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# Correlation risk<sup>☆</sup>

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### ABSTRACT

Investors hold portfolios of assets with different risk-reward profiles for diversification benefits. Conditional on the volatility of assets, diversification benefits can vary over time depending on the correlation structure among asset returns. The correlation of returns between assets has varied substantially over time. To insure against future "low diversification" states, investors might demand securities that offer higher payouts in these states. If this is the case, then investors would pay a premium for securities that perform well in regimes in which the correlation is high. We empirically test this hypothesis and find that correlation carries a significantly negative price of risk, after controlling for asset volatility and other risk factors.

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## 1. Introduction

Correlations are important for diversification. There is now considerable evidence that the correlation of returns between assets has varied substantially over time. Goetzmann, Li and Rouwenhorst (2005), for example, examine the major world equity markets and find that correlations vary considerably through time, and, because of the time-varying nature of correlations, diversification benefits are also time varying. There is also evidence that inter-asset correlations generally increase during financial crises, and, more generally, in bear markets (see, for example, Longin and Solnik, 2001, and Ang and Bekaert, 2002).

An increase in asset correlations can lower diversification benefits for investors, and increase market volatility. If diversification opportunities diminish in states of nature when they are most needed, investors would want to hedge against such states. If correlation between assets is a systematic risk factor, investors would pay a premium for securities that offer higher payouts in states of high asset correlations. Our paper empirically tests this hypothesis. Specifically, we investigate whether inter-asset time-varying correlation carries a significant price of risk in the cross-section of stock returns.

Correlations are covariances scaled by the product of asset return volatilities. If returns follow a one-factor model, correlations are increasing in asset betas and market variance and decreasing in idiosyncratic asset volatility, everything else equal. Therefore, it is important to control for market variance and asset volatility when examining the price of correlation risk.<sup>3</sup> We control for both of these in our study. Like Ang, Hodrick, Xing and Zhang (2006b), we find that asset volatility is significantly priced in the cross-section of expected stock returns, and that the price of asset volatility is significant even in the presence of aggregate market

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<sup>&</sup>lt;sup>3</sup> Campbell, Lettau, Malkiel and Xu (2001) show that market volatility changes over time but shows no discernable long-term trend. Therefore, it is necessary to control for the level of market volatility when analyzing changes in asset correlations. Bekaert, Hodrick, and Zhang (in press) make a similar point.

volatility. As Ang, Hodrick, Xing and Zhang (2006b) admit, this is somewhat of a puzzle. However, we find that when inter-asset correlation is included in the model, the price of asset volatility is no longer significant.

We analyze the inter-asset correlation structure over the period from July 1963 through December 2007, and then examine whether inter-asset correlation is priced in the cross-section of asset returns. We find that correlation carries a significantly negative price of risk. The negative price of correlation risk suggests that investors prefer stocks that perform well in states in which a portfolio is effectively less diversified. The demand for such stocks is then reflected in lower expected returns. We find that the significantly negative market price of correlation risk persists after controlling for well-known risk factors, higher moments, different test assets, and time variation in factor loadings.

In our empirical analysis, we face several important issues. First, we need to construct a correlation risk factor in a way that summarizes the pair-wise correlations among many assets. In theory, it would be desirable to measure the pair-wise correlations of all securities. In practice, this is not feasible and so we select assets that represent different risk-return profiles, namely the 25 Fama-French portfolios. We compute the time series of monthly pair-wise correlations for these assets using daily returns. Then we derive the unexpected component of each correlation series by controlling for the effects of past correlations, HML and SMB, the market return, expected inflation, unanticipated inflation, the growth rate of industrial production, and the real interest rate. It is important to control for these macroeconomic variables since we want to construct a correlation factor that represents diversification benefits rather than business-cycle risk. It is also important to purge the correlations of other well-known systematic risk factors. Next, we reduce the dimensionality of inter-asset correlation innovations by extracting the principal component that account for most of the variability in pair-wise correlation innovations, and use it as our correlation risk factor.

Second, even though we control for asset and market volatility, as well as for macroeconomic variables, correlation may be related to the fact that investors who are more sensitive to downside losses relative to upside gains may require a premium for holding assets that covary strongly when the market falls. Extensions of the CAPM that incorporate the asymmetric treatment of risk involve lower partial moments (Bawa and Lindenberg, 1977), higher moments and comoments (Kraus and Litzenberger, 1983; Harvey and Siddique, 1999), and downside risk (Ang et al., 2006a). We account for these possible asymmetric responses by incorporating factors for coskewness and cokurtosis, and variables that proxy for market movements. We find that correlation risk continues to be priced significantly in the cross section.

To the best of our knowledge, no previous study has addressed whether time-varying correlation between stocks carries a significant price of risk in the cross-section of stock returns. The one study that is most closely related to ours is Driessen, Maenhout and Vilkov (in press), who examine correlation risk using option-implied estimates of correlations and variances. They develop a model for equity prices with priced correlation risk, and empirically measure the magnitude of the risk premium. In their model, asset prices follow geometric Wiener processes where the correlations are connected through a simple one factor model. The dynamics of this common factor under the risk neutral measure are identified after specifying the structure for the correlation risk premium. They observe that the price of correlation risk is significantly negative and plays an important role in removing well-known biases in option models.<sup>4</sup>

The rest of the paper is organized as follows. In Section 2, we describe our data and the empirical methods we use to extract the time series of correlations. In Section 3, we test our factor model using cross-sectional regressions. In Section 4, we perform robustness checks. Section 5 concludes.

### 2. Data and empirical methods

### 2.1. Data

The sample period we study is from July 1963 to December 2007. The start date of our sample period is similar to that examined in other papers in the literature (see, for example, Fama and French, 1993 and Ang et al., 2001). We collect data on three groups of variables: (i) assets used to construct the correlation factor, (ii) test assets used in cross-sectional regressions, and (iii) control variables that proxy for the business cycle and represent other risk factors in the cross-section of average returns. We describe each of these groups below.

We use 25 portfolios sorted by size and book-to-market to compute inter-asset correlation risk. The daily returns of these asset classes come from Professor Ken French's web site. We use portfolios as opposed to individual stocks to reduce the dimensionality of the problem. In addition, these portfolios represent broad asset classes that follow popular investment styles.

We use the excess returns of all NYSE, AMEX and Nasdaq stocks from the monthly Center for Research in Security Prices (CRSP) database as test assets. Alternatively, we use the excess returns on 30 portfolios sorted by industry and the excess returns on 25 portfolios sorted by loadings on the market return and correlation innovation as test assets. The construction of 25 portfolios sorted by loadings on the market return and correlation innovation is detailed later in the paper. The returns on the industry portfolios are also obtained from Ken French's web site.

We collect data on the following control variables:

### 2.1.1. Fama-French factors

We collect data on the Fama and French (1993) book-to-market and size factors, HML and SMB. We use these factors for two purposes. First, we use them to purge the inter-asset correlations of these two well-known systematic risk factors. Second, we use them as control variables in our cross-sectional regressions to determine the price of correlation risk. The monthly returns of HML and SMB come from Ken French's web site.

<sup>&</sup>lt;sup>4</sup> Pollet and Wilson (2007) express the variance of the market portfolio as a function of the average correlation between individual stocks and the average stock variance. They find that the average correlation forecasts market returns, while the average variance does not.

#### 2.1.2. Default risk premium

Following Fama and French (1993), who show that bond returns have explanatory power for stock returns, we collect data on default risk premium, DEF, to control for the returns on Government and risky corporate bonds. DEF is defined as

$$DEF_t = R_{LG,t} - R_{LT,t}$$

where  $R_{LG,t}$  is the total return on a low grade corporate bond and  $R_{LT,t}$  is the total return on a long term Treasury bond from the lbbotson database.

### 2.1.3. Unanticipated inflation

Following Chen, Roll and Ross (1986) and Brennan, Wang, and Xia (2004), we collect data on unanticipated inflation as an additional risk factor. We first obtain the Consumer Price Index (CPI) data from the FRED database of the Federal Reserve Bank of St. Louis. Then, we define unanticipated inflation as

$$UI_t = I_t - EI_t$$

where  $I_t$  is the realized monthly first difference of the natural logarithm of CPI for period t, and  $EI_t$  is the date t-1 expected inflation. The expected inflation series are the fitted values of an AR(3) process on monthly CPI.

