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Journal of Financial Economics 63 (2002) 443–494

JOURNAL OF
Financial
ECONOMICS

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Asymmetric correlations of equity portfolios[☆]

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Received 11 May 2000; received in revised form 4 April 2001

Abstract

Correlations between U.S. stocks and the aggregate U.S. market are much greater for downside moves, especially for extreme downside moves, than for upside moves. We develop a new statistic for measuring, comparing, and testing asymmetries in conditional correlations. Conditional on the downside, correlations in the data differ from the conditional correlations implied by a normal distribution by 11.6%. We find that conditional asymmetric correlations are fundamentally different from other measures of asymmetries, such as skewness and co-skewness. We find that small stocks, value stocks, and past loser stocks have more asymmetric movements. Controlling for size, we find that stocks with lower betas exhibit greater correlation asymmetries, and we find no relationship between leverage and correlation asymmetries. Correlation asymmetries in the data reject the null hypothesis of multivariate normal distributions at daily, weekly, and monthly frequencies. However, several empirical models with greater flexibility, particularly regime-switching models, perform better at capturing correlation asymmetries. © 2002 Elsevier Science B.V. All rights reserved.

JEL classification: C12; C15; C32; G12

Keywords: Stock return asymmetries; Correlation; Dispersion; Model bias; GARCH; Jump model; Regime-switching

[☆] Any errors or omissions are the responsibility of the authors. The authors wish to thank Larry Glosten, Charlie Himmelberg, Harrison Hong, and seminar participants at Columbia University, the Federal Reserve Board, Ohio State University, University of California at Riverside, University of Colorado at Boulder, University of Southern California, Vanderbilt University, and Washington University. We are especially grateful for suggestions from Geert Bekaert, Bob Hodrick, and Ken Singleton. We also thank an anonymous referee whose comments and suggestions greatly improved the paper.

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1. Introduction

Correlations conditional on “downside” movements, which occur when both a U.S. equity portfolio and the U.S. market fall, are, on average, 11.6% higher than correlations implied by a normal distribution. In contrast, correlations conditional on “upside” movements, which occur when both an equity portfolio and the market rise, cannot be statistically distinguished from those implied by a normal distribution. Asymmetric correlations are important for several applications. For example, in optimal portfolio allocation, if all stocks tend to fall together as the market falls, the value of diversification may be overstated by those not taking the increase in downside correlations into account. Asymmetric correlations have similar implications in risk management. In this paper, we examine this correlation asymmetry in several ways.

We begin by formally defining downside correlations as correlations for which both the equity portfolio and the market return are below a pre-specified level. Similarly, upside correlations occur when both the equity portfolio and the market return are above a pre-specified level. Downside correlations in U.S. markets are much larger than upside correlations as shown by the plots of downside and upside correlations presented in Longin and Solnik (2001). These graphs demonstrate that, on the downside, portfolios are much more likely to move together with the market.

Second, we measure this asymmetry by developing a summary statistic, H . The H statistic quantifies the degree of asymmetry in correlations across downside and upside markets relative to a particular model or distribution. This measurement of asymmetry is different from other measurements established in the literature. Covariance asymmetry has usually been interpreted within a particular generalized autoregressive conditional heteroskedasticity (GARCH) model, where covariance asymmetry is defined to be an increase in conditional covariance resulting from past negative shocks in returns.¹ In contrast, our statistic measures correlation asymmetry by looking at behavior in the tails of the distribution. Our statistic is not specific to any model. Hence, we can apply the statistic to evaluate several different models. We show that conditional correlations differ from other measures of higher moments, such as skewness and co-skewness, and from risk measured by beta.

The H statistic corrects for conditioning biases. Boyer et al. (1999), Forbes and Rigobon (1999), and Stambaugh (1995) note that calculating correlations conditional on high or low returns, or high or low volatility, induces a conditioning bias in the correlation estimates. For example, for a bivariate normal distribution with a given unconditional correlation, the conditional correlations calculated on joint upside or downside moves are different from the unconditional correlation. Ignoring these conditioning biases may lead to spurious findings of correlation asymmetry.

¹ Authors such as Cho and Engle (2000), Bekaert and Wu (2000), Kroner and Ng (1998), and Conrad et al. (1991) document the covariance asymmetry of domestic stock portfolios using multivariate asymmetric GARCH models.

Third, we establish several empirical facts about asymmetric correlations in the U.S. equity market. We find the level of asymmetry, measured at the daily, weekly, and monthly frequencies, produces sufficient evidence to reject the null hypothesis of a normal distribution. To investigate the nature of these asymmetric movements, we examine the magnitudes of correlation asymmetries using portfolios sorted on various characteristics. Returns on portfolios of either small firms, value firms, or low past return firms exhibit greater correlation asymmetry. We find significant correlation asymmetry in traditional defensive sectors, such as petroleum and utilities. We also find that riskier stocks, as reflected in higher beta, have lower correlation asymmetry than lower beta stocks. After controlling for size, the magnitude of correlation asymmetry is unrelated to the leverage of a firm. Previous work focuses on asymmetric movements of leverage-sorted portfolios of Japanese stocks (Bekaert and Wu, 2000), and size-sorted portfolios of U.S. stocks (Kroner and Ng, 1998; Conrad et al., 1991) using asymmetric GARCH models.

Finally, we analyze asymmetric correlations by asking if several reduced-form empirical models of stock returns can reproduce the asymmetric correlations found in the data. These candidate models are used by various authors to capture the increase in covariances on downside movements. We discuss four models that allow asymmetric movements between upside and downside movements in returns. These models are an asymmetric GARCH-in-Mean (GARCH-M) model, a Poisson Jump model, for which jumps are layered on a bivariate normal distribution, a regime-switching normal distribution model, and a regime-switching GARCH model. We find the most successful models in replicating the empirical correlation asymmetry are regime-switching models. However, none of these models completely explain the extent of asymmetries in correlations.

Our study of asymmetric correlations is related to several areas of finance. There is a long literature documenting the negative correlation between a stock's return and its volatility of returns.² Other studies analyze patterns of asymmetries in the covariances of stock returns in domestic equity portfolios.³ This literature concentrates on documenting covariance asymmetry within a GARCH framework. Our approach uses a different methodology to document asymmetric correlations, interpreting asymmetries more broadly than simply within the class of GARCH models. We examine a much wider range of portfolio groups than previously used in the literature, and investigate if other classes of empirical models can replicate the correlation asymmetry found in data.

Our approach of creating portfolios sorted by firm characteristics creates a very different view of the determinants of conditional correlations than previously obtained in the literature. The H statistic uses the full sample of observations

²For example see, among others, French et al. (1987), Schwert (1989), Cheung and Ng (1992), Campbell and Hentschel (1992), Glosten et al. (1993), Engle and Ng (1993), Hentschel (1995), and Duffee (1995). Bekaert and Wu (2000) provide a summary of recent GARCH model applications with asymmetric volatility.

³Some papers documenting asymmetric betas are Ball and Kothari (1989), Braun et al. (1995), and Cho and Engle (2000). Conrad et al. (1991), Kroner and Ng (1998), and Bekaert and Wu (2000) document asymmetric covariances in multivariate GARCH models.

measured over time to calculate the correlation at the extreme tails of the joint distribution. By employing time-series data, we use as many observations as possible to calculate correlations for events for which there are relatively few observations. We also focus on the cross-sectional determinants of correlation asymmetry in stock returns, whereas Erb et al. (1994) and Dumas et al. (2000) use conditioning on instrumental variables such as business cycle indicators, rather than on the observations, to determine the characteristics of time-varying correlations.

Work in international markets has found that the correlations of international stock markets tend to increase conditional on large negative, or “bear market”, returns.⁴ Longin and Solnik (2001) use extreme value theory to show that the correlation of large negative returns is much larger than the correlation of positive returns. However, in their work, Longin and Solnik do not provide distribution-specific characterizations of downside and upside correlations. Our paper uncovers strong correlation asymmetries that exist in domestic markets and emphasizes that such asymmetries are more than an international phenomenon in aggregate markets. In our domestic focus we examine which individual firm characteristics are most related to the magnitude of correlation asymmetry.

Other related studies by Campbell et al. (2001), Bekaert and Harvey (2000), and Duffee (1995) examine cross-sectional dispersion of individual stocks, which has increased in recent periods. Duffee (2000) and Stivers (2000) document an asymmetric component in the cross-sectional dispersion. Chen et al. (2001) and Harvey and Siddique (2000) analyze cross-sectional differences in conditional skewness of stock returns. However, these authors have not examined the relationships between firm characteristics and asymmetric correlations. We find that stocks which are smaller, have higher book-to-market ratios, or have low past returns exhibit greater asymmetric correlations. Stocks with higher beta risk show fewer correlation asymmetries. We also show that correlation asymmetry is different from skewness and co-skewness measures of higher moments.

The remainder of this paper is organized as follows. Section 2 demonstrates the economic significance of asymmetries in correlations within a portfolio allocation framework. Section 3 shows that correlation asymmetries exist in domestic U.S. equity data. We define and characterize conditional upside and downside correlations and betas of a bivariate normal distribution in closed-form, and discuss how to correct for conditioning bias. Section 4 measures the correlation asymmetries, and analyzes their cross-sectional determinants. In Section 4, we develop the H statistic measure of correlation asymmetry, and demonstrate asymmetric correlations in equity portfolios using the normal distribution as the benchmark. In Section 5, we ask if several models incorporating asymmetry into the conditional covariance structure can replicate the asymmetry found empirically in the data. Section 6 contains our conclusions. Proofs are reserved for the appendices.

⁴See Erb et al. (1994), Lin et al. (1994), Longin and Solnik (1995, 2001), Karolyi and Stulz (1996), De Santis et al. (1999), Forbes and Rigobon (1999), Boyer et al. (1999), Stărică (1999), Ang and Bekaert (2000), Bae et al. (2000), and Das and Uppal (2001).

2. Economic significance of asymmetric correlations

In this section, we demonstrate the economic significance of asymmetric correlations using a simple asset allocation problem. Appendix A details the solution and the calibration method used in this example. Suppose an investor can hold amounts α_1 and α_2 of two assets with continuously compounded returns x and y , respectively. The remainder of her wealth is held in a riskless asset. Let \tilde{x} and \tilde{y} denote the standardized transformations of x and y , respectively.⁵ The agent maximizes her expected end-of-period constant relative risk aversion (CRRA) utility as follows:

$$\max_{\alpha_1, \alpha_2} E \left[\frac{W^{1-\gamma}}{1-\gamma} \right]. \quad (1)$$

In Eq. (1), the end-of-period wealth is given by $W = e^{r_f} + \alpha_1(e^x - e^{r_f}) + \alpha_2(e^y - e^{r_f})$, $r_f = 0.05$ is a constant continuously compounded risk-free rate, and γ is the agent's coefficient of risk aversion. We set γ equal to 4.

To abstract from the effects of means and variances on portfolio weights, suppose both assets have the same mean and volatility. We denote the expected continuously compounded excess return of both x and y as $\mu = 0.07$, and the volatility of the continuously compounded excess return as $\sigma = 0.15$. For illustration, we set the unconditional correlation of x and y to be $\rho = 0.50$.

Suppose that the agent believes x and y are normally distributed. Since each asset has the same mean and volatility, the investor holds equal amounts of either asset. Let α^\dagger denote this portfolio position. With normal distributions, lower unconditional correlations imply greater benefits from diversification.

We examine the joint behavior of the two assets conditional on downside moves, which can also be called bear-market moves. We define this bear-market move to be a draw that is below each asset's mean by more than one standard deviation. If x and y are normally distributed with unconditional correlation $\rho = 0.5$, the correlation conditional on $x < \mu - \sigma$ and $y < \mu - \sigma$ is

$$\begin{aligned} \bar{\rho} &= \text{corr}(x, y | x < \mu - \sigma, y < \mu - \sigma) \\ &= \text{corr}(\tilde{x}, \tilde{y} | \tilde{x} < -1, \tilde{y} < -1) = 0.1789. \end{aligned} \quad (2)$$

Note that the downside correlation for a normal distribution is less than the unconditional correlation. This difference arises from the conditioning bias of viewing returns based on contemporaneous events of both x and y being below a fixed level. Appendix B demonstrates how to calculate this conditional correlation in closed-form.

Suppose the actual distribution of x and y is a regime-switching (RS) model, although the agent erroneously believes that x and y are normally distributed. Under

⁵To standardize a variable x , we perform the transformation $\tilde{x} = (x - \mu)/\sigma$, where μ is the unconditional mean of x and σ is the unconditional standard deviation of x . Throughout the paper, we use tildes to denote standardized returns. Variables without tildes are not standardized.

the RS model, returns $X = (x, y)$ are given by

$$X \sim N(\mu_{s_t}, \Sigma_{s_t}), \quad s_t \in \{1, 2\}. \quad (3)$$

For regime $s_t = i$, we denote μ_i as the mean returns and Σ_i as the covariance matrix. The transitions between the regimes $s_t = 1$ and $s_t = 2$ are given by a Markov chain with transition probabilities:

$$\begin{pmatrix} P & 1 - P \\ 1 - Q & Q \end{pmatrix}. \quad (4)$$

In Eq. (4), $P = \Pr(s_t = 1 | s_{t-1} = 1)$ and $Q = \Pr(s_t = 2 | s_{t-1} = 2)$. We calibrate the RS model to have the same unconditional mean, μ , the same unconditional volatility, σ , and the same unconditional correlation, ρ , as the normal distribution.

Instead of the downside correlation $\bar{\rho}$ being 0.1789, suppose that the true downside correlation $\bar{\rho}$ is H percent higher. That is,

$$\text{corr}(\tilde{x}, \tilde{y} | \tilde{x} < -1, \tilde{y} < -1) = 0.1789 + H. \quad (5)$$

This magnitude, represented by H , reflects the statistic we develop in Section 4. This increase is an effect which cannot be captured by using the normal distribution, since it is determined only by its first two moments. However, the increase in correlation on downside moves relative to the normal distribution can be captured by the RS model.

Let the RS model have parameters $\mu_1 = \mu_2 = (0.14, 0.14)'$, $P = \frac{2}{3}$, and $Q = \frac{2}{3}$. This specification implies that the stable probabilities of the Markov Chain are $\pi = \Pr(s_t = 1) = \frac{1}{2}$. We express the covariance matrices, Σ_i , as

$$\Sigma_i = \sigma_i^2 \begin{pmatrix} 1 & \rho_i \\ \rho_i & 1 \end{pmatrix}, \quad i = 1, 2. \quad (6)$$

In Eq. (6), ρ_i is the correlation of returns in regime i . We set $\sigma_1 = \sigma_2 = 0.15$ to focus on the effect of regime-dependent correlations. The correlations ρ_1 and ρ_2 are chosen so that the RS model has the same unconditional correlation ρ as the normal distribution. We choose $\rho_1 > \rho_2$ such that $\frac{1}{2}(\rho_1 + \rho_2) = \rho$. The resulting RS model has the same first two unconditional moments as the normal distribution, but its correlation conditional on downside moves is higher than that implied by the normal distribution.

The asset allocations from the RS model can be shown to be dependent on the regime. Since x and y have the same moments, the optimal holdings in each asset are the same, but the proportion held in x and y differ across the regimes. We denote the optimal portfolio holdings in each asset as $\alpha_{s_t}^*$ for regime s_t . In regime 1, with ρ_1 greater than ρ , the investor, who holds α^* based on her belief that x and y are normally distributed, holds much more equity as a proportion of her investment compared to the optimal weight α_1^* . In regime 2, with ρ_2 less than ρ the investor holds too little equity compared to the optimal α_2^* . The higher ρ_1 in the first regime causes downside correlations to increase relative to the normal distribution. Since the normal distribution cannot incorporate the asymmetries

in conditional correlations, the investor over-estimates the benefits of diversification on the downside in regime 1, and over-invests in risky assets. Similarly, she under-estimates the benefits of diversification in regime 2, and under-invests in risky assets.

We calculate the utility loss, which represents the monetary compensation required for an investor to use the non-optimal normal weights α^\dagger instead of the optimal RS model weights $\alpha_{s_t}^*$. This loss is the compensation, in cents per dollar of wealth, that the investor should receive in order to hold α^\dagger instead of $\alpha_{s_t}^*$. This estimate is given by $w = 100 \times (\bar{w} - 1)$, where

$$\bar{w} = \left(\frac{Q_{s_t}^*}{Q_{s_t}^\dagger} \right)^{1/(1-\gamma)}. \quad (7)$$

In Eq. (7), $Q_{s_t}^*$ is the indirect CRRA utility under the RS model, with optimal weights $\alpha_{s_t}^*$ conditional on being in regime s_t , and $Q_{s_t}^\dagger$ is the indirect CRRA utility under the RS model distribution, with sub-optimal weights α^\dagger conditional on being in regime s_t . That is,

$$Q_{s_t}^* = E[(W_{s_t}^*)^{1-\gamma} | s_t] \quad \text{and} \quad Q_{s_t}^\dagger = E[(W_{s_t}^\dagger)^{1-\gamma} | s_t] \quad (8)$$

for which $W_{s_t}^* = e^{r_f} + \alpha_{s_t}^*(e^x - e^{r_f}) + \alpha_{s_t}^*(e^y - e^{r_f})$, $W_{s_t}^\dagger = e^{r_f} + \alpha^\dagger(e^x - e^{r_f}) + \alpha^\dagger(e^y - e^{r_f})$, and both expectations are taken under the RS model.

Fig. 1 graphs the advance monetary compensation the investor should have received to compensate for choosing the sub-optimal normal distribution weights instead of the optimal RS model weights. The compensation required per dollar of wealth is not small. In Regime 1, for $H = 0.10$ the investor requires more than 120 basis points in compensation. In Regime 2, the investor requires around 100 basis points in compensation. This simple example shows that potential utility losses are economically large if correlations increase on the downside relative to a standard normal distribution. In Fig. 1, H measures the difference between the true downside correlation and the downside correlation implied by a normal distribution.

3. Calculating upside and downside moments

We now formally develop the H statistic, show and correct for a bias in measuring it, and use it to characterize the nature of asymmetric correlations in stock portfolios.

