

Lab #4 – Path tracing (part 2)

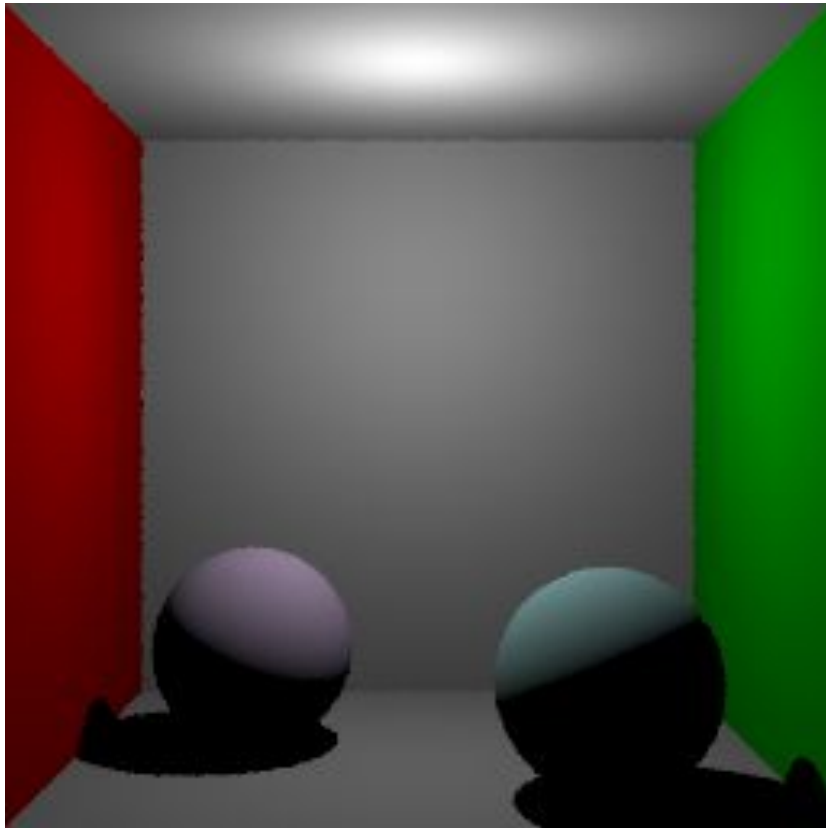
Informática Gráfica

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Pablo Luesia - J. Daniel Subías – Óscar Pueyo

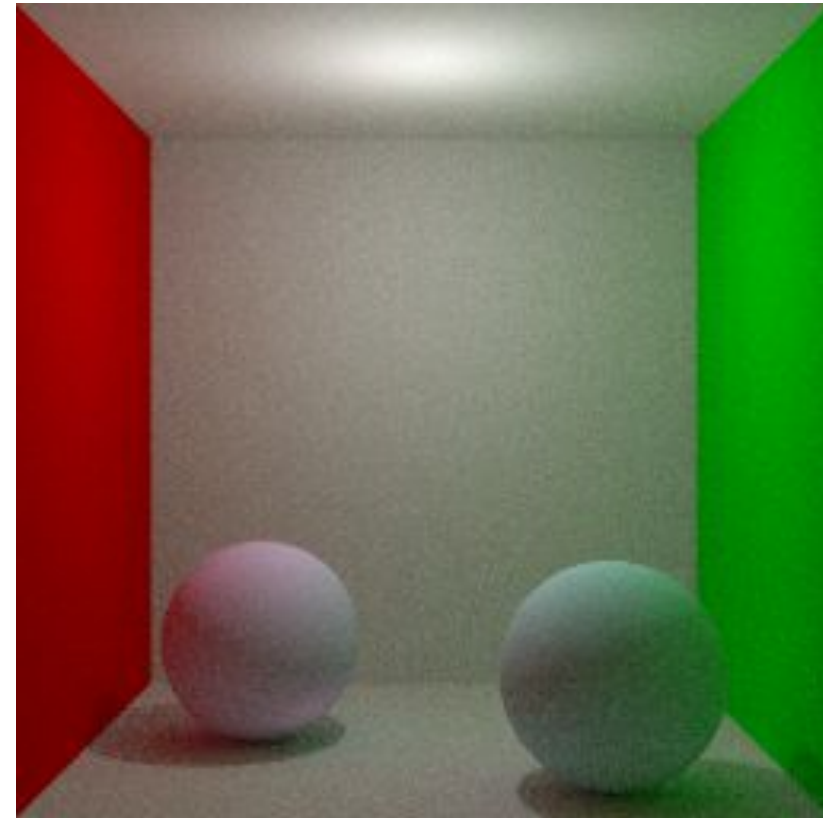


Before we begin...

- Today: calculate direct illumination **and** indirect illumination



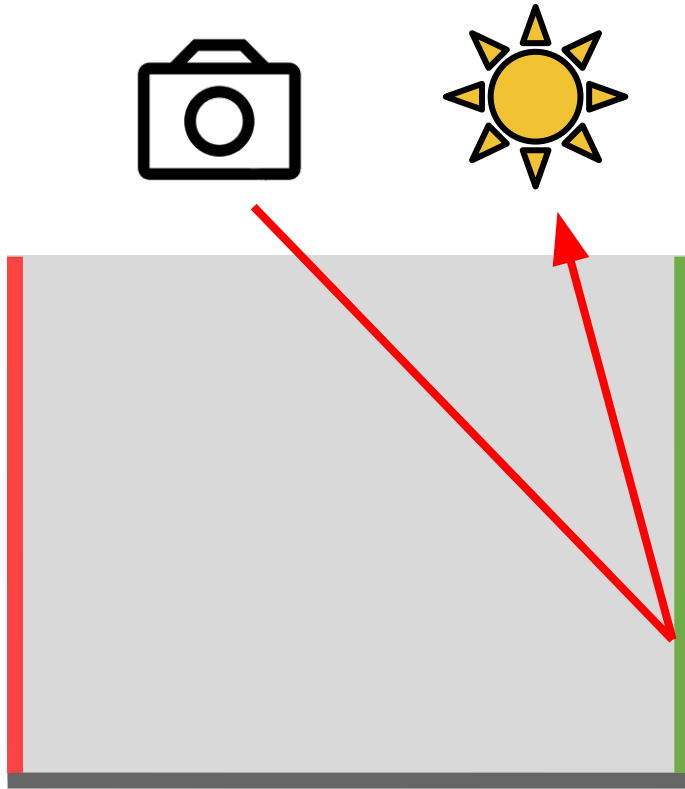
Previous session (direct light)



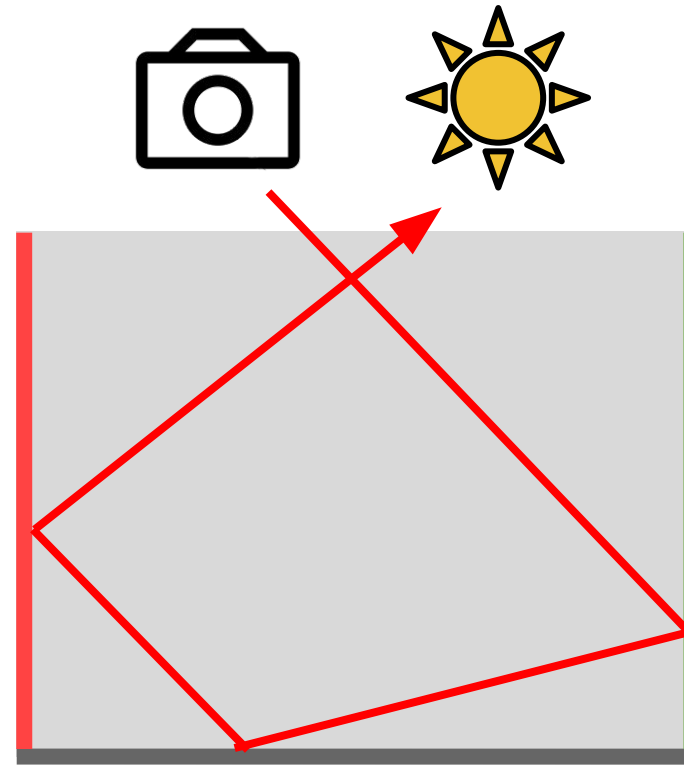
Full path tracing (+ indirect light)

Before we begin...

- Today: calculate direct illumination **and** indirect illumination



Previous session (direct light)

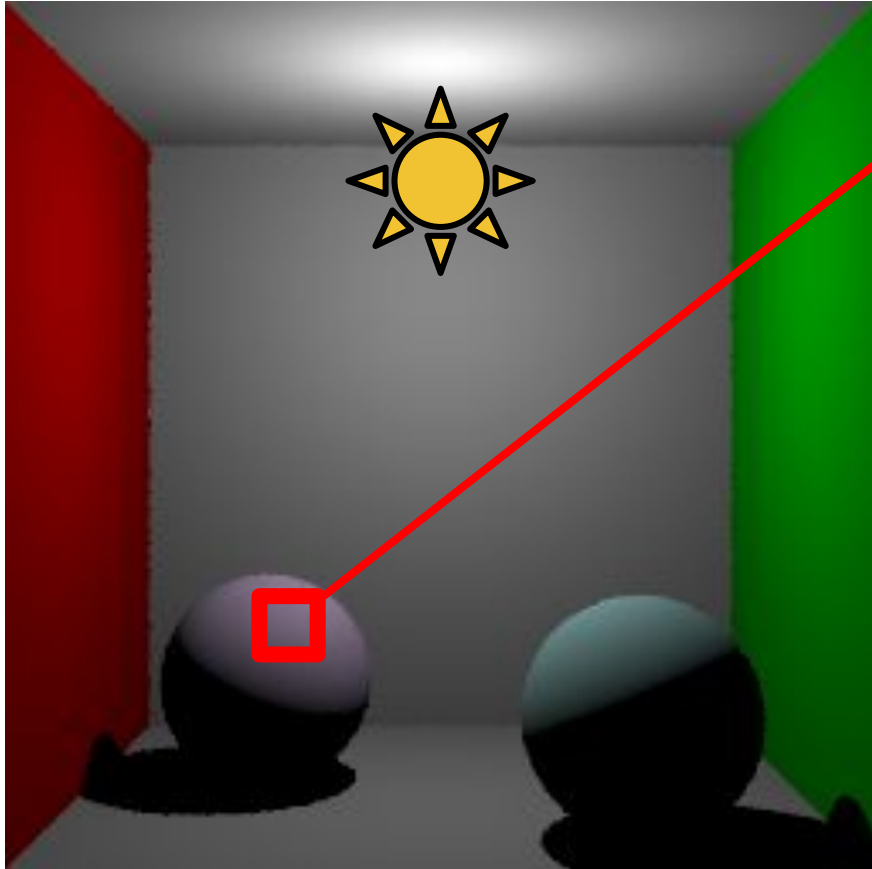


Full path tracing (+ indirect light)

Before we begin...

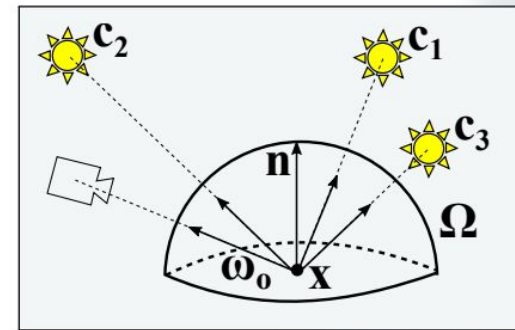
- Warning: there is a lot of theory on these slides, handle with care
 - If you want to go straight to the point/programming, go to the end
- Lab 4 (path tracing) **is the first submitted work**
 - Recommended deadline: November 13th
 - Moodle: January 11th
 - You will use most of today's code for Lab 5 (photon mapping) too
- Remember: Final work is 80% of the final grade

Previously: only direct illumination



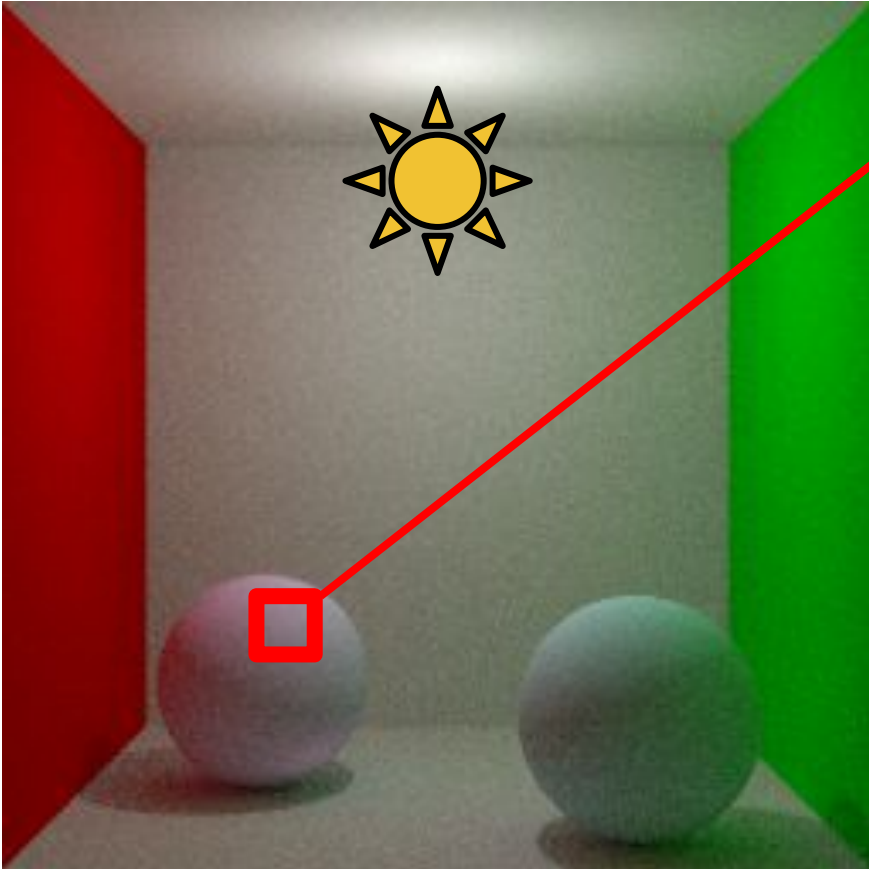
$$L_o(\mathbf{x}, \omega_o) = \cancel{L_e(\mathbf{x}, \omega_o)} + \int_{\Omega} L_i(\mathbf{x}, \omega_i) f_r(\mathbf{x}, \omega_i, \omega_o) |\mathbf{n} \cdot \omega_i| d\omega_i$$