### 2.1.4. Growth in industrial production

Vassalou (2003) uses GDP as an explanatory variable for stock returns. However, the data on GDP is available only quarterly, while we conduct our analysis on a monthly basis. Therefore, we collect data on the growth rate of US industrial production, GIP. This data is obtained from the FRED database of the Federal Reserve Bank of St. Louis. If  $IP_t$  denotes the rate of industrial production in month t, then the monthly growth rate is

$$GIP_t = ln(IP_t) - ln(IP_{t-1}).$$

### 2.1.5. Aggregate market volatility

We follow French, Schwert, and Stambaugh (1987) and calculate the monthly volatility of the CRSP value-weighted portfolio by using daily returns each month. Schwert (1989) relates stock market volatility to a number of economic variables, including inflation rates and industrial production growth. Accordingly, we regress realized market volatility on a vector of variables, including the market return, inflation, growth rate of industrial production, the real rate, and past volatility realizations. The residual from the regression, MVOL, is used as a control factor in our cross-sectional tests.

### 2.1.6. Liquidity and real rate

Pastor and Stambaugh (2003) show that stocks with high liquidity betas have higher average returns. Since they show that liquidity is a systematic risk factor, we control for this factor in our cross-sectional analysis. We use the monthly returns of the aggregate liquidity factor, LIQ.<sup>5</sup> We collect monthly data on the real interest rate, defined as the one-month T-bill rate minus expected inflation. The CRSP value-weighted portfolio is our proxy for the market portfolio. Whenever we use excess returns, we define them as simple returns minus the nominal one-month T-Bill rate. Data on the one-month T-bill rate comes from CRSP.

Our test of whether aggregate correlation is priced in the cross-section of returns proceeds in three main steps. The first step involves estimating the time series of pair-wise correlations between different assets, and constructing an aggregate inter-asset correlation factor without look-ahead bias. The second step involves estimating individual asset's loadings with respect to correlation risk, controlling for multiple risk factors. The third step examines whether asset loadings with respect to correlation risk are important determinants of average returns.

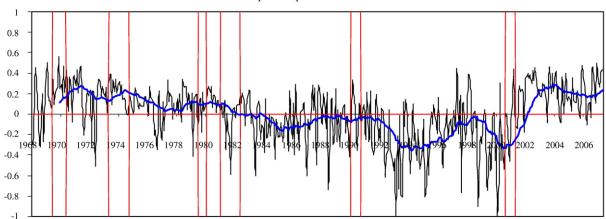
# 2.2. Construction of correlation and volatility factors

Since computing the correlation between each pair of stocks is virtually impossible, we reduce the dimensionality of the problem by choosing the 25 Fama-French portfolios, that are double-sorted by size and book-to-market, as representative assets. These 25 portfolios represent the well-known value, growth, small-capitalization, and large-capitalization investment styles. We compute the monthly pair-wise correlations of these 25 portfolios from July 1963 through December 2007. The correlation for each month between a pair of portfolios is computed as the sample correlation of daily returns within the month. The advantage of this method is that the correlation is not model-dependent. We compute all 300 time series of correlations. Our goal is to summarize the information contained in these series into one factor that captures correlation risk. In addition, it is important to extract a factor that is free of lookahead bias. At the same time, the process should not require a long pre-estimation period that would significantly shorten the time series available for analysis. With these goals in mind, we first average correlations related to the portfolios of a particular size quintile, since it is the basis of initial sorting of our representative assets, and then perform a principal components analysis. The principal components of inter-asset correlations for the month of July 1968 are then computed using the time series of correlations from July

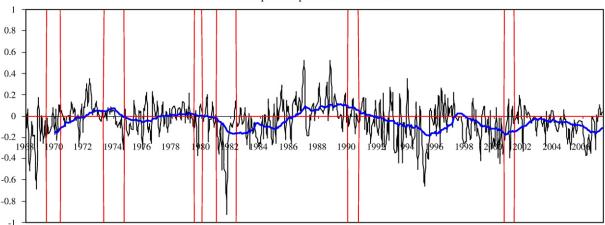
 $<sup>^{\</sup>rm 5}\,$  The LIQ data comes from CRSP and is available until December 2006.

<sup>&</sup>lt;sup>6</sup> Collin-Dufresne, Goldstein, and Martin (2001) use a similar method.

# Panel A First Principal Component of Correlations



# Panel B Second Principal Component of Correlations



**Fig. 1.** Correlation Plots. The figures plot our measures of aggregate inter-asset correlation. The base assets used are 25 portfolios sorted by size and book-to-market. Our measures of aggregate inter-asset correlation are derived by performing a principal component analysis, without any look-ahead bias, on the matrix of pairwise correlations. Panel A plots the 1st principal component and Panel B plots the 2nd principal component. The 2-year moving average is also shown. The vertical lines correspond to NBER recession periods. The sample period is from July 1968 to December 2007.

1963 through June 1968. The principal components for each subsequent month are computed using the time series of correlations up to that month. Thus, the principal components are free of any look-ahead bias.<sup>7</sup>

Panels A and B of Fig. 1 plot the first two principal components derived from the matrix of pair-wise correlations. The first principal component explains 61% of the variation in correlations, while the second principal component explains an additional 16%. Fig. 1 also highlights the NBER recession periods. The figure shows that correlation tends to increase in recessionary periods. In Panel A, the difference in average correlation between recessions and expansions is 0.093, which is statistically significant at the 1 percent level.

The top two rows of Table 1 show the sample correlations of different risk factors with our summary measures of correlation. The factors include the market return (MKT), HML, SMB, liquidity (LIQ), aggregate market volatility innovation (MVOL), default premium (DEF), unanticipated inflation (UI), growth rate of industrial production (GIP), and the NBER recession dummy (CYCLE) that takes the value of 1 in recessions and 0 otherwise. The first principal component of inter-asset correlations is significantly positively related to market volatility, default premium, and CYCLE, indicating that asset correlations tend to go up in recessionary periods. Both principal components of inter-asset correlations are also significantly related to MKT, HML and SMB. The adjusted  $R^2s$  from regressing each principal components on all factors are 21% and 9%, respectively. This indicates that a large portion of the variation in the principal components remains unexplained by well-known business-cycle variables.

The significant comovement between correlation and several macroeconomic variables raises the concern that inter-asset correlation is just another proxy for the business cycle. To alleviate this concern, we purge each pair-wise correlation series of the

 $<sup>^7</sup>$  In Section 4, we cross-check our results by using the simple average correlation of any month, and find that our results are robust.

**Table 1** Factor correlations.

Correlation factor	Portion of variance explained by principal component	MKT	HML	SMB	LIQ	MVOL	DEF	UI	GIP	CYCLE	Adjusted R <sup>2</sup>	F-statistic
1st principal component of correlations	61.09%	-0.35***	0.10*	-0.15***	0.06	0.19***	0.26**	0.09*	-0.02	0.11**	20.81%	12.02***
2nd principal component of correlations	15.81%	-0.23***	0.13**	-0.16***	-0.01	0.08	0.05	0.02	0.01	0.02	8.71%	4.21***
1st principal component based on correlation innovations	60.11%	_	_	_	0.05	0.16***	0.22***	_	-	0.08	9.09%	5.03***
2nd principal component based on correlation innovations	14.75%	_	_	_	0.00	0.05	0.05	_	_	-0.01	1.32%	0.67

This table reports the correlation coefficients between the aggregate correlation factor and other risk factors. The aggregate correlation factor is computed based on a set of 25 stock portfolios sorted by size and book-to-market. We consider two measures of aggregate correlation: (a) the first and second principal components based on pair-wise asset correlations (first and second rows of the table), and (b) the first and second principal components based on pair-wise correlation innovations, which are obtained after controlling for past correlation levels, the market return, HML, SMB, expected inflation, unexpected inflation, real rate, and growth rate of industrial production (third and fourth rows of the table). The other variables that we consider in this table are the excess market return, MKT, HML, SMB, liquidity (LIQ), market volatility (MVOL), default premium (DEF), defined as a low-grade bond return minus a long-term Treasury bond return, unexpected inflation (UI), growth rate of industrial production (GIP), and the NBER recession dummy (CYCLE). The notation '—' denotes that the correlation is almost zero because each inter-asset correlation is purged of this factor. The last two columns show the adjusted  $R^2$  values and the F-statistics from regressions of each correlation measure on MKT, HML, SMB, LIQ, MVOL, DEF, UI, GIP, and CYCLE. Also reported is the portion of variance explained by each principal component. The sample period is from July 1973 to December 2007.

