Commodities and Equities: A "Market of One"?

Bahattin Büyükşahin   **Michael S. Haigh**   **Michel A. Robe**[1]

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Abstract

Amidst a sharp rise in commodity investing, many have asked whether commodities nowadays move in sync with traditional financial assets. We provide evidence that challenges this idea. Using dynamic correlation and recursive cointegration techniques, we find that the relation between the returns on investable commodity and U.S. equity indices has not changed significantly in the last fifteen years. Importantly, we provide the first analysis of co-movement during periods of extreme returns. Again, we find no evidence of a secular increase in co-movement between the returns on commodity and equity investments during extreme events.

``*As more money has chased (...) risky assets, correlations have risen. By the same logic, at moments when investors become risk-averse and want to cut their positions, these asset classes tend to fall together. The effect can be particularly dramatic if the asset classes are small -- as in commodities. (...) This marching-in-step has been described (...) as a 'market of one'."* The Economist, March 8th, 2007.

1. <u>Introduction</u>

In the past decade, investors have sought an ever greater exposure to commodity prices – by directly purchasing commodities, by taking outright positions in commodity futures, or by acquiring stakes in exchange-traded commodity funds (ETFs) and in commodity index funds. This pattern has accelerated in the last few years. To wit, Standard and Poor's GSCI index was created by Goldman Sachs in 1991. This world-production weighted index tracks the prices of major physical commodities for which there are active, liquid futures markets. As recently as 1999, the sums invested in investment vehicles tracking this index were estimated at less than 5 billion dollars. Nowadays, however, investments linked to the GSCI or to one of five other prominent commodity indices likely exceed 140 billion dollars. In a similar vein, the first-ever commodity exchange traded fund (the streetTRACKS Gold Shares ETF) was started in November 2004. As of July 2008, its market capitalization approaches 20 billion dollars, and it has been joined by numerous commodity ETF competitors.

One naturally wonders whether this sharp increase in investor appetite for commodities has had a significant impact on the pricing of related financial instruments. On the one hand, one might expect commodities and equities not to move in sync because risk factors that explain the cross sectional variations in equity returns have historically had no forecasting power in commodity markets. On the other hand, extant studies of commodity returns (e.g., Erb and Harvey, 2006; Chong and Miffre, 2007) are based on data series that end in late 2003 or early 2004, i.e., precisely at the time when the new kinds of market participants – hedge funds and swap dealers – had begun to greatly increase their participation in commodity markets.

One reason why the large-scale arrival of financial institutions in commodity markets could matter for pricing is if it has led to a reduced scope for cross-market arbitrage opportunities (as in Basak and Croitoru, 2006) and, in the process, has more closely linked commodity and equity markets. Another channel for tighter links between commodity and equity markets is if financial institutions respond differently from commercial traders to extreme stock market movements – in particular, if sharp downward movements in one market force financial investors to liquidate positions in commodity markets so as to raise cash for margin calls. In this paper, we investigate the relation (or lack thereof) between ordinary as well as between extreme returns on passive investments in commodity and equity markets.

Because much of the new commodity exposure has been achieved through direct or indirect participation in futures markets, it should be reflected in the magnitude and composition of commodity futures trading. Haigh, Harris, Overdahl, and Robe (2007; henceforth, HHOR) confirm this intuition, using proprietary data on trader positions in the world's largest-volume futures contract on a physical commodity – the New York

Mercantile Exchange's WTI sweet crude oil futures. HHOR show that greater market participation by commodity swap dealers and hedge funds has been accompanied by a change in the relation between crude oil futures prices at different maturities and greater price efficiency. Specifically, the prices of one-year and two-year futures have become cointegrated with the price of near-month futures, for the first time ever, since mid-2004.

Whereas this previous research documents that the prices of different-maturity commodity futures have recently become much more closely linked, we use dynamic correlation and recursive cointegration techniques to show that the degree of co-movement between benchmark commodity- and equity-investment returns has not changed materially over the course of the last fifteen years. In particular, notwithstanding the surge in commodity investment flows, the already very low correlation between the rates of return on passive investments in these two asset classes has become negative in the last five years. Our results are similar in spirit to the finding that, despite increased capital flows to emerging markets in the years following their financial liberalization and despite greater integration with world equity markets, cross-market return correlations did not increase enough to diminish the benefit, to U.S. investors, of diversifying into emerging-market stocks (Bekaert and Harvey, 2000; Carrieri, Errunza and Hogan, 2007).

We use Standard and Poor's S&P 500 return and GSCI total return data to proxy for the rates of return on representative unlevered investments in, respectively, U.S. equities and commodities (we obtain qualitatively similar results with two other widely-used indices: Dow Jones' DJIA equity and DJ-AIG total return commodity indices[2]). Since much of the commodity investment boom is still quite new, any change in pricing relationships is likely to be a recent phenomenon. HHOR, for example, do not find pricing efficiency changes across crude oil futures maturities until September 2003 (for one-year contracts) or May 2004 (for both one- and two-year contracts). It is therefore important to utilize recent data. Accordingly, we use daily, weekly and monthly returns from January 1991 (when GSCI products first became available) through May 2008.[3]

To identify possible changes in the co-movements between the asset return series, we run all of our analyses on the entire sample period and then focus in particular on three successive sub-periods: June 1992 to May 1997; June 1997 to May 2003; and, June 2003 to June 2008. The first sub-period predates the commodity investment boom, while the third sub-period covers the interval of time during which financial traders greatly

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[2] Unlike the GSCI, which uses weights that reflect world-production figures and is consequently heavily tilted toward energy commodities, the DJ-AIG commodity index is specifically designed to provide a "diversified benchmark for the commodity futures market." In particular, it assigns a weight of only about 30% to energy commodities, including about 13% to crude oil. By comparison, as of mid-July, 2007, the GSCI assigned a weight of more than 70% to energy commodities, including 36% to crude oil (WTI nearby contract). Other GSCI competitors include the Deutsche Bank Liquid Commodity Index, Rogers International Commodity Index, and Reuters-CRB.

[3] To our knowledge, the only analysis of commodity returns that uses data more recent than May 2004 is Chong and Miffre (2008), which updates through 2006 the dynamic conditional correlation (DCC) analysis of commodity and equity returns in Chong and Miffre (2007). In addition to dynamic conditional correlation analyses that cover the full five years characterized by the large-scale presence of financial intermediaries in commodity markets (including the last year and a half of turbulence in asset markets), our paper differs from that contemporaneous study by assessing equity-commodity co-movements over the long-term (by means of co-integration analyses) and during weeks and days of extreme returns.

increased their participation in commodity futures markets (HHOR). These two sub-periods correspond to times of economic expansion. The second sub-period, in contrast, includes an economic contraction as defined by the National Bureau of Economic Research (NBER). More importantly, June 1997 to May 2003 captures the Asian crisis; the Russian and Argentinian sovereign defaults; and, the U.S. stock-market bubble and its direct aftermath. By choosing these specific sub-periods, we can thus establish that our results are not driven by the relation between commodities and equities (or the relation between their volatilities) during times of stock-market or macro-economic turbulence.

We find statistically significant differences in the means and standard deviations of the rates of returns across the two asset classes and, for each asset class, across the three sub-periods. By contrast, we find only small differences in cross-asset correlations for the three sub-periods: the simple correlation between equities and commodities, which was slightly positive between 1992 and 1997, is slightly negative between 2003 and 2008. We obtain qualitatively similar results at all return frequencies.

Notwithstanding the fact that the simple cross-correlations are almost zero in all three sub-periods, we find that rolling measures of the correlations between equity and commodity return series fluctuate substantially throughout the sample period. The *pattern* of fluctuations, however, does not appear to change during the entire sample period. We confirm these analyses using the dynamic conditional correlation (DCC) methodology proposed by Engle (2002). The range of values taken by DCC estimates is indeed quite wide: weekly values, for example, can be as low as -0.5 or as high as $+0.5$. Most of the time, however, the DCC estimates are close to 0. What is more important, we find no evidence of a secular increase in correlations in the last few years.

Correlation estimates are relevant for short-term investors. For long-term investors, however, the key issue is whether there exists a long-term relation between the prices of commodity and equity investment even though these prices may diverge in the short term (Kasa, 1992). To answer this question, we apply recursive cointegration techniques (Johansen, 1998, 1991; Johansen and Juselius, 1990) to examine the stability and the possible strengthening over time of the relation between equity- and commodity-investment price series. This analysis complements our other results: except for a short period in the late 1990's, we find little statistical evidence of cointegration – and none in the last eight years. The implication is that passive investors can still achieve substantial gains by diversifying portfolios across the two asset classes.

Although there is no evidence of a structural shift in correlation and cointegration levels, it could still be the case that financial and commodity markets have become a "market of one" *during extreme events*. Hartmann, Straetmans and de Vries (2004), for example, find evidence of cross-asset extreme linkages in the case of bond and equity returns from the G-5 countries. Using a different approach, Longin and Solnik (2001) provide evidence that international equity-market correlations increase in bear markets.

Here, we first identify the days and weeks during which returns on equity indices were at least one or two standard deviations away from their means, and then analyze the contemporaneous returns on investable commodity indices. Contrary to extant findings on extreme linkages between other asset markets, this first analysis finds little relation between exceptionally large returns on equities and those on commodities. This is true for the whole sample period as well as for all three of our sub-periods; for positive as well as for negative exceptional returns; and, for periods of stock market upturns as well

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as downturns. We then use a technique similar to Longin and Solnik's (2001) to analyze equity-commodity linkages when the returns on equity and commodity indices both take values in the tails of their respective distributions. We only find weak extreme-event correlations. Interestingly, whereas Longin and Solnik find high positive correlations across international equity markets in down markets, we document that the equity-commodity correlation is mildly positive in the upper tail but is *negative* in the lower tail. That is, conditional on both equity and commodity returns being very poor, the two return series are negatively correlated. This finding highlights yet another aspect of the diversification benefits that commodities provided to equity investors between 1991 and 2008.

In sum, the lack of greater return co-movement across equities and commodities suggests that commodities have retained their role as a portfolio diversification tool. The import of this conclusion cannot be overstated, since many academics and practitioners have long called for substantial allocations to commodities as an asset class for the purposes of return generation and portfolio diversification.[4]

The remainder of the paper proceeds as follows. Section 2 describes the data. Section 3 presents the correlation analyses. Section 4 shows the robustness of our results to alternative methodological choices. Section 5 concludes.

2. **Data and descriptive statistics**

This section discusses the data and gives summary statistics for the return series.

2.1 Returns data

We take the perspective of a passive investor on the relation between commodities and traditional financial investments. Modern portfolio theory suggests that the relevant information matrix for such an investor includes the expected asset returns, the variances of these returns, as well as cross-asset correlations.

To assess short-term correlations, we use daily, weekly, and monthly returns and return volatilities on four commodity and equity indices that are widely used to construct portfolios.[5] We focus on results for weekly (Tuesday to Tuesday) holding-period returns, and provide a brief discussion of our (qualitatively similar) findings for daily and

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[4] See, e.g., Ankrim and Hensel (1993), Froot (1995), Huberman (1995), and Satyanarayan and Varangis (1996) for early work on how commodities help reduce an investor's unconditional portfolio risk. See also Erb and Harvey (2006), Gorton and Rouwenhorst (2006), and Miffre and Rallis (2007) for evidence on the strategic and tactical values of commodity investments. The data series in these newer papers end in 2004 and, hence, do not cover the period during which took place much of the growth in financial traders' positions in commodity futures markets.

[5] We measure the percentage rate of return on the Ith investable index in period t as $r^I_t = 100 \, \mathrm{Log}(P^I_t / P^I_{t-1})$, where P^I_t is the value of index I at time t. We measure the volatility of an index in period t as $(r^J_t - \bar{a})^2$, where $\bar{a}$ is the mean value of r^I_t over the relevant sample (sub-)period.

monthly returns. To analyze long-term cointegration, we use the Tuesday close prices for the same four indices.

For equities, we use Standard and Poor's S&P 500 index; in robustness checks, we use Dow-Jones's DJIA index.[6] For commodities, we focus on the unlevered total return on Standard and Poor's S&P GSCI (formerly, the Goldman Sachs Commodity Index), i.e., the return on a "fully collateralized commodity futures investment that is rolled forward from the fifth to the ninth business day of each month." While the GSCI includes twenty-four nearby commodity futures contracts, it is heavily weighted toward energy. For robustness checks, we use total (unlevered) returns on the second most widely used investable benchmark, Dow-Jones's DJ-AIG total-return commodity index (henceforth, DJ-AIG). This rolling index, which is composed of futures contracts on nineteen physical commodities, was designed to provide a "diversified benchmark for the commodity futures market."

We also analyze potential changes in the relation between the rates of returns on specific types of commodities. For this purpose, we use daily, weekly, and monthly total returns on several investable sub-indices representing key components of the GSCI: Energy, Non-Energy, Industrial Metals, Precious Metals, Agriculture, and Livestock.

We obtain the price data for all four indices (GSCI, DJ-AIG, S&P 500 and DJIA) and the GSCI commodity sub-indices from Bloomberg. Our data cover more than seventeen years, from January 2 1991 through May 2008. We also provide results for three successive sub-periods: June 1, 1992 to May 31, 1997; June 1, 1997 to May 31, 2002; and, June 1, 2003 to May 26, 2008.

2.2 Returns on Equity and Commodity Indices: Summary Statistics

Table 1 presents some descriptive statistics, for the entire sample period, for the two equity- and the two commodity-return series. Panels 1, 2 and 3 present statistics for, respectively, daily, weekly and monthly returns.[7]

From January 1991 through mid-2008, the mean weekly total rate of return on the GSCI was 0.16% (or 8.43% in annualized terms), with a minimum of -14.59% and a maximum of 7.78%. The typical rate of return varies sharply across the sample period: it averaged 0.14% in 1992-1997 (7.45 % annualized); 0.045% in 1997-2003 (or a mere 2.36% annualized); and, jumped to 0.35% in 2002-2007 (19.79% annualized). The corresponding figures are very similar for the DJAIG total return index – with two exceptions. One exception is the first sub-period (1992-1997), when the average weekly return was 0.205% for the DJAIG *versus* 0.138% for the GSCI; Figure 1, which plots the levels of the four indices, indeed shows that the GSCI did not start appreciating until the end of 1996 whereas the DJAIG started appreciating in 1994. The second exception is in

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[6] We use returns on both of these equity indices that are exclusive of dividend yields. This approach leads to an underestimation of the expected returns on equity investments (Shoven and Sialm, 2000). However, insofar as large U.S. corporations smooth dividend payments over time (Allen and Michaely, 2002), the correlation estimates that are the focus of our paper should be essentially unaffected.

[7] For weekly returns, tables presenting the corresponding statistics for each of the three successive sub-periods are available from the authors upon request.

the third sub-period, when the DJAIG yielded 0.29% per week on average, vs. 0.35% on average for the GSCI. Figure 1 shows that, while the two indices did grow in sync during much of that period (except in the second half of 2006), the GSCI indeed grew more.

During the entire sample period, the mean weekly rate of return on the S&P 500 was only slightly higher than that on commodities: 0.17% (or 8.99% in annualized terms), with a minimum of -12.17% and a maximum of 12.37%. While the lowest weekly rate of return on all four indices is found in the second sub-period, the median weekly rate of return on the commodity indices was very low (0.023% on the GSCI and 0.014% on the DJAIG) between June 1997 and May 2003, whereas the S&P 500 equity index had its highest median weekly rate of return during the same period (+0.37%). In a similar vein, whereas the highest equity returns took place in the first sub-period, the peak for commodities is observed in the third sub-period. These observations suggest that equities and commodities do not move together.

Consistent with the fact that the DJAIG is by construction more diversified than is the GSCI, the standard deviation of the weekly rates of return is much lower for the DJ-AIG (1.80% for the whole sample) than for the GSCI (2.68%). This pattern of approximately 45% greater GSCI volatility is observed in all three sub-periods: 1.81% *vs.* 1.26% in 1992-1997; 2.87% *vs.* 1.88% in 1997-2003; and, 3.17% *vs.* 2.20% in 2003-2008. Standard deviations increase throughout the sample for commodities, while they peak in the second sub-period for equities.