But just the direct light



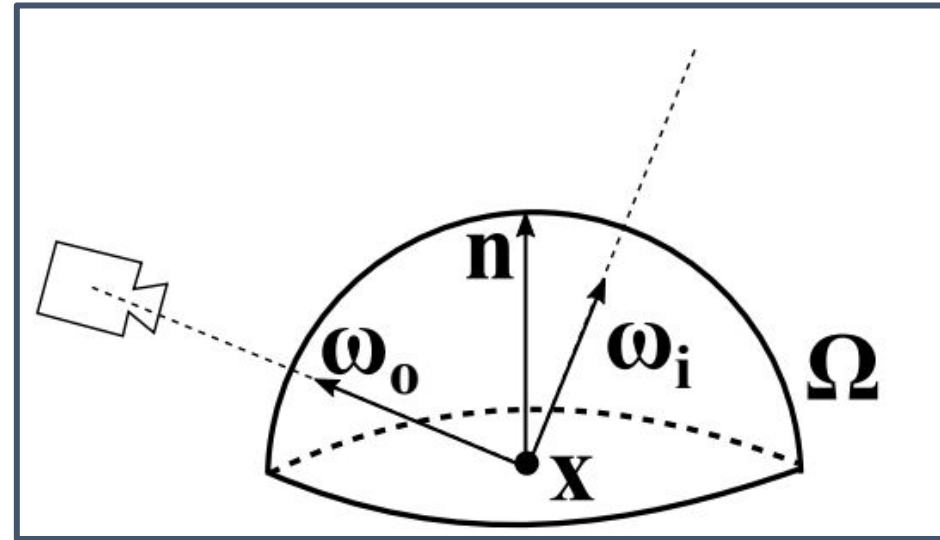
$$L_o(\mathbf{x}, \omega_o) = \sum_{i=1}^n \frac{p_i}{|\mathbf{c}_i - \mathbf{x}|^2} f_r \left(\mathbf{x}, \frac{\mathbf{c}_i - \mathbf{x}}{|\mathbf{c}_i - \mathbf{x}|}, \omega_o \right) \left| \mathbf{n} \cdot \frac{\mathbf{c}_i - \mathbf{x}}{|\mathbf{c}_i - \mathbf{x}|} \right|$$

Which color do we fill each pixel with?



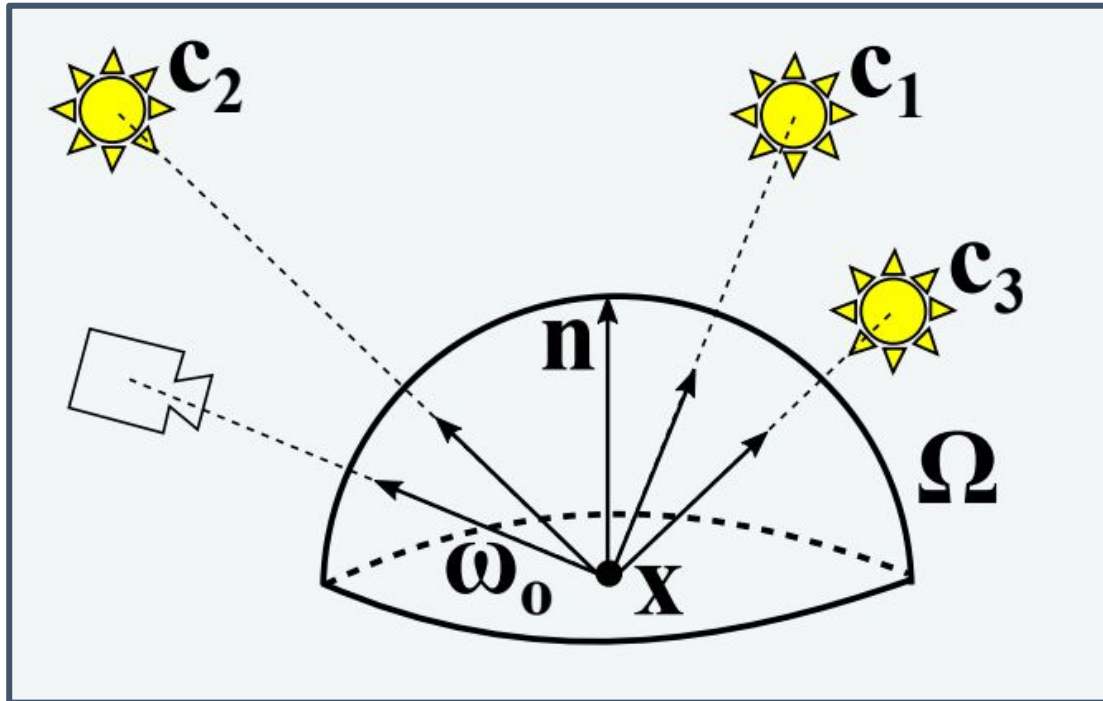
$$L_o(\mathbf{x}, \omega_o) = \cancel{L_e(\mathbf{x}, \omega_o)} + \int_{\Omega} L_i(\mathbf{x}, \omega_i) f_r(\mathbf{x}, \omega_i, \omega_o) |\mathbf{n} \cdot \omega_i| d\omega_i$$

The full integral



Computing the path integral

- How do we compute $\int_{\Omega} L_i(\mathbf{x}, \omega_i) f_r(\mathbf{x}, \omega_i, \omega_o) |\mathbf{n} \cdot \omega_i| d\omega_i$?
- On the previous session (direct light):



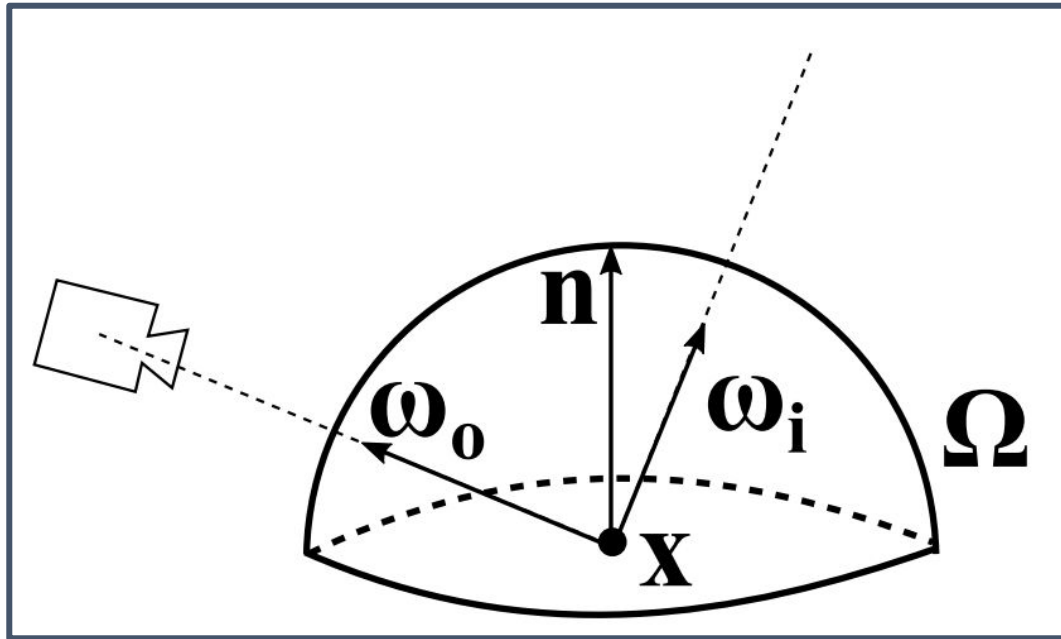
$$= \sum_{i=1}^n \frac{p_i}{|\mathbf{c}_i - \mathbf{x}|^2} f_r \left(\mathbf{x}, \frac{\mathbf{c}_i - \mathbf{x}}{|\mathbf{c}_i - \mathbf{x}|}, \omega_o \right) \left| \mathbf{n} \cdot \frac{\mathbf{c}_i - \mathbf{x}}{|\mathbf{c}_i - \mathbf{x}|} \right|$$

- Paths have exactly one bounce
- Closed-form solution

Computing the path integral

- How do we compute $\int_{\Omega} L_i(\mathbf{x}, \omega_i) f_r(\mathbf{x}, \omega_i, \omega_o) |\mathbf{n} \cdot \omega_i| d\omega_i$?

- Today (direct light + indirect light):



$$= \int_{\Omega_1} L_i(\mathbf{x}_1, \omega_{i1}) f_r(\mathbf{x}_1, \omega_{i1}, \omega_{o1}) |\mathbf{n}_1 \cdot \omega_{i1}| d\omega_{i1}$$

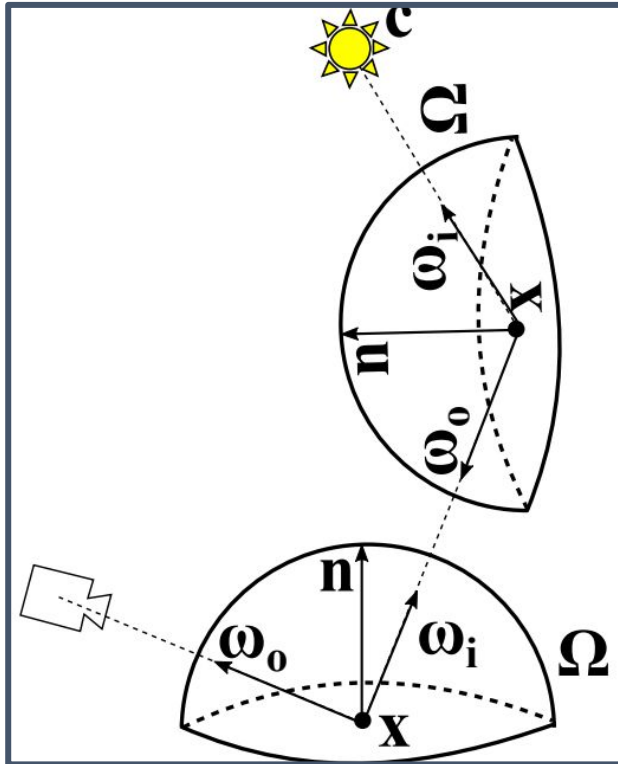
- Integrate over the whole hemisphere

Computing the path integral

- How do we compute

$$\int_{\Omega} L_i(\mathbf{x}, \omega_i) f_r(\mathbf{x}, \omega_i, \omega_o) |\mathbf{n} \cdot \omega_i| d\omega_i \quad ?$$

- Today (direct light + indirect light):



$$= \int_{\Omega_1} \left(\int_{\Omega_2} L_i(\mathbf{x}_2, \omega_{i2}) f_r(\mathbf{x}_2, \omega_{i2}, -\omega_{i1}) |\mathbf{n}_2 \cdot \omega_{i2}| d\omega_{i1} \right) f_r(\mathbf{x}_1, -\omega_{i1}, \omega_{o1}) |\mathbf{n}_1 \cdot \omega_{i1}| d\omega_{i1}$$

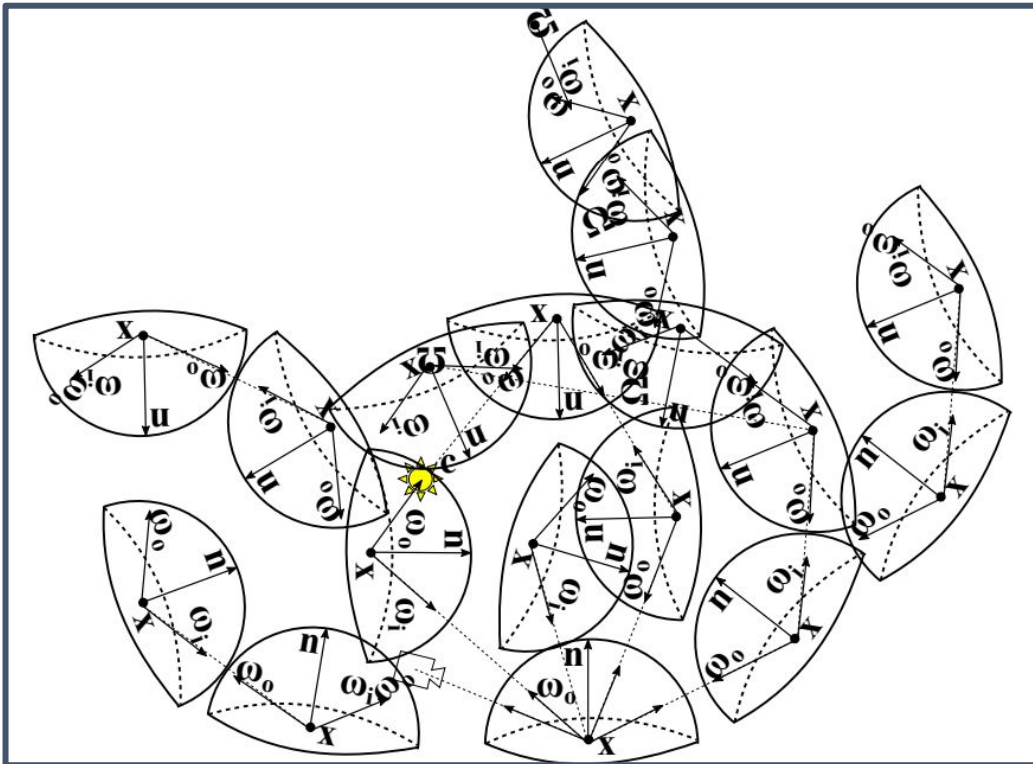
- Integrate over the whole hemisphere²

Computing the path integral

- How do we compute

$$\int_{\Omega} L_i(\mathbf{x}, \omega_i) f_r(\mathbf{x}, \omega_i, \omega_o) |\mathbf{n} \cdot \omega_i| d\omega_i \quad ?$$

- Today (direct light + indirect light):