\*, \*\*, and \*\*\* denote significance at the 10%, 5% and 1% levels respectively.

effects of macroeconomic variables and lagged correlation realizations. Therefore, we compute correlation innovations. To do this, we transform the inter-asset correlations,  $\{r_{it}|i=1,...300; t=1,...T\}$  which by construction are bounded by [-1,1], into Fisher correlations,  $\{\rho_{it}|i=1,...300; t=1,...T\}$  defined as:

$$\rho_{it} = 0.5 \ln \left( \frac{1 + r_{it}}{1 - r_{ir}} \right). \tag{1}$$

This function is continuous and monotonic, and there is a one-to-one mapping between the actual and Fisher-transformed correlations.

Next, we regress each pair-wise correlation series on its lagged value, the market return, HML and SMB, expected inflation, unanticipated inflation, the growth rate of industrial production, and the real interest rate:

$$\rho_{i,t} = b_{i,0} + b_{i,1}\rho_{i,t-1} + b_{i,2}R_{m,t} + b_{i,3}R_{HML,t} + b_{i,4}R_{SMB,t} + b_{i,5}EI_t + b_{i,6}UI_t + b_{i,7}GIP_t + b_{i,8}RealRate_t + u_{i,t}. \tag{2}$$

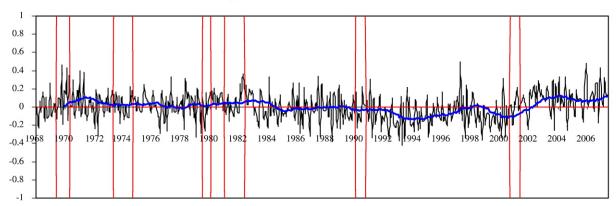
When this regression includes only macroeconomic variables, the average adjusted  $R^2$  is 7.98%. Judging from the F-statistics, the effect of macroeconomic variables on the correlation between assets is statistically significant but economically small. When HML and SMB are included in the specification, the average adjusted  $R^2$  increases to 8.87%. When lagged correlation is included in the model, the average adjusted  $R^2$  increases further to 24.51%. Therefore, lagged correlation adds a substantial explanatory power to the model over and above the other variables. However, the portion of the variation in return correlations not explained by the specified model remains large.

We store the time series of residuals,  $u_{i,t}$ , from Eq. (2) for each asset pair. Following the same procedure discussed above, we perform a principal component analysis, without look-ahead bias, on the matrix of correlation innovations. We find that the first principal component explains 60% of the variation in correlation innovations, while the second principal component explains an additional 15%. These two principal components represent our measures of correlation risk.

Panels A and B of Fig. 2 show the time series of the first two principal components based on correlation innovations. Comparing Panel A of Fig. 2 with Panel A of Fig. 1, it is clear that the variation in correlations is reduced once we control for the effects of past correlations and macroeconomic variables. For instance, during recessions, the difference between the first principal component of correlations in Panel A of Fig. 1 and the first principal component of correlation innovations in Panel A of Fig. 2 is 0.056, which is significant at the 10% level. In other words, once we control for variables related to the business cycle, correlations in bad times decrease in magnitude. Nevertheless, correlation innovations still show a time-varying pattern. Pindyck and Rotemberg (1993) also document excess correlation in stock price movements that is not explained by current or future values of macroeconomic variables. Kallberg and Pasquariello (2008) show that the comovement between US industry indexes that cannot be explained by fundamental factors is high and statistically significant.

The bottom two rows of Table 1 correspond to the principal components based on correlation innovations. They show that once the effects of business cycle variables have been removed, the correlation innovations are not significantly correlated with the CYCLE variable. This suggests that correlation innovations carry information independent of the business cycle and other well-known risk factors. The adjusted  $R^2$ s and F-statistics from multiple regressions of each correlation measure on the set of risk factors confirm this. The results show that while the effects of the variables examined are statistically significant, their explanatory power is small. This indicates that our correlation factors could represent separate risk factors in the cross-section of returns.

# Panel A First Principal Component of Correlation Innovations



# Panel B Second Principal Component of Correlation Innovations

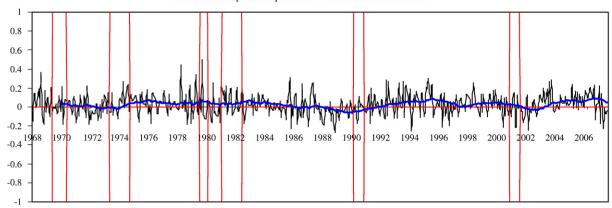


Fig. 2. Correlation Innovation Plots. The figures plot our measures of aggregate inter-asset correlation innovations. The base assets used are 25 portfolios sorted by size and book-to-market. Our measures of aggregate inter-asset correlation innovations are derived by first regressing asset correlations on past correlations, the market return, HML, SMB, expected inflation, unexpected inflation, real rate, and growth rate of industrial production and then performing a principal component analysis, without look-ahead bias, on the matrix of pair-wise correlation innovations. Panel A plots the 1st principal component and Panel B plots the 2nd principal component. The 2-year moving average is also shown. The vertical lines correspond to the NBER recession periods. The sample period is from July 1968 to December 2007.

As discussed earlier, it is important to control for asset volatilities. We follow French, Schwert, and Stambaugh (1987) and calculate the monthly volatility of the 25 size and book-to-market portfolios using daily returns within each month. Following the structure of Eq. (2), we remove the effects of macroeconomic variables by regressing each volatility series on past volatility, the market return, HML, SMB, inflation, growth rate of industrial production, and the real interest rate. We run 25 such regressions since there are 25 portfolios under consideration. Next, we perform a principal component analysis, without a look-ahead bias, on the residuals from these regressions. The first principal component explains 79% of the variation in asset volatility innovations, while the second one explains 14%. Therefore, we derive the representative asset volatility innovations in a manner consistent with the way we derive the representative correlation innovations. The first two principal components are the asset volatility factors, AVOL, that we use as control variables in our subsequent analysis.

# 3. Main results

### 3.1. Factor model specification

The model specification we consider is of the form:

$$R_{i,t} = \alpha_i + \beta_{i,F} F_t + \beta_{i,X} P_{CX,t} + e_{i,t}, \tag{3}$$

where  $R_{i,t}$  is the return on asset i in excess of the risk-free rate at the end of period t,  $F_t$  is a vector of realizations for control risk factors at the end of period t, and  $P_{\text{CX},t}$  is a vector of principal components. X refers to either correlation innovations or asset

volatility innovations. Our goal is to determining whether the factor related to correlation innovations is priced in the cross-section of returns.

The unconditional expected excess return on asset i is given by:

$$E(R_i) = \gamma_F \beta_{i,F} + \gamma_X \beta_{i,X} \tag{4}$$

where  $\gamma_F$  is a vector of risk prices associated with the control variables, and  $\gamma_X$  is a vector of risk prices related to correlation (or volatility) risk. The betas are the slope coefficients from the return-generating process in Eq. (3). The implication of the factor model in Eq. (4) is that assets with different loadings with respect to the risk factor have different average returns.

Thus, we conduct our tests in two steps: first, we examine a time series regression for each test asset to estimate the factor loadings; second, we conduct cross-sectional regressions to compute the factor prices of risk. We use the standard Fama and MacBeth (1973) regression analysis. This model assumes away time variation in the factor loadings of the assets. We relax this assumption later in the paper and examine time-varying risk loadings.

Since the betas are estimated from the time series regression in Eq. (3), they represent generated regressors in the empirical tests of Eq. (4). This is the classical errors-in-variables problem, arising from the two-pass nature of the Fama-MacBeth approach. Following Shanken (1992), we use a correction procedure that accounts for the errors-in-variables problem. Shanken's correction is designed to adjust for the overstated precision of the Fama-MacBeth standard errors.

