Lab Report

Measuring Solar Motion in a Day

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PRA2023 - Astronomical Observing Techniques

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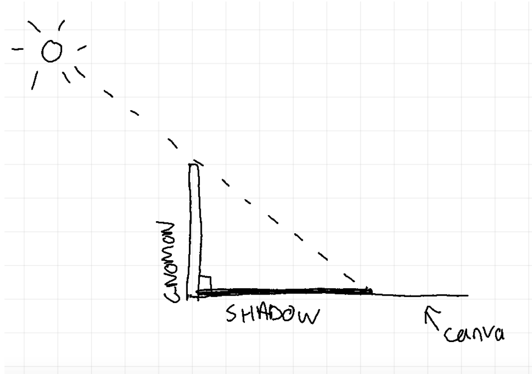

Figure 1: Diagram of setup, viewed from the side

1 Objective

In this experiment, the goal is to study the diurnal motion of the Sun to infer several of its key features, namely identifying local noon, extrapolating when sunrise occurred and when sunset will occur, and estimating the cardinal directions. Moreover, the azimuth of the Sun can also be measured during the sunset, sunrise and local noon to identify the current season.

2 Procedure

In order to perform these measurements, a gnomon was placed vertically atop a horizontal surface and the tip of its shadow was measured in 15-30 min interval over the course of 3 hours centered around local noon. For this experiment, measurements were taken from 11:45 to 15:00. Then the length of the shadow was measured and using the length of the gnomon and some trigonometry, the Sun's azimuth and elevation were estimated.

For this experiment, some key materials as well as ideal conditions were required.

2.1 Materials

The key part of the experiment relied on having a good gnomon. One that is long enough to give more accurate readings, but also short enough such that its shadow stayed within the canvas (see Figures 1 2). The gnomon chosen for this experiment was a metal rod and a base from the PASCO experiment set (see Figure 3). However the width of the rod was not ideal, this will be explored later in the analysis. The rod measured 43.6 cm high and 1.25 cm thick.

Then for the analysis, a meter was used to measure the lengths of the shadows and a protractor to measure the angle of the shadows. For the setup a level and a plumb bob was used.

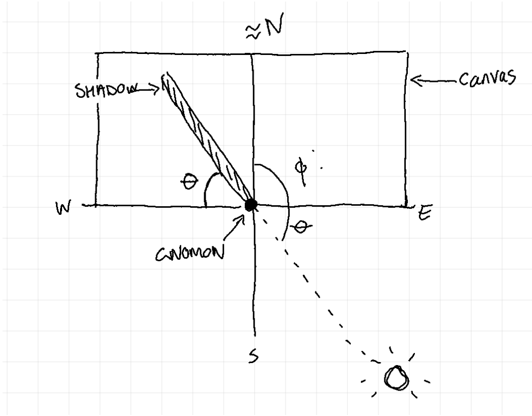

Figure 2: Diagram of setup, viewed from the top

Figure 3: PASCO rod, the gnomon for this experiment

2.2 Methods

Setting up the experiment was the most crucial part. There were several criteria that needed to be met for optimal results. Namely, the surface of the canvas needed to be perfectly flat and horizontal, while the gnomon needed to be perfectly vertical. The makeshift plumb bob allowed to align the gnomon as vertical as possible with the limits of the setup. The canvas was made flat using sticks and books to prop up the corners until the level on top showed a flat surface. For the sake simplifying the analysis later on, the canvas was roughly oriented along East to West, with the gnomon setup at the southern end, such that the shadow lay fully in the canvas.

The data collection in this experiment consisted of marking the tip of the gnomon shadow every 15-30 min when there was enough direct sunlight to cast a shadow. This step, although trivial sounding allowed for some ambiguity. Indeed, as the shadow hit the top of the gnomon, it was hard to discern the exact point to mark. When possible the umbra was marked instead of the penumbra, but occasionally there was ambiguity. Due to the wind, paper weights were made out of rocks to keep the canvas flat and in the same locations.

