Lab #4 — Path tracing (part 3)

Informática Gráfica

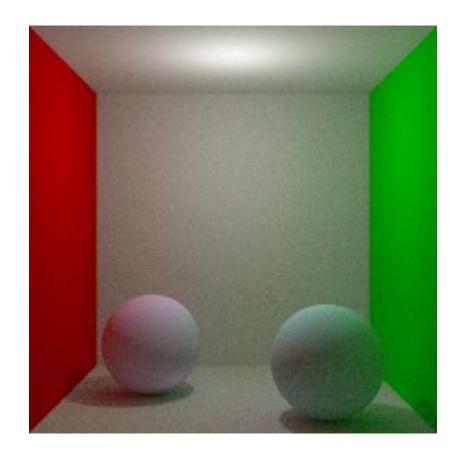
Adolfo Muñoz - Julio Marco Pablo Luesia - J. Daniel Subías — Óscar Pueyo



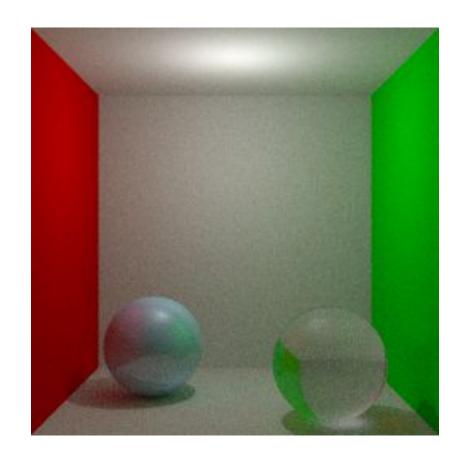
Before we begin...



Today: add more materials to the path tracing algorithm



Previous session (diffuse)



Perfect specular and refraction

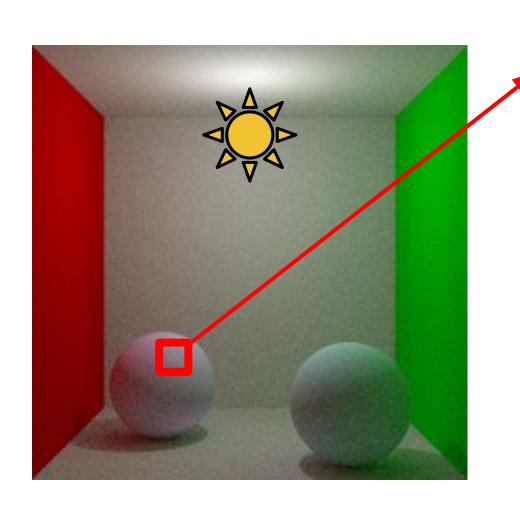
Before we begin...



- Lab 4 (path tracing) is the first submitted work
 - Recommended deadline: November 13th
 - Moodle: January 11th
 - You will use most of today's code for Lab 5 (photon mapping) too
- Remember: Final work is 80% of the final grade

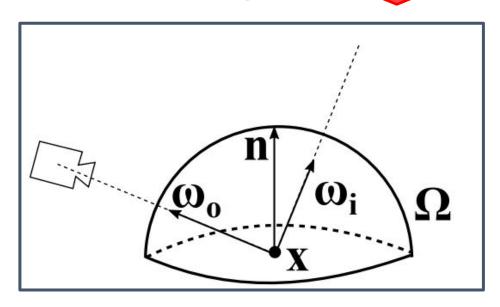
Recap: Which color do we fill each pixel with?





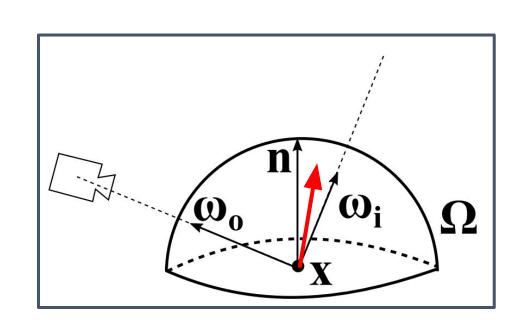
$$L_o(\mathbf{x}, \omega_o) = L_o(\mathbf{x}, \omega_o) + \int_{\Omega} L_i(\mathbf{x}, \omega_i) f_r(\mathbf{x}, \omega_i, \omega_o) |\mathbf{n} \cdot \omega_i| d\omega_i$$

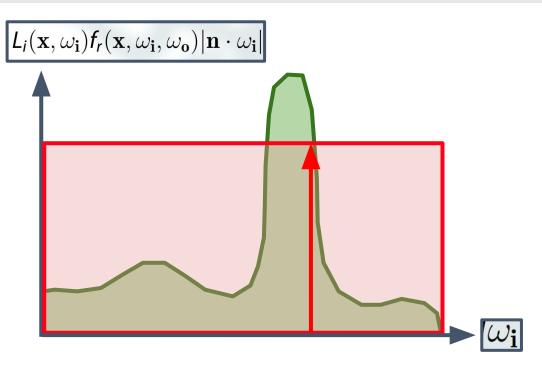
The full integral



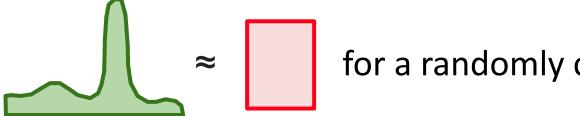
Recap: Approximating one integral







- There are infinite values for $\omega_{\mathbf{i}}$
- Idea 3: Monte Carlo estimator, use the mean of N = 1 random sample

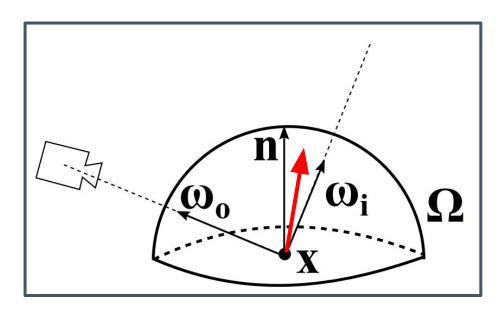


for a randomly chosen direction ____

Recap: Approximating one integral in practice

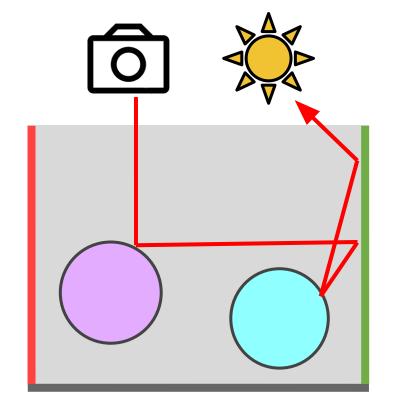


Monte Carlo estimation for the path integral



One random ω_i on each bounce

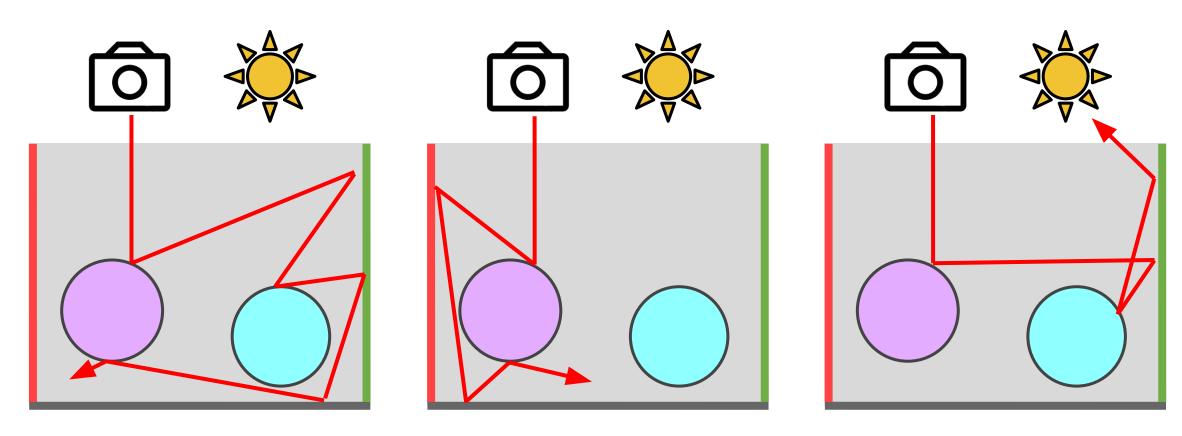
One random path



Recap: Approximating one integral in practice

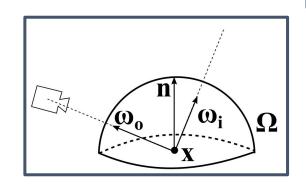


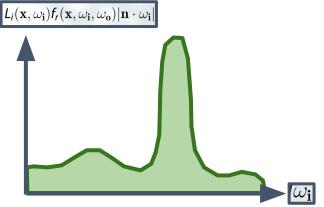
- Monte Carlo estimation for the path integral
 - Sum of multiple random paths
 - \circ More paths \rightarrow better approximation of the integral (better result)





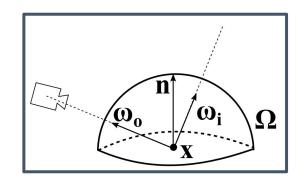
- How to generate a value for ω_i ?
- Use the sampling method (probability distribution of ω_i) that benefits you

