How to Look Clever and Have Envious Neighbors: Average Volatility Managed Leverage Timing

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ABSTRACT

While taking leverage to invest may be seen as risky, timing leverage can produce higher returns and timing by the right signal can produce significantly higher returns. Compared to a portfolio that manages investment in the market by the variance of daily market returns, a portfolio that manages by the average variance of the individual assets in the market portfolio produces utility gains by generating significantly higher average returns with significantly better Sharpe, Sortino, and Kappa ratios. The average variance managed portfolio is also cheaper and more practically implementable than the variance managed portfolio.

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While Warren Buffet may not be an enthusiastic supporter of taking leverage to invest[1], even at fixed levels of portfolio volatility leveraged investing can be used to increase returns. Moreira and Muir (2017) show across investment strategies and asset classes that merely managing leverage in a portfolio by that portfolio's volatility produces greater expected returns and performance ratios. As Rutterford and Sotiropoulos (2016) document, the discussion of the risk-return tradeoff is at least a centurty old and central to modern financial theory, however Moreira and Muir (2017) demonstrate that managing investment in the market portfolio by the inverse of the prior period volatility generates greater returns and better performance ratios. Unfortunately, the volatility managed portfolio requires extreme levels of leverage. I show that managing investment in the market portfolio by the inverse of the average variance of individual daily asset returns is not only a better performer, but cheaper and more practically implementable.

Since the early identification of a low-risk anomaly, the absence of a need to take on more risk for higher returns as in Haugen and Heins (1972), a large number of researchers have sought to identify a positive relationship between return variance and expected returns. Campbell (1987), French, Schwert, and Stambaugh (1987), Glosten, Jagannathan, and Runkle (1993) and many, many others have found insufficient evidence identifying a positive relationship between return variance and future returns. Some investors seek to exploit aspects of this disconnection through risk management and risk-parity funds. As of 2016, at least \$150 and as much as \$400 billion sat in these funds.(Steward, 2010; Cao, 2016) The Moreira and Muir (2017) results even hold for positions which already seek to exploit the low-risk anomaly like the "betting-against-beta" strategy of Frazzini and Pedersen (2014). Additionally, other researchers have applied similar variance or volatility management to specific assets or trading styles.[2] The generation of higher expected returns without a commensurate increase in portfolio volatility implies that the representative investor, holding the market, requires less equity premium in time of high volatility. In short, it seems risk does not equal reward. However, as with pigs, some volatility is "more equal" than other volatility.[3]

While the total variance of stock market returns is a function of both the variance of individual asset returns and the covariance between assets in the market, Pollet and Wilson (2010) show that the average pair-wise correlation of portfolio assets is the component of portfolio volatility most related to systemic changes in the economy and is the risk component compensated with higher returns. By managing the market portfolio by the other comoponent of total volatility, the average variance of the individual asset returns, I generate higher average annualized monthly returns and significantly better Sharpe, Sortino, and Kappa ratios. In addition to identifying a better strategy for investors, decoupling the average variance of individual returns from the that of the market portfolio sheds light on the risk-return trade-off dynamics of the market and allows leveraging investment into times when higher volatility will be compensated and pulling out when it will not.

Using daily market returns from the Center for Research in Securities Prices (CRSP), I extend Pollet and Wilson (2010) generating monthly time series of stock market variance, SV, average correlation, AC, and average variance, AV. From June 1962 to the end of 2016, encompassing the sample of Pollet and Wilson (2010), asset average variance is a significant in-sample predictor of higher daily market return variance, average asset variance, pairwise correlation, and lower log excess market returns at the monthly frequency. A one standard deviation increase in annualized average variance, from .77 to 1.62, is related to an increase in next months annualized market return variance of .545 of a standard deviation or a .22 increase. This makes next months expected market variance more than double the mean. AV remains a significant predictor or next month's SV even when this month's SV is included. A one standard deviation increase in AV also anticipates a .13 standard deviation, .58 percentage point, lower log excess return. This makes the following month's expected return negative. When both AV and SV are used to predict next month's return, AV is significant but SV is not. However, over the full, 1926 to 2016, sample average variance is a signficant predictor of higher daily market return variance, average asset variance, pairwise correlation, but not log excess market returns, as shown in table IV. This evidence supports the use of average variance as a leverage management signal. Scaling investment in the market by the inverse of average asset variance in the current month will pull funds out when the following month will have high market variance without sacrificing higher expected returns. It may, in fact, avoid negative returns. These results support the intuition from the work on volatility management in Moreira and Muir (2017) and portfolio average variance and correlation in Pollet and Wilson (2010) but in-sample regression use all available information and do not necessarily identify tradeable strategies.(Welch and Goyal, 2008)

Investors can only make decisions using the limited information available to them at a given time. For example in June of 2007 investors and investment models could only use historical information up to that month; the effects of November 2008 on the regression coefficients cannot not affect the predictions for July 2007. Moreira and Muir (2017) demonstrate that market volatility is an effective market portfolio management technique across the CRSP data set from 1926 to 2015. To motivate average variance as a better market portfolio leverage signal, I run expanding window out-of-sample regressions using AV on market volatility, average variance, average correlation, and log excess returns. From June 1926 to December 2016 and using the predictions from SV as a benchmark, AV is a significantly better predictor or next months AV, AC and SV. It generates better Diebold and Mariano (1995) test statistics, significantly lower mean squared forecast errors judging by the MSE-F statistic from McCracken (2007) and the emcompassing test of Harvey, Leybourne, and Newbold (1998) show that average variance contains all of the predictive information in market variance. As with the in-sample results, average variance serves investors at least as well as market variance and likely better in avoiding risk without giving up return. Out-of-sample testing always raises questions about model specification, recursive expansion versus rolling window parameter estimation, choices of the training period and prediction window and the influence of specific periods. Using the techniques in Rossi and Inoue (2012), the Diebold and Mariano (1995) and Harvey et al. (1998) measures can be calculated robust to concerns on window selection for either an expanding or rolling specification. The Rossi and Inoue (2012) robust statistics show that AV is a significantly better predictor than SV robust to the choice of window or regression specification.

Thus, I expect managing leverage in the market portfolio by AV will produce substantially better return performance as compared to management by SV.

As promised by the out-of-sample results, AV is substantially better than SV as a leverage management signal. Targeting the volatility of the buy and hold market portfolio return, as in Moreira and Muir (2017), an investor without borrowing constraints earns an annualized average monthly return of 9.7% from the average variance managed portfolio. This return is a statistically significant increase of more than 1% over the SV managed portfolio; the difference in annualized average monthly returns grows to more than 2% when practical leverage constraints are applied. With unconstrained borrowing, the AV managed portfolio has significantly better performance ratios like the symmetric Sharpe ratio, .52, and more asymmetric risk-return measures, e.g., Kappa 3 and Kappa 4 at .15 and .11 respectively. The advantage of managing with AV grows with risk aversion. The most risk-averse, $\gamma = 5$, constrained investor sees a certainty equivalent return (CER) gain of more than 2% annualized; this return represents a 27.4% increase in utility nearly as substantial as the utility gains seen in return timing strategies. (Campbell, Lo, MacKinlay, et al., 1997) Targeting the volatility of the buy and hold return requires seeing into the future and knowing the buy and hold return volatility. However, this look-ahead does not affect performance ratios like the Sharpe ratio, moreover the significantly better performance of AV is robust to other choices of target volatility.

The AV and SV portfolio management strategies each generate a weight in the market portfolio as a function of the daily returns of the market the prior month. Some of the investments demanded by the AV mangaged portfolio maybe difficult to achieve, however the investments needed to make the SV management strategy work are simply problematic. The SV managed portfolio takes far more extreme leverage far more often. In addition to higher average monthly borrowing costs, 15.107 vs 11.411 basis points, the SV managed portfolio has higher turnover. The AV managed portfolio generates lower transactions costs with a break even transaction cost more than 2.5 times higher than the SV managed portfolio. This makes the AV managed portfolio cheaper for the investor in both borrowing and transaction costs while generating higher returns. The AV managed portfolio is also cheaper to insure. Drawdowns are shallower and shorter for the AV managed portfiolio at 9% vs 11.2% and 10.5 vs 15 months on average. The drawdown profiles for the AV and SV managed portfolios also reveal that the AV managed portfolio benefits fund managers in addition to fund investors. The SV managed portfolio exposes a fund manager to twice the risk of a drawdown sever enough to threaten their job, or possibly shutter the fund, and would be nearly 91.7% more expensive to insure.

Using the decomposition of market variance, I identify a better portfolio leverage management signal. Weighting investment leverage by the inverse of the average of individual asset variance, AV, rather than overall return variance, is a new addition to the portfolio management literature letting investors capture better performance as measured by expected annualized returns, performance ratios, costs and utility gains. The returns to the AV managed portfolio serve as another example in the risk-return dynamics literature exposing the returns to correlation risk obscured when risk is measured by portfolio variance. These developments come from the formation and analysis of the AV signal. This work requires a few publically available datasets and a few considerations for the calculations at the monthly frequency. Most of the work is in the calculations required to show significant regression and portfolio performance.

I. Data

To calculate stock market variance, average asset variance, and average asset correlation, I use daily return data from CRSP and calculate the variance of daily returns monthly. To simplify the analysis of individual assets, I require that the asset be traded on each in the month which mitigates any liquidity effects and ensures consistent variance, covariance and correlation calculations. These conditions make the calculation of asset variance:

$$\sigma_{m,t}^2 = \frac{1}{T-1}\sum_{\tau=1}^{T}\left(R_{m,\tau} - \frac{\sum_{\tau=1}^{T} R_{m,\tau}}{T}\right)^2. \tag{1}$$

where $R_{m,\tau}$ is the daily return, including dividends, on an asset for day τ in month t. When the asset is the market portfolio, so $R_{m,\tau} = R_{s,\tau}$, the result is the variation of market returns, SV. The standard Pearsons correlation where the correlation of assets m and n for month t is:

$$\rho_{m,n,t} = \frac{\sum_{\tau=1}^{T}\left(R_{m,\tau} - \frac{\sum_{\tau=1}^{T} R_{m,\tau}}{t}\right)\left(R_{n,\tau} - \frac{\sum_{\tau=1}^{T} R_{n,\tau}}{t}\right)}{\sqrt{\left(R_{m,\tau} - \frac{\sum_{\tau=1}^{T} R_{m,\tau}}{t}\right)^2 \sum_{\tau=1}^{T}\left(R_{n,\tau} - \frac{\sum_{\tau=1}^{T} R_{n,\tau}}{t}\right)^2}}. \tag{2}$$

Unfortunately, for samples as small as the monthly series of daily returns Pearsons correlation is not an unbiased estimator of the true correlation, even if the returns are normal. Hotelling (1953) The average month in my sample has 22 trading days however the number commonly drops into the teens.[4] For samples of these sizes, the bias causes an underestimation of the correlation which is worse the lower the true correlation. I employ an approximate correction from Olkin and Pratt (1958) such that the monthly correlation between two assets m and n is:

$$\rho_{m,n,t} = \widehat{\rho_{m,n,t}}\left(1 + \frac{1 + \widehat{\rho^{m,n}}^2}{2(t-3)}\right) \tag{3}$$

where $\widehat{\rho_{m,n,t}}$ is the Pearson correlation between a and b.[5] Average variance and average correlation are value-weighted so each month I calculate market capitalization for all of the assets available in CRSP. The capitalization used in month t for asset m is the product of the end of month price (PRC) and common shares outstanding (SHROUT) values for asset m in month t-1.

$$MCAP_{m,t} = PRC_{m,t-1} \times SHROUT_{m,t-1} \tag{4}$$

To make the analysis more computationally trackable I use only, at most, the top 500 assets in CRSP by market capitalization for a given month.[6] Given this restriction, an assets market capitalization weight is defined by:

$$w_{m,t} = \frac{MCAP_{m,t}}{\sum_{j=1}^{J} MCAP_{n,t}} \tag{5}$$

with j $\leq$ 500. Thus, the other series of interest, market variance, SV, average variance, AV, and average correlation, AC, are defined by:

$$SV_t = \frac{1}{t-1}\sum_{\tau=1}^{T}\left(R_{s,\tau} - \frac{\sum_{\tau=1}^{T} R_{s,\tau}}{t}\right)^2 \tag{6}$$

$$AV_t = \sum_{m=1}^{M} w_{m,t}\sigma_{m,t}^2 \tag{7}$$

$$AC_t = \sum_{m=1}^{M}\sum_{m\neq n}^{N} w_{m,t}w_{n,t}\rho_{m,n,t} \tag{8}$$

Figure 1 shows the time series behavior of market and average variance, in percent, as well as average correlation. With the easily noticeable exception of October 1987, spikes in both market and average variance concentrate around NBER defined recessions.