3.1. Upside and downside correlations

Conditioning on upside or downside moves and calculating correlations induces a “conditioning bias”. For a bivariate normal with unconditional correlation ρ , the correlation calculated conditioning on a subset of observations (for example taking observations above or below a certain level) differs from the unconditional

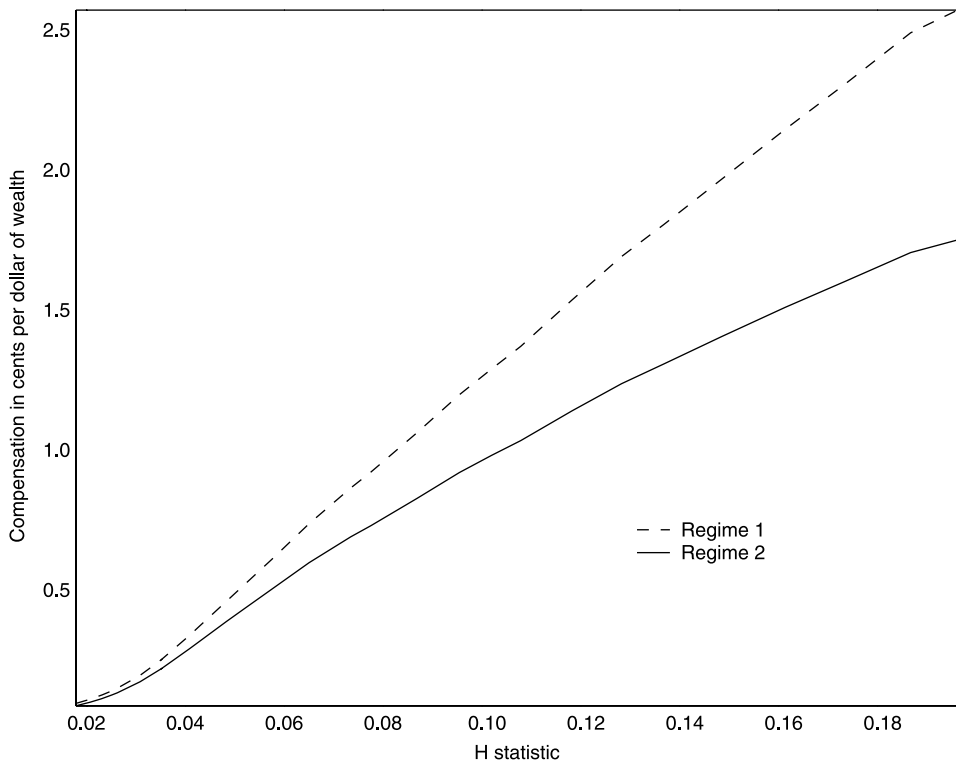


Fig. 1. Economic costs of downside asymmetric correlations. Fig. 1 shows the effects of ignoring increasing correlation on the downside in a hypothetical portfolio allocation problem. A constant relative risk aversion (CRRA) investor with risk aversion $\gamma = 4$ allocates her portfolio among two risky assets and a riskless asset. She believes the assets are lognormally distributed, and chooses asset holdings α^\dagger . Under the normal distribution, the correlation, conditional on downside movements of both assets by more than 1 standard deviation from the mean, is given by $\bar{\rho}$. The true distribution of the continuously compounded returns is given instead by a regime-switching (RS) model with identical unconditional means, variances, and correlation. This distribution produces a true correlation of $\bar{\rho} + H$ conditional on a downside move of more than 1 standard deviation from the mean, where $H > 0$. The optimal portfolio weights, implied by the RS model, are given by $\alpha_{s_t}^*$ for regime $s_t = 1, 2$. The regime-dependent correlations of the RS models are chosen to produce various H statistics. The plot shows ex-ante utility losses, in cents per dollar of wealth, created since the investor holds sub-optimal weights α^\dagger instead of $\alpha_{s_t}^*$ for regime s_t .

correlation. Appendix B calculates this bias in closed-form for a bivariate normal distribution.⁶ In this section, we show that the conditioning bias for a bivariate normal distribution exists, and that ignoring this bias can lead to incorrect inferences from tests of correlation asymmetry.

⁶Related work by Forbes and Rigobon (1999) looks at the correlation of returns conditioning on different volatilities. Boyer et al. (1999) derive correlations for a bivariate normal conditioning on events for one variable. In a discussion of Karolyi and Stulz (1996), Stambaugh (1995) demonstrates the conditioning bias by simulation.

We consider pairs of standardized returns $(\tilde{x}, \tilde{y}) \sim N(0, \Sigma)$, where Σ has unit variances and unconditional correlation ρ . We define

$$\hat{\rho}(h_1, h_2, k_1, k_2) = \text{corr}(\tilde{x}, \tilde{y} | h_1 < \tilde{x} < h_2, k_1 < \tilde{y} < k_2; \rho) \quad (9)$$

as the correlation between \tilde{x} and \tilde{y} , conditional on observations for which $h_1 < \tilde{x} < h_2$ and $k_1 < \tilde{y} < k_2$. The variable, $\hat{\rho}(h_1, h_2, k_1, k_2)$, represents the correlation of a doubly truncated bivariate normal. In Eq. (9), \tilde{x} and \tilde{y} have unconditional correlation ρ .

There are several special cases of this specification. When h_2 and k_2 are infinite, we obtain the one-sided truncation case specified in Rosenbaum (1961). Another special case is the Longin and Solnik (2001) exceedance correlation. A correlation at an exceedance level ϑ is defined as the correlation between two variables when both variables register increases or decreases of more than ϑ standard deviations away from their means, such that

$$\bar{\rho}(\vartheta) = \begin{cases} \hat{\rho}(\vartheta, \infty, \vartheta, \infty) = \text{corr}(\tilde{x}, \tilde{y} | \tilde{x} > \vartheta, \tilde{y} > \vartheta; \rho) & \text{if } \vartheta \geq 0, \\ \hat{\rho}(-\infty, \vartheta, -\infty, \vartheta) = \text{corr}(\tilde{x}, \tilde{y} | \tilde{x} < \vartheta, \tilde{y} < \vartheta; \rho) & \text{if } \vartheta \leq 0. \end{cases} \quad (10)$$

For a bivariate normal distribution, these variables are the same, by symmetry. Longin and Solnik discuss the limiting behavior of exceedance correlations using extreme value theory, but do not give distribution-specific characterizations of exceedance correlations.

For an exceedance level ϑ , we calculate the empirical exceedance correlation $\bar{\rho}(\vartheta)$ as follows. For pairs of standardized observations $\{(\tilde{x}, \tilde{y})\}$, we select a subset of observations such that $\{(\tilde{x}, \tilde{y}) | \tilde{x} > \vartheta \text{ and } \tilde{y} > \vartheta\}$ for $\vartheta \geq 0$, and $\{(\tilde{x}, \tilde{y}) | \tilde{x} < \vartheta \text{ and } \tilde{y} < \vartheta\}$ for $\vartheta \leq 0$. The correlation of the observations in this subset is the empirical exceedance correlation at ϑ . For $\vartheta = 0$, we calculate both $\text{corr}(\tilde{x}, \tilde{y} | \tilde{x} > 0, \tilde{y} > 0)$ and $\text{corr}(\tilde{x}, \tilde{y} | \tilde{x} < 0, \tilde{y} < 0)$. In theory, these correlations are the same for a symmetric distribution, but may differ in the data. In calculating the exceedance correlation, ϑ determines the cutoff points for the conditioning sample, which are expressed in multiples or fractions of standard deviations from the observed mean values. For $\vartheta = -1$, the exceedance correlation between an equity portfolio and the market return is calculated on a subset of observations for which both the equity portfolio and the market return are more than 1 standard deviation below their empirical means.

Panel A of Fig. 2 shows graphs of conditional correlations of a bivariate normal distribution, conditional on returns above or below a certain level. Panel A shows exceedance correlations $\bar{\rho}(\vartheta)$ for various unconditional ρ . These correlations are calculated using Eqs. (B.11)–(B.13) shown in Appendix B. For a given ρ , the exceedance correlations are tent-shaped. Intuitively, the exceedance correlations tend to zero as ϑ approaches infinity, either positive or negative, because the tails of the bivariate normal are flat. The exceedance correlations are calculated over a quadrant of \tilde{x} and \tilde{y} , with origin at point (ϑ, ϑ) . As ϑ increases, the quadrant is pushed further into the tails of the bivariate normal, where the distribution becomes flatter. One way to determine if correlations are different for upside ($\vartheta > 0$) or downside ($\vartheta < 0$) moves is to compare positive or negative exceedance correlations in the data with those implied from a particular distribution, such as the normal distribution. Fig. 2 shows

that comparing correlations conditional on high or low absolute returns cannot be done without taking into account the conditioning bias.

We can also construct correlations that are conditional on levels of a single variable, \tilde{x} . Panel B of in Fig. 2 shows conditional correlations $\hat{\rho}(h_1, h_2, -\infty, +\infty) = \text{corr}(\tilde{x}, \tilde{y} | h_1 < \tilde{x} < h_2; \rho)$ over different intervals (h_1, h_2) . The truncation points h_1 and h_2 are chosen to correspond to abscissae from an inverse cumulative normal, which we denote by $\Phi^{-1}(\cdot)$. In Fig. 2, h_1 and h_2 correspond to the abscissae intervals of probabilities [0.0 0.2 0.4 0.6 0.8 1.0]. That is, the intervals (h_1, h_2) correspond to

$$\begin{aligned} (\Phi^{-1}(0.0), \Phi^{-1}(0.2)) &= (-\infty, -0.8146) \\ (\Phi^{-1}(0.2), \Phi^{-1}(0.4)) &= (-0.8146, -0.2533) \\ &\vdots \\ (\Phi^{-1}(0.8), \Phi^{-1}(1.0)) &= (0.8146, +\infty). \end{aligned} \tag{11}$$

The conditional correlations $\hat{\rho}(h_1, h_2, -\infty, +\infty)$ are plotted at the inverse cumulative normal abscissae corresponding to the midpoints [0.1 0.3 0.5 0.7 0.9]. The conditional correlations produced this way lie in a U-shape.⁷ Hence, comparing conditional correlations constructed from samples where one variable has large absolute returns to conditional correlations constructed from samples where the same variable has small absolute returns must also be done taking into account the conditioning bias. In particular, calculating conditional correlations when the conditioning information set consists of exogenous instrumental variables, such as macroeconomic variables, may also induce a bias, if these conditioning variables are correlated with returns.

In our empirical work, we take \tilde{x} to be standardized returns of a stock portfolio and \tilde{y} to be standardized market returns. We can look at movements in \tilde{x} and \tilde{y} conditional on large movements in both the market and the stock portfolio as analyzed in Longin and Solnik (2001), or look at movements in \tilde{x} and \tilde{y} conditional only on large market moves (Butler and Joaquin, 2000). In both cases, we cannot simply compare conditional correlations of high or low return periods. We concentrate on the analysis based on the exceedance correlations of Longin and Solnik (2001). This characterization has the advantage of succinctly describing the conditional correlations with one parameter, the exceedance level ϑ , rather than a series of truncation intervals, as is done in Panel B of Fig. 2. The exceedance conditioning of both \tilde{x} and \tilde{y} also focuses attention on joint “downside” and “upside” moves. This demonstration is particularly relevant given past episodes of market crashes when stocks have made simultaneous extreme moves on the downside.

⁷ A similar exercise in showing conditional correlation bias over different intervals is done by Boyer et al. (1999). A plot of conditional correlations $\text{corr}(x, y | h_1 < x < h_2; \rho)$ where h_1 and h_2 values are chosen with equal intervals, would show a picture very similar to the plots of Panel A, which has a tent shape. This relation also applies if we show correlations conditioning only on x , such as $\text{corr}(x, y | x > \vartheta; \rho)$. In this case, we produce a tent similar to the top plot of Fig. 2.

3.2. *Asymmetric correlations in the returns data*

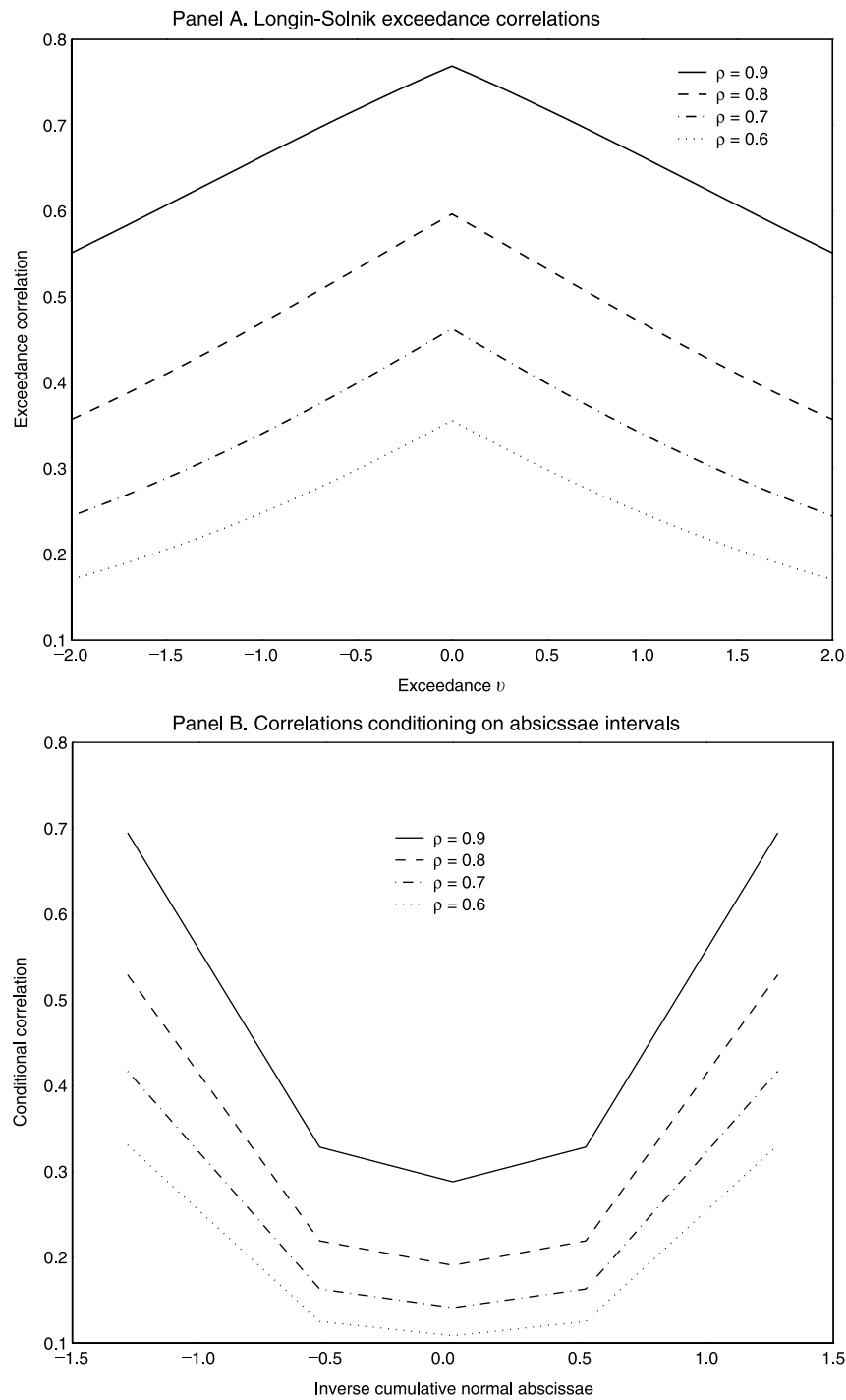
We focus on portfolio returns of stocks sorted by industry classifications, size, value, and momentum. We use market capitalizations to represent size, book-to-market ratios to represent value, and past returns to represent momentum. Stocks are sorted on market capitalization, book-to-market ratios, and lagged past 6-month returns and grouped into quintiles to form size, book-to-market, and momentum portfolios (smallest to largest, growth to value, and losers to winners, respectively).

We focus on these portfolio groups for the following reasons. Industries have varying exposures to economic factors (see Ferson and Harvey, 1991). The Fama and French (1993) model, using size and value-based factors is very popular. The momentum effect has received recent attention, largely because it cannot be explained by the Fama and French model (see Fama and French, 1996). We also study portfolios formed by other cross-sectional characteristics, such as beta and co-skewness, and portfolios formed by other firm characteristics, such as leverage. These portfolios are also divided into quintiles. To control for possible interaction between market capitalization (size) and other characteristics, we also construct two sets of doubly sorted portfolios: one on size and beta, and another on size and leverage.

For our empirical analysis, we use data from the Center for Research in Security Prices (CRSP) and Standard & Poor's COMPUSTAT to construct portfolios based on various firm and distributional characteristics. We use both daily and monthly returns from CRSP for the period covering July 1st, 1963 to December 31st, 1998. We use COMPUSTAT's annual files to obtain information about book values and financial leverage. We follow standard conventions, and restrict our universe to common stocks listed on NYSE, AMEX, or NASDAQ of companies incorporated in the United States. For the risk-free rate, we use the one-month Treasury Bill rate provided by Ibbotson Associates. We take CRSP's value-weighted return of all stocks to be used as the market portfolio.

We first construct a set of value-weighted industry portfolios grouped by their two-digit Standard Industrial Classification (SIC) codes. We use the same classification of these industries as Ferson and Harvey (1991). In addition, group all stocks that do not fall into this classification scheme into a miscellaneous industry category. The industries analyzed are miscellaneous, petroleum, finance, durables, basic industries, food and tobacco, construction, capital goods, transportation, utilities, textile and trade, service, and leisure.

Within each month, for each portfolio, we calculate daily returns of a buy-and-hold strategy using the CRSP daily file. At the beginning of every month, each portfolio is re-balanced and re-formed according to the strategy. The returns are aggregated into weekly frequency by calculating the total buy-and-hold return of each strategy from the end of every Wednesday to the end of the following Wednesday. At a weekly frequency, this action yields 1,852 observations. The monthly returns are calculated directly from the CRSP monthly file, and are also rebalanced and reformed at the beginning of every month. Finally, all returns are



converted into continuously compounded yields and expressed as returns in excess of the one-month T-bill rate.

The second set of portfolios we construct are value-weighted, size-sorted portfolios. At the beginning of every month, we determine the breakpoints on market capitalization for our stocks based on the quintile breakpoints of stocks listed on the NYSE. Hence, our first size-sorted portfolio contains all the stocks listed on the combined NYSE/AMEX/NASDAQ listings that are smaller than the 20th percentile NYSE stock.

The third set of portfolios we construct are value-weighted book-to-market portfolios. At the beginning of every month, our universe of stocks is once again sorted based on quintile breakpoints of stocks listed on the NYSE. The sorting variable is the book-to-market ratio calculated using the most recently available fiscal year-end balance sheet data on COMPUSTAT. Following Fama and French (1993), we define “book value” as the value of common stockholders’ equity, plus deferred taxes and investment tax credit, minus the book value of preferred stock. The book value is then divided by the market value on the day of the firm’s fiscal year-end.

The next set of portfolios consists of the “6-6” momentum strategy portfolios of Jegadeesh and Titman (1993). For this set, we sort our stocks based upon the past six-months returns of all stocks in our universe, rather than just on NYSE stocks. To avoid market microstructure effects, we require a one-month lag between when the returns are realized and when the portfolios are formed. Hence, each month, an equal-weighted portfolio is formed based on six-months returns ending one month prior. Similarly, equal-weighted portfolios are formed based on past returns that ended two months prior, three months prior, and so on, up to six months prior. We then take the simple average of six such portfolios. Hence, our first momentum portfolio consists of $\frac{1}{6}$ of the returns of the worst performers one month ago, plus $\frac{1}{6}$ of the returns of the worst performers two months ago, etc.

The next two sets of portfolios are based on distributional characteristics of past returns. The beta with respect to the market is estimated as the regression coefficient of monthly excess portfolio returns on monthly excess market returns over the past 60 months. Standardized co-skewness is estimated for every stock using past one-year daily stock returns. As with size and book-to-market portfolios, value-weighted portfolios based on NYSE quintile breakpoints are formed over the following month.