Panels E and F show similar patterns for daily and monthly returns that Panels A to D showed for weekly returns, i.e.:

- Between 1991 and 2008, the rates of return on commodity indices were similar to those on equity indices. However, the rank-ordering of returns on these two asset classes fluctuates dramatically over the course of the sample period. For example, equity returns trounce commodity returns in 1992-1997, but commodity returns are approximately double equity returns in 2003-2008.
- The rates of return on equities are somewhat more volatile than those on a well-diversified basket of commodities (represented by the DJAIG), except in the last five years (2003-2008).
- The rates of return on the GSCI are the most volatile throughout the entire sample period. Of note, the GSCI returns are approximately 40-50% more volatile than those on the DJAIG.

2.3 Simple Cross-Asset Correlations: Returns and Return Volatilities

Figure 1 gives some preliminary insights into the co-movements between the commodity and equity indices. This graph allows the reader to visualize which sub-periods help determine the co-movements between the index returns that are summarized by the return correlations presented in Table 2. In particular, it suggests a high correlation between the two equity indices; a positive, but somewhat weaker, correlation between the two commodity indices; and, a weak or possibly negative correlation between the equity and commodity indices, especially during the second sub-period (June 1997 - May 2003).

Table 2 quantifies these first impressions by providing an overview of the simple correlations between the two four benchmark asset-return series. This summary table is helpful for the interpretation of the empirical results in Section 3. Panels 1, 2 and 3 of Table 2 are for, respectively, daily, weekly and monthly returns. For weekly returns, Tables 2B, 2C and 2D present the corresponding statistics for each of the three successive five-year sub-periods.

As one would expect, the simple correlation between the returns on the DJIA and S&P 500 equity indices is very high (more than 0.93 from 1991 to 2008), especially in the last five years (0.96). Likewise, the rates of return on the two commodity indices are strongly positively correlated. At all three return frequencies, their simple correlation is about 0.89 for the whole sample; it is strongest in the second sub-period (0.93) and is slightly weaker in 1992-1997 at the weekly return frequency.

In sharp contrast, equity-commodity cross-correlations are typically very low or even negative:

- In the case of **daily returns**, Panel 1 shows that the rates of return on the commodity indices exhibit very little correlation with either of the equity returns, with the coefficient estimates ranging from -0.0615 to 0.0014 depending on the index pair and the time period.
- For **weekly returns**, Tables 2B and 2C show that the "highest" weekly correlations, a mere 0.09 to 0.13 depending on the index, were observed in the first (1992-1997) and second (1997-2003) sub-periods.
- For **monthly returns**, equity-commodity correlations are not statistically significantly different from zero for the entire sample period.[8]

In short, despite a commonly-expressed view that both equity and commodity *prices* have increased in tandem since 2003, the correlation between commodity and equity *returns* is almost zero in our third sub-period – indeed, the total returns on the GSCI are negatively correlated with the returns on both equity indices during that period. Figure 1 suggests that, to the extent that the correlations were at all positive prior to 2002, the likely reasons are joint run-ups in commodity and equity prices in 1995-1997 and again in the eighteen month period from late 1998 through Spring 2000.

One clear conclusion from Table 2 is that the simple correlation between the rates of return on commodities and equities are typically not statistically significantly different from zero. Even if there is little correlation between the first moments of the return series, though, it may still be the case that the second moments of the returns are related. Table 3 shows the correlation between return volatilities, at the weekly frequency, for the full sample period (Panel A) and our three sub-periods (Panels B, C and D). Similar to the evidence in Table 2 regarding return correlations, return-volatility correlations are always extremely close to zero. In the last five years of the data series, likewise, correlations between equity and commodity return volatilities are even slightly negative (though not statistically significantly different from zero).

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[8] At the monthly frequencey, the GSCI's correlation with the S&P 500 and the DJIA was 0.27 in 1992-1997 (statistically significantly positive at the 5% level); this correlation became statistically significantly *negative* in 2002-2007 (-0.25 with the DJIA and -0.3 with the S&P 500). Tables for sub-periods and sub-indices are available from the authors upon request.

2.4 Returns on Specific Categories of Commodities

2.4.1 Summary Statistics

Table 4 provides summary statistics for the unlevered (total) rates of return on six investable sub-indices representing key components of the S&P GSCI index: the GSCI Energy, Non-Energy, Industrial Metals, Precious Metals, Agriculture, and Livestock investable indices. We focus on Panel 2 (weekly returns) for the sake of brevity, because the results are similar for daily and monthly return series (Panels 1 and 3, respectively).

Over the entire sample period, individuals who invested in Energy or Metal sub-indices experienced greater average returns (but also more volatility) than investors in other commodity sub-indices. However, performance rankings vary significantly from period to period. Industrial as well as Precious Metals, for example, both underperform all other commodity sub-indices between 1992 and 1997, but beat all but Energy between 2003 and 2008. In a similar vein, Agriculture outperforms all other sub-indices in 1992-1997 but is the worst performer in 2003-2008.

2.4.2 Simple Correlations

Table 5 shows the simple correlations between the unlevered rates of return on the S&P 500 equity index, the S&P GSCI, and the six narrow commodity benchmarks introduced in Table 4. Again, Table 5 focuses on weekly returns. Table 5A presents statistics for the entire sample period, while Tables 5B, 5C and 5D present the corresponding statistics for each of our three successive sub-periods. Four patterns emerge from Table 5:

- Equity returns exhibit very little correlation with the returns on *any* of the commodity sub-indices. The highest weekly individual correlation is for Industrial Metals, but even that figure is a mere 0.13 over the whole sample (Table 5A), peaking at 0.20 in 2003-2008 (Table 5D). All the other cross-correlations are less than 0.17, and quite a few are slightly negative.
- There is no evidence of a material increase, over time, of the correlation between the returns on equities and those on either the Agriculture or the Livestock sub-indices.
- Consistent with the fact that the GSCI is a value-weighted index and is consequently heavily weighted toward energy (as energy contracts make up the world's largest commodity futures markets), the unlevered returns on the GSCI and on the Energy sub-index are very highly positively correlated – between 0.94 and 0.98 depending on the sample period. In contrast, the correlation between the returns on the entire GSCI index and those on the Non-Energy sub-index range from 0.39 to 0.43 depending on the sub-period.
- The returns on the Non-Energy sub-index are strongly positively correlated with the returns on all the other GSCI sub-indices (but not with the Energy

sub-index). This finding suggests the possibility of a common economic variable driving the returns on most types of commodities.

In addition to the return correlations between equity and commodity sub-indices, a relevant piece of information for some investors is the correlation between the return *volatilities* of those assets. The numbers in Column 1 in Table 6 show these return-volatility correlations, from 1991 to 2008. These numbers strengthen our conclusion that the correlation between the volatilities of equities and commodities are always very low and almost never statistically significantly different from zero. Indeed, Table 6 shows that the correlations between return volatilities are if anything even lower than the correlation between the returns themselves.

3. **Short-Term Co-Movements**

Tables 1B, 1C and 1D, and Tables 3B, 3C and 3D, in the Appendix show that the unconditional return volatilities, as measured by the standard deviations of returns, vary a lot over time. In particular, the weekly rates of return on equities were 50% more volatile in the third sub-period (2003-2008) than in the first (1992-1997). Even more strikingly, the standard deviation of the returns on commodity investments almost doubled over the course of our entire sample period.

In contrast, although the unconditional correlations between the rates of returns on equity and commodity investments vary somewhat from sub-period to sub-period, Tables 2B, 2C and 2D suggest that these fluctuations are quite mild and that the correlations are always close to zero. Put differently, the analysis in Section 2 seemed to suggest that commodity returns exhibit consistently low correlations with their equity counterparts.

Before concluding that commodities provide a good hedge for equity portfolios, however, one should account for possible time variations in these correlation measures. In this Section, we provide estimates of the intensity of co-movements (or the lack thereof) that account for time variations in the various moments of the return series.

3.1 Methodology

Measuring the relationship between variables at various points in time, rather than using a single correlation coefficient over the entire sample period, provides information on the evolution of the relationship over time. For this purpose, simple correlation measures such as rolling historical correlations and exponential smoothing are widely used in the literature.

Rolling historical correlations take into account the time-varying nature of the re-lationship between variables straightforwardly, by calculating the correlation at any point in time as the estimate for a specified window (say, k observations) that does not overlap with the full sample. The correlation is first estimated over sub-periods 1 to k, then over sub-periods 2 to $k+1$, and so on. The rolling historical correlation estimator is thus:

$$\hat{\rho}_{12,t+1} = \frac{\sum\limits_{s=t-k}^{t} x_{1,s} x_{2,s}}{\sqrt{\left(\sum\limits_{s=t-k}^{t} x_{1,s}^2\right)\left(\sum\limits_{s=t-k}^{t} x_{2,s}^2\right)}} \tag{1}$$

where x_1 and x_2 are the deviations from the means of the two random variables of interest, with mean zero.

Although this simple estimation technique provides some information on the evolution of relationship between two variables, it suffers from assigning an equal weight to all observation in the estimation window and zero weight to older observations. It also raises the issue of window-length determination. On the one hand, if the window is too narrow, one runs the risk of ignoring important observations in the data by giving zero weight to these observations. On the other hand, if the window is too wide, old observations will be given weight even though they may not be relevant to the analysis.

To overcome these problems, exponential smoothing techniques assign declining weights to older observations based on a parameter, λ, without any prior determination on the amount of past data to be used in the analysis. The exponential-smoothing estimator can be written as

$$\hat{\rho}_{12,t+1} = \frac{\sum\limits_{s=1}^{t} \lambda^{t-s} x_{1,s} x_{2,s}}{\sqrt{\left(\sum\limits_{s=1}^{t} \lambda^{t-s} x_{1,s}^2\right)\left(\sum\limits_{s=1}^{t} \lambda^{t-s} x_{2,s}^2\right)}} \tag{2}$$

One drawback of this second approach is that the user must adopt an *ad hoc* approach to choose smoothing parameter λ. Engle (2002) used $\lambda = 0.94$ to analyze daily returns on the major equity indices. We use the same value for monthly and weekly returns, but set $\lambda = 0.97$ for daily returns.

More importantly, like the rolling historical correlation, the exponential-smoothing technique cannot adequately account for changes in volatility. The sensitivity of the estimated correlation to volatility changes restricts inferences about the true nature of the relationship between variables. Since the estimated correlations are subject to volatility shocks, interpreting these correlations becomes more difficult especially during high volatility periods.

The Dynamic Conditional Correlation methodology (DCC) developed by Engle (2002) helps to remedy this problem. The DCC model is based on a two-steps approach to estimating the time-varying correlation between two series. In the first step, time-varying variances are estimated using a GARCH model. In the second step, a time-varying correlation matrix is estimated using the standardized residuals from the first-stage estimation.

More formally, consider a $n \times 1$ vector of normally-distributed with mean zero and covariance matrix H_t returns series r_t of n assets are assumed the have the following structure:

$$r_t \sim N(0, H_t) \tag{3}$$

$$H_t = D_t R_t D_t \tag{4}$$

where, H_t is the conditional covariance matrix; R_t is the time varying correlation matrix; and, D_t is a diagonal matrix of time-varying standard deviations given by $D_t = \text{diag}\sqrt{E_{t-1}(r_{i,t}^2)} = \text{diag}\sqrt{h_{i,t}}$. The $h_{i,t}$ can be thought of as univariate GARCH models, so the standardized disturbance can be expressed as $\varepsilon_{i,t} = r_{i,t}/\sqrt{h_{i,t}} = D_t^{-1}r_{i,t}$, where $\varepsilon_{i,t} \sim N(0, R_t)$. Consider the following conditional correlations:

$$\rho_{ij,t} = \frac{E_{t-1}[r_{i,t}r_{j,t}]}{\sqrt{E_{t-1}[r_{i,t}^2 r_{j,t}^2]}} \tag{5}$$

Re-writing these conditional correlation in terms of standardized residuals from GARCH estimates yields:

$$\rho_{ij,t} = E_{t-1}\varepsilon_{i,t}\varepsilon_{j,t} \tag{6}$$

This implies the equivalence of conditional correlation of returns and conditional covariance between the standardized disturbances. Therefore, the matrix R represent the time-varying conditional correlation matrix of returns as well as the conditional covariance matrix of the standardized residuals (Engle ,2002).

The DCC model of Engle (2002) suggest the following dynamics of the correlation matrix:

$$R_t = Q_t^{*-1} Q_t Q_t^{*-1} \tag{7}$$

$$Q_t = (1-\alpha-\beta)\overline{Q} + \alpha(\varepsilon_{i,t-1}\varepsilon_{j,t-1}) + \beta Q_{t-1} \tag{8}$$

where $\overline{Q}$ is the unconditional correlation matrix of standardized residuals and Q_t^* is a diagonal matrix composed of square root of the diagonal elements of Q_t. The correlation estimator is given by the typical element of R_t in the form of

$$\rho_{ij,t} = \frac{q_{ij,t}}{\sqrt{q_{ii,t}q_{jj,t}}}$$

This specification ensures the mean reversion as long as $\alpha + \beta < 1$. The resulting estimator is called DCC by loglikelihood with mean reverting model. The log-likelihood of the DCC model outlined above is given by :

$$L = -\frac{1}{2}\sum_{t=1}^{T}(n\log(2\pi) + 2\log(|D_t|) + \log(|R_t|) + \varepsilon' R_T^{-1}\varepsilon)$$

In essence, the log-likelihood function has two components: the volatility part, which contains terms in D_t; and the correlation part, which contains terms in R_t. In the first stage of the estimation, n univariate GARCH(1,1) estimates are obtained, which produces consistent estimates of time-varying variances (D_t). In the second stage, the

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correlation part of the log-likelihood function is maximized, conditional on the estimated D_t from the first stage.

We use rolling historical correlation, exponential smoother, and dynamic conditional correlation by log-likelihood for mean-reverting model estimation to analyze the dynamic properties of the relevant variables.

3.2 Equities and Commodities

Figures 2 to 5 plot the estimates of the time-varying correlation between the unlevered rates of return on investable equity and commodity indices over the sample period. Figure 2, 3, 4, and 5 provide information on the correlations between, respectively, the S&P 500 and GSCI; S&P 500 and DJ-AIG; DJIA and GSCI; and, DJIA and DJ-AIG. Figure 6 provides similar plots for the correlation between the two equity indices (S&P 500 and DJIA). Note that, in Figures 2 and 3, three panels are provided: Figures 2A and 3A are for weekly returns; Figures 2B and 3B, for daily returns; and Figures 2C and 3C, for monthly returns.

For weekly returns, each panel or Figure contains three plots, one for each of the estimation methods outlined above: rolling historical correlation; exponential smoother with smoothing parameter 0.94; and dynamic conditional correlation (DCC) by log-likelihood for mean-reverting process. For the sake of brevity, only the DCC estimates are presented in the case of daily and monthly returns. The straight line running through each graph shows the relevant simple correlation from Table 2, which is not an average of any of the four time-varying correlation estimates. Several facts are immediately apparent from these graphs.

- The correlation between equity and commodity returns fluctuates notably over time. This finding holds regardless of the choice of equity or commodity indices, in that the correlation time-patterns are the same for all four pairs of investable indices.
- There is little evidence that correlations are any higher after 2002 than they were in prior years. If anything, consistent with the results obtained with simple correlations (see Table 2, in particular Table 2D), the time-varying correlation graphs show that correlations are lower since 2003 than before.
- Notwithstanding some amount of fluctuation over time, the correlations between equities and commodities are not often greater than 0.3. In contrast, Figure 6 shows that the correlation between the two equity indices is very high, typically well above 0.9.

In sum, equity-commodity return pairwise correlations fluctuate over the sample period. Quite often, the correlation estimates are even negative. These findings underline the importance of accurate measures of co-movement between asset returns necessary for portfolio optimization.

3.3 Commodity Sub-Indices

Figures 7 and 8 complement the analysis of the previous subsection, by plotting estimates of the time-varying correlations between the unlevered rates of return on benchmark equity indices and on specific categories of investable commodity indices. Figure 7 focuses on the difference between "Energy" and "Non-Energy" commodity baskets. Figure 8 refines Figure 7 by breaking down the Non-Energy index further into several investable sub-indices: Precious Metals, Industrial Metals, Agriculture, and Livestock. All of the plots in Figures 7 and 8 are directly comparable, in that they are all drawn using dynamic conditional correlations estimated by log-likelihood for mean-reverting model. Figures 7 and 8 highlight four facts:

- There is a substantial amount of time variation in the correlations between returns on equities and on both the Energy and Non-Energy commodity sub-indices. Depending on the time sub-period, these correlations fluctuate between -0.4 and 0.45. By contrast, the unconditional correlations are close to zero across the entire sample period.
- Both the GS and DJ Energy sub-indices present patterns of correlation with equity indices that are extremely close to the corresponding patterns for the comprehensive GSCI and DJ-AIG indices.
- Figure 8 suggests, however, that indices based on more narrow categories of commodities exhibit less correlation with equities than the overall non-energy index – suggesting possible diversification opportunities.[9]
- Finally, and importantly for the purpose of the present paper, it is readily apparent from both Figures 7 and 8 that there is no obvious secular pattern toward an increase in correlations in the last few years.