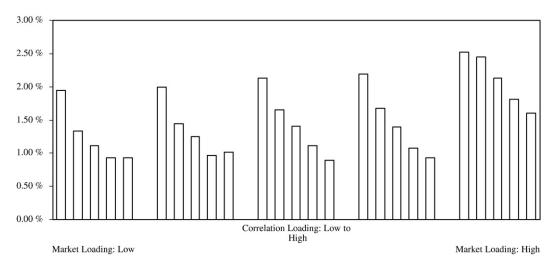
#### 3.2. Test assets

The first set of test assets that we use consists of all stocks trading on the NYSE, AMEX, and Nasdaq. In a recent paper, Ang, Liu and Schwarz (2008) show that using individual stocks leads to more powerful tests of whether factors are priced. This is the case since the most important determinant of the variance of factor risk premia is the cross-sectional distribution of factor loadings. This dispersion is greatest when individual stocks are used.

As a robustness check, we also use portfolios as test assets. Our set of portfolios consists of 30 portfolios sorted by industry classification and 25 portfolios sorted by risk loadings. To construct the risk-sorted portfolios, we use the returns of all NYSE, AMEX and Nasdaq stocks. Each month, each stock's risk loadings are computed from a multiple regression of the stocks excess returns over the previous 60 months on two factors: the excess market return and the excess return on a portfolio that mimics the first principal component of correlation innovations. A simple way to construct a mimicking portfolio is to regress our measure of aggregate inter-asset correlation on a set of base asset returns, as suggested in Breeden, Gibbons and Litzenberger (1989). We use this approach in creating a correlation-mimicking portfolio, using:

$$\rho_t = c' B_t + e_t. \tag{5}$$

Here  $B_t$  represents the set of base portfolio returns in excess of the risk free rate. The return on the mimicking portfolio,  $R_{\rho,t}$ , is equal to  $c'B_t$ . The base assets are six value-weighted portfolios, taken from Ken Frenchs web site, from the intersection of two size



**Fig. 3.** Portfolio Returns Sorted by Market and Correlation Loadings. This figure shows plots of the average returns of 25 portfolios of stocks for the period from July 1973 to December 2007. These 25 portfolios are constructed as follows. Each month, each stock's risk loadings are computed from a multiple regression of returns over the previous 60 months on the factors over the same 60 months. The factors are the excess market return, and the excess return on a portfolio that mimics the first principal component of correlation innovations. The assets used to obtain the mimicking portfolio returns are 6 portfolios sorted on size and book-to-market. 5 groups are formed by sorting stocks on market beta. Each market beta group is further sorted into 5 groups based on correlation betas. Thus, the 5 major groups of bars in the figure below are based on market returns loadings, while the 5 bars in each major group are based on correlation loadings.

and three book-to-market portfolios. These portfolios are created from a separate sorting of the assets relative to the 25 size and book-to-market portfolios.

Our objective is to form portfolios that have a large spread in their betas with respect to the market and the correlation factor. To accomplish this, we perform a double sort on market beta and on the beta with respect to the first principal component of correlation innovations. That is, we first form 5 groups by sorting all stocks on market beta, and then sort stocks in each market beta group into 5 groups on correlation beta. This produces a total of 25 portfolios. The value-weighted returns of these portfolios are recorded for the period from July 1973 to December 2007 (we start in July 1973 since we need 60 months to compute the risk loadings).

Fig. 3 shows the average returns of the set of 25 risk portfolios. In this figure, the five major groups of bars are based on market betas, while the bars in each major group are based on correlation loadings. The average returns of stocks with different correlation loadings follow a regular pattern: within each market beta group, average returns decrease in the correlation loadings. Thus, assets that do well when correlation is high provide a hedge in states with poor diversification benefits. Therefore, they are more valuable and have lower average returns.

Within each market beta group, we compute the return difference between the extreme portfolios based on correlation loadings. Each return difference represents the monthly return on a zero-investment strategy that is based on correlation risk. This investment strategy produces positive and significant alphas relative to the Fama and French (1993) model.

# 3.3. Cross-sectional regressions

We first examine whether asset volatility is priced. We begin with the following equation, in which the test assets are individual stock returns. Specifically, for any stock i:

$$R_{i,t} = \alpha_i + \beta_{i,m} R_{m,t} + \beta_{i,HML} R_{HML,t} + \beta_{i,SMB} R_{SMB,t} + \beta_{i,MVOL} R_{MVOL,t} + \beta_{i,AVOL} PC_{AVOL,t} + e_{i,t}, \tag{6}$$

where the left-hand-side variables represent returns in excess of the riskfree rate and  $PC_{AVOL}$  is a vector of the first two principal components of volatility innovations.

**Table 2**Cross-sectional price of risk: is asset variance or inter-asset covariance risk priced?

Factor	Specification 1	Specification 2
MKT	0.0077	0.0160
	(0.92)	(1.37)
HML	0.0037	0.0044
	(1.67)*	(2.89)***
SMB	0.0019	0.0018
	(1.05)	(0.88)
MVOL	0.0030	0.0051
	(1.72)*	(1.97)**
AVOL 1st principal component	- 1.9568	
	(-1.98)**	
AVOL 2nd principal component	− 1.3357	
	(-1.75)*	
COV 1st principal component		-0.9655
		(-1.38)
COV 2nd principal component		-0.6472
		(-0.61)
Intercept	0.0110	-0.0201
	(1.29)	(-1.64)*
Adjusted R <sup>2</sup>	47.74	52.74

The first column reports the cross-sectional  $\gamma$  coefficients of the following Fama-MacBeth regression

$$R_{i,t} = \gamma_0 + \gamma_m \hat{\beta}_{i,m} + \gamma_{HML} \hat{\beta}_{i,HML} + \gamma_{SMB} \hat{\beta}_{i,SMB} + \gamma_{MVOL} \hat{\beta}_{i,MVOL} + \gamma_{AVOL} \hat{\beta}_{i,AVOL} + e_{i,t},$$

while the second column reports the cross-sectional  $\gamma$  coefficients of the following Fama-MacBeth regression:

$$R_{i,t} = \gamma_0 + \gamma_m \hat{\beta}_{i,m} + \gamma_{HML} \hat{\beta}_{i,HML} + \gamma_{SMB} \hat{\beta}_{i,SMB} + \gamma_{MVOL} \hat{\beta}_{i,MVOL} + \gamma_{COV} \hat{\beta}_{I,COV} + e_{i,t}.$$

The dependent variables are the excess returns of all NYSE, AMEX and Nasdaq listed stocks from the monthly Center for Research in Security Prices (CRSP) database. The coefficients are as follows:  $\gamma_m$  is the market risk premium,  $\gamma_{HML}$  and  $\gamma_{SMB}$  are the risk premia associated with HML and SMB,  $\gamma_{MVOL}$  is the risk premia associated with market volatility innovation,  $\gamma_{AVOL}$  is the premium associated with asset volatility innovations, and  $\gamma_{COV}$  is the premium associated with covariance innovations. To compute aggregate variance and covariance, we use 25 portfolios sorted by size and book-to-market as base assets. Our asset variance (covariance) factors are the first 2 principal components of the matrix of asset variance innovations (pair-wise covariance innovations) that are computed after controlling for past level and macroeconomic variables. The full-sample factor loadings, which are the independent variables in the regressions, are computed in one multiple time series regression. The coefficients are expressed in decimals per month. The adjusted  $R^2$  follows Jagannathan and Wang (1996) and is reported as a percentage. The t-statistics are adjusted for errors-in-variables, following Shanken (1992), and are reported in parenthesis. The sample period is from July 1973 to December 2007.