[Place Figure 1 about here]

Table I shows the summary statistics for the calculated variables. Despite the use of the actual number of trading days, the restriction to assets that trade every trading day, and the adjustment to the calculation of correlation, the quarterly calculated values are almost identical to those in Pollet and Wilson (2010) over the same sample. Over the expanded the period, the annualized monthly average variance has a mean value of .88%. The annualized stock market variance mean is much lower at .25% monthly. Average correlation is relatively consistent at .23 quarterly in the Pollet and Wilson (2010) sample, .261 monthly in the same sample and .276 over the full time period. Average variance is more volatile than the stock market variance, more than twice as much. In each sample average variance has the highest autocorrelation. While average correlation is also persistent, the stock market variance is only strongly persistent at the monthly frequency with autocorrelation of .61. All three time series are stationary rejecting the unit root null in the tests of Dickey and Fuller (1979), Ng and Perron (2001), and Elliott, Rothenberg, and Stock (1996).

[Place Table I about here]

As my primary interest is in the use of average variance versus market variance in the management of leverage in the CRSP market portfolio, I test AV and SV against CRSP log excess returns. Specifically, I take the difference between the natural log of one plus the CRSP market return and the natural log of one plus the risk-free rate using:

$$r_t = \log R_{m,t} - \log R_{f,t} \tag{9}$$

where $R_{f,t}$ the risk-free return for which I use the 1-month treasury bill rate from Ken French's website[7]. As shown in table II, over the full data period, each variance and correlation time series are contemporaneously correlated to lower log excess returns. Average variance, AV, is significantly correlated with next month's market variance AV, SV, and AC. Surprisingly, over the full data set, this month's AV is even nominally more correlated with next month's SV than this month's SV is, 0.625 versus .612. Over the basis period, AV is time series most negatively correlated with next months log excess return at -.129, while it is entirely unrelated to next month's return over the whole data set.

[Place Table II about here]

As in the prior literature, July 1963 serves as the start of the basis data period for the regression analysis.[8] For in and out-of-sample tests, I regress market and average variance, and average correlation against these excess log return values. Out-of-sample regressions require an in-sample training period which, I set this at 15% of the available time series for consistent calculation of robust out of sample statistics later in the analysis. This training window means that out of sample regressions, analysis begins at the end of July 1970 in the basis sample and December 1939 in the full sample.

II. Regression Analysis

A. In Sample

The effectiveness of either market variance or average variance as an investment management signal will be driven primarily by their relationship with future risk and return. It is the trade-off which is key to the leverage management strategy. Assuming that investors hold a portfolio with whose risk-return ratio they are indifferent. When risk increases but expected returns do not, the risk-return ratio become more unattractive, and any risk-averse investor would like to decrease there position. Conversely, when the risk of the portfolio goes down without a decrease in return there is an opportunity to leverage into the position returning to the previous level of risk but now with a magnified return.

To get an understanding of the relationship between stock market or average variance and returns, I begin with in-sample regressions. In each of these regressions, all of the information available in the sample is used to estimate the parameters. In general, the regressions take this form:

$$y_{t+1} = \alpha + \beta x_t + \epsilon_t. \tag{10}$$

The cotemporaneous regressions decomposing market variance are left unreported. The results show the same relationships found in Pollet and Wilson (2010) table 2. The only difference of note is that the relationship between average correlation, AC, and next months log excess return is weaker monthly than quarterly if average variance is not also included. For all in-sample regressions, the series are standardized to a mean of zero and standard deviation of one.

[Place Table III about here]

Table III contains the results of regressions run on the basis data set which spans 1962 to 2016. Panel A shows that AV is a significant predictor of next months SV in all specifications. A one standard deviation increase in AV means a .545 standard deviation increase in next months market variance. This change represents an increase from the mean of .2 to .42 or an increase from an annualized standard deviation of .45% to .65%. There does not appear to be any real advantage to using this months SV to predict next months market variance over AV. The coefficient values are nearly identical, .551 versus .545, as are the adjusted R^2 values, 30.3% versus 29.6%. This months AV even remains significant in the specification including this months SV. Holding this months SV constant, a one standard deviation increase in AV still signals a .257 standard deviation increase in next months SV. Again, the inclusion of SV appears to be of little to no help as the adjusted R^2 only increases from 29.6% to 31.5%. Panels B shows predictive regressions of next months average variance. Here, unlike the prior relationship, there is a definite advantage to using this months AV in the prediction of next months average variance. The adjusted R^2 of this month's AV, 44.5%, is nearly twice this month's SV, 27.2%, and the inclusion of SV with AV does not appear to be a significant improvement.

Pollet and Wilson (2010) argue that average correlation is a better measure of systemic market risk. Increases in average correlation are related to a higher covariance between labor income growth and the stock market, and the stock market and bond returns. Also, they show that in the context of Rolls Critique, average correlation should be a better indicator of risk for the true market portfolio rather than stock market variance. (Roll, 1977) Thus, the results in panel C of table III make the argument that leverage management using AV is a better idea. While both AV and SV are positively related to next months AC, SV is more highly related. The adjusted R^2 value for the regression of next months AC on current SV is more than twice that of current AV. Additionally, each signal is related to higher levels of both AV and SV next month. Hence, the use of SV as a leverage signal means taking on less weight in the market portfolio when next months systemic risk is higher relative to idiosyncratic risk. Theoretically, this is avoiding times of higher compensated versus uncompensated risk. Panel D presents the results of directly regressing next months log excess return on the market variance, average variance, and average correlation series. Indeed, AV is a significant predictor or lower returns in the next month and a better predictor than SV. A one standard deviation increase in this months AV means a .13 standard deviation, .58 percentage points, lower log excess return next month. So, a one standard deviation increase in AV means an expected negative return next month. Average variance remains a significant predictor of lower returns next month even when SV is included while the coefficient on SV is insignificant.

[Place Table IV about here]

Table IV presents the results for the in-sample regressions across the whole CRSP data set, from 1926 to 2016. Across the whole data set, AV is an even better predictor of next months SV than current SV. The panel A results here are even better than in table III. The results in panel B show the same. The prior results are not only supported but appear better with AV being the best predictor of next months AV and SV providing no help and becoming insignificant with both variables are included. As before, across the whole data set stock market variance is more highly related to next months average correlation than is average variance. Panel C in table IV supports panel C in table III. Panel D is the first place the results appear different in a meaningful way. There is no relationship between AV and next months return in panel D of table IV. The coefficient on AV is insignificantly positive with an adjusted R^2 of -0.1%. Over the full sample, SV is a better predictor of next month's log excess return. A one standard deviation increase in this month's stock market variance indicates a .056 standard deviation, .3 percentage points, lower return. However, with an adjusted R^2 of .2%, there are better return predictors. While it is no longer obvious AV is better than SV at both risk and return anticipation across the whole data set, the results still suggest that investors are likely to be better off using AV than SV as it is a better risk anticipation measure and at least unrelated to future returns. AV should capture more returns for the same level of risk, if not avoid negative returns.

A.1. Robustness

Average variance is an autocorrelated time series; this opens the possibility that predictive regressions using AV have estimation bias as highlighted in Stambaugh (1999). Campbell and Hentschel (1992) show that the Stambaugh bias in predictive regressions involving volatility measures and future returns can be particularly severe because of a volatility feedback effect. To eliminate the Stambaugh bias in the estimated coefficients on AV in the regressions above, I follow the methodology in Amihud and Hurvich (2004) and furthur make the p-values used for coefficient significance robust through wild-bootstraping as detailed in MacKinnon (2002). Table V shows that the relationships demonstrated above are unaffected by robust bias correction. Average variance is a predictor of higher average correlation and higher stock market variance across data sets. While AV is a significant predictor of lower returns in the 1962 forward period, it is unrelated to the next months log excess returns in the whole data set. So long as the relationships hold with the limited information that investors have available at the time they make investment decisions, AV is likely to be a better leverage management signal than SV.

[Place Table V about here]

B. Out of Sample

Average variance is a good in-sample signal, however the out-of-sample performance remains in doubt. As Welch and Goyal (2008) definitively show, out-of-sample performance is not guaranteed by in-sample performance and is essential to any investment strategy which hopes to generate positive returns. To determine the out-of-sample relationships between market and average variance, average correlation and returns, I run regressions of the standard form

$$y_{t+1} = \alpha_t + \beta_t x_t + \epsilon_t \tag{11}$$

where α_t and β_t are estimated with from the data available only until time t. That is, I estimate α_t and β_t by regressing $\{y_{s+1}\}_{s=1}^{t-1}$ on a constant and $\{x_s\}_{s=1}^{t-1}$. In all the reported results, I follow an expanding window approach so that for the next period $t+2$, y_{t+2} is estimated as $\alpha_{t+1} + \beta_{t+1}x_{t+1}$, where α_{t+1} and β_{t+1} by regressing $\{y_{s+1}\}_{s=1}^{t}$ on a constant and $\{x_s\}_{s=1}^{t}$. I follow this process for all subsequent months. However, as part of a test on the robustness of the out-of-sample results, I demonstrate that the results do not depend on the use of an expanding window. Most critically, equation (11) prevents any look-ahead bias. The out-of-sample prediction tests use the same set of variables as the in sample tests. Each out-of-sample test requires an in-sample training period in which parameters are estimated using all the data up to the time period before the first out-of-sample quarter or month.