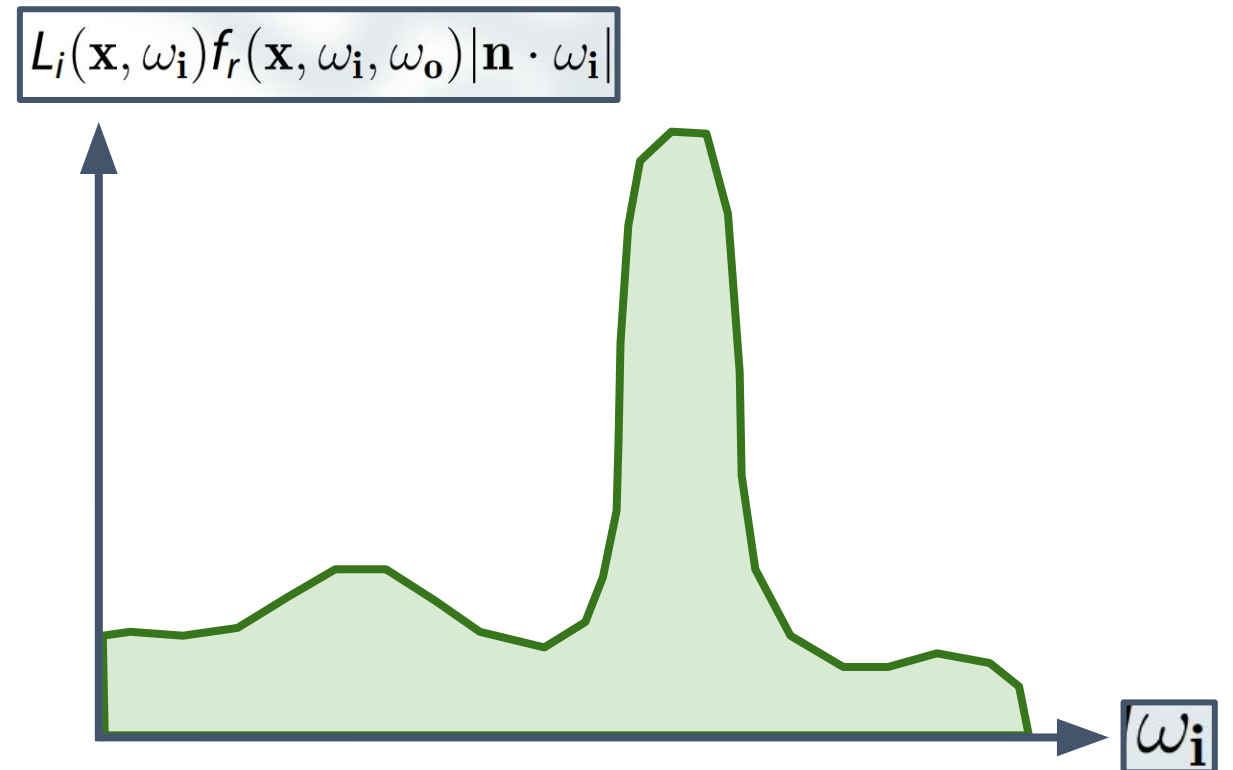
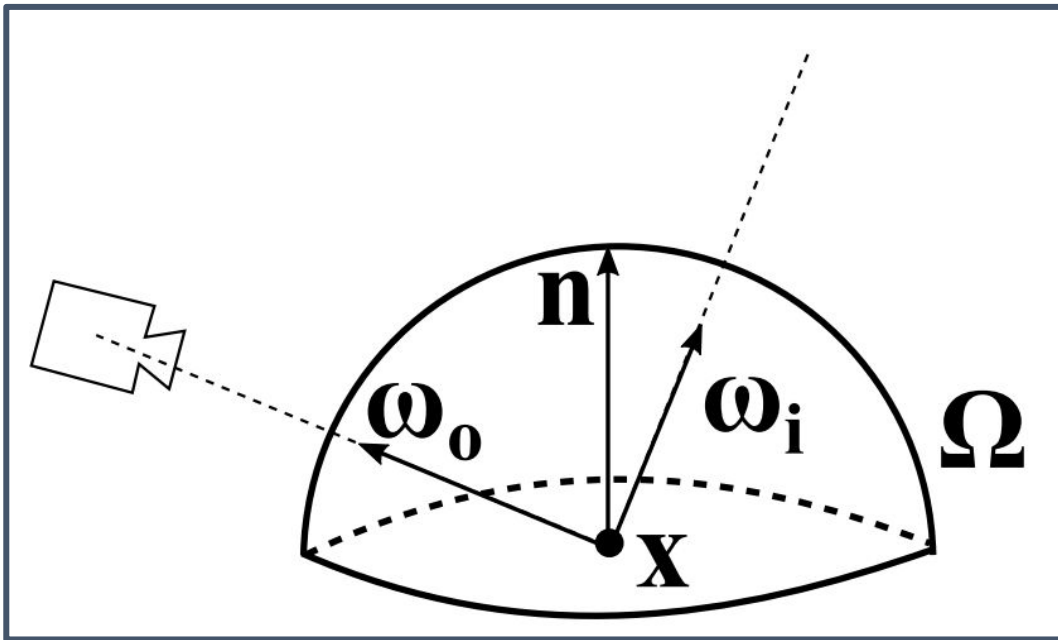
$$= \int_{\Omega_1} \cdots \int_{\Omega_N} L_i(\mathbf{x}_N, \omega_{N2}) \left(\prod_{k=2}^N f_r(\mathbf{x}_k, \omega_{ik}, -\omega_{i(k-1)}) \right) f_r(\mathbf{x}_1, \omega_{i1}, \omega_{o1})$$
$$\left(\prod_{k=1}^N |\mathbf{n}_k \cdot \omega_{ik}| \right) d\omega_{iN} \cdots d\omega_{i1}$$

- Paths can have 1..N bounces
- How do we solve this integral?

Approximating one integral

- How do we compute

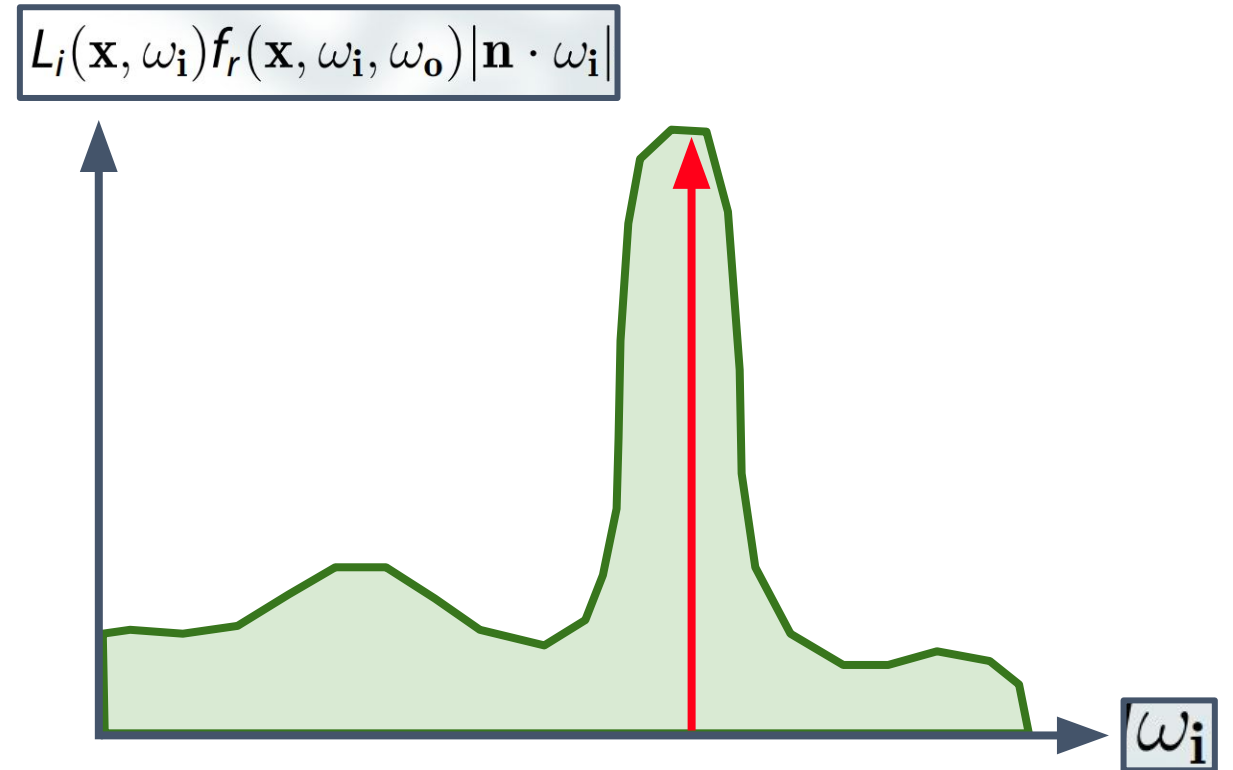
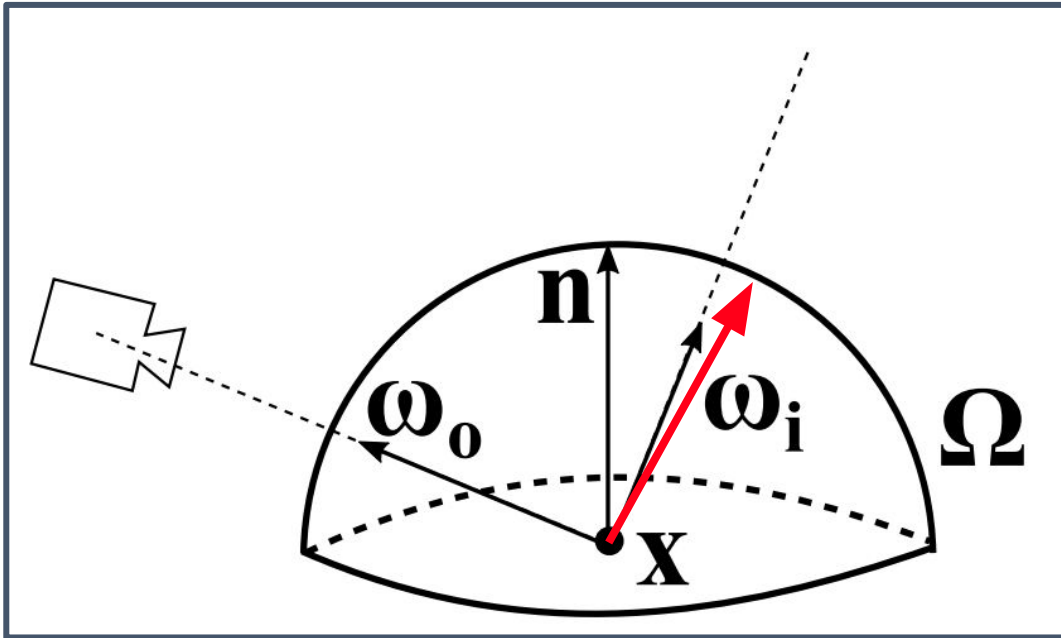
$$\int_{\Omega} L_i(\mathbf{x}, \omega_i) f_r(\mathbf{x}, \omega_i, \omega_o) |\mathbf{n} \cdot \omega_i| d\omega_i \quad ?$$



Approximating one integral

- How do we compute

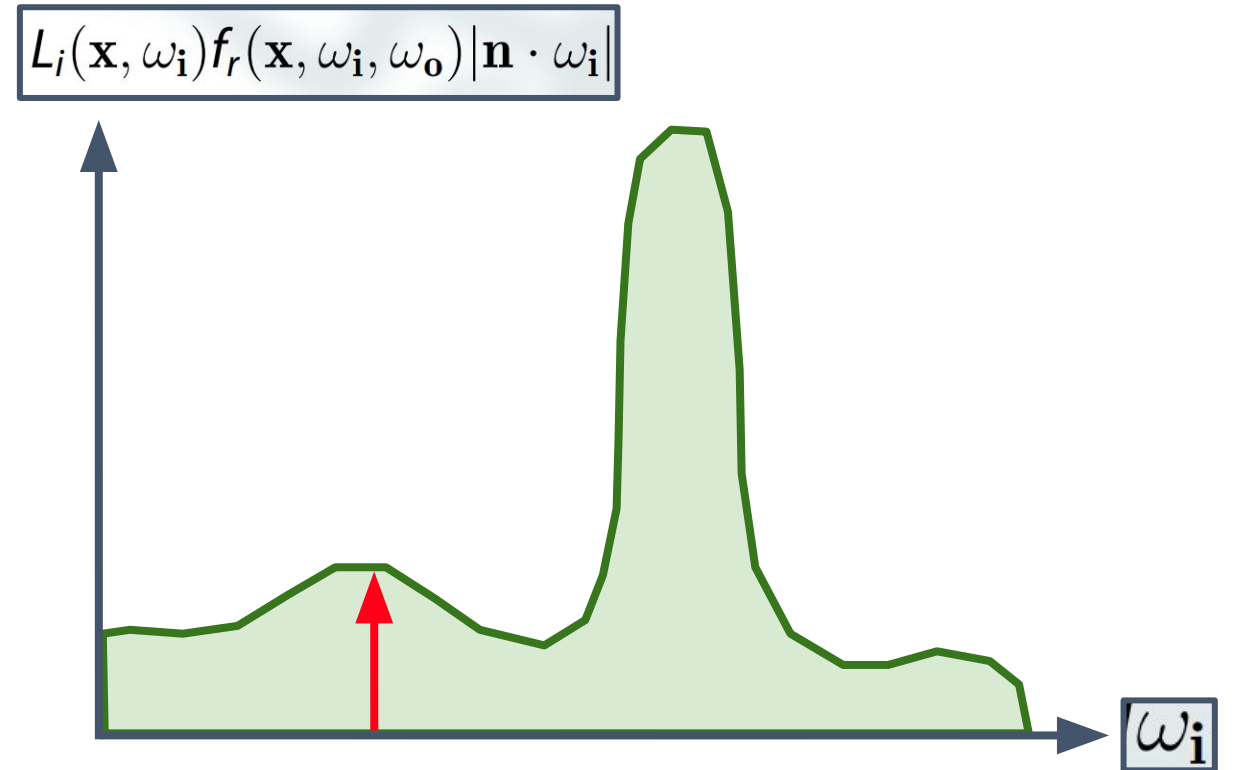
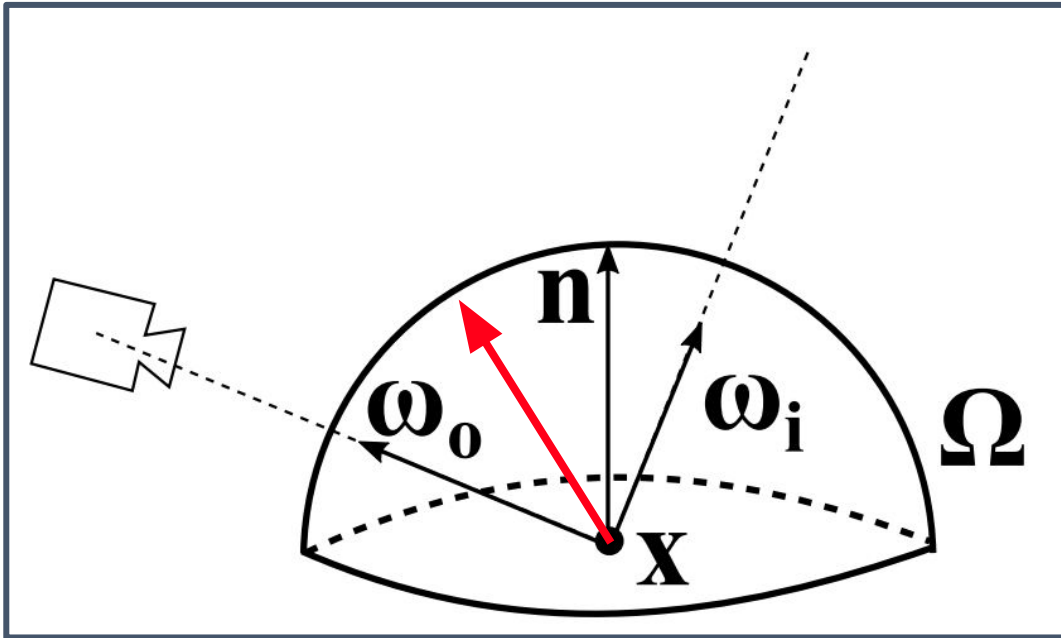
$$\int_{\Omega} L_i(\mathbf{x}, \omega_i) f_r(\mathbf{x}, \omega_i, \omega_o) |\mathbf{n} \cdot \omega_i| d\omega_i \quad ?$$



