overall, the results suggest that many industries are significantly exposed to average variance.

In summary, the $\triangle AV$ factor captures the book-to-market and momentum effects by producing insignificant pricing errors in the cross section of portfolios sorted by these characteristics. Although $\triangle AV$ does not explain the cross section of industry portfolios perfectly, there is evidence that $\triangle AV$ is a useful state variable that outperforms the Fama-French factors.

4.2 Cross section of individual stock returns

In a recent article, Ang et al. (2010) argue that although forming portfolios produces more precise estimates of factor loadings, it also reduces the precision of the estimates of the factor risk premia. They suggest that using individual stocks increases the cross-sectional dispersion in factor loadings and this helps in estimating more precise factor risk premia. Therefore, in this section, we turn to individual stock returns to examine the robustness of our previous results.

Column (10) of Table 6 presents estimates of factor risk premia from Equation (17) using individual stocks and the Fama-MacBeth method with rolling betas. The factor loadings are estimated from a time-series regression, using the previous 60 months of data. At least 24 months of monthly observations are required. Innovations in average variance have a significantly negative price of risk at -0.22%. For individual stocks, the 1st-percentile $\triangle AV$ beta is -3.36, while the 99th-percentile $\triangle AV$ beta is 4.10.[14] Since the price of $\triangle AV$ risk is -0.22%, if $\triangle AV$ beta increases from the 1st to the 99th percentile, expected return will decrease by 1.7%. This number is similar to the one derived from the set of 25 size-IV portfolios.

[14] The distribution of $\triangle AV$ loadings for stocks is more dispersed than the one for 25 size-IV portfolios. However, the means of the two distributions are similar. The average $\triangle AV$ beta of 25 size-IV portfolios is 0.02, while the average $\triangle AV$ beta for a stock is 0.07 (with a median of 0.02).

In Column (11) of Table 6, we examine whether firm-level IV keeps its power in determining expected returns in the presence of loadings from the model in Column (10). We also control for other stock characteristics that predict stock returns: book-to-market, size, and momentum. The book-to-market ratio of each stock is the ratio available six months prior, size is the log market capitalization of the firm at the end of the previous month, and momentum is the stock return over the previous six months. The results in Column (11) show that innovations in average variance still have a negative and significant price of risk. The coefficient in front of firm-level IV is not significant.

5. Interpreting the Pricing of Average Variance

In this section, we present additional results to help with the interpretation of the average variance component of market variance. For this purpose, we examine further the identity of stock with high IV, as well as the relation between AV, measures of aggregate macroeconomic uncertainty, and measures of aggregate growth options.

5.1 Interpreting the sign of the $\triangle AV$ loadings

In general, we would expect that when average variance goes up overall market variance increases and drives up the expected market risk premium (Merton 1980). This in turn should increase the discount rate of firms and decrease their values. Thus, there should be a negative contemporaneous relation between stock returns and positive shocks to average variance. However, while all stocks may be negatively affected by increasing market-wide volatility, high IV stocks are less so. Conditional on their Fama-French betas, high IV stocks are good hedges for times of high volatility. Therefore, for the types of stocks concentrated in these portfolios there must be an additional effect of average variance on returns that is opposite to the discount rate effect mentioned above.

To understand the source of such an effect, we measure the research and development (R&D) expenditures of the 25 size-IV portfolios. R&D is computed as R&D investment divided by total assets. The R&D ratio of a portfolio is computed as an equally weighted average of the ratios of the stocks within the portfolio. We drop observations with missing values for R&D. In month t, we match IV estimated in month $t-1$ with R&D available for fiscal year ending in month $t-14$ to $t-3$.

In Figure 3, we report the time-series averages of R&D ratios for the period from July 1966 to December 2009 for 25 size-IV portfolios. Stocks with high IV have significantly higher R&D expenditure than stocks with low IV. This effect is largest in the smallest quintile, but it holds for all size groups. Combining this result with the previous observation on factor loadings with respect to average variance, it follows that high R&D stocks tend to be less negatively affected by increases in average variance than low R&D stocks.