Fig. 2. Conditional correlations of a bivariate normal distribution. Panel A shows the exceedance correlations, $\text{corr}(\tilde{x}, \tilde{y} | \tilde{x} > \vartheta, \tilde{y} > \vartheta; \rho)$, for exceedance $\vartheta > 0$ of \tilde{x} and \tilde{y} drawn from a bivariate normal with zero means, unit variances, and unconditional correlation ρ . For $\vartheta < 0$, the exceedance correlation is $\text{corr}(\tilde{x}, \tilde{y} | \tilde{x} < \vartheta, \tilde{y} < \vartheta; \rho)$. Panel B gives conditional correlations $\text{corr}(\tilde{x}, \tilde{y} | h_1 < \tilde{x} < h_2; \rho)$, where h_1 and h_2 are chosen to correspond to abscissae from an inverse cumulative normal. We choose h_1 and h_2 to correspond to the abscissae intervals of probabilities [0.0 0.2 0.4 0.6 0.8 1.0]. That is, we choose the first $(h_1, h_2) = (\Phi^{-1}(0), \Phi^{-1}(0.2))$ for which $\Phi^{-1}(\cdot)$ is an inverse cumulative normal. We plot these at the inverse cumulative normal abscissae corresponding to the midpoints [0.1 0.3 0.5 0.7 0.9], such that the x -axis points are $\Phi^{-1}(0.1)$, $\Phi^{-1}(0.3)$, etc.

The final set of portfolios are formed according to firm leverage. Leverage is calculated annually as total assets divided by book value, where book value is defined as above. Leverage for a given month is defined as the most recently reported value at the beginning of the month. As with size and book-to-market portfolios, we compute quintile breakpoints based on stocks listed on NYSE and value-weighted portfolios are formed.

In addition, we create two sets of doubly sorted portfolios: one sorted on size and beta, and another sorted on size and leverage. For both sets, we first sort every stock in our universe by size into quintiles using NYSE breakpoints. Then, within each size quintile, we further sort stocks into quintiles based on beta. The breakpoints for beta within each size quintile are also calculated using only NYSE stocks. We then form value-weighted portfolios according to the 5×5 groupings. Size and leverage portfolios are formed the same way, except that we use leverage rather than beta.

Table 1 presents the summary statistics of the market, industry, size, book-to-market, and momentum portfolios at the weekly frequency.⁸ The mean and standard deviation of the excess portfolio returns are annualized by multiplying the mean by 52 and the standard deviation by $\sqrt{52}$. The size effect, value effect, and momentum effect are clearly depicted by the mean returns of these portfolios across quintiles.

Non-synchronous trading can cause a bias in the estimation of covariance, and hence correlation. Our portfolio constructions rebalance portfolios at the end of every month, minimizing micro-structure bid–ask bounce effects. We focus on the weekly frequency since this frequency represents the best trade-off to avoid the market microstructure biases at daily frequencies, yet provide a large number of observations. We also focus on value-weighted portfolios for the industry, size, and book-to-market portfolios to avoid putting too much weight on small illiquid stocks. As a check, the last two columns of Table 1 list the sample unconditional correlation with the market portfolio at both the weekly and the monthly frequencies. The unconditional correlations calculated using weekly data and monthly data are very similar. This evidence suggests our results are not plagued by errors in the estimation of correlations induced by non-synchronous trading.

Table 2 lists the ten largest positive and negative excess weekly returns of the market portfolio. The information in Table 2 is not annualized. The table shows that, aside from a large negative return attributable to the October 1987 crash, the top ten largest weekly moves in absolute magnitude of the market are approximately the same for both positive and negative moves. This finding suggests that our results on asymmetric correlations are not due to under-sampling of either the downside or upside movements relative to each other at the weekly frequency. Our results of asymmetric correlations are also robust to excluding the October 1987 crash.

If equity and market returns are normally distributed, their exceedance correlations would exhibit the tent-shaped distributions shown in Fig. 2. To construct plots of empirical exceedance correlations, we take \tilde{x} to be the standardized excess return of an equity portfolio, and \tilde{y} to be the standardized

⁸For brevity, we do not report the statistics of other portfolios. Additional summary statistics of the other portfolios and other frequencies are available from the authors.

Table 1

Summary statistics for the market and equity portfolios

This table shows the summary statistics of the market portfolio and the equity portfolios. Data is sampled weekly, or monthly for the last column, from July 1963 to December 1998. The number of observations is 1852, or 426 for the last column. The mean and the standard deviation have been annualized by multiplying the mean and standard deviation in the data by 52 and $\sqrt{52}$, respectively. The columns Auto 1 and Auto 2 give the first and the second autocorrelations. The last two columns show the unconditional correlation of the portfolios with the market at weekly and monthly frequencies. All returns are log-returns in excess of the annualized 1-month T-bill rate.

The market portfolio is the value-weighted index of all stocks in CRSP. Panel A shows the summary statistics of the value-weighted industry portfolios. Panels B and C show the summary statistics of the value-weighted portfolios formed by sorting on market capitalizations and book-to-market ratios, respectively. Panel D presents the summary statistics of the equal-weighted portfolios of stocks sorted by their lagged past six-months returns, with one to six months of lags.

Portfolio	Mean	Standard deviation	Auto 1	Auto 2	Unconditional correlation with the market	
					Weekly	Monthly
Market portfolio	0.066	0.146	0.068	0.004		
<i>Panel A. Industry portfolios (value-weighted)</i>						
Misc.	0.031	0.188	0.137	0.046	0.860	0.883
Petroleum	0.054	0.174	0.033	−0.001	0.706	0.708
Finance	0.062	0.160	0.120	0.024	0.920	0.917
Durables	0.052	0.177	0.084	0.025	0.935	0.934
Basic ind	0.056	0.157	0.049	0.004	0.946	0.944
Food/tobacco	0.081	0.143	0.034	0.050	0.866	0.867
Construction	0.050	0.185	0.108	0.011	0.895	0.902
Capital goods	0.048	0.179	0.064	0.000	0.918	0.905
Transportation	0.032	0.200	0.102	0.012	0.854	0.851
Utilities	0.048	0.115	0.092	0.023	0.798	0.792
Textile/trade	0.060	0.181	0.101	0.035	0.872	0.859
Service	0.074	0.209	0.144	0.033	0.895	0.896
Leisure	0.065	0.187	0.136	0.096	0.891	0.888
<i>Panel B. Size portfolios (value-weighted)</i>						
1 Smallest	0.054	0.163	0.332	0.154	0.823	0.830
2	0.066	0.169	0.242	0.080	0.894	0.903
3	0.064	0.162	0.193	0.054	0.931	0.934
4	0.063	0.156	0.140	0.026	0.966	0.967
5 Largest	0.053	0.145	0.013	−0.009	0.988	0.987
<i>Panel C. Book-to-market portfolios (value-weighted)</i>						
1 Growth	0.048	0.167	0.037	0.004	0.961	0.954
2	0.049	0.153	0.086	0.009	0.966	0.971
3	0.050	0.141	0.099	0.003	0.938	0.942
4	0.073	0.132	0.082	0.021	0.917	0.913
5 Value	0.091	0.143	0.116	0.058	0.875	0.859
<i>Panel D. Momentum portfolios (equal-weighted)</i>						
1 Past losers	0.023	0.171	0.371	0.169	0.783	0.743
2	0.060	0.140	0.319	0.145	0.864	0.831
3	0.078	0.131	0.291	0.126	0.901	0.873
4	0.092	0.136	0.252	0.103	0.911	0.887
5 Past winners	0.111	0.163	0.224	0.086	0.882	0.857

Table 2

Ten largest weekly negative and positive moves of the market portfolio

We present the ten largest positive and negative moves for the value-weighted market portfolio in excess of the risk-free rate. Data is sampled weekly, from July 1963 to December 1998. Dates reported are end-of-period. Returns are not annualized.

Ranking	Date	Excess Return
<i>Panel A. Largest negative moves</i>		
1 Largest move	21-Oct-87	−0.194
2	28-Oct-87	−0.108
3	2-Sep-98	−0.089
4	20-Nov-74	−0.067
5	22-Aug-90	−0.066
6	29-Oct-97	−0.062
7	4-Aug-74	−0.062
8	31-Jul-74	−0.060
9	10-Dec-80	−0.060
10	7-Oct-98	−0.058
<i>Panel B. Largest positive moves</i>		
1 Largest move	3-Jun-70	0.097
2	13-Oct-82	0.089
3	25-Aug-82	0.086
4	29-Jan-75	0.086
5	21-Oct-98	0.075
6	4-Nov-87	0.070
7	1-Dec-71	0.067
8	26-Aug-70	0.064
9	7-Jan-87	0.064
10	9-Oct-74	0.062

excess return of the market. Fig. 3 shows the exceedance correlations for the equity portfolios at the weekly frequency.⁹ The figure provides clear pictorial representations of the asymmetric movements between the equity portfolios and the market. There are two main features of the plots. First, we observe that, far from being symmetric, the exceedance correlations for negative exceedance levels are always greater than the exceedance correlations for positive exceedances. There is a sharp break evident at $\vartheta = 0$, where the conditioning changes from calculating $\text{corr}(\tilde{x}, \tilde{y} | \tilde{x} > 0, \tilde{y} > 0)$ using the positive quadrant to $\text{corr}(\tilde{x}, \tilde{y} | \tilde{x} < 0, \tilde{y} < 0)$ using the negative quadrant. Second, instead of tapering off to zero, as in the case of a bivariate normal distribution, the negative exceedances are either flat, or tend to increase as ϑ becomes more negative. The positive exceedance correlations are more variable than the negative ones, but there is some evidence that, as ϑ increases, these correlations taper off to zero for some portfolios.

⁹Plots for daily and monthly frequencies and for equal-weighted market returns are available on request. Both the daily and the monthly frequencies exhibit the same highly asymmetric patterns as documented here for the weekly frequency. The H statistic in the legend is the measure of this asymmetry we develop in Section 4.

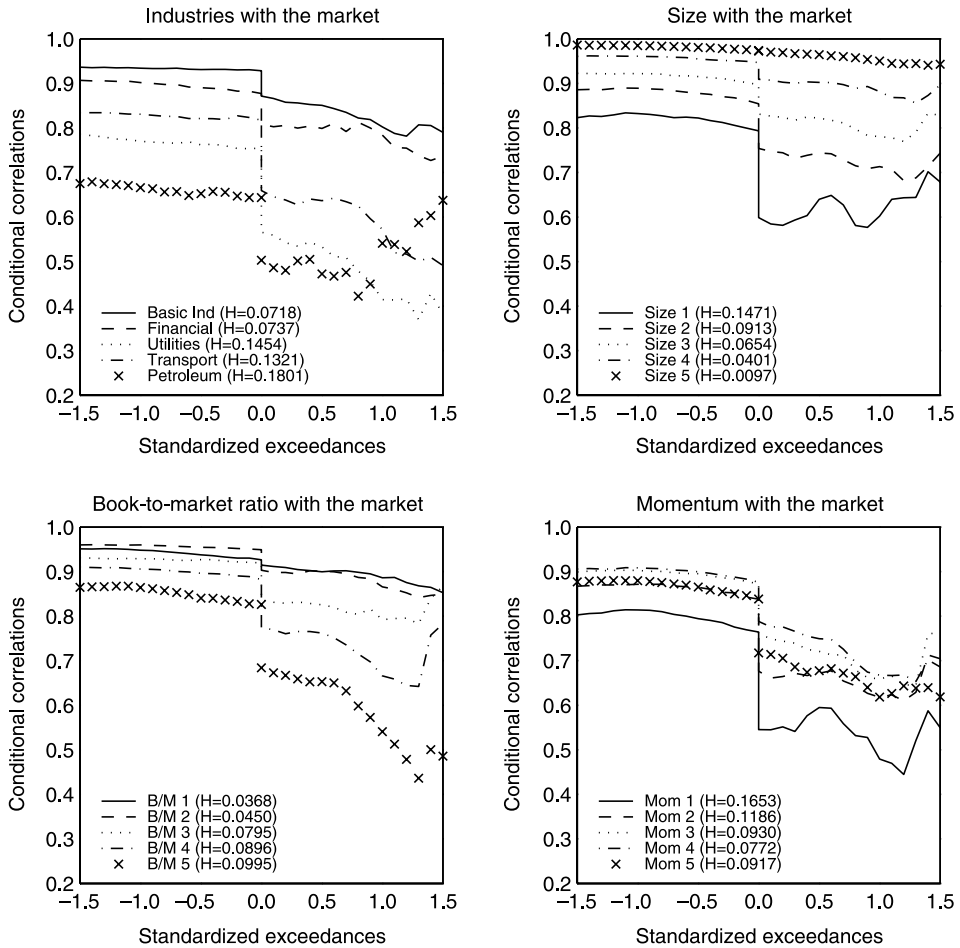


Fig. 3. Exceedance correlations of industry, size, book-to-market, and momentum portfolios. We plot exceedance correlations with the market portfolio for selected industry, size, book-to-market, and momentum portfolios. These are the conditional correlations $\text{corr}(\tilde{x}|\tilde{x} > \vartheta, \tilde{y} > \vartheta; \rho)$ for exceedance $\vartheta > 0$ for normalized portfolio \tilde{x} and the normalized market portfolio \tilde{y} . For $\vartheta < 0$, the exceedance correlation is defined as $\text{corr}(\tilde{x}, \tilde{y}|\tilde{x} < \vartheta, \tilde{y} < \vartheta; \rho)$. Exceedance correlations are calculated at the weekly frequency. The H statistic in the legend is the measure of correlation asymmetry developed in Section 4.

Fig. 4 shows the exceedance correlations of two representative equity portfolios. It plots the exceedance correlations of the first and fifth quintiles of the size portfolios with the market. The implied exceedance correlations from a bivariate normal distribution with the same unconditional correlation as the equity portfolio and market pairs appear on the same plot. Fig. 4 demonstrates that the negative exceedance correlations for both portfolios do not tend towards zero, and are substantially greater than the exceedance correlations of the bivariate normal distributions. This pattern indicates that correlations between the market and the portfolios are significantly

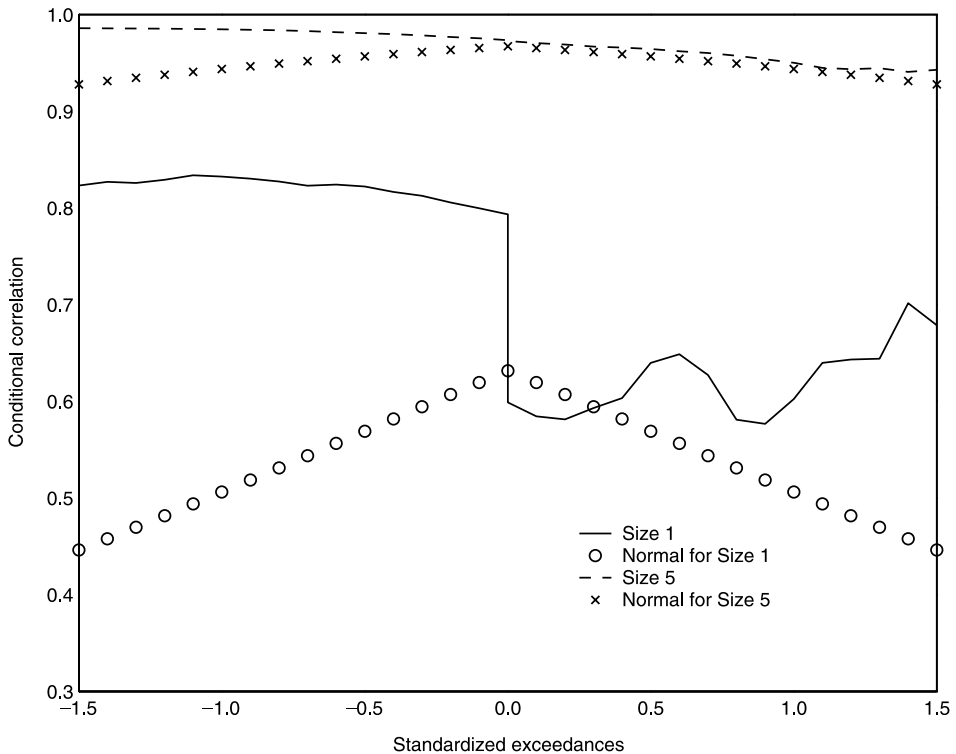


Fig. 4. Exceedance correlations: empirical versus bivariate normal. This figure shows the exceedance correlations with the market portfolio for the quintile 1 and quintile 5 size portfolios. Data is sampled weekly, from July 1963 to December 1998. The theoretical exceedance correlations from a bivariate normal with the same unconditional correlation is also shown on the plot for each portfolio.

higher in falling markets than a normal distribution would imply. The correlations implied by a bivariate normal distribution presents a good approximation of the positive exceedances for the fifth size quintile, while the empirical exceedances lie above those implied by the bivariate normal for the first size quintile. Fig. 4 suggests that, while a bivariate normal distribution cannot match the negative exceedances from the data, it may approximate positive exceedances for some portfolios.

The exceedance plots in Figs. 3 and 4 provide a graphical representation of the asymmetric movements in equity portfolios. They show that correlation asymmetries exist in the data.

3.3. Upside and downside betas

Analogous to the upside and downside exceedance correlations, we can define upside and downside beta coefficients.¹⁰ For simplicity, we measure upside and

¹⁰We thank an anonymous referee for suggesting this analysis.

downside betas relative to the means μ_x and μ_y of the portfolio excess return x and market excess return y , respectively. We define an upside beta, β^+ , as

$$\beta^+ = \frac{\text{cov}(x, y | x > \mu_x, y > \mu_y)}{\text{var}(y | x > \mu_x, y > \mu_y)} = \frac{\sigma_x^+}{\sigma_y^+} \text{corr}(\tilde{x}, \tilde{y} | \tilde{x} > 0, \tilde{y} > 0), \quad (12)$$

such that $\sigma_x^+ = \sqrt{\text{var}(x | x > \mu_x, y > \mu_y)}$ and $\sigma_y^+ = \sqrt{\text{var}(y | x > \mu_x, y > \mu_y)}$. Similarly, we can define a downside beta, β^- , as

$$\beta^- = \frac{\text{cov}(x, y | x < \mu_x, y < \mu_y)}{\text{var}(y | x < \mu_x, y < \mu_y)} = \frac{\sigma_x^-}{\sigma_y^-} \text{corr}(\tilde{x}, \tilde{y} | \tilde{x} < 0, \tilde{y} < 0), \quad (13)$$

for which $\sigma_x^- = \sqrt{\text{var}(x | x < \mu_x, y < \mu_y)}$ and $\sigma_y^- = \sqrt{\text{var}(y | x < \mu_x, y < \mu_y)}$.

