.... RESEARCH NOTES AND COMMUNICATIONS

CONTENTS

Theil's Forecast Accuracy Coefficient: A Clarification

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INTRODUCTION

Theil's coefficient of inequality is one of the statistical forecasting evaluators frequently cited in the literature [1–7]. However, there seems to be some confusion about this coefficient which may stem from the fact that Theil himself proposed two different formulae at different times under the same name. Both are called "coefficient of inequality" and labeled with the symbol "U." In addition, there are two different interpretations possible for the formula first proposed.

THEIL'S OWN PROPOSALS

In *Economic Policy and Forecast,* Theil proposes the following formula as a measure of forecast accuracy:

$$(1) \qquad \mathrm{U} = \frac{\left[\frac{1}{n}\sum_{i=1}^{n}(A_i - P_i)^2\right]^{1/2}}{\left[\frac{1}{n}\sum_{i=1}^{n}A_i^2\right]^{1/2} + \left[\frac{1}{n}\sum_{i=1}^{n}P_i^2\right]^{1/2}} = \mathrm{UI}$$

"where A_i are the actual observations and P_i are the corresponding predictions" [6, p. 32]. This coefficient is denoted as UI in this article. In *Applied Economic Forecasting,* Theil suggests as a measure of forecast quality:

$$(2) \qquad \mathrm{U} = \frac{\left[\sum_{i=1}^{n}(P_i - A_i)^2\right]^{1/2}}{\left[\sum_{i=1}^{n}A_i^2\right]^{1/2}} = \mathrm{UII}$$

"where (A_i, P_i) stands for a pair of predicted and observed *changes*" [7, p. 28]. This coefficient is denoted here as UII.

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The formal difference between UI and UII is the absence or presence of the P-term in the denominator. This P-term causes UI to be bounded between 0 and 1. There is no finite upper boundary for UII, where the P-term is missing.

Theil states for UI:

> "We have $U = 0$ in the case of equality: $P_i = A_i$ for all i. This is clearly the case of perfect forecast. We have $U = 1$ (the "maximum inequality") if there is either a negative proportionality or if one of the variables is identically zero" [6, p. 32].

In comparison, the corresponding statement for UII would be:

> "We have $U = 0$ in the case of equality as above: $P_i = A_i$ for all i. This is again the case of perfect forecast. We have $U = 1$ when the prediction method is naive no-change extrapolation or when it leads to the same standard deviation of forecast error as that method."

THE UI FORMULA

Interpretation Problems and Their Consequences

Theil's formulation that "A_i are the actual observations and P_i are the corresponding predictions" does not make it clear whether absolute values (e.g., next year we will sell 20,000 trucks) or changes in absolute values (e.g., next year we will sell 1,000 more trucks than this year) are forecasted. In some secondary literature about Theil's coefficient one finds that same ambiguous formulation [2, 5], while other writers specify which interpretation they prefer to use [3, 4]. This ambiguous and multiple sort of interpretation of a "standard" formulation should be avoided, especially in view of the fact that Theil himself uses UI only on change data.

Interpretation A and Its Implications

If one interprets P_i and A_i as the absolute[1] values for forecast and outcomes (interpretation A) and computes UI under this assumption, then the following implications must be kept in mind:

1. UI will be a proxi-variable for the standard error of the forecast, where this error is expressed as a fraction of the sum of the standard values for the forecast series and the series of actual outcomes. For example, a value of .06 for UI would mean that the standard error of the forecast is .06 times as large as the standard value of all forecasts plus the standard value of all outcomes added together. This is indeed a complicated relationship. Its information value seems doubtful when comparing forecasting methods with each other, since the error generated by a particular forecasting method is scaled down by the variation in the predictions generated by the same method.
2. UI is, of course, bounded by 0 and 1. Here, the lower boundary is the ideal case of perfect forecast, while the upper boundary stands for a number of situations which are trivial or impossible for all practical purposes of sales forecasting, namely: (a) $A_i = 0$ for all A_i. This would mean we try repeatedly to forecast sales where there are none now and were none in the past; (b) $P_i = 0$ for all P_i. This would mean we repeatedly forecast zero sales where sales have been made both now and in the past; and (c) A_i and P_i are negative proportional. This would mean that it is possible to have negative sales or forecasts of negative sales.