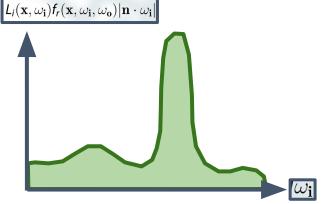






- How to generate a value for ω_i ?
- Use the sampling method (probability distribution of ω_i) that benefits you





Uniform solid angle sampling

$$p(\theta_i) = \sin \theta_i$$

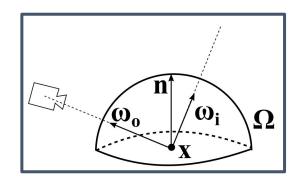
$$p(\phi_i) = \frac{1}{2\pi}$$

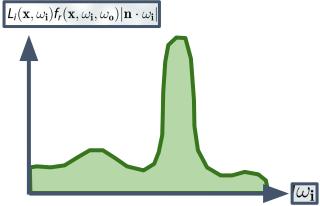
$$p(\theta_i) = 2\sin\theta_i\cos\theta_i$$

$$p(\phi_i) = \frac{1}{2\pi}$$



- How to generate a value for ω_i ?
- Use the sampling method (probability distribution of ω_i) that benefits you





Uniform solid angle sampling

$$p(\theta_i) = \sin \theta_i$$

$$p(\phi_i) = \frac{1}{2\pi}$$

$$\boldsymbol{c}^{-1}(\xi_{\theta_i}) = \arccos \xi_{\theta_i}$$

$$\boldsymbol{c}^{-1}(\xi_{\phi_i}) = 2\pi \xi_{\phi_i}$$

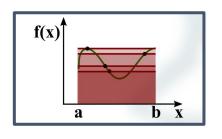
$$p(\theta_i) = 2\sin\theta_i\cos\theta$$

$$p(\phi_i) = rac{1}{2\pi}$$

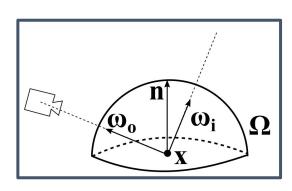
$$p(\theta_i) = 2\sin\theta_i\cos\theta_i$$
 $c^{-1}(\xi_{\theta_i}) = \arccos\sqrt{1-\xi_{\theta_i}}$

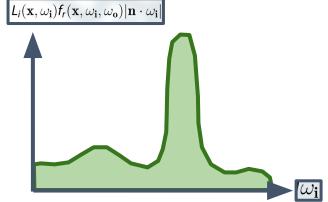
$$\boldsymbol{c}^{-1}(\xi_{\phi_i}) = 2\pi \xi_{\phi_i}$$





$$\int_{a}^{b} f(x)dx \approx \frac{1}{N} \sum_{i=1}^{N} \frac{f(x_{i})}{p(x_{i})}$$





Uniform solid angle sampling

$$p(\theta_i) = \sin \theta_i$$

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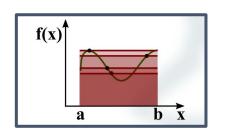
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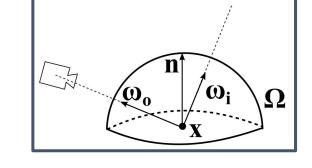
$$\boldsymbol{c}^{-1}(\xi_{\phi_i}) = 2\pi \xi_{\phi_i}$$

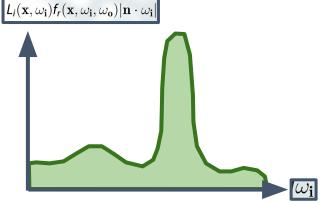




Remember to simplify

$$\int_{a}^{b} f(x)dx \approx \frac{1}{N} \sum_{i=1}^{N} \frac{f(x_i)}{p(x_i)}$$



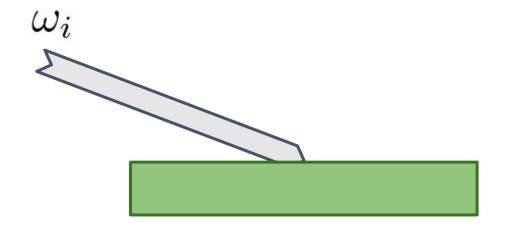


$$L_o(\mathbf{x}, \omega_o) \approx \sum_{i=1}^{N} \frac{2\pi L_i(\mathbf{x}, \omega_i) f_r(\mathbf{x}, \omega_i, \omega_o) \cos \theta_i \sin \theta_i}{\sin \theta_i}$$

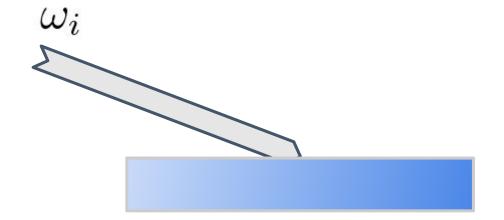
$$L_o(\mathbf{x}, \omega_o) \approx \sum_{i=1}^N \frac{2\pi L_i(\mathbf{x}, \omega_i) f_r(\mathbf{x}, \omega_i, \omega_o) \cos \theta_i \sin \theta_i}{2\sin \theta_i \cos \theta_i}$$



Diffuse material



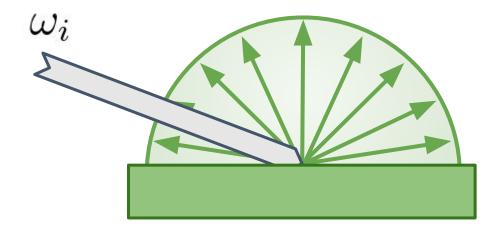
Perfect specular material



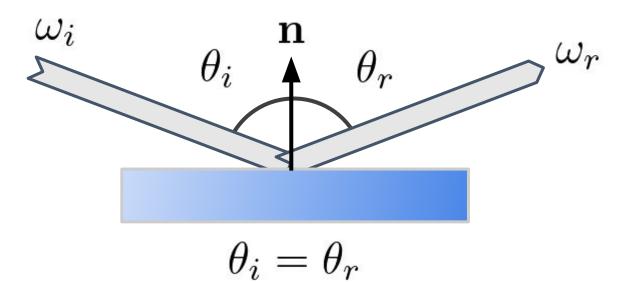


Diffuse material

Light is reflected in all directions equally



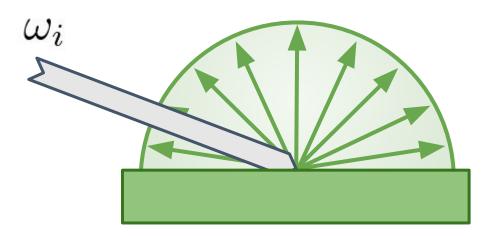
Perfect specular material





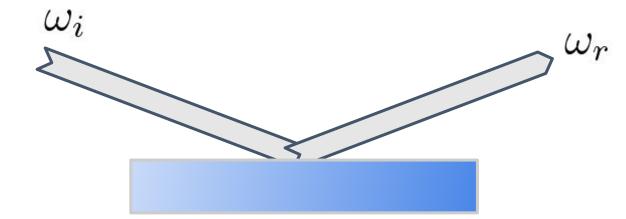
Diffuse material

Light is reflected in all directions equally



$$f_r(\mathbf{x}, \omega_i, \omega_o) = \frac{k_d}{\pi}$$

Perfect specular material

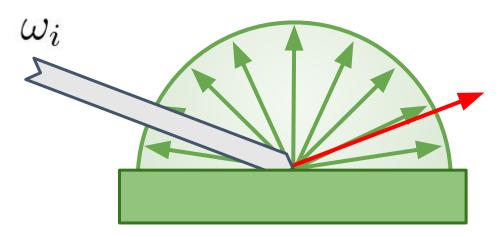


$$f_r(\mathbf{x}, \omega_i, \omega_o) = \frac{\delta_{\omega_r}(\omega_i)}{(\omega_i \cdot \mathbf{n})}$$



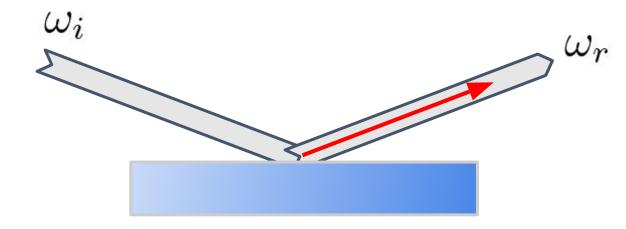
Diffuse material

Light is reflected in all directions equally



$$f_r(\mathbf{x}, \omega_i, \omega_o) |\mathbf{n} \cdot \omega_i| = \frac{k_d}{\pi} |\mathbf{n} \cdot \omega_i|$$