Denoting $k^+ = \sigma_x^+ / \sigma_y^+$ and $k^- = \sigma_x^- / \sigma_y^-$ we can write β^+ and β^- as

$$\beta^+ = k^+ \times \bar{\rho}(0)^+,$$

and

$$\beta^- = k^- \times \bar{\rho}(0)^-, \quad (14)$$

where $\bar{\rho}(0)^+ = \text{corr}(\tilde{x}, \tilde{y} | \tilde{x} > 0, \tilde{y} > 0)$ is the positive exceedance correlation at $\vartheta = 0$, and $\bar{\rho}(0)^- = \text{corr}(\tilde{x}, \tilde{y} | \tilde{x} < 0, \tilde{y} < 0)$ is the negative exceedance correlation at $\vartheta = 0$. The term k^+ is the ratio of upside portfolio volatility to market volatility, and the term k^- is the ratio of downside portfolio volatility to market volatility.

For a bivariate normal distribution, β^+ must equal β^- by symmetry. The proposition in Appendix B can be used to calculate β^+ and β^- in closed-form. Note that for a bivariate normal distribution, k^- equals k^+ . Betas increase on the downside if the downside exceedance correlation increases, or if portfolios become more volatile on the downside relative to the market. In order for the latter condition to hold, the conditional $\text{var}(\tilde{x} | \tilde{x} < 0, \tilde{y} < 0)$ must increase relative to $\text{var}(\tilde{y} | \tilde{x} < 0, \tilde{y} < 0)$, when compared to their upside counterparts.

The upside and downside betas examined here are related to, but different from, the asymmetric betas defined by Ball and Kothari (1989), Braun et al. (1995), and Cho and Engle (2000). The asymmetric betas previously defined in this literature depend on the sign of unanticipated shocks realized at a given time. Our upside and downside betas are calculated conditional on the tails of the distribution, analogous to the upside and downside exceedance correlations. Forbes and Rigobon (1999) study the relationship described in Eq. (14) and note that if betas are constant, asymmetric covariances may be driven by asymmetries in volatility. Bawa and Lindenberg (1977) use upside and downside betas, calculated conditional only on the market return, while we calculate betas conditional on quadrants, analogous to the exceedance correlations.

3.4. Asymmetric betas in the returns data

We investigate whether empirically different upside and downside betas are the result of asymmetries either in volatility or of correlations. Under the normal distribution, upside and downside betas are equal. Table 3 reports β^- and β^+ for

Table 3

Asymmetries in beta and volatility

This table shows the level of asymmetries in the betas among equity portfolios. The first column of this table shows the unconditional beta observed in the data. The second column shows the beta conditional on an upside or downside move under the normal distribution. The third and fourth columns show the conditional betas observed in the data. The fifth and sixth columns show k^- and k^+ where $k^- = \sigma_x^- / \sigma_y^-$, $k^+ = \sigma_x^+ / \sigma_y^+$, $\sigma_x^- = \sqrt{\text{var}(x|x < \mu_x, y < \mu_y)}$, $\sigma_y^- = \sqrt{\text{var}(y|x < \mu_x, y < \mu_y)}$, $\sigma_x^+ = \sqrt{\text{var}(x|x > \mu_x, y > \mu_y)}$ and $\sigma_y^+ = \sqrt{\text{var}(y|x > \mu_x, y > \mu_y)}$. The last column shows the p -value of testing $k^- = k^+$. * Indicates rejection at the 5% confidence level. ** Indicates rejection of a test that the observed value equal the theoretical value at the 1% confidence level. Tests for the observed β^- and β^+ determine if β^- or β^+ equals the theoretical value implied by a normal distribution. p -values for the test of $k^- = k^+$ are calculated using bootstrap methodology with 1000 simulated samples. Data is sampled weekly from July 1963 to December 1998. The market portfolio is the value-weighted index of all stocks in CRSP. Panel A shows the level of asymmetries in the betas of the value-weighted industry portfolios. Panels B and C show the level of asymmetries in the betas of the value-weighted portfolios formed by sorting on market capitalizations and book-to-market ratios, respectively.

Portfolio	Unconditional	Theoretical	Observed		Observed		$k^- = k^+$
	β	$\beta^- = \beta^+$	β^-	β^+	k^-	k^+	p -value
<i>Panel A. Industry portfolios (value-weighted)</i>							
Misc	1.10	0.89	1.16**	0.81	1.42	1.21	0.0286*
Petroleum	0.84	0.56	0.74**	0.61	1.16	1.22	0.5503
Finance	1.01	0.89	0.97**	0.87	1.10	1.08	0.6163
Durables	1.13	1.02	1.09**	0.95**	1.21	1.16	0.3070
Basic ind	1.01	0.93	1.03**	0.88*	1.10	1.01	0.0026**
Food/tobacco	0.85	0.69	0.85**	0.66	1.02	0.93	0.2309
Construction	1.14	0.96	1.19**	0.91	1.37	1.22	0.0099**
Capital goods	1.12	0.99	1.08**	0.97	1.23	1.18	0.2629
Transportation	1.17	0.94	1.12**	0.86	1.37	1.31	0.3237
Utilities	0.63	0.47	0.57**	0.46	0.75	0.82	0.3037
Textile/trade	1.08	0.89	1.06**	0.88	1.28	1.21	0.3002
Service	1.28	1.09	1.25**	1.03	1.47	1.35	0.1471
Leisure	1.14	0.96	1.13**	0.90	1.35	1.22	0.1454
<i>Panel B. Size portfolios (value-weighted)</i>							
1 Smallest	0.92	0.71	1.01**	0.65	1.27	1.08	0.0232*
2	1.03	0.88	1.11**	0.81*	1.30	1.07	0.0032**
3	1.03	0.92	1.11**	0.84*	1.24	1.02	0.0003**
4	1.03	0.98	1.07**	0.91**	1.13	1.00	0.0002**
5 Largest	0.98	0.96	0.98	0.92*	1.01	0.95	0.0766
<i>Panel C. Book to market portfolios (value-weighted)</i>							
1 Growth	1.10	1.03	1.08*	1.01	1.16	1.10	0.3015
2	1.01	0.96	1.04**	0.89	1.09	0.98	0.0003**
3	0.90	0.82	0.95**	0.75	1.03	0.90	0.0000**
4	0.83	0.73	0.82**	0.67	0.93	0.86	0.1304
5 Value	0.85	0.70	0.81**	0.64	0.98	0.94	0.5116

industry, size, and book-to-market portfolios. The first column of Table 3 lists the unconditional beta of each portfolio. The second column gives the theoretical value of $\beta^- = \beta^+$ assuming the null hypothesis of a normal distribution. In all portfolios, $\beta^- > \beta > \beta^+$, where β is the unconditional beta. In all cases except one, we reject the hypothesis that β^- is equal to its theoretical value implied by a normal distribution. However, on the upside, we usually fail to reject the hypothesis that β^+ is equal to its theoretically implied value.

Volatility is well-known to be asymmetric and increasing on the downside. For the market, the downside volatility, σ_y^- equals 0.0148, and the upside volatility, σ_y^+ equals 0.0129. The theoretical value implied by a normal distribution is $\sigma_y^- = \sigma_y^+ = 0.0122$. We reject the hypothesis that the observed σ_y^- equals this value at a 1% confidence interval, but fail to reject the hypothesis that σ_y^+ equals this value at a 5% confidence interval. However, the ratio of the downside portfolio volatility to the market $k^- = \sigma_x^-/\sigma_y^-$ is roughly the same as the ratio of upside portfolio volatility to the market $k^+ = \sigma_x^+/\sigma_y^+$. The last three columns of Table 3 show k^- , k^+ and a p -value of the test that k^- and k^+ are equal. The table shows that, in most cases, we cannot reject that the upside and downside volatilities are equal. Hence, the statistically significant increase in downside betas is largely driven by the increase in downside correlations relative to upside correlations, as shown in Eq. (14).

4. A formal characterization of asymmetric correlations

We develop a summary H statistic of correlation asymmetries which quantitatively measures asymmetric correlations. Previously, we have concentrated on asymmetric correlations relative to a normal distribution as the null distribution, but our analysis can handle more general distributions. The H statistic has several advantages over graphical approaches. First, the statistic formally summarizes the magnitudes of correlation asymmetries by providing a succinct numerical measure. That is, the degree of asymmetry can be measured and compared across different portfolios and different frequencies. The H statistic can be used to rank portfolios, allowing us to examine whether various characteristics of equity portfolios are related to the degree of correlation asymmetry. Second, we can numerically compare empirical exceedance correlations with those implied by a null distribution. By doing so, we account for the conditioning bias in the exceedance correlations. Finally, we can formally test if exceedance correlations in the data can be produced by candidate null distributions.

4.1. Description of the H statistic

As in Eq. (10), we denote the exceedance correlation for a given exceedance level ϑ_i as $\bar{\rho}(\vartheta_i)$ for standardized data (\tilde{x}, \tilde{y}) . We choose N exceedance levels $\theta = (\vartheta_1, \vartheta_2, \dots, \vartheta_N)$. These exceedance levels are set exogenously. Suppose we wish to test if a distribution $\xi(\phi)$, characterized by parameters ϕ , can produce the empirical

exceedances $\bar{\rho}(\vartheta_i)$ in the data. We denote the exceedance correlations implied by distribution $\xi(\phi)$ as $\check{\rho}(\vartheta_i, \phi)$.

If $\xi(\phi)$ were to perfectly explain the degree of correlation asymmetry in the data, then, on average, we would have $\bar{\rho}(\vartheta_i) - \check{\rho}(\vartheta_i, \phi) = 0$. We create a quadratic statistic based on this difference. The statistic $H = H(\phi)$ is defined as

$$H = \left[\sum_{i=1}^N w(\vartheta_i) \cdot (\check{\rho}(\vartheta_i, \phi) - \bar{\rho}(\vartheta_i))^2 \right]^{1/2}, \quad (15)$$

for which the weights $w(\vartheta_i) \geq 0$ satisfy

$$\sum_{i=1}^N w(\vartheta_i) = 1. \quad (16)$$

This statistic measures a weighted average of the squared differences of the exceedance correlations implied by a model and those given by data. For example, an $H = 0.116$ means that, on average, the exceedance correlations in the data lie 11.6 percentage points away from the exceedance correlations implied by the model. To briefly preview our results, while a normal distribution would imply a conditional downside correlation of around 76% on average, the conditional downside correlation in the data is around 87.6%. Note that H is a non-linear function of parameters ϕ for a fixed set of ϑ . To look at correlations jointly over upside and downside movements, we set the exceedance levels $\theta = [-1.5, -1.0, -0.5, 0.0, 0.0, 0.5, 1.0, 1.5]$. The repeated zero is necessary because we calculate exceedance correlations $\text{corr}(\tilde{x}, \tilde{y} | \tilde{x} < 0, \tilde{y} < 0)$ and $\text{corr}(\tilde{x}, \tilde{y} | \tilde{x} > 0, \tilde{y} > 0)$. We use a set of positive exceedances $\theta^+ = [0.0, 0.5, 1.0, 1.5]$ to look at correlations on the upside and a set of negative exceedances $\theta^- = [-1.5, -1.0, -0.5, 0.0]$ to assess correlations on the downside. We denote the H statistics calculated from θ^- and θ^+ as H^- and H^+ , respectively. Note that $H^2 = (H^+)^2 + (H^-)^2$, so H represents a non-linear average of H^+ and H^- .

In addition to a quadratic distance, we also consider the weighted sum of the differences between the exceedance correlations in the data and those implied by the model.¹¹ We define the statistic AH as

$$AH = \sum_{i=1}^N w(\vartheta_i) \cdot (\check{\rho}(\vartheta_i, \phi) - \bar{\rho}(\vartheta_i)). \quad (17)$$

The AH statistic may be zero or even negative if some of the exceedance correlations in the data are less than the exceedance correlations implied by the model. In contrast, by construction the H statistic is strictly positive.

The weights $w(\vartheta_i)$ are exogenously set, and are related to the level of sampling error associated with a particular exceedance correlation. The more accurately estimated the exceedance correlation for exceedance level ϑ_i , the higher we set $w(\vartheta_i)$. Below, we discuss various choices for the weights $w(\vartheta_i)$.

¹¹ We thank an anonymous referee for suggesting this analysis.

The H statistic can be written in matrix notation. We denote $\bar{\rho}(\theta)$ as the N vector of exceedances from data, and $\check{\rho}(\theta, \phi)$ as the N vector of exceedances implied by distribution $\xi(\phi)$:

$$\bar{\rho}(\theta) = \begin{pmatrix} \bar{\rho}(\mathfrak{g}_1) \\ \bar{\rho}(\mathfrak{g}_2) \\ \dots \\ \bar{\rho}(\mathfrak{g}_N) \end{pmatrix} \quad \text{and} \quad \check{\rho}(\theta, \phi) = \begin{pmatrix} \check{\rho}(\mathfrak{g}_1, \phi) \\ \check{\rho}(\mathfrak{g}_2, \phi) \\ \dots \\ \check{\rho}(\mathfrak{g}_N, \phi) \end{pmatrix}. \quad (18)$$

Following Eq. (18), H can be expressed as

$$H = \sqrt{(\bar{\rho} - \check{\rho}(\phi))' \Omega^{-1} (\bar{\rho} - \check{\rho}(\phi))}, \quad (19)$$

in which we suppress the dependence on θ . In Eq. (19), $\Omega = \Omega(\theta)$ is a fixed diagonal weighting matrix dependent only on θ , which takes the form

$$\Omega = \begin{pmatrix} w(\mathfrak{g}_1)^{-1} & 0 & \dots & 0 \\ 0 & w(\mathfrak{g}_2)^{-1} & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \dots & w(\mathfrak{g}_N)^{-1} \end{pmatrix}. \quad (20)$$

If ξ is normally distributed, the implied exceedance correlations $\check{\rho}(\theta, \phi)$ can be calculated in closed-form using the proposition in Appendix B. We detail the calculation of the standard errors for H in Appendix C.

4.2. Choices of weights

The H statistic can be interpreted as the square root of a quadratic statistic. For the quadratic statistic J , if we suppress θ , J can be written as

$$J = (\bar{\rho} - \check{\rho}(\phi))' \Omega^{-1} (\bar{\rho} - \check{\rho}(\phi)). \quad (21)$$

In this form, the efficient choice for Ω , Ω_E , is

$$\Omega_E = \text{var}(\bar{\rho}) - 2 \text{cov}(\bar{\rho}, \check{\rho}(\phi)) + \text{var}(\check{\rho}(\phi)), \quad (22)$$

for the case that N is less than the number of parameters in ϕ . We choose not to use the efficient weighting matrix for two reasons.

First, if the data are fixed, or we estimate $\bar{\rho}(\theta)$ without error, then $\Omega_E = \text{var}(\check{\rho}(\theta, \phi))$ and J would have a conventional χ_N^2 distribution. For a normal distribution, there is only one degree of freedom, $\phi = \rho$, in the parameters of the bivariate normal, which determines the exceedance correlation. Hence, this approach would mean only one exceedance correlation can be incorporated in J . In the case of a normal distribution with $N > 1$, $\Omega_E = \check{D}' \Gamma \check{D}$, where $\check{D} = (\partial/\partial\phi)\check{\rho}(\theta, \phi)$ is singular because there are more restrictions imposed by exceedance correlations than degrees of freedom allowed by the parameters. However, we can capture the notion of using weights inversely proportional to the sample variance of $\check{\rho}(\theta, \phi)$, $\sigma^2(\check{\rho}(\mathfrak{g}_i, \phi))$, by

using a standardized measure of the inverse of $\sigma^2(\check{\rho}(\mathcal{G}_i, \phi))$:

$$w(\mathcal{G}_i) = \frac{\sigma^{-2}(\check{\rho}(\mathcal{G}_i, \phi))}{(\sum_{j=1}^N \sigma^{-2}(\check{\rho}(\mathcal{G}_j, \phi)))}. \quad (23)$$

The larger the sampling variance of $\check{\rho}(\mathcal{G}_i, \phi)$, the smaller the weight placed on that exceedance. We calculate $\sigma^2(\check{\rho}(\mathcal{G}_i, \phi))$ using the δ -method:

$$\sigma^2(\check{\rho}(\mathcal{G}_i, \phi)) = D'_{2i} \Gamma D_{2i} \quad (24)$$

for which \check{D}_i is

$$\check{D}_i = \frac{\partial}{\partial \phi} \check{\rho}(\mathcal{G}_i, \phi). \quad (25)$$

The difference between this choice of Ω and the efficient generalized method of moments (GMM) choice is that Ω is diagonal, to avoid singularities, and is normalized to unity.

The second reason we choose not to use the efficient weighting matrix is that each different model or distribution ξ implies a different weighting matrix. The first choice of weights above is not immune to this critique. Since each model implies a different set of weights, the H statistics are not directly comparable across models. Like the constant weighting matrix of Hansen and Jagannathan (1997) used to compare different pricing kernels with the same data, we would like to use a constant weighting matrix to compare different models with the same data. The next two choices of weights do not depend on any particular distribution, and therefore can be used to compare different models regarding their estimates of correlation asymmetry.

The second set of weights is held constant across models, and takes into account some notion of sampling error. We note that increasing the number of observations increases the accuracy of the estimate. For the normal distribution, covariance sampling error is of the order $1/\sqrt{T}$, where T is the sample size. One way to account for sampling error is to set the weights proportional to the number of observations used to calculate the exceedance correlations. Hence, a second choice for $w(\mathcal{G}_i)$ uses weights

$$w(\mathcal{G}_i) = \frac{T_i}{(\sum_{j=1}^N T_j)}, \quad (26)$$

such that T_i is the number of observations used in calculating $\bar{\rho}(\mathcal{G}_i)$, the sample exceedance correlation at the exceedance level \mathcal{G}_i . This choice of weights places more emphasis on exceedance correlations for which more data are available.

Finally, equal weights may be used

$$w(\mathcal{G}_i) = \frac{1}{N}. \quad (27)$$

This choice places greater weight on observations in the extreme tails of the distribution than the previous choice of weights.

Our preferred form of the H statistic uses the weights presented in Eq. (26). However, we show all of our results to be robust to different choices of weights.

4.3. Magnitudes and tests of asymmetric correlations

For various pairs of standardized excess returns of the market and stock portfolios (\tilde{x}, \tilde{y}), we estimate the unconditional correlation ρ and calculate H under the null hypothesis of a bivariate standard normal distribution with unconditional correlation ρ . We estimate the standard errors of H , H^- , H^+ , and AH by GMM using six Newey-West lags as described in Newey and West (1987).

The H statistics capture the same features as the exceedance plots. H statistics are reported in the legends of Fig. 3 corresponding to the various portfolios. The larger the difference in positive and negative exceedance correlations, the larger the H statistic. With the H statistic, a numerical measure of the correlation asymmetry can now be assigned to each portfolio.

4.3.1. Impact of weights and frequencies

Table 4 presents H statistics using the three choices of weights for the five size-sorted portfolios at daily, weekly, and monthly frequencies. The size portfolios are representative of the results obtained for all the portfolios. Columns 1 and 2 of Table 4 present the H statistics weighted by the variances in the normal distribution using Eq. (23). Columns 3 and 4 are weighted by the number of observations used to construct the sample exceedances using Eq. (26). The last two columns present equal-weighted H statistics using Eq. (27).

There are two major results of Table 4 which we present as Empirical Facts.

Empirical Fact 1. Asymmetric correlations in the data lead us to reject the null hypothesis of a normal distribution.