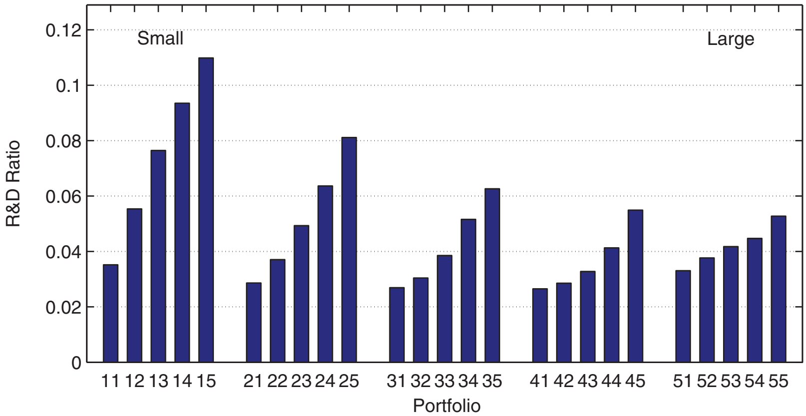

Figure 3
Portfolio R&D ratios
This figure reports the time-series averages of R&D expenditure ratios for 25 portfolios sorted by size and idiosyncratic volatility. Each two-digit number represents a separate portfolio. The first digit refers to the size quintile (1 being the smallest and 5 the largest), while the second digit refers to the idiosyncratic risk quintile (1 being the lowest and 5 the highest). For each stock, R&D expenditure is measured as R&D investment divided by total assets. Portfolio-level R&D is an equally weighted average of the R&D ratios of the stocks within the portfolio. The sample period is from July 1966 to December 2009.

Several authors have suggested that firms with large R&D expenditure have many real options. Therefore, our finding that high R&D stocks have positive loadings with respect to average variance is consistent with predictions from the real options literature. Theoretical models in this literature predict that the value of a real option should be increasing in the volatility of the underlying asset (e.g., McDonald and Siegel 1986). Therefore, the value of a firm with a lot of real options should also be positively related to increasing volatility, both systematic and idiosyncratic.[15] Note that the HML betas of high (low) IV portfolios tend to be negative (positive), which suggests that they might be weighted toward growth (value) stocks.[16]

The size of R&D is likely to be correlated with the proportion of firm value due to investment opportunities, and their growth rates are likely to be more uncertain than those of existing assets. Thus, the positive $\triangle AV$ betas of high R&D portfolios are also consistent with a model by Pastor and Veronesi (2003). Pastor and Veronesi show that there is a positive relation between returns and changes in volatility, especially for firms with a lot of growth opportunities.

---

[15] Grullon, Lyandres, and Zhdanov (2012) find empirical evidence that stock returns are contemporaneously positively correlated with changes in individual stock volatility. This relation is stronger for firms that are more likely to have more real options and for firms with more irreversible investment opportunities.

[16] Cao, Simin, and Zhao (2008) documented that high IV stocks tend to have a lot of growth options. They show that accounting for growth options eliminates or reverses the upward trend in aggregate idiosyncratic volatility. Their study focuses on the time series of aggregate idiosyncratic risk, while we examine the cross-sectional pricing of individual idiosyncratic volatility.

We use another common measure of growth options, the market-to-book ratio of the firm. Untabulated results for this measure show that portfolios with high IV have higher market-to-book ratios than portfolios with low IV.

The evidence in this section suggests that portfolios with high IV have option-like characteristics. Therefore, the previous finding that correlation risk is not priced in the cross section of IV portfolios is in line with Driessen et al. (2009), who show that individual options are not exposed to correlation risk. The value of growth options should increase when aggregate market volatility increases. According to Driessen et al. (2009), the increase in the individual growth option value is driven mostly by the increase in the variance of the average stock.

To further examine whether the positive contemporaneous relation between a portfolio return and $\triangle AV$ is partly due to growth options whose value is increasing in volatility, we look at a cross section of 49 industry portfolios. We identify industries in which growth options are more likely to represent larger proportions of firm values. These industries are Fama and French (1997) industry 22 (electrical equipment), 32 (telecommunications), 35 (computers), 36 (computer software), 37 (electronic equipment), 38 (measuring and control equipment), 12 (medical equipment), and 13 (pharmaceutical products).[17] We form an equally weighted portfolio of these industries. Similarly, we form an equally weighted portfolio of the remaining industries. Industries with growth options should be less negatively affected when average variance increases. Untabulated results show that the $\triangle AV$ beta of the industry portfolio with growth options is 0.06 with a t-statistic of 2.91, while the $\triangle AV$ beta of the other portfolio is -0.01 with a t-statistic of -0.75.

5.2 The idiosyncratic volatility effect during the recent financial crisis

During the recent financial crisis, market volatility increased dramatically. Our previous discussion suggests that during such times a strategy that invests in high IV stocks and shorts low IV stocks would have provided a good hedge against rising volatility. In this section, we examine the validity of this prediction. To do that, we use daily returns for the 25 size-IV portfolios for the period from January 2008 to June 2009 (377 daily observations). This period is classified as the last recession experienced by the U.S. market according to NBER. We split this recessionary period into two parts. The first part is from January 2008 to September 2008, and it is characterized by relatively lower values of the VIX index. The second part is from October 2008 to June 2009, and it is characterized by very high levels of VIX. We compute the average daily returns and volatilities of 25 size-IV portfolios in each period. The results are reported in Table 7.

---

[17] We use theoretical and empirical studies to identify potential industries with growth options (e.g., Majd and Pindyck 1987; Bollen 1999).