Approximating one integral

- How do we compute

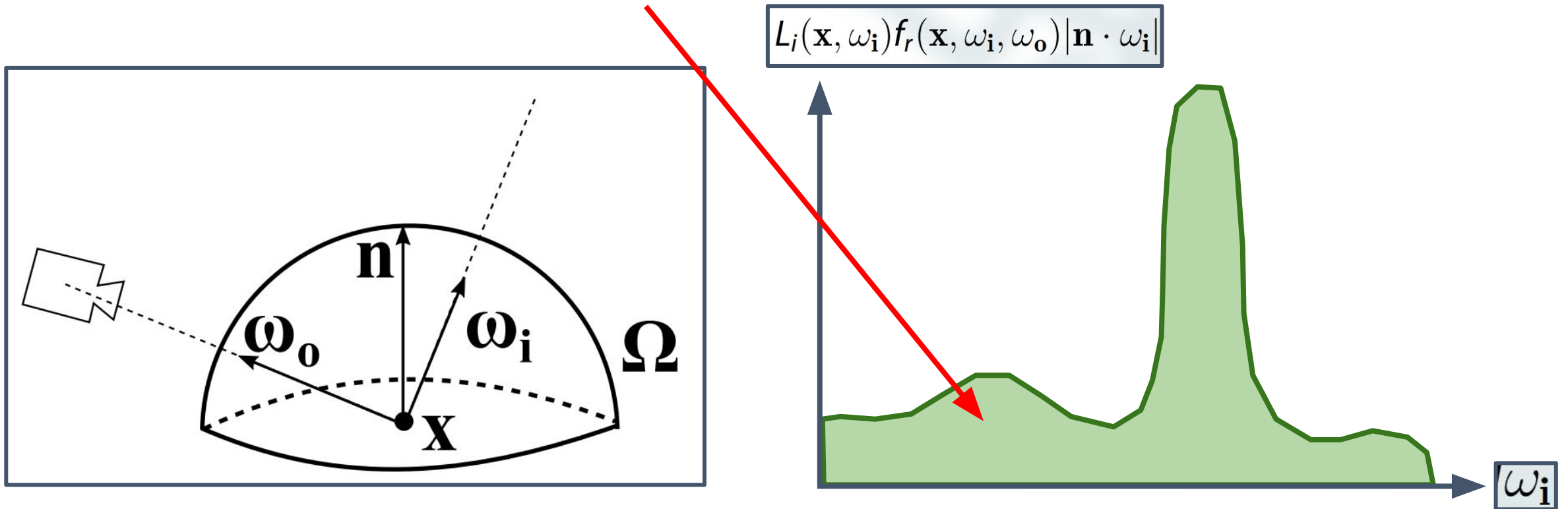
$$\int_{\Omega} L_i(\mathbf{x}, \omega_i) f_r(\mathbf{x}, \omega_i, \omega_o) |\mathbf{n} \cdot \omega_i| d\omega_i \quad ?$$



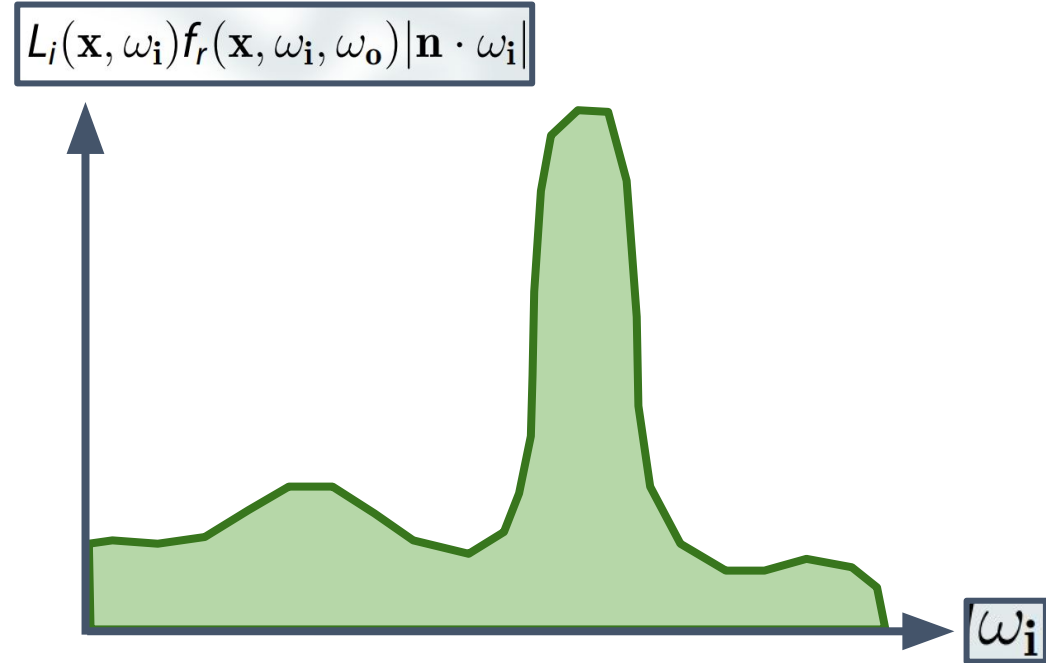
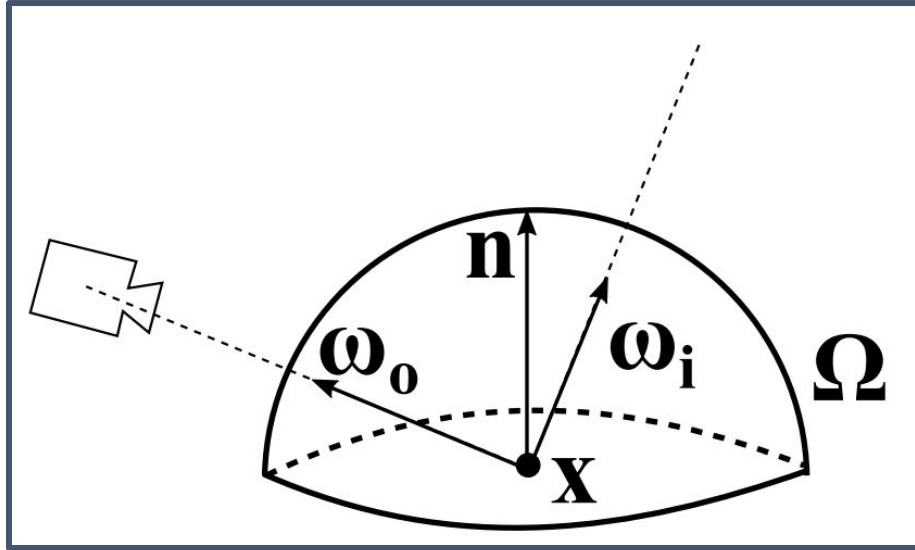
Approximating one integral

- How do we compute $\int_{\Omega} L_i(\mathbf{x}, \omega_i) f_r(\mathbf{x}, \omega_i, \omega_o) |\mathbf{n} \cdot \omega_i| d\omega_i$?

- Compute this green area under the curve

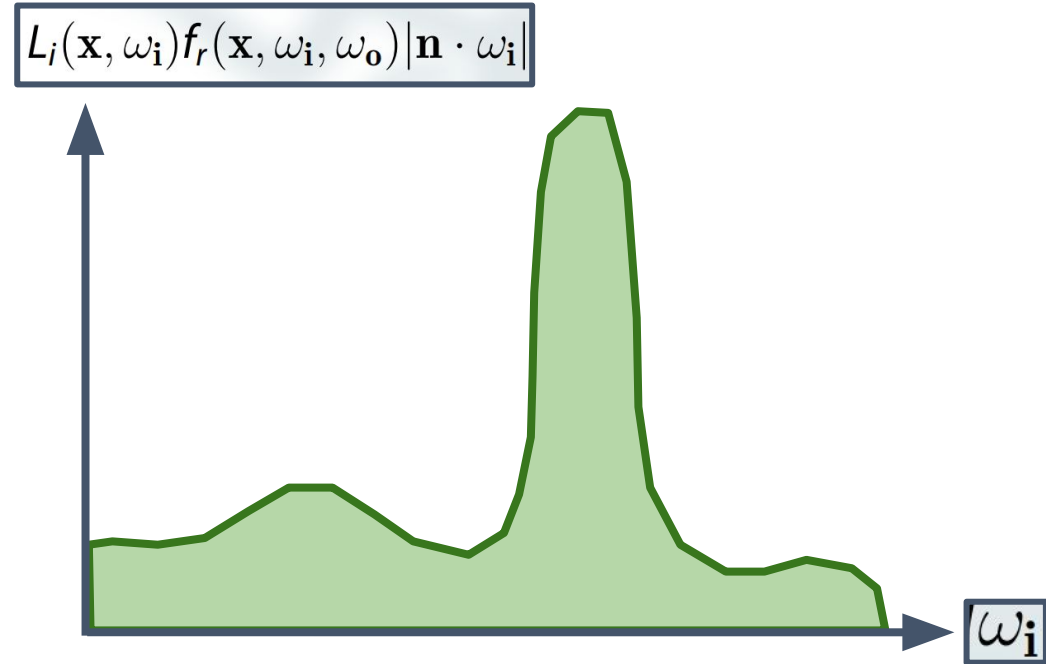
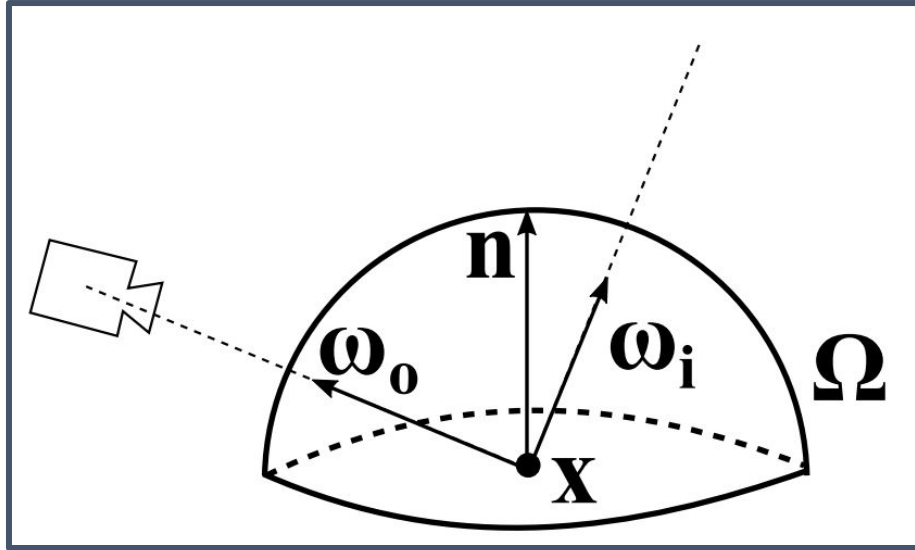