**Table 3** Cross-sectional price of risk: is correlation risk priced?

Factor	Specification 1	Specification 2
MKT	0.0090	0.0060
	(2.08)**	(0.15)
HML	0.0068	0.0008
	(2.90)***	(0.41)
SMB	0.0011	0.0061
	(0.57)	(1.71)*
MVOL	0.0011	0.0011
	(0.67)	(0.81)
AVOL (1st principal component)	<b>–</b> 1.5777	-0.8860
	(-1.13)	(-0.50)
AVOL (2nd principal component)	-0.4864	0.3415
	(-0.92)	(0.44)
$\rho$ (1st principal component)	-0.6503	-0.2733
	(-2.56)***	(-2.50)**
$\rho$ (2nd principal component)	0.0852	-0.0267
	(1.04)	(-0.13)
Intercept	0.0039	0.0054
	(0.37)	(1.51)
Adjusted R <sup>2</sup>	68.09	64.74

$$R_{i,t} = \gamma_0 + \gamma_m \hat{\beta}_{i,m} + \gamma_{HML} \hat{\beta}_{i,HML} + \gamma_{SMB} \hat{\beta}_{i,SMB} + \gamma_{MVOL} \hat{\beta}_{i,MVOL} + \gamma_{AVOL} \hat{\beta}_{i,AVOL} + \gamma_{\rho} \hat{\beta}_{i,\rho} + e_{i,t}$$

In Specification 1, the dependent variables are the excess returns of all NYSE, AMEX and Nasdaq listed stocks from the monthly Center for Research in Security Prices (CRSP) database while in Specification 2, the dependent variables are the excess returns on 30 portfolios sorted by industry plus 25 portfolios sorted by loadings on the market return and correlation innovation,  $\gamma_m$  is the market risk premium,  $\gamma_{HML}$  and  $\gamma_{SMB}$  are the risk premia associated with HML and SMB,  $\gamma_{MVOL}$  is the risk premia associated with market volatility innovation,  $\gamma_{AVOL}$  is the premium associated with asset volatility innovation, and  $\gamma_p$  is the premium associated with correlation innovation. To compute aggregate correlation we use 25 portfolios sorted by size and book-to-market as base assets. Our correlation factors are the first 2 principal components of the matrix of pair-wise correlation innovations (computed after controlling for past correlations and other macroeconomic variables). The adjusted  $R^2$  follows Jagannathan and Wang (1996) and is reported as a percentage. The t-statistics are adjusted for errors-in-variables, following Shanken (1992), and are reported in parenthesis. The sample period is from July 1973 to December 2007.

\*, \*\*, and \*\*\* denote significantly different from zero at the 10%, 5% and 1% levels respectively.

The second step of the Fama-MacBeth procedure involves relating the average excess returns of all assets to their exposures to the risk factors in the model. We specify the cross-sectional relation:

$$R_{i,t} = \gamma_0 + \gamma_m \hat{\beta}_{i,m} + \gamma_{HML} \hat{\beta}_{i,HML} + \gamma_{SMB} \hat{\beta}_{i,SMB} + \gamma_{MVOL} \hat{\beta}_{i,MVOL} + \gamma_{AVOL} \hat{\beta}_{i,AVOL} + e_{i,t}, \tag{7}$$

for all i each month. If assets' loadings with respect to the risk factors are important determinants of average returns, then the  $\gamma$  terms that represent the prices of risk for each factor should be significant.

The first column of Table 2 presents the results from the estimation of Eq. (7). As the table shows, the price of risk associated with the first principal component of volatility innovations is negative in magnitude and statistically significant. This result is robust to the errors-in-variables adjustment. The price of risk for the second principal component (which is orthogonal to the first principal component) is significant only at the 10% level. The risk premia associated with aggregate market volatility and HML are positive and marginally significant.

To examine whether covariance risk is priced in the cross-section of asset returns, we estimate Eq. (6) by replacing the first two principal components of AVOL with the first two principal components of covariance innovations, COV, that are also computed without a look-ahead bias. The second column of Table 2 presents the results, using individual stock returns as test assets. The principal components of covariance innovations are not significantly priced.

Next we turn to our main specification which examines the price of market-wide correlation risk, after controlling for the effects of market and asset volatility. Specifically, we look at the cross-sectional relation:

$$R_{i,t} = \gamma_0 + \gamma_m \hat{\beta}_{i,m} + \gamma_{HML} \hat{\beta}_{i,HML} + \gamma_{SMB} \hat{\beta}_{i,SMB} + \gamma_{MVOL} \hat{\beta}_{i,MVOL} + \gamma_{AVOL} \hat{\beta}_{i,AVOL} + \gamma_{\rho} \hat{\beta}_{i,\rho} + e_{i,t}, \tag{8}$$

where the left-hand-side variables represent returns in excess of the riskfree rate and  $\rho$  refers to the first two principal components of correlation innovations. We use two different sets of test assets: individual stocks (Specification 1) and 30 industry portfolios plus 25 risk-sorted portfolios (Specification 2).

Table 3 presents the results from the estimation of Eq. (8). The first column reports on Specification 1. The price of risk associated with the first principal component of correlation innovations is negative in magnitude and statistically significant. This result is robust to the errors-in-variables adjustment. The price of risk for the second principal component of correlation innovations is not significant. Based on the observation that the second principal component of correlation innovations is not priced in the cross-section of returns, we do not consider the second principal component in further analysis. Neither of the first two principal components of volatility innovations is significant. The market risk premium and the risk premium associated

**Table 4**Cross-sectional price of risk: controlling for other risk factors.

Factor	Specification 1	Specification 2	Specification 3	Specification 4
MKT	0.0070	0.0092	0.0180	0.0220
	(0.97)	(1.17)	(0.47)	(0.52)
HML	0.0047	0.0055	0.0012	0.0015
	(2.83)***	(3.04)***	(0.63)	(0.73)
SMB	0.0024	0.0027	0.0071	0.0058
	(1.32)	(1.37)	(1.91)*	(1.65)*
MVOL	0.0018	0.0017	0.0012	0.0006
	(1.42)	(1.34)	(1.25)	(0.92)
LIQ		0.0475		0.0996
		(1.49)		(0.43)
DEF	-0.0018	-0.0018	-0.0017	-0.0017
	(-1.77)*	(-1.80)*	(-1.95)*	(-1.64)*
UI	-0.0009	-0.0007	-0.0002	-0.0004
	(-1.25)	(-0.82)	(-0.67)	(-1.28)
GIP	0.0046	0.0047	0.0005	0.0010
	(1.85)*	(1.79)*	(0.64)	(1.11)
AVOL (1st principal component)	-0.4060	-0.3419	-0.4180	-0.4430
	(-0.55)	(-0.44)	(-0.65)	(-0.74)
$\rho$ (1st principal component)	-0.6043	-0.6630	-0.2778	-0.3912
	(-2.61)***	(-2.70)***	(-2.55)***	(-2.73)***
Intercept	0.0018	-0.0015	0.0060	0.0022
	(0.26)	(-0.20)	(1.61)	(0.59)
Adjusted R <sup>2</sup>	75.06	75.45	75.23	77.03

$$R_{i,t} = \gamma_0 + \gamma_m \hat{\beta}_{i,m} + \gamma_{HML} \hat{\beta}_{i,HML} + \gamma_{SMB} \hat{\beta}_{i,SMB} + \gamma_{MVOL} \hat{\beta}_{i,MVOL} + \gamma_{LIQ} \hat{\beta}_{i,LIQ} + \gamma_{DEF} \hat{\beta}_{i,DEF} + \gamma_{UI} \hat{\beta}_{i,UI} + \gamma_{GIP} \hat{\beta}_{i,GIP} + \gamma_{AVOL} \hat{\beta}_{i,AVOL} + \gamma_{\rho} \hat{\beta}_{i,\rho} + e_{i,t}.$$

In Specifications 1 and 2, the dependent variables are the excess returns of all NYSE, AMEX and Nasdaq listed stocks from the monthly Center for Research in Security Prices (CRSP) database while in Specifications 3 and 4, the dependent variables are the excess returns on 30 portfolios sorted by industry plus 25 portfolios sorted by loadings on the market return and correlation innovation,  $\gamma_m$  is the market risk premium,  $\gamma_{\text{HML}}$ ,  $\gamma_{\text{SMB}}$ ,  $\gamma_{\text{MNOL}}$ ,  $\gamma_{\text{LIQ}}$ ,  $\gamma_{\text{DEF}}$ ,  $\gamma_{\text{UI}}$ , and  $\gamma_{\text{CIP}}$  are the risk premia associated with HML, SMB, default market volatility innovation, liquidity, default spread, unexpected inflation, and the growth rate of industrial production, respectively,  $\gamma_{\text{AVOL}}$  is the premium associated with asset volatility innovation, and  $\gamma_{\text{P}}$  is the coefficient associated with correlation innovation. To compute aggregate correlation we use 25 portfolios sorted by size and book-to-market as base assets. Our correlation factors are the first two principal components of the matrix of pair-wise correlation innovations (computed after controlling for past correlations and other macroeconomic variables). The full-sample factor loadings, which are the independent variables in the regressions, are computed in one multiple time series regression. The coefficients are expressed in decimals per month. The adjusted  $R^2$  follows Jagannathan and Wang (1996) and is reported as a percentage. The *t*-statistics are adjusted for errors-in-variables, following Shanken (1992), and are reported in parenthesis. The sample period is from July 1973 to December 2007 for specifications 1 and 3, and July 1973 to December 2006 for specifications 2 and 4 that include the LIO factor.