For consistency, the first one-fourth of the data is used as the initial parameter estimation period with the remaining three-fourths of observations moved through recursively generating out-of-sample predictions. Three measures of out-of-sample performance are estimated. I use the Diebold and Mariano (1995) statistic and McCracken (2007) MSE-F as measures of the increased accuracy of AV based forecasts compared to forecasts from SV as a benchmark. The DM statistic is defined as:

$$DM = \frac{\bar{d}}{\sqrt{\frac{2\pi f_d(0)}{T}}} \tag{12}$$

where $\bar{d}$ is the mean difference in the loss differential. The loss differential is the function used to measure the difference between the forecasted and actual values. I use the squared forecast error, $(y_t - \hat{y}_t)^2$. So, $\bar{d}$ is the mean value of the difference between the squared error using AV and the squared error using the benchmark forecast from SV.

$$\bar{d} = \frac{1}{T}\sum_{T}^{\tau=1}((y_t - \hat{y}_{AV,t})^2 - (y_t - \hat{y}_{SV,t})^2) \tag{13}$$

I use the same consistent estimator for the mean loss differential, $f_d(0)$ as in Diebold and Mariano (1995). The statistic is normally distributed under the null hypothesis of no difference in accuracy between the benchmark and proposed model. The standard postive critial values from the normal Z-table serve as cutoffs to establish a significant improvement in forecast accuracy. $MSFE_{SV}$ is mean squared forecast error when a benchmark model is used to generate out-of-sample predictions. Mean squared forecast error is defined as

$$MSFE_x = \frac{1}{T}\sum_{\tau=t}^{T}(y_\tau - \hat{y}_\tau^x)^2 \tag{14}$$

where $\hat{y}_t^x$ is the out-of-sample prediction of y_t generated from the a model using variable x, t is the first out-of-sample prediction time period, and T is the total number of out-of-sample time periods. The F-statistic in McCracken (2007) is calculated by

$$MSE - F = T\frac{MSFE_x - MSFE_b}{MSFE_b}. \tag{15}$$

The significance of the F-statistic is determined from bootstrapped values provided in McCracken (2007). Each of these two tests depends on the reduction of average squared error by the predictor x relative to a benchmark model. The final measure is a forecast encompassing statistic.

Encompassing tests the more stringent requirement that the benchmark forecasts contain no useful information absent in the forecasts of variable x. Forecast encompassing tests come from the literature on optimal forecast combination. (Chong and Hendry, 1986; Fair and Shiller, 1990) An optimal forecast as a convex combination of two forecasts for time period t +1 defined as

$$\hat{y}_t^* = (1-\lambda)\hat{y}_t^b + \lambda\hat{y}_t^x \tag{16}$$

where $\hat{y}_t^x$ are predicted values generated from the model using variable x and $\hat{y}_t^b$ are forecasts from the benchmark model. I use the forecast encompassing test of Harvey et al. (1998), ENC-HLN. The encompassing test of Harvey et al. (1998) directly tests the value and significance of the forecast combination λ. The test procedure rests on the calculation of a modification to the Diebold and Mariano (1995) test statistic and the consistent estimation of the long-run covariance between the difference in forecast error between the benchmark model and a model based on a competing variable, x. As such there is no one line equation that sums up the statistic used to judge the significance of λ. However, intuitively λ must be significantly different from zero for AV to have information above and beyond the forecasting information in SV and values close to one indicate that AV has all of the relevant information in SV and is optimal by itself.

[Place Table VI about here]

The results in table VI show that AV is a significantly better out-of-sample predictor of AV and SV. For both the variables, all three measures of out-of-sample performance show significant imporvement in both the out-of-sample period starting in 1970 and starting in 1939. The forecast encompassing tests also show that AV contains all the forecasting information in SV and is optimal on its own. This means that investors concerned about the variance in the returns on their investment in the market are better off using this month's level of average asset variance to hedge next month's stock market variance than using this month's stock market variance. AV is also a significantly better predictor of next month's log excess return for the out-of-sample period starting in 1970. The DM statistic, 1.278, is nearly significant at the 10% level and both the MSE-F test and encompassing tests show significant imporvement for AV. Again the encompassing test shows that AV is optimal alone and no weight needs to be given to SV in the prediction of log excess returns. Over the period starting July 1970, investors attempting to predict next month's returns would have been better off using this month's average asset variance rather than total market variance and investors deleveraging based on high values of AV would have done better avoiding negative returns rather than deleveraging based on high values of SV. Supporting the results seen in the in-smaple tests, across the longer out-of-sample period, starting in 1939, AV is not a significant improvement over SV in the prediction of next month's log excess return. It will be important to look at the asset allocation performance of AV versus SV over the entire CRSP sample to see if portfolio management by AV does generate significantly higher returns than management by SV.

B.1. Robustness

Out of sample estimation always raises issues with the choices made in the specification of the model and how to split the data into in and out of sample windows. Bluntly speaking, there are no good answers. The standard practice as in Rapach and Zhou (2013), Rapach, Strauss, and Zhou (2010), Rapach, Ringgenberg, and Zhou (2016), and Huang, Jiang, Tu, and Zhou (2015), and many others, is to show performance in a few subsamples split by dates that the authors choose for unknown reasons. One of the concerns with subsample selection is that the window may be "ad-hoc" and the selection may mask significant results that would appear if the subsamples had been constructed differently. A second, more cynical, concern is that the presented subsample may represent significant performance that has been found either by chance or as the result of analyzing many subsample and only presenting the significant results. In any case, evaluation of the differences in performance across subsamples is often left to the imagination of the reader and whatever importance they place on the first half of the sample versus the second, the middle third versus the first and last thirds or however the data has been separated. While the selection of 1962 is not arbitary as the daily return data is of much higher quality after, we have seen already a difference in return prediction performance for AV between the period after 1962 and the whole data set starting in 1926 which raises the question of the robustness of the out-of-sample results.

To address the robustness of the out of sample results and avoid the use of subsampling completely, I present out-of-sample statistics robust to both the specification of the prediction model, either expanding or rolling, and the choice of prediction window. Rossi and Inoue (2012) presents out of sample statistics robust to the choice of split between in and out of sample periods. The paper presents the calculation of the Diebold and Mariano (1995) statistic and the Harvey et al. (1998) encompassing test such that the choice of out-of-sample starting period is eliminated as a nuisance parameter and the asymptotic behavior of the statistics can be used to measure their observed significance. Fundamentally, this involves the calculation of each of the statistics for all feasible out-of-sample windows and in the case of rolling regression specification all feasible window sizes. The modifications are different for each of the statistics and the calculation of the robust statistic is different depending both on which statistic and which concern is being addressed. When the conern is that the choosen window could be unrepresentatively optimal, perhaps the best results of many tests, then it is possible that the null of no imporovement is rejected based on the calculated statistics when in general it is true. To eliminate this possibility, Rossi and Inoue (2012) provide the R_T measure which essentially insures that the highest calculated statistics are so extreme that they could not occur without a underlying significant improvement in forecast accuracy from the benchmark to the proposed model. The A_T measure insures that the average calculated statistics are large enough that an arbitrarily selected out-of-sample starting period would not lead to the failure to reject the null of no accuracy improvement when it was indeed false. This two measures tackle the type I and II error questions. Given the results thus far, we will be looking for significant R_T values to support the significant ability of AV to predict stock market and average variance across the data set, and significant A_T values to tell us that the significant ability of AV to predict log excess returns, seen from 1970 forward, is indicative of a real accuracy improvement while the lack of performance when including predictions from 1939 forward is simply a noisy period of poor performance obfuscating the superiority of the AV based model.

[Place Table VII about here]

Table VII shows the robust out-of-sample statistics. Panel A has the R_T and A_T statistics for the comparision of all possible expanding window forecasting models using AV with in-sample training windows of at least 15% of the data and out-of-sample forecasting periods of at least 15% of the data against a benchmark model using SV with the same specifictions. The porportional data cut offs are necessary to use the critical values provided in the Rossi and Inoue (2012) paper, 15% is the smallest, and mean that the first feasible specification starts forecasting in December 1939 as in the out-of-sample results shown above and forecasts for at least April 2003 to December 2016 are made. Every DM statistic is significant indicating that the AV model is a significant improvment in forecasting accuracy for all variable of interest. Every encompassing test statistic is significant, however these are not λ values directly so while we know that AV contains information over and above SV they do not directly indicate it is optimal alone. Panel B has the results for comparisons of rolling window specification tests of AV and SV models, again using the 15% proportional cut-offs. While all DM statistics are significant, not all encompassing tests are. There are too many rolling window specifications in which SV contains significant forecasting information which is not captured by AV. A forecast combining the values from rolling window models using AV and SV will almost surely be better than using either rolling window forecast alone while using the expanding window AV model forecast alone may very well be better compared to any combination of that prediction with an expanding window SV forecast.

III. Asset Allocation

Of course, the most direct and practically relevant measure of AV as a portfolio leverage management tool is whether or not it generates portfolio gains. And while superior out-of-sample predictability usually translates into better portfolio performance, here timing leverage to risk, it is important to make adjustments for the riskiness of the managed portfolio and compare performance across dimensions for a full investment picture. The AV managed portfolio may well generate higher annualized returns, but will it have a better Sharpe ratio than the SV managed portfolio?

To measure portfolio performance, in addition to annualized monthly log excess return, I will calculate each portfolios Sharpe ratio, Sortino ratio, and two Kappa ratios. The classic Sharpe ratio is a symmetric measure of risk and is defined as the ratio of the expected excess portfolio return over the standard deviation of portfolio returns.

$$\frac{\mathbb{E}[r_x]}{\sigma(r_x)} \tag{17}$$

While Sharpe measures each dollar of expected return for dollar of risk, the Sortino ratio attempts to more directly measure the risk most investors worry about. By using only downside deviation in the denominator, the Sortino quantifies each dollar of expected return for each dollar of loss. This downside is measured relative to a target return. (Sortino and Price, 1994) As the log returns are already excess of the risk free rate, I set the Sortino target to 0 which makes the Sortino formula:

$$\frac{\mathbb{E}[r_x - 0]}{\sqrt{\int_{-\infty}^{0}(0 - r_x)^2 f(r_x)\, dr}} \tag{18}$$

The Sortino is just a specific instance of a more general risk measurement ratio formula. The Kappa ratio keeps the expected return relative to a target in the numerator but allows any lower partial moment in the denominator. (Kaplan and Knowles, 2004) With the target again set to 0 the general formula is of the form:

$$\frac{\mathbb{E}[r_x - 0]}{\sqrt[n]{LPM_n}} \tag{19}$$

The Sortino ratio is the Kappa_2 ratio. I calculate Kappa_3 and Kappa_4 also to see relative performance of AV and SV management adjusted for negative return skew and kurtosis. All of the measures are annualized, e.g., log returns are multipled by 12 and raitos, like Sharpe, are multiplied by the square root of 12.