Table 7
The idiosyncratic volatility effect during the crisis: January 2008–June 2009

	Low IV	2	3	4	High IV	Low IV	2	3	4	High IV
	Panel A: Period of low VIX (2008:01-2008:09)									
	$\overline{R}$					σ				
Small	−0.02	−0.07	−0.09	−0.12	−0.12	1.36	1.72	1.82	1.92	1.91
2	0.00	−0.01	−0.05	−0.09	−0.13	1.52	1.87	1.93	2.08	2.29
3	0.00	−0.05	−0.05	−0.10	−0.11	1.44	1.67	1.82	1.97	2.30
4	−0.05	−0.05	−0.09	−0.12	−0.14	1.20	1.58	1.78	2.04	2.55
Large	−0.06	−0.08	−0.10	−0.08	−0.19	1.37	1.52	1.56	2.20	2.78
	Panel B: Period of high VIX (2008:10-2009:06)									
	$\overline{R}$					σ				
Small	−0.02	0.07	0.06	0.11	0.24	2.75	3.68	3.96	4.02	4.30
2	−0.07	−0.08	−0.03	−0.01	0.13	2.86	3.45	3.75	4.10	4.91
3	−0.07	−0.11	−0.09	0.00	0.10	2.67	3.17	3.53	3.88	4.91
4	−0.07	−0.04	−0.05	−0.05	0.07	2.48	2.90	3.24	3.79	4.84
Large	−0.06	−0.03	−0.06	−0.06	0.00	2.44	2.63	3.04	3.37	4.70

This table presents average daily returns and daily volatilities of 25 portfolios sorted by size and idiosyncratic volatility. Panel A refers to the period from January 2008 to September 2008 when the VIX index was below its average value for the 2008–2009 period. Panel B refers to the period from October 2008 to June 2009 when the VIX index was above its average value for the 2008–2009 period.

Panel A of Table 7 shows that during the low VIX period, high IV stocks underperformed low IV stocks in terms of average daily returns. In contrast, Panel B shows that during the high VIX period, high IV stocks did better than low IV stocks in terms of average daily returns. In both cases, it is hard to judge the difference in performance given the large volatilities of daily returns. However, the evidence suggests that high IV stocks did better than low IV stocks in a state of the world characterized by rising variance. Untabulated results show that high (low) IV stocks have positive (negative) loadings with respect to VIX, after controlling for their Fama-French betas, for the period January 2008 to June 2009.

Finally, in Figure 4, we plot the cumulative returns of the market portfolio and the small high-minus-low IV portfolio for the period January 2008 to June 2009, using their daily returns. The small high-minus-low IV return is constructed as the difference between the high and low IV return in the smallest quintile. The results show that an investment of 1 dollar in the market portfolio in January 2008 would have decreased to 67 cents by the end of June 2009, for a total return of -33%. In contrast, a similar investment in the small high-minus-low IV portfolio would have appreciated to 1.3 dollars, for a total return of 30%. The figure shows that when the VIX index increased dramatically, the IV strategy did not experience as large of a decline as the market strategy.

5.3 Relation between average variance and other state variables

Next, we suggest an economic interpretation behind the cross-sectional pricing of average variance. In particular, we relate AV to aggregate liquidity, the variance of consumption growth, and the aggregate market-to-book ratio. These

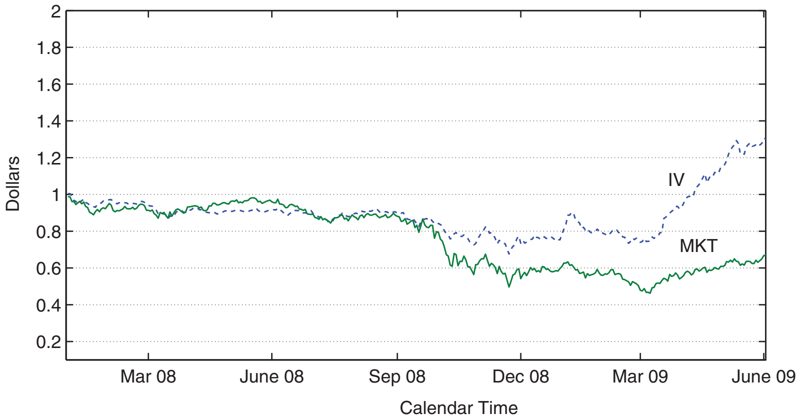

Figure 4
Cumulative returns: January 2008–June 2009
This figure plots the cumulative returns of the market portfolio (solid line) and the small high-minus-low IV portfolio (dotted line) for the period January 2008 to June 2009 using daily returns. The small high-minus-low IV return is constructed as the difference between the high IV return in the smallest size group and the low IV return in the smallest size group.

are important state variables for stock returns, and there are reasons to believe that these variables are related to average variance.

Variance and liquidity could be related since high volatility indicates uncertainty, which in turn implies information asymmetry. As a result, adverse selection costs and inventory risk in trading increase and risk-bearing capacity decreases. In the models of Gromb and Vayanos (2002) and Brunnermeier and Pedersen (2009), higher variance predicts less available risk-bearing capacity. In these models, lower risk-bearing capacity leads to lower liquidity. Motivated by these models, we examine the relation between AV and aggregate liquidity. We compute the Pastor and Stambaugh (2003) measure of market-wide liquidity for the period from July 1963 to December 2009 and denote it as LIQ. High values of LIQ correspond to high market-wide liquidity.