In total 11 measurements were made within the time frame. When the measurements were made, the exact time was written in a logbook for future analysis.

3 Data

Table 1 shows the data collected from the canvas and compiled into a table. The data collected was the length of the shadow (from the base of the gnomon to the tip of the shadow) and the angle of the shadow with respect to the East-West axis.

	Time	Shadow Length (cm)	Shadow Angle (degrees)
0	11:48:36	48.40	51.25
1	12:00:26	46.45	55.50
2	12:19:00	44.40	61.10
3	12:46:30	42.25	69.00
4	12:57:30	41.20	74.00
5	13:20:45	40.20	83.00
6	13:37:50	39.80	89.00
7	14:02:30	40.45	98.00
8	14:26:00	41.60	106.00
9	14:45:00	42.60	113.00
10	15:09:30	45.50	121.00

Table 1: Raw data collected from experiment

Table 2 shows a collection of important data for further analysis, including the exact date and location where the measurements were taken, and the height and width of the gnomon used for the experiment.

Gnomon Length (cm)	Gnomon Width (cm)	Date	Coordinates
43.6	1.25	14/04/2023	50°50’23.7 N, 5°42’27.3 E

Table 2: Important data for analysis

4 Analysis

4.1 Sunrise, Local Noon and Sunset

The local noon can be estimated using the collected data and by with the formula from Figure 4:

$$\theta = arctan(\frac{length of gnomon}{length of shadow})$$

Then plotting the elevation of the Sun as a function of time and fitting a curve to it allows us to investigate when local noon occurred, this will be when the elevation is highest, and when sunrise and sunset occurred, this will be when the elevation is 0°. For the sake of simplifying the analysis, time has been converted to UNIX format. Furthermore, all the analysis was done in python using Matplolib, Pandas and Numpy libraries on a Jupyter Notebook. The Notebook can be found here: GitHub $(https://github.com/jerbeario/Solar_Motion/tree/main)$

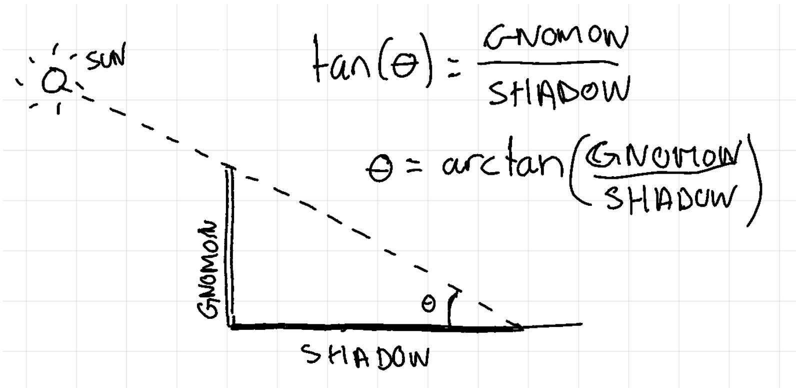

Figure 4: Diagram of setup and equation used to calculate elevation angle of the Sun