Approximating one integral



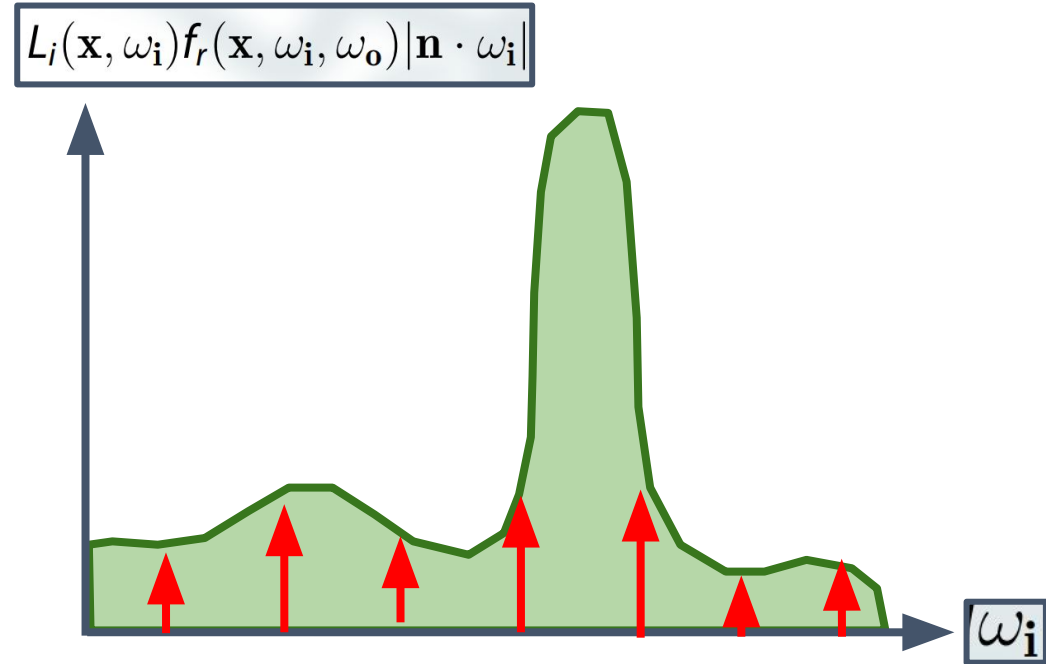
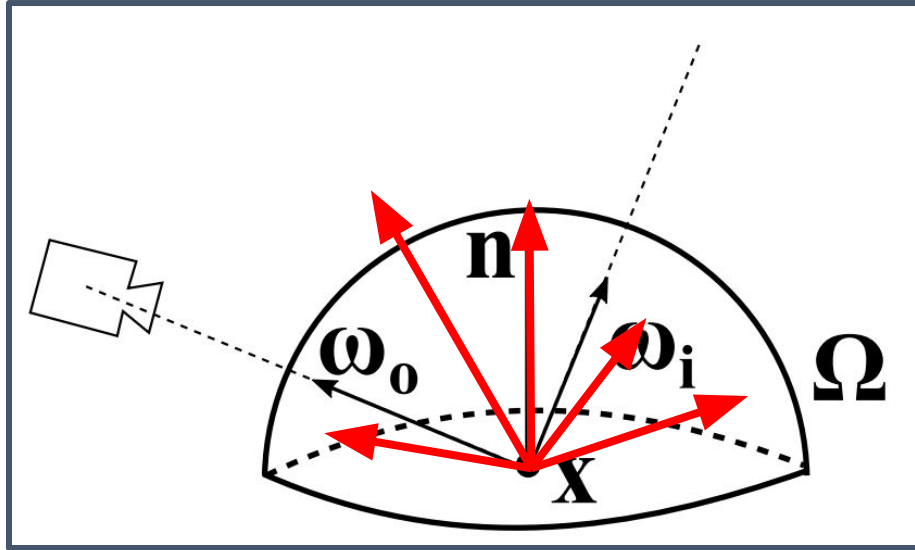
- There are infinite values for ω_i

Approximating one integral



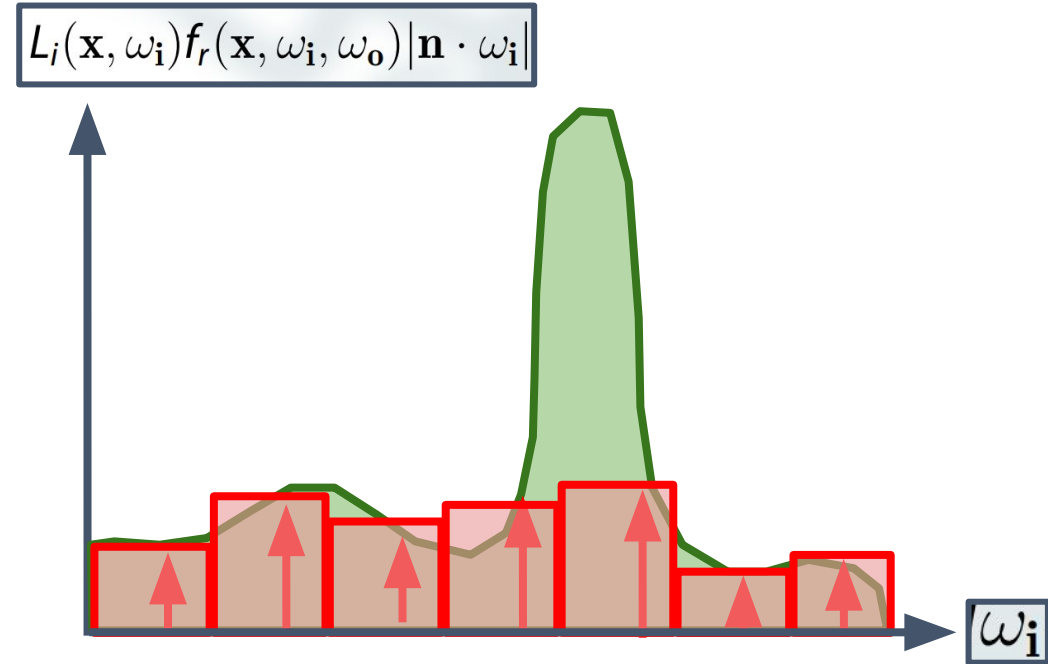
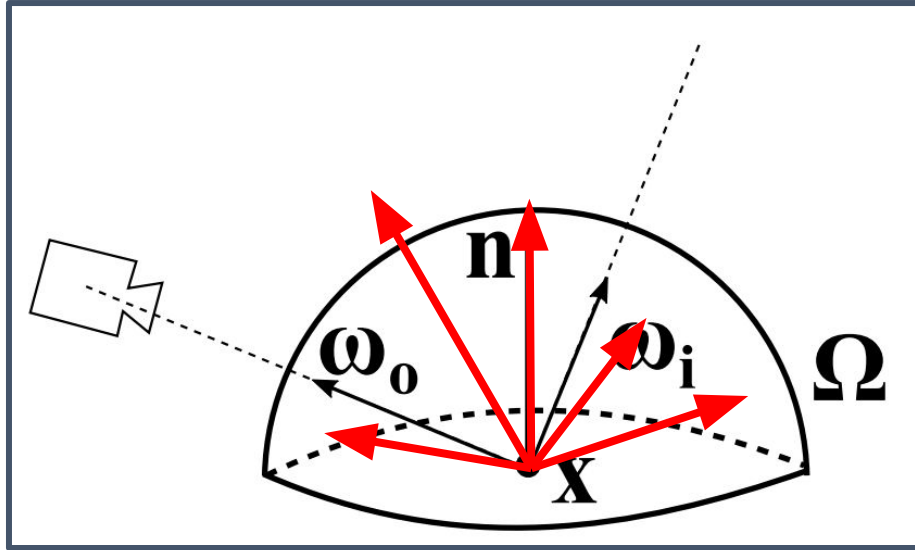
- There are infinite values for ω_i
- Idea 1: approximate using a finite amount of (evenly spaced) samples

Approximating one integral



- There are infinite values for ω_i
- Idea 1: approximate using a finite amount of (evenly spaced) samples

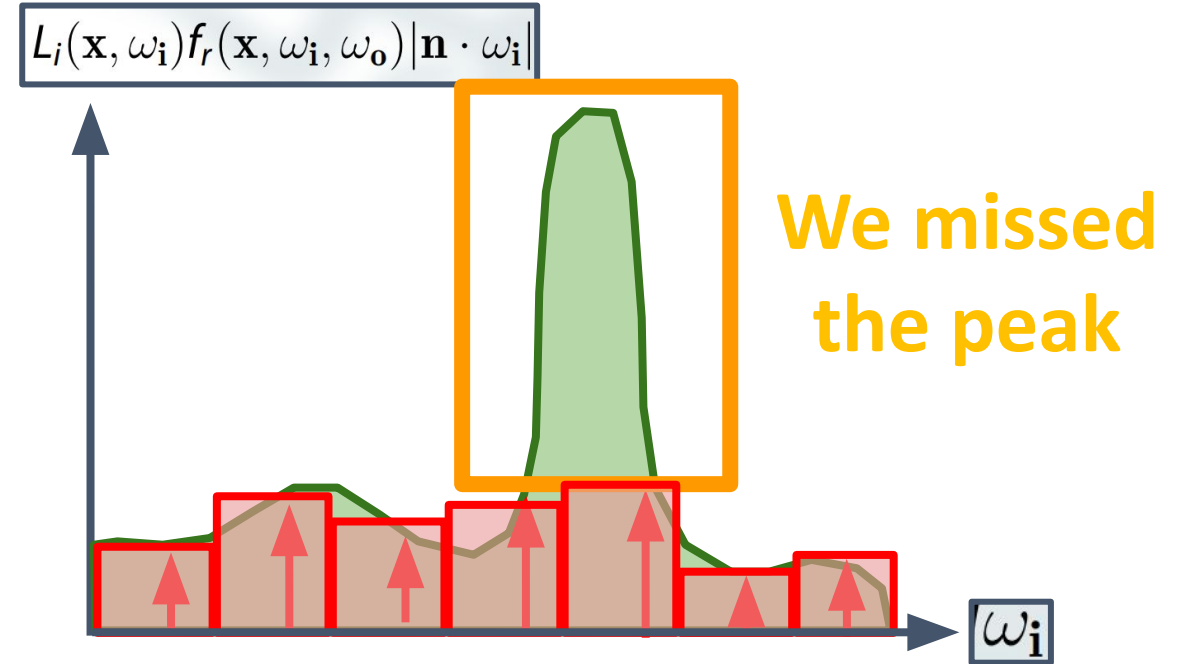
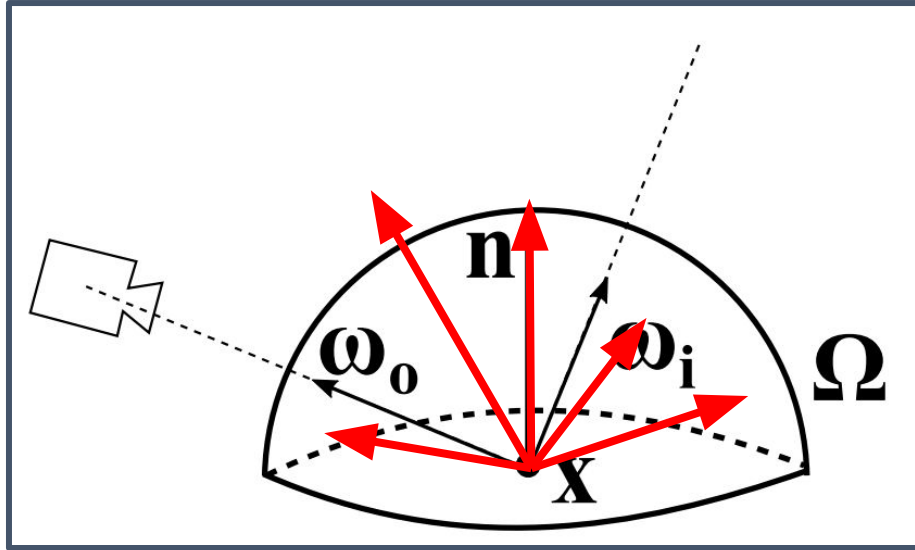
Approximating one integral



- There are infinite values for ω_i
- Idea 1: approximate using a finite amount of (evenly spaced) samples

A diagram illustrating the approximation of the integral. On the left, a green curve represents the function. To its right is an approximation symbol \approx followed by a sum of seven red rectangles of varying widths and heights, representing the function's value at discrete samples.

Approximating one integral

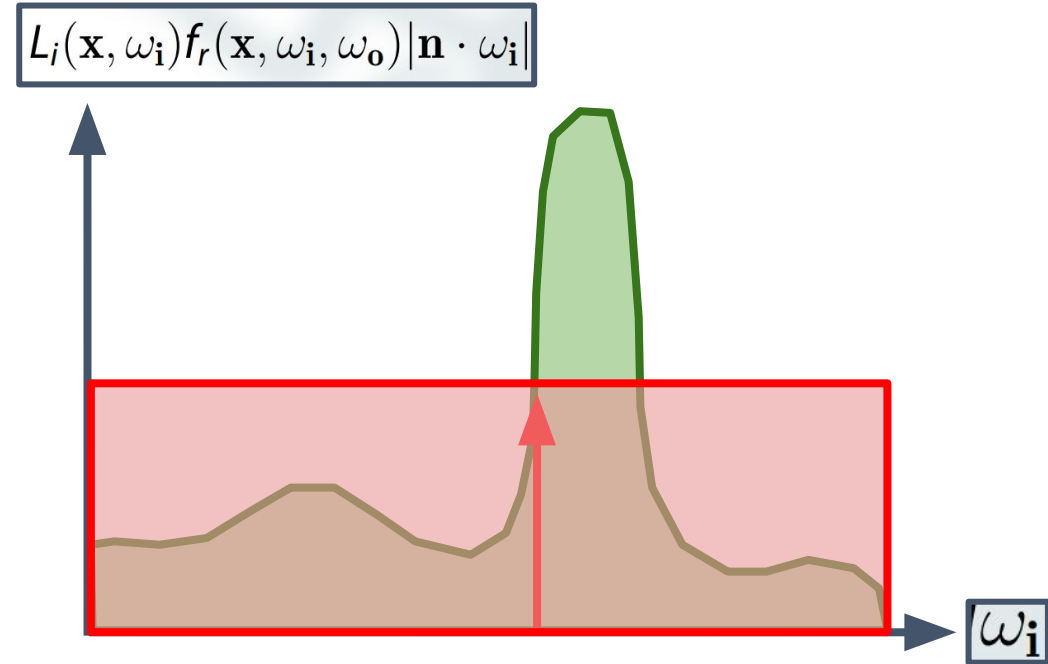
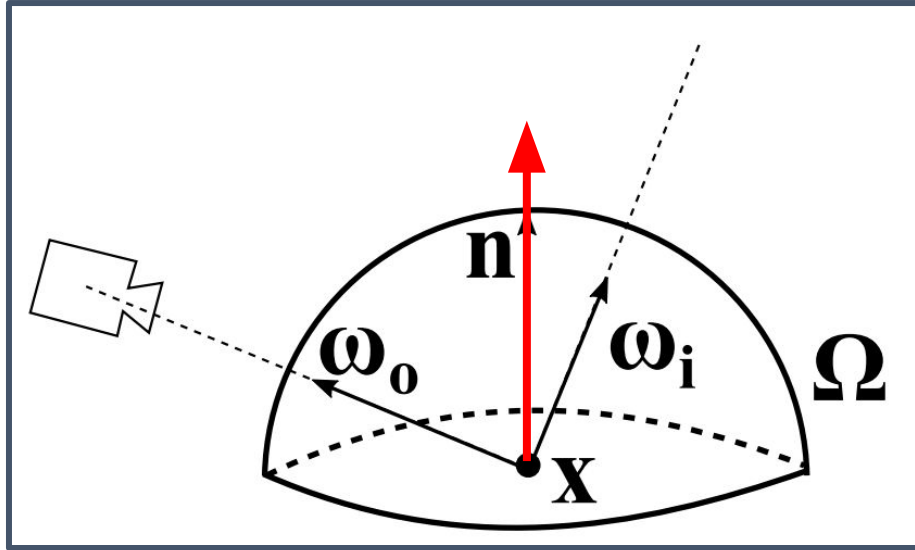


