Topic 4

Representation and Reasoning with Uncertainty

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4.0 Representation and Reasoning with Uncertainty

Reasoning can be uncertain in several ways:

- Uncertainty in data (facts)
- Uncertainty in inference processes (rules)

Uncertainty in data (facts)

- **Data not available** (e.g., doctor does not know if patient has allergy to a drug.
- **Data unreliable**: data is available, but not trustworthy, e.g.,
  - **errors of measurement**, two measurements gave different results, etc.
  - The data is in an **imprecise representation**.
  - The data represents a **guess** by the expert,
    - e.g., `Patient is in good health?   True (0.9)`
  - The data may be just a **default value**, and there may be exceptions to default values.

4.0 Representation and Reasoning with Uncertainty

Uncertainty in knowledge (rules)

- Rule may represent just a statistical association observed by experts, e.g.,.

```
IF    primary infection is bacteriacea
      AND site of infection is sterile
      AND entry point of infection
             is gastrointestinal tract
THEN organism is “bacteroid” (0.7)
```

- Rule may not be appropriate in all cases
  - e.g., rules designed for the average person may not be suitable for someone who is totally atypical

Issues in managing uncertainty

1. **How to represent uncertain data and knowledge**.
2. **How to combine two or more items of uncertain data** (e.g., if both premises to a rule are uncertain, how do we combine the uncertainty)
3. **How to draw inferences (conclusions) using uncertain data and knowledge** (e.g., how do we combine the uncertainty of premises and rules to estimate the uncertainty of the conclusion)

Some ways for representing uncertainty:

1. As Probability (Bayesian Logic): what is the probability of X being True
   - P(x)=0: X is false
   - P(x)=1: X is true
2. As a Certainty Factor: how certain are we that X is true or not true.
   - CF(x)= 0: I have no knowledge if x is true or false
   - CF(x)= -1: I believe x is not true
   - CF(x)= 1: I believe x is true
3. As Fuzzy Logic: to what degree x is in various sets.
   - a given cup of coffee can be member of cold-drink and hot-drink sets at the same time.

Topic 4

Representation and Reasoning with Uncertainty

4.1 Probabilistic methods (Bayesian)

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4.1 Probabilistic methods (Bayesian)

Definitions

- **Random Variable:** an aspect of the problem for which a value is not initially known.

  Examples: patient temperature, patient has viral infection?
- **Domain**: the range of possible values a variable can take. Can be boolean (true/false), discreet (black/red/green) or continuous values.
- **Atomic event (outcome)**: A complete specification of the state of the world about which the agent is uncertain.
  - E.g., if the world consists of only two Boolean variables Cavity and Toothache, then there are 4 distinct atomic events:

    Cavity = false & Toothache = false

    Cavity = false & Toothache = true

    Cavity = true & Toothache = false

    Cavity = true & Toothache = true

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Example

Tossing a coin.

Random Variable X: the result of the toss

Domain: {head, tail}

Outcomes: {X=head, X=tail}

Two tosses of the same coin

Random Variable : X (result of the first toss),
Y (result of the second toss)

Domain of both X and Y: {head, tail}

Outcomes: { {X=head, Y=head}, {X=head, Y=tail},
{X=tail, Y=head}, {X=tail, Y=tail} }

Example II

Patient being diagnosed with Flu base based on cough, temperature, age, fever .

Random Variables and their domains:

Cough:	{ yes, no}	(boolean)
Temperature	10-45ºC	(discrete, if the thermometer has resolution of 0.5 ºC)
Age:	0-100	(discrete)
Fever:	{yes, no}	(boolean)
Flu :	{yes, no}	(boolean)

More definitions

- **Event:** set of zero, one or various atomic events.
  It represents something that can happen .
  Example : In the problem involving two tosses of the coin, the event "has at least one head" is the set :
  { {X=head, Y=head}, {X=head, Y=tail},
  {X=tail, Y=head} }

  The event “the second toss is different to the first” is:
  { {X=head, Y=tail}, {X=tail, Y=head} }.