4. Long-term Co-Movements

The foregoing analysis indicates that the correlations between the equity and commodity return series may have fallen amidst the commodity investment boom. The very fact that these correlation estimates fluctuate significantly over time, however, is evidence of their short-term nature. If there is a reason to suspect that equity and commodity return should move together in the long run, however, a complementary technique is required.

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[9] See Chong and Miffre (2008) and references cited therein for discussions of diversification strategies using individual commodity futures.

4.1 Cointegration Analysis

A large volume of research evaluates the degree of interconnectivity between prices from different markets by employing time-series techniques that are appropriate for non-stationary and co-integrated data. In particular, much work on applied co-integration analysis has relied on Johansen's multivariate approach (Johansen, 1988, 1991; Johansen and Juselius, 1990).

Johansen (1988) proposes and implements a unified vector autoregressive system approach for testing cointegration. Johansen derived the maximum likelihood estimator of the space of cointegration vectors and the likelihood ratio test of the hypothesis that it has a given number of dimensions. The procedure involves the following stages:

- Model checking, determination of lag length;
- Determination of cointegration rank, trace and maximum eigenvalue statistic;
- Estimation of the cointegration space;

The first step of the model building involves the choice of lag order. The most common procedure is to estimate a vector autoregression using the undifferenced data. Then we can use different information criteria to select the number of lag lengths. In our analysis, we use Schwarz (SC) criteria to determine the optimal lag – 2 in our case.

After selecting the lag length, the Johansen procedure estimates a vector error correction model (VECM) to determine the number of cointegrating vectors. According to Johansen (1988), a general polynomial distributed lag process, x_t, involving up to k lags, can be written as:

$$x_t = \Pi_1 x_{t-1} + ... + \Pi_k x_{t-k} + u_t \tag{9}$$

where x_t is a vector of n variables of interest, Π_i is an $(n \times n)$ matrix of parameters, and u is n-dimensional Gaussian independently distributed random variables with zero mean and variance matrix (Λ). This equation can be reformulated into VECM form:

$$\Delta x_t = \Gamma_1 \Delta x_{t-1} + ... + \Gamma_{k-1} \Delta x_{t-k+1} + \Theta x_{t-k} + u_t \tag{10}$$

where $\Gamma_i = -\Sigma_{j=i+1}^{k} \Pi_j$, $(i = 1,2,...,k-1)$, and $\Theta = \Sigma_{i=1}^{k}(\Pi_i - I)$. This way of specifying the system contains information on both the short and long run adjustments to changes in x_t, via the estimates of $\hat{\Gamma}_i$ and $\hat{\Theta}$, respectively. Assuming that x_t is $I(1)$, while r linear combinations of x_t are stationary, we can write

$$\Theta = \alpha \beta', \tag{11}$$

where α is the vector of adjustment coefficients; β is the cointegrating vector; and both are $(n \times r)$ matrices. The approach of Johansen is based on the estimation of system (10) by maximum likelihood, while imposing the restriction in (11) for a given value of r.

Johansen (1988) demonstrates that β can be estimated by regressing ΔX_t and X_{t-k} on the lagged differences.

The next step in the Johansen approach involves testing the hypothesis about the rank of the long run matrix Θ, or equivalently the number of columns in β. The likelihood ratio test for the determination of the rank r is discussed in Johansen (1992). In general, tests of the hypothesis that $r \leq q$ use the likelihood ratio test statistics:

$$\lambda_{trace}(q) = -T\Sigma_{j=q+1}^{k}log(1-\hat{\lambda}_j) \qquad (12)$$

This test is called the trace test. It checks whether the smallest $k - q$ eigenvalues are significantly different from zero. Furthermore, we can test $H_0 : r \leq q$ versus the more restrictive alternative $H_1 : r = q+1$ using

$$\lambda_{max}(q) = -Tlog(1-\hat{\lambda}_{q+1}) \qquad (13)$$

This alternative test is the so-called maximum eigenvalue test, as it is based on the estimated (q+1)th largest eigenvalue.

Most of the existing Monte Carlo studies on the Johansen methodology point out that dimension of the data series for a given sample size may pose particular problems since the number of parameters of the underlying VAR models grows very large as the dimension increases. Likewise, difficulties often arise, when a given lag length of the system is either over or under parameterized. Reimers (1992) argues that for small samples, the Johansen procedure over-rejects when the null is true. To correct this bias, he suggests an adjustment in the degrees of freedom in the trace statistics and the maximum eigenvalue test statistics by replacing T by $T - nk$ for small samples. The corresponding degrees of freedom adjusted trace and maximum eigenvalue test statistics can be written as:

$$\lambda_{trace}^{a}(q) = -(T-nk)\Sigma_{j=q+1}^{k}log(1-\hat{\lambda}_j)$$

$$\lambda_{max}^{a}(q) = -Tlog(1-\hat{\lambda}_{q+1})$$

We first perform a univariate Augmented Dickey Fuller unit root test to determine the order of integration for each variable. Both variables (S&P GSCI total return index and S&P 500 index) appear to be integrated of order one; that is to say, our variables are nonstationary and they only become stationary after we take first differences. This finding suggests that we cannot rely on standard regression procedures, since OLS estimators have sampling distributions that are very different from those derived under the assumption of stationarity. Therefore, we proceed with the Johansen cointegration approach to determine whether there exists a long run relationship between our variables. Table 7 shows the cointegration rank between S&P's 500 and GSCI indices for the full sample period. Trace statistics suggest the absence of any cointegrating vector between commodity and equity indices. This result is consistent with the low-correlation that we observe from our dynamic conditional correlation estimation.

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4.2 Recursive Cointegration Analysis

To obtain an understanding of the dynamics of the relationship, which the foregoing analysis cannot provide, we examine the dynamics and extent of relationship, if any, between our indices using recursive cointegration method outlined in Hansen and Johansen (1993). Recursive cointegration techniques allow us to test for the level of co-integration among indices during our sample period. The recursive technique allows us to recover two ECM representations. In the "Z-representation," all the parameters of the ECM (β and α) are re-estimated during the recursions, while under the "R-representation" the short-run parameters (α) are kept fixed to their full sample values and only the long run parameters (β) are re-estimated.

The logic behind the recursive cointegration technique is similar to Johansen's (1988) multivariate cointegration approach. Instead of using all observations, we start with an initial sample period from t_0 to t_j to perform Johansen (1988) cointegration approach and calculate the corresponding trace statistics for this sub-sample. Then, we increase the sample size by 1 from t_0 to t_{j+1} and calculate the relevant trace statistics for this sample period. This process continues until we exhaust all the observations and, in the final stage, we perform the cointegration analysis for the full sample and calculate the trace statistics. Of course, the trace statistic calculated in the final stage is equal to standard static trace statistics calculated with the Johansen (1998) method. The recursive method, however, allows us to see the *dynamics* of the trace statistic.

We start with 52 weeks of observations and add one more week in each step until we exhaust all our observations.[10] We re-scale our trace statistics by the 95% quantile of the trace distribution derived for the selected model without exogenous variables or dummies. Re-scaled trace statistics suggest the rejection of null hypothesis of no cointegration if it is above 1. In addition, to see whether there exist a cointegrating vector among our variables, the slope of re-scaled trace statistic determines the direction of co-movements between our variables. An upward slope indicates rising co-movement, while a downward slope for the trace statistics reveals declining co-movement between our variables.

Figure 9 shows the R-1 form of the trace statistic, recursively calculated and scaled by the 5% critical value. The dark blue line gives the estimate calculated using data from the whole sample (i.e., from January 1991 through May 2008).[11] Although there is a large variation in trace statistics during our sample period, we can divide our sample period into three distinctive periods.

- The first period, from January 1992 to May 1997, is characterized by a relatively stable trace statistics that is generally below the threshold level of 1 (implying no cointegrating relationship).

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[10] We also use three years of observation in our initial estimation to see the robustness of our results. Three years of initial estimation did not change our qualitative results.

[11] We utilize weekly price data from the year prior to a given estimation period to start the recursive procedure for that period. Our results are robust to using more weeks for the prior period.

- The second period, from June 1997 to May 1999, is charaterized by instability in the trace test statistics. The latter is generally above 1, constituting some support for co-movement between our indices. However, during that two-year period, the extent of co-movement varies and statistical significance of the trace are mixed.
- The last period starts in June 1999 and continues up to the end of the sample. During this period, there is no statistical evidence of *any* long-run relationship between the benchmark commodity and equity indices.
- These results are robust, in that we obtain similar findings with our alternative commodity index (DJ-AIG total return index) and with the Energy as well as the Non-Energy commodity sub-indices.

In sum, there is little evidence of a common long-term trend between investable commodity and equity indices, and no evidence of a possible secular strengthening of any such trend.

5. <u>Extreme Events</u>

Sections 2 and 3 provide evidence that neither the average levels of correlation between equity and commodity returns, nor the pattern of variation of these correlations over time, have been qualitatively very different in the last five years than in the foregoing ten years. The widespread perception that financial markets nowadays move much more in lock-step, however, could be due not as much to changes in average levels and patterns but, instead, to the joint behavior of financial markets on "stressful days." In this Section, we provide evidence that there has been no increase in cross-market co-movements during periods of exceptionally large returns on commodities and equities.

5.1 Summary Statistics

To assess whether cross-asset extreme linkages exist in the case of commodities, we first identify the days and weeks during which the returns on the benchmark S&P 500 equity index was at least one or two standard deviations above or below its sample mean, and then analyze the contemporaneous unlevered returns on the benchmark investable commodity index, the S&P GSCI. Implicit in this approach is the notion that, if changes in extreme linkages have taken place because of commodity investment flows, then the fact that equity markets are much larger than commodity markets suggests that ripple effects are more likely to emanate from the former than from the latter. For the same reason, liquidity problems or panic reactions should be more likely to spread from stock to commodity markets than the reverse.

As in the rest of the paper, we look at joint commodity-equity return behaviors for the whole sample as well as for three successive sub-periods. Tables 8 and 9 summarize our findings for weekly and daily returns, respectively. Table 8A (*resp. 9A*) tallies the episodes when the weekly (*resp. daily*) return on the S&P 500 equity index was "large,"

i.e., at least one standard deviation away from its mean during a given period. Table 8B (*resp. 9B*) tallies what happens on weeks (*resp. days*) of "extreme" stock returns, i.e., when these returns were at least two standard deviations away from the relevant mean. Tables 8 and 9 show in italics the number of times when the unlevered return on the GSCI index was positive or negative, for a given direction of the large (Tables 8A and 9A) or extreme (Tables 8B and 9B) S&P 500 return. It also shows in **bold** the number of times when the contemporaneous GSCI return itself was also more than one (Tables 8A and 9A) or two (Tables 8B and 9B) standard deviations away from its own sample mean.

For the sake of brevity, we focus on weekly results (Table 8) because the daily results are qualitatively similar (Table 9). Between June 1, 1991 and May 26, 2008, there were 126 weeks (71+55) when the rate of return on the S&P 500 equity index was below its sample mean by one standard deviation or more, and 21 weeks (15+6) when the same return was below its mean by more than two standard deviation. During the 126 weeks of large, poor S&P 500 returns, the total return on the GSCI was *positive* (though not necessarily large or extreme) 55 times and negative 71 times. Of those 126 times, the GSCI return deviated from its mean by more than one standard deviation a total of 39 times – 17 times below the mean but 22 times *above* the mean. In other words, when the S&P 500 drops a lot, it is not clear which way the GSCI return will go – neither in terms of its sign nor in comparison to its mean. A similar pattern emerges when equities do very well. To wit, in the 96 (40+56) weeks when the S&P 500 return exceeded its sample mean by one standard deviation or more, the GSCI total return was positive only 56 times. Likewise, in the 13 weeks when the S&P 500 return exceeded its mean by more than two standard deviation, the GSCI total return was extremely bad just twice and extremely good only once.

In sum, contrary to extant findings that there exist extreme linkages between other asset markets (e.g., Hartman et al., 2004; Solnik and Longin, 2001), our evidence is suggestive of little relation between exceptionally large returns on commodities and equities. This negative result holds for the whole sample period as well as for all three of the sub-periods; for positive as well as for negative exceptional returns; and, for periods of stock market upturns as well as for downturns.

5.2 Extreme Correlation Analysis

To further investigate possible links between equity and commodity investments in periods of market stress, we compute the excedence correlation between the two series. We define correlation at an exceedence level q as the correlation between the two series when both of them exceed predefined threshold level of q. We choose empirical distribution of each series to determine threshold levels. This construction allows us to calculate the cross-correlation between the weekly returns on unlevered passive equity and commodity investments for each percentile of the joint return distribution. This technique is similar to that used by Longin and Solnik (2001) to assess pairwise U.S.-international equity market linkages during periods of extreme returns.

Figure 10 summarizes our results. Of particular interest is, naturally, what happens when both equity and commodity returns are either very high or very low. Figure 10 shows that, while equity-commodity correlations are typically very low, they

strengthen during extreme events. Figure 10 further shows that, whereas the equity-commodity correlation is mildly positive in the upper tail of the joint distribution, the cross-correlation is *negative* in the lower tail. That is, conditional on both equity and commodity returns being very poor, the two returns series are negatively correlated.

6. Conclusion

Amidst sharp rises in commodity prices and in commodity investing, many commentators have wondered whether commodities nowadays move in sync with traditional financial assets. We provide evidence that challenges this idea. Using dynamic correlation and recursive cointegration techniques, we find that the relation between the returns on investable commodity and equity indices has not changed significantly in the last fifteen years. We also find no evidence of an increase in co-movement during periods of extreme returns. Our findings are consistent with the notion that commodities have continued to provide benefits in terms of portfolio diversification.

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Table 1: Rates of Return – Summary Statistics

	DJIA	S & P500	DJAIG	GSCI
Panel 1: Daily Rates of Return (%, January 1991 through May 2008)				
Mean	0.0362	0.0334	0.0340	0.0334
Median	0.0490	0.0457	0.0436	0.0320
Maximum	6.1547	5.5744	4.8512	6.5670
Minimum	-7.4549	-7.1127	-9.1524	-18.4322
Std. Dev.	0.9879	1.0092	0.8284	1.2380
Skewness	-0.2243	-0.1066	-0.3412	-0.7931
Kurtosis	7.5442	6.7521	8.0619	16.1457
Jarque-Bera	3774.96***	2556.98***	4723.10***	31741.20***
Observations	4345	4345	4345	4345
Panel 2: Weekly Rates of Return (%, January 1991 through May 2008)				
Mean	0.1801	0.1657	0.1610	0.1557
Median	0.2771	0.3142	0.1912	0.2094
Maximum	11.9501	12.3746	5.3854	7.7770
Minimum	-9.5373	-12.1706	-7.3817	-14.5915
Std. Dev.	2.1272	2.1525	1.7955	2.6755
Skewness	-0.0539	-0.1553	-0.2428	-0.5455
Kurtosis	6.1518	6.4341	3.9157	5.1168
Jarque-Bera	375.45***	448.84***	40.56***	214.09***
Observations	906	906	906	906
Panel 3: Monthly Rates of Return (%, January 1991 through May 2008)				
Mean	0.7342	0.6717	0.7357	0.7268
Median	1.0877	1.0264	0.7925	0.7953
Maximum	10.0792	10.5790	11.5861	15.6086
Minimum	-16.4073	-15.7586	-7.8447	-15.5615
Std. Dev.	3.9575	3.8928	3.5994	5.4184
Skewness	-0.6483	-0.6306	0.0448	-0.0436
Kurtosis	4.8059	4.3408	3.0807	3.2343
Jarque-Bera	42.83***	29.37***	0.13	0.54
Observations	208	208	208	208

Notes: Table 1 provides summary statistics for the unlevered rates of return on the Dow Jones Industrial Average (DJIA) and the S&P 500 equity indices (excluding dividends), as well as on the Dow Jones DJAIG and S&P GSCI commodity indices (total return). Table 1 uses sample moments computed using rates of return (precisely, changes in log prices multiplied by 100) from January 2, 1991 to May 26, 2008. One, two or three stars indicate, respectively, that normality of the return distribution is rejected at the 10%, 5% or 1% level of statistical significance.