Empirical Fact 2. The magnitude of the correlation asymmetries is unrelated to the horizon.

In Table 4, the p -values of the H statistics are all less than 2.5%, across all choices of weights and frequencies, and therefore are not reported. There is also no discernable pattern across the sampling frequencies. For the smallest and the largest size portfolios, correlation asymmetries with the market portfolio are the greatest at the monthly frequency with all three weight choices.

The equally weighted H statistic is always larger than the other two choices of weights. This result occurs because the largest sampling error in the normal distribution and the smallest number of observations occur at the largest absolute value exceedance levels at $\mathfrak{g} = \pm 1.0$, and 1.5. At these exceedances, in particular for the negative exceedances, the largest discrepancies between the normal distribution and the sample exceedance correlations arise (see Figs. 3 and 4). These discrepancies are given more weight in the equally weighted H statistic.

These results extend to other portfolios. Since the rejection of the normal distribution and the patterns of asymmetries are robust to the weighting choice and the frequency of observations, we concentrate on using weights proportional to the

Table 4

H Statistics for the size portfolios with the market

We present the *H* statistics under the null hypothesis of a bivariate normal distribution for the value-weighted size-sorted portfolios. A different bivariate normal is fitted for each pair of (*x*, *y*) observations, where *x* is the normalized excess market return and *y* is a normalized excess stock portfolio return. The market portfolio is the value-weighted index of all stocks in CRSP and the stocks portfolios are the value-weighted size portfolios, formed by sorting on market capitalizations. Panels A, B, and C report results at the daily, weekly, and monthly frequencies, respectively.

Columns labeled SE display the standard error of the model. The first two columns reflect weights constructed using the variances of the exceedance correlations $\hat{\rho}(\theta, \phi)$ implied by a bivariate normal distribution, as in Eq. (23). In Columns 3 and 4, the weights are proportional to the number of observations used to construct each $\hat{\rho}(\theta)$, the sample exceedance, shown in Eq. (26). The last two columns use equal weights. The null hypothesis of a bivariate normal is rejected at the 2.5% confidence level for every portfolio, at all frequencies, by the *H* statistics (*p*-values are not reported). All standard errors are calculated using GMM and 6 Newey and West (1987) lags. Data is from July 1963 to December 1998.

Portfolio	Weighted by normal distn $\sigma^2(\hat{\rho})$		Weighted by number of observations		Equally weighted	
	<i>H</i>	SE	<i>H</i>	SE	<i>H</i>	SE
<i>Panel A. Daily frequency</i>						
1 Smallest	0.152	0.036	0.150	0.034	0.185	0.053
2	0.141	0.028	0.132	0.025	0.178	0.038
3	0.122	0.023	0.109	0.020	0.151	0.029
4	0.081	0.017	0.068	0.014	0.096	0.020
5 Largest	0.023	0.004	0.017	0.004	0.026	0.004
<i>Panel B. Weekly frequency</i>						
1 Smallest	0.150	0.057	0.147	0.055	0.199	0.070
2	0.098	0.043	0.091	0.040	0.126	0.053
3	0.074	0.032	0.065	0.029	0.092	0.039
4	0.049	0.020	0.040	0.017	0.059	0.022
5 Largest	0.012	0.006	0.010	0.004	0.014	0.007
<i>Panel C. Monthly frequency</i>						
1 Smallest	0.214	0.048	0.214	0.045	0.252	0.064
2	0.146	0.036	0.135	0.033	0.175	0.049
3	0.120	0.028	0.096	0.027	0.162	0.031
4	0.057	0.021	0.046	0.019	0.067	0.023
5 Largest	0.043	0.010	0.037	0.008	0.047	0.011

number of observations in each sample exceedance as suggested in Eq. (26) and analyze the weekly frequency for the rest of the paper.

4.3.2. Characterizing asymmetric correlations

In order to further characterize the asymmetric correlations in equity portfolios, we examine the relationship between different portfolio sortings and their *H* statistics. To estimate extreme correlations requires us to focus on the observations lying in the tails where there are relatively few data points. The *H* statistic uses the full time series sample of returns to measure correlation asymmetries. To maintain

the use of the full sample, we sort portfolios of stocks by various cross-sectional characteristics and examine their correlation asymmetries.

Table 5 presents the H statistics across a wide selection of portfolios, assuming the null hypothesis of a bivariate normal distribution. Panels A and B examine the properties of portfolios formed by industry classifications, size, book-to-market and momentum. Panels E through G investigate the asymmetry properties of portfolios formed by past beta, co-skewness, and leverage. For all panels, the first four columns of Table 5 show the H , H^- , H^+ , and AH statistics. The H statistics for all portfolios have p -values smaller than the 2.5% level of significance, just as Table 4 show for the size portfolios. The AH statistics also have p -values smaller than 2.5%. The average H^- statistic across all portfolios is 0.1161, while the average H^+ statistic is only 0.0300. We therefore observe the following:

Empirical Fact 3. Correlation asymmetries are greater for extreme downward moves.

Further, only nine portfolios out of 43 portfolios reject at the 5% significance level that the upside correlations can be reproduced by a theoretical bivariate normal distribution. In contrast, all H^- statistics reject this hypothesis at the 1% level of significance. In calculating the average H statistics across portfolios in Table 5, we observe:

Empirical Fact 4. Conditional on downside and upside moves, on average the observed correlations between a portfolio and the market differ from the correlations implied by a normal distribution by 8.48%. Conditional on just downside moves, the average difference is 11.61%.

The sixth and seventh columns of Table 5 report standardized measures of skewness and co-skewness, and their standard errors. In Table 5, skewness and co-skewness are defined as

$$\text{skewness} = \frac{E[\dot{x}^3]}{(E[\dot{x}^2])^{3/2}} \quad (28)$$

$$\text{and co-skewness} = \frac{E[\dot{x}\dot{y}^2]}{\sqrt{E[\dot{x}^2]E[\dot{y}^2]}}. \quad (29)$$

In Eqs. (28) and (29), \dot{x} is the de-meaned excess return of the portfolio x , $\dot{x} = x - E(x)$ and \dot{y} is the de-meaned excess return of the market y , $\dot{y} = y - E(y)$. All standard errors are calculated by GMM using 6 Newey-West lags.

Table 5 also shows that, at the weekly frequency, each of the portfolios are both negatively skewed and are negatively co-skewed with the market. This finding may indicate that there is some common component among all three asymmetry statistics. To ensure that we are not capturing the same information in H as skewness and co-skewness, we present the correlation among these statistics across the 43 portfolios in Table 6. The correlation of H with skewness is 0.243, and with co-skewness is only 0.150. We also find similar correlation results using AH instead of H . This finding

Table 5

H Statistics from a bivariate normal distribution

This table presents asymmetry statistics for equity portfolios assuming the null hypothesis of a bivariate normal distribution. Weights proportional to the number of observations in each sample exceedance are used (Eq. (26)) to construct the *H* statistics. Data is sampled weekly from July 1963 to December 1998. The market portfolio is the value-weighted index of all stocks in CRSP. Panel A shows the statistics of the value-weighted industry portfolios. Panels B and C show the statistics of the value-weighted portfolios formed by sorting on market capitalizations and book-to-market ratios, respectively. Panel D presents the statistics of the equal-weighted portfolios of stocks sorted by their lagged past six-months returns, with one to six months of lags. Panels E, F, and G show the statistics of the value-weighted portfolios formed by sorting on beta, co-skewness, and leverage, respectively.

The second column of this table shows the mean returns of the portfolios, annualized by multiplying the weekly mean by 52. The third, fourth, fifth, and sixth columns show the *H*, *H*[−], *H*⁺, and *AH* statistics, respectively. For the *H*, *H*[−], *H*⁺, and *AH* statistics, * and ** indicate that the model cannot be rejected at the 5% and 1% confidence levels, respectively. The seventh and eighth columns show skewness and co-skewness, respectively. For skewness and co-skewness, *** indicates rejection of the hypothesis that the statistic is not different from zero at the 5% confidence level. The last column shows the beta of the portfolios. All standard errors are calculated using GMM and 6 Newey and West (1987) lags.

Portfolio	Mean	<i>H</i>	<i>H</i> [−]	<i>H</i> ⁺	<i>AH</i>	Skewness	Co-skewness	β
<i>Panel A. Industry Portfolios (value-weighted)</i>								
Misc	0.031	0.125	0.174	0.033*	0.080	−0.881	−0.624	1.104
Petroleum	0.054	0.180	0.237	0.094**	0.146	−0.148	−0.353	0.839
Finance	0.062	0.074	0.099	0.033*	0.055	−0.378	−0.451	1.006
Durables	0.052	0.078	0.109	0.016*	0.043	−0.528	−0.540	1.134
Basic ind	0.056	0.072	0.101	0.015*	0.052	−0.670	−0.580	1.014
Food/tobacco	0.081	0.117	0.163	0.032*	0.087	−0.500***	−0.477	0.847
Construction	0.050	0.119	0.165	0.037**	0.083	−0.875	−0.649	1.136
Capital goods	0.048	0.087	0.116	0.043**	0.068	−0.509	−0.515	1.123
Transportation	0.032	0.132	0.185	0.020*	0.082	−0.572	−0.573	1.172
Utilities	0.048	0.145	0.202	0.026*	0.089	−0.115	−0.401	0.630
Textile/trade	0.060	0.125	0.165	0.064**	0.099	−0.568	−0.527	1.080
Service	0.074	0.094	0.132	0.027*	0.072	−0.583	−0.522	1.280
Leisure	0.065	0.078**	0.110	0.019*	0.055	−0.539***	−0.499	1.138
<i>Panel B. Size portfolios (value-weighted)</i>								
1 Smallest	0.054	0.147	0.198	0.066**	0.102	−0.893	−0.654	0.920
2	0.066	0.091**	0.124	0.039*	0.068	−0.953	−0.629	1.035
3	0.064	0.065**	0.088	0.031*	0.049	−0.935	−0.623	1.033
4	0.063	0.040	0.054	0.018*	0.028	−0.717	−0.576	1.031
5 Largest	0.053	0.010**	0.012	0.007**	0.008	−0.530	−0.502	0.982
<i>Panel C. Book to market portfolios (value-weighted)</i>								
1 Growth	0.048	0.037	0.045	0.027	0.032	−0.454	−0.463	1.100
2	0.049	0.045	0.063	0.012*	0.028	−0.662	−0.569	1.014
3	0.050	0.080	0.108	0.035*	0.053	−0.903	−0.652	0.904
4	0.073	0.090	0.122	0.038**	0.050	−0.531	−0.542	0.831
5 Value	0.091	0.100	0.136	0.038*	0.048**	−0.398	−0.495	0.855
<i>Panel D. Momentum portfolios (equal-weighted)</i>								
1 Past losers	0.023	0.165	0.224	0.058*	0.120	−0.112	−0.486	0.914
2	0.060	0.119	0.163	0.041*	0.079	−0.590	−0.568	0.825
3	0.078	0.093**	0.129	0.028*	0.062	−1.038	−0.676	0.807

Table 5 (continued)

Portfolio	Mean	H	H^-	H^+	AH	Skewness	Co-skewness	β
4	0.092	0.077**	0.110	0.012*	0.052	-1.345	-0.743	0.848
5 Past winners	0.111	0.092**	0.130	0.016*	0.057	-1.348***	-0.745	0.986
<i>Pabel E. Beta portfolios (value-weighted)</i>								
1 Low beta	0.071	0.123	0.171	0.038*	0.072	-0.762	-0.633	0.612
2	0.074	0.053	0.074	0.015*	0.037	-0.733	-0.583	0.864
3	0.071	0.057	0.078	0.019*	0.035	-0.719	-0.586	0.977
4	0.063	0.056	0.076	0.025*	0.039	-0.762	-0.588	1.109
5 High beta	0.040	0.068**	0.091	0.033*	0.050	-0.499	-0.501	1.330
<i>Panel F. Co-skewness portfolios (value-weighted)</i>								
1 Low/neg. Coskew	0.079	0.066	0.092	0.022*	0.037	-0.686***	-0.576	0.994
2	0.080	0.054	0.075	0.020*	0.034	-0.759	-0.596	1.005
3	0.085	0.057	0.077	0.026*	0.033**	-0.614	-0.561	0.990
4	0.055	0.053	0.072	0.020*	0.033	-0.872	-0.623	0.988
5 High/pos. Coskew	0.058	0.067**	0.090	0.031*	0.048	-0.440	-0.482	0.924
<i>Panel G. Leverage portfolios (value-weighted)</i>								
1 Low debt	0.057	0.063	0.086	0.025**	0.042	-0.821	-0.618	1.003
2	0.064	0.043	0.059	0.018*	0.028	-0.472	-0.489	0.957
3	0.062	0.050**	0.068	0.021*	0.030	-0.601	-0.544	0.967
4	0.061	0.059	0.081	0.022*	0.031	-0.735	-0.599	0.964
5 High debt	0.058	0.101	0.140	0.030*	0.067	-0.613	-0.577	1.024

Table 6

Correlations among asymmetry statistics

We present the correlations among the asymmetry statistics calculated in Table 5. The correlations are calculated using the 43 estimates of H , H^- , H^+ , and AH statistics, skewness, co-skewness, and beta, as presented in Table 5.

	H	H^-	H^+	AH	Skewness	Coskewness	β
H	1.000	0.998	0.751	0.964	0.243	0.150	-0.274
H^-		1.000	0.714	0.953	0.220	0.123	-0.280
H^+			1.000	0.824	0.342	0.321	-0.167
AH				1.000	0.272	0.222	-0.177
Skewness					1.000	0.951	-0.054
Coskewness						1.000	-0.003
β							1.000

indicates that H is capturing something that is fundamentally different from skewness or co-skewness. Skewness and co-skewness are much more highly correlated at 0.951, as are H and AH (correlation of 0.964).

The final column of Table 5 reports the betas of the portfolios with respects the market. The correlations between the H statistics, skewness, co-skewness, and beta

are also reported in Table 6. All measures of return asymmetries appear to have little positive relation with systematic risk. In particular, the H statistics are negatively correlated (-0.274) with the beta.

Table 5 reveals that certain portfolios exhibit greater asymmetric correlations than others, leading to:

Empirical Fact 5. Petroleum and utility industries have the most asymmetric correlations, while financial firms and basic industries exhibit the lowest asymmetric correlations.

Among industries, petroleum ($H = 0.180$) and utilities ($H = 0.145$) are the most asymmetric, while financials and basic industries exhibit the least asymmetric correlations. Petroleum and utilities have low betas (0.839 and 0.630, respectively), suggesting that investing in these traditional defensive sectors may be less beneficial than popularly believed. Note that these industries have the least negative skewness and co-skewness, and would appear, by these measures, to be the most normal.

Among size-sorted stock portfolios, we observe the following:

Empirical Fact 6. Decreasing size increases the correlation asymmetry.

This pattern has been previously documented in a GARCH specification by Kroner and Ng (1998) and Conrad et al. (1991). The book-to-market portfolios also display an increasing pattern of H statistics going from growth to value stocks, leading to the following:

Empirical Fact 7. Value stocks are more asymmetric than growth stocks.

While Fama and French (1993) observe size and value premia, portfolios formed on these characteristics may be more risky by their greater correlation asymmetry than by measuring risk only by second moments. In both the size and book-to-market portfolio sortings, the H statistics are monotonic, unlike the point statistics of the skewness and co-skewness measures. Moreover, the latter two measures do not display any discernable pattern.

Turning to the momentum quintiles, we observe:

Empirical Fact 8. The past loser portfolio has greater correlation asymmetry than the past winner quintile.

In the momentum strategy postulated by Jegadeesh and Titman (1993), investors sell short past loser stocks and invest in past winner stocks. In periods of extreme downside moves, the loser portfolio is more likely to lose more money than estimated under constant correlations, thus affording momentum players even greater rewards in down markets. This effect exacerbates the puzzle posed by the momentum effect. Like Chen et al. (2001) and Harvey and Siddique (2000), we find that the past winner portfolio is more negatively skewed than the past loser

Table 7

H Statistics across size/beta portfolios

The table presents the *H* statistics for equity portfolios assuming the null hypothesis of a bivariate normal distribution. We first stocks by size into quintiles. Then, within each size quintile, we further sort stocks into quintiles based on beta. For each size and beta grouping, we form a value-weighted equity portfolio. Frequency of the data is weekly. Weights proportional to the number of observations in each sample exceedance are used (see Eq. (26)) to construct the *H* statistic. The null hypothesis of a bivariate normal is rejected at the 2.5% confidence level for every portfolio at all frequencies by the *H* statistic (*p*-values are not reported).

Size	Beta				
	1 Low	2	3	4	5 High
1 Smallest	0.185	0.162	0.167	0.159	0.145
2	0.168	0.118	0.119	0.118	0.100
3	0.149	0.100	0.094	0.088	0.083
4	0.157	0.088	0.077	0.058	0.067
5 Largest	0.127	0.082	0.057	0.056	0.056

portfolio, which is consistent with a premium associated with skewness. However, the relationship between *H* and skewness or co-skewness goes in the opposite direction, such that the past losers are the least skewed or co-skewed, and are the most asymmetric.

In Panels E–G of Table 5, we search for additional determinants of asymmetries.¹² We first characterize the correlation asymmetries of portfolios sorted by systematic risk, measured by the beta. The portfolio of lowest beta stocks is the portfolio that exhibits the greatest correlation asymmetry. Lower risk firms exhibit more correlation asymmetries than higher risk firms. Note that co-skewness monotonically increases with beta, while skewness has no discernable pattern.¹³

The relationship between beta and correlation asymmetry is robust to size controls. In Table 7, we sort the stocks twice, to examine the interaction between size, systematic risk, and correlation asymmetries. For each month, we first sort stocks in our universe into quintiles by size. Then within each size quintile, we perform a second sort of stocks into quintiles by past estimates of beta. We construct value-weighted portfolios within this 5×5 grouping. We find that by controlling for size, riskier firms have fewer correlation asymmetries than less risky firms. In Table 7, we observe that *H* statistics decrease going down the rows, where we sort by size. Going across the columns, where we control for size and sort by beta, the lowest beta stocks, which appear in the first column of data, have the highest *H* statistics. Thus, we conclude that:

¹² We also calculated *H* statistics for portfolios sorted by volatility (no relationship), skewness (results similar to co-skewness), turnover (lower *H* for low turnover stocks), and earnings yield (results similar to book-to-market).

¹³ We also sorted on Scholes and Williams (1977) betas to alleviate potential concerns over non-synchronous trading. We found slightly lower *H* statistics, but the qualitative results were unchanged.

Empirical Fact 9. Increasing beta decreases correlation asymmetry.

When the sorting criteria is individual stock's past co-skewness in Panel B of Table 5, we do not find any pattern between past co-skewness and correlation asymmetry. This results obtained from sorting stocks by co-skewness suggest that co-skewness measure presented in by Harvey and Siddique (2000) is not related to the degree of correlation asymmetry in the data. There is also no pattern in the skewness or co-skewness of portfolios formed by past conditional co-skewness. The risk measured by beta of stocks sorted by past co-skewness is near market risk across all quintiles.