Perfect specular material

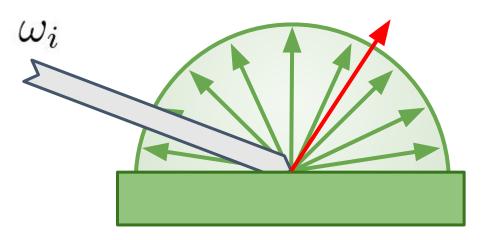


$$f_r(\mathbf{x}, \omega_i, \omega_o) |\mathbf{n} \cdot \omega_i| = 1$$



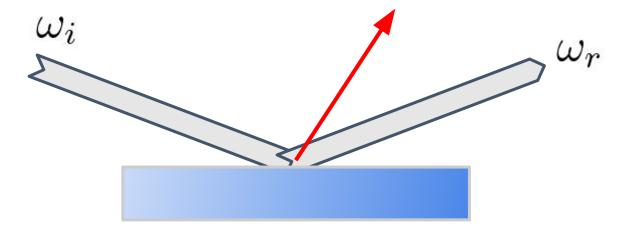
Diffuse material

Light is reflected in all directions equally



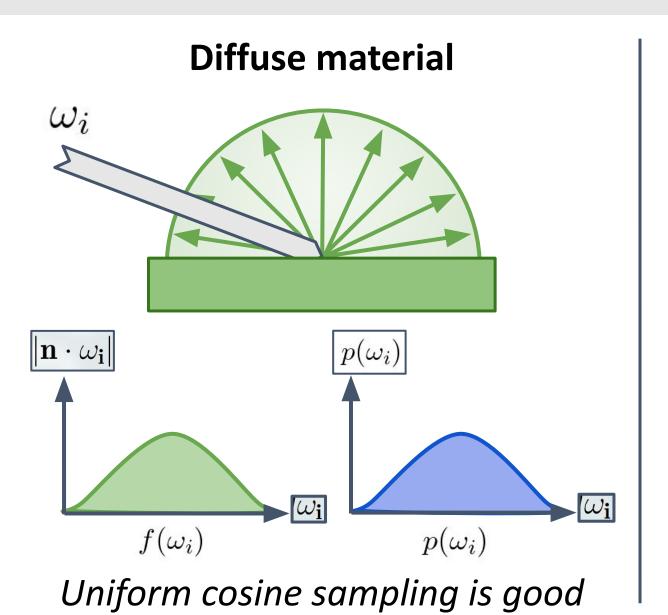
$$f_r(\mathbf{x}, \omega_i, \omega_o) |\mathbf{n} \cdot \omega_i| = \frac{k_d}{\pi} |\mathbf{n} \cdot \omega_i|$$

Perfect specular material

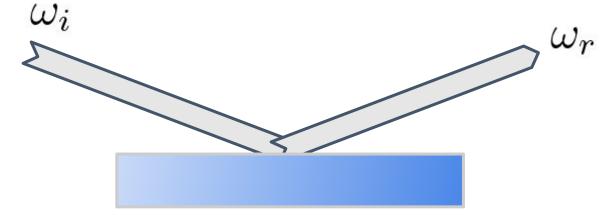


$$f_r(\mathbf{x}, \omega_i, \omega_o) |\mathbf{n} \cdot \omega_i| = 0$$

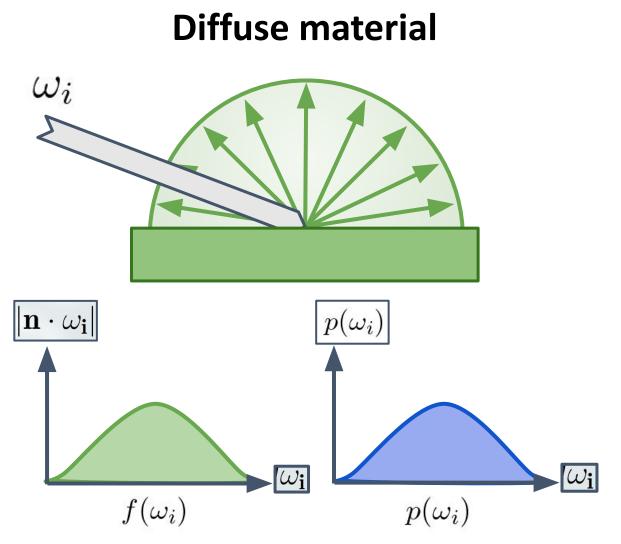




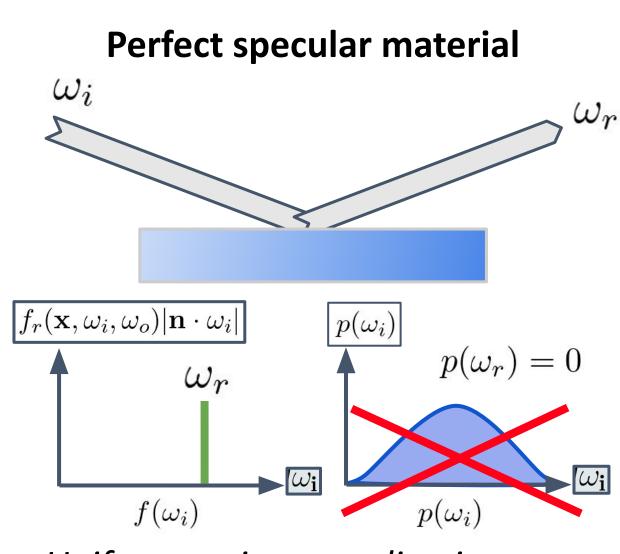
Perfect specular material





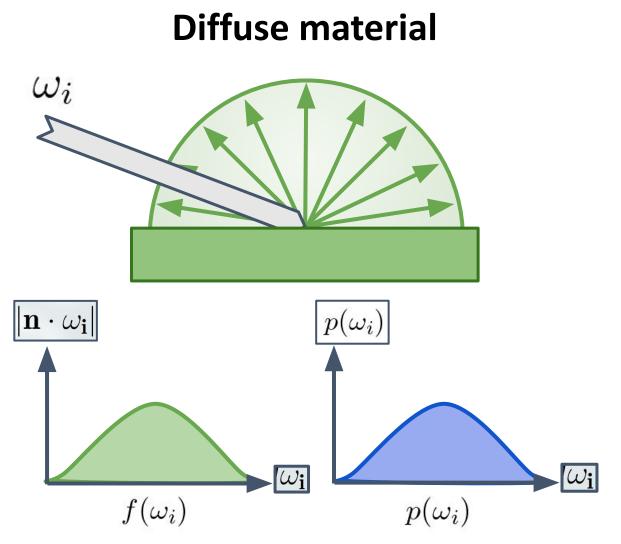


Uniform cosine sampling is good

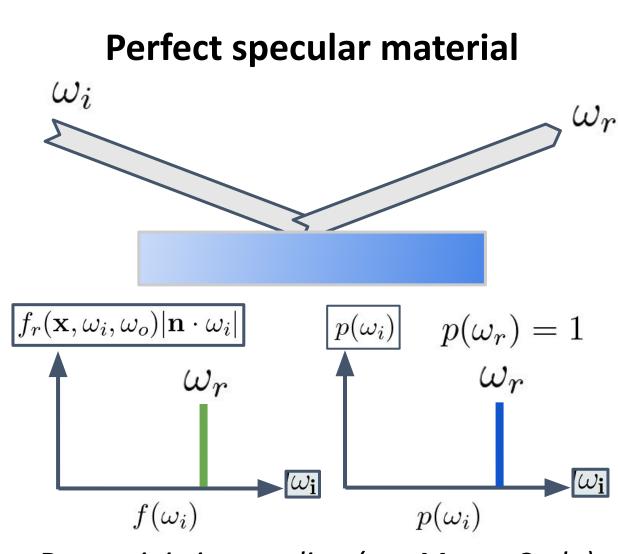


Uniform cosine sampling is wrong





Uniform cosine sampling is good

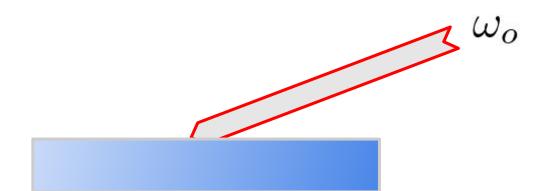


Deterministic sampling (not Monte Carlo)

Perfect specular (delta BRDF) sampling



Remember: paths go from the camera to the light source



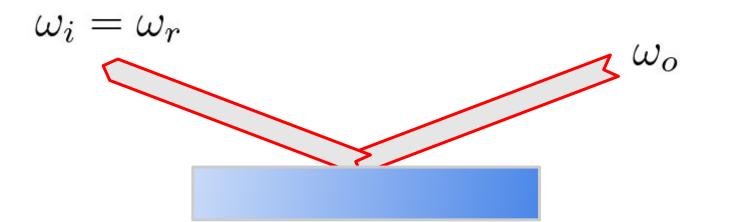
$$f_r(\mathbf{x}, \omega_i, \omega_o) = \frac{\delta_{\omega_r}(\omega_i)}{(\omega_i \cdot \mathbf{n})}$$

Always sample the perfect specular direction ω_r

Perfect specular (delta BRDF) sampling



Remember: paths go from the camera to the light source



$$f_r(\mathbf{x}, \omega_i, \omega_o) = \frac{\delta_{\omega_r}(\omega_i)}{(\omega_i \cdot \mathbf{n})}$$