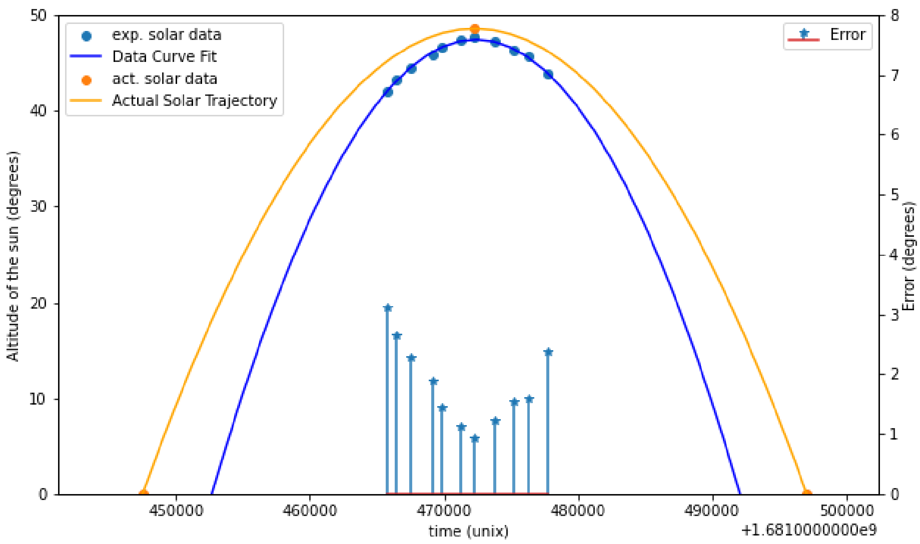

Figure 5: Elevation of the Sun as a function of time on 14/04/2023 (plotted from 5am to 10pm)

The resulting plot of the elevation as a function of time is shown in Figure 5, where the best fit curve for the data is in blue and the actual trajectory of the Sun is in orange. The path for the sun was done by using three known points taken from suncalc.org, namely the time of sunrise (6:45) and sunset (20:30) with the corresponding elevation of 0° and the local noon time (13:37) with the elevation of 48.54°. The main assumption here is that the best fit curve is a 2nd order polynomial of the form $ax^2 + bx + c$.

Based on the experimental data, sunset and sunrise can be calculated by finding the roots of the best fit curve:

$$-1.2218570508443105e - 07x^2 + 410.90377609635175x - 345461674650.076 = 0$$

Which yields $x1 = 1.68149207e + 09, x2 = 1.68145268e + 09$, which converted from UNIX back to local time yields sunrise: 2023-04-14 08:11:20 and sunset: 2023-04-14 19:07:54. The experimental Local Noon can be calculated by taking the derivative of the experimental best fit polynomial and taking its root. This gives a Local Noon time of 2023-04-14 13:39:37. Comparing these results to the actual values in Table 3 shows a clear error for the rise and set times of the Sun, which can also be seen in Figure 5. The Local Noon time calculated experimentally matches the actual time within a couple minutes, how ever there seems to be a systematic error of around 90 minutes between the rise and set times. This can be explained in two ways: firstly, the data collection occurred only in the time frame around Local Noon, within the range of 40-50° thus extrapolating the best fit curve to the end range of 0° will propagate any errors. Secondly, the fact that the experimental curve is too narrow compared to the actual trajectory might signify that there was a problem with the experimental setup. Indeed, upon further investigation, it was found that the surface on which the canvas was resting during the measurements was slightly concave. This means that the shadows appeared to be longer in the tails of the data which would account for some of the error.

Moreover, there were many other sources of error in the data collection, from the difficulty to discern the exact end of the shadow to the surface not being perfectly horizontal and the gnomon not perfectly vertical. However, the estimation for Local Noon only being off by 2 minutes suggest that the experiment still had some validity to it. Finally, one way to increase the accuracy of this experiment would be the perform some measurements in the early morning and in the late evening, after sunrise and before sunset such that the end range of the trajectory is more certain. Another way to improve this experiment is to perform it on a much larger gnomon such as an obelisk or a flag pole.

	Sunrise	Local Noon	Sunset
Experimental	08:11	13:39	19:07
Actual	06:45	13:37	20:30

Table 3: Experimental vs. Actual times of interest.