Table 2: Return Correlations, 1991-2008

	DJIA	S & P500	DJAIG	GSCI
Panel 1: Daily Returns, January 1991 to May 2008				
DJIA	1.0000			
S & P500	0.9455***	1.0000		
DJAIG	-.0211	0.0014	1.0000	
GSCI	-.0615***	-.0347**	0.8979***	1.0000
Panel 2: Weekly Returns, January 1991 to May 2008				
DJIA	1.0000			
S & P500	0.9387***	1.0000		
DJAIG	0.0613*	0.0753**	1.0000	
GSCI	0.0065	0.0335	0.8970***	1.0000
Panel 3: Monthly Returns, January 1991 to May 2008				
DJIA	1.0000			
S & P500	0.9263***	1.0000		
DJAIG	0.0827	0.0735	1.0000	
GSCI	-.0244	-.0018	0.8770	1.0000

Notes: Table 2 provide simple cross-correlation tables for the unlevered rates of return (precisely, for the changes in log prices) on four investable indices: the Dow Jones Industrial Average (DJIA) and the S&P 500 equity indices, as well as on the Dow Jones DJAIG and S&P GSCI commodity indices. Panels 1, 2 and 3 use, respectively, daily, weekly, and monthly return data from January 2, 1991 to May 26, 2008. One, two or three stars indicate that an estimate is statistically significantly different from zero at the 10%, 5% or 1% level, respectively.

Table 2B: Weekly Return Correlations, June 1992 to May 1997

	DJIA	S & P500	DJAIG	GSCI
DJIA	1.0000			
S & P500	0.9200***	1.0000		
DJAIG	0.0960	0.0578	1.0000	
GSCI	0.1265**	0.0950	0.8221***	1.0000

Table 2C: Weekly Return Correlations, June 1997 to May 2003

	DJIA	S & P500	DJAIG	GSCI
DJIA	1.0000			
S & P500	0.9376***	1.0000		
DJAIG	0.0915	0.1132**	1.0000	
GSCI	0.0490	0.0816	0.9346***	1.0000

Table 2D: Weekly Return Correlations, June 2003 to May 2008

	DJIA	S & P500	DJAIG	GSCI
DJIA	1.0000			
S & P500	0.9587***	1.0000		
DJAIG	-0.0050	0.0399	1.0000	
GSCI	-0.1179*	-0.0553	0.8897***	1.0000

Notes: Panels B, C and D of Table 2 provide simple cross-correlation tables for the **weekly** unlevered rates of return (precisely, for the changes in log prices) on four investable indices: the Dow Jones Industrial Average (DJIA) and the S&P 500 equity indices, as well as on Dow Jones' DJAIG and S&P's GSCI commodity indices. Tables 2B, 2C and 2D provide cross-correlations for three successive sub-periods: June 2, 1992 to May 31, 1997; June 1, 1997 to May 31, 2003; and, June 1, 2003 to May 26, 2008. One, two or three stars indicate that an estimate is statistically significantly different from zero at the 10%, 5% or 1% level, respectively.

Table 3: Correlations of Weekly Adjusted-Return Volatilities, January 1991 to May 2008

	DJIA	S & P500	DJAIG	GSCI
DJIA	1.0000			
S & P500	0.9283***	1.0000		
DJAIG	0.0616*	0.0540	1.0000	
GSCI	0.0820**	0.0654**	0.8410***	1.0000

Table 3B: Correlations of Weekly Adjusted-Return Volatilities, June 1992 to May 1997

	DJIA	S & P500	DJAIG	GSCI
DJIA	1.0000			
S & P500	0.8654***	1.0000		
DJAIG	-.0290	-.0542	1.0000	
GSCI	-.0215	-.0295	0.6871***	1.0000

Table 3C: Correlations of Weekly Adjusted-Return Volatilities, June 1997 to May 2003

	DJIA	S & P500	DJAIG	GSCI
DJIA	1.0000			
S & P500	0.9249***	1.0000		
DJAIG	0.0740	0.0522	1.0000	
GSCI	0.0880	0.0526	0.8831***	1.0000

Table 3D: Correlations of Weekly Adjusted-Return Volatilities, June 2003 May 2008

	DJIA	S & P500	DJAIG	GSCI
DJIA	1.0000			
S & P500	0.9298***	1.0000		
DJAIG	-.0272	-.0189	1.0000	
GSCI	-.0589	-.0458	0.8020***	1.0000

Notes: Table 3 provides simple cross-correlation tables for the volatilities of the **weekly** unlevered rates of return on four investable indices: the Dow Jones Industrial Average (DJIA) and the S&P 500 equity indices, as well as on Dow Jones' DJAIG and S&P's GSCI commodity indices. Volatilities are computed by squaring the weekly changes in log prices, net of the relevant period's mean change, and mutiplying the result by 10,000) Table 3A uses weekly return data from January 2, 1991 to May 26, 2008. Tables 3B, 3C and 3D provide the corresponding cross-correlations for three successive sub-periods: June 1, 1992 to May 31, 1997; June 1, 1997 to May 31, 2002; and, June 1, 2003 to May 26, 2008. One, two or three stars indicate that an estimate is statistically significantly different from zero at the 10%, 5% or 1% level, respectively.

Table 4: Return on Commodity Sub-Indices – Summary Statistics, 1991-2008

	S&P 500	GSCI	Agri-culture	Energy	Industr. Metals	Live-stock	Non-Energy	Precious Metals
Panel 1: Daily Rate of Return								
Mean	0.0334	0.0334	0.0038	0.0408	0.0293	0.0024	0.0136	0.0243
Median	0.0457	0.0320	-0.0039	0.0593	0.0186	0.0059	0.0126	0.0258
Maximum	5.5744	6.5670	4.4942	12.0280	7.5886	3.2544	2.9706	8.5586
Minimum	-7.1127	-18.4322	-5.6174	-34.8297	-9.0147	-5.4679	-4.4016	-8.2391
Std.Dev.	1.0092	1.2380	0.9982	1.9136	1.1392	0.8382	0.6663	0.9342
Skewness	-0.1066	-0.7931	0.0357	-1.3868	-0.1463	-0.1515	-0.1900	-0.2861
Kurtosis	6.7521	16.1457	4.7495	30.2431	7.2698	4.0611	5.5057	10.6445
Jarque-Bera	2,556[***]	31,741[***]	555.0[***]	135,759[***]	3,316[***]	220.4[***]	1,162[***]	10,639[***]
Observations	4345	4345	4345	4345	4345	4345	4345	4345
Panel 2: Weekly Rates of Return								
Mean	0.1657	0.1557	0.0183	0.1897	0.1448	0.0079	0.0640	0.1151
Median	0.3142	0.2094	0.0414	0.2810	0.0638	-0.0272	0.0471	0.1095
Maximum	12.3746	7.7770	8.8851	13.6955	9.1229	7.4351	5.2744	15.5192
Minimum	-12.1706	-14.5914	-7.0900	-24.2736	-10.5801	-11.8773	-5.2802	-12.2636
Std.Dev.	2.1525	2.6755	2.1972	4.0563	2.3197	1.8965	1.4373	2.0438
Skewness	-0.1553	-0.5455	0.2045	-0.5958	0.0183	-0.2380	0.1088	0.0874
Kurtosis	6.4341	5.1168	3.9186	5.4258	4.1182	5.4304	3.7282	8.8973
Jarque-Bera	448.84[***]	214.10[***]	38.176[***]	275.74[***]	47.25[***]	231.55[***]	21.81[***]	1314.03[***]
Observations	906	906	906	906	906	906	906	906
Panel 3: Monthly Rates of Return								
Mean	0.6717	0.7268	0.0717	0.9141	0.6552	0.0265	0.2766	0.5381
Median	1.0264	0.7953	-0.2500	1.1302	0.4524	0.2472	0.0593	0.0774
Maximum	10.5790	15.6086	14.2744	29.7686	19.3373	10.2632	10.8357	14.4768
Minimum	-15.7586	-15.5615	-13.9058	-25.0302	-13.8004	-15.8318	-10.1024	-11.6927
Std.Dev.	3.8928	5.4184	4.5145	8.3505	4.9569	4.0416	2.9316	3.9364
Skewness	-0.6306	-0.0436	-0.0542	0.0268	0.3944	-0.5404	0.1585	0.3950
Kurtosis	4.3408	3.2343	3.4727	3.4298	3.6518	3.5580	3.8975	3.6966
Jarque-Bera	29.37[***]	0.54	2.04	1.63	9.08[**]	12.82[***]	7.85[**]	9.61[***]
Observations	208	208	208	208	208	208	208	208

Notes: Table 4 provides summary statistics for the unlevered (or "total") rates of return (precisely, changes in log prices multiplied by 100) on the S&P 500 equity and GSCI commodity indices, as well as on six investable GSCI sub-indices: Agriculture (Wheat, Red Wheat, Corn, Soybeans, Cotton, Sugar, Coffee, and Cocoa); Energy (WTI Crude Oil, Brent Crude Oil, RBOB Gas, Heating Oil, GasOil, and Natural Gas); Industrial Metals (Aluminium, Copper, Lead, Nickel, and Zinc); Precious Metals (Gold and Silver); Livestock (Live Cattle, Feeder Cattle, and Lean Hogs); and, Non-Energy. Table 4 uses sample moments computed from January 2, 1991 to May 26, 2008.

Table 5: Return Correlations – Commodity Sub-Indices, 1991-2008

	S&P 500	GSCI	Agri-culture	Energy	Indust. Metals	Live-stock	Non-Energy	Precious Metals
Panel 1: Daily Rates-of-Return Correlations, January 1991 to May 2008								
S&P 500	1.0000							
GSCI	-.0615***	1.0000						
Agriculture	0.0382**	0.2756***	1.0000					
Energy	-.0754***	0.9673***	0.1116***	1.0000				
Ind.Metals	0.1062***	0.1987***	0.1270***	0.0941***	1.0000			
Livestock	0.0290*	0.1269***	0.0411***	0.0432***	0.0526***	1.0000		
NonEnergy	0.0713***	0.3602***	0.7994***	0.1587***	0.5525***	0.3744***	1.0000	
Prec.Metals	-.0935***	0.2733***	0.1872***	0.2068***	0.2990***	0.0379**	0.3836***	1.0000

	S&P 500	GSCI	Agri-culture	Energy	Indust. Metals	Live-stock	Non-Energy	Precious Metals
Panel 2: Weekly Rates-of-Return Correlations, January 1991 to May 2008								
S&P 500	1.0000							
GSCI	0.0335	1.0000						
Agriculture	0.0018	0.2382***	1.0000					
Energy	0.0311	0.9673***	0.1189***	1.0000				
Ind.Metals	0.1352***	0.2473***	0.1270***	0.1479***	1.0000			
Livestock	0.0391	0.1601***	0.0713***	0.0687**	0.0542	1.0000		
NonEnergy	0.0615*	0.3913***	0.8168***	0.1923***	0.5170***	0.4065***	1.0000	
Prec.Metals	-.0087	0.2576***	0.2131***	0.1818***	0.3552***	0.0316	0.4166***	1.0000

	S&P 500	GSCI	Agri-culture	Energy	Indust. Metals	Live-stock	Non-Energy	Precious Metals
Panel 3: Monthly Rates-of-Return Correlations, January 1991 to May 2008								
S&P 500	1.0000							
GSCI	-.0244	1.0000						
Agriculture	0.1607**	0.1855***	1.0000					
Energy	-.0561	0.9681***	0.0326	1.0000				
Ind.Metals	0.2314***	0.2166***	0.1295*	0.1179	1.0000			
Livestock	0.0311	0.0910	0.0419	0.0183	-.0022	1.0000		
NonEnergy	0.1986***	0.2963***	0.8153***	0.1084	0.5012***	0.3818***	1.0000	
Prec.Metals	-.0486	0.2129***	0.1624**	0.1352*	0.3114***	-.0394	0.3421***	1.0000

Notes: Table 5 shows the simple cross-correlations between the unlevered rates of return (precisely, changes in log prices) on eight investable indices: the S&P 500 equity and GSCI commodity indices, as well as six GSCI sub-indices: Agriculture (Wheat, Red Wheat, Corn, Soybeans, Cotton, Sugar, Coffee, and Cocoa); Energy (WTI Crude Oil, Brent Crude Oil, RBOB Gas, Heating Oil, GasOil, and Natural Gas); Industrial Metals (Aluminium, Copper, Lead, Nickel, and Zinc); Precious Metals (Gold and Silver); Livestock (Live Cattle, Feeder Cattle, and Lean Hogs); and, Non-Energy. Table 5 uses sample moments computed from January 2, 1991 to May 26, 2008. One, two or three stars indicate that an estimate is statistically significantly different from zero at the 10%, 5% or 1% level, respectively.

Table 6: Correlations of Return Volatilities for Equities and Commodity Sub-Indices, 1991-2008

	S&P 500	GSCI	Agri-culture	Energy	Indust. Metals	Live-stock	Non-Energy	Precious Metals
Panel 1: Correlations of Daily Return Volatilities, 1991-2008								
S&P 500	1.0000							
GSCI	0.1102***	1.0000						
Agriculture	0.0574***	0.0670***	1.0000					
Energy	0.1038***	0.9758***	0.0169	1.0000				
Ind.Metals	0.0008	0.0615***	0.0983***	0.0211	1.0000			
Livestock	0.0495***	0.0317**	0.0588***	0.0218	0.0445***	1.0000		
NonEnergy	0.0354**	0.1105***	0.6781***	0.0302**	0.5378***	0.1588***	1.0000	
Prec.Metals	0.0724***	0.2756***	0.1181***	0.2555***	0.2193***	0.0241	0.3201***	1.0000

	S&P 500	GSCI	Agri-culture	Energy	Indust. Metals	Live-stock	Non-Energy	Precious Metals
Panel 2: Correlations of Weekly Return Volatilities, 1991-2008								
S&P 500	1.0000							
GSCI	0.0654**	1.0000						
Agriculture	0.0159	0.0549*	1.0000					
Energy	0.0634*	0.9308***	0.0079	1.0000				
Ind.Metals	0.0124	0.0454	0.0337	-.0064	1.0000			
Livestock	-.0225	0.0873***	0.0348	0.0751**	0.0257	1.0000		
NonEnergy	0.0028	0.1577***	0.6971***	0.0630*	0.3702***	0.1807***	1.0000	
Prec.Metals	0.0147	0.0900***	0.0646**	0.0473	0.2405***	-.0002	0.3091***	1.0000

Panel 3: Correlations of Monthly Return Volatilities, 1991-2008								
	S&P 500	GSCI	Agri-culture	Energy	Indust. Metals	Live-stock	Non-Energy	Precious Metals
S&P 500	1.0000							
GSCI	0.0452	1.0000						
Agriculture	0.0422	0.1143	1.0000					
Energy	0.0339	0.9069	0.0341	1.0000				
Ind.Metals	-.0393	-.0049	0.0421	-.0519	1.0000			
Livestock	0.0445	0.0853	0.0478	0.0863	0.0577	1.0000		
NonEnergy	0.0916	0.0949	0.7535***	-.0106	0.3179***	0.1835***	1.0000	
Prec.Metals	0.0055	0.0221	0.1269*	-.0030	0.2798***	0.0290	0.2783***	1.0000

Notes: Table 6 shows the simple cross-correlations between the volatilities of the unlevered rates of return on the S&P 500 equity and GSCI commodity indices, as well as six investable GSCI sub-indices: Agriculture (Wheat, Red Wheat, Corn, Soybeans, Cotton, Sugar, Coffee, and Cocoa); Energy (WTI Crude Oil, Brent Crude Oil, RBOB Gas, Heating Oil, GasOil, and Natural Gas); Industrial Metals (Aluminium, Copper, Lead, Nickel, and Zinc); Precious Metals (Gold and Silver); Livestock (Live Cattle, Feeder Cattle, and Lean Hogs); and, Non-Energy. Table 6 uses sample moments for the period January 2, 1991 to May 26, 2008. Volatilities are computed by squaring the changes in log prices, net of the relevant period's mean change, and mutiplying the result by 10,000.