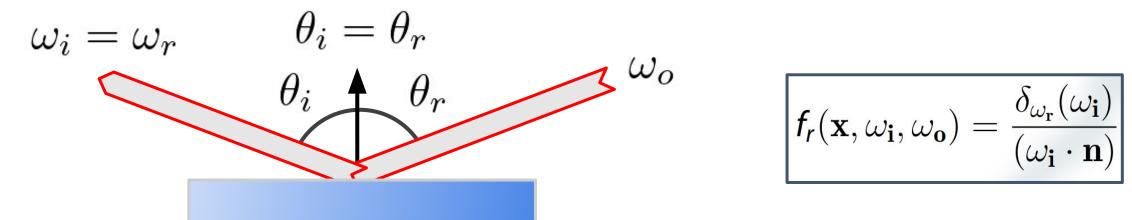
Always sample the perfect specular direction ω_r

$$\omega_i = \omega_r = \omega_o - 2\mathbf{n}(\omega_o \cdot \mathbf{n})$$

Perfect specular (delta BRDF) sampling Graphics and Imaging Lab



Remember: paths go from the camera to the light source

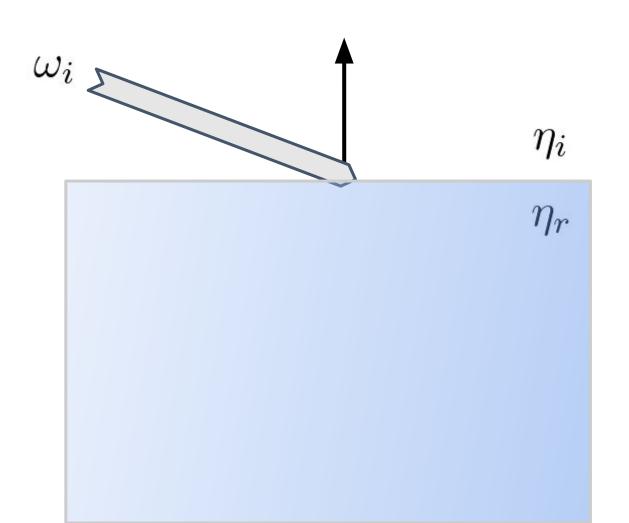


Always sample the perfect specular direction ω_r

$$\omega_i = \omega_r = \omega_o - 2\mathbf{n}(\omega_o \cdot \mathbf{n})$$
$$p(\omega_r) = 1$$



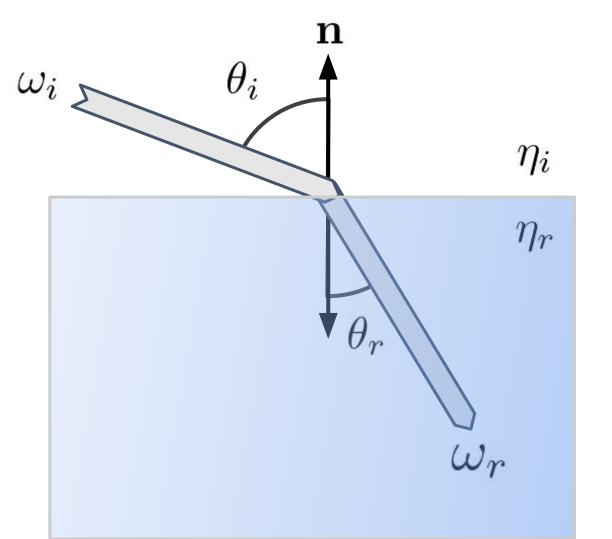
The material does not **reflect** light, instead it (perfectly) **refracts** light



$$f_r(\mathbf{x}, \omega_i, \omega_o) = \frac{\delta_{\omega_i}(\omega_i)}{(\omega_i \cdot \mathbf{n})}$$



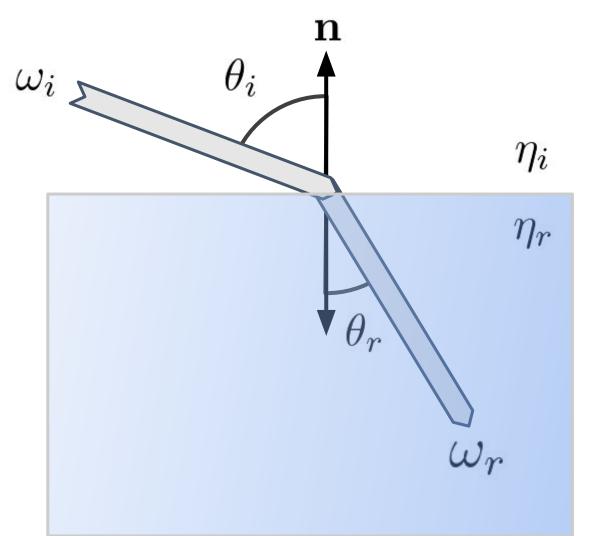
• The material does not **reflect** light, instead it (perfectly) **refracts** light



$$f_r(\mathbf{x}, \omega_i, \omega_o) = \frac{\delta_{\omega_t}(\omega_i)}{(\omega_i \cdot \mathbf{n})}$$



The material does not **reflect** light, instead it (perfectly) **refracts** light



$$f_r(\mathbf{x}, \omega_i, \omega_o) = \frac{\delta_{\omega_i}(\omega_i)}{(\omega_i \cdot \mathbf{n})}$$

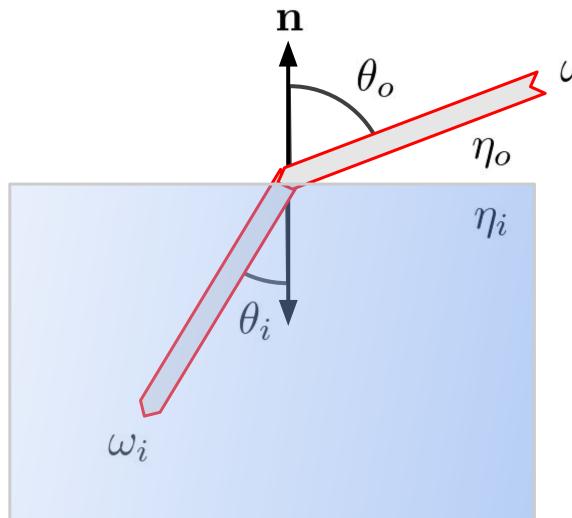
Snell's law:

 ω_r depends on the index of refraction η_i , η_r of the two media

$$\eta_i \sin \theta_i = \eta_r \sin \theta_r$$



The material does not **reflect** light, instead it (perfectly) **refracts** light



$$f_r(\mathbf{x}, \omega_i, \omega_o) = \frac{\delta_{\omega_i}(\omega_i)}{(\omega_i \cdot \mathbf{n})}$$

Snell's law:

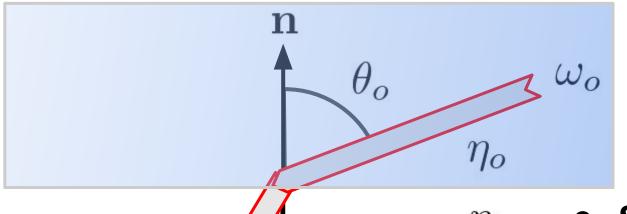
 ω_r depends on the index of refraction η_i , η_r of the two media

$$\sin \theta_i = \sin \theta_r = \frac{\eta_o}{\eta_i} \sin \theta_o$$

$$p(\omega_r) = 1$$



The material does not **reflect** light, instead it (perfectly) **refracts** light



$$f_r(\mathbf{x}, \omega_i, \omega_o) = \frac{\delta_{\omega_i}(\omega_i)}{(\omega_i \cdot \mathbf{n})}$$

• Snell's law: η_i

 ω_r depends on the index of refraction η_i , η_r of the two media

$$\sin \theta_i = \sin \theta_r = \frac{\eta_o}{\eta_i} \sin \theta_o$$

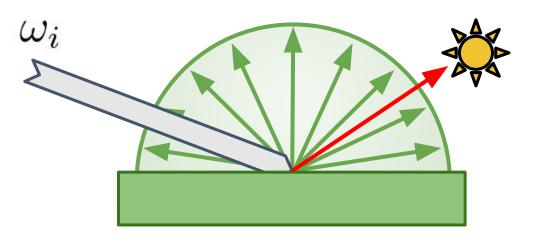
$$p(\omega_r) = 1$$

Next-event estimation and delta BRDF



Diffuse material

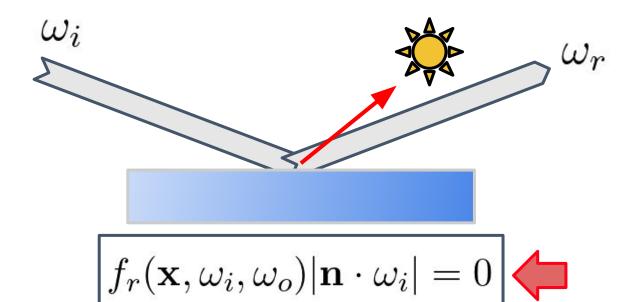
Light is reflected in all directions equally



$$\left| f_r(\mathbf{x}, \omega_i, \omega_o) | \mathbf{n} \cdot \omega_i \right| = \frac{k_d}{\pi} |\mathbf{n} \cdot \omega_i|$$

