- There is an online tool to test your abilities with Bayes, CFs, etc.
- http://arantxa.ii.uam.es/~ic/teoria/pi/

4.1 Certainty Factors (CF)

- The MYCIN developers realized that a Bayesian approach was intractable, as too much data and/or suppositions/estimates are required.
- In addition, medical diagnosis systems based on Bayesian methods were not acepted because the systems did not provide simple explanations of how it has reached its conclusion.
- Doctors reason more in terms of gathering evidences that supports/contradicts a particular hypothesis. The MYCIN developers thus developed a logic which worked this way: **certainty factors**.

4.1 Certainty Factors (CF)
Certainty Factors vs. probabilities

- Certainty Factors are similar to conditional probabilities, but somewhat different.
  - Rather than representing the degree of probability of an outcome, they represent a measure of belief in the outcome.
  - Where probabilities range from 0 (false) to 1 (true), CFs range from:
    - -1 believed not to be the case
    - 1 believed to be the case
  - The absolute size of the CF measures the degree of belief
  - The sign indicates belief vs disbelief.

4.1 Certainty Factors (CF)
Certainty Factors and Facts and Rules

- We can associate CFs with facts:
  - E.g., padre(John, Mary) with CF .90
- We can also associate CFs with rules:
  - (if (sneezes X) then (has_cold X)) with CF 0.7
  - where the CF measures our belief in the conclusion given the premise is observed.

4.1 Certainty Factors (CF)
Calculating Certainty Factors

- CFs are calculated using two other measures:
  1. MB(*H, E*) – Measure of Belief: value between 0 and 1 representing the degree to which belief in the hypothesis *H* is supported by observing evidence *E*.

$$MB(H,E) = \begin{cases} 1 & \text{si } p(H) = 1 \\ \frac{p(H|E)-p(H)}{1-p(H)} & \text{si } p(H) < 1 \end{cases}$$

  2. MD(*H, E*) – Measure of Disbelief: value between 0 and 1 representing the degree to which disbelief in the hypothesis *H* is supported by observing evidence *E*.

$$MD(H,E) = \begin{cases} 1 & \text{si } p(H) = 0 \\ \frac{p(H)-p(H|E)}{p(H)} & \text{si } p(H) > 0 \end{cases}$$

4.1 Certainty Factors (CF)
Calculating Certainty Factors

- CFs are calculated using two other measures:
  1. MB(*H, E*) – Measure of Belief: value between 0 and 1 representing the degree to which belief in the hypothesis *H* is supported by observing evidence *E*.

$$MB(H,E) = \begin{cases} 1 & \text{si } p(H) = 1 \\ \frac{p(H|E)-p(H)}{1-p(H)} & \text{si } p(H) < 1 \end{cases}$$

  - This is an invented formula (not theoretically derived)
  - It is intended to capture the degree to which the evidence increases probability: p(H|E)-p(H) in proportion to the maximum possible increase in probability: 1-p(H)

4.1 Certainty Factors (CF)

Calculating Certainty Factors

MB:

$$MB(H,E) = \begin{cases} 1 & \text{si } p(H) = 1 \\ \frac{p(H|E) - p(H)}{1 - p(H)} & \text{si } p(H) < 1 \end{cases}$$

- To avoid negative values, the following modification is used:

$$MB(H,E) = \begin{cases} 1 & \text{si } p(H) = 1 \;\; \text{si } p(H) = 1 \\ \frac{max(p(H|E), p(H)) - p(H)}{1 - p(H)} < 1 & \text{si } p(H) < 1 \end{cases}$$

4.1 Certainty Factors (CF)

Calculating Certainty Factors

1. MD(*H, E*) – Measure of Disbelief: value between 0 and 1 representing the degree to which disbelief in the hypothesis *H* is supported by observing evidence *E*.

$$MD(H,E) = \begin{cases} 1 & \text{si } p(H) = 0 \\ \frac{p(H) - p(H|E)}{p(H)} & \text{si } p(H) > 0 \end{cases}$$

- Again, to avoid negative values, the following modification is used:

$$MD(H,E) = \begin{cases} 1 & \text{si } p(H) = 0 \\ \frac{min(p(H|E), p(H)) - p(H)}{0 - p(H)} & \text{si } p(H) > 0 \end{cases}$$

4.1 Certainty Factors (CF)
Calculating Certainty Factors

- CF is calculated in terms of the difference between MB and MD:

$$CF(H,E) = MB(H,E) - MD(H,E)$$

▷ All this implies that the range of CF values goes from $-1{,}0$ to $1{,}0$

▷ ¡Surprise?: the same E can contribute MB and MD toward the same H

▷ However, if we used probabilities to compute CF, that is impossible:

◇ it is not possible that $p(H|E) > p(H)$ and also $p(H|E) < p(H)$

◇ thus at least one of MB and MD must be zero (review the formulae).