Measuring a difference in each of these measures for a pair of portfolios is not difficult; measuring a significant difference is. When evaluating the difference in Sharpe ratios, the method in Memmel (2003) seems to be popular. However, when returns are not normally distributed or autocorrelated this method is not valid. AV and SV managed returns, like market returns, are weakly autocorrelated, slightly skewed, and have much fatter tails when compared to normally distributed returns. Moreover, current period returns to either the AV or SV managed portfolio depend on the prior period variance of the market return strongly questioning the i.i.d assumption made in most hyptothesis testing methodologies. Studentized time series bootstrap sampling preserves the time series properties of the AV and SV managed returns which is critical; for example, Scherer (2004) demonstrates that methods which loose the time dependence in the calculation of differences in Sortino ratios fail to properly estimate the sampling distribution and critical values. Time seires bootstrap methods preserve the structure from the original data and allow for efficient robust hypothesis testing. (Politis and Romano, 1994; Davison and Hinkley, 1997) Ledoit and Wolf (2008) show that this method is even more efficient than using Newey and West (1987) or Andrews and Monahan (1992) heteroskedasticity and autocorrelation corrected standard errors for testing the significance of differences between two portfolio Sharpe ratios. I follow the p-value estimation method in Ledoit and Wolf (2008) to determine the significance of the difference between the portfolio performance ratio measures of the AV and SV mangement strategies. This uses circular block bootstrapping of the return time series, robust centered studentized statistics computed from the bootstrap samples and is proven to be the most efficient hypothesis testing method. Politis and Romano (1992); Ledoit and Wolf (2008)

As in Moreira and Muir (2017), investment weight in the market portfolio is a function of the variance of daily market returns, SV, or the average daily return asset variance, AV, scaled by a constant, c. Moreira and Muir (2017) use a constant that scales the variance of the volatility managed portfolio equal to the buy and hold market portfolio. In the basic portfolio weighting specification, I use the same approach so that the returns of both the SV and AV managed portfolios have the same variance as the buy and hold strategy. This constant is denoted c_{053} and it takes different values for SV and AV. This scaling requires knowing the full sample buy and hold return variance. While this does not distortions in the performance ratios, to insure robustness two other specifications for the scaling targets are used. Annual volatilities of 12% and 10% are common in academic literature and fund management so c_{035} and c_{029} approximately target those levels. Barroso and Santa-Clara (2015); Morrison and Tadrowski (2013); Verma (2018); Fleming, Kirby, and Ostdiek (2002); Hocquard, Ng, and Papageorgiou (2013) Using each of these constant an investor reblances at the end of month t investing in the market portfolio with weight:

$$w_{x,t} = \frac{c}{x(t)} \tag{20}$$

where $x(t)$ is either SV_t or AV_t and hold for month t + 1.

[Place Table VIII about here]

Table VIII shows summary statistics for the resulting investment weights for AV and SV for the three volatility targets. When targeting the volatility of the market portfolio, both portfolios are leveraged into the market on averge with investment weights of 1.3 indicating 30% leverage. Regardless of the volatility target the SV managed portfolio calls for extreme levels of leverage. Figure 2 shows that the SV strategy targeting the buy and hold volatility calls for investment weights above the maximum AV weight in several peiords. More than 500% leverage is needed at the end of the 1920s, throughout the 1960s, and in the 1990s. Given that these levels of leverage are unrealistic for most investors, it will be important to see if there is a difference in performance for the AV and SV strategies under real-world investment constraints and to investigate the associated costs generated by the trading needed for the SV managed portfolio.

[Place Figure 2 about here]

A. Portfolio Performance

Before exaiming the constrained portfolio performance, I present the results for SV and AV strategies targeting the buy and hold volatility without investment constraints in table IX. As in Moreira and Muir (2017) the portfolio perforamnce is measured across the whole CRSP data set, however the relative performance is the same or better across the basis data set.

[Place Table IX about here]

Table IX presents the performance ratios for the SV and AV managed portfolios targeting the buy and hold volatility without investment constraints. The buy and hold market strategy is included for reference. However since Moreira and Muir (2017) establish that the SV managed portfolio out performs the buy and hold, statistical significance results are only presented for the comparison of the SV and AV managed portfolios. The AV managed portfolio generates a statistically significant 1.08 percentage points higher average annualized log excess return. As shown in the bottom panel of figure 3 the AV strategy builds its performance advantage slowly but consistently starting from the early 1950s and from that time the SV managed portfolio is never a better investment. As both strategies are targeting the same volatility, the significant difference in return translates into a significant difference in Sharpe ratio. At .520 versus .462, for the SV managed portfolio, the AV managed Sharpe ratio is 12.6% higher. AV also generates significantly higher Kappa_3 and Kappa_4 ratios. So while the overall payment for downside risk, measured by Sortino ratio, may not be significantly higher, the payment for downside skewness and extreme downside return is.

[Place Figure 3 about here]

Like out-of-sample statistics, portfolio performance numbers always bring forward questions on performance in subsamples. While divisions of the sample by date are largely arbitrary and do not automatically convey the importance of the subsample, divisions along business cycles call out specific periods of investor sensitivity. Panels (b) and (c) in table IX present the performance of the buy and hold, SV and AV managed portfolios. The AV managed portfolio is a significantly better performer across all measures. In business cycle expansion, the AV managed portfolio provides signficantly more compensation and significantly more compensation for every measure of risk.

The results for NBER constractions are as stark in the other direction. The SV managed portfolio appears to be so much better that it might be a more desirable option given that we cannot know periods of extended contractions before they begin and investors may desire a portfolio that protects value through downturns more than one that maximizes returns during market upswings. However, as panel (d) shows, the better performance of the SV managed portfolio is due to one, albeit a rather important, data point. The significantly better performance of the SV managed portfolio through NBER contractions depends enitirely on the 1929 to 1933 Great Depression. Excluding that time period, The SV managed portfolio still provides higher returns overall but there is no significant difference in performance by any portfolio risk measure. Additionally, investors that are concerned about the loss of portfolio value, regardless of when it occurs, will value the AV managed portfolio more than the SV managed portfolio.

Drawdowns, the peak-to-trough decline in the value of a portfolio, may be the most natural measure of real market risk. (Magdon-Ismail and Atiya, 2006) Maximum drawdown, the largest peak-to-trough decline in portfolio value, in particular is often used in place of return variance as a portfolio risk measure. (Johansen and Sornette, 2000; Vecer, 2006, 2007; Sornette, 2003) Drawdowns play a significant role in the lives of fund managers as deep losses not only rob the fund of capital but motivate investors to withdraw funds making drawdowns a significant determinant of fund survival. (Baba and Goko, 2009; Papaioannou, Park, Pihlman, and Hoorn, 2013; Lang and Prestbo, 2006) To compare SV and AV managed portfolios against the market buy and hold, I consider drawdowns longer than one month so two consecutive months of negative returns will start a drawdown. The drawdown continues until the portfolio regains the value it had at the beginning of the first month of the drawdown.

[Place Table XI about here]

Table XI presents the drawdown statistics for the buy and hold, SV managed, and AV managed portfolios. The AV managed portfolio has more discrete drawdown events, 87, than either the buy and hold or SV managed portfolio. However, the drawdowns are much less severe. The buy and hold strategy losses a maximum of 84.8% of its value at the deepest point of its maximum drawdown, Max DD. AV and SV lose only 60.3% and 63.6% at the bottom of their worst drawdowns. The SV managed portfolio has the worst average loss during a drawdown, Avg DD, at 11.2% of the portfolio value while AV's average loss is only 9%. SV also stays "underwater" the longest both on average, 15 months, and during its longest drawdown, 246 months. From figure 4, the deepest losses for AV and the buy and hold occur during and after the Great Depression. However, the deepest and longest sustained losses of value for the SV managed portfolio start in the 1960s and SV does not recover until 1989. The notion that there is a drawdown so severe that it causes the collapse of the fund, or at least a management turnover, is known as the "knockout" drawdown. (Pav, 2016) Given a knockout drawdown value, it is possible to estimate the likelihood of the knockout occuring, the fund or manager not surviving, by fitting a binomial distribution to the drawdown observations using the knockout drawdown level as a cutoff to create binary values indicating a drawdown exceeding that level, 1, or not, 0. (Pav, 2016) Setting the knockout drawdown at 45%, a loss of nearly half the current value, in any given month the SV managed and AV managed portfolios have probabilities of 1.06% and .55% of incurring a knockout drawdown in the next 12 months. The AV managed portfolio is far less risky in these terms as the SV managed portfolio would be nearly twice as likely to fail and 91.7% more expensive, in theory, to insure using the max drawdown insurance of Carr, Zhang, and Hadjiliadis (2011).[9]

[Place Figure 4 about here]

B. Leverage

More practical analysis of portfolio performance requires the incorporation of limits on the level of investment taken in the market portfolio. Leverage of 50%, a coefficient of 1.5 on the market, is a common constraint meant to mimic real market leverage constraints for the average investor based in part on the Reg. T margin requirement[10]. (Campbell and Thompson, 2008; Rapach et al., 2010; Rapach and Zhou, 2013; Huang et al., 2015; Rapach et al., 2016; Moreira and Muir, 2017; Deuskar, Kumar, and Poland, 2017) There are at least two exchange traded funds, ETFs, which three times the return of the SP500.[11] So, I take a market coefficient of three as the maximum feasible investment a typical investor can make in the market portfolio.

[Place Table X about here]

Table X panel (c) presents the results from applying investment constraints after calculating the

weights for AV and SV targeting the buy and hold volatility. While both portfolios still outperform the buy and hold, The seperations in average annualized excess return, 1.71% and 2.07%, are even greater when investment constraints are applied. Panels (a) and (b) in figure 3 show the effects of the growing separation. While SV is barely able to clear the buy and hold strategy under typical brokerage constraints, returns to the AV managed portfolio remain clearly above. Investors that use a leveraged SP500 ETF to impliment the AV managed portfolio strategy are rewarded with returns significantly higher than the SV managed portfolio suggested weights. The brokerage investment restrictions pull the volatility of the AV managed portfolio too far from the SV returns to generate significant differences in performance ratios. However, investors using the leveraged ETFs are rewarded not only with higher returns but significantly better performance ratios across the board with the exception of the Rachev ratio which is at least no longer signficantly better for the SV managed portfolio. The ETF leverage constrained AV strategy even generates better Sharpe and Sortino ratios than the unconstrained strategy. The results in Panels (a) and (b) demonstrate that better performance of AV is not a result of or contingent on looking to the volatility of the buy and hold strategy. As the targeted volatility is lowered the difference in performance between AV and SV becomes more significant.

The differences in investment weight profiles show in table VIII not only generate differences in returns but also in costs. As show in table XII the AV managed portfolio generates less than half the turnover of the SV managed portfolio. The average monthly absolute change in investment weight is .752 for the SV managed portfolio and only .317 for AV. Table XII also shows the Fama-French five factor and Fama-French five factor with momentum annualized alphas for the SV and AV managed portfolios. AV management results in higher annualized alphas, 3.036% and 3.152% vs 2.833% and 2.769%, than SV. To capture the effect of these difference in annualized alphas and turnover, I calculate the transaction costs needed to zero the strategy alphas so the portfolio only breaks even. (Frazzini, Israel, and Moskowitz, 2015; Moreira and Muir, 2017) Seen in table XII, the break even transaction costs are more than 2.5 time higher for the AV managed portfolio. The SV managed portfolio breaks even at 31.436 and 30.725 basis points while it takes costs of 79.992 and 83.061 basis points to zero out the AV managed annualized alphas. However, transaction costs are only the only expense incurred by the leveraged portfolios. To estimate the borrowing costs for each strategy, I assume that any month a strategy requires a position greater than one in the market the difference between the investment weight and one is borrowed. The average monthly cost of borrowing to invest for the AV managed strategy is nearly 25% lower than for the SV managed portfolio. Using the broker call money lending rates available in Bloomberg from September 1988 to October 2016, SV incurs an average monthly cost of 15.107 basis points while AV costs only 11.411 basis points. This ignores the possibility of the investor using saved gains rather than borrowing and the necessity of borrowing when the investor has lost money, however, as the AV managed portfolio has greater returns and better drawdown statistics included gains and losses would only further the seperation.