Finally, we observe that the most leveraged stocks have the greatest correlation asymmetry. This effect is weakly monotonic, and not reflected in either the skewness or the co-skewness measures. Bekaert and Wu (2000) find that the leverage effect accounts for only a small proportion of asymmetric covariance. In Table 8 we examine the effect of leverage on correlation asymmetry when controlling for size. We observe, as expected from Empirical Fact 6, that H statistics decrease as stocks become larger. This pattern is most noticeable when making comparisons going down rows. However, when size is held constant, we observe the following:

Empirical Fact 10. There is no relation between leverage and correlation asymmetries controlling for size.

The lack of a pattern between leverage and correlation asymmetry within size groups may account for the weak support Bekaert and Wu (2000) uncover for the leverage effect as an explanation for covariance asymmetry.

4.3.3. Summary of empirical facts

We find that correlation asymmetries in equity portfolios are not fully explained by traditional skewness and co-skewness measures. These correlation asymmetries

Table 8

H Statistics across size/leverage portfolios

This table presents the H statistics for equity portfolios assuming the null hypothesis of a bivariate normal distribution. We first stocks by size into quintiles. Then, within each size quintile, we further sort stocks into quintiles based on leverage. For each size and leverage grouping, we form a value-weighted equity portfolio. Frequency of the data is weekly. Weights proportional to the number of observations in each sample exceedance are used (see Eq. (26)) to construct the H statistic. The null hypothesis of a bivariate normal is rejected at the 2.5% confidence level for every portfolio at all frequencies by the H statistic (p -values are not reported).

Size	Leverage				
	1 Low	2	3	4	5 High
1 Smallest	0.158	0.151	0.156	0.156	0.162
2	0.115	0.116	0.110	0.119	0.117
3	0.104	0.084	0.085	0.106	0.086
4	0.083	0.064	0.072	0.092	0.095
5 Largest	0.079	0.057	0.065	0.066	0.074

persist across daily, weekly, and monthly frequencies, and are greatest for downside moves. Correlation asymmetries are larger for small size, high book-to-market ratios, and low past return portfolios. This observation suggests that size and value strategies are exposed to more contemporaneous downside moves with the market, which is not reflected in measures that solely capture second moments, such as volatility. Momentum strategies are more profitable than they first appear, because in times of market distress, loser stocks, are more likely to fall with the market than past winners. High beta portfolios are less asymmetric than low beta portfolios. Once we have controlled for size, there is no discernable pattern between correlation asymmetries and the leverage of firms.

5. Empirical models of asymmetric correlations

The previous section examines the characteristics of asymmetric correlations relative to a normal distribution. We now seek to explain the correlation asymmetries in the data by using richer models of stock returns which can potentially capture the asymmetric movements. We evaluate several empirical reduced-form models by using the H metric, which measures how closely each model can match the correlation asymmetries in the data. Section 5.1 describes the models, Section 5.2 presents the empirical results of the H statistics using these models as the null distribution, and Section 5.3 provides some intuition behind the rejection patterns.

5.1. Description of models

Our choice of models is motivated by examining several popular models used to capture asymmetries between upside and downside movements in stock returns. We use weekly data, and following Braun et al. (1995), Cho and Engle (2000), and others, we work with independent pairs of stock portfolio and aggregate market observations.

The first model is the GARCH-M model with asymmetry. The GARCH-M model is a time-varying expected returns model, in which volatility risk is priced in the expected return, with the conditional covariances set according to a GARCH process. The GARCH process incorporates asymmetry which allows covariances to increase on the downside. The second model is the Jump model. This model layers negative jumps, which are perfectly correlated in time for both returns, on top of a bivariate normal distribution to produce larger downside correlations. The last two models are regime-switching (RS) models. The RS normal model mixes two different bivariate normal distributions. This process allows returns to switch to a regime with lower conditional means, higher volatility, and higher correlations. Transitioning into this regime increases downside correlations. The RS-GARCH model combines elements of the switching behavior of pure RS normal models with the volatility persistence of GARCH processes.

We note that other empirical models capable of producing asymmetric correlations are available. One large class of models that we do not pursue here are continuous-time stochastic volatility models, in which shocks to conditional mean and conditional volatility factors may be correlated, with jumps in either prices or volatility. This class of models is very hard to estimate (see e.g., Pan, 2001) particularly on multivariate series, and it is not clear that these models would produce markedly different results from the discrete-time weekly data. Our Jump model captures jumps in returns, but without stochastic volatility. The regime-switching models we estimate can both capture stochastic volatility and jump effects through regime switches.

Other models we do not investigate involve residuals drawn from distributions that reflect higher moments. One such model is Harvey and Siddique (1999), which draws from a non-central t -distribution to capture skewness and kurtosis. In a multivariate application, this model is computationally intensive because maximum likelihood methods cannot be used. However, the mixture of normal distributions we employ can also match any degree of conditional skewness and kurtosis, as noted by Bekaert et al. (1998).

5.1.1. An asymmetric GARCH-M model

As before, we denote the excess returns of the equity portfolio by x , and the excess market returns by y . We model the pair (x_t, y_t) as

$$x_t = \delta \text{cov}_{t-1}(x_t, y_t) + \varepsilon_{1,t} \quad (30)$$

and

$$y_t = \delta \text{var}_{t-1}(y_t) + \varepsilon_{2,t}. \quad (31)$$

We take ε_t as a bivariate normal distribution with zero means and variances equal to H_t . The coefficient δ is the price of risk and is positive in the capital asset pricing model (CAPM). We can model the conditional covariances H_t of (x_t, y_t) as a GARCH model, and introduce asymmetry using a multivariate version of Glosten et al. (1993)

$$H_t = C'C + A'H_{t-1}A + B'\varepsilon_{t-1}\varepsilon'_{t-1}B + D'\eta_{t-1}\eta'_{t-1}D \quad (32)$$

for which

$$\eta_{t-1} = \varepsilon_t \odot \mathbf{1}_{\{\varepsilon_{t-1} < 0\}}. \quad (33)$$

The symbol \odot is a Hadamard product representing element by element multiplication, and $\mathbf{1}_{\{\varepsilon_{t-1} < 0\}}$ is a vector of individual indicator functions for the sign of the errors for x and y . The matrices A , B , C , and D are symmetric to ensure that H_t is positive definite. Shocks on the downside increase the variance, as well as the covariance through the asymmetric term in H_t , but they also increase the conditional mean, by allowing H_t to enter the conditional mean shown in Eqs. (30) and (31). Eq. (32) is the asymmetric Baba, Engle, Kraft, and Kroner (BEKK) model of Engle and Kroner (1995), and its multivariate form of asymmetry is a special case of the nomenclature system of Kroner and Ng (1998). Similar GARCH-M models with

asymmetry are estimated by Bekaert and Harvey (1997), De Santis et al. (1999), and Bekaert and Wu (2000).

5.1.2. Bivariate normal distribution with Poisson jumps

Das and Uppal (2001) recommend a model in which returns are drawn from a bivariate normal distribution that allows negative jumps. The jumps occur simultaneously in time for both variables, but the size of the jumps can differ. This jump allowance induces higher correlation with downward moves. The model is given by

$$X_t = \mu + \Sigma^{1/2} \varepsilon_t + \sum_{i=1}^{n_t} Y_t \quad (34)$$

with $X_t = (x_t, y_t)'$. This model incorporates a Poisson jump process with intensity λ , with jump distribution Y_t , which is a bivariate normal distribution with means equal to μ_j and variances equal to H_j . There are n_t actual jumps during each period. Das and Uppal discuss how this model can produce unconditional skewness and kurtosis which match equity data.

5.1.3. Regime-switching bivariate normal distribution

The regime-switching bivariate normal (RS normal) model draws the portfolio returns $X_t = (x_t, y_t)'$ from one of two bivariate normal distributions of returns, depending on the prevailing regime $s_t = 1, 2$ at time t :

$$X_t = \mu(s_t) + \Sigma^{1/2}(s_t) \varepsilon_t. \quad (35)$$

In this model, the error term ε_t is independently and identically distributed as a bivariate normal distribution with zero means and variances equal to I . Following Hamilton (1989), s_t follows a Markov Chain with transition probability matrix Π , given by:

$$\Pi = \begin{pmatrix} P & 1 - P \\ 1 - Q & Q \end{pmatrix}. \quad (36)$$

In Eq. (36), $P = \Pr(s_t = 1 | s_{t-1} = 1)$ and $Q = \Pr(s_t = 2 | s_{t-1} = 2)$.

This model has been used by Ang and Bekaert (2000) to review international asset allocation under higher correlations with downside moves in country returns. Ang and Bekaert show that this model captures a large part of the asymmetric correlations in international equity markets of developed countries. In this model, asset returns are allowed to switch into a regime with higher correlations and volatility, reflecting potentially lower means.

5.1.4. A regime-switching GARCH model

In the regime-switching GARCH (RS-GARCH) model, portfolio returns $X_t = (x_t, y_t)'$ follow the process:

$$X_t = \mu(s_t) + \varepsilon_t \quad (37)$$

with two regimes $s_t = 1, 2$ and the error term ε_t distributed as a bivariate normal distribution with zero means and variances equal to $H_t(s_t)$. The regime variable s_t follows the same Markov Chain with transition probability matrix Π given by Eq. (36). The conditional covariance $H_t(s_t)$ is given by

$$H_t(s_t) = C(s_t)'C(s_t) + A(s_t)'H_{t-1}A(s_t) + B(s_t)'\varepsilon_{t-1}^*\varepsilon_{t-1}^{*'}B(s_t). \quad (38)$$

In Eq. (38), the forecast error ε_{t-1}^* is given by

$$\begin{aligned} \varepsilon_{t-1}^* &= X_{t-1} - E_{t-2}(X_{t-1}) \\ &= X_{t-1} - (p_{t-1}\mu_1 + (1 - p_{t-1})\mu_2) \end{aligned} \quad (39)$$

for which $\mu_i = \mu(s_t = i)$, and p_{t-1} is the ex-ante probability $p_{t-1} = p(s_{t-1} = 1 | \mathcal{I}_{t-2})$. Following Gray (1996) H_{t-1} , is given by

$$\begin{aligned} H_{t-1} &= E_{t-2}(X_{t-1}X_{t-1}') - E_{t-2}(X_{t-1})E_{t-2}(X_{t-1})' \\ &= p_{t-1}(\mu_1\mu_1' + H_{t-2,1}) + (1 - p_{t-1})(\mu_2\mu_2' + H_{t-2,2}) \\ &\quad - [p_{t-1}\mu_1 + (1 - p_{t-1})\mu_2][p_{t-1}\mu_1 + (1 - p_{t-1})\mu_2]', \end{aligned} \quad (40)$$

such that $H_{t-2,i} = H_{t-2}(s_t = i)$. The matrix $C(s_t)$ is symmetric, but for reasons of parsimony we restrict $A(s_t)$ and $B(s_t)$ to be diagonal.

This RS-GARCH model uses an RS version of the Engle and Kroner (1995) BEKK multivariate GARCH model. It uses a multivariate generalization the algorithm presented in Gray (1996), and reflected in Eq. (40) to re-combine the lagged RS conditional covariance term. The model combines the switching character of the RS normal model, with the volatility persistence of GARCH. One of the features of this model is that the volatility can also switch to a regime which reflects both higher volatility and less persistence, with a switch in the conditional mean. Glosten et al. (1993) discuss the pure asymmetric GARCH specifications, cannot easily capture this feature.

5.2. Model performance

In this section, we use the H statistic as a criterion to judge the adequacy of a model to match the asymmetric correlation found in data. We consider a model to do an adequate job of capturing the correlation asymmetry in the data if that model's H statistic cannot be statistically rejected. As a second measure, since the H statistic measures the difference between the empirical conditional correlations and the conditional correlations implied by the models, we consider the average magnitude of H statistics across portfolio pairs. We calculate the H statistics from the models using fixed weights from Eq. (26), which place more weight on sample exceedance correlations that have been calculated with more observations. These weights ensure that the same weighting matrix is used across all four models. In this section, we focus our analysis on the portfolios formed by industry classifications, size, book-to-market ratios, and past returns.

Table 9 summarizes the rejections across the 28 portfolios. The GARCH-M model is rejected by 6 portfolios at the 5% level, the Jump model is rejected by 21

Table 9

Summary of rejections from Table 10

We present a summary of rejections from Table 10. We list the number of rejections, M , out of a possible N number of portfolios as M/N in the table.

	GARCH-M	Jump model	RS normal	RS-GARCH
<i>Panel A. Rejections at 5% confidence level</i>				
Industry	2/13	11/13	3/13	2/13
Size	1/5	3/5	0/5	1/5
Book to market	1/5	3/5	2/5	0/5
Momentum	2/5	4/5	1/5	1/5
Overall	6/28	21/28	6/28	4/28
<i>Panel B. Rejections at 1% confidence level</i>				
Industry	1/13	6/13	3/13	1/13
Size	0/5	1/5	0/5	1/5
Book to market	0/5	2/5	0/5	0/5
Momentum	1/5	2/5	0/5	0/5
Overall	2/28	11/28	3/28	2/28

portfolios, and the RS normal model is rejected by 6 portfolios. At the 5% level, the RS-GARCH model is rejected by 4 out of 28 portfolios, giving the RS-GARCH model the best performance by this criterion. However, the model still leaves some amount of the correlation asymmetry unexplained. The full details of the H statistics on the four empirical models summarized by Table 9 are listed in Table 10.

Table 10 reports the H statistic for each portfolio. To summarize each model's performance, we tabulate how many times a particular model produces the smallest H statistic out of all five models. In all cases, the normal distribution's H statistic is higher than the best-performing empirical model presented in Section 5.1. The Jump model never produces smallest statistic, the GARCH-M model produces the smallest statistic once, and the RS-GARCH model produces the smallest statistic in five cases. The RS-normal model presents the strongest alternative, producing the lowest H statistic in 22 cases.

Reviewing the magnitudes of the H statistics in Table 10, we find that, while the RS-GARCH model rejects the null hypothesis in the fewest cases, it can be a very poor fit of the data for some portfolios. The H statistic for the RS-GARCH model is greater than 0.13 for 6 out of 28 portfolios. The average H statistic across all 28 portfolios for the RS normal model is 0.0564, while the averages for the GARCH-M model, the Jump model, and the RS-GARCH model are larger: 0.085, 0.090, and 0.096, respectively. In comparison, the average H statistic for the normal distribution is 0.095. Hence, while the H statistic rejects the RS-GARCH model the least number of times, the RS normal model provides the best fit of exceedance correlations.

Table 10

H Statistics assessing alternate models

The table reports the *H* statistics for equity portfolios under the null hypothesis of other distributions: a GARCH-M model, a Poisson Jump model, a regime-switching (RS) normal distribution model and a regime-switching GARCH model. The weights used are proportional to the number of observations used to calculate the sample exceedance correlations (see Eq. (26)). Frequency of the data is weekly. * Indicates rejection of the model at the 5% confidence level. ** Indicates rejection at the 1% confidence level.

The market portfolio is the value-weighted index of all stocks in CRSP. Panel A shows the *H* statistics of the value-weighted industry portfolios. Panels B and C show the *H* statistics of the value-weighted portfolios formed by sorting on market capitalizations and book-to-market ratios, respectively. Panel D presents the *H* statistics of the equal-weighted portfolios of stocks sorted by their lagged past six-months returns, with one to six months of lags.

Portfolio	GARCH-M		Jump model		RS normal		RS-GARCH	
	<i>H</i>	SE	<i>H</i>	SE	<i>H</i>	SE	<i>H</i>	SE
<i>Panel A. Industry portfolios (value-weighted)</i>								
Misc	0.114	0.249	0.117**	0.038	0.076	0.066	0.044	0.049
Petroleum	0.107	0.280	0.212**	0.050	0.127**	0.045	0.168**	0.031
Finance	0.060	0.040	0.067*	0.030	0.034	0.125	0.070	0.138
Durables	0.073	0.040	0.074**	0.028	0.056	0.052	0.076	0.163
Basic ind	0.056	0.051	0.077**	0.021	0.048**	0.008	0.046	0.161
Food/tobacco	0.130	0.322	0.127**	0.031	0.073	0.066	0.106	0.061
Construction	0.087*	0.036	0.111*	0.054	0.067	0.261	0.076	0.107
Capital goods	0.051	0.031	0.058*	0.029	0.037	0.022	0.030	0.098
Transportation	0.114	0.119	0.102*	0.050	0.082	0.094	0.187	0.097
Utilities	0.156	0.115	0.138**	0.030	0.109	0.117	0.197*	0.099
Textile/trade	0.072	0.066	0.108	0.102	0.061**	0.023	0.102	0.160
Service	0.079**	0.025	0.064*	0.028	0.035	0.027	0.129	0.115
Leisure	0.068	0.041	0.061	0.036	0.039	0.041	0.054	0.031
<i>Panel B. Size portfolios (value-weighted)</i>								
1 Smallest	0.124*	0.053	0.127**	0.042	0.084	0.119	0.069	0.067
2	0.073	0.044	0.075*	0.032	0.039	0.062	0.092	0.100
3	0.046	0.034	0.044	0.024	0.026	0.020	0.064	0.093
4	0.030	0.018	0.030	0.024	0.016	0.033	0.055	0.074
5 Largest	0.008	0.007	0.016*	0.007	0.003	0.004	0.030**	0.005
<i>Panel C. Book-to-market portfolios (value-weighted)</i>								
1 Growth	0.023	0.017	0.042	0.036	0.010	0.052	0.060	0.054
2	0.039*	0.018	0.036**	0.010	0.024*	0.011	0.036	0.033
3	0.059	0.116	0.083**	0.024	0.048	0.030	0.110	0.085
4	0.080	0.052	0.093	0.068	0.061	0.120	0.122	0.179
5 Value	0.114	0.072	0.115*	0.057	0.078*	0.038	0.175	0.106
<i>Panel D. Momentum portfolios (equal-weighted)</i>								
1 Past losers	0.239	0.129	0.157**	0.033	0.101*	0.044	0.090	0.220
2	0.105*	0.043	0.117**	0.015	0.080	0.058	0.100	0.104
3	0.094**	0.028	0.096*	0.039	0.061	0.292	0.138*	0.068
4	0.081	0.044	0.074	0.044	0.046	0.036	0.115	0.109
5 Past winners	0.099	0.075	0.097*	0.046	0.059	0.109	0.149	0.125

The same portfolios whose empirical correlation asymmetry proved difficult to match using the normal distribution, tend to make a difficult fit across all four models. In general, the petroleum and utility industries have the highest H statistics across models. Portfolios formed of small stocks, value stocks, and past loser stocks also tend to have the highest H statistics.

5.3. *Explaining the model performance*

In explaining the performance in matching the correlation asymmetries, it is instructive to examine a portfolio which no model appears to fit. Fig. 5 shows the exceedance correlations from the third momentum quintile, which rejects all four empirical models. The sample exceedance correlations are given by the solid line. Taking each model in turn, Fig. 5 shows that the GARCH-M model produces exceedance correlations which are asymmetric but go the wrong way. That is, the sample exceedance correlations increase on the downside, for negative ϑ , but the GARCH-M model exceedance correlations are higher on the upside, for positive ϑ . The Jump model produces exceedance correlations which have a tent-shape, much like the normal distribution. The RS normal model produces exceedance correlations with the correct asymmetry, but decay too quickly on the downside. Finally, the RS-GARCH model produces a small amount of correlation asymmetry in the right direction, but is too persistent on both the downside and the upside.