▷ But if we only use (subjective) expertise to compute CF, we may have to combine MB and MD

▷ CF is more flexible and intuitive (for the experts) than probability

4.1 Certainty Factors (CF)
Calculating Certainty Factors: example

- Calculate CF(Jac, Hip) given the following data:

	Con Jaqueca	Sin Jaqueca
Con Hipertensión	0.21	0.39
Sin Hipertensión	0.33	0.07

$$MB(H,E) = \begin{cases} 1 & \text{si } p(H) = 1 \text{ si } p(H) = 1 \\ \frac{max(p(H|E),p(H)) - p(H)}{1-p(H)} < 1 & \text{si } p(H) < 1 \end{cases}$$

$$MD(H,E) = \begin{cases} 1 & \text{si } p(H) = 0 \\ \frac{min(p(H|E),p(H)) - p(H)}{0-p(H)} & \text{si } p(H) > 0 \end{cases}$$

$$CF(H,E) = MB(H,E) - MD(H,E)$$

4.1 Certainty Factors (CF)
example

	Con Jaqueca	Sin Jaqueca
Con Hipertensión	0.21	0.39
Sin Hipertensión	0.33	0.07

- p(H|E)=0.21/0.54=0.388   p(H)=0.6
- MB(H,E) = 0
- MD(H,E)=(0.6-0.388)/0.6
  = 0.3519

$$MB(H,E) = \begin{cases} 1 & \text{si } p(H) = 1 \text{ si } p(H) = 1 \\ \frac{max(p(H|E),p(H))-p(H)}{1-p(H)} < 1 & \text{si } p(H) < 1 \end{cases}$$

$$MD(H,E) = \begin{cases} 1 & \text{si } p(H) = \\ \frac{min(p(H|E),p(H))-p(H)}{0-p(H)} & \text{si } p(H) > \end{cases}$$

$$CF(H,E) = MB(H,E) - MD(H,E)$$

4.1 Certainty Factors (CF)

- **EXAM TIP: Bring your calculator!!!**

4.1 Certainty Factors (CF)

COMBINING Certainty Factors

- Multiple sources of evidence produce CFs for the same fact.
- For instance, two rules may provide evidence for the same conclusion:
  - if P1 Then C CF=0.8
  - if P2 Then C CF=0.7
- We need to know how to combine the CFs in such cases
- If two rules both support a hypothesis,then that should increase our belief in the hypothesis.

4.1 Certainty Factors (CF)

COMBINING Certainty Factors

- We use an incremental approach as with Bayes
- Take: X = CF(E), Y = CF(E')

$$CF(H, E \wedge E') = \begin{cases} X + Y(1 - X) & \text{if } X, Y > 0 \\ X + Y(1 + X) & \text{if } X, Y < 0 \\ \frac{X+Y}{1-min(|X|,|Y|)} & \text{if } sign(X) \neq sign(Y) \end{cases}$$

- In the first case, the bigger the CF(E), the less of CF(E') we need to add.
- Note: X+Y.(1-X) = Y+X(1-Y)

4.1 Certainty Factors (CF)
COMBINING Certainty Factors

- Example:
  - A jurors beliefs:
    1. if defendent's fingerprints on gun then GUILTY (0.75)
    2. if defendent has motive then GUILTY (0.6)
    3. if defendent has alibi then GUILTY (-0.8)
  - The court established beyond doubt that the defendant's fingerprints were on the gun, had no motive, and that the defendent had an alibi.

$$CF(H, E \wedge E') = \frac{X+Y}{1-min(|X|,|Y|)} \quad \text{if } sign(X) \neq sign(Y)$$

  - CF=(0.75+-0.8)/(1-0.75)= -0.05/0.25 = -0.20

4.1 Certainty Factors (CF)
Rules with uncertain evidence

- In the previous slides, we dealt with rules like:

  if P1 Then C CF=0.8
- We saw that if P1 is observed, then we can conclude C with CF 0.8.
- However, what happens if P1 is itself uncertain? (has an associated CF), E.g,
  - `if defendent's fingerprints on gun then GUILTY (0.75)`

    But expert only .9 certain that the fingerprints are defendants

    -> CF(defendent's fingerprints on gun) =0.9
  - `if defendent has motive then GUILTY (0.6)`

    Witness says victim was sleeping with defendant´s wife, but later it was revealed the witness had lied in other court cases.

    -> CF(`defendent has motive`) =0.5

4.1 Certainty Factors (CF)

Rules with uncertain evidence

- Evidence may also be uncertain when it itself is gained from applying a rule:
  - IF pregnant(X) THEN has_diabetes(X)   CF=0.3
  - IF has_diabetes(X) THEN kidney_damaged(X) CF=0.2
  - If we know absolutely that Mary is pregnant, then the fact: diabetes(Mary) is estamated with a CF of 0.3.
  - So, when we go to apply the second rule, we need to take into account that the premise is not certain.