[Place Table XII about here]

C. Investor Utility

One part still missing from the analysis of the difference in performance between AV and SV is a measure of the impact on different investors. Investors with different risk aversion will experience different utility effects to the constrained returns of AV and SV[12]. I consider a mean-variance investor as in Kandel and Stambaugh (1996), Campbell and Thompson (2008), Ferreira and Santa-Clara (2011), Rapach et al. (2016), and others. To measure changes to investor utility from switching between the SV and AV portfolios and due to leverage constraints, I use the difference in certainty equivalent return, CER. The CER change will measure the change in riskless return an investor demands, given their risk aversion, to use a given portfolio versus another given investment conditions. This can equivalently be though of as a measure of the management fee an investor, given a specific risk aversion, would be willing to pay to have access to an investment fund. CER change is calculated as the difference in risk adjusted return using:

$$\Delta \text{ CER} = \left(\hat{\mu}_{r_x} - \frac{1}{2}\gamma\hat{\sigma}^2_{r_x}\right) - \left(\hat{\mu}_{r_y} - \frac{1}{2}\gamma\hat{\sigma}^2_{r_y}\right) \tag{21}$$

where $\hat{\mu}_{r_x}$, $\hat{\mu}_{r_y}$, $\hat{\sigma}^2_{r_x}$, and $\hat{\sigma}^2_{r_y}$ are the means and variances of the returns to the x and y portfolios and γ is the investor risk aversion coefficient. I multiply the gains by 12 to annualize them. All investor risk aversion coeffiecients from 1 to 5 are tested for investors subject to investment constraints from a limit of 1 to 3 on the market portfolio, no leverage to 200% leverage.

[Place Figure 5 about here]

As shown in figure 5, CER losses due to leverage constraints accumulate for the SV managed portfolio sooner than for the AV managed portfolio. This is due to the extreme leverage positions needed for the SV managed portfolio. Across risk aversion coefficients, leverage constraints bite sooner and cut deeper into the SV managed portfolio until both are driven together when no leverage is allowed. The CER of the AV starts higher at 7.95%, 8.98%, and 9.33% vs 6.87%, 7.90%, and 8.25% for investors with risk aversion coefficients of 1, 3, and 5. When those investors are subject to 200% leverage constraint the CERs are 7.96%, 8.99%, 9.33%, versus 6.22%, 7.05%, and 7.33%. The AV manged portfolio provides the same investor utility while the benefit from the SV managed portfolio decreases by 9.46%, 10.76%, and 11.15%. To incorporate the starting difference in utility I look at the gains to moving from the SV to AV managed portfolio.

[Place Figure 6 about here]

As shown in figure 6, CER gains for the market variance targeting AV managed portfolio are increasing in both risk aversion and leverage use for constrained risk averse mean-variance investors. An investor with a risk aversion coefficient of 2 would capture an annualized CER gain of 1.49% using 50% leverage and 1.91% implementing the AV strategy through the 200% leveraged ETFs. The most risk averse investors subject to a 20% leverage limit see a CER gain of 1.35% while the most risk tolerant see only 1.08%. The most risk averse investor, using the highest feasible leverage, realizes a CER gain of over 2% which translates to a utility increase of 27.4%. This increase is in the neighborhood of those typically seen from return timing strategies. (Campbell et al., 1997; Moreira and Muir, 2017) Risk averse, mean-variance investors see substaincial utility gains switching from the SV to AV managed portfolio and these gains increase with leverage usage.

The return on the market portfolio is the return on the value-weighted representative average portfolio held by all investors. However, any investor subject to investment constraints or not, risk averse or tolerant benefits from switching from the SV managed portfolio to the AV managed portfolio. They benefit from switching from the market buy and hold to the AV managed portfolio even more. For the same level of variance risk they get far more return with better risk adjusted properties. How is it that this holds over nine decades of market history? Why is the average investor accepting the lessor return of the market portfolio?

IV. Conclusion

I analyze the relationship between AV, the average variance of individual asset returns, and future risk and future returns. AV is a significant predictor of future portfolio variance, controlling for current variance, both in and out of sample. In contrast, AV is not significantly related to future returns and thus serves as a leverage timing signal. By using the decomposition of market variance, I add a better portfolio leverage management signal to the literature. Weighting investment by the inverse of the average asset variance, AV, rather than SV, increases market investment when total variance is expected to be low and decreases when higher total variance and lower returns are expected. The results are better Sharpe, Sortino and Kappa ratios with better Fama-French five factor and five factor plus momentum alphas. With better access to leverage timing on compensated risk, investors capture more utility with lower costs. AV portfolio investors, as well as fund managers, are more protected agains drawdowns. These investment benefits manifest because the AV managed portfolio takes advantage of the correlation risk and return dynamics rather than the portfolio variance and return dynamics. Hence, the AV managed portfolio adds another dimension to the risk-return literature.

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Notes

[1]"When leverage works, it magnifies your gains. Your spouse thinks you're clever, and your neighbors get envious,...but leverage is addictive. Once having profited from its wonders, very few people retreat to more conservative practices. And as we all learned in third grade and some relearned in 2008 any series of positive numbers, however impressive the numbers may be, evaporates when multiplied by a single zero. History tells us that leverage all too often produces zeroes, even when it is employed by very smart people." (McWhinnie, 2014)

[2]See, for example, Barroso and Santa-Clara (2015) and Kim, Tse, and Wald (2016) for discussions of volatility managment of the momentum portfolio.

[3](Orwell, 1946)

[4]The shortest trading month in the sample is September 2001 with 15 trading days while 17 is a common number in the months with holidays.

[5]The exact correction suggested in Olkin and Pratt (1958) is too computationally taxing for the equiptment to which I have access.

[6]The least number of assets which trade every day in a given month is 392 in August of 1932. There are regularly 500 qualifying assets by the end of the 1930s.

[7]http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html#Research

[8]CRSP has, as of 2005, backfilled NYSE daily returns to 1926, however the pre-1962 data is very different from the post-1962 data. The earlier data is much shallower having months with fewer than 400 assets total that meet the data requirements. Twice as many firms covering twice as many industries are available at the end of 1962 as compared to the end of 1961. And, as documented in Jones (2002) the pre-1962 period is significantly and persistantly more illiquid.

[9]Calculation of actual insruance costs require prices on the zero coupon bond, however given this common price the digital call option on the knockout value of the SV managed portfolio is 1.9166 times the price of the AV managed portfolio.

[10]Federal Reserve Board Regulation T (Reg T) establishes a baseline requirement that investors deposit 50% of an investment position in their margin trading accounts, however a brokerage house may set a higher equity requirement.

[11]The Direxion Daily S&P 500 Bull 3x Shares ETF, symbol SPXL, and ProShares Ultra Pro

S&P 500 ETF, symbol UPRO, are two such funds.

[12]As Moreira and Muir (2017) note, "With no leverage limit, percentage utility gains are the same regardless of risk-aversion because investors can freely adjust theiraverage risk exposure."

Figure 1. Time Series of Market Statistics: The time series of the total variance of market returns, AV is the average variance of the daily returns of individual assets in percentage; SV is the total variance of the daily market return in percentage, and AC is the average pairwise correlation of daily asset returns in the market.

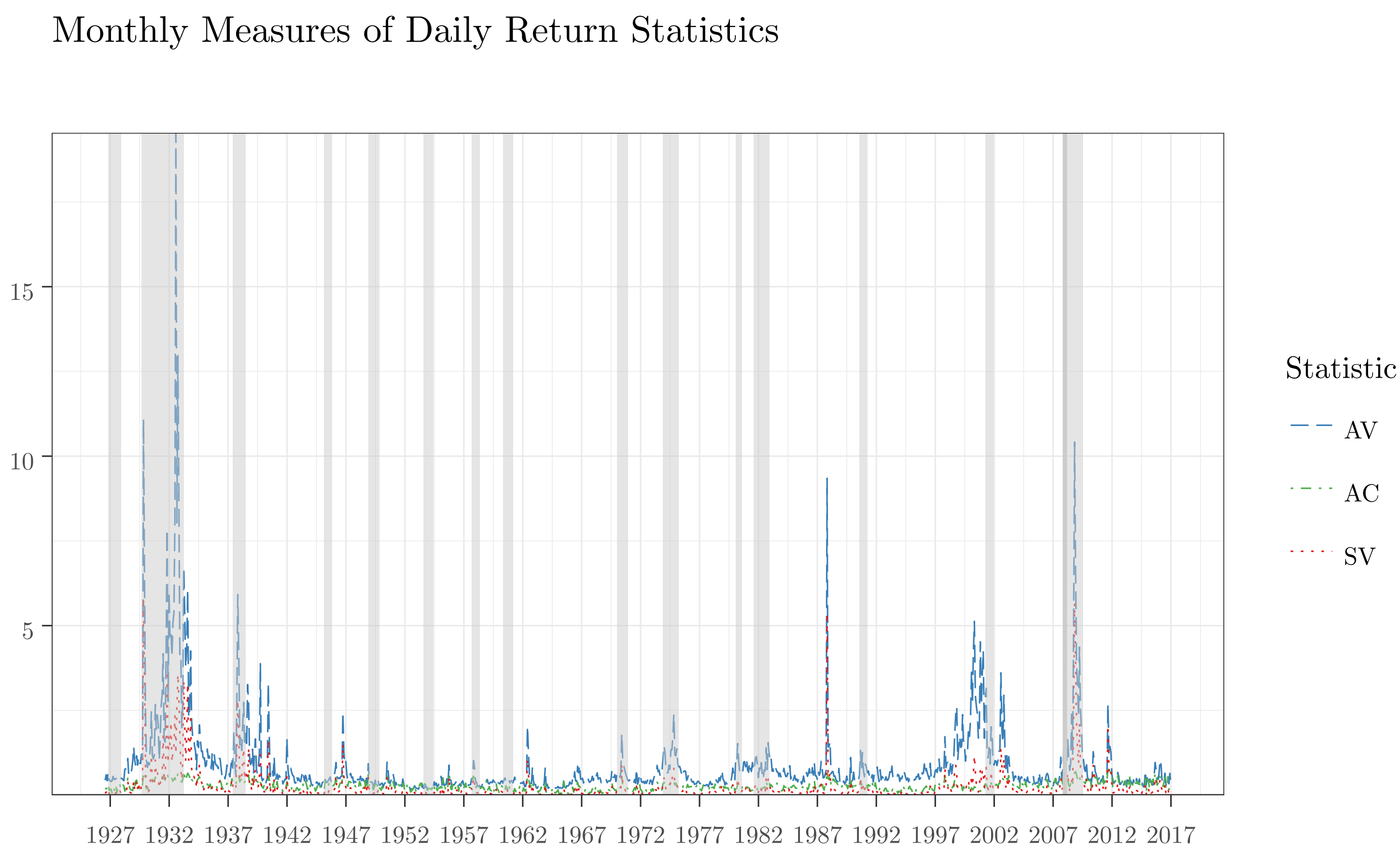

Figure 2. Time Series of Investment Weights: The time series of the investment weight into the market portfolio for SV and AV managed portfolios. c_{029} and c_{035} represents the weights when the SV and AV strategies target 10% and 12% annual volatilities while c_{053} represent targeting the buy and hold annual volatility over the 1926-2016 holding period.

Figure 3. Cummulative Log Excess Returns: The time series of cummulative log excess returns for the buy and hold market investment as well as the AV and SV managed portfolios. Panel a limits the coefficient on the market portfolio between 0 and 1.5 for the AV and SV strategies; panel b limit them to weights from 0 to 3 and they are unconstrained in panel c.