\*, \*\*, and \*\*\* denote significantly different from zero at the 10%, 5% and 1% levels respectively.

with HML are significantly positive. The adjusted  $R^2$  value shows that the model explains a substantial portion of the cross-sectional variation in average returns. The results for Specification 2 look similar.

Since the dependent variables in the cross-sectional regression are excess returns, the intercept term,  $\gamma_0$ , should be equal to zero if the model is well-specified model. This hypothesis cannot be rejected in the case of the model presented in Eq. (8). Overall the results indicate that correlation risk is priced in the cross-section of average returns. The estimated negative price of risk suggests that assets that pay more when correlation is high have high values and hence, lower expected returns.

### 3.4. Controlling for other risk factors

Does the price of correlation risk continue to be significantly negative even in the presence of other well-known risk factors? To examine this, we include additional risk factors in the basic model. These are factors that have been shown to be important determinants of the cross-section of returns. They include default premium (DEF), unexpected inflation (UI), and the growth rate of industrial production (GIP). We examine the following equation:

$$R_{i,t} = \gamma_0 + \gamma_m \hat{\beta}_{i,m} + \gamma_{HML} \hat{\beta}_{i,HML} + \gamma_{SMB} \hat{\beta}_{i,SMB} + \gamma_{MVOL} \hat{\beta}_{i,MVOL} + \gamma_{DEF} \hat{\beta}_{i,DEF} + \gamma_{UI} \hat{\beta}_{i,UI} + \gamma_{GIP} \hat{\beta}_{i,GIP} + \gamma_{AVOL} \hat{\beta}_{i,AVOL} + \gamma_{\rho} \hat{\beta}_{i,\rho} + e_{i,t}. \quad (9)$$

The results are presented in Table 4.

The first column uses the excess returns of individual stocks. The price of correlation risk continues to be negative in magnitude and significant even after the Shanken adjustment. The risk premium of HML is positive and significant. Among the other variables, the default risk premium is significantly negative and the premium associated with the growth in industrial production is significantly positive. The intercept term is not significant and the adjusted  $R^2$  value indicates that the explanatory power of the model has increased relative to the basic model.

Next, we include Pastor and Stambaugh's (2003) liquidity factor, LIQ, as an additional risk factor in the above regression specification. As discussed before, correlation tends to be high in bear markets, and these are the very times when liquidity tends to be low. Therefore, high aggregate correlation could potentially capture the effects of liquidity risk. Hence, we explicitly control for the

**Table 5**Cross-sectional price of risk: controlling for co-skewness and co-kurtosis.

Factor	Specification 1	Specification 2	Specification 3	Specification 4
MKT	0.0044	0.0013	0.0120	0.0180
	(1.19)	(0.23)	(0.30)	(0.46)
HML	0.0031	0.0046	0.0017	0.0017
	(2.00)**	(2.92)***	(0.83)	(0.88)
SMB	0.0015	0.0020	0.0065	0.0069
	(0.85)	(1.16)	(1.64)*	(1.47)
MVOL	0.0010	0.0011	0.0020	0.0017
	(1.13)	(1.24)	(1.72)*	(1.32)
DEF		-0.0014		-0.0009
		(-1.54)		(-0.92)
UI		-0.0006		-0.0001
		(-1.01)		(-0.28)
GIP		0.0036		0.0002
		(1.89)*		(0.27)
Co-skewness	0.0011	0.0014	0.0008	0.0006
	(0.55)	(1.16)	(1.15)	(0.92)
Co-Kurtosis	0.0007	0.0010	0.0010	0.0010
	(0.13)	(0.61)	(0.92)	(0.87)
AVOL 1st principal component	<b>- 1.4555</b>	-0.3361	-0.5980	-0.2167
	(-1.61)	(-0.56)	(-0.86)	(-0.35)
ho 1st principal component	-0.6091	-0.6638	-0.2505	-0.2169
	(-2.48)**	(-2.98)***	(-2.33)**	(-2.12)**
Intercept	0.0088	0.0019	0.0051	0.0057
-	(1.18)	(0.34)	(1.47)	(1.57)
Adjusted R <sup>2</sup>	69.56	75.96	68.80	67.45

$$R_{l,t} = \gamma_0 + \gamma_m \hat{\beta}_{l,m} + \gamma_{HMI} \hat{\beta}_{l,HML} + \gamma_{SMB} \hat{\beta}_{l,SMB} + \gamma_{MVOl} \hat{\beta}_{l,MVOl} + \gamma_{DEF} \hat{\beta}_{l,DEF} + \gamma_{Ul} \hat{\beta}_{l,Ul} + \gamma_{GP} \hat{\beta}_{l,GIP} + \gamma_{AVOl} \hat{\beta}_{l,AVOL} + \gamma_{SKEW} \hat{\beta}_{l,SKEW} + \gamma_{KEW} \hat{\beta}_{l,SKEW} + \gamma_{KWR} \hat{\beta}_{l,KURT} + \gamma_{\rho} \hat{\beta}_{l,Q} + \varepsilon_{l,1}$$

In Specifications 1 and 2, the dependent variables are the excess returns of all NYSE, AMEX and Nasdaq listed stocks from the monthly Center for Research in Security Prices (CRSP) database while in Specifications 3 and 4, the dependent variables are the excess returns on 30 portfolios sorted by industry plus 25 portfolios sorted by loadings on the market return and correlation innovation,  $\gamma_m$  is the market risk premium,  $\gamma_{HML}$ ,  $\gamma_{SMB}$ ,  $\gamma_{MVOL}$ ,  $\gamma_{DEF}$ ,  $\gamma_{UI}$ , and  $\gamma_{GIP}$  are the risk premia associated with HML, SMB, default market volatility innovation, default spread, unexpected inflation, and the growth rate of industrial production, respectively,  $\gamma_{AVOL}$  is the premium associated with asset volatility innovations,  $\gamma_{SKEW}$  and  $\gamma_{KURT}$  are the risk premia associated with inter-asset co-skewness and co-kurtosis, and  $\gamma_P$  is the coefficient associated with correlation innovation. Our correlation factor is the first principal component of the matrix of pair-wise correlation innovations (computed after controlling for past correlations and other macroeconomic variables). The coefficients are expressed in decimals per month. The adjusted  $R^2$  follows Jagannathan and Wang (1996) and is reported as a percentage. The t-statistics are adjusted for errors-in-variables, following Shanken (1992), and are reported in parenthesis. The sample period is from July 1973 to December 2007.

\*, \*\*, and \*\*\* denote significantly different from zero at the 10%, 5% and 1% levels respectively.

presence of aggregate liquidity. The second column in Table 4 reports results after including the liquidity factor as an additional control variable. The price of correlation risk continues to be negative in magnitude and significant after the Shanken adjustment. The intercept term from the regression is not significant and the adjusted  $R^2$  value indicates that the explanatory power of the model is high.

We replicate the analysis above using the 30 industry portfolios plus 25 risk-sorted portfolios as test assets. The results are presented in the last 2 columns of Table 4. The price of correlation risk continues to be negative in magnitude and significant after the errors-in-variables adjustment, controlling for other systematic risk factors and liquidity (as reported in the last column). HML is not significantly priced while SMB is weakly significant. The default risk premium continues to be negative and significant. The intercept term from the regression is not significant.