Perfect specular material

All light is (perfectly) reflected towards one direction ω_r



Next-event estimation will always return 0

The full BSDF



Combining these three properties:

$$f_r(\mathbf{x}, \omega_{\mathbf{i}}, \omega_{\mathbf{o}}) = k_d \frac{1}{\pi} + k_s \frac{\delta_{\omega_{\mathbf{r}}}(\omega_{\mathbf{i}})}{\mathbf{n} \cdot \omega_{\mathbf{i}}} + k_t \frac{\delta_{\omega_{\mathbf{t}}}(\omega_{\mathbf{i}})}{\mathbf{n} \cdot \omega_{\mathbf{i}}}$$

- Each geometry primitive/material must have three BSDF coefficients:
 - $\circ |k_d|$: Lambertian diffuse
 - $\circ |k_s|$: Perfect specular reflectance (delta BRDF, law of reflection)
 - \circ $|k_t|$: Perfect refraction (delta BTDF, Snell's law)

The full BSDF



Combining these three properties:

$$f_r(\mathbf{x}, \omega_{\mathbf{i}}, \omega_{\mathbf{o}}) = k_d \frac{1}{\pi} + k_s \frac{\delta_{\omega_{\mathbf{r}}}(\omega_{\mathbf{i}})}{\mathbf{n} \cdot \omega_{\mathbf{i}}} + k_t \frac{\delta_{\omega_{\mathbf{t}}}(\omega_{\mathbf{i}})}{\mathbf{n} \cdot \omega_{\mathbf{i}}}$$

- For physical correctness:
 - $k_d + k_s + k_t \le 1$ on all channels (RGB)
- Recommended, not mandatory:
 - Emissive objects do not reflect/refract, and vice versa
 - \blacksquare Area lights with emission coefficient ($k_e > 0$)

Material coefficients



Each primitive/material must have four coefficients in total:

- $\left|k_d\right|$ lambertian diffuse

- k_e emission coefficient \longrightarrow $\iota_{e(\mathbf{x},\omega_{\mathbf{o}})}$

$$L_o(\mathbf{x}, \omega_o) = L_e(\mathbf{x}, \omega_o) + \int_{\Omega} L_i(\mathbf{x}, \omega_i) f_r(\mathbf{x}, \omega_i, \omega_o) |\mathbf{n} \cdot \omega_i| d\omega_i$$

Material coefficients



Each primitive/material must have four coefficients in total:

- $\left|k_d
 ight|$ lambertian diffuse

- k_e emission coefficient \longrightarrow $L_e(\mathbf{x}, \omega_{\mathbf{o}})$

$$\boxed{k_s \text{ perfect specular}}$$
 perfect refraction
$$\boxed{k_t \text{ perfect refraction}}$$

$$L_o(\mathbf{x}, \omega_o) = L_e(\mathbf{x}, \omega_o) + \int_{\Omega} L_i(\mathbf{x}, \omega_i) f_r(\mathbf{x}, \omega_i, \omega_o) |\mathbf{n} \cdot \omega_i| d\omega_i$$

Combining coefficients



You can combine coefficients to get different materials

Diffuse



$$k_d > 0$$

Combining coefficients



You can combine coefficients to get different materials

Diffuse



 $k_d > 0$

Specular

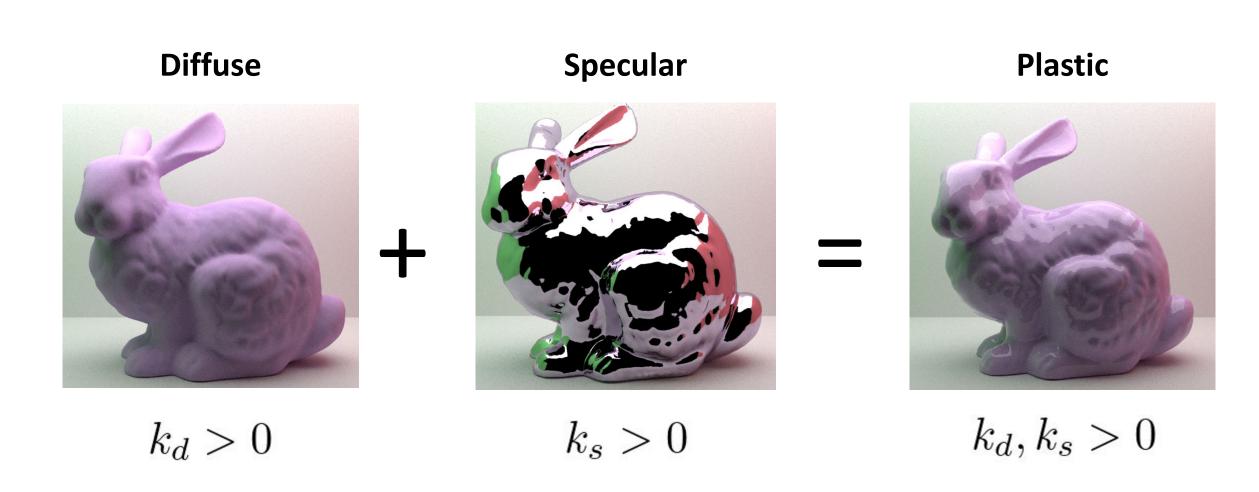


$$k_s > 0$$

Combining coefficients



You can combine coefficients to get different materials



Combining coefficients



• How do we sample ω_i when there is more than one strategy?

Diffuse Specular Plastic



Uniform cosine sampling

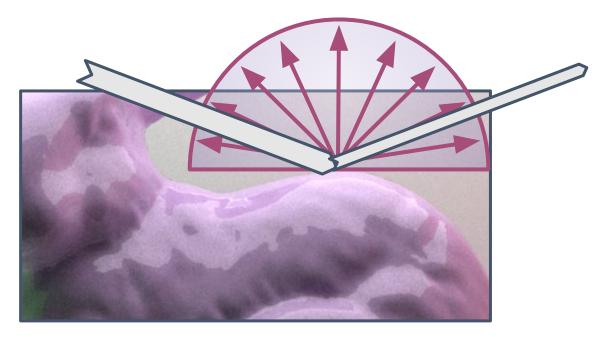


Deterministic (delta BRDF)

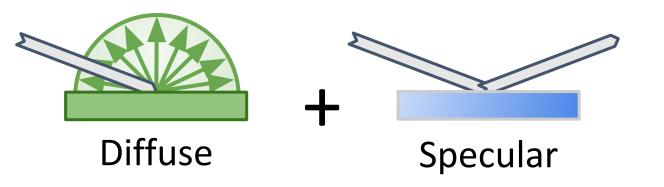


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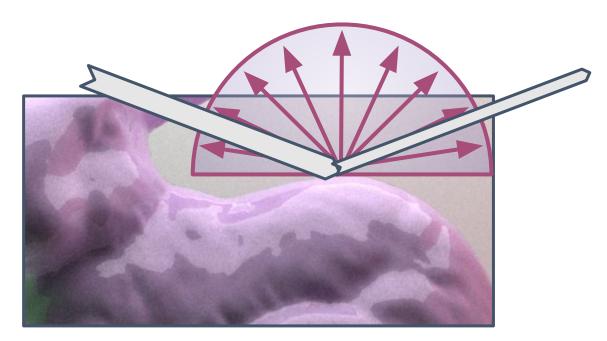




Plastic

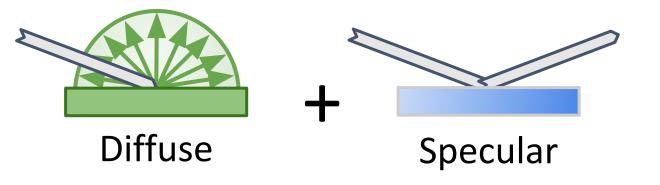




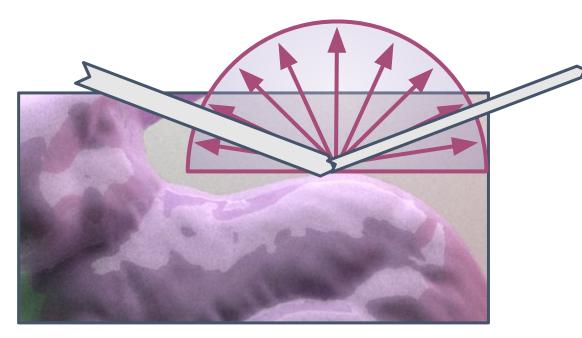


Plastic

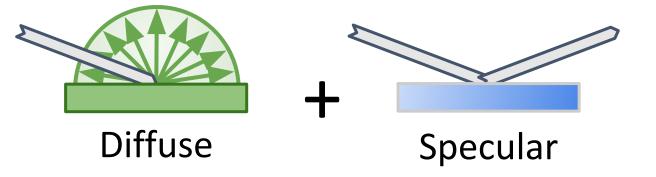
- Only sample diffuse
 - Ignores specular part
- Only sample specular
 - Ignores diffuse part





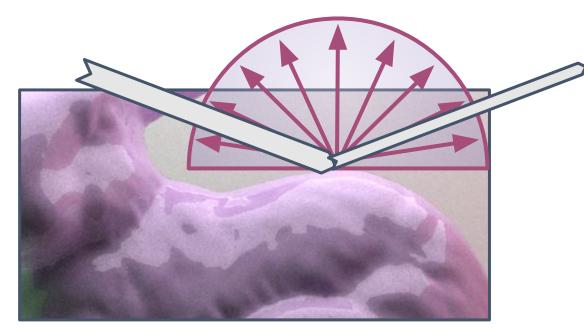


Plastic

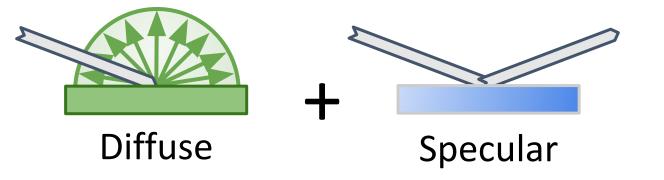


- Only sample diffuse
 - Ignores specular part
- Only sample specular
 - Ignores diffuse part
- Sample both at the same time with two rays
 - Grows exponentially