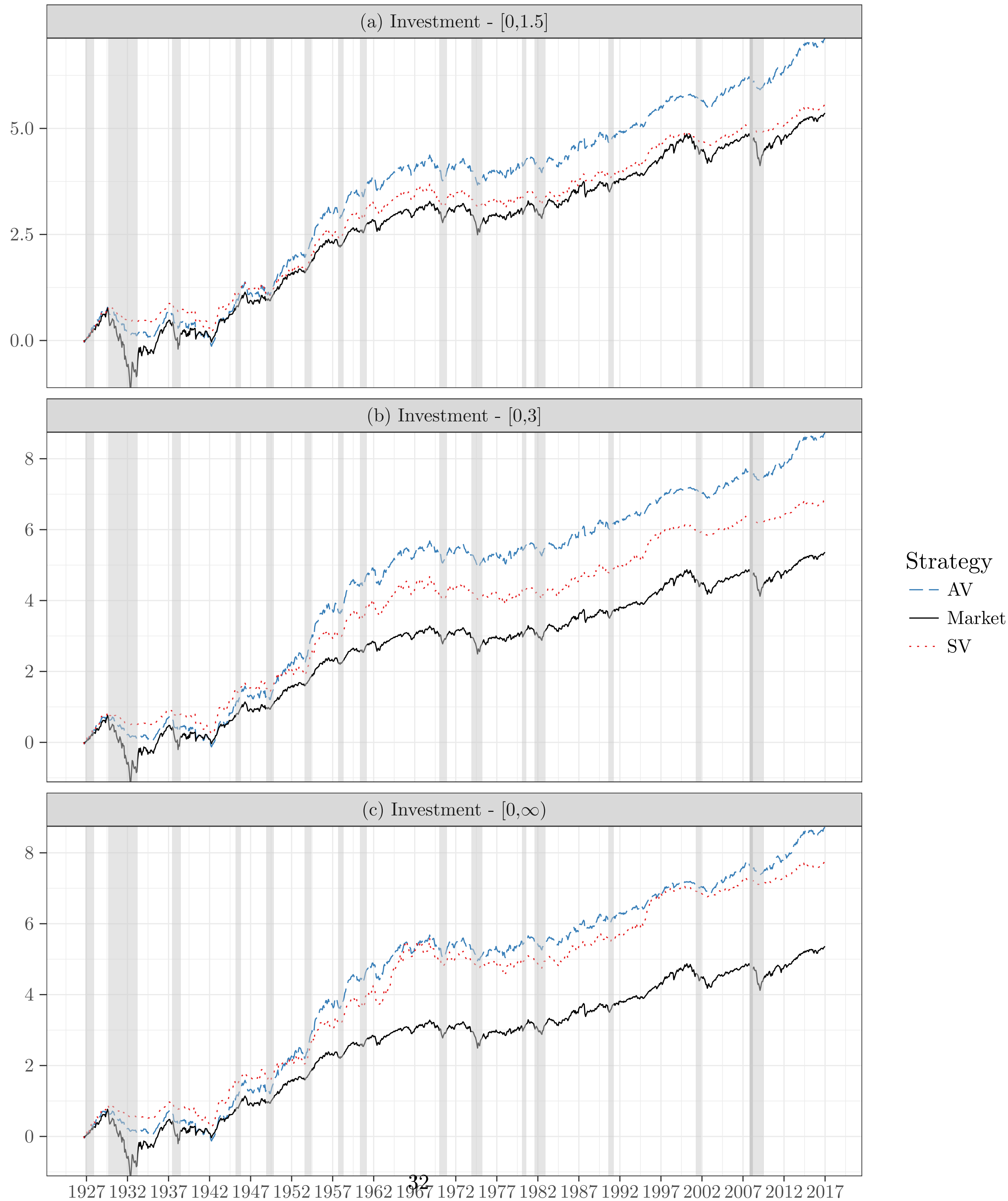

Figure 4. Portfolio Drawdowns: The time series of cummulative drawdowns for the buy and hold market investment as well as the AV and SV managed portfolios. SV and AV managed portfolios are targeting the buy and hold market volatility and have no investment constraints. See section III for details.

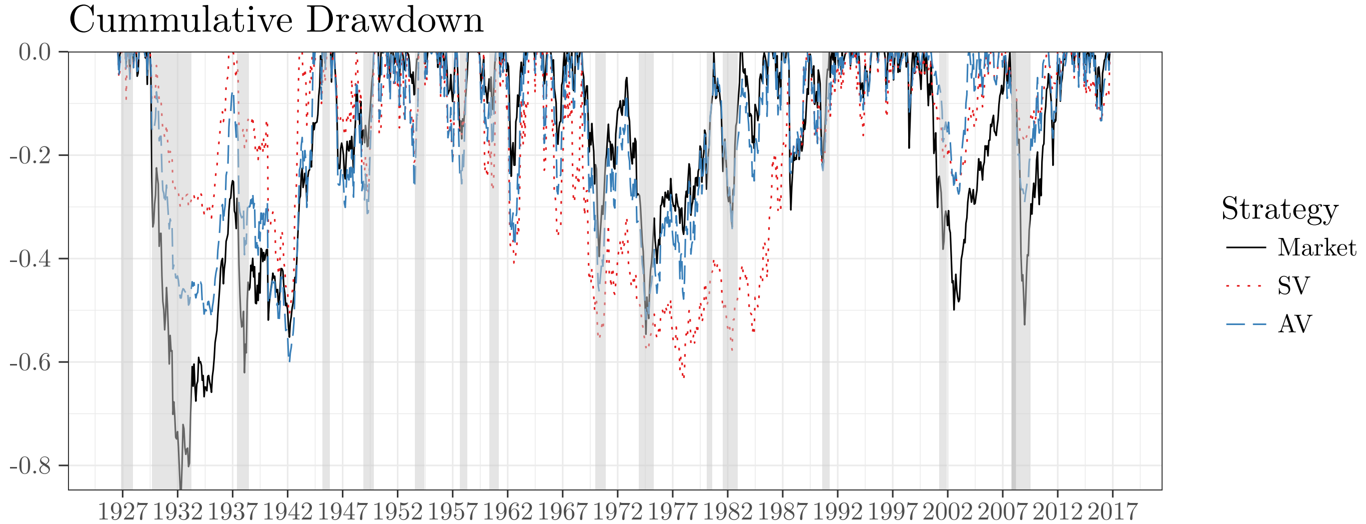

Figure 5. CER Losses: This figure displays the loss of certainty equivalent return as leverage constraints are applied to the AV and SV managed portfolios. The losses are expressed in percentage point differences of the constrained returns from the unconstrained returns for investors with γ risk aversion coefficients of 1, 2.5, and 5.

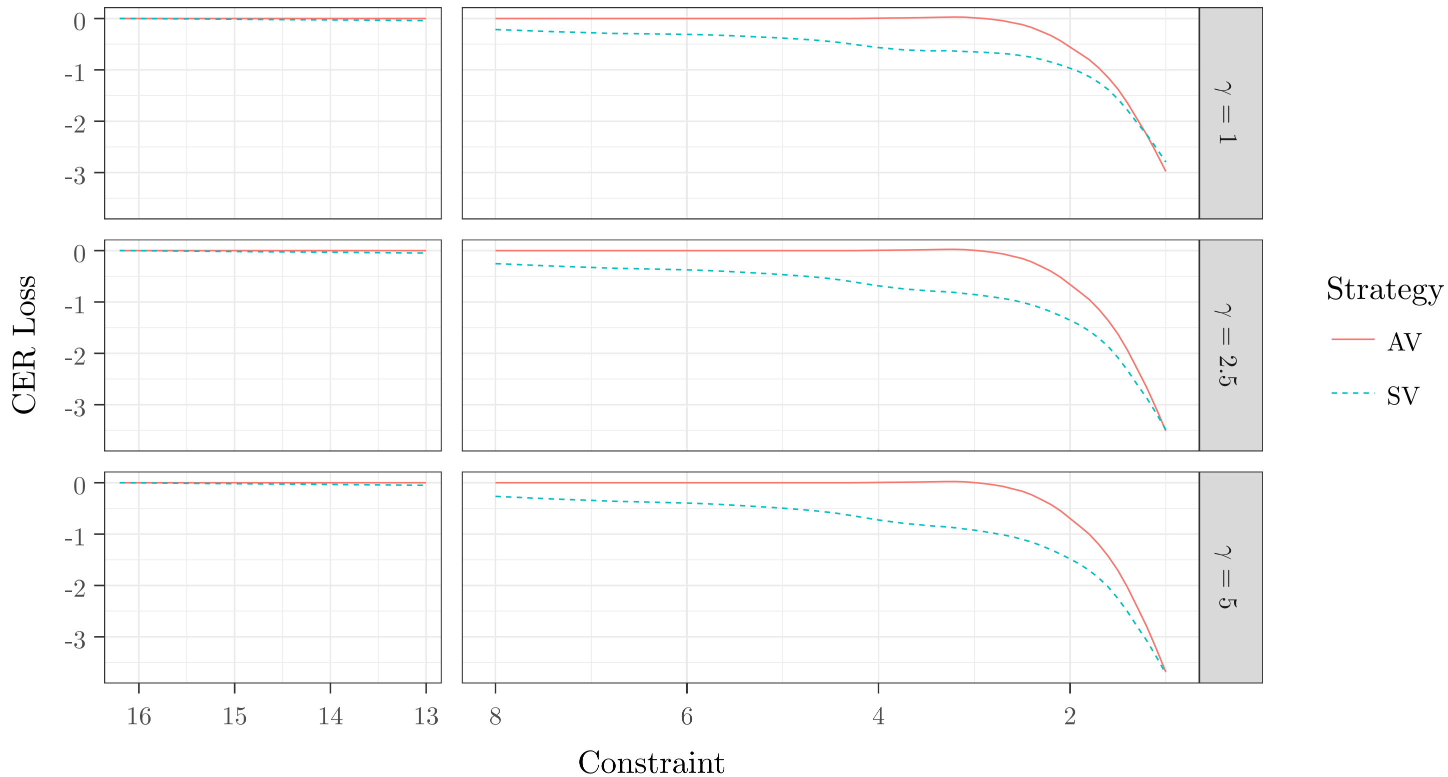

Figure 6. CER Gains: Certainity Equivalent Return gains for mean variance investors with risk aversion coefficients ranging from 1 to 5 and subject to investment constraints ranging from 1 to 3. See section III for details.