4.1 Certainty Factors (CF)

Rules with uncertain evidence: 1 premise

- When a rule has a single premise, the certainty of the conclusion is the PRODUCT of the certainty of the premise multiplied by the certainty of the rule:
- I.E. `R1 if P then C`

  CF(C) = CF(P) * CF(R1)

- Example:
  - R1: IF pregnant(X) THEN has_diabetes(X)   CF=0.3
  - R2: IF has_diabetes(X) THEN kidney_damaged(X) CF=0.2
  - CF(pregnant(Mary)) =0.9
  - Thus: CF(diabetes(Mary)) = CF(pregnant(Mary)) * CF(R1) = 0.27
  - Thus: CF(kidney_dam(Mary)) = CF(diabetes(Mary)) * CF(R2) = 0.054

4.1 Certainty Factors (CF)

Rules with uncertain evidence: negative evidence

- A rule is only applicable if you believe the premise to be true
- Thus, if the CF of the Premises is negative (you do not believe them) then the rule does not apply.
- E.g.
  IF has_fever(X) THEN has_COLD(X) CF 0.7
  But has_fever(John) has CF -0.2
  Then I cannot say anything about John having a cold.

THUS:

CF(C) = CF(P) * CF(RULE) if CF(P)>0
= 0 otherwise

CF of 0 indicates we know nothing as the result of applying the rule (we neither believe or disbelieve. Thus, our knowledge does not change!

4.1 Certainty Factors (CF)

Rules with uncertain evidence: more than one premise

- Our belief in a conjunction of premises is simply the smallest belief of the set:
- Given beliefs:
  cf(color(snow, white)) = 1.0
  cf(color(coal, black)) = 1.0
  cf(color(blood, green)) = -1.0

So, what is your CF in:

- color(snow, white) & color(coal, black) and color(blood, green)
- Simply, the minimum value. -> -1

4.1 Certainty Factors (CF)

Rules with uncertain evidence: more than one premise

- If a rule has more than one premise:
  IF P1&P2&P3 THEN C
- We find the CF of the set of premises
  - WHICH is just the MIN
- CF(C) is thus calculated:

CFPs = MIN(CF(P1), CF(P2), CF(P3))		
CF(C)	= CFPs * CF(RULE)	if CFPs>0
	= 0 otherwise	

4.1 Certainty Factors (CF)

Rules with uncertain evidence: more than one premise

Implementation shortcut:

- Note: IF the CF of any one premise is ≤ 0
  THEN the CF of the set is ≤ 0

  THUS: The rule does not apply!

- THUS, when evaluating the premises of a rule, one can stop processing if a premise has CF ≤ 0

Problem 1: example with CFs to judge case of murder

Example:

- **A jurors reasoning**:
  1. if defendent's fingerprints on gun then GUILTY (0.75)
  2. if defendent has motive then GUILTY (0.6)
  3. if defendent has alibi then GUILTY (-0.8)
- **Evidence Presented**:
  1. A CSI expert testifies (0.9) that the homicida weapon has the fingerprints of the defendant.
  2. The mother-in-law of the defendant testifies that the defendant wanted to cash the victim's life insurance, but since it is later shown that she hates the defendant, the credibility of this supposed motive for the murder is not high (0.5).
  3. A judge of high prestige provides an excellent alibi for the defendant (0.95).
- **Verdict**: innocent or guilty?

Problem 2:

A dental clinic has analyzed the existence or not of *decay* in all his patients, and have obtained the following probabilities distributed according to

	Con Dolor	**Sin** Dolor	
Con Caries	0.25	0.08	0.33
Sin Caries	0.35	0.32	0.67
	0.60	0.40	1.00

	Con Flemón	**Sin** Flemón	
Con Caries	0.20	0.13	0.33
Sin Caries	0.10	0.57	0.67
	0.30	0.70	1.00

- Observed a patient with flemón, we know its *CF* to have decay
- If in addition, we observed that **it has pain teeth**, which is now its *CF* **to have decay**?
- For these and other problems on uncertainty, consult:
- http://www.ii.uam.es/~ic/teoria/pi

Problem 3:
Given following initial *CF* s:

$$CF(A) = 0'3, CF(B) = 0'0, CF(C) = 0'0, CF(D) = 0'4,$$
$$CF(E) = 0'0, CF(F) = -0'5, \text{ y } CF(G) = 0'8$$

And given the following rules with its *CF*:

$$A \rightarrow C, \text{con } CF(regla) = 0'2$$
$$B \rightarrow C, \text{con } CF(regla) = 1'0$$
$$D \rightarrow E, \text{con } CF(regla) = 0'6$$
$$E \rightarrow C, \text{con } CF(regla) = 1'0$$
$$F \rightarrow B, \text{con } CF(regla) = 0'9$$
$$G \rightarrow E, \text{con } CF(regla) = -0'7$$

Which are final *CF* s of *C*, *and*, *B*…?

Advantages of CF:

- **Model of simple calculation**
- **It allows effects + and – manifold on each hypothesis**
- **Easy to integrate with rules**
- **Easy to extract of the expert**

Easy to extract via statistics

Critics (disadvantages) to CF:

- **Supposition of independence (p.ej.: multiple alibis, related symptoms)**
- **To review the knowledge base is more complex**

Surprise: MYCIN without CF, still correct!