More definitions

- **Mutually exclusive events:** events that do not have any atomic event in common.
- This implies that they cannot happen simultaneously.
  - Example: The event "*the second toss is different to the first*" and the event "*the second toss is the same as the first*" are mutually exclusive.
  - The event “*there is at least one head”* and the event “*there is at least one tail”* are not mutually exclusive.

- **Exhaustive events:** a set of events which together cover all the atomic events of the problem.

4.1 Probabilistic methods (Bayesian)

Probabilities

- The probability of event s_i is given as $p(s_i)$
- The probability of each event s_i is between 0 and 1

$0<p(s_i)<1.0$

- If **s** is an event which must occur, $p(\mathbf{s}) = 1$.
  If **s** cannot occur, $p(\mathbf{s}) = 0$
- The sum of probabilities of all events is 1.0:

$$\sum_{i=1}^{n} p(s_i) = 1.0$$

- The probability of ~s is 1- $p(s)$:

$$p(\sim s_i) = 1 - p(s_i)$$

$$p(s_i) + p(\sim s_i) = 1$$

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4.1 Probabilistic methods (Bayesian)

Combined Probability

- If s_1 and s_2 are two outcomes ('U' = 'v' = 'or'; ∩ = & = ∧ = 'and') :

$$p(s_1 \cup s_2) = p(s_1) + p(s_2) - p(s_1 \cap s_2)$$

e.g. p(dice<4) = 1/2, p(dice=even)=1/2,
p(dice<4 or dice=even) = 1/2 + 1/2 - 1/6 = 5/6

- If s_1 and s_2 are mutually exclusive, this simplifies :

$$\mathbf{p(s_i \vee s_j) = p(s_i) + p(s_j)}$$

e.g. p(dice=3) = 1/6, p(dice=4)=1/6,
p(dice = 3 or 4) = 2/6

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4.1 Probabilistic methods (Bayesian)

Joint Probability

- If x_i and y_j are independent events:

$$p(x_i \,\&\, y_j) = p(x_i)\, p(y_j)$$

e.g. p(dice1=3) = 1/6, p(dice2=4)=1/6,
p(dice1 = 3 & dice2=4) = 1/36

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4.1 Probabilistic methods (Bayesian)

Conditional Probability

- The probability of s_1 given outcome s_2: $p(s_1 | s_2)$
- The conditional probability can be calculated

$$p(s_1 \mid s_2) \equiv \frac{p(s_1 \cap s_2)}{p(s_2)}$$

- E.g., p(dice=4 | dice=even) = 1/6 / 1/2
  = 1/3
  - given we roll an even dice, we have 1/3 chance of rolling a 4.

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4.1 Probabilistic methods (Bayesian)

Conditional Probability with independent events

- If x and *y* are totally independent events:

  $p(x \& y) = p(x) . p(y)$

- Thus:

  $p(x | y) = p(x \& y) / p(y)$

  $= p(x) . p(y) / p(y)$

  $= p(x)$

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4.1 Probabilistic methods (Bayesian)

Conditional Probability: alternative formula

Given: $p(y | x) = p(y \& x) / p(x)$

Then: $p(y \& x) = p(y | x) * p(x)$

Substituing this into our prior formula:

$p(x | y) = p(x \& y) / p(y)$

Gives:

$p(x | y) = p(y | x) * p(x) / p(y)$

THIS IS CALLED BAYES THEOREM!!!

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4.1 Probabilistic methods (Bayesian)

Bayes Rule: example

Given that p(dice=2|dice=even) = 1/3

Calculate p(dice=even|dice=2)

$$p(\text{dice=even}|\text{dice=2}) = \frac{p(\text{dice=2}|\text{dice=even}) * p(\text{dice=even})}{p(\text{dice=2})}$$

$$= \frac{1/3 * 1/2}{1/6} = \frac{1/6}{1/6} = 1$$