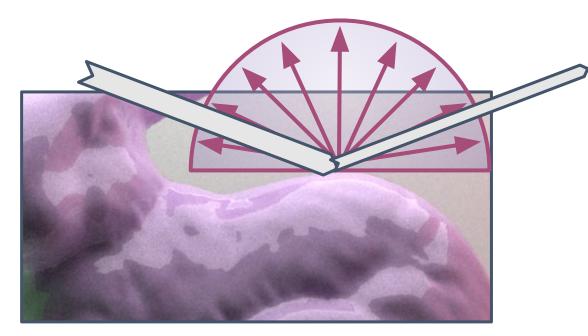
Plastic



- Only sample diffuse
 - Ignores specular part
- Only sample specular
 - Ignores diffuse part
- Sample both at the same time with two rays
 - Grows exponentially
- Russian Roulette: sample one random event (diffuse / specular)

Russian Roulette: explanation





Plastic

 Russian Roulette: sample one random event (diffuse / specular)

Random number $\xi_{RR} \subseteq [0, 1]$:





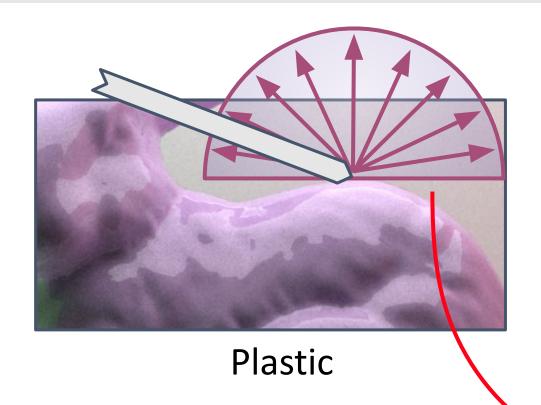
Diffuse



Specular

Russian Roulette: explanation

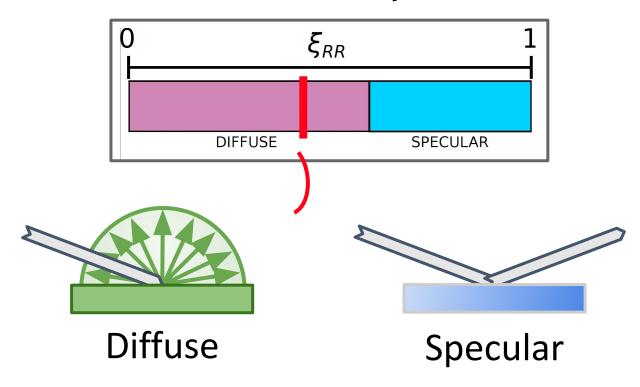




Sample ω_i based on your strategy for **diffuse** BRDFs

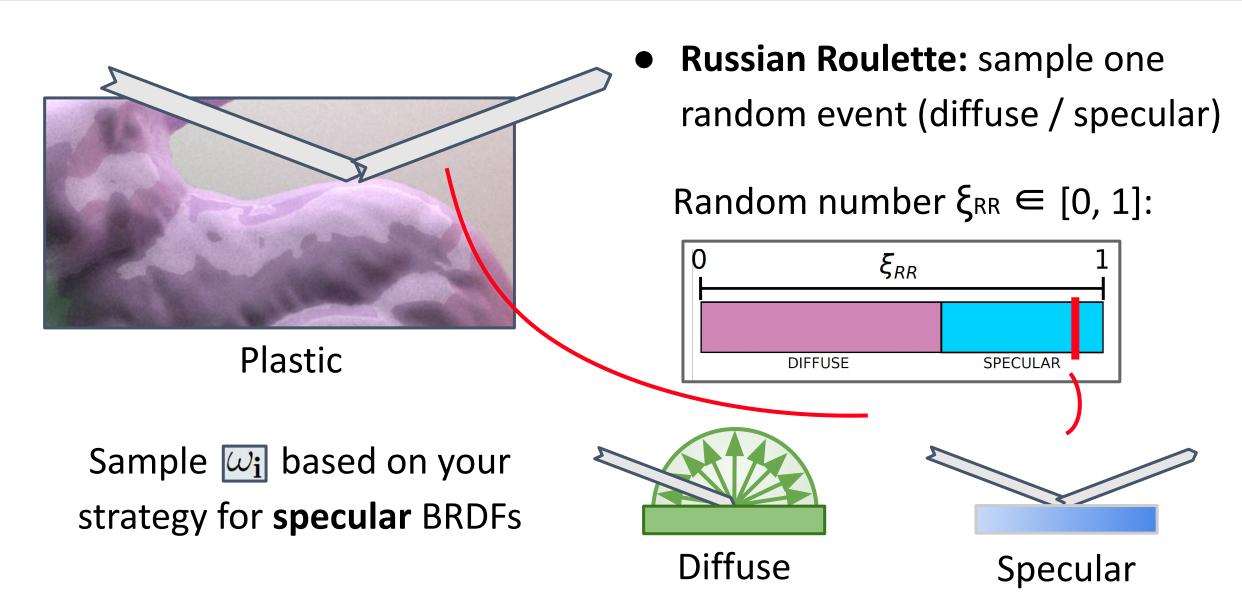
 Russian Roulette: sample one random event (diffuse / specular)

Random number $\xi_{RR} \subseteq [0, 1]$:

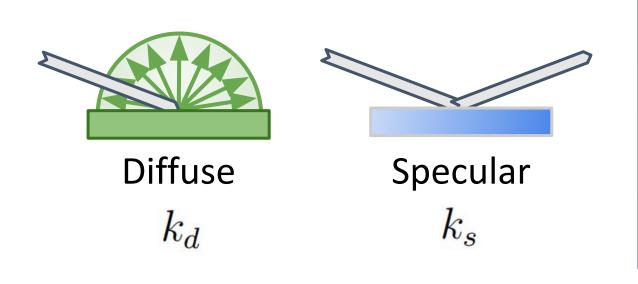


Russian Roulette: explanation

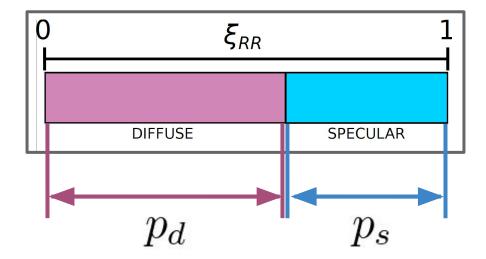




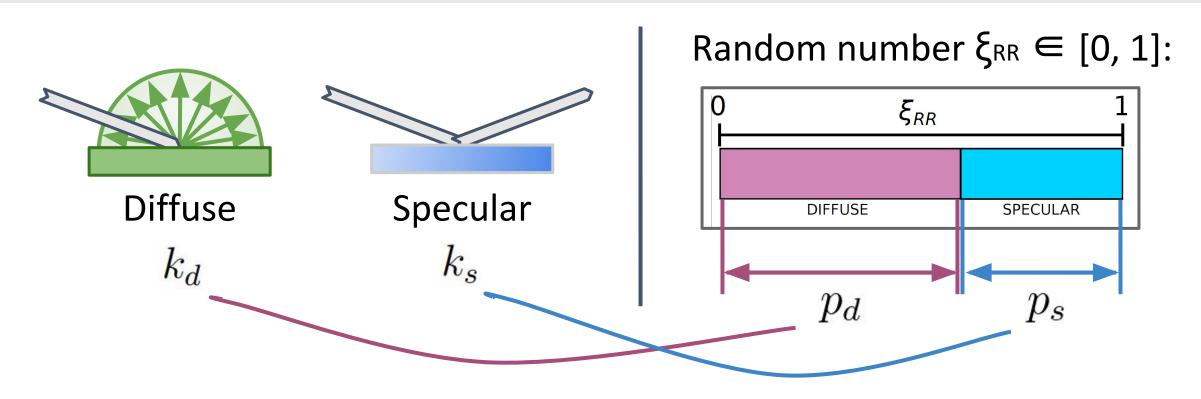




Random number $\xi_{RR} \subseteq [0, 1]$:

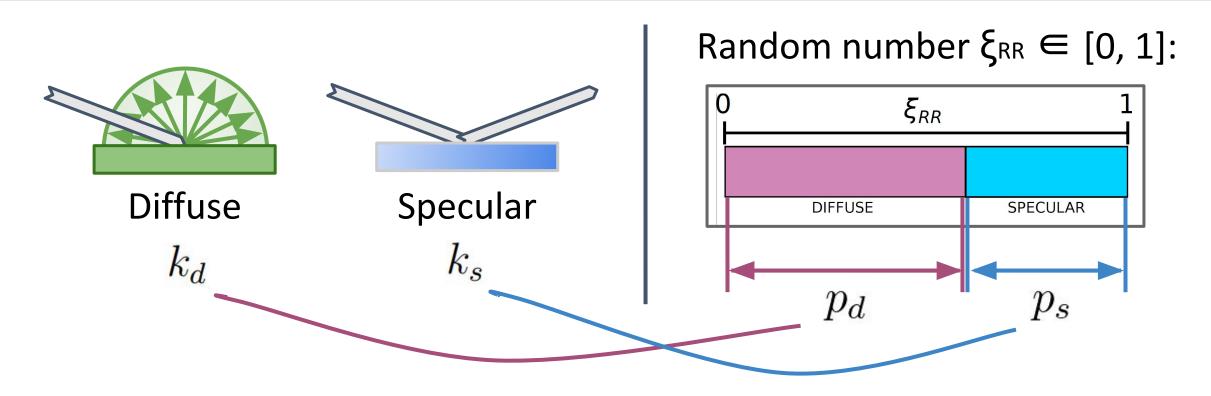






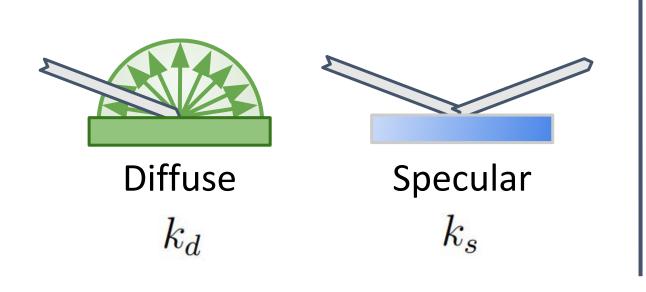
ullet How to obtain probabilities p_d , p_s based on RGB coefficients k_d , k_s ?



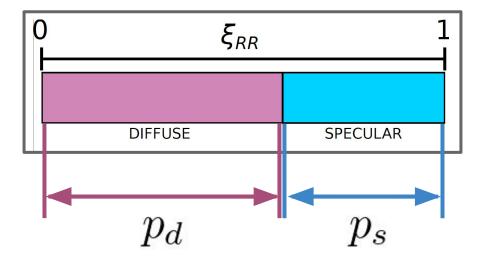


- ullet How to obtain probabilities p_d , p_s based on RGB coefficients k_d , k_s ?
 - \circ Example: $p_i = \max k_i$ (maximum of RGB channels)
 - \circ Remember: $k_d + k_s + k_t \le 1$ on all channels (RGB)





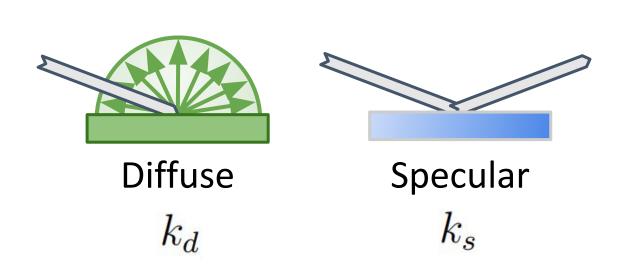
Random number $\xi_{RR} \subseteq [0, 1]$:



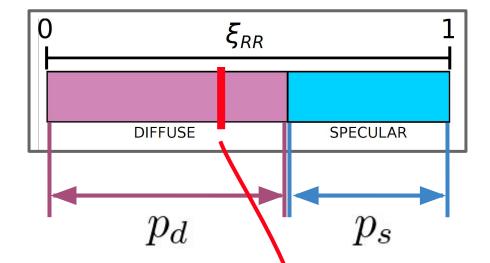
After evaluating each event, divide by its probability:

$$f_r(\mathbf{x}, \omega_{\mathbf{i}}, \omega_{\mathbf{o}}) = k_d \frac{1}{\pi} + k_s \frac{\delta_{\omega_{\mathbf{r}}}(\omega_{\mathbf{i}})}{\mathbf{n} \cdot \omega_{\mathbf{i}}} + k_t \frac{\delta_{\omega_{\mathbf{t}}}(\omega_{\mathbf{i}})}{\mathbf{n} \cdot \omega_{\mathbf{i}}}$$





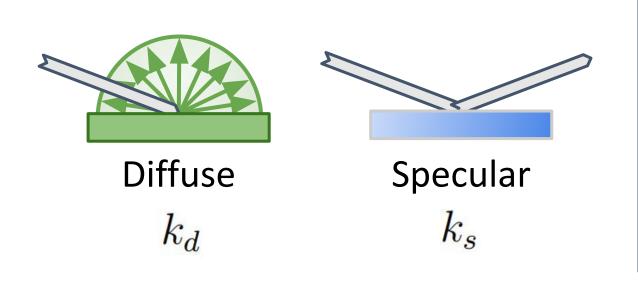
Random number $\xi_{RR} \in [0, 1]$:



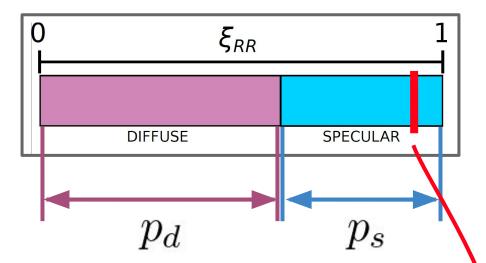
After evaluating each event, divide by its probability:

$$f_r(\mathbf{x}, \omega_{\mathbf{i}}, \omega_{\mathbf{o}}) = k_d \frac{1}{\pi} \cdot \frac{1}{p_d}$$





Random number $\xi_{RR} \subseteq [0, 1]$:

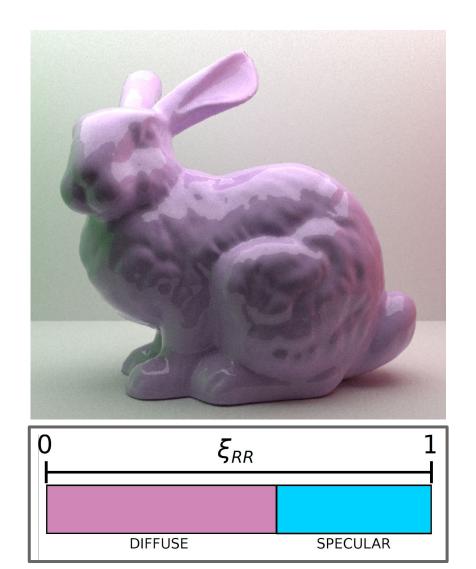


After evaluating each event, divide by its probability:

$$f_r(\mathbf{x}, \omega_{\mathbf{i}}, \omega_{\mathbf{o}}) = k_s \frac{\delta_{\omega_{\mathbf{r}}}(\omega_{\mathbf{i}})}{\mathbf{n} \cdot \omega_{\mathbf{i}}} \cdot \frac{1}{p_s}$$

Material combinations



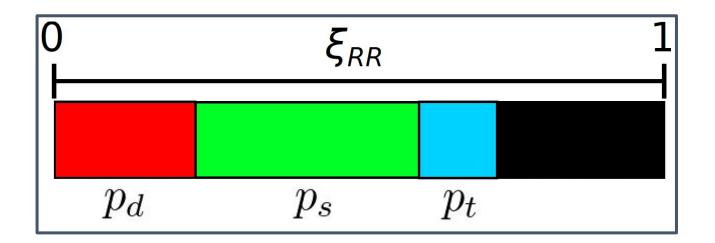








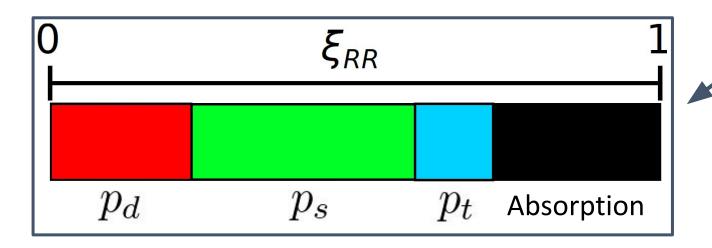
- Until now we have defined three events for Russian Roulette
 - Diffuse, perfect specular, perfect refractive



$$f_r(\mathbf{x}, \omega_{\mathbf{i}}, \omega_{\mathbf{o}}) = k_d \frac{1}{\pi} + k_s \frac{\delta_{\omega_{\mathbf{r}}}(\omega_{\mathbf{i}})}{\mathbf{n} \cdot \omega_{\mathbf{i}}} + k_t \frac{\delta_{\omega_{\mathbf{t}}}(\omega_{\mathbf{i}})}{\mathbf{n} \cdot \omega_{\mathbf{i}}}$$



- Until now we have defined three events for Russian Roulette
 - Diffuse, perfect specular, perfect refractive
- New: "absorption" event as a path termination condition