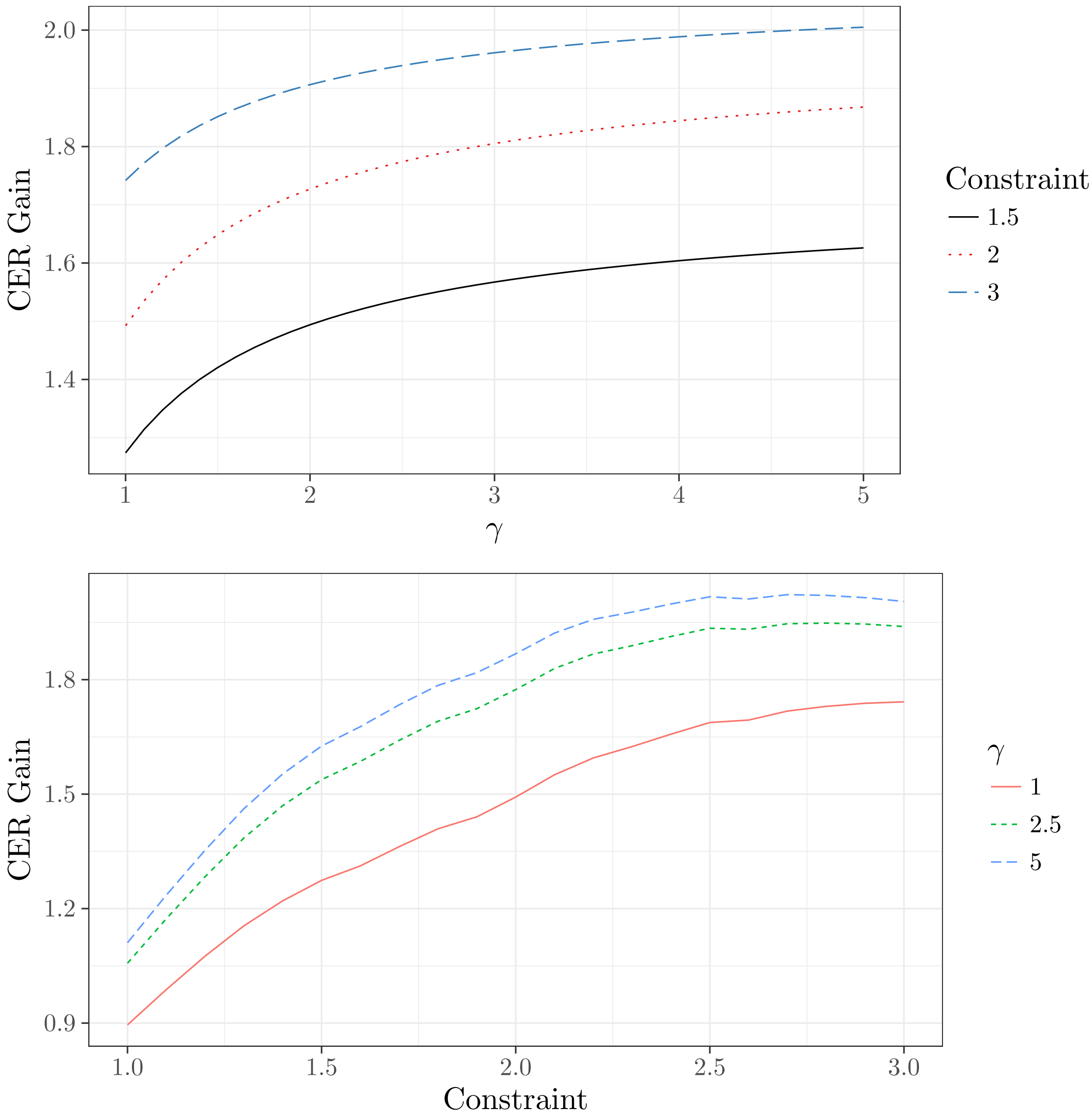

Table I:Summary statistics

The table displays summary statistics for the market variance and correlation statistics, and returns. RET is the log excess return of the CRSP market portfolio. AC is the average pairwise correlation of the daily returns of the 500 largest firms in the CRSP data set over the month or quarter. AV is the average of the individual variances of daily returns for the 500 largest firms in the CRSP data set. SV is the variance of daily CRSP market returns. See section I for details on construction.

(a) Pollet and Wilson Sample 1963Q1:2006Q4

Statistic	N	Mean	St. Dev.	Min	Max	Autocorrelation
RET	176	1.169	8.372	−30.072	19.956	0.000
AC	176	0.231	0.091	0.034	0.648	0.556
AV	176	2.221	1.827	0.634	12.044	0.695
SV	176	0.484	0.615	0.029	6.397	0.310

(b) Sample 1962M6:2016M12

Statistic	N	Mean	St. Dev.	Min	Max	Autocorrelation
RET	655	0.409	4.453	−25.985	14.515	0.081
AC	655	0.261	0.129	0.019	0.762	0.620
AV	655	0.770	0.849	0.198	10.416	0.667
SV	655	0.200	0.406	0.006	5.664	0.551

(c) Full Sample 1926M7:2016M12

Statistic	N	Mean	St. Dev.	Min	Max	Autocorrelation
RET	1,085	0.495	5.371	−34.523	33.188	0.106
AC	1,085	0.276	0.134	0.019	0.762	0.610
AV	1,085	0.881	1.281	0.154	19.540	0.718
SV	1,085	0.248	0.502	0.006	5.808	0.612

Table II:Correlations

The table displays Pearson correlation statistics for the market variance and correlation statistics, and returns. RET is the log excess return of the CRSP market portfolio. AC is the average pairwise correlation of the daily returns of the 500 largest firms in the CRSP data set over the month or quarter. AV is the average of the individual variances of daily returns for the 500 largest firms in the CRSP data set. SV is the variance of daily CRSP market returns. See section I for details on construction.

(a) Sample 1962M6:2016M12

	RET	AC	AV	SV	RET_{t+1}	AC_{t+1}	AV_{t+1}	SV_{t+1}
RET	1	0	0	0	0	0	0	0
AC	-0.244	1	0	0	0	0	0	0
AV	-0.284	0.352	1	0	0	0	0	0
SV	-0.353	0.529	0.899	1	0	0	0	0
RET_{t+1}	0.081	0.049	-0.129	-0.107	1	0	0	0
AC_{t+1}	-0.223	0.622	0.240	0.360	-0.239	1	0	0
AV_{t+1}	-0.268	0.218	0.667	0.522	-0.283	0.351	1	0
SV_{t+1}	-0.289	0.332	0.545	0.552	-0.351	0.528	0.899	1

(b) Full Sample 1926M7:2016M12

	RET	AC	AV	SV	RET_{t+1}	AC_{t+1}	AV_{t+1}	SV_{t+1}
RET	1	0	0	0	0	0	0	0
AC	-0.295	1	0	0	0	0	0	0
AV	-0.136	0.467	1	0	0	0	0	0
SV	-0.279	0.619	0.857	1	0	0	0	0
RET_{t+1}	0.106	0.011	0	-0.057	1	0	0	0
AC_{t+1}	-0.229	0.610	0.383	0.453	-0.295	1	0	0
AV_{t+1}	-0.191	0.358	0.718	0.607	-0.136	0.467	1	0
SV_{t+1}	-0.259	0.416	0.625	0.612	-0.279	0.619	0.857	1

Table III:In Sample Results - Post 1962

The table displays in sample regression results for monthly market variance, correlation and return statistics. SV is the annualized monthly variance of daily CRSP market returns. AV and AC are the monthly average variance and average pairwise correlation of daily returns for the top 500 assets in the CRSP market, as in Pollet and Wilson (2010). RET is the log return of the CRSP market portfolio minus the log return on the 1 month treasury bill. The sample period is from 1962:06 to 2016:12. The series are standardized to a mean of zero and standard deviation of one.

(a) Market Return Variance - SV_{t+1}

AV	0.545***			0.489***	0.257***
	p = 0.000			p = 0.000	p = 0.001
AC		0.332***		0.160***	
		p = 0.000		p = 0.00001	
SV			0.551***		0.320***
			p = 0.000		p = 0.00002
Constant	−0.0005	−0.0001	−0.0003	−0.0005	−0.0004
	p = 0.989	p = 0.999	p = 0.993	p = 0.989	p = 0.991
R^2	0.297	0.110	0.304	0.320	0.317
Adjusted R^2	0.296	0.109	0.303	0.318	0.315

(b) Average Asset Return Variance - AV_{t+1}

AV	0.667***			0.674***	1.030***
	p = 0.000			p = 0.000	p = 0.000
AC		0.218***		−0.019	
		p = 0.00000		p = 0.544	
SV			0.522***		−0.403***
			p = 0.000		p = 0.000
Constant	−0.001	−0.00004	−0.0003	−0.001	−0.001
	p = 0.985	p = 1.000	p = 0.994	p = 0.984	p = 0.981
R^2	0.445	0.048	0.273	0.446	0.477
Adjusted R^2	0.445	0.046	0.272	0.444	0.475

(c) Average Asset Return Correlation - AC_{t+1}

AV	0.239***			0.024	−0.438***
	p = 0.000			p = 0.470	p = 0.00000
AC		0.621***		0.613***	
		p = 0.000		p = 0.000	
SV			0.360***		0.753***
			p = 0.000		p = 0.000
Constant	−0.0002	−0.0001	−0.0002	−0.0001	−0.00003
	p = 0.996	p = 0.998	p = 0.996	p = 0.997	p = 1.000
R^2	0.057	0.387	0.130	0.387	0.167
Adjusted R^2	0.056	0.386	0.128	0.385	0.164

(d) Log Excess Market Return - RET_{t+1}

AV	−0.130***			−0.168***	−0.173*
	p = 0.001			p = 0.0001	p = 0.052
AC		0.049		0.108***	
		p = 0.212		p = 0.010	
SV			−0.107***		0.048
			p = 0.006		p = 0.588
Constant	−0.000	−0.000	−0.000	−0.000	−0.000
	p = 1.000	p = 1.000	p = 1.000	p = 1.000	p = 1.000
N	655	655	655	655	655
R^2	0.017	0.002	0.012	0.027	0.017
Adjusted R^2	0.015	0.001	0.010	0.024	0.014

Notes: ***,**, and * Significant at the 1, 5, and 10 percent levels.

Table IV:Full In Sample Results

The table displays in sample regression results for monthly market variance, correlation and return statistics. SV is the annualized monthly variance of daily CRSP market returns. AV and AC are the monthly average variance and average pairwise correlation of daily returns for the top 500 assets in the CRSP market, as in Pollet and Wilson (2010). RET is the log return of the CRSP market portfolio minus the log return on the 1 month treasury bill. The sample period is from 1926:07 to 2016:12.

(a) Market Return Variance - SV_{t+1}

AV	0.625***			0.551***	0.379***
	p = 0.000			p = 0.000	p = 0.000
AC		0.416***		0.159***	
		p = 0.000		p = 0.000	
SV			0.612***		0.288***
			p = 0.000		p = 0.000
Constant	−0.0003	−0.0001	−0.0002	−0.0003	−0.0003
	p = 0.991	p = 0.998	p = 0.993	p = 0.991	p = 0.991
R^2	0.391	0.173	0.375	0.410	0.413
Adjusted R^2	0.390	0.173	0.374	0.409	0.412

(b) Average Asset Return Variance - AV_{t+1}

AV	0.718***			0.704***	0.745***
	p = 0.000			p = 0.000	p = 0.000
AC		0.358***		0.029	
		p = 0.000		p = 0.232	
SV			0.606***		−0.031
			p = 0.000		p = 0.445
Constant	−0.0003	−0.0001	−0.0002	−0.0003	−0.0003
	p = 0.989	p = 0.998	p = 0.993	p = 0.989	p = 0.989
R^2	0.515	0.128	0.368	0.516	0.516
Adjusted R^2	0.515	0.127	0.367	0.515	0.515

(c) Average Asset Return Correlation - AC_{t+1}

AV	0.383***			0.125***	−0.018
	p = 0.000			p = 0.00001	p = 0.738
AC		0.610***		0.551***	
		p = 0.000		p = 0.000	
SV			0.453***		0.468***
			p = 0.000		p = 0.000
Constant	−0.0002	−0.0001	−0.0002	−0.0002	−0.0002
	p = 0.996	p = 0.996	p = 0.996	p = 0.995	p = 0.996
R^2	0.147	0.372	0.205	0.385	0.205
Adjusted R^2	0.146	0.372	0.204	0.384	0.204

(d) Log Excess Market Return - RET_{t+1}

AV	0.0002			−0.006	0.182***
	p = 0.996			p = 0.857	p = 0.002
AC		0.011		0.014	
		p = 0.724		p = 0.692	
SV			−0.056*		−0.213***
			p = 0.064		p = 0.0003
Constant	−0.00000	−0.00001	0.00002	−0.00001	0.00002
	p = 1.000	p = 1.000	p = 1.000	p = 1.000	p = 1.000
N	1,085	1,085	1,085	1,085	1,085
R^2	0.00000	0.0001	0.003	0.0001	0.012
Adjusted R^2	−0.001	−0.001	0.002	−0.002	0.010

Notes: ***,**, and * Significant at the 1, 5, and 10 percent levels.