Standard models about the behavior of aggregate stock prices identify changes in the conditional variance of fundamentals as a major source of fluctuations in asset prices. For example, Bekaert et al. (2009) show that countercyclical changes in the variance of consumption growth drive the countercyclical volatility of aggregate returns.[18] Motivated by their model and the observation that AV tends to increase in recessions (see Figure 1), we examine the relation between average variance and the variance of consumption growth.

[18] Other studies that examine the effects of macroeconomic uncertainty on asset prices and equity premia include Kandel and Stambaugh (1990) and Bansal and Yaron (2004), among others.

We obtain quarterly data on seasonally adjusted real consumption from the NIPA tables of the Bureau of Economic Analysis. Aggregate consumption is defined as expenditures on non-durables and services. The growth rate of consumption for quarter t, $\triangle c_t$, is constructed by taking the first difference of the log consumption series. We follow the approach of Bekaert et al. (2009) and estimate the following system for the conditional mean and variance of consumption growth:

$$\begin{aligned} E_{t-1}(\triangle c_t) &= a_0 + a_1 X_{t-1} \\ Var_{t-1}(\triangle c_t) &= b_0 + b_1 X_{t-1}, \end{aligned} \tag{20}$$

where X_{t-1} is a vector of explanatory variables known at the end of quarter $t-1$, which includes the T-bill rate, dividend yield, and term spread, estimated at quarterly frequency.[19] The system of equations in (20) is estimated via GMM. The fitted value from the second equation in (20) is the estimate of the conditional variance of consumption growth, CV. To match the quarterly estimates of CV with monthly data, each month within the same quarter we use repeated values equal to $1/3$ of the quarterly variance of consumption growth. Alternatively, we compute quarterly portfolio returns and run quarterly regression tests, which produce similar results.

Several authors show that both systematic and idiosyncratic variance are related to the growth opportunities of the firm (e.g., Berk, Green, and Naik 1999; Cao, Simin, and Zhao 2008; Bekaert, Hodrick, and Zhang 2010). Motivated by these studies, we examine the relation between AV and the aggregate market-to-book ratio, MB. The MB ratio is a proxy for corporate growth options since the market value of assets captures expectations of future growth opportunities within the firm while book value does not. We compute MB as the value-weighted average of firm-level market assets over book assets.

Next, we run contemporaneous time-series regressions of AV on the three variables discussed above, LIQ, CV, and MB, to determine the strength of their relationships. The results indicate that movements in AV are positively related to variation in CV and MB, and negatively related to variation in LIQ. The relation between AV and MB is the strongest. More details are available in Appendix C.

Average variance increases when aggregate liquidity drops and macroeconomic uncertainty rises. It also increases when the aggregate value of growth options goes up. Therefore, high AV could arise from factors that are generally considered to decrease welfare, as well as from factors that are related to innovation and opportunities for growth.[20] The latter channel may explain the

---

[19] Bekaert et al. (2009) use the same variables in the variance equation, but they use only the consumption-dividend ratio (or just a constant) in the mean equation. We have tried a similar specification and found the resulting $Var_{t-1}(\triangle c_t)$ series to be very highly correlated with the main one we use.

[20] Bartram, Brown, and Stulz (2011) argue that return volatility can be high for reasons that contribute positively or negatively to shareholder wealth and economic growth. Therefore, they distinguish between good and bad volatility.

Table 8
Cross-sectional regressions: Liquidity, variance of consumption growth, and market-to-book

	Full-sample betas		Rolling betas	
	(1)	(2)	(3)	(4)
γ_0	−0.23	−0.19	0.23	0.19
	(−2.48)	(−2.09)	(3.59)	(2.96)
γ_M	0.63	0.57	0.23	0.27
	(2.80)	(2.57)	(1.05)	(1.23)
γ_{HML}	0.93	0.84	0.33	0.40
	(4.13)	(3.41)	(1.94)	(2.47)
γ_{SMB}	0.22	0.27	0.04	0.06
	(1.34)	(1.72)	(0.25)	(0.40)
$\gamma_{\triangle AV*}$	−3.23		−2.58	
	(−2.51)		(−3.88)	
$\gamma_{\triangle AV**}$		−5.06		−2.31
		(−4.54)		(−3.45)
$\gamma_{\triangle AC}$	−5.94	−2.21	0.91	−0.55
	(−1.12)	(−0.96)	(1.85)	(−1.07)
R^2	0.61	0.76	0.59	0.55