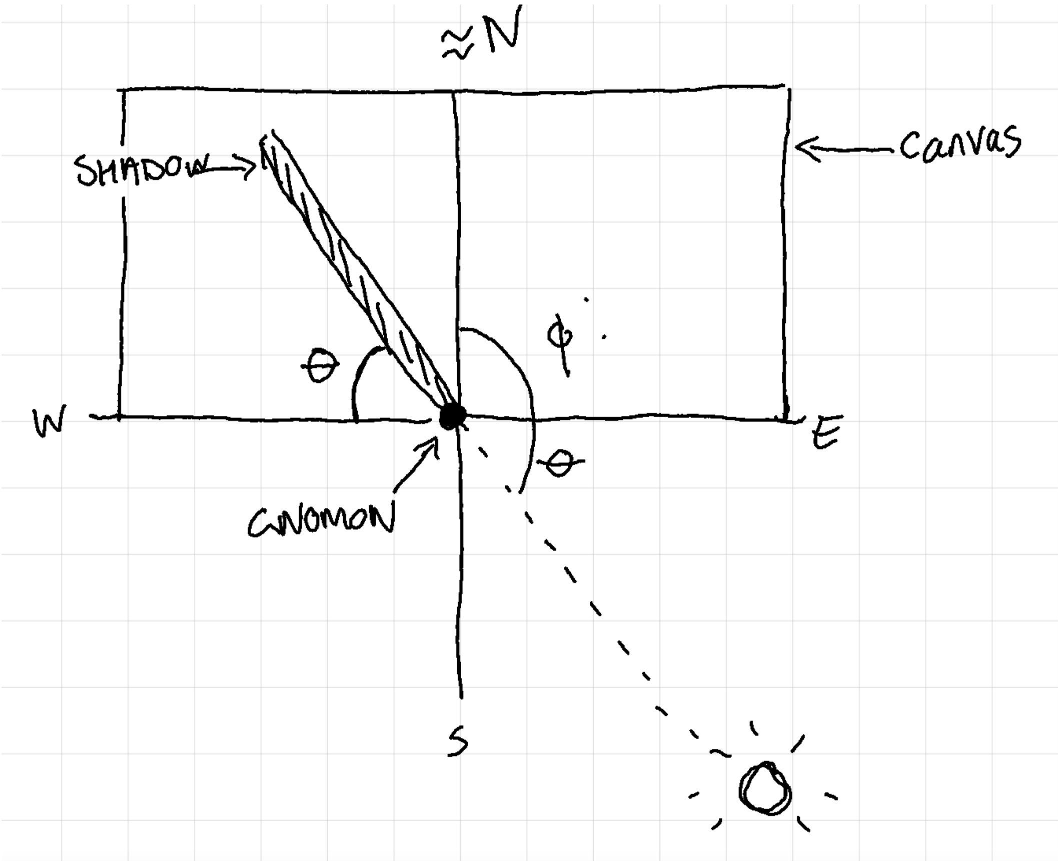

Figure 6: Diagram of azimuth angle ϕ and measured angle θ

4.2 Azimuth Estimation

To experimentally measure the Azimuth angle of the Sun, we can use the angle of the shadow of the stick along with some trigonometry. In Figure 6, the azimuth angle can be calculated using the alternate interior angle theorem, and yield a value $\phi = \theta + 90°$. Using this value we can plot the azimuth against time in Figure 7, where the blue data points are fitting to linear best fit blue curve. Also plotted is a best fit curve passing through the Azimuths from Sunrise (74°), Local Noon (180°), and Sunset (286°). Using our best fit curve, the experimental Azimuth at Sunrise, Local Noon, and Sunset are estimated to be 33.822512708604336°, 178.79921576939523° 324.1569120269269°, respectively. Once again the Local Noon estimation is only with a small percent error $\frac{180-178.799}{180} * 100 = 0.667$ percent. However both the Sunrise and Sunset have much stronger errors: $\frac{74-33.822}{74} * 100 = 54.32$ percent and $\frac{324-286}{286} * 100 = 13.286$ percent respectively. This is likely another manifestation of the concave nature of the surface coupled with systematic error similar to the previous measurement. Another possible source of error is the fact that the North we chose when orienting the canvas may not have been perfect, yet this would only account for a small percent error.