- There are infinite values for ω_i
- Idea 1: approximate using a finite amount of (evenly spaced) samples

A diagram illustrating the approximation of a function (represented by a green curve) using a sum of rectangles (represented by red boxes). The equation is:


$$\text{Function} \approx \text{rectangle}_1 + \text{rectangle}_2 + \text{rectangle}_3 + \text{rectangle}_4 + \text{rectangle}_5 + \text{rectangle}_6 + \text{rectangle}_7$$

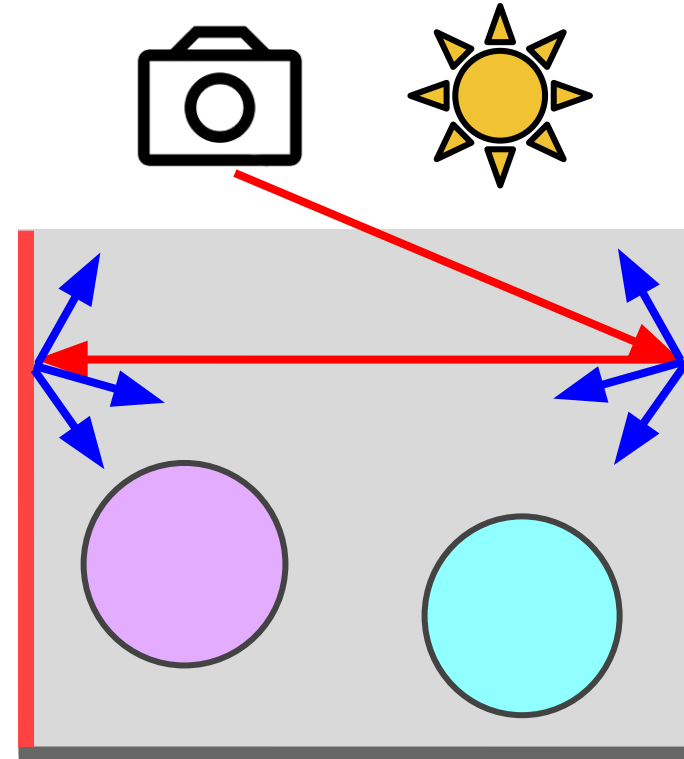
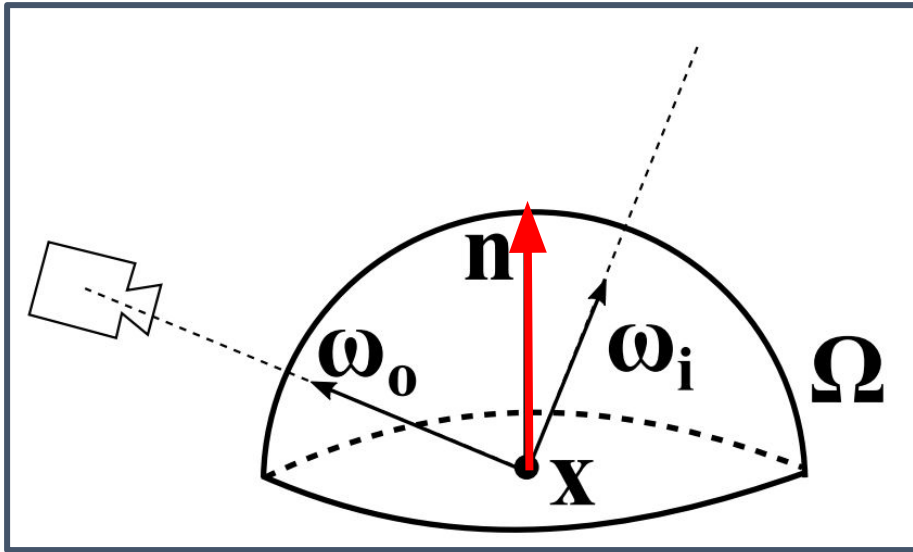
Approximating one integral



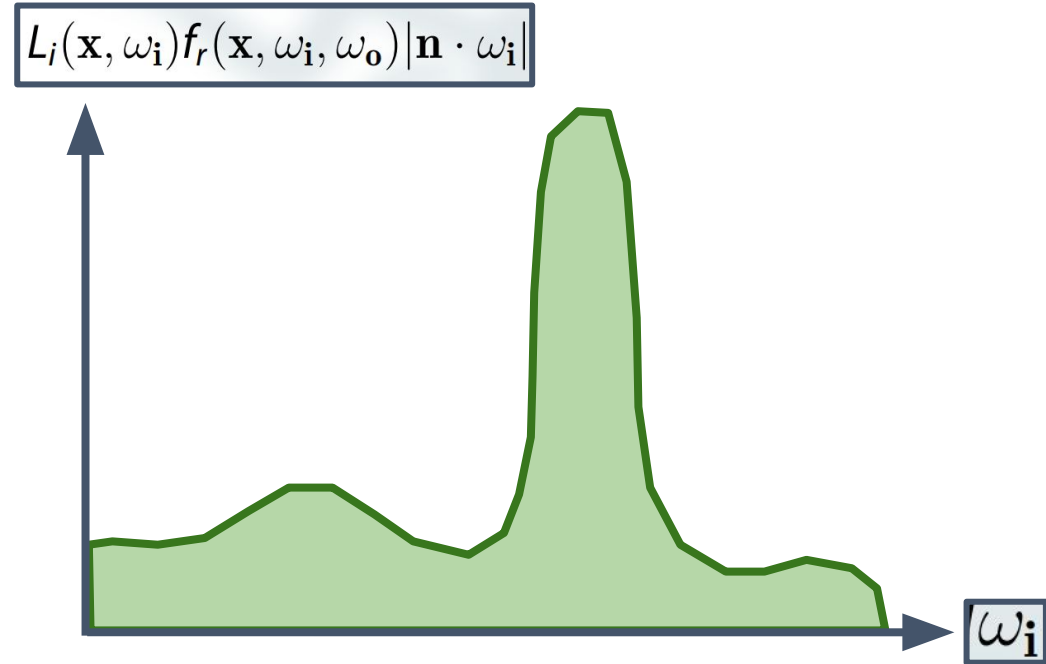
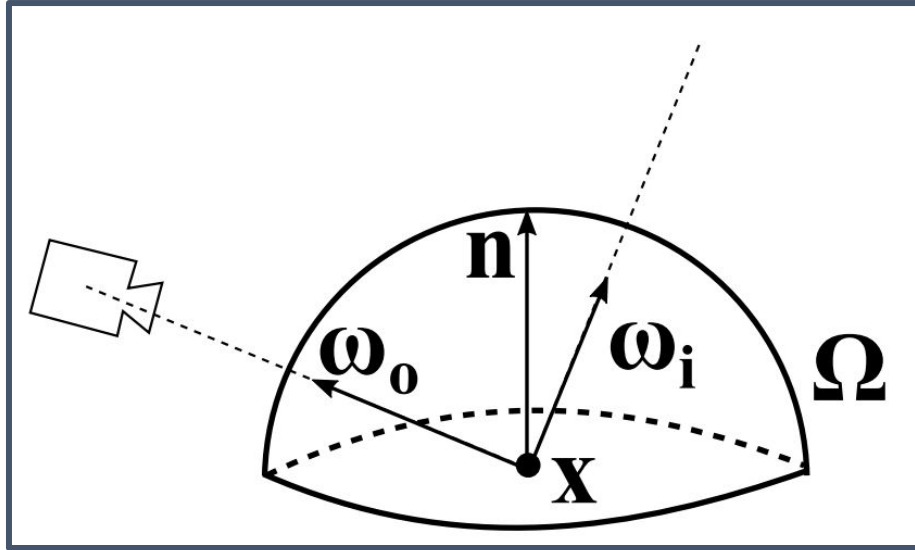
- There are infinite values for ω_i
- Idea 1: approximate using a finite amount of (evenly spaced) samples
 - Example of one deterministic sample

Approximating one integral in practice

- If all rays go towards \mathbf{n} , you ignore the rest of the hemisphere
 - Notice this will also happen with N samples
- Produces what's called a biased result (ignores )

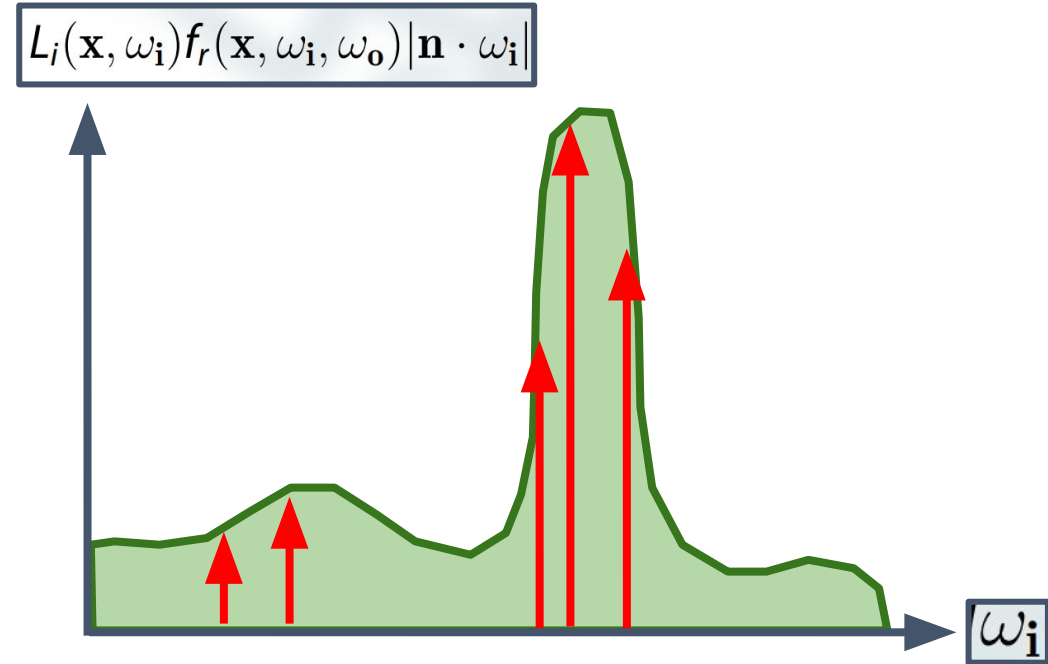
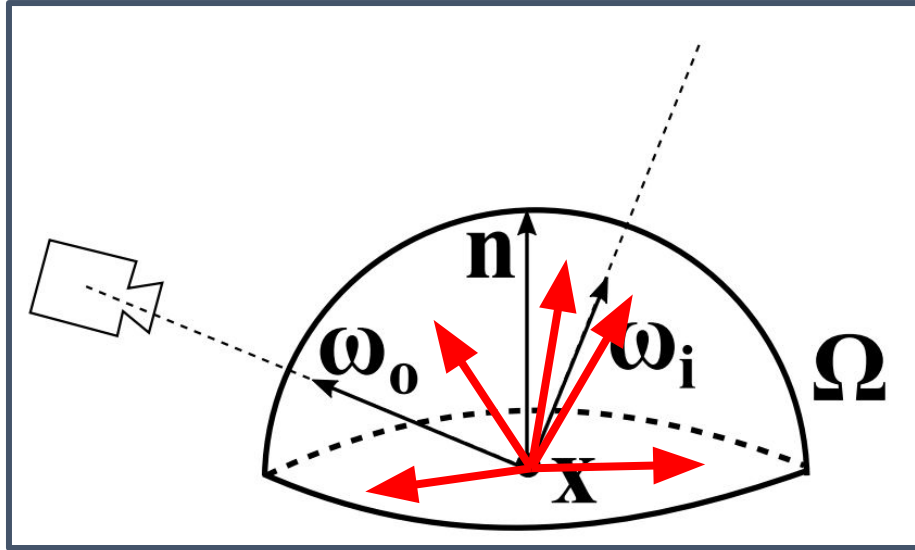


Approximating one integral



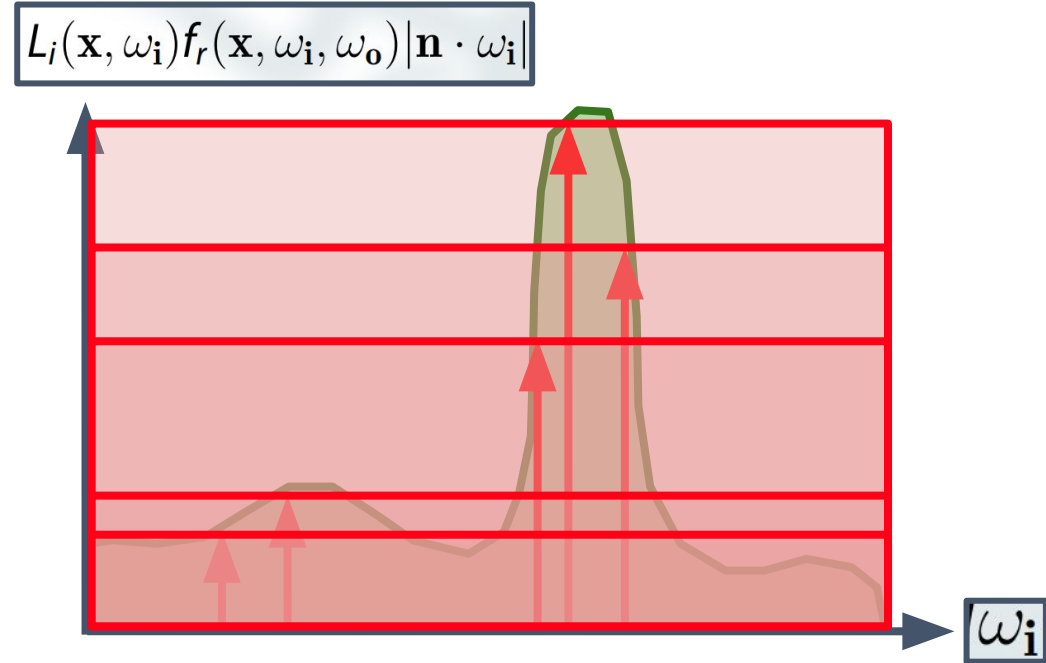
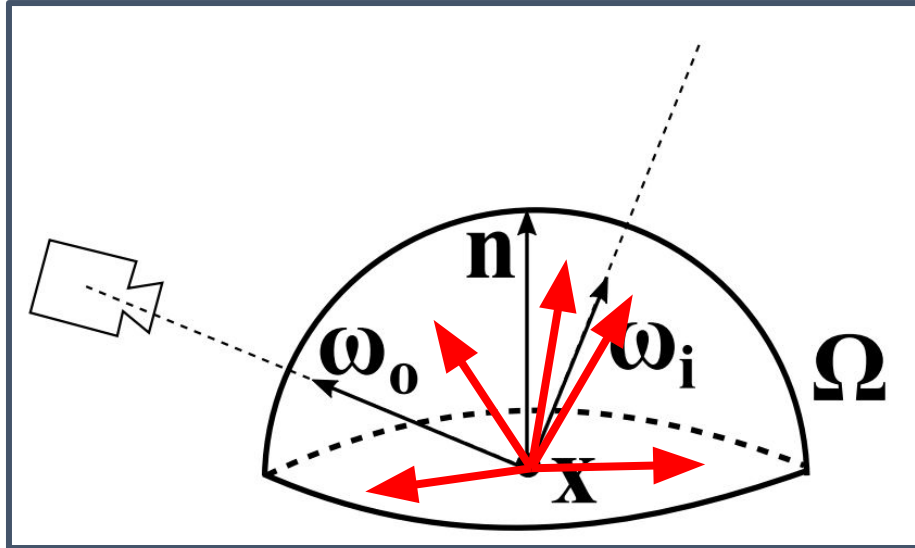
- There are infinite values for ω_i
- ~~Idea 1: approximate using a finite amount of (evenly spaced) samples~~
- Idea 2: **Monte Carlo estimator**, use the mean of N random samples