This table presents Fama-MacBeth regressions using the excess returns of 25 portfolios sorted by size and idiosyncratic volatility. The factor betas, which are the independent variables in the regressions, are computed either over the full sample (full-sample betas) or in 60-month rolling regressions (rolling betas). Columns (1) and (3) present results for the Fama-French model augmented with innovations in average variance ($\triangle AV^*$) and average correlation ($\triangle AC$). The AV^* factor is the component of average variance, AV, projected on the aggregate market-to-book ratio. Columns (2) and (4) present results for the Fama-French model augmented with innovations in average variance ($\triangle AV^{**}$) and average correlation ($\triangle AC$). The AV^{**} factor is the component of total AV projected on aggregate liquidity, the variance of consumption growth, and the aggregate market-to-book ratio. The adjusted R^2 follows Jagannathan and Wang (1996). The t-statistics are in parentheses and adjusted for errors-in-variables, following Shanken (1992). All coefficients are multiplied by 100, and the market portfolio, HML, and SMB are included among the test assets. The sample period is from July 1966 to December 2009.

partial negative relation between AV and expected market returns documented in Table 1. The high positive correlation between AV and aggregate growth options may be the reason why AV is a negative predictor of future market returns.

Finally, we examine whether the pricing ability of AV is due to its correlation with liquidity, the variance of consumption growth, and the aggregate market-to-book ratio. We compute the projection of AV on MB and denote it as AV^*, and we compute the projection of AV on CV, LIQ, and MB and denote it as AV^{**}. We choose to examine AV^* first because of the high correlation between average variance and market-to-book. We estimate the system of Equations (16)–(17) for 25 size-IV portfolios, replacing $\triangle AV$ with the newly constructed $\triangle AV^*$ or $\triangle AV^{**}$.

Table 8 presents the results. Column (1) reveals that for full-sample betas, the price of risk for $\triangle AV^*$ is significantly negative. However, the explanatory power of the model is lower than that when the $\triangle AV$ factor is used. Therefore, the relation between MB and AV contributes to the pricing ability of the $\triangle AV$ factor, but does not explain it completely.

Column (2) of Table 8 shows that the price of risk for $\triangle AV^{**}$ is also significantly negative. The cross-sectional R^2 is close to the one reported previously for the main model (Column (3) of Table 3). This finding suggests that the relation between AV and CV and LIQ contributes to the pricing ability of the $\triangle AV$ factor over and above MB.[21]

Columns (3) and (4) of Table 8 present similar results for rolling betas. However, the pricing errors of the models are significant and the R^2s are lower than the ones reported when total $\triangle AV$ is used. This suggests that average stock variance subsumes and exceeds the pricing abilities of liquidity, consumption variance, and MB. There seems to be additional information in AV that matters for stock returns.

6. Alternative Explanations for the Idiosyncratic Volatility Puzzle

Recent studies have offered alternative explanations for the IV effect documented by AHXZ. Here, we review these studies and compare their results to ours. We summarize only the main findings. Tabulated results are in Appendix D.

6.1 Lagged and contemporaneous idiosyncratic volatility

Our main results are about the negative relation between idiosyncratic volatility at time t and returns at time $t+1$. A recent article by Sonmez (2009) claims that it is the change in IV from t to $t+1$ that predicts returns at time $t+1$. More specifically, stocks that move from low IV quintiles at time t to high IV quintiles at time $t+1$ earn high average returns. Stocks that move from high IV quintiles at t to low IV quintiles at $t+1$ earn low average returns. For stocks that stay in the same IV quintile at times t and $t+1$, idiosyncratic volatility is positively related to average returns. Therefore, Sonmez (2009) suggests that it might be realized IV at time $t+1$ that is related to returns.

In this section, we examine more closely the relation between IV_t (lagged IV) and IV_{t+1} (contemporaneous IV) and stock returns at $t+1$. AHXZ (2009) state that estimates of the realized mean and realized variance of returns are positively correlated because stock returns are positively skewed. Therefore, to study the relation between contemporaneous IV and stock returns, we have to look at log returns. The predictive relation between lagged IV and returns is not affected by using log returns to measure IV. It is only the contemporaneous relation between realized returns and realized volatility that is affected by the skewness of stock returns.

Because of the effect of skewness, we consider the IV of log returns. We form two sets of portfolios. First, we sort stocks by IV_{t+1} and then by IV_t. Second, we sort stocks by IV_t and then by IV_{t+1}. The two dependent sorts

[21] Appendix C examines the separate pricing of CV and LIQ in the cross section of returns.

disentangle the effects of lagged versus contemporaneous IV on stock returns. Overall, the results suggest that controlling for IV_{t+1} does not explain the IV_t puzzle.

We examine whether the Fama-French model augmented with $\triangle AV$ and $\triangle AC$ can explain the difference in average returns for the two sets of portfolios discussed above. The results show that for both sets of portfolios, $\triangle AV$ betas are significant determinants of their expected returns. Portfolios with high (low) IV_t and IV_{t+1} have positive (negative) $\triangle AV$ betas.