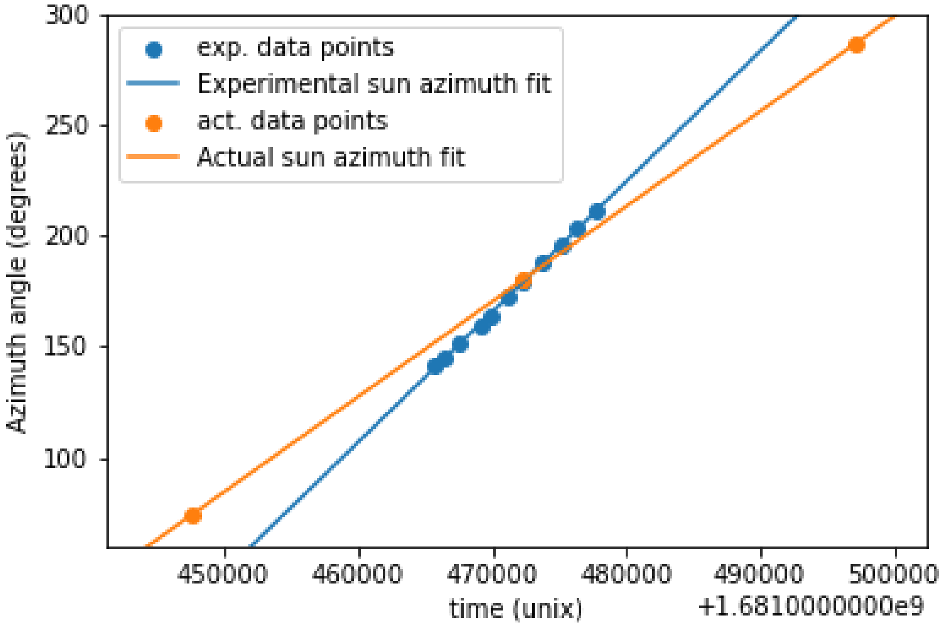

Figure 7: Plot of the Azimuth against time on 14/04/2023 (from 5am to 10pm)

5 Conclusion

In conclusion, the data acquired while measuring the length of a gnomon allowed us to estimate and log the Elevation of the Sun on the 14/04/2023. We estimated the elevation at Local Noon to be 47.40594482421875°, which has an error of $\frac{48.539°-47.405°}{48.539°=2.336}$ percent. We also estimated the time of Local Noon, which turned out to be 1681472377.6910515 s in UNIX time, or 2023-04-14 13:39:37 (2 minutes off of actual Local Noon which is 13:37).

Interestingly this value 13:37 is so far from what the 'noon' that is 12:00 because of two important factors: one is the Summer Hour which accounts for 1 hour ahead of the clock. The last 37 minutes is explained by, the position of Maastricht in its Timezone; according to table 2, the coordinates of the experiment indicate that we were located almost $5\frac{3}{4}°$ East of the Greenwhich Meridian. The Sun rotates an approximate 360° in 24 hours, meaning $\frac{360°}{24h} = 15°$ per hour. Our Local Timezone coordinates is an hour later than Greenwhich (UTC+1) thus it is $1h * 15°/h = 15°E$, meaning we are $15° - 5.75° = 9.25°$ away from the Sun which is $9.25° * \frac{1}{15} = 0.617h$ or $37min$ later.

In the second part of the Analysis, we explored the use of the data is estimating the Azimuth angle of the sun, which although being significantly off, still point to the fact that the Sun is setting in the North-Eastern part of the sky which is typical of this season.

Finally improvements were addressed which focus namely of trying to minimize the experimental error arising from improper alignment of the canvas and gnomon, or ambiguous shadow measurements and irregular surfaces. Repeating the same protocol with these issues in mind and using a taller gnomon would generate more accurate results.