Correlations are related to higher movements. To examine whether correlation risk continues to be significantly priced in the cross-section of stocks after controlling for standard higher moments, we include inter-asset co-skewness and co-kurtosis in our multivariate regression analysis.

We follow the method we use to compute correlation risk, and compute the inter-asset co-skewness and co-kurtosis from the time series of returns of the 25 Fama-French portfolios. We purge each time series of co-skewness and co-kurtosis of past levels, HML, SMB, and macroeconomic variables to derive co-skewness and co-kurtosis innovations. We compute the first principal components (without a lookahead bias) of co-skewness and co-kurtosis innovations and use them as additional factors.

We include these two higher moments in an expanded specification:

$$R_{i,t} = \gamma_0 + \gamma_m \hat{\beta}_{i,m} + \gamma_{HML} \hat{\beta}_{i,HML} + \gamma_{SMB} \hat{\beta}_{i,SMB} + \gamma_{MVOL} \hat{\beta}_{i,MVOL} + \gamma_{DEF} \hat{\beta}_{i,DEF} + \gamma_{UI} \hat{\beta}_{i,UI} + \gamma_{GIP} \hat{\beta}_{i,GIP} + \gamma_{AVOL} \hat{\beta}_{i,AVOL} + \gamma_{\rho} \hat{\beta}_{i,\rho}$$
(10)  
+  $\gamma_{COSKEW} \hat{\beta}_{i,COSKEW} + \gamma_{COSURT} \hat{\beta}_{i,COSKIRT} + e_{i,t}$ .

<sup>&</sup>lt;sup>8</sup> In this specification, our sample ends in December 2006 rather than in December 2007 because LIQ data is available until December 2006.

**Table 6**Cross-sectional price of risk: using average correlation.

Factor	Specification 1	Specification 2
MKT	0.0020	0.0038
	(0.31)	(0.48)
HML	0.0046	0.0055
	(2.73)***	(2.98)***
SMB	0.0028	0.0030
	(1.44)	(1.43)
MVOL	0.0018	0.0018
	(1.36)	(1.32)
LIQ		0.0480
		(1.35)
DEF	-0.0034	-0.0031
	(-1.97)**	(-1.57)
UI	-0.0008	-0.0006
	(-1.19)	(-1.32)
GIP	0.0063	0.0060
	(2.15)**	(2.06)**
AVOL (Average)	-0.4774	-0.5369
	(-1.04)	(-1.12)
ho (Average)	-0.6014	-0.6013
	(-3.10)***	(-3.05)***
Intercept	0.0030	-0.0004
	(0.43)	(-0.05)
Adjusted R <sup>2</sup>	74.43	73.90

$$R_{i,t} = \gamma_0 + \gamma_m \hat{\beta}_{i,m} + \gamma_{HML} \hat{\beta}_{i,HML} + \gamma_{SMB} \hat{\beta}_{i,SMB} + \gamma_{MVOL} \hat{\beta}_{i,MVOL} + \gamma_{LIQ} \hat{\beta}_{i,LIQ} + \gamma_{DEF} \hat{\beta}_{i,DEF} + \gamma_{UI} \hat{\beta}_{i,UI} + \gamma_{GIP} \hat{\beta}_{i,GIP} + \gamma_{AVOL} \hat{\beta}_{i,AVOL} + \gamma_{\rho} \hat{\beta}_{i,\rho} + e_{i,t}.$$

The dependent variables are the excess returns of all NYSE, AMEX and Nasdaq listed stocks from the monthly Center for Research in Security Prices (CRSP) database,  $\gamma_m$  is the market risk premium,  $\gamma_{HML}$ ,  $\gamma_{SMB}$ ,  $\gamma_{MVOL}$ ,  $\gamma_{LIQ}$ ,  $\gamma_{DEF}$ ,  $\gamma_{UI}$ , and  $\gamma_{GIP}$  are the risk premia associated with HML, SMB, default market volatility innovation, liquidity, default spread, unexpected inflation, and the growth rate of industrial production, respectively,  $\gamma_{AVOL}$  is the premium associated with average asset volatility innovation, and  $\gamma_P$  is the coefficient associated with correlation innovation. To compute the aggregate correlation we start with a base set of 25 Fama-French portfolios sorted by size and book-to-market. Our correlation factor is the average of asset correlation innovations (computed after controlling for past correlations and other macroeconomic variables). The full-sample factor loadings, which are the independent variables in the regressions, are computed in one multiple time series regression. The coefficients are expressed in decimals per month. The adjusted  $R^2$  follows Jagannathan and Wang (1996) and is reported as a percentage. The *t*-statistics are adjusted for errors-in-variables, following Shanken (1992), and are reported in parenthesis. The sample period is from July 1973 to December 2007 for specification 1 and July 1973 to December 2006 for specification 2 that includes the LIQ factor.

\*, \*\*\*, and \*\*\* denote significantly different from zero at the 10%, 5% and 1% levels respectively.

The results are presented in Table 5.

The first two columns report results of two specifications with the excess returns of individual stocks as test assets. The last two columns report results of two specifications with the excess returns of 30 industry portfolios plus 25 risk-sorted portfolios as test assets. The co-skewness and co-kurtosis factors are not significant. The price of correlation risk continues to be negative and significant after the Shanken adjustment. Therefore, the correlation factor does not appear to be capturing the effects of higher moments.

We also construct factors that account for asset's co-skewness relative to the market following Harvey and Siddique (1999), and co-kurtosis in a similar fashion (see Ang et al., 2001 and Dittmar, 2002). We check the price of correlation risk in the presence of these control variables. Our results (unreported) indicate that these factors are not significant in the cross-section of asset returns in the presence of the correlation risk factor. The price of correlation risk continues to be negative and significant. Since we purge the correlation factor of the effects of business-cycle variables, and find that it is significantly priced in the presence of higher order moments, we conjecture that the correlation factor is picking up diversification risk.

Overall, the results reveal that correlation innovation risk is significantly priced in the cross-section of expected returns. This result is robust to the use of different sets of test assets, to the presence of other well-known risk factors, and to errors-in-variables adjustment.

# 4. Robustness checks

## 4.1. Using average correlation innovations

We examine whether our procedure of extracting principal components, rather than simply taking the average of the correlation innovations, is driving our results. Using the excess returns of individual stocks as test assets, we examine two different specifications of the following model:

$$\begin{split} R_{i,t} &= \gamma_0 + \gamma_m \hat{\beta}_{i,m} + \gamma_{HML} \hat{\beta}_{i,HML} + \gamma_{SMB} \hat{\beta}_{i,SMB} + \gamma_{MVOL} \hat{\beta}_{i,MVOL} + \gamma_{LIQ} \hat{\beta}_{i,LIQ} + \gamma_{DEF} \hat{\beta}_{i,DEF} + \gamma_{UI} \hat{\beta}_{i,UI} + \gamma_{GIP} \hat{\beta}_{i,GIP} + \gamma_{AVOL} \hat{\beta}_{i,AVOL} \ (11) \\ &+ \gamma_\rho \hat{\beta}_{i,\rho} + e_{i,t}, \end{split}$$