6.2 Lagged and conditional idiosyncratic volatility

Fu (2009) points out that AHXZ's IV_t might not be a good proxy for $E_t(IV_{t+1})$ since it is not a random walk. AHXZ (2009) use a different estimate of expected IV, based on a model that contains lagged IV, size, book-to-market, past six-month return, skewness, and turnover. They show that, controlling for expected IV, lagged IV still predicts future returns.

Following up on AHXZ (2009), we construct a different measure of IV based on rolling 36-month Fama-French regressions. That is, each month, returns are matched to IV measured as the standard deviation of the residuals from the Fama-French model run over the previous 36 months using monthly data. This estimate of IV, denoted as IV_{36}, does look like a random walk, and so, it avoids Fu's criticism. We find that IV_{36} is still negatively related to average returns in each size quintile. The cross-sectional pricing of the size-IV_{36} portfolios reveals that their $\triangle AV$ betas are significant variables in the cross section.

6.3 Idiosyncratic volatility and maximum daily return

Bali et al. (2011) show that the maximum daily return over the past one month, MAX, is negatively related to stock returns in the cross section. Since stocks with high MAX in a given month also have high IV measured over the same month, Bali et al. (2011) test whether MAX is a proxy for the IV effect. For value-weighted portfolios, they show that after controlling for MAX, high IV stocks still have lower average returns than low IV stocks, but the magnitude of the IV effect is significantly reduced. However, for equally weighted portfolios, high IV stocks have higher average returns than low IV stocks after controlling for MAX.

First, we show that the findings in Bali et al. (2011) for equally weighted portfolios do not appear to be robust. In particular, we exclude penny stocks from the sample and construct 25 equally weighted portfolios sorted by MAX and then sorted by IV. The IV effect is still negative and significant in the two highest MAX quintiles. We test whether the $\triangle AV$ factor is priced in the cross section of these portfolios and find that it is negative and significant in the case of rolling betas.

Second, the existence of the MAX effect is not necessarily inconsistent with our explanation for the IV puzzle. According to Bali et al. (2011), firms with high MAX have a relatively small probability of a large payoff. Therefore, they could also be firms that have very uncertain growth rates. Such firms are likely to have more investment opportunities relative to existing assets than firms with more certain growth rates. The size of R&D expenditure is likely to be correlated with the proportion of firm value due to investment opportunities. Therefore, we compute the R&D expenditures for five portfolios sorted by MAX. Untabulated results show that high MAX stocks have higher R&D than low MAX stocks. This suggest that the variable MAX may be another way of identifying stocks with uncertain growth rates and many growth options.

6.4 Idiosyncratic volatility and return reversals

Fu (2009) and Huang et al. (2010) show that return reversals from stocks with high IV in the last month lead to AHXZ's results. If the IV puzzle is driven by short-term return reversals, it is likely that the IV effect will be much weaker or even non-existent one or two months after portfolio formation. We show that the IV effect continues for about seven months after portfolio formation. Furthermore, we find that the $\triangle AV$ loadings of high (low) IV stocks continue to be positive (negative) several months after portfolio formation. Overall, the results suggest that short-term return reversals cannot explain the IV puzzle and its persistence several months after portfolio formation.

6.5 Idiosyncratic volatility and skewness

Boyer, Mitton, and Vorking (2010) find that expected idiosyncratic skewness is negatively related to stock returns. They show that expected idiosyncratic skewness and IV seem to have independent effects on average returns. Furthermore, their estimate of expected idiosyncratic skewness contains IV as an explanatory variable. This suggests that it is not entirely clear how to disentangle the two measures. To the extent that the presence of growth options induces positive skewness in returns (e.g., Andres-Alonso et al. 2006; Haanappel and Smit 2007), our explanation for the IV puzzle is not necessarily inconsistent with the results reported by Boyer et al. (2010). Firms with high skewness are likely to have growth options, and therefore, positive loadings in $\triangle AV$. This makes them good hedges of volatility risk and lowers their expected returns.

6.6 Innovations in idiosyncratic volatility

Grullon, Lyandres, and Zhdanov (2011) and Bali, Scherbina, and Tang (2011) find that innovations in idiosyncratic volatility, $\triangle IV_{t+1}$, are positively related to contemporaneous stock returns. Grullon et al. (2011) show that this result is stronger for firms with a lot of growth options. Bali et al. (2011) find that the relationship reverses in the future. Both of these findings are consistent with our explanation for the existence of the IV puzzle. An increase in volatility (both systematic and idiosyncratic) is likely to increase the value of growth options, leading to the positive contemporaneous relation between $\triangle IV_{t+1}$ and R_{t+1}. Firms with growth options are therefore hedges for rising volatility, leading to their lower expected returns.