Approximating one integral



- There are infinite values for ω_i
- ~~Idea 1: approximate using a finite amount of (evenly spaced) samples~~
- Idea 2: **Monte Carlo estimator**, use the mean of N random samples

Approximating one integral

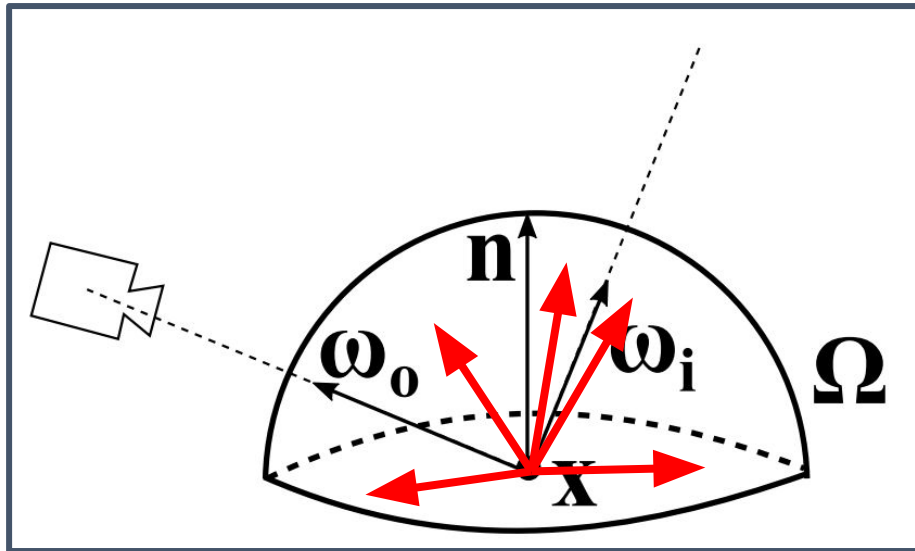


- There are infinite values for ω_i
- Idea 2: **Monte Carlo estimator**, use the mean of N random samples

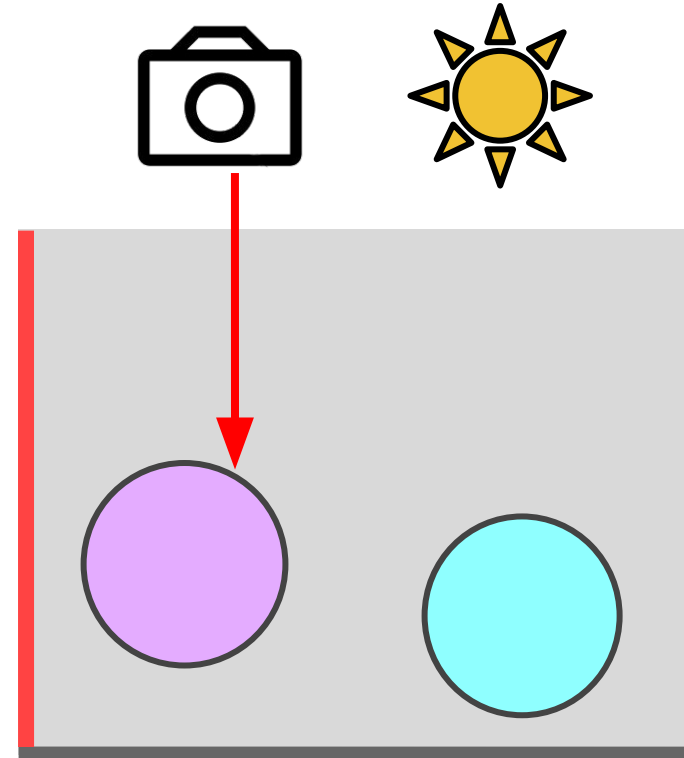
$$\approx (\text{bin}_1 + \text{bin}_2 + \text{bin}_3 + \text{bin}_4 + \text{bin}_5) / N$$

Approximating one integral in practice

- What happens if you approximate integrals using N samples?

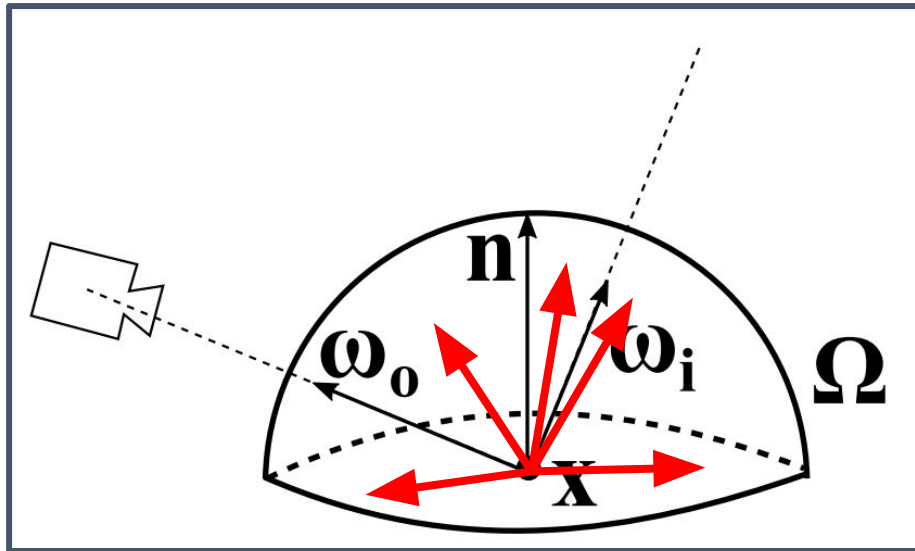


One ray from the camera

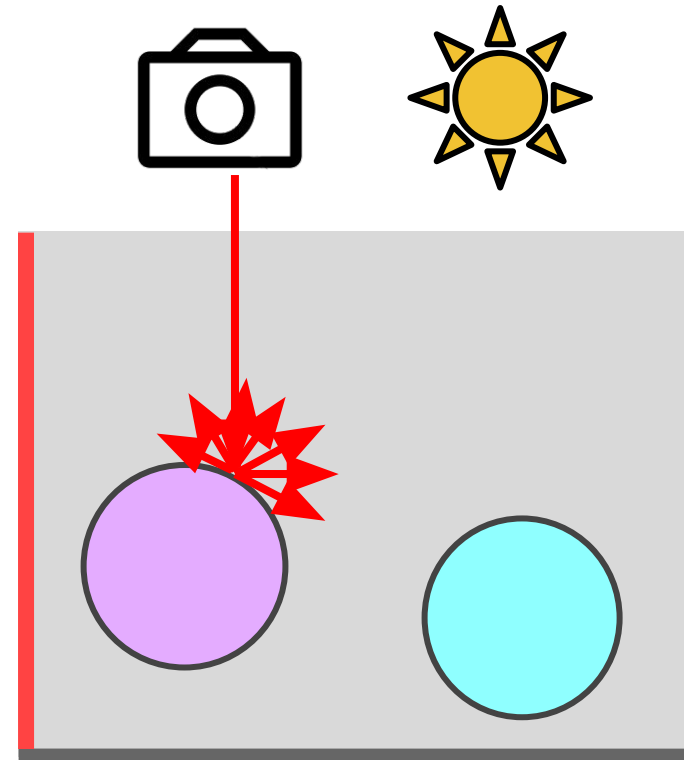


Approximating one integral in practice

- What happens if you approximate integrals using N samples?

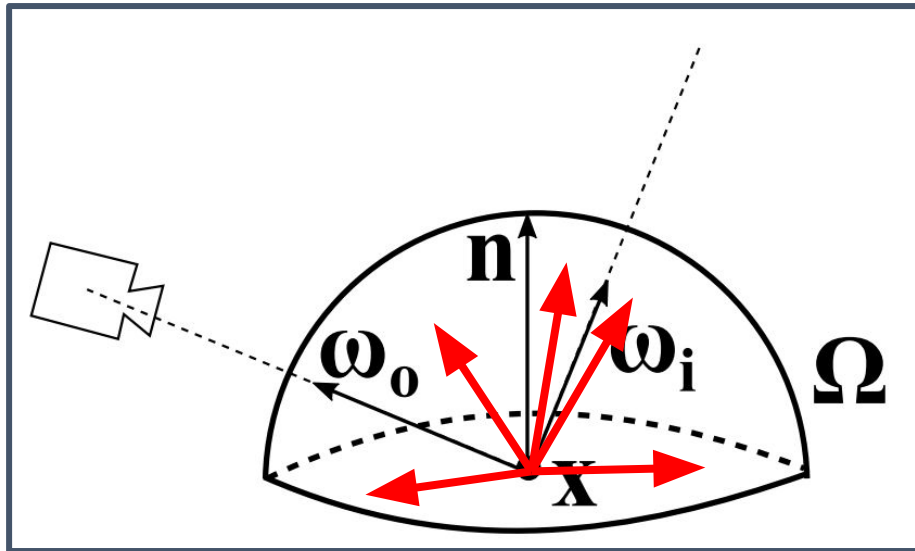


N rays for the first bounce

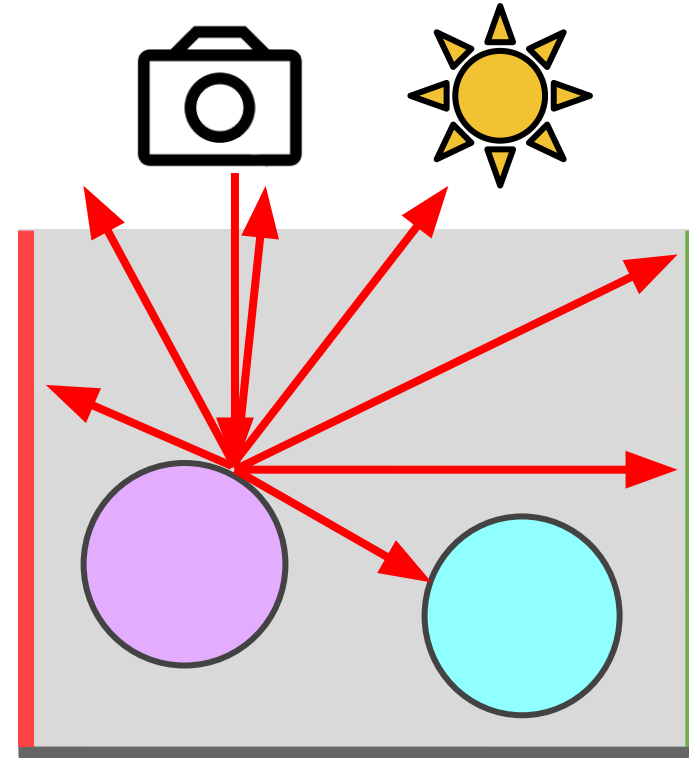


Approximating one integral in practice

- What happens if you approximate integrals using N samples?



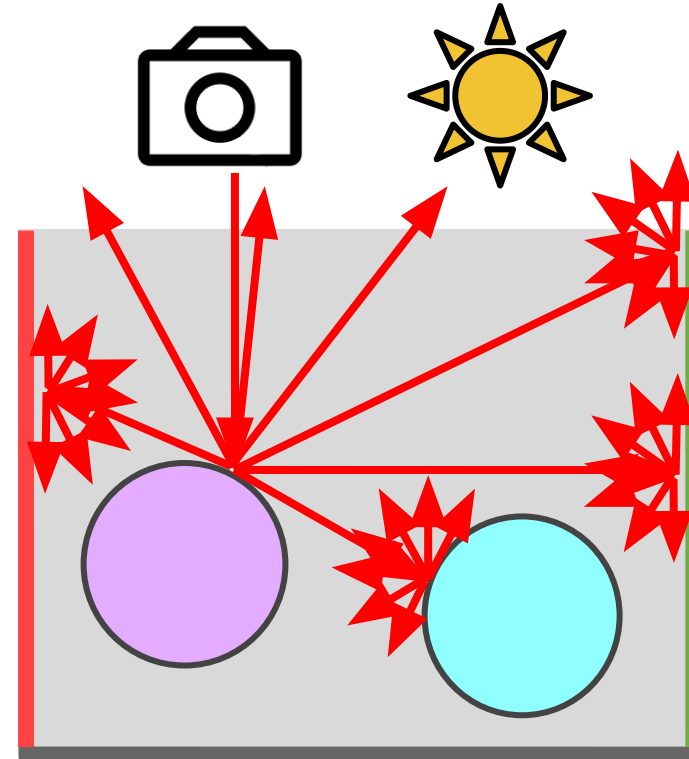
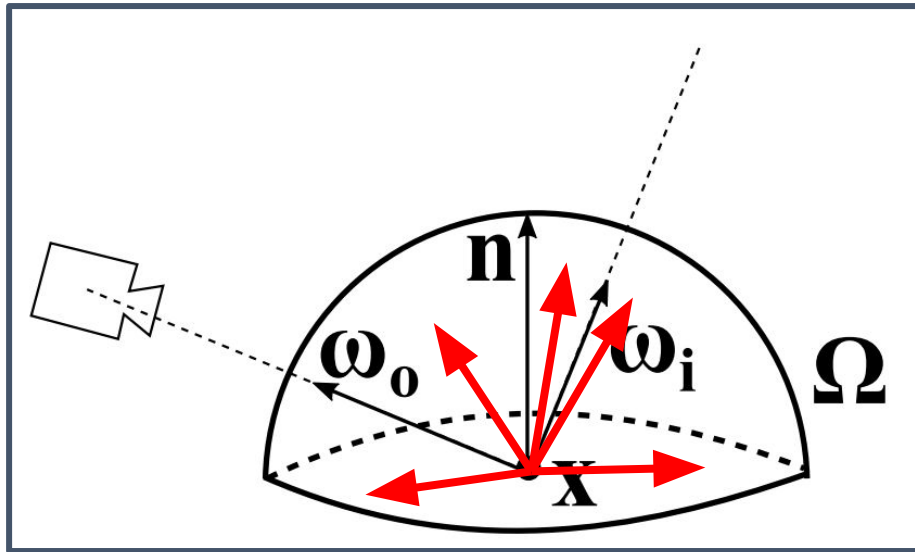
N rays for the first bounce



Approximating one integral in practice

- What happens if you approximate integrals using N samples?