Table V:In Sample Robust Results

The table displays in-sample regression results of AC_{t+1}, SV_{t+1} and RET_{t+1} on AV_t. The coeffiecients and standard errors calculated robust to Kandel and Stambaugh (1996) bias using the correction in Amihud and Hurvich (2004). Robust p-values are calculated through t-statistic wild-bootstrap simulation, as in MacKinnon (2002).

(a) AV_t: Sample 1962:06 to 2016:12

	β	t.stat	p
AC_{t+1}	0.241	6.567	0.000
SV_{t+1}	0.550	32.442	0.000
RET_{t+1}	-0.131	-3.494	0.166

(b) AV_t: Sample 1926:07 to 2016:12

	β	t.stat	p
AC_{t+1}	0.384	14.342	0.000
SV_{t+1}	0.627	40.002	0.000
RET_{t+1}	0.000	-0.016	0.562

Table VI:Full Out-of-Sample Results

The table displays out-of-sample expanding window regression results for monthly market variance, correlation and return statistics. SV is the annualized monthly variance of daily CRSP market returns. AV and AC are the monthly average variance and average pairwise correlation of daily returns for the top 500 assets in the CRSP market, as in Pollet and Wilson (2010). RET is the log return of the CRSP market portfolio minus the log return on the 1 month treasury bill. DM is the Diebold and Mariano (1995) statistic measuring for cast accuracy. MSE-F is the mean squared error improvement F-test of in McCracken (2007) and ENC-HLN is the forecast encompassing test of Harvey et al. (1998). In each panel the benchmark forecasts come from a model which uses SV_t to predict the independent variable.

(a) Sample 1970:07 to 2016:12

	DM	MSE-F	ENC-HLN
AC_{t+1}	1.074	109.736***	1
SV_{t+1}	1.53*	29.252***	1**
AV_{t+1}	2.286**	109.333***	1***
RET_{t+1}	1.278	11.801***	1*

(b) Sample 1939:12 to 2016:12

	DM	MSE-F	ENC-HLN
AC_{t+1}	1.604*	46.251***	1**
SV_{t+1}	1.041	21.57***	0.956**
AV_{t+1}	3.104***	198.267***	1***
RET_{t+1}	-2.027	-8.702	0

Notes: ***,**, and * Significant at the 1, 5, and 10 percent levels.

Table VII:Out of Sample Robust Results

The table displays out-of-sample regression results of forecasts using AV_{t+1} as a predictor. Rossi and Inoue (2012) provides the methodology to make the calculations of the out-of-sample accuracy improvements of Diebold and Mariano (1995) and McCracken (2007) and the encompassing test of Harvey et al. (1998). In each panel the benchmark forecasts come from a model which uses SV_t to predict the independent variable.

(a) Robust Expanding Window Results

Stat	Variable	DM	ENC-HLN
R_T	AC_{t+1}	28.532***	6.769***
R_T	SV_{t+1}	8.874***	1.838***
R_T	AV_{t+1}	34.347***	18.197***
R_T	RET_{t+1}	29.124***	4.871***
A_T	AC_{t+1}	19.867***	1.828***
A_T	SV_{t+1}	2.647***	0.949***
A_T	AV_{t+1}	21.751***	10.7***
A_T	RET_{t+1}	13.347***	1.68***

(b) Robust Rolling Window Results

Stat	Variable	DM	ENC-HLN
R_T	AC_{t+1}	27.398***	8.706**
R_T	SV_{t+1}	21.92***	3.973
R_T	AV_{t+1}	34.292***	29.804***
R_T	RET_{t+1}	15.964***	3.884
A_T	AC_{t+1}	8.08***	1.542
A_T	SV_{t+1}	8.218***	2.062
A_T	AV_{t+1}	21.631***	19.449***
A_T	RET_{t+1}	9.209***	1.78

Notes: ***,**, and * Significant at the 1, 5, and 10 percent levels.

Table VIII:Investment Weights

This table displays summary statistics for the time series of investment weights used by both the AV and SV managed portfolio strategies with differenct volatility targets. c_{053} represents targeting the annual volatility of the buy and hold market portfolio over the whole data set, 1926 to 2016. c_{029} and c_{035} target, approximately, 10% and 12% annual return volatility for the AV and SV managed portfolios.

Portfolio	Target	Mean	St. Dev.	Min	Pctl(25)	Median	Pctl(75)	Max
SV	c_{029}	0.697	0.762	0.009	0.246	0.512	0.874	8.743
AV	c_{029}	0.702	0.383	0.018	0.425	0.667	0.915	2.296
SV	c_{035}	0.841	0.920	0.011	0.297	0.618	1.055	10.552
AV	c_{035}	0.848	0.463	0.022	0.513	0.805	1.104	2.772
SV	c_{053}	1.290	1.412	0.017	0.455	0.948	1.619	16.193
AV	c_{053}	1.301	0.710	0.033	0.787	1.235	1.694	4.253

Table IX:Portfolio Performance - Unconstrained

This table displays portfolio performance measures for the AV and SV managed portfolio strategies using c_{053} to target the annual volatility of the buy and hold market portfolio over the whole data set, 1926 to 2016, and over NBER business cycle expansions, contractions and contractions excluding the Great Depression. RET is the average annualized monthly log excess return. Sharpe and Sortino are the Sharpe and Sortino ratios respectively; Kappa_3 and Kappa_4 are the lower parital skewness and lower partial kurtosis Kappa measures. See section III for details. No constraints are plased on the level of investment in the market portfolio for the AV and SV managed portfolio; the buy and hold strategy always has an investment weight of one in the market. Stars on the lines for the AV and SV managed portfolios indicate a significant postive performance difference between those two portfolios.

(a) Full Sample

	Return	Sharpe	Sortino	Kappa_3	Kappa_4
BH	5.932	0.319	0.129	0.082	0.061
SV	8.598	0.462	0.208	0.132	0.097
AV	9.677***	0.520*	0.225	0.150*	0.112**

(b) NBER Expansions

	Return	Sharpe	Sortino	Kappa_3	Kappa_4
BH	7.947	0.524	0.221	0.144	0.105
SV	10.046	0.52	0.236	0.149	0.11
AV	11.869***	0.639**	0.282**	0.184**	0.136**

(c) NBER Contractions

	Return	Sharpe	Sortino	Kappa_3	Kappa_4
BH	-4.065	-0.134	-0.0520	-0.036	-0.029
SV	1.216***	0.084**	0.035***	0.025***	0.021**
AV	-1.467	-0.080	-0.031	-0.022	-0.018

(d) NBER Contractions x1929:09-1933:03

	Return	Sharpe	Sortino	Kappa_3	Kappa_4
BH	5.031	0.225	0.088	0.060	0.047
SV	4.65***	0.298	0.130	0.093	0.077
AV	3.978	0.207	0.084	0.061	0.050

Notes: ***,**, and * Significant at the 1, 5, and 10 percent levels.

Table X:Portfolio Performance - Constrained

This table displays portfolio performance measures for the AV and SV managed portfolio strategies using c_{029}, c_{035}, and c_{053} to target the annual volatilities of 10%, 12% and equal to the buy hold market portfolio over the whole data set, 1926 to 2016. Performance ratios are calculated for investment constraints of a maximum of 1.5 and 3, 50% and 200% leverage. RET is the average annualized monthly log excess return. Sharpe and Sortino are the Sharpe and Sortino ratios respectively; Kappa_3 and Kappa_4 are the lower parital skewness and lower partial kurtosis Kappa measures. See section III for details. No constraints are plased on the level of investment in the market portfolio for the AV and SV managed portfolio; the buy and hold strategy always has an investment weight of one in the market. Stars on the lines for the AV and SV managed portfolios indicate a significant postive performance difference between those two portfolios.

(a) 10% Volatility Target

	Constraint - 1.5					Constraint - 3				
Portfolio	Return	Sharpe	Sortino	Kappa_3	Kappa_4	Return	Sharpe	Sortino	Kappa_3	Kappa_4
SV	4.065	0.461	0.201	0.130	0.097	4.396	0.454	0.200	0.127	0.094
AV	5.196***	0.522**	0.225**	0.150**	0.966	5.225***	0.520*	0.225*	0.150**	0.112**

(b) 12% Volatility Target

	Constraint - 1.5					Constraint - 3				
Portfolio	Return	Sharpe	Sortino	Kappa_3	Kappa_4	Return	Sharpe	Sortino	Kappa_3	Kappa_4
SV	4.735	0.470	0.205	0.133	0.098	5.219	0.452	0.198	0.127	0.094
AV	6.081***	0.516*	0.221*	0.147*	0.945	6.306***	0.520**	0.225**	0.150**	0.112**

(c) Buy and Hold Volatility Target

	Constraint - 1.5					Constraint - 3				
Portfolio	Return	Sharpe	Sortino	Kappa_3	Kappa_4	Return	Sharpe	Sortino	Kappa_3	Kappa_4
BH	5.932	0.319	0.129	0.082	0.061	5.932	0.319	0.129	0.082	0.061
SV	6.171	0.467	0.200	0.128	0.091	7.606	0.456	0.199	0.129	0.096
AV	7.885***	0.486	0.204	0.133	0.097	9.677***	0.522**	0.226**	0.150**	0.112**

Notes: ***,**, and * Significant at the 1, 5, and 10 percent levels.

Table XI:Portfolio Drawdowns

This table displays drawdown statistics for the buy and hold, AV, and SV managed portfolio strategies targeting the buy and hold strategy volatility without investment constraints. N represents the number of drawdowns longer than one month. Max DD and Avg DD are the maximum and mean drawdown in return percentage terms. Max Length and Avg Length are the lengths in months of the maximum drawdown and mean drawdown. Max Recovery and Avg Recovery are the maximum and mean times to recover back to the peak portfolio value at the start of the drawdown. See section III for details.

Strategy	N	Max DD	Avg DD	Max Length	Avg Length	Max Recovery	Avg Recovery
BH	82	-84.803	-8.069	188	11.549	154	7.207
SV	65	-63.637	-11.196	246	14.954	135	7.446
AV	87	-60.264	-9.026	205	10.851	135	5.034

Table XII:Costs

This table displays the average change in absolute market investment weight for the buy and hold volatility targeting, unconstrained, SV and AV managed portfolio strategies and the costs associated with trading and borrowing. Fama-French five factor and five factor with Momentum alphas are calculated using the factor portfolio return data from Ken French's website. Strategy break-even points are calculated in basis points as the cost to investment weight turnover which drives the alphas to zero. Borrowing costs are calculated in basis points as the average monthly cost incurred borrowing to take a position in the market greater than one for the SV and AV manged portfolios at the Bloomberg broker call money rate, 1984-2014. See section III.B for details.

		FF-5		FF-5 + Mom		
Strategy	$\lvert\Delta\omega\rvert$	α	Break Even	α	Break Even	Borrowing
SV	0.752	2.833	31.436	2.769	30.725	15.107
AV	0.317	3.036	79.992	3.152	83.061	11.411