7. Conclusion

We provide a rational explanation for the existence of the idiosyncratic volatility puzzle documented by AHXZ (2006, 2009). Since idiosyncratic volatility is usually measured as the standard deviation of the residuals from the Fama-French model, it is model dependent. If a risk factor is missing from the model, idiosyncratic volatility may appear to be priced simply because it proxies for a risk exposure with respect to the missing factor. We identify the factor missing from the Fama-French model as average stock variance. Investors are willing to pay an insurance premium for high idiosyncratic volatility stocks since their payoff is high when return variance is large, conditional on their market betas.

We show that innovations in average stock variance represent a priced risk factor in the cross section of stock returns. The price of risk for average variance is negative. This implies that rising variance signals deterioration in investment opportunities. We find that portfolios with high (low) idiosyncratic volatility relative to the Fama-French model have positive (negative) loadings with respect to innovations in average variance. This difference in the loadings, combined with a negative price of risk for average variance, explains the idiosyncratic volatility puzzle of AHXZ. In the presence of loadings with respect to innovations in average variance, individual idiosyncratic volatility does not affect expected returns.

We provide an economic interpretation for the pricing of average variance. The results suggest that it is related to aggregate liquidity, the variance of consumption growth, and the aggregate market-to-book ratio. Therefore, average variance could be interpreted as a risk factor measuring economic uncertainty, and also an indicator for the prevalence of aggregate growth options.

AHXZ (2009) show that the idiosyncratic volatility puzzle is present across 23 developed markets. In addition, they document a strong comovement in the low returns to high idiosyncratic volatility stocks across countries, suggesting that broad, not easily diversifiable, factors may lie behind this phenomenon. A possible extension of the current article is to examine whether the source of these common movements across countries is economic uncertainty as measured by average stock variance.

References

Adrian, T., and J. Rosenberg. 2008. Stock Returns and Volatility: Pricing the Short-run and Long-run Components of Market Risk. *Journal of Finance* 63:2997–3030.

Andres-Alonso, P., V. Azofra-Palenzuela, and G. de la Fuente-Herrero. 2006. The Real Options Component of Firm Market Value: The Case of Technological Corporation. *Journal of Business Finance and Accounting* 33:203–19.

Ang, A., J. Liu, and K. Schwarz. 2010. Using Stocks or Portfolios in Tests of Factor Models. Working Paper, Columbia University.

Ang, A., R. Hodrick, Y. Xing, and X. Zhang. 2006. The Cross-section of Volatility and Expected Returns. *Journal of Finance* 61:259–99.

Ang, A., R. Hodrick, Y. Xing, and X. Zhang. 2009. High Idiosyncratic Volatility and Low Returns: International and Further U.S. Evidence. *Journal of Financial Economics* 91:1–23.

Bali, T., and N. Cakici. 2008. Idiosyncratic Volatility and the Cross-section of Expected Returns. *Journal of Financial and Quantitative Analysis* 43:29–58.

Bali, T., N. Cakici, and R. Whitelaw. 2011. Maxing Out: Stocks as Lotteries and the Cross-section of Expected Returns. *Journal of Financial Economics* 99:427–46.

Bali, T., A. Scherbina, and Y. Tang. 2011. Unusual News Events and the Cross-section of Stock Returns. Working Paper, Georgetown University.

Bansal, R., and A. Yaron. 2004. Risks for the Long Run: A Potential Resolution of Asset Pricing Puzzles. *Journal of Finance* 59:1481–509.

Barinov, A. 2010. Idiosyncratic Volatility, Growth Options, and the Cross-section of Returns. Working Paper, University of Georgia.

Bartram, S. M., G. Brown, and R. Stulz. 2011. Why Are U.S. Stocks More Volatile? *Journal of Finance* Forthcoming.

Bekaert, G., R. Hodrick, and X. Zhang. 2010. Aggregate Idiosyncratic Volatility. *Journal of Financial and Quantitative Analysis* Forthcoming.

Bekaert, G., E. Engstrom, and Y. Xing. 2009. Risk, Uncertainty, and Asset Prices. *Journal of Financial Economics* 91:59–82.

Berk, J., R. Green, and V. Naik. 1999. Optimal Investment, Growth Options, and Security Returns. *Journal of Finance* 54:1553–607.

Black, F., M. Jensen, and M. Scholes. 1972. The Capital Asset Pricing Model: Some Empirical Tests. In M. Jensen (ed.), *Studies in the Theory of Capital Markets*, 79–121. New York: Praeger.

Bollen, N. 1999. Real Options and Product Life Cycles. *Management Science* 45:670–84.

Boyer, B., T. Mitton, and K. Vorkink. 2010. Expected Idiosyncratic Skewness. *Review of Financial Studies* 23:169–202.

Breeden, D., M. Gibbons, and R. Litzenberger. 1989. Empirical Tests of the Consumption-Oriented CAPM. *Journal of Finance* 44:231–62.

Brunnermeier, M., and L. Pedersen. 2009. Market Liquidity and Funding Liquidity. *Review of Financial Studies* 22:2201–38.

Campbell, J. 1993. Intertemporal Asset Pricing Without Consumption Data. *American Economic Review* 83:487–512.