**Table 7**Cross-sectional price of risk: the conditional beta approach.

Factor	
MKT	0.0086
	(1.89)*
$MKT \times Z_{t-1}$	0.0003
	(0.24)
HML	0.0040
	(1.70)*
$HML{ imes Z_{t-1}}$	0.0019
MVOL	(0.97) 0.0020
WIVOL	(1.55)
$MVOL \times Z_{t-1}$	0.0003
MV0L^2t-1	(0.24)
AVOL (1st principal component)	-0.4878
(	(-0.52)
$AVOL \times Z_{t-1}$	-0.0009
	(-0.06)
ho (1st principal component)	-0.6548
	(-3.17)***
$ ho  imes Z_{t-1}$	-0.0007
	(-0.87)
$Z_{t-1}$	0.0145
Total and the second se	(1.97)**
Intercept	0.0020
Adjusted P <sup>2</sup>	(0.20)
Adjusted $R^2$ p-value	77.89 0.03
p-value	0,03

$$R_{l.t} = \gamma_0 + \gamma_2 Z_{t-1} + \gamma_m \hat{\beta}_{l.m} + \gamma_{mz} \hat{\beta}_{l.m^* Z_{t-1}} + \gamma_{HML} \beta_{l.HML} + \gamma_{HML2} \hat{\beta}_{l.HML^* Z_{t-1}} + \gamma_{MVOL} \hat{\beta}_{l.MVOL} + \gamma_{MVOL2} \hat{\beta}_{MVOL^* Z_{t-1}} + \gamma_{AVOL} \hat{\beta}_{l.AVOL} + \gamma_{AVOL2} \hat{\beta}_{l.AVOL^* Z_{t-1}} + \gamma_\rho \hat{\beta}_{l.\rho} + \gamma_{\rho z} \hat{\beta}_{l.\rho^* Z_{t-1}} + e_{l.t-1} + e_{l.t-1} \hat{\beta}_{l.MVOL} + \gamma_{MVOL2} \hat{\beta}_{l.AVOL^* Z_{t-1}} + \gamma_{AVOL2} \hat{\beta}_{l.AVOL^* Z_{t-1}} + \gamma_\rho \hat{\beta}_{l.\rho} + \gamma_{\rho z} \hat{\beta}_{l.\rho^* Z_{t-1}} + e_{l.t-1} \hat{\beta}_{l.AVOL^* Z_{t-1}} + \gamma_\rho \hat{\beta}_{l.\rho} + \gamma_{\rho z} \hat{\beta}_{l.\rho^* Z_{t-1}} + e_{l.t-1} \hat{\beta}_{l.AVOL^* Z_{t-1}} + \gamma_\rho \hat{\beta}_{l.\rho} + \gamma_\rho \hat$$

The dependent variables are the excess returns of all NYSE, AMEX and Nasdaq listed stocks from the monthly Center for Research in Security Prices (CRSP) database, the conditioning variable  $Z_{t-1}$  is the term premium, defined as the yield spread between a 10-year and a 1-year Government bond.  $\gamma_m$  is the market risk premium,  $\gamma_{mx}$  is the price of risk related to time-variation in the market risk premium,  $\gamma_{HML}$  is the risk premium of HML,  $\gamma_{HMLz}$  is the price of risk related to time-variation in HML,  $\gamma_{MVOL}$  is the premium associated with market volatility innovation,  $\gamma_{MVOLz}$  is the coefficient associated with time-variation in market volatility innovation,  $\gamma_{AVOLz}$  is the premium associated with correlation innovation, and  $\gamma_{pz}$  is the price of risk related to time-variation in correlation innovation. The last row reports the p-value for the test that  $\gamma_p$  and  $\gamma_{pz}$  are jointly equal to zero. The sample period is from July 1973 to December 2007.

where the correlation factor is measured as the average correlation innovation rather than the first principal component. Correspondingly, we use average asset volatility innovations instead of the first principal component. The results are presented in Table 6.

Table 6 shows that the price of correlation risk continues to be negative in magnitude and significant after the Shanken adjustment, in both specifications reported. The risk premia of HML and GIP are positive and significant in both specifications. The intercept term from the regression is not significant and the adjusted  $R^2$  value indicates that the explanatory power of the model is high. Thus, our results do not appear to be driven by the way we reduce the dimensionality of inter-asset correlation innovations.

### 4.2. Time-varying factor loadings

The models examined thus far assume that the betas with respect to the risk factors remain unchanged over the entire sample period. As shown by Jagannathan and Wang (1996), Lettau and Ludvigson (2001), and Ferson and Harvey (1999), among others, asset betas tend to vary over time. As a result, we relax the assumption of constant betas by estimating factor loadings that vary through time. The time-varying betas are estimated using a standard approach. With this approach, we account for time variation in the factor loadings of the assets by following Shanken (1990) and Ferson and Harvey (1999). They impose a simple structure on the time variation of the assets' factor loadings. In particular, let

$$\alpha_{i,t-1} = a_{i0} + a_{i1}Z_{t-1}, \beta_{i,t-1} = b_{i0} + b_{i1}Z_{t-1},$$

where  $Z_{t-1}$  is a conditioning variable available to investors at time t. Our conditioning variable is the term premium, defined as the spread in yields between a 10-year and a 1-year Treasury bond, the data from which are taken from the Federal Reserve Bank of St. Louis. The choice of this variable is motivated by the time series literature on return predictability. Fama and French (1989), for example, document that the term spread is able to track the short-term fluctuations in the business cycle.

 $<sup>^{\</sup>rm 9}$  Also see Keim and Stambaugh (1986) and Campbell (1987).

Excluding the SMB factor (as it is not found to be significant when we use excess returns of individual stocks), we get the following return-generating process:

$$\begin{split} R_{i,t} &= a_{i0} + a_{i1}Z_{t-1} + b_{i0,m}R_{m,t} + b_{i1,m*Z} \left(R_{m,t}Z_{t-1}\right) + b_{i0,HML}R_{HML,t} + b_{i1,HML*Z} \left(R_{HML,t}Z_{t-1}\right) + b_{i0,MVOL}R_{MVOL,t} + b_{i1,MVOL*Z} \left(R_{MVOL,t}Z_{t-1}\right) \ (12) \\ &+ b_{i0,AVOL}R_{AVOL,t} + b_{i1,AVOL*Z} \left(R_{AVOL,t}Z_{t-1}\right) + b_{i0,\rho}PC_{\rho,t} + b_{i1,\rho*Z} \left(PC_{\rho,t}Z_{t-1}\right) + e_{i,t}. \end{split}$$

This return-generating process corresponds to the following cross-sectional regression:

$$R_{i,t} = \gamma_0 + \gamma_Z Z_{t-1} + \gamma_m \hat{b}_{i0,m} + \gamma_{mz} \hat{b}_{i1,m*Z} + \gamma_{HML} \hat{b}_{i0,HML} + \gamma_{HMLz} \hat{b}_{i1,HML*Z} + \gamma_{MVOL} \hat{b}_{i0,MVOL} + \gamma_{MVOLz} \hat{b}_{i1,MVOL*Z} + \gamma_{AVOL} \hat{b}_{i0,AVOL}$$
(13)
$$+ \gamma_{AVOLz} \hat{b}_{i1,AVOL*Z} + \gamma_{\rho} \hat{b}_{i0,\rho} + \gamma_{\rho \rho} \hat{b}_{i1,\rho*Z} + e_{i,t}.$$

The results are presented in Table 7. The term premium is significantly positive (see Campbell, 1987). The price of correlation risk continues to be negative in magnitude and statistically significant. The two terms pertaining to correlation innovation risk are jointly significant, as indicated by the p-value shown in the last row of the table. The market and HML risk premia are positive and significant. The terms associated with asset volatility innovations are not significant. Table 7 shows that the price of correlation risk is negative and significant, even after allowing for time-varying betas.

#### 5. Conclusion

We address the following research questions in this paper: are innovations in return correlations an important determinant of expected returns? Is the correlation factor priced even in the presence of well-known risk factors? The answer to these two research questions is an emphatic yes. Controlling for asset and market volatility, correlation carries a significantly negative price of risk that cannot be explained by the market return, size and book-to-market factors, default spread, inflation, liquidity, and other risk factors. The market price of correlation risk is significant after accounting for macroeconomic variables that are known to influence the dynamics of asset correlations. Correlation is a complicated function of higher-order moments, and so might serve as a better proxy for downside risk under kinked utility functions or under binding short-sales constraints or wealth constraints. However, the market price of correlation risk is significant even after controlling for standard higher moments. Since we purge our correlation factor of the effects of macroeconomic variables, well-known risk factors, and higher-order moments, we conjecture that our correlation factor is picking up changing diversification benefits. The market price of correlation risk is robust to whether we use individual stocks or portfolios as test assets. The market price of correlation risk continues to be significantly negative when we allow for time variation in the factor loadings of the assets. Finally, the market price of correlation risk is robust to different specifications for the correlation factor.

As the correlation between assets that span the risk-return spectrum increases, investors lose at least part of the diversification benefit. Stocks that perform well in states where asset correlations are high are more attractive and the expected returns on these securities are lower. Therefore, the market price of correlation risk is significantly negative.

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