The exceedance correlation asymmetry for the GARCH-M model in Fig. 5 is shared by all other portfolios. Although this model allows the conditional covariance to increase in response to an unanticipated shock in returns, the expected return of both the market and the portfolio also increase in this model. Eq. (30) shows that, for a positive price of risk δ , both the conditional mean of the market and the stock portfolio may increase when the conditional covariance increases. Therefore, while the conditional covariance increases through a negative shock in expected returns, the expected return also increases, making it more likely to draw returns on the upside. However, the GARCH effect induces persistence in the exceedance correlations across increasing or decreasing ϑ , which the normal distribution cannot capture.

To illustrate the impact of the negative price of risk on the results of the GARCH-M model, we turn to Fig. 6. This figure shows exceedance correlations for the smallest size portfolio for all four models in each panel, against the sample exceedance correlations. The top left-hand panel shows the exceedance correlations for the GARCH-M model. The estimated exceedance correlations implied by the model are given by circles. If we make the price of risk to be negative, the GARCH-M model closely matches the sample exceedance correlations. This effect is the main failing of the GARCH-M model: asymmetric exceedance correlations can be produced, but the asymmetry goes the wrong way unless a negative price of risk is employed. Economic models do not necessarily rule out negative prices of risk, but the economic plausibility of negative prices of risk and empirical estimates of the market risk premium of the U.S. market weigh heavily against this assumption.

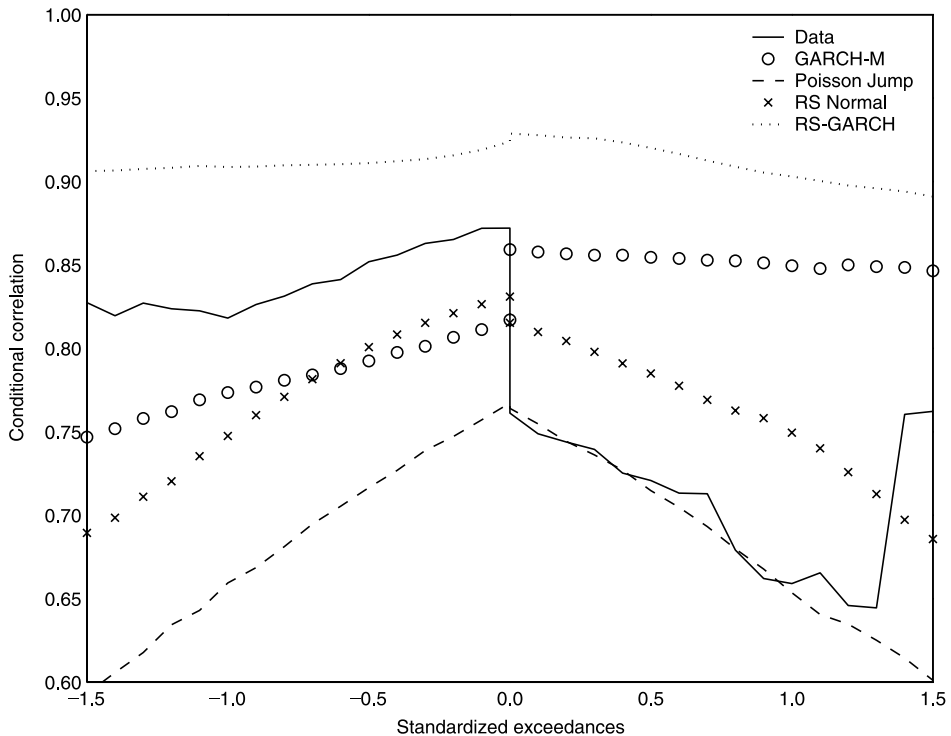


Fig. 5. Exceedance correlations for the third momentum portfolio. This figure shows the correlations for the third quintile momentum portfolio with the value-weighted market portfolio. Data is sampled weekly, from July 1963 to December 1998. The theoretical exceedance correlations from an asymmetric GARCH-in-Mean (GARCH-M) model, a Poisson Jump model, a regime-switching (RS) normal distribution, and a regime-switching GARCH (RS-GARCH) model are presented on the same plot together with the empirical exceedance correlations found in the data.

As in Fig. 5, the Jump model in Fig. 6 produces a tent shape. This general result is the reason behind the poor performance of this model. The Jump model performs poorly because it fails to capture the persistence in volatility. The other three models capture this feature of the data. The Jump model can be interpreted to be a special case of the RS normal model such that one regime can be interpreted as a jump regime. The probability of entering this regime is positive, but the probability of remaining in this regime is zero. Examining international data, Ang and Bekaert, (2000) find this crash-like regime to be persistent, but it cannot be captured in a jump model, which assumes an immediate exit from this regime.

To understand why the Jump model produces mostly tent shapes in the exceedance plot, consider the following. Ordinarily, returns are drawn from a normal distribution, which has a tent shape. Occasionally, when a jump occurs, returns are drawn from another normal distribution. These jumps are not persistent, and the effect mirrors the tent shapes of an ordinary normal distribution. The model

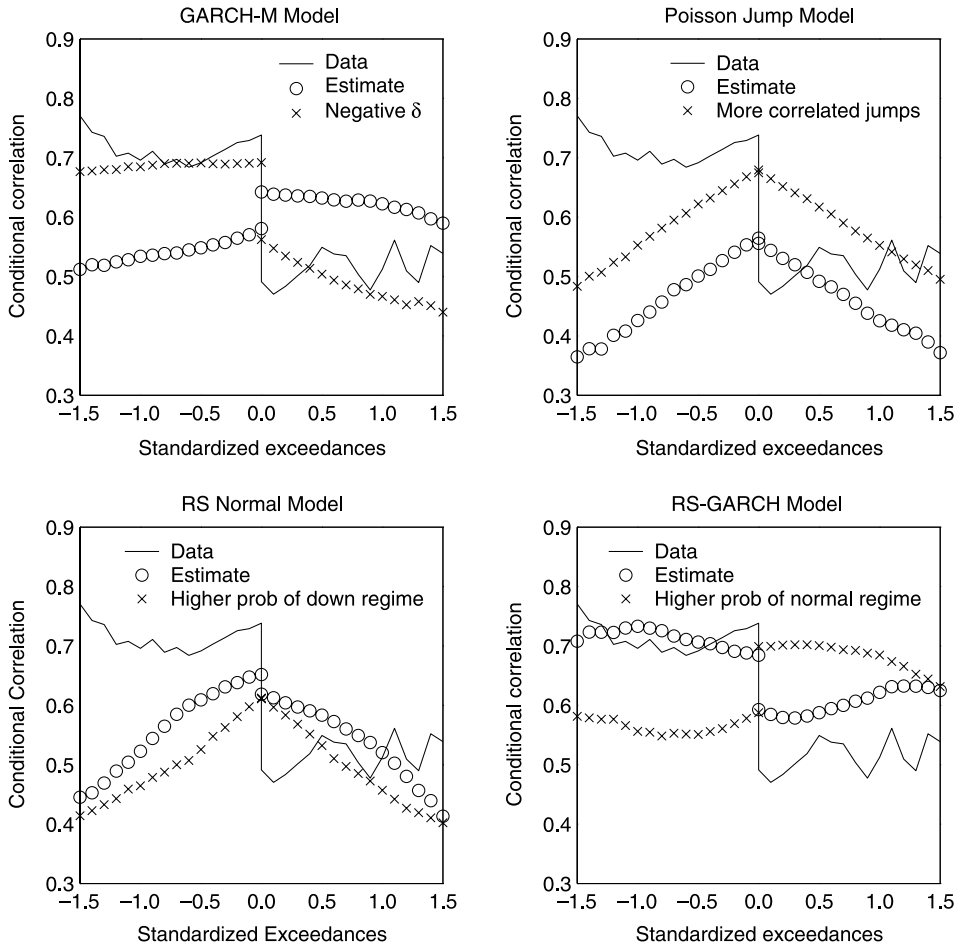


Fig. 6. Exceedance correlations for the smallest size portfolio. We plot the exceedance correlations for the smallest size portfolio with the value-weighted market at the weekly frequency. We show the exceedance correlations from the data (solid lines) and those implied by various models. From top left clockwise, we have a GARCHM model, a Jump model, a RSGARCH model and a RS normal model. Within each panel, we also plot an exceedance correlation of a comparative static, that is, altering one parameter of the models and re-calculating the exceedance correlations. For the GARCHM model, we make the price of risk negative. For the Jump model, we provide jumps with greater correlation between the market and the equity portfolio. For the RS normal model, we increase the probability of entering a downside regime. For the RS-GARCH model, we increase the probability of entering a normal regime.

produces a correlation asymmetry, which is very small and not persistent across exceedance levels. Changing the parameters of the Jump model has little effect on the tent-shape of its exceedance correlations. The top right hand panel of Fig. 6 shows what happens when the correlation between the market and stock portfolio increases in the jump distribution. In this case, the tent shape has moved upwards but retained its shape. A similar effect occurs when increasing the jump intensity.

Fig. 5 demonstrates that RS normal model may produce exceedance correlations that decline too fast when the exceedance levels \mathfrak{J} approach positive or negative infinity. Exceedance correlations can be too persistent across \mathfrak{J} for the RS normal model to mimic. Empirical estimates of this model produce both a “normal regime”, with high expected returns, low volatilities, and low correlations, and a “downside” regime, with low expected returns, high volatilities, and high correlations.

The bottom left panel of Fig. 6 shows that merely increasing the probability of staying in the down-regime does not necessarily increase the degree of asymmetry. The down-regime corresponds to $s_t = 2$, and the probability of staying in this regime can be isolated as Q in the Markov Chain of Eq. (36). If the down-regime is not at all persistent ($Q = 0$), the RS normal model performs like the Jump model, producing tent-shapes. Also, the case of $Q = 1 - P$ is a simple switching model, in which the regimes have no persistence. This case is shown in the bottom left panel of Fig. 6. The persistence through time of the two regimes drives the persistence across exceedance levels \mathfrak{J} of the exceedance correlations. Unfortunately, the persistence across the exceedances, when \mathfrak{J} approaches positive or negative infinity, may not be matched by the RS normal model.

The final model, the RS-GARCH model, employs persistent covariance and allows regime switching, leading to a more successful match of the persistence in the exceedance correlations across the exceedance levels. The bottom right panel of Fig. 6 shows the RS-GARCH model exceedance correlations plotted against the sample exceedance correlations. The panel also shows what happens to the exceedance correlation when the probability of staying in the normal regime increases, given that we are in the normal regime (or P in Eq. (36) if $s_t = 1$). In this case, the exceedance correlations switch sign, allowing to increase on the upside. In general, the superior performance of this model arises from its ability to produce asymmetries of the right direction, as does the RS normal model, and adding the ability to match exceedance correlation persistence across \mathfrak{J} .

In summary, of the four discrete-time models we consider, no single model captures all of the asymmetry in correlations observed in the data. The GARCH-M model produces correlation asymmetry which is persistent across the exceedance levels, but this correlation asymmetry goes the wrong way except with a negative price of risk. The Jump model is rejected almost uniformly across all the portfolios, showing the importance of allowing for persistent volatility and covariance effects. Volatility persistence cannot be captured in a pure jump model. The RS normal model can produce the correct sign of correlation asymmetry and provides the best fit with the data. It generally produces the lowest H statistics across all the models considered here. However, this model may not match the persistence of the asymmetries across exceedance levels. The RS-GARCH model is rejected by the data least frequently, and is able to match the persistence of the asymmetries across exceedance levels. Our results point to the need for the development of more sophisticated empirical models to capture the empirical asymmetric correlations. These models must capture persistent volatility effects, as well as capture more asymmetric correlation patterns than the models presented here.

6. Conclusion

Correlations between domestic equity portfolios and the aggregate market are greater in downside markets than in upside markets. To quantify these effects, we develop an H statistic to measure the asymmetries in correlations. Unlike previous literature, which examines covariance asymmetry in the context of the class of asymmetric GARCH models, we can assess the extent of correlation asymmetry in the data relative to any particular model. Moreover, the statistic we develop has the advantage of allowing us to succinctly measure correlation asymmetries, easily compare the degree of asymmetries across portfolios, frequencies, and null distributions, and formally conduct statistical tests of asymmetries.

Asymmetries between upside and downside correlations exist between stocks in a single market, as well as across markets internationally. We find that correlation asymmetries are fundamentally different from other measures of asymmetries, such as skewness and co-skewness, and tend to be inversely related to systematic market risk. We examine the sources of correlation asymmetries and find greater asymmetries among smaller stocks, value stocks, and recent loser stocks. Correlation asymmetry is the largest among traditional defensive sectors, such as petroleum and utilities. We find that riskier stocks, as measured by a higher beta, have lower correlation asymmetry, and, controlling for size, the degree of correlation asymmetry is unrelated to leverage. Overall, a typical portfolio exhibits correlations conditional on the downside that differ from those of a normal distribution by 11.6%.

We examine several empirical models to see if they can account for the correlation asymmetries in the data. Normal distributions are, not surprisingly, rejected by the data. We estimate an asymmetric GARCH-M model, a Poisson jump model, a regime-switching normal distribution model, and a regime-switching GARCH model. Of these, the regime-switching normal model is the best able to match the magnitude of empirical correlation asymmetries, while the regime-switching GARCH model is statistically rejected least often. The popular CAPM-based GARCH-M models can produce asymmetric correlations, but these correlations go the wrong way unless a negative price of risk is used. Our Jump model fails to capture the persistence of covariance dynamics in the data, and captures almost no asymmetric correlation effects. While regime-switching models perform best in explaining the amount of correlation asymmetry reflected in the data, these models still leave a significant amount of correlation asymmetry in the data unexplained.

Our results have implications for empirical and theoretical asset pricing. Harvey and Siddique (2000) demonstrate that non-linearities in third moments are priced. Since asymmetric correlations are different from skewness or co-skewness, asymmetric correlations may also play a role in an asset-pricing model. One example where these effects would arise is an economy with a representative agent with first order risk aversion (see Ang et al., 2000) or Loss Aversion (see Barberis et al., 2001) preferences. Such an investor treats gains and losses asymmetrically and is very averse to downside risks. Our H statistic quantifies asymmetric correlation risk on the downside, which may also be priced. Further, asymmetric correlations also have implications for portfolio allocation and risk management.

Our work raises the question: why do asymmetric movements in asset returns arise in the first place? They may reflect some particular structure of the macro-economy or some intricate interactions of economic agents in equilibrium. While Dumas et al. (2000) show that aggregate characteristics affect returns across countries, we show that cross-sectional firm characteristics are related to the magnitudes of asymmetric correlations within a domestic market. Modern equilibrium models with either noise traders and frictions (Kyle and Xiong, 2001), or disparately informed agents with frictions (Hong and Stein, 2001) explain little about the relation between firm characteristics and asymmetric movements. These authors do not model cross-sectional differences between individual asset characteristics. Our work shows that these differences in firm characteristics are related to the asymmetries in asset returns.

Appendix A. Solution of the asset allocation problem

The first order conditions (FOC) of the investor's investment problem are

$$E_t(W^{-\gamma}x) = 0, \quad (\text{A.1})$$

where $W = e^{rf} + \alpha(e^x - e^{rf}) + \alpha(e^y - e^{rf})$.

Since x and y have the same distribution, the portfolio holding in each asset is identical, even though these assets are correlated. This expectation can be computed by numerical quadrature, described in Tauchen and Hussey (1991), as follows:

$$\sum_{s=1}^M (W_s^{-\gamma} x_s p_s) = 0, \quad (\text{A.2})$$

where the M values of the risky asset returns ($\{x_s\}_{s=1}^M$ and $\{y_s\}_{s=1}^M$) and associated probabilities are chosen by an optimal quadrature rule. W_s represents the investor's terminal wealth when the risky asset returns are x_s and y_s . Tauchen and Hussey (1991) demonstrate that quadrature is very accurate using few optimally chosen points. The FOC in Eq. (A.2) can be solved over α by a non-linear root solution.

When x and y are bivariate normally distributed, Gaussian quadrature is used with 5 points to approximate the distribution of x and y . Hence, we use $M = (5 \times 5)$, or 25 quadrature points. Correlation is achieved by using a Cholesky decomposition transformation.

When $X = (x, y)'$ is drawn from the RS model, we approximate the joint distribution as follows. For regime $s_t = 1$, we approximate the normal distribution, $N(\mu_1, \Sigma_1)$, using 25 quadrature points. For regime $s_t = 2$, another 25 quadrature points are used. Conditional on regime $s_t = 1$, we use weights P and $1 - P$, where $P = \Pr(s_t = 1 | s_{t-1} = 1)$, to mix the associated probabilities of the quadrature points of regimes 1 and 2 to produce $M = 50$ quadrature point approximation to the RS model conditional on regime 1. Conditional on regime $s_t = 2$, we use weights $1 - Q$ and Q , where $Q = \Pr(s_t = 2 | s_{t-1} = 2)$, to mix the associated probabilities of the quadrature points of regimes 1 and 2.

To match the first and second moments of the RS model to the unconditional means, volatilities, and correlation of the normal distribution, we note that the unconditional mean of the RS model is given by

$$\pi\mu_1 + (1 - \pi)\mu_2, \quad (\text{A.3})$$

where $\pi = \Pr(s_t = 1)$ is the stable probability of the RS model. This probability in Eq. (A.3) is

$$\pi = \frac{1 - Q}{2 - P - Q} \quad (\text{A.4})$$

and the unconditional covariance is given by

$$\begin{aligned} & \pi(\Sigma_1 + \mu_1\mu_1') + (1 - \pi)(\Sigma_2 + \mu_2\mu_2') \\ & - (\pi\mu_1 + (1 - \pi)\mu_2)(\pi\mu_1 + (1 - \pi)\mu_2)'. \end{aligned} \quad (\text{A.5})$$

By exogenous choices of $P = Q = \frac{2}{3}$, $\mu_1 = \mu_2 = (0.07, 0.07)'$, $\sigma_1 = \sigma_2 = 0.15$, and the stable probability $\pi = \frac{1}{2}$, the unconditional means of x and y using the RS model are both 0.07, and unconditional volatilities of x and y using the RS model are both 0.15. We can choose ρ_1 and ρ_2 to produce the unconditional correlation ρ by setting $\frac{1}{2}(\rho_1 + \rho_2) = \rho$.

We produce a particular H as follows. We choose ρ_2 to determine ρ_1 . For example, ρ_2 set at 0.35 will yield ρ_1 of 0.65. This relation gives the RS model the same unconditional means, volatilities, and correlation as the bivariate normal distribution. Then we calculate $\text{corr}(\tilde{x}, \tilde{y} | \tilde{x} < -1, \tilde{y} < -1)$, for x and y drawn from the RS model, by using simulation with 100,000 draws. This calculation will be greater than the correlation with the same conditioning calculated from the bivariate normal, which is given in Appendix B in closed-form. The difference between $\text{corr}(\tilde{x}, \tilde{y} | \tilde{x} < -1, \tilde{y} < -1)$ calculated from the RS model and from the bivariate normal gives H . To produce Fig. 1, we choose $\rho_2 \in \{0.19, 0.20, \dots, 0.48\}$.