Thus the information obtained from UI under interpretation A is not very enlightening. A UI-value computed for a forecasting series would tell the reader little or nothing which could give him a feeling for the relative reliability of the applied forecasting method.

Interpretation B and Its Implications

If one means by A_i and P_i the observed *changes* and the predicted *changes* and computes UI under this assumption, then the following implication arises: *The values for UI become inconclusive and thus their information content is limited. We can easily show this in a general way of reasoning as below.*[2] UI is also bounded by 0 and 1. The lower boundary is, as before, the case of ideal forecast. The upper boundary is reached in the case that all P_i are zero.[3] Under interpretation B, that case would be equivalent to the most naive forecasting model, the no-change model, which says: There will be no change; the forecast for the next period is equal to the actual sales of this period. While it is nice to have this case of the simple no-change model as a reference point, it is also desirable that the value of UI indicates whether a forecasting method gives results better or worse than the no-change model. But precisely that information cannot be found from UI. The coefficient assumes the upper boundary of "maximal inequality" when using the no-change model. Any other forecasting method, regardless whether it is better or worse than the no-change model, will yield lower UI values, lower or equal to unity. Thus, UI should not be used at all for ranking alternative forecasting methods.[4, 5]

[1] The term "absolute" is used in the meaning defined in the previous paragraph.

[2] When proposing UII, Theil himself expressed the drawbacks of UI in a footnote by stating that the denominator of UI depends on the forecasts and that it is therefore not true that UI is uniquely determined by the mean square prediction error.

[3] Other cases, which need not be treated here, show the inconclusiveness of UI.

[4] Hirsch and Lowell demonstrate the limitedness of UI by the following example (paraphrased): "Suppose that the forecaster knows that $A = 0$ and also knows the standard deviation S_A, and nothing more. He might simply forecast no change, in which case he would have a mean square error of S_A^2 and a $UI = 1$. Alternatively, he might draw his forecasts randomly from a dis-

UII, THE COEFFICIENT WITHOUT PROBLEMS

The second version, namely UII, which apparently has not yet been adopted to a great extent by scholarly writers, is unambiguous and easy to understand and to interpret. UII reaches its lower boundary of UII = 0 at perfect forecasts. It assumes the value of 1 when a forecasting method delivers forecasts with the same standard error as the naive no-change extrapolation. It increases monotonically as the standard error forecasting improves over the no-change extrapolation. If UII is larger than 1, the forecasting method applied is to be rejected because it cannot beat the most simple no-change extrapolation. Because of the superiority of these properties, it is advisable to use only UII (and not UI) when assessing the reliability of sales forecasting methods.

tribution with mean zero and a variance of S_A^2. Although the second strategy is manifestly worse than the first (it yields a mean square error of 2 S_A^2), it receives a better rating in terms of UI, namely UI = $\sqrt{1/2}$ rather than unity."

[5] One might argue against this recommendation by proposing that good forecasting models will be better than the naive no-change forecasting model, and thus UI will be appropriate for comparing "better" models in the region where UI is small. This proposition implies that indeed one is sure from other indicators that the forecasting models are better than the no-change model. In view of the state of art in forecasting, "better" forecasts do not seem warranted. Evidence to this account is given by Zarnovitz [8]. He compared forecasts made by professional teams to the no-change alternative by computing R_1, the ratio of mean square error of the experts' forecasts to the mean square error from the no-change model. For sectoral economic forecasts made during 1958–63, Zarnovitz found: "While no more than one-eighth of them exceeds the ratio of 1.0, only about one-fourth are less than 0.6" [8, p. 40].

SUMMARY

It has been shown that one version of Theil's coefficient of inequality has little or no value as an index to assess forecast accuracy, although the use of this version is still suggested by many scholarly writers. In contrast, another and even simpler version of Theil's coefficient of inequality gives more meaningful information about the accuracy of forecasting methods. It is suggested that only this latter version be applied.

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