Campbell, J. 1996. Understanding Risk and Return. *Journal of Political Economy* 104:298–345.

Cao, C., T. Simin, and J. Zhao. 2008. Can Growth Options Explain the Trend in Idiosyncratic Risk? *Review of Financial Studies* 21:2599–2633.

Chen, J. 2003. Intertemporal CAPM and the Cross-section of Stock Returns. Working Paper, University of California, Davis.

Da, Z., and E. Schaumburg. 2011. The Pricing of Volatility Risk Across Asset Classes and the Fama-French Factors. Working Paper, Federal Reserve Bank of New York.

Driessen, J., P. Maenhout, and G. Vilkov. 2009. The Price of Correlation Risk: Evidence from Equity Options. *Journal of Finance* 64:1377-–1406.

Fama, E., and K. French. 1993. Common Risk Factors in the Returns on Stocks and Bonds. *Journal of Financial Economics* 33:3–56.

Fama, E., and K. French. 1997. Industry Costs of Equity. *Journal of Financial Economics* 43: 153–93.

Fama, E., and J. MacBeth. 1973. Risk, Return, and Equilibrium: Empirical Tests. *Journal of Political Economy* 71:607–36.

Ferson W., and R. C. Harvey. 1999. Conditioning Variables and the Cross-section of Stock Returns. *Journal of Finance* 54:1325–60.

French, K., W. Schwert, and R. Stambaugh. 1987. Expected Stock Returns and Volatility. *Journal of Financial Economics* 19:3–29.

Fu, F. 2009. Idiosyncratic Risk and the Cross-section of Expected Stock Returns. *Journal of Financial Economics* 91:24–37.

Goyal, A., and P. Santa-Clara. 2003. Idiosyncratic Risk Matters! *Journal of Finance* 58: 975–1007.

Gromb, D., and D. Vayanos. 2002. Equilibrium and Welfare in Markets with Financially Constrained Arbitrageurs. *Journal of Financial Economics* 66:361–407.

Grullon, G., E. Lyandres, and A. Zhdanov. 2012. Real Options, Volatility, and Stock Returns. *Journal of Finance* Forthcoming.

Guo, H., and R. Savickas. 2008. Average Idiosyncratic Volatility in G7 Countries. *Review of Financial Studies* 21:1259–96.

Haanappel, H., and H. Smit. 2007. Return Characteristics of Strategic Options. *Annals of Operations Research* 151:57–80.

Huang, W., Q. Liu, G. Rhee, and L. Zhang. 2010. Return Reversals, Idiosyncratic Risk, and Expected Returns. *Review of Financial Studies* 23:147–68.

Jagannathan, R., and Z. Wang. 1996. The Conditional CAPM and the Cross-section of Expected Returns. *Journal of Finance* 51:3–53.

Kandel, S., and R. Stambaugh. 1990. Expectations and Volatility of Consumption and Asset Returns. *Review of Financial Studies* 3:207–32.

Lewellen, J., S. Nagel, and J. Shanken. 2010. A Skeptical Appraisal of Asset Pricing Tests. *Journal of Financial Economics* 96:175–94.

Liu, X., and L. Zhang. 2008. Momentum Profits, Factor Pricing, and Macroeconomic Risk. *Review of Financial Studies* 21:2417–48.

MacKinlay, A. C. 1995. Multifactor Models Do Not Explain Deviations from the CAPM. *Journal of Financial Economics* 38:3–28.

MacKinlay, A. C., and L. Pastor. 2000. Asset Pricing Models: Implications for Expected Returns and Portfolio Selection. *Review of Financial Studies* 13:883–916.

Majd, S., and R. Pindyck. 1987. Time to Build, Option Value, and Investment Decisions. *Journal of Financial Economics* 18:7–27.

McDonald, R., and D. Siegel. 1986. The Value of Waiting to Invest. *Quarterly Journal of Economics* 101: 707–27.

Merton, R. 1980. On Estimating the Expected Return on the Market: An Exploratory Investigation. *Journal of Financial Economics* 8:323–61.

Merton, R. 1987. A Simple Model of Capital Market Equilibrium with Incomplete Information. *Journal of Finance* 42:483–510.

Moise, C. 2010. Volatility Pricing in the Stock and Treasury Markets. Working Paper, Case Western Reserve University.

Pastor, L., and R. Stambaugh. 2003. Liquidity Risk and Expected Stock Returns. *Journal of Political Economy* 111:642–85.

Pastor, L., and P. Veronesi. 2003. Stock Valuation and Learning About Profitability. *Journal of Finance* 58:1749–89.

Pollet, J., and M. Wilson. 2010. Average Correlation and Stock Market Returns. *Journal of Financial Economics* 96:364–80.

Shanken, J. 1992. On the Estimation of Beta-pricing Models. *Review of Financial Studies* 5:1–34.

Sonmez, F. 2009. Rethinking Idiosyncratic Volatility: Is It Really a Puzzle? Working Paper, Queen's University.