Table 7: Johansen (1988) Cointegration Analysis
between S&P GSCI Total Return Index and S&P 500 Index, Full Sample (1991-2008)

p-r	H$_0$=r	Eigenvalue	Trace	Trace*	Trace 95%	P-Value	P**-Value
2	0	0.014	16.043	15.997	20.164	0.175	0.178
1	1	0.004	3.603	2.280	9.142	0.486	0.722

Notes: VAR specification includes unrestricted constant and two lags. * Small sample corrected trace test statistic. ** The approximate *p*-value using the small sample corrected trace statistic.

Table 8A: Large Weekly Co-Movements: S&P 500 *versus* GSCI

	S&P 500 Down		S&P 500 Up	
Sample	GSCI Down	GSCI Up	GSCI Down	GSCI Up
Full Sample	*71*(**17**)	*55* (**22**)	*40* (**11**)	*56* (**21**)
1992-1997	*9* (**0**)	*11* (**1**)	*7* (**0**)	*8* (**1**)
1997-2003	*47* (**13**)	*30* (**12**)	*25* (**7**)	*35* (**12**)
2003-2008	*15* (**4**)	*14* (**9**)	*8* (**4**)	*13*(**8**)

Table 8B: Extreme Weekly Co-Movements: S&P 500 *versus* GSCI

	S&P 500 Down		S&P 500 Up	
Sample	GSCI Down	GSCI Up	GSCI Down	GSCI Up
Full Sample	*15* (**0**)	*6* (**0**)	*5* (**2**)	*8* (**1**)
1992-1997	*0* (**0**)	*0* (**0**)	*0* (**0**)	*0* (**0**)
1997-2003	*14* (**0**)	*5* (**0**)	*5* (**2**)	*8* (**1**)
2003-2008	*1* (**0**)	*1* (**0**)	*0* (**0**)	*0* (**0**)

Note: Table 8 focuses on the episodes when the weekly return on the S&P 500 index was at least 1 standard deviation (``Large'' returns, Table 8A) or at least 2 standard deviations (``Extreme'' returns, Table 8B) away from its mean during a given period. Table 8 shows in *Italics* the number of times when the unlevered return on the GSCI index was positive or negative, for a given direction of the large (Table 8A) or extreme (Table 8B) S&P return. It also shows in **Bold** the number of times when the contemporaneous GSCI return itself was also more than one (Table 8A) or two (Table 8B) standard deviations away from its own sample mean. For example, the first line in Table 8A shows that, between June 1, 1992 and May 20, 2008, there were 126 weeks (71+55) when the rate of return on the S&P 500 equity index was below its sample mean by one standard deviation or more, while the corresponding line in Table 8B shows that there were 21 weeks (15+6) when the same return was below its mean by more than two standard deviation. During the 126 weeks listed in Table 8A, the total return on the GSCI was positive (though not necessarily large or extreme) 55 times, and negative the other 71 times. Of those 126 times, the GSCI return deviated from its mean by more than one standard deviation a total of 39 times – 17 below the mean and 22 above the mean.

**Table 9A: Large Daily Co-Movements: S&P 500 *versus* GSCI**

	S&P 500 Down		S&P 500 Up	
Sample	GSCI Down	GSCI Up	GSCI Down	GSCI Up
Full Sample	*262* (**67**)	*232* (**85**)	*228* (**74**)	*237* (**68**)
1992-1997	*28* (**2**)	*37* (**11**)	*36* (**8**)	*32* (**7**)
1997-2003	*164* (**46**)	*136* (**50**)	*138* (**44**)	*151*(**43**)
2003-2008	*70* (**19**)	*59* (**24**)	*54* (**22**)	*54* (**18**)

Table 9B: Extreme Daily Co-Movements: S&P 500 *versus* GSCI

|	S&P 500 Down		S&P 500 Up	
Sample	GSCI Down	GSCI Up	GSCI Down	GSCI Up
Full Sample	*78* (**8**)	*46* (**5**)	*48* (**10**)	*61* (**4**)
1992-1997	*6* (**0**)	*3* (**0**)	*3* (**0**)	*1*(**0**)
1997-2002	*53* (**7**)	*41* (**5**)	*39* (**9**)	*50* (**3**)
2002-2007	*19* (**1**)	*2* (**0**)	*6* (**1**)	*10* (**1**)

Note: Table 9 focuses on the episodes when the daily return on the S&P 500 index was at least 1 standard deviation (``Large'' returns, Table 9A) or at least 2 standard deviations (``Extreme'' returns, Table 9B) away from its mean during a given period. Table 9 shows in *Italics* the number of times when the unlevered return on the GSCI index was positive or negative, for a given direction of the large (Table 9A) or extreme (Table 9B) S&P return. It also shows in **Bold** the number of times when the contemporaneous GSCI return itself was also more than one (Table 9A) or two (Table 9B) standard deviations away from its own sample mean. For example, the first line in Table 9A shows that, between January 2, 1991 and May 26, 2008, there were 494 days (262+232) when the rate of return on the S&P 500 equity index was below its sample mean by one standard deviation or more, while the corresponding line in Table 9B shows that there were 124 days (78+46) when the same return was below its mean by more than two standard deviation. During the 494 days listed in Table 9A, the total return on the GSCI was positive (though not necessarily large or extreme) 232 times, and negative the other 262 times. Of those 494 times, the GSCI return deviated from its mean by more than one standard deviation a total of 152 times – 67 below the mean and 85 above the mean).

Figure1: Major Commodity and Equity Indices, 1991-2008
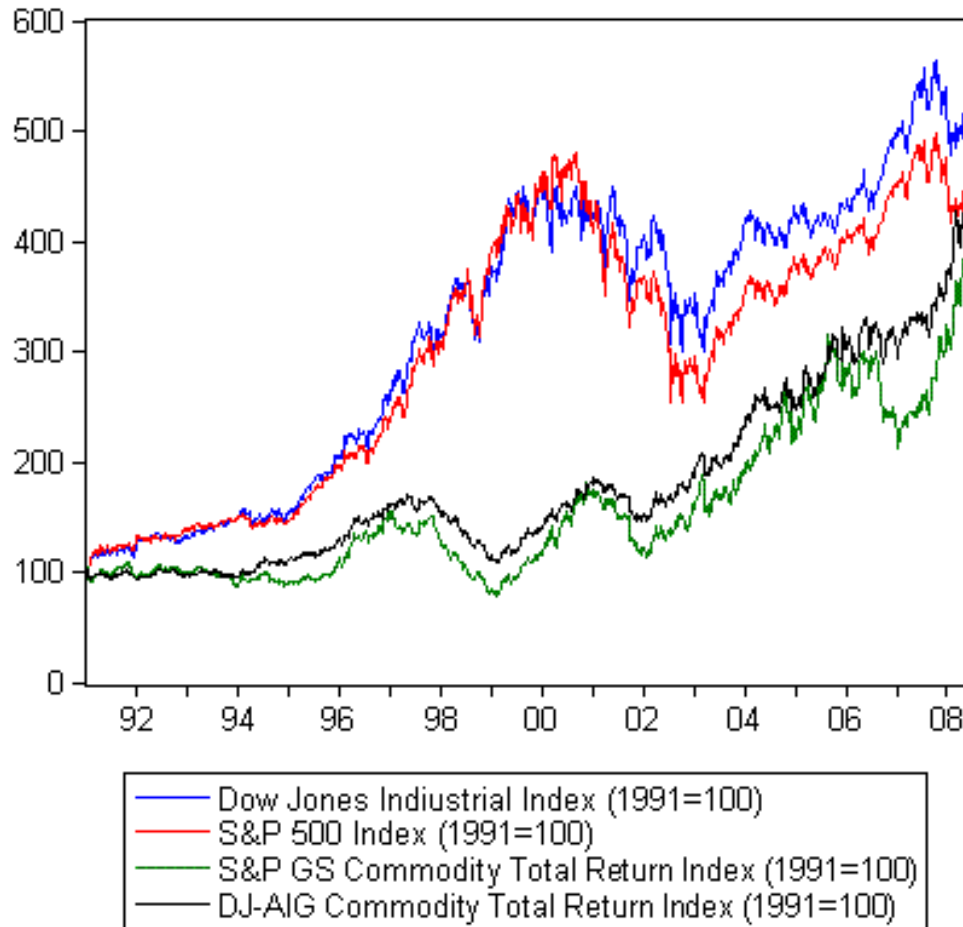
Note: Figure 1 plots the levels of four indices from January 2, 1991 to May 26, 2008: the Dow Jones Industrial Average (DJIA) and the S&P 500 equity indices, and the Dow Jones DJ-AIG and S&P GSCI commodity indices. The base level is set for January 2, 1991. The two equity indices (top two trends) tend to move closely together. So do the two commodity indices – most of the time. Two exceptions are 1994-1995 and 2006-2007, when the DJ-AIG index rose while the GSCI either stagnated or outright dropped in value.

Figure 2A: Equity and Commodity Weekly Return Correlations: S&P 500 *vs.* GSCI, 1991-2008

EXP_SP_GSTR

MOVAV_SP_GSTR

DCC_MR_SP_GSTR
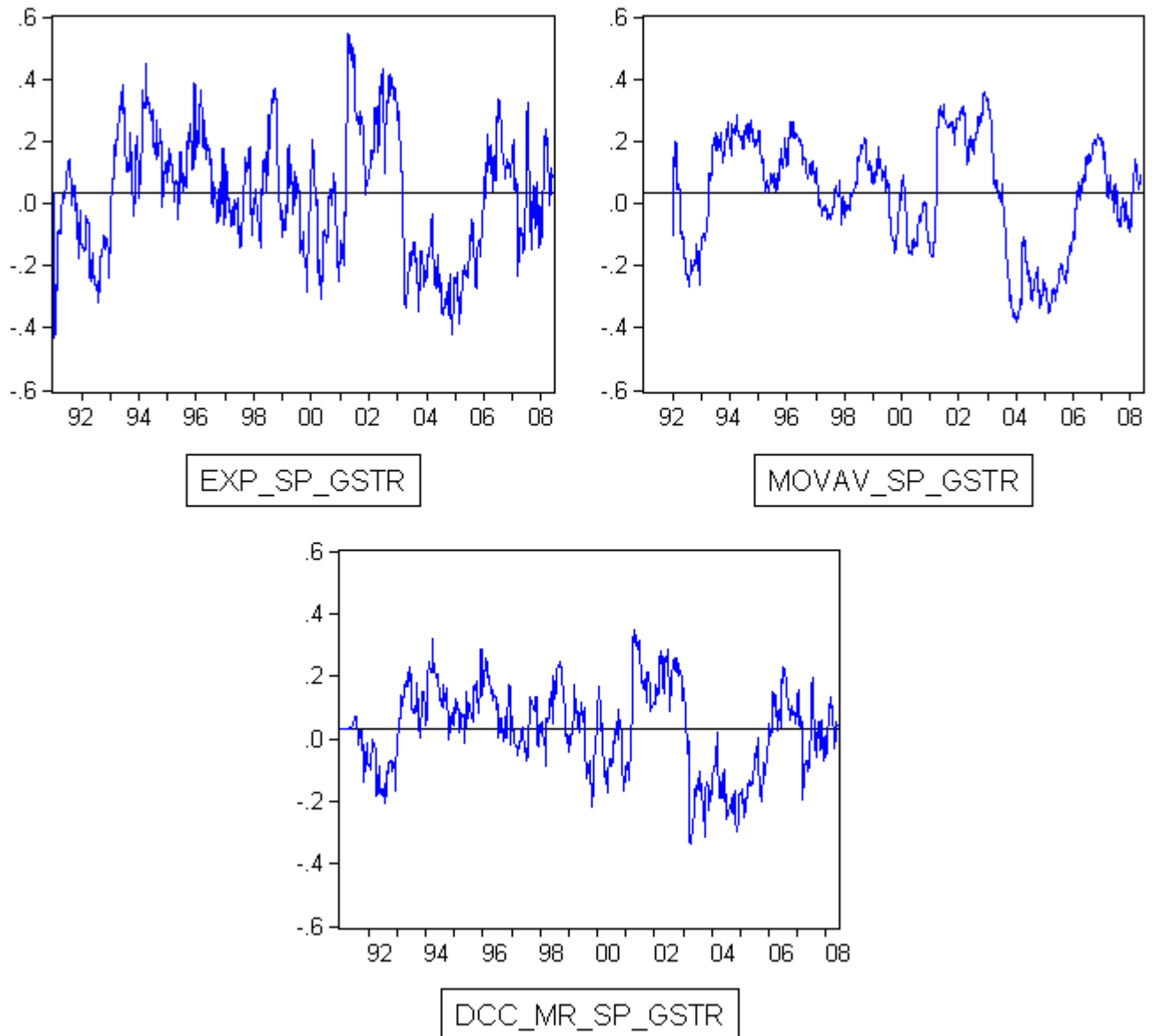
Note: Figure 2A depicts estimates of the time-varying correlation between the **weekly** unlevered rates of return (precisely, changes in log prices) on the S&P 500 (SP) and GSCI total return (GSTR) indices from January 2, 1991 to May 26, 2008. The Figure provides plots for the following three estimation methods: exponential smoother with 0.94 smoothing parameter (top left panel), rolling historical correlation (top right) and dynamic conditional correlation by log-likelihood for mean reverting model estimation (bottom panel). The straight line running through each graph is the unconditional correlation from Table 2A.
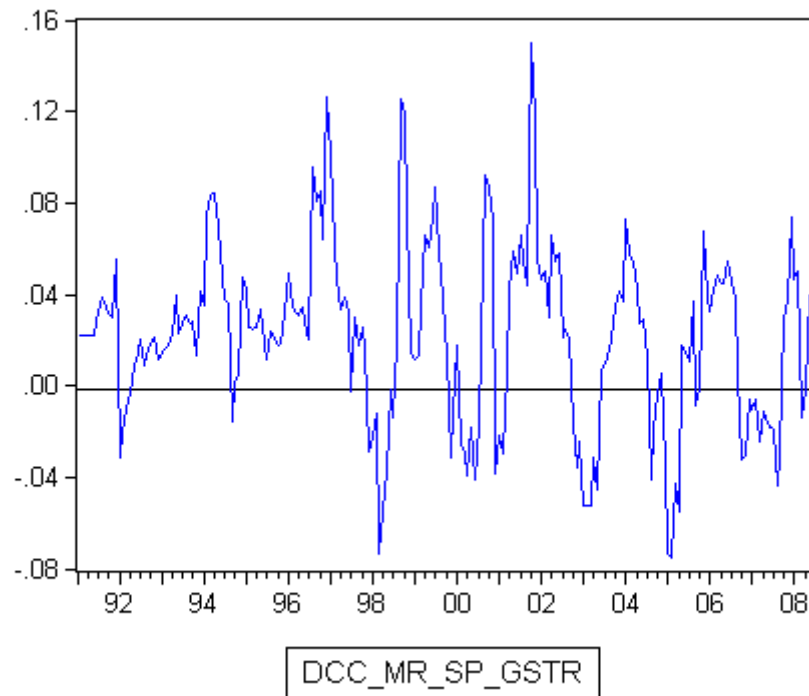
Figure 2B:

Equity and Commodity Daily Return Correlations: S&P 500 *vs.* GSCI, 1991-2008

DCC_MR_SP_GSTR
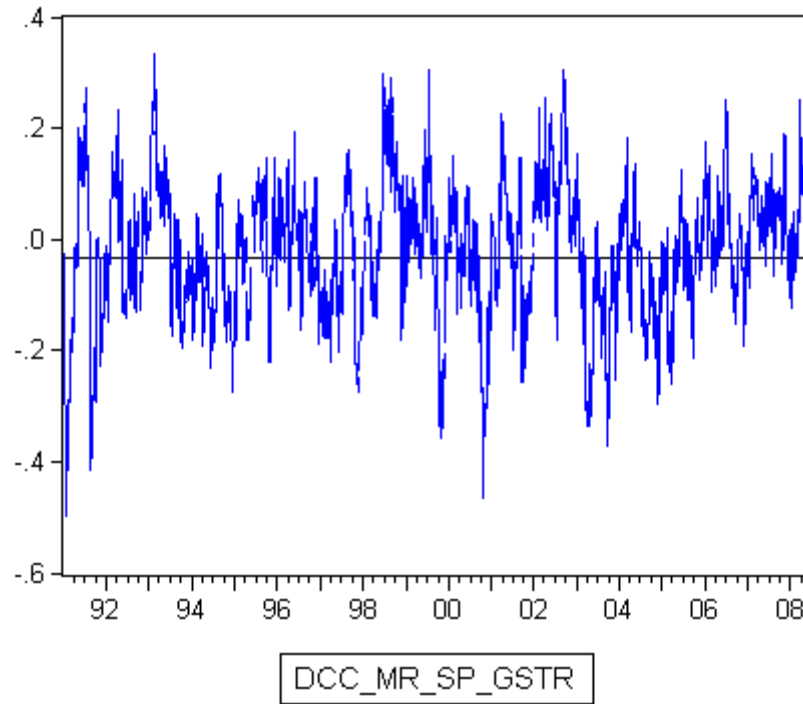
Note: Figure 2B depicts estimates of the time-varying correlation between the **daily** unlevered rates of return (precisely, changes in log prices) on the S&P 500 (SP) and GSCI total return (GSTR) indices from January 2, 1991 to May 26, 2007. The method used to compute estimates is dynamic conditional correlation by log-likehood for mean reverting model estimation. The straight line running through each graph is the unconditional correlation.
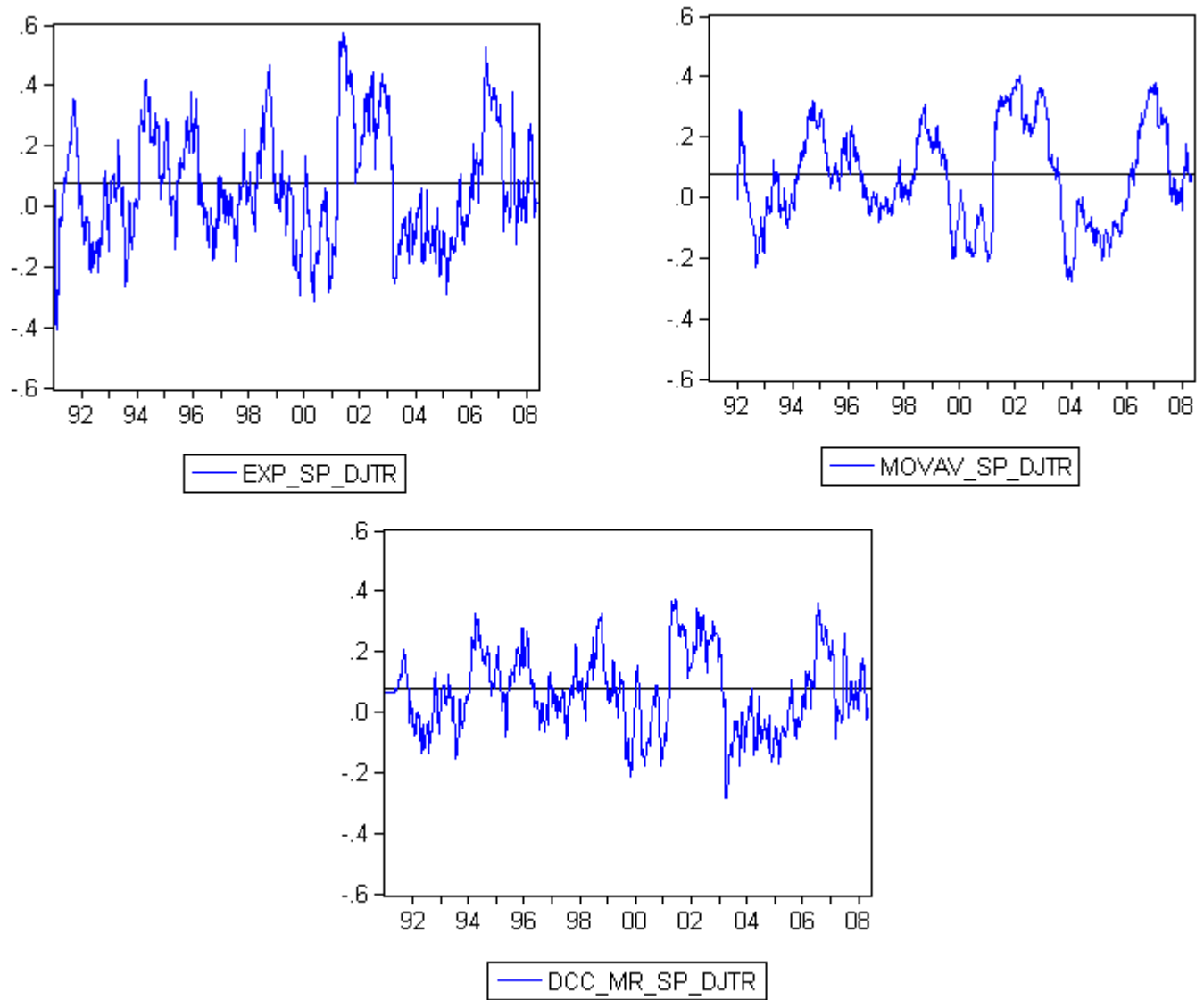
Figure 2C:

Equity and Commodity Monthly Return Correlations: S&P 500 *vs.* GSCI, 1991-2008

DCC_MR_SP_GSTR

Note: Figure 2C depicts estimates of the time-varying correlation between the **monthly** unlevered rates of return (precisely, changes in log prices) on the S&P 500 (SP) and GSCI total return (GSTR) indices from January 2, 1991 to May 26, 2008. The method used to ompute estimates is dynamic conditional correlation by log-likehood for mean reverting model estimation. The straight line running through each graph is the unconditional correlation.
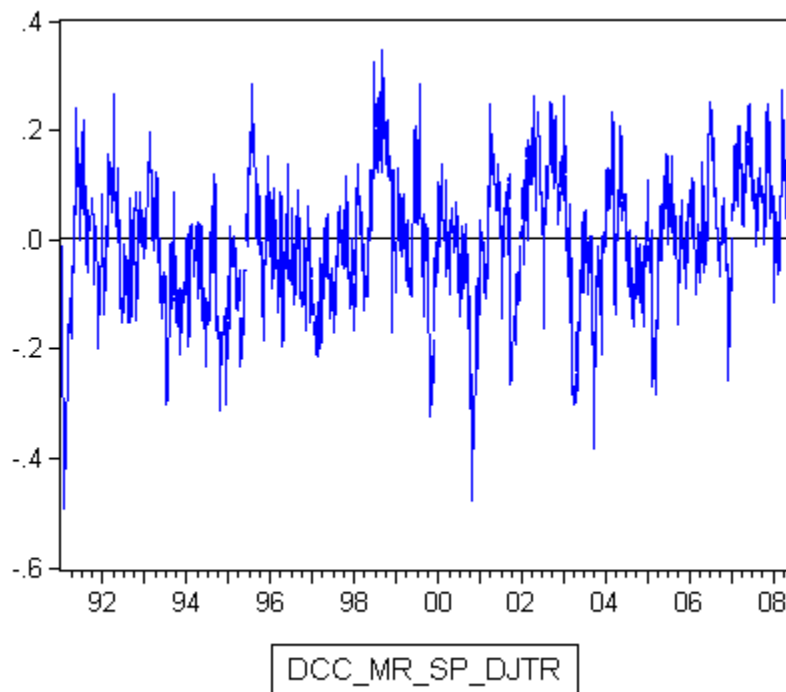
Figure 3A:

Equity and Commodity Weekly Return Correlations: S&P 500 *vs.* DJ-AIG, 1991-2008

Note: Figure 3A depicts estimates of the time-varying correlation between the **weekly** unlevered rates of return (precisely, changes in log prices) on the S&P 500 (SP) and DJ-AIGCI total return (DJTR) indices from January 2, 1991 to May 26, 2008. The Figure provides plots for the following three estimation methods: exponential smoother with 0.94 smoothing parameter (top left panel), rolling historical correlation (top right), and dynamic conditional correlation by log-likehood for mean reverting model estimation (bottom panel). The straight line running through each graph is the unconditional correlation.
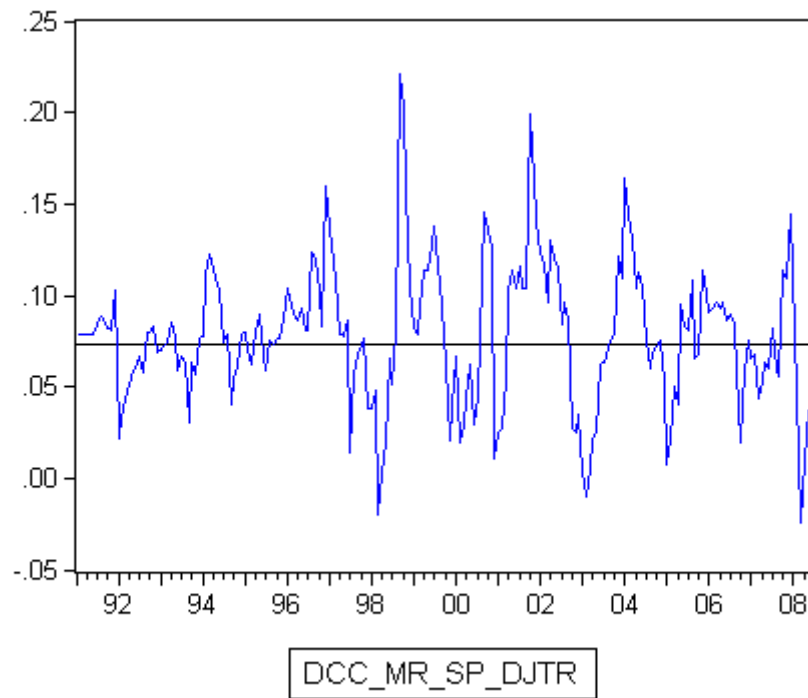
Figure 3B:

Equity and Commodity Daily Return Correlations: S&P 500 *vs.* DJ-AIG, 1991-2008

DCC_MR_SP_DJTR

Note: Figure 3B depicts estimates of the time-varying correlation between the **daily** unlevered rates of return (precisely, changes in log prices) on the S&P 500 (SP) and DJ-AIGCI total return (DJTR) indices from January 2, 1991 to May 26, 2008. The method used to compute estimates is dynamic conditional correlation by log-likehood for mean reverting model estimation. The straight line running through each graph is the unconditional correlation.

Figure 3C:

Equity and Commodity Monthly Return Correlations: S&P 500 *vs.* DJ-AIG, 1991-2008

Note: Figure 3C depicts estimates of the time-varying correlation between the **monthly** unlevered rates of return on the S&P 500 (SP) and DJ-AIGCI total return (DJTR) indices from January 2, 1991 to May 26, 2008. The method used to compute estimates is dynamic conditional correlation by log-likelihood for mean reverting model estimation. The straight line running through each graph is the unconditional correlation.
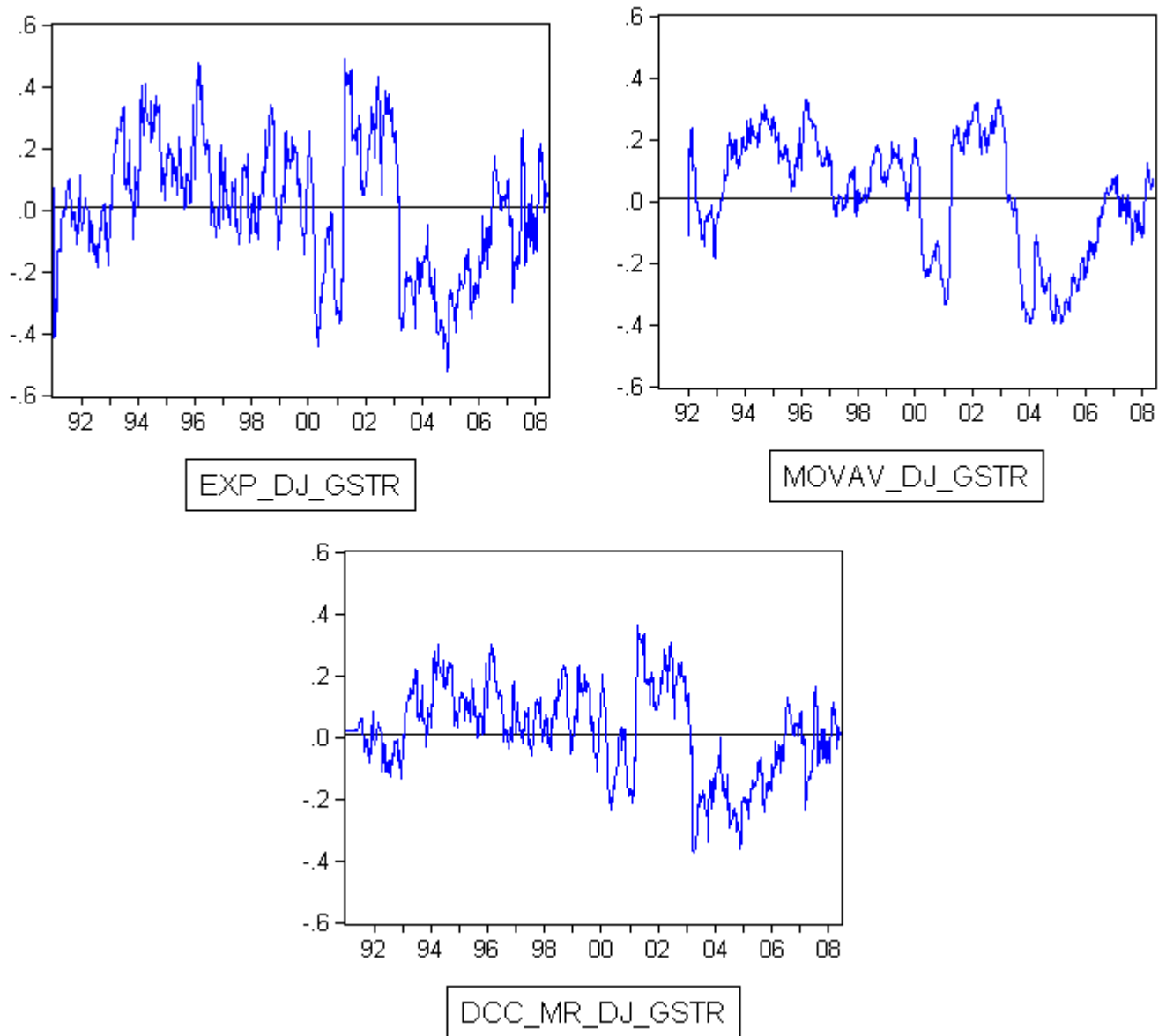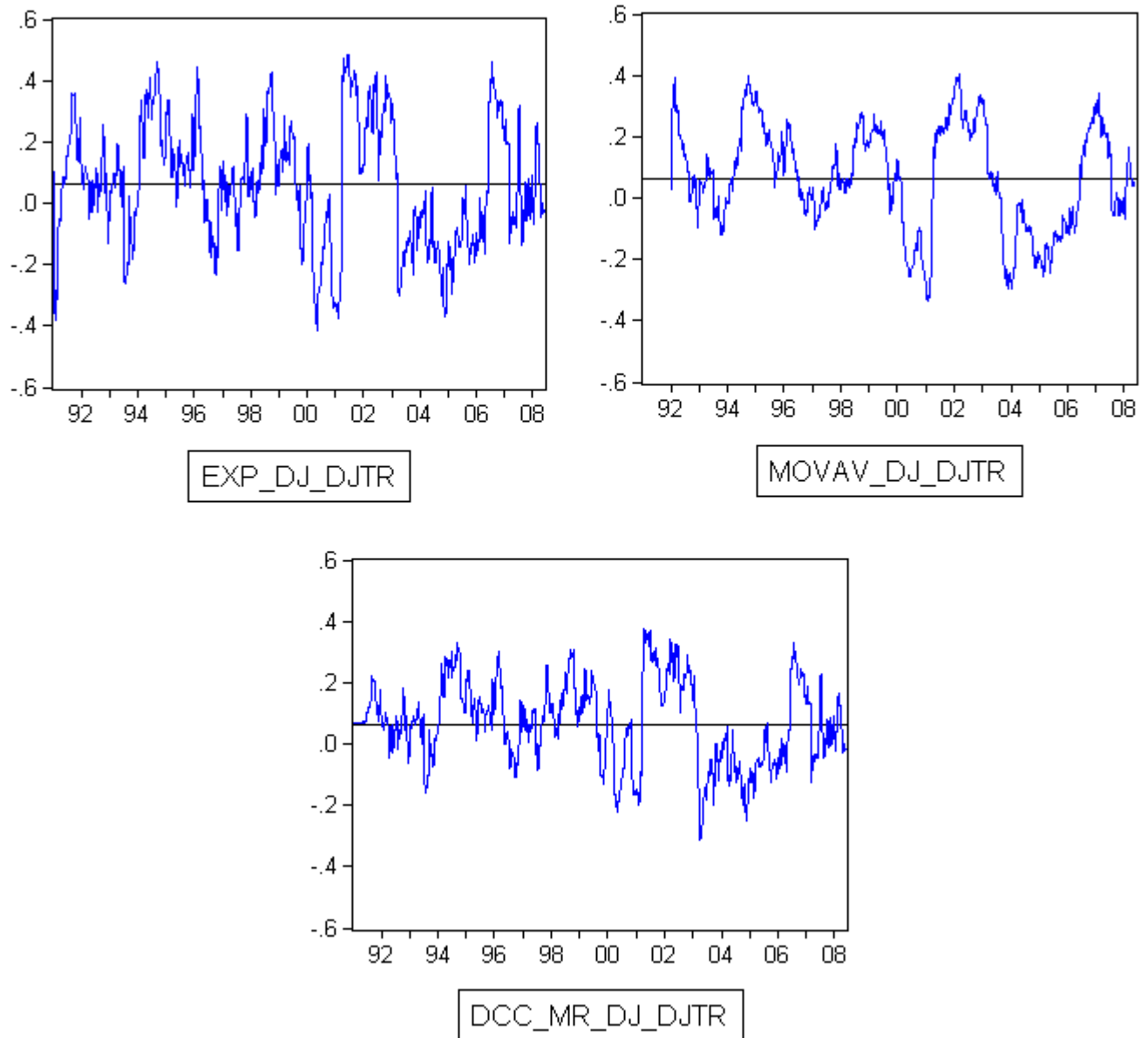
Figure 4:

Equity and Commodity Weekly Return Correlations: DJIA *vs.* GSCI, 1991-2008

EXP_DJ_GSTR

MOVAV_DJ_GSTR

DCC_MR_DJ_GSTR

Note: Figure 4 depicts estimates of the time-varying correlation between the **weekly** unlevered rates of return (precisely, changes in log prices) on the Dow Jones Industrial Average (DJ) and GSCI total return (GSTR) indices from January 2, 1991 to May 26, 2008. The Figure provides plots for the following three estimation methods: exponential smoother with 0.94 smoothing parameter (top left panel), rolling historical correlation (top right) and dynamic conditional correlation by log-likelhood for mean reverting model estimation (bottom panel). The straight line running through each graph is the unconditional correlation from Table 2A.

Figure 5:

Equity and Commodity Weekly Return Correlations: DJIA *vs.* DJ-AIG, 1991-2008

EXP_DJ_DJTR

MOVAV_DJ_DJTR

DCC_MR_DJ_DJTR

Note: Figure 5 depicts estimates of the time-varying correlation between the **weekly** unlevered rates of return (precisely, changes in log prices) on the Dow Jones Industrial Average (DJ) and DJ-AIGCI total return (DJTR) indices from January 2, 1991 to May 26, 2008. The Figure provides plots for the following three estimation methods: exponential smoother with 0.94 smoothing parameter (top left panel), rolling historical correlation (top right) and dynamic conditional correlation by log-likelihood for mean reverting model estimation (bottom panel). The straight line running through each graph is the unconditional correlation from Table 2A.
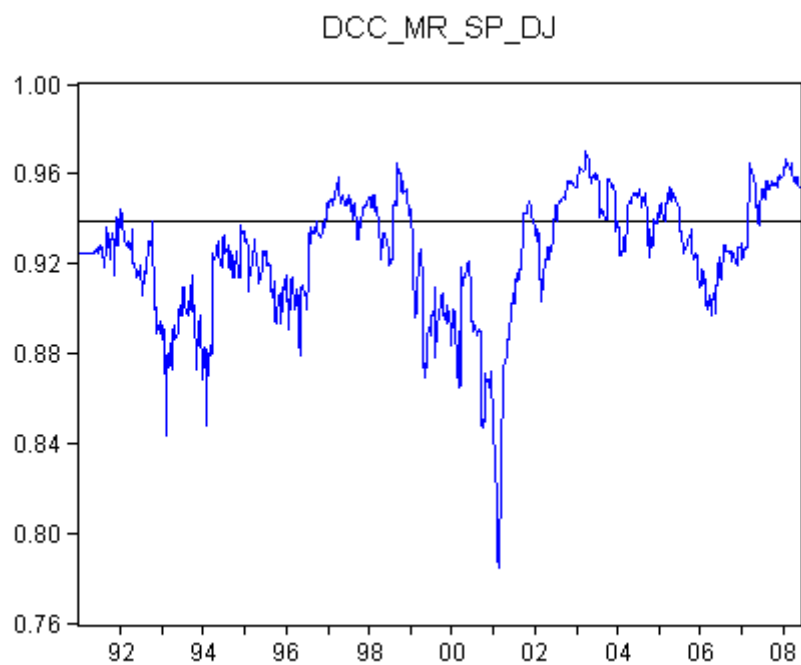
Figure 6: S&P 500 and DJIA Weekly Equity Returns Correlations, 1991-2008

DCC_MR_SP_DJ

Note: Figure 6 depicts estimates of the time-varying correlation between the **weekly** unlevered rates of return (precisely, changes in log prices) on the Dow Jones Industrial Average (DJ) and S&P 500 (SP) equity indices from January 2, 1991 to May 26, 2008. The method to compute the estimates is dynamic conditional correlation by log-likehood for mean reverting model estimation. The straight line running through each graph is the unconditional correlation from Table 2A.

Figure 7: S&P 500 vs. Energy and Non-Energy Indices – Return Correlations, 1991-2008
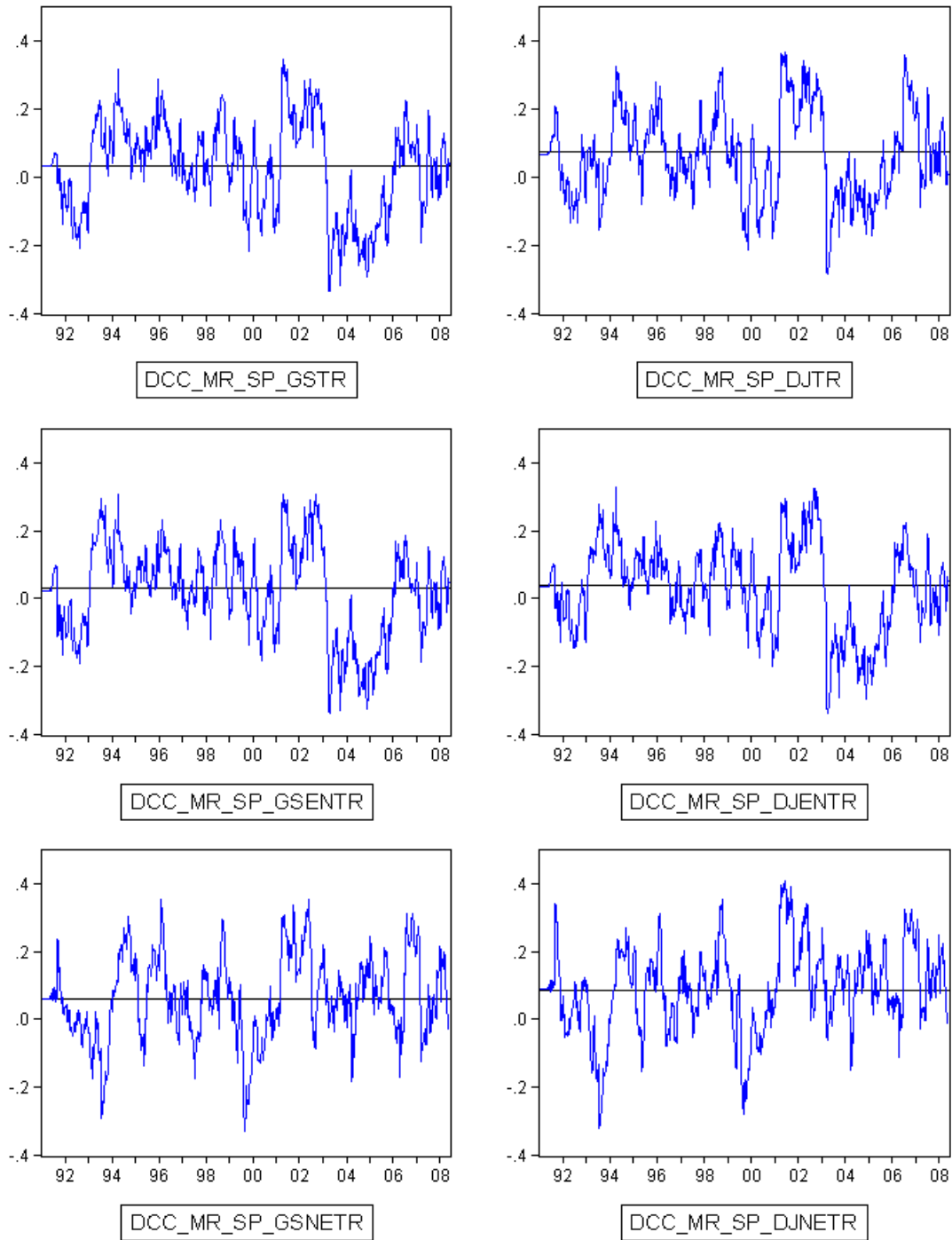
Note: Figure 7 plots the time-varying correlations between the unlevered ("total") **weekly** rates of return on the S&P 500 (SP) equity index and six investable commodity products. Dynamic conditional correlation are estimated by log-likelihood for mean-reverting model (Engle, 2002). The straight lines through the graphs show the unconditional correlations from January 2, 1991 to May 26, 2008. Clockwise from top right: GSCI (GSTR); GSCI Energy (GSENTR); GSCI Non-energy (GSNETR); DJ-AIG Non-energy (DJNETR); DJ-AIG energy (DJENTR); and, DJ-AIG (DJTR) total return indices.

Figure 8: S&P 500 and GSCI Weekly Sub-Index Returns Correlations, 1991-2008
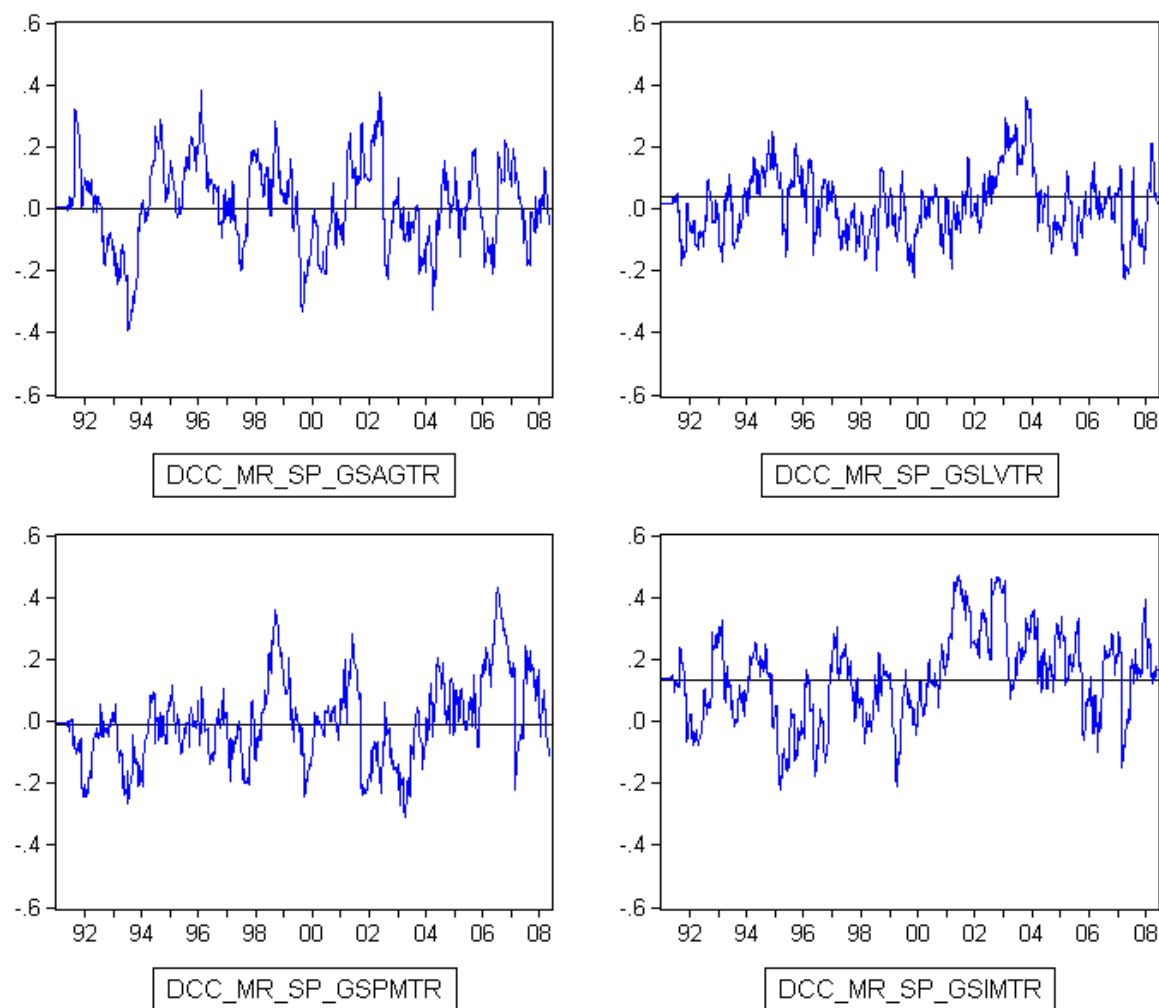
Note: Figure 8 depicts estimates of the time-varying correlation between the unlevered weekly rates of return on the S&P 500 (SP) equity index and various investable GSCI total return commodity sub-indices. Counter-clockwise from the top left corner: GSCI total return index (GSTR); GSCI Industrial-Metals total return index (GSIMTR); GSCI Precious-Metals total return index (GSPMTR); GSCI Livestock total return index (GSLVTR); and, GSCI Agriculture total return index (GSAGTR). Dynamic conditional correlation are estimated by log-likelihood for mean-reverting model (Engle, 2002). The straight lines through the graphs show the unconditional correlations from January 2, 1991 to May 26, 2008.