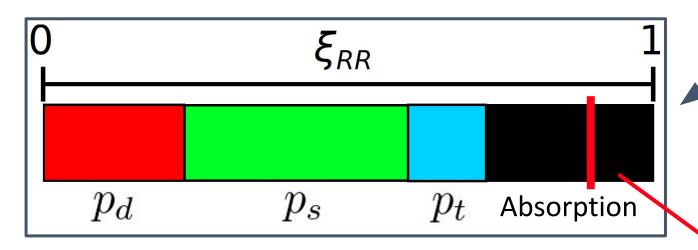


$$\int 1 - (p_d + p_s + p_t)$$

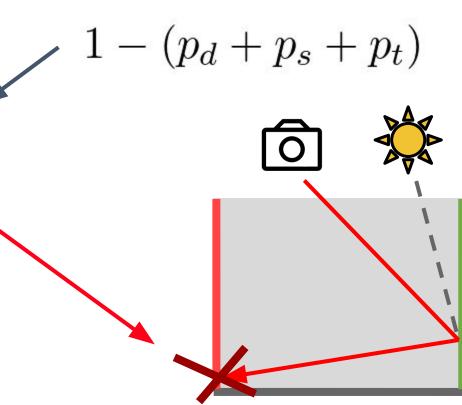
$$f_r(\mathbf{x}, \omega_{\mathbf{i}}, \omega_{\mathbf{o}}) = k_d \frac{1}{\pi} + k_s \frac{\delta_{\omega_{\mathbf{r}}}(\omega_{\mathbf{i}})}{\mathbf{n} \cdot \omega_{\mathbf{i}}} + k_t \frac{\delta_{\omega_{\mathbf{t}}}(\omega_{\mathbf{i}})}{\mathbf{n} \cdot \omega_{\mathbf{i}}}$$



- Until now we have defined three events for Russian Roulette
 - Diffuse, perfect specular, perfect refractive
- New: "absorption" event as a path termination condition



$$f_r(\mathbf{x}, \omega_{\mathbf{i}}, \omega_{\mathbf{o}}) = k_d \frac{1}{\pi} + k_s \frac{\delta_{\omega_{\mathbf{r}}}(\omega_{\mathbf{i}})}{\mathbf{n} \cdot \omega_{\mathbf{i}}} + k_t \frac{\delta_{\omega_{\mathbf{t}}}(\omega_{\mathbf{i}})}{\mathbf{n} \cdot \omega_{\mathbf{i}}}$$

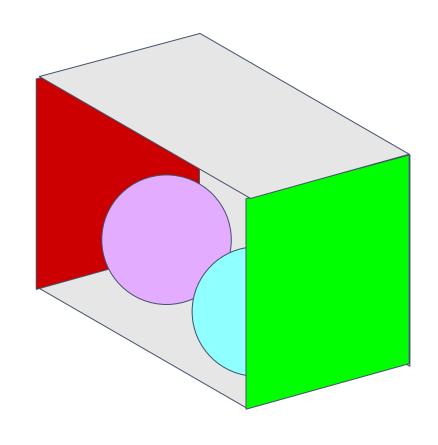




- Until now we have defined three events for Russian Roulette
 - Diffuse, perfect specular, perfect refractive
- New: "absorption" event as a path termination condition
 - Remember previous termination path conditions:
 - Ray does not intersect
 - Ray intersects with an area light
 - Number of bounces > N (not required with Russian Roulette)



Geometry



Planes defined by normal (n) and distance (d)

Left plane n = (1, 0, 0), d = 1

Right plane n = (-1, 0, 0), d = 1

Floor plane n = (0, 1, 0), d = 1

Ceiling plane n = (0, -1, 0), d = 1

Back plane n = (0, 0, -1), d = 1

Spheres defined by center (c) and radius (r)

Left sphere c = (-0.5, -0.7, 0.25), r = 0.3

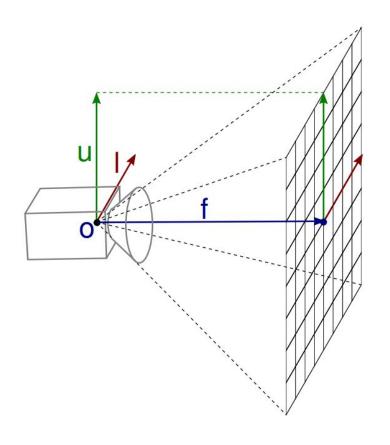
Mix of blue diffuse + specular

Right sphere c = (0.5, -0.7, -0.25), r = 0.3

• Mix of specular and refraction, $\eta = 1.5$



Camera & light sources



Camera and image plane defined by

Origin O = (0, 0, -3.5)

Left L = (-1, 0, 0)

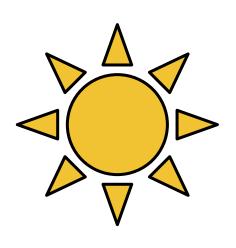
Up U = (0, 1, 0)

Forward F = (0, 0, 3)

Size 256x256 pixels



Light sources



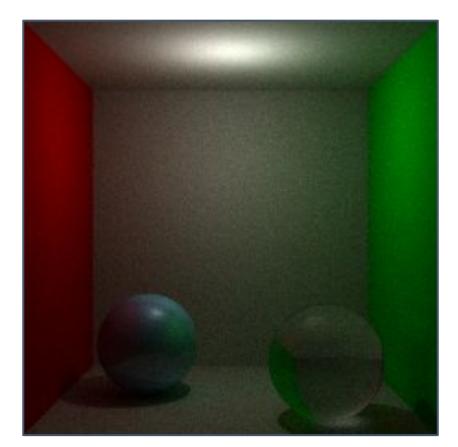
Center and power (emission)

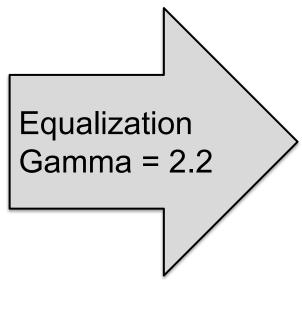
```
Center c = (0, 0.5, 0)
Power can be any number e.g. p = (1, 1, 1)
Just be careful with the #MAX
```

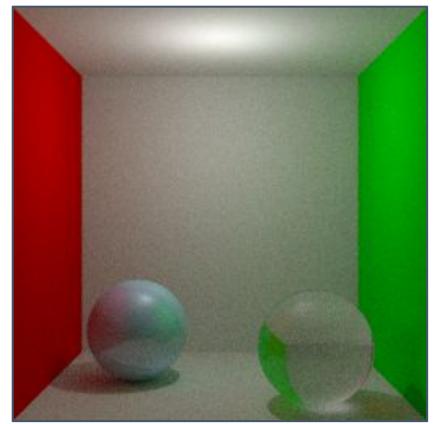
```
1 P3
2 # feep.ppm
3 #MAX=<maximum of your RGB memory values>
4 4 4
5 15
6 0 0 0 0 0 0 0 0 0 15 0 15
```



Results (no area lights + point light)





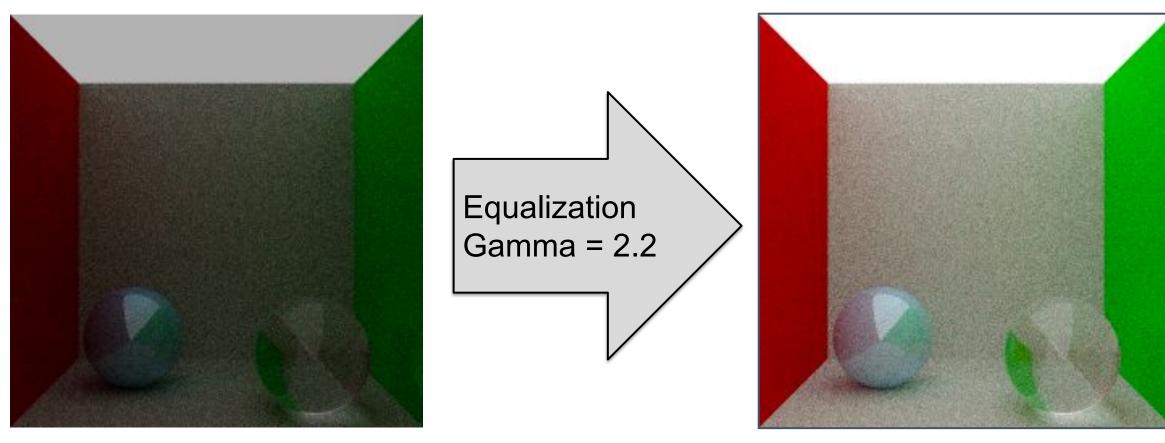


Using a point light

With tone mapping



Results (no point light + ceiling plane is an area light)



Using an area light

With tone mapping

Questions



DO ASK questions, either now or after the lab

But be reasonable, please:)

<u>pluesia@unizar.es</u> | <u>dsubias@unizar.es</u> | <u>o.pueyo@unizar.es</u>

What to expect from this session



In the programming language of your choice implement:

- Perfect specular and refractive BSDFs (add ks and kt for each material)
 - Extend "evaluate" and "sample" functions
- Apply Russian Roulette at each intersection to select an event:
 - (1) Diffuse, (2) Perfect specular, (3) Perfect refractive, (4) Absorption
 - Sample BSDF based on selected event, evaluate and divide by event's probability
- Recommended deadline: November 13th (moodle: January 11th)
 - Extensions (do not count towards recommended deadline):
 - Recommended to finish base path tracer for next week before any optionals
 - Realistic coefficients: Modulate specular/refractive coefficients ks and kt using Fresnel equations

 \mathbf{X}

- **Textures:** make material coefficients on hit position
- Fresnel effects: make material coefficients depend on viewing direction