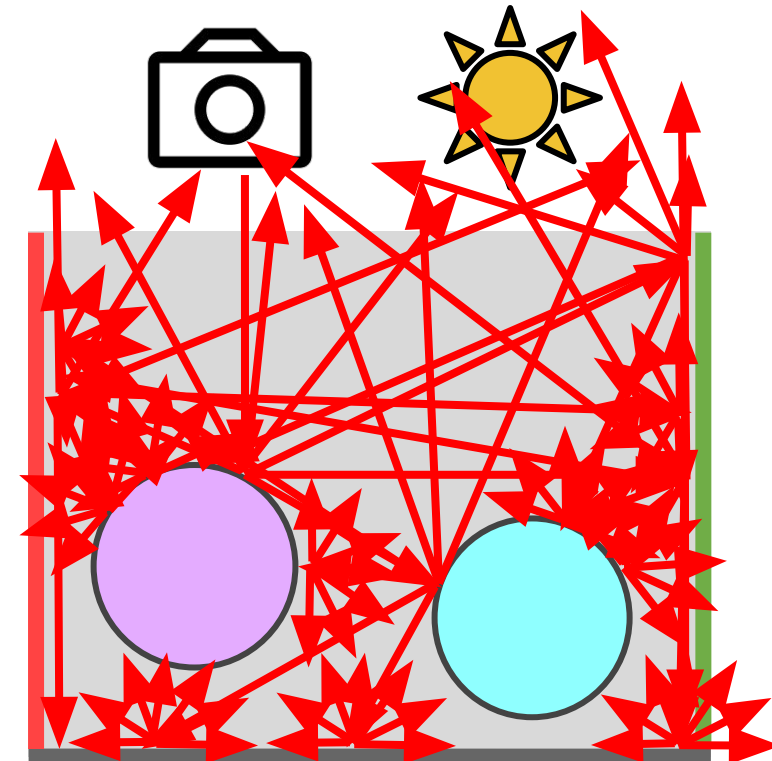
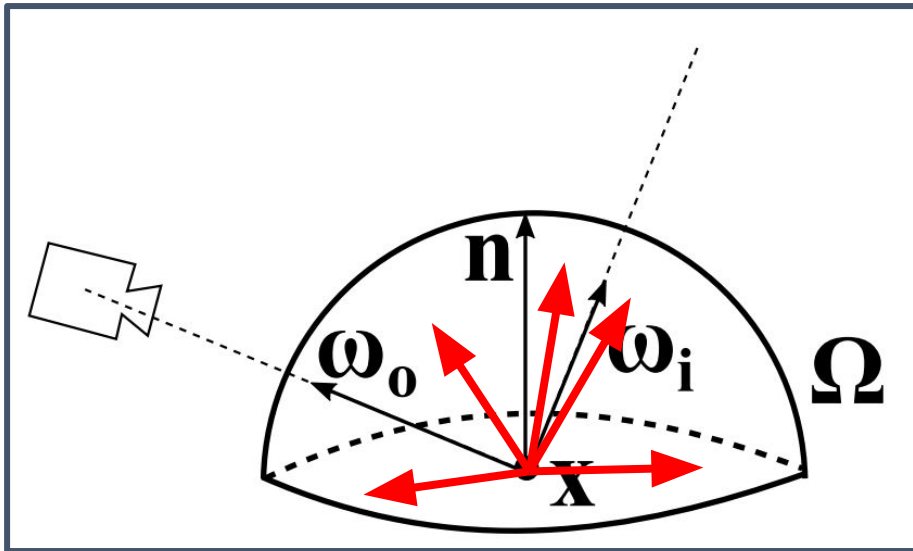
N^2 rays for the second bounce



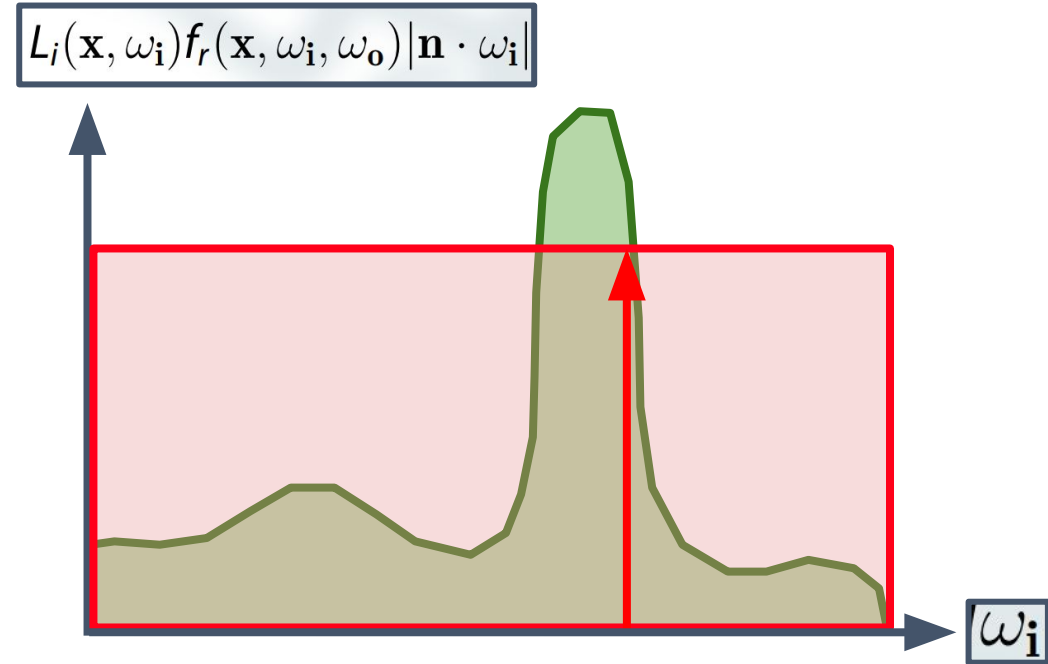
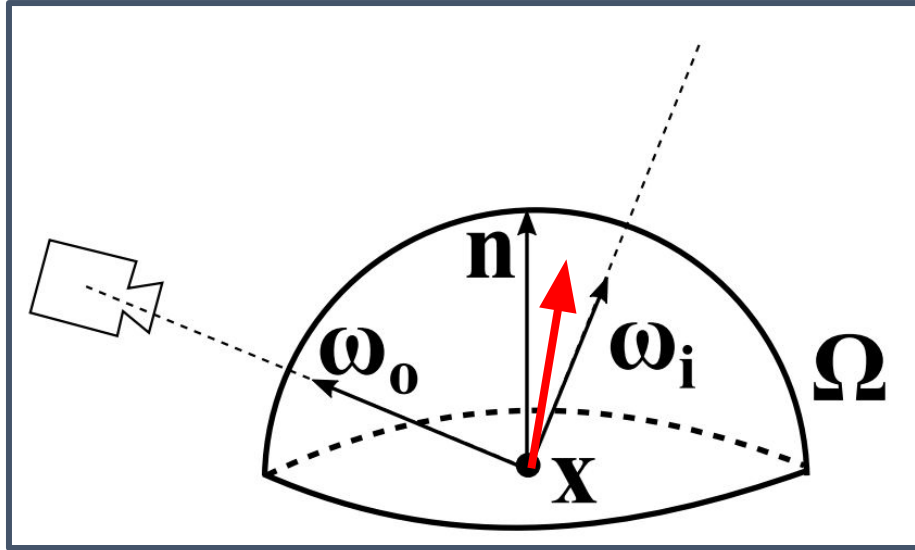
Approximating one integral in practice

- What happens if you approximate integrals using N samples?

N^N rays for the N th bounce



Approximating one integral

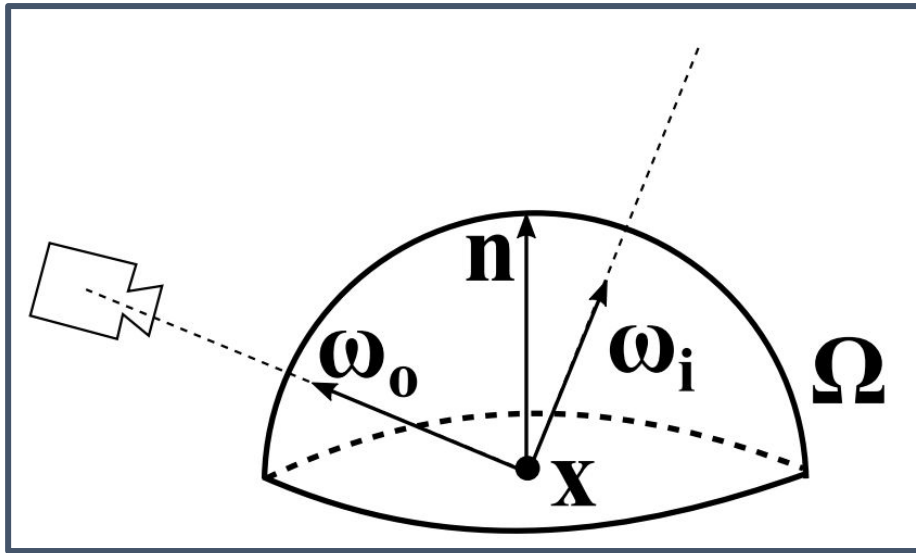


- There are infinite values for ω_i
- Idea 3: **Monte Carlo estimator**, use the mean of $N = 1$ random sample



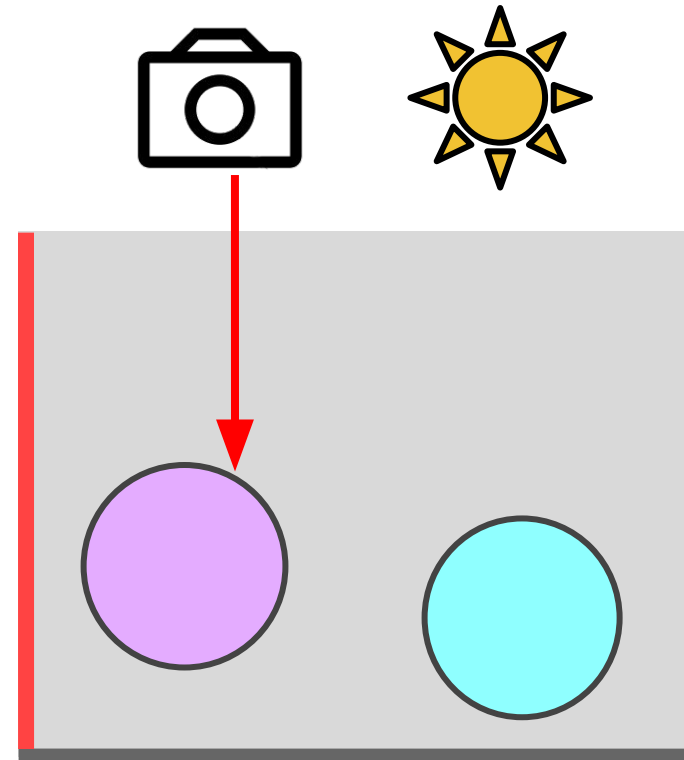
Approximating one integral in practice

- Monte Carlo estimation for the path integral



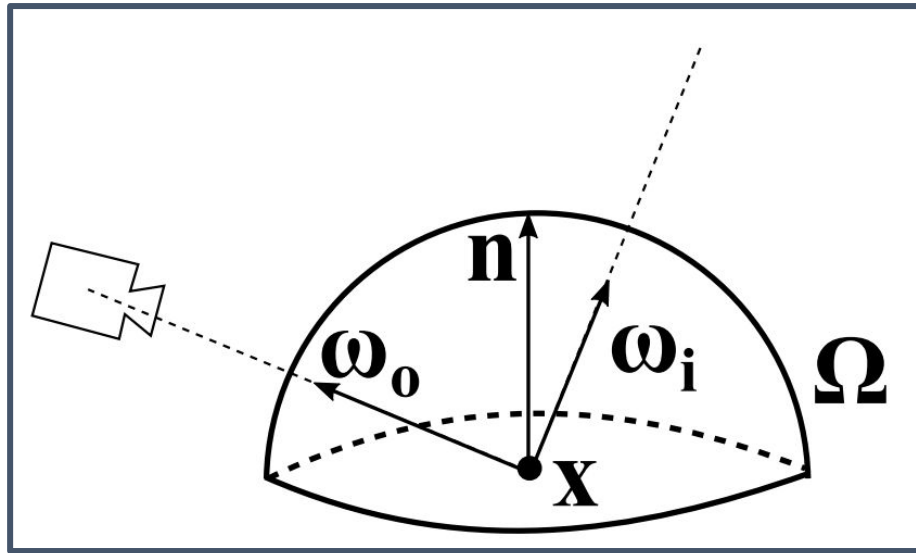
One random ω_i on each bounce

One ray from the camera