Appendix B. Proposition

Let $X = (x, y) \sim N(0, \Sigma)$, where Σ has unit variances and unconditional correlation ρ . We define

$$\hat{\rho}(h_1, h_2, k_1, k_2) = \text{corr}(x, y | h_1 < x < h_2, k_1 < y < k_2; \rho) \quad (\text{B.6})$$

as the correlation of x and y conditional on observations for which $h_1 < x < h_2$ and $k_1 < y < k_2$, where x and y have unconditional correlation ρ .

Let $L(\cdot)$ denote the cumulative density of a doubly truncated bivariate normal distribution:

$$L(h_1, h_2, k_1, k_2) = \int_{h_1}^{h_2} \int_{k_1}^{k_2} g(x, y; \rho) \, dx \, dy, \quad (\text{B.7})$$

where

$$g(x, y; \rho) = \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left(-\frac{1}{2(1-\rho^2)}(x^2 - 2\rho xy + y^2)\right)$$

is the density function of X . $L(\cdot)$ can be evaluated by numerical methods.

The following proposition allows us to obtain a closed-form solution for $\hat{\rho}$:

Proposition 1. Let $m_{ij} = E(x^i y^j | h_1 < x < h_2, k_1 < y < k_2)$. Then

$$m_{10} = \left(\frac{1}{L(\cdot)}\right) [\psi(h_1, h_2, k_1, k_2; \rho) + \rho \psi(k_1, k_2, h_1, h_2; \rho)], \quad (\text{B.8})$$

$$m_{20} = \left(\frac{1}{L(\cdot)}\right) [L(\cdot) + \chi(k_1, k_2, h_1; \rho) - \chi(k_1, k_2, h_2; \rho) + \rho^2 \chi(h_1, h_2, k_1; \rho) - \rho^2 \chi(h_1, h_2, k_2; \rho)] \quad (\text{B.9})$$

and

$$m_{11} = \left(\frac{1}{L(\cdot)}\right) [\rho L(\cdot) + \rho \Upsilon(h_1, h_2, k_1; \rho) - \rho \Upsilon(h_1, h_2, k_2; \rho) + \rho \Upsilon(k_1, k_2, h_1; \rho) - \rho \Upsilon(k_1, k_2, h_2; \rho) + \Lambda(h_1, h_2, k_1; \rho) - \Lambda(h_1, h_2, k_2; \rho)]. \quad (\text{B.10})$$

In Eqs. (B.8)–(B.10), $\psi(\cdot)$, $\chi(\cdot)$, $\Upsilon(\cdot)$ and $\Lambda(\cdot)$ are given in the proof. The moments m_{01} and m_{02} are obtained by interchanging (h_1, h_2) and (k_1, k_2) in the formulae for m_{10} and m_{20} .

From Proposition 1:

$$\text{var}(x | h_1 < x < h_2, k_1 < y < k_2) = m_{20} - m_{10}^2, \quad (\text{B.11})$$

$$\text{var}(y | h_1 < x < h_2, k_1 < y < k_2) = m_{02} - m_{01}^2, \quad (\text{B.12})$$

$$\text{and } \text{cov}(x, y | h_1 < x < h_2, k_1 < y < k_2) = m_{11} - m_{10}m_{01}. \quad (\text{B.13})$$

Eqs. (B.11)–(B.13) allow us to calculate $\hat{\rho}(h_1, h_2, k_1, k_2)$ as

$$\begin{aligned} & \hat{\rho}(h_1, h_2, k_1, k_2) \\ &= \frac{\text{cov}(x, y | h_1 < x < h_2, k_1 < y < k_2)}{\sqrt{\text{var}(x | h_1 < x < h_2, k_1 < y < k_2)} \sqrt{\text{var}(y | h_1 < x < h_2, k_1 < y < k_2)}}. \end{aligned} \quad (\text{B.14})$$

Proof of Proposition 1. Let

$$\phi(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) \quad (\text{B.15})$$

denote the $N(0, 1)$ density, and

$$\Phi(x) = \int_{-\infty}^x \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{w^2}{2}\right) dw \quad (\text{B.16})$$

denote the cumulative distribution function of $N(0, 1)$.

First moment: The first moment m_{10} is obtained from the definition

$$m_{10} = \frac{1}{2\pi\sqrt{1-\rho^2}L(\cdot)} \int_{k_1}^{k_2} \int_{h_1}^{h_2} x \exp\left(-\frac{1x^2 - 2\rho xy + y^2}{2(1-\rho^2)}\right) dx dy. \quad (\text{B.17})$$

The equation for m_{01} is similar, by symmetry. Make the change of variable $z = (x - \rho y)/(\sqrt{1-\rho^2})$, and let $v_1 = (h_1 - \rho y)/(\sqrt{1-\rho^2})$ and $v_2 = (h_2 - \rho y)/(\sqrt{1-\rho^2})$. We re-write Eq. (B.17) as

$$\begin{aligned} m_{10}L(\cdot) &= \frac{\sqrt{1-\rho^2}}{2\pi} \int_{k_1}^{k_2} \left[-\exp\left(-\frac{1}{2}(z^2 + y^2)\right) \right]_{z=v_1}^{z=v_2} dy \\ &\quad + \frac{\rho}{2\pi} \int_{k_1}^{k_2} y \exp\left(-\frac{y^2}{2}\right) \left[\int_{v_1}^{v_2} \exp\left(-\frac{z^2}{2}\right) dz \right] dy. \end{aligned} \quad (\text{B.18})$$

The second term of Eq. (B.18), $\rho m_{01}L(\cdot)$, and the first term can be written, after a further change of variable and integration by parts, as $(1-\rho^2)\psi(h_1, h_2, k_1, k_2)$, where

$$\begin{aligned} \psi(h_1, h_2, k_1, k_2; \rho) &= \phi(h_1) \left[\Phi\left(\frac{k_2 - \rho h_1}{\sqrt{1-\rho^2}}\right) - \Phi\left(\frac{k_1 - \rho h_1}{\sqrt{1-\rho^2}}\right) \right] \\ &\quad - \phi(h_2) \left[\Phi\left(\frac{k_2 - \rho h_2}{\sqrt{1-\rho^2}}\right) - \Phi\left(\frac{k_1 - \rho h_2}{\sqrt{1-\rho^2}}\right) \right]. \end{aligned} \quad (\text{B.19})$$

By symmetry we have

$$m_{10}L(\cdot) = (1-\rho^2)\psi(h_1, h_2, k_1, k_2; \rho) + \rho m_{01}L(\cdot),$$

and

$$m_{01}L(\cdot) = (1-\rho^2)\psi(k_1, k_2, h_1, h_2; \rho) + \rho m_{10}L(\cdot) \quad (\text{B.20})$$

hence,

$$m_{10}L(\cdot) = \psi(h_1, h_2, k_1, k_2; \rho) + \rho\psi(k_1, k_2, h_1, h_2; \rho). \quad (\text{B.21})$$

Variable m_{01} is given by interchanging the order of h_1, h_2, k_1 , and k_2 .

Second moment: By definition:

$$m_{20} = \frac{1}{2\pi\sqrt{1-\rho^2}L(\cdot)} \int_{k_1}^{k_2} \int_{h_1}^{h_2} x^2 \exp\left(-\frac{1x^2 - 2\rho xy + y^2}{2(1-\rho^2)}\right) dx dy. \quad (\text{B.22})$$

Using the same change of variables as above, we have

$$\begin{aligned} m_{20}L(\cdot) &= \frac{1}{2\pi} \int_{k_1}^{k_2} \left[(z(1-\rho^2) + 2\rho\sqrt{1-\rho^2}y) \exp\left(-\frac{z^2}{2}\right) \right]_{z=v_1}^{z=v_2} \exp\left(-\frac{y^2}{2}\right) dy \\ &\quad + \frac{1}{2\pi} \int_{k_1}^{k_2} [(1-\rho^2) + \rho^2 y^2] \int_{v_1}^{v_2} \exp\left(-\frac{z^2}{2}\right) dz \exp\left(-\frac{y^2}{2}\right) dy. \end{aligned} \quad (\text{B.23})$$

The first term equals $(1-\rho^2)L(\cdot) + \rho^2 m_{02}L(\cdot)$, and the second term, after a further change of variables and integration by parts, can be written as

$$(1-\rho^4)(\chi(k_1, k_2, h_1; \rho) - \chi(k_1, k_2, h_2; \rho)), \quad (\text{B.24})$$

where

$$\begin{aligned} \chi(k_1, k_2, h_1; \rho) = & h_1 \phi(h_1) \left[\Phi \left(\frac{k_2 - \rho h_1}{\sqrt{1 - \rho^2}} \right) - \Phi \left(\frac{k_1 - \rho h_1}{\sqrt{1 - \rho^2}} \right) \right] \\ & + \frac{\rho \sqrt{1 - \rho^2}}{\sqrt{2\pi}(1 + \rho^2)} \left[\phi \left(\frac{\sqrt{k_1^2 - 2\rho k_1 h_1 + h_1^2}}{\sqrt{1 - \rho^2}} \right) \right. \\ & \left. - \phi \left(\frac{\sqrt{k_2^2 - 2\rho k_2 h_1 + h_1^2}}{\sqrt{1 - \rho^2}} \right) \right]. \end{aligned} \quad (\text{B.25})$$

By symmetry, we have

$$m_{20}L(\cdot) = L(\cdot)((1 - \rho^2) + \rho^2 m_{02}) + (1 - \rho^4)(\chi(k_1, k_2, h_1; \rho) - \chi(k_1, k_2, h_2; \rho)),$$

and

$$\begin{aligned} m_{02}L(\cdot) = & L(\cdot)((1 - \rho^2) + \rho^2 m_{20}) + (1 - \rho^4)(\chi(h_1, h_2, k_1; \rho) \\ & - \chi(h_1, h_2, k_2; \rho)). \end{aligned} \quad (\text{B.26})$$

Solving Eq. (B.26) gives

$$\begin{aligned} m_{20}L(\cdot) = & L(\cdot) + \chi(k_1, k_2, h_1; \rho) - \chi(k_1, k_2, h_2; \rho) + \rho^2 \chi(h_1, h_2, k_1; \rho) \\ & - \rho^2 \chi(h_1, h_2, k_2; \rho). \end{aligned} \quad (\text{B.27})$$

Cross moment: By definition:

$$m_{11} = \frac{1}{2\pi\sqrt{1 - \rho^2}L(\cdot)} \int_{k_1}^{k_2} \int_{h_1}^{h_2} xy \exp\left(-\frac{1x^2 - 2\rho xy + y^2}{2(1 - \rho^2)}\right) dx dy. \quad (\text{B.28})$$

Using the same change of variables as above, we have

$$\begin{aligned} m_{11}L(\cdot) = & \frac{1}{2\pi} \int_{k_1}^{k_2} \int_{v_1}^{v_2} \rho y^2 \exp\left(-\frac{1}{2}(y^2 + z^2)\right) dz dy \\ & + \frac{1}{2\pi} \int_{k_1}^{k_2} (\sqrt{1 - \rho^2}y) \left[-\exp\left(-\frac{z^2}{2}\right) \right]_{z=v_1}^{z=v_2} \exp\left(-\frac{y^2}{2}\right) dy. \end{aligned} \quad (\text{B.29})$$

The first term in Eq. (B.29) is $\rho m_{02}L(\cdot)$, and the second term can be written, after a change of variables and integration by parts, as

$$\begin{aligned} & \rho(1 - \rho^2)(\lambda(k_1, k_2, h_1; \rho) - \lambda(k_1, k_2, h_2; \rho)) \\ & + \frac{(1 - \rho^4)}{(1 + \rho^2)}(\lambda(k_1, k_2, h_1; \rho) - \lambda(k_1, k_2, h_2; \rho)), \end{aligned} \quad (\text{B.30})$$

where

$$\begin{aligned} \Upsilon(k_1, k_2, h_1; \rho) &= h_1 \phi(h_1) \left[\Phi \left(\frac{k_2 - \rho h_1}{\sqrt{1 - \rho^2}} \right) - \Phi \left(\frac{k_1 - \rho h_1}{\sqrt{1 - \rho^2}} \right) \right] \\ \text{and } \Lambda(k_1, k_2, h_1; \rho) &= \frac{\sqrt{1 - \rho^2}}{\sqrt{2\pi}} \left[\phi \left(\frac{\sqrt{k_1^2 - 2\rho k_1 h_1 + h_1^2}}{\sqrt{1 - \rho^2}} \right) \right. \\ &\quad \left. - \phi \left(\frac{\sqrt{k_2^2 - 2\rho k_2 h_1 + h_1^2}}{\sqrt{1 - \rho^2}} \right) \right]. \end{aligned} \quad (\text{B.31})$$

Note that $\chi(a, b, c; \rho) = \Upsilon(a, b, c; \rho) + \rho/(1 + \rho^2)\Lambda(a, b, c; \rho)$. Also note that $(\Lambda(k_1, k_2, h_1; \rho) - \Lambda(k_1, k_2, h_2; \rho)) = (\Lambda(h_1, h_2, k_1; \rho) - \Lambda(h_1, h_2, k_2; \rho))$. After simplification, we can write m_{11} as

$$\begin{aligned} m_{11}L(\cdot) &= \rho L(\cdot) + \rho \Upsilon(h_1, h_2, k_1; \rho) - \rho \Upsilon(h_1, h_2, k_2; \rho) + \rho \Upsilon(k_1, k_2, h_1; \rho) \\ &\quad - \rho \Upsilon(k_1, k_2, h_2; \rho) + \Lambda(h_1, h_2, k_1; \rho) - \Lambda(h_1, h_2, k_2; \rho). \end{aligned} \quad (\text{B.32})$$

Appendix C. Calculating H statistics

To calculate the H statistics using the null distribution of the empirical models in Section 5, we need to calculate the implied exceedance correlations $\check{\rho}(\theta, \phi)$ by simulation. Denote the distribution under the null as $\xi(\phi)$, such that ξ represents one of the models from Section 5 with parameters, ϕ . For each equity portfolio, we estimate the parameters, ϕ , of the model. At the estimated parameters, we then create a simulated time series with 100,000 observations. We take the exceedance correlations of the simulated time series as the exceedance correlation implied by the distribution $\check{\rho}(\theta, \phi)$.

To calculate standard errors for the H statistic, we use GMM and the δ -method. To illustrate, we first take a single exceedance correlation, $\bar{\rho}(\vartheta_i)$, corresponding to the exceedance level ϑ_i . For expositional simplicity, we assume ϑ_i to be positive. The exceedance correlation $\bar{\rho}(\vartheta_i)$ can be estimated using the following moment conditions $g_{1i}(\eta_i)$:

$$g_{1i}(\eta_i) = \text{E} \begin{bmatrix} x_t \mathbf{1}_{\{x_t > \vartheta_i, y_t > \vartheta_i\}} - \eta_{1i} \\ y_t \mathbf{1}_{\{x_t > \vartheta_i, y_t > \vartheta_i\}} - \eta_{2i} \\ x_t^2 \mathbf{1}_{\{x_t > \vartheta_i, y_t > \vartheta_i\}} - \eta_{3i} \\ y_t^2 \mathbf{1}_{\{x_t > \vartheta_i, y_t > \vartheta_i\}} - \eta_{4i} \\ x_t y_t \mathbf{1}_{\{x_t > \vartheta_i, y_t > \vartheta_i\}} - \eta_{5i} \end{bmatrix}. \quad (\text{C.1})$$

In Eq. (C.1), $\eta_i = (\eta_{1i}, \eta_{2i}, \eta_{3i}, \eta_{4i}, \eta_{5i})'$ and $\mathbf{1}$ is an indicator function. Note that $\bar{\rho}(\vartheta_i)$ is a nonlinear function of η_i .

We can set up moment conditions similar to Eq. (C.1) for all exceedance correlations $\bar{\rho}(\vartheta_i)$ in the N vector of exceedance correlations $\bar{\rho}(\theta)$ corresponding to

the vector of exceedances $\theta = (\vartheta_1, \dots, \vartheta_N)'$. Denote these moment conditions as $g_1(\eta) = \{g_{1i}(\eta_i)\}_{i=1}^N$, such that $\eta = (\eta'_1 \dots \eta'_N)'$. Let G_1 denote $(\partial/\partial\eta)g_1$ and S_1 denote the estimate of the covariance matrix of $g_1(\eta)$, which can be obtained with the estimator of Newey and West (1987), or another similar estimate. Then, by the δ -method, the conditional moment estimates η have covariance matrix $\Gamma_1 = \{G_1 S_1^{-1} G_1'\}^{-1}$.

Suppose that the parameters of $\xi(\phi)$ are estimated by setting GMM orthogonality conditions $g_2(\phi)$ to zero. Let G_2 denote $(\partial/\partial\phi)g_2$, and S_2 denote the estimate of the covariance matrix of $g_2(\phi)$. In the case of maximum likelihood, the orthogonality conditions $g_2(\phi)$ are the scores at the optimum, and the covariance matrix S_2 can be estimated by a White (1980) outerproduct of the scores. The parameters, ϕ , are estimated with covariance matrix, Γ_2 . Using the δ -method, $\Gamma_2 = \{G_2 S_2^{-1} G_2'\}^{-1}$.

Let S denote the joint covariance matrix of $[g'_1 g'_2]'$. Furthermore, let

$$G = \begin{bmatrix} G_1 & 0 \\ 0 & G_2 \end{bmatrix}, \quad (\text{C.2})$$

with $D_1 = (\partial/\partial\eta)H$ and $D_2 = (\partial/\partial\phi)H$. Then, using the δ -method

$$\text{var}(H) = D\{GS^{-1}G'\}^{-1}D', \quad (\text{C.3})$$

such that $D = [D'_1 \ D'_2]'$. The square root of Eq. (C.3) is the standard error of H .

If ξ is a normal distribution, then the derivative D_2 used for the calculation of the standard errors of H can be obtained analytically. For more complex distributions of ξ , $\tilde{\rho}(\theta, \phi)$ will not be in closed-form. For these cases, D_2 must be calculated by simulation, as follows. Note that $\tilde{\rho}(\vartheta, \phi)$, the exceedance correlation implied by $\xi(\phi)$, is a function of ϕ which can be computed by simulation. Holding fixed the simulated errors involved in computing $\tilde{\rho}(\vartheta, \phi)$, we change the i th parameter in $\hat{\phi}$ by $\varepsilon = 0.0001$, and re-compute the simulated time series at the new parameters. This new time series is used to calculate a new implied exceedance correlation, which we denote $H_i(\theta, \phi)$. The i th element of D_2 can be estimated with the directional derivative (Gateaux derivative) for an increment of ε in the i th parameter of $\hat{\phi}$, given by $(H_i(\theta, \phi) - H(\theta, \phi))/\varepsilon$.

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