Figure 9:

Recursively Calculated Trace Test Statistic Scaled by the 5% Critical value, 1992-2008
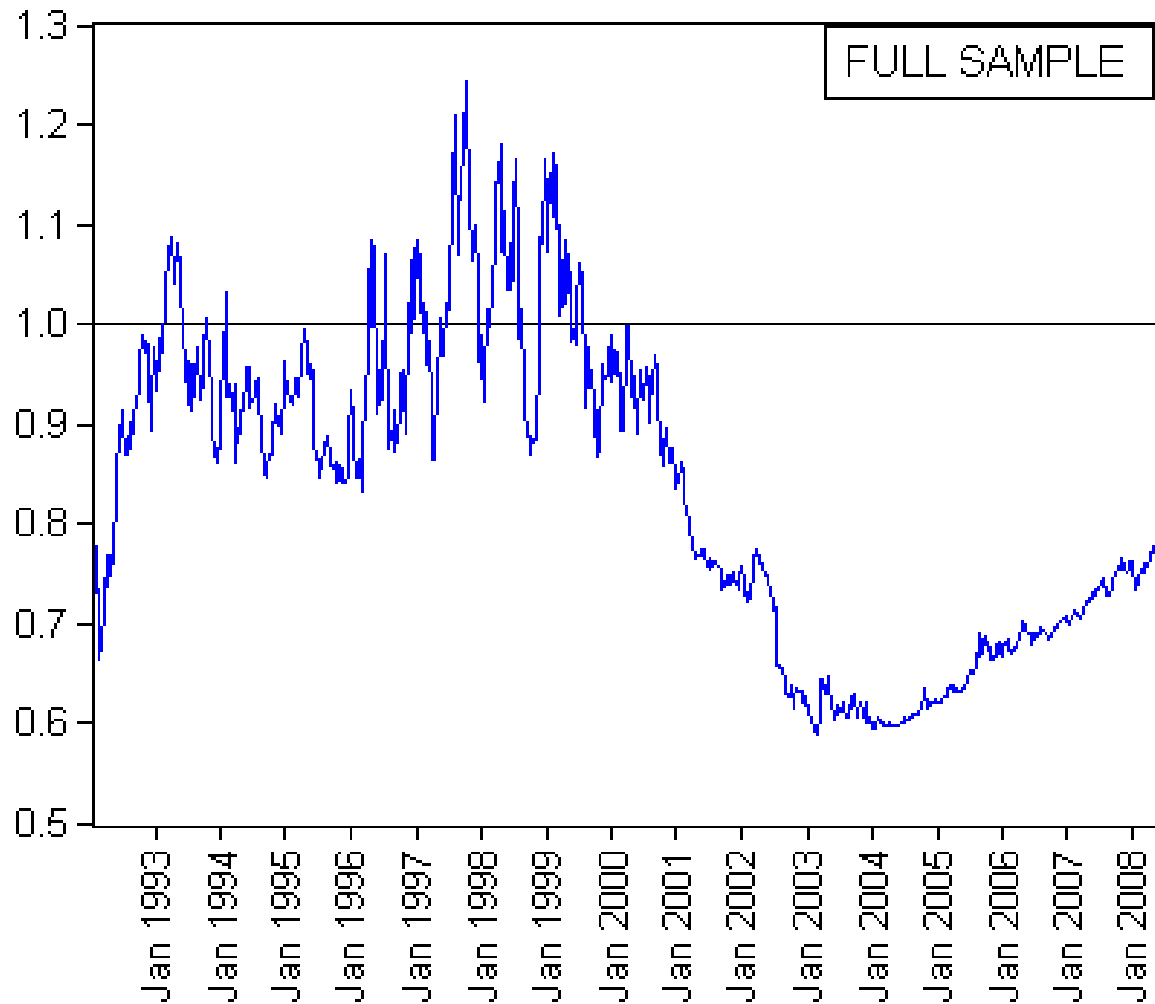
Note: Figure 9 shows the R-1 form of the trace statistic. The 5% critical value is represented by the solid (horizontal) black line. The dark blue graph shows the estimate calculated recursively using data from the whole sample, i.e., from January 1991 through May 2008. Weekly price data from the year prior to a given estimation period are utilized to start the recursive procedure for that period.

Figure 10: Extreme-Event Cross-Correlations, 1991-2008
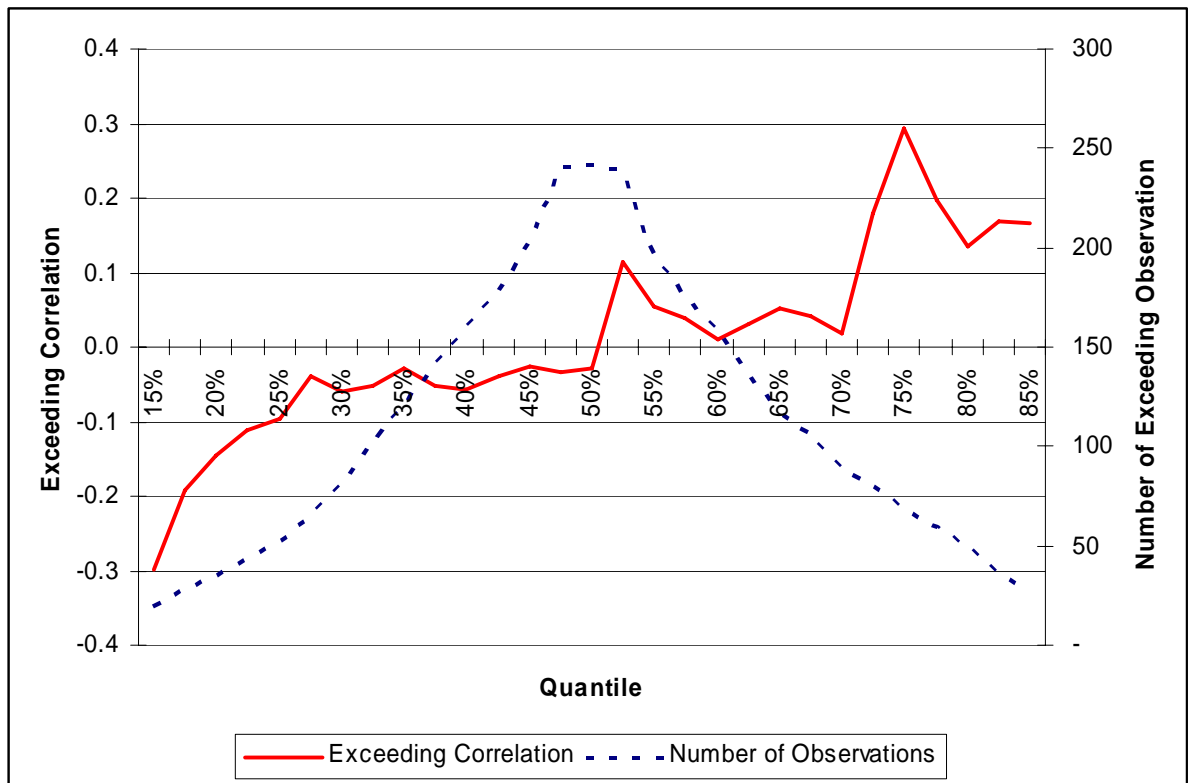
Note: Figure 10 plots, for each percentile of the joint return distribution, the cross-correlation between the weekly returns on unlevered passive equity and commodity investments. The dotted line shows the number of observations in each percentile. The absolute value of the correlation is virtually nil (less than 0.1) for the vast majority of stock returns, namely, those between the 25th and the 75 percentiles of the joint return distribution. However, the solid line shows that, between January 1991 and May 2008, the cross-correlation was negative when equity and commodity returns were both poor, and positive when these returns were both strongly positive.

Appendix: Supplemental Data

Table 1B: Weekly Rates of Return: %, June 1992 through May 1997

	DJIA	S & P500	DJAIG	GSCI
Mean	0.2962	0.2723	0.2046	0.1382
Median	0.3366	0.3456	0.2320	0.1487
Maximum	3.8326	4.1943	3.5111	5.3406
Minimum	-4.3260	-4.1124	-4.2086	-9.2089
Std. Dev.	1.5328	1.4406	1.2625	1.8115
Skewness	-0.2793	-0.2580	-0.2529	-0.3951
Kurtosis	3.2419	3.3717	3.3123	5.4266
Jarque-Bera	4.04	4.41	3.86	71.10***
Observations	262	262	262	262

Table 1C: Weekly Rates of Return: %, June 1997 through May 2003

	DJIA	S & P500	DJAIG	GSCI
Mean	0.0587	0.0391	0.0558	0.0449
Median	0.1551	0.3670	0.0136	0.0230
Maximum	11.9501	12.3746	5.3787	7.4794
Minimum	-9.5373	-12.1706	-7.3817	-14.5914
Std. Dev.	2.8678	2.9435	1.8830	2.8769
Skewness	0.0871	-0.0255	-0.0099	-0.4457
Kurtosis	4.8033	4.7886	3.5417	4.8887
Jarque-Bera	42.94***	41.89***	3.84	57.06***
Observations	314	314	314	314

Table 1D: Weekly Rates of Return: %, June 2003 through May 2008

	DJIA	S & P500	DJAIG	GSCI
Mean	0.1581	0.1646	0.2895	0.3478
Median	0.2853	0.2947	0.4355	0.5384
Maximum	4.1648	4.2537	5.3854	7.7770
Minimum	-4.5635	-5.2363	-7.0995	-12.2813
Std. Dev.	1.5739	1.6136	2.2032	3.1762
Skewness	-0.3688	-0.4353	-0.3803	-0.4976
Kurtosis	3.2348	3.2821	3.0614	3.2654
Jarque-Bera	6.52**	9.11**	6.33**	11.54***
Observations	261	261	261	261

Notes: Tables 1B, 1C and 1D provide summary statistics for the **weekly** unlevered rates of return (precisely, changes in log prices multiplied by 100) on the Dow Jones Industrial Average (DJIA) and the S&P 500 equity indices (excluding dividends), as well as on the Dow Jones DJAIG and S&P GSCI commodity indices (total returns). Tables 1B, 1C and 1D use weekly rates of returns for three successive sub-periods: June 1, 1992 to May 31, 1997; June 1, 1997 to May 31, 2003; and, June 1, 2003 to May 26, 2008. One, two or three stars indicate, respectively, that normality of the return distribution is rejected at the 10%, 5% or 1% level of statistical significance.

Table 3B: Weekly Returns on Commodity Sub-Indices (%, June 1992 to May 1997)

	S&P 500	GSCI	Agri-culture	Energy	Industr. Metals	Live-stock	Non-Energy	Precious Metals
Mean	0.2723	0.1382	0.2134	0.0934	0.0912	0.1174	0.1544	0.0289
Median	0.3456	0.1487	0.1884	0.0548	0.1521	0.0854	0.1261	0.1118
Maximum	4.1943	5.3406	8.8851	8.6179	5.5859	5.4744	4.8813	5.0754
Minimum	-4.1124	-9.2089	-5.8188	-15.7337	-7.8761	-3.8916	-3.1343	-6.9975
Std.Dev.	1.4406	1.8115	1.9414	3.1753	2.0263	1.6141	1.2063	1.5047
Skewness	-0.2580	-0.3951	0.3298	-0.3488	-0.3407	0.3171	0.3086	-0.1818
Kurtosis	3.3717	5.4266	5.0862	4.9000	3.8469	3.3302	4.1443	6.6266
Jarque-Bera	4.4140	71.10***	52.26***	44.72***	12.90***	5.58*	18.45***	145.02***
Observations	262	262	262	262	262	262	262	262

Table 3C: Weekly Returns on Commodity Sub-Indices (%, June 1997 to May 2003)

	S&P 500	GSCI	Agri-culture	Energy	Industr. Metals	Live-stock	Non-Energy	Precious Metals
Mean	0.0391	0.0449	-0.2343	0.1552	-0.0836	-0.1103	-0.1518	0.0578
Median	0.3670	0.0230	-0.1797	0.2623	-0.1593	-0.1601	-0.1299	-0.0479
Maximum	12.3746	7.4794	6.6448	13.6955	9.1229	7.4351	5.2744	15.5192
Minimum	-12.1706	-14.5914	-6.9088	-21.5174	-5.3059	-7.7096	-3.6933	-5.3964
Std.Dev.	2.9435	2.8769	2.0720	4.4716	1.9433	1.9606	1.3351	1.9736
Skewness	-0.0255	-0.4457	0.1833	-0.4062	0.3919	-0.1719	0.2821	1.6838
Kurtosis	4.7886	4.8887	3.2898	4.3512	4.2542	4.8277	3.6380	14.7282
Jarque-Bera	41.89***	57.06***	2.86	32.52***	28.62***	45.25***	9.49***	1,948***
Observations	314	314	314	314	314	314	314	314

Table 3D: Weekly Returns on Commodity Sub-Indices (%, June 2003 to May 2008)

	S&P 500	GSCI	Agri-culture	Energy	Industr. Metals	Live-stock	Non-Energy	Precious Metals
Mean	0.1646	0.3478	0.0840	0.3963	0.5401	0.0186	0.2261	0.3516
Median	0.2947	0.5384	0.1165	0.7441	0.6263	-0.0591	0.2418	0.5926
Maximum	4.2537	7.7770	8.4111	10.2195	8.0036	6.2184	5.0991	7.8871
Minimum	-5.2363	-12.2813	-7.0900	-16.1318	-10.5801	-11.8773	-5.2802	-12.2636
Std.Dev.	1.6136	3.1762	2.6132	4.1887	3.0010	2.1747	1.7996	2.6405
Skewness	-0.4353	-0.4976	0.1282	-0.4770	-0.2525	-0.5085	-0.1697	-0.8237
Kurtosis	3.2821	3.2654	3.3066	3.1970	3.3773	5.9116	3.0390	5.1292
Jarque-Bera	9.10**	11.54***	1.74	10.32***	4.32	103.44***	1.27	78.81***
Observations	261	261	261	261	261	261	261	261

Note: Table 3 provides descriptive statistics for the unlevered (or "total") **weekly** rates of return (precisely, changes in log prices multiplied by 100) on the S&P 500 equity and GSCI commodity indices, as well as on six investable GSCI sub-indices: Agriculture (Wheat, Red Wheat, Corn, Soybeans, Cotton, Sugar, Coffee, and Cocoa); Energy (WTI Crude Oil, Brent Crude Oil, RBOB Gas, Heating Oil, GasOil, and Natural Gas); Industrial Metals (Aluminium, Copper, Lead, Nickel, and Zinc); Precious Metals (Gold and Silver); Livestock (Live Cattle, Feeder Cattle, and Lean Hogs); and, Non-Energy. Tables 3B, 3C and 3D provide sample moments for three successive sub-periods: June 1, 1992 to May 31, 1997; June 1, 1997 to May 31, 2002; and, June 1, 2003 to May 26, 2008.

Table 4B: Weekly Rates-of-Return Correlations, 1992-1997

	S&P 500	GSCI	Agri-culture	Energy	Indust. Metals	Live-stock	Non-Energy	Precious Metals
S&P 500	1.0000							
GSCI	0.0950	1.0000						
Agriculture	-.0388	0.3359***	1.0000					
Energy	0.1117*	0.9423***	0.0644	1.0000				
Ind.Metals	0.0303	0.1254**	0.0896	0.0066	1.0000			
Livestock	0.0354	0.1925***	0.0914	0.0386	0.0358	1.0000		
NonEnergy	-.0047	0.4141***	0.8347***	0.0978	0.3524***	0.5122***	1.0000	
Prec.Metals	-.0949	0.0724	0.2066***	-.0140	0.2459***	0.0174	0.2645***	1.0000

Table 4C: Weekly Rates-of-Return Correlations, 1997-2003

	S&P 500	GSCI	Agri-culture	Energy	Indust. Metals	Live-stock	Non-Energy	Precious Metals
S&P 500	1.0000							
GSCI	0.0816	1.0000						
Agriculture	-0.0171	0.3430***	1.0000					
Energy	0.0810	0.9755***	0.1908***	1.0000				
Ind.Metals	0.1931***	0.2905***	0.1118**	0.2187***	1.0000			
Livestock	0.0526	0.1672***	0.1498***	0.0626	0.1295**	1.0000		
NonEnergy	0.0499	0.4274***	0.8705***	0.2492***	0.4169***	0.5170***	1.0000	
Prec.Metals	-0.0597	0.2027***	0.0826	0.1700***	0.1915***	0.0973*	0.2480***	1.0000

Table 4D: Cross-correlations of Weekly Index and Sub-index Returns, 2003-2008

	S&P 500	GSCI	Agri-culture	Energy	Indust. Metals	Live-stock	Non-Energy	Precious Metals
S&P 500	1.0000							
GSCI	-0.0553	1.0000						
Agriculture	-.0220	0.2494***	1.0000					
Energy	-.0787	0.9859***	0.1284**	1.0000				
Ind.Metals	0.1962***	0.2902***	0.1504**	0.1927***	1.0000			
Livestock	0.0200	0.1522**	-.0111	0.1184*	0.0191	1.0000		
NonEnergy	0.1012	0.3897***	0.7816***	0.2389***	0.6649***	.2407***	1.0000	
Prec.Metals	0.1733***	0.3460***	0.3405***	0.2560***	0.5092***	-.0264	0.6080***	1.0000

Notes: Table 4 provides simple cross-correlation tables for the unlevered **weekly** rates of return (i.e., changes in log prices) on eight investable indices: the S&P 500 equity and GSCI commodity indices, as well as six GSCI sub-indices: Agriculture (Wheat, Red Wheat, Corn, Soybeans, Cotton, Sugar, Coffee, and Cocoa); Energy (WTI Crude Oil, Brent Crude Oil, RBOB Gas, Heating Oil, GasOil, and Natural Gas); Industrial Metals (Aluminium, Copper, Lead, Nickel, and Zinc); Precious Metals (Gold and Silver);Livestock (Live Cattle, Feeder Cattle, and Lean Hogs); and, Non-Energy. Tables 4B, 4C and 4D provide sample moments for three successive sub-periods: June 1, 1992 to May 31, 1997; June 1, 1997 to May 31, 2002; and, June 1, 2003 to May 26, 2008. One, two or three stars indicate that an estimate is statistically significantly different from zero at the 10%, 5% or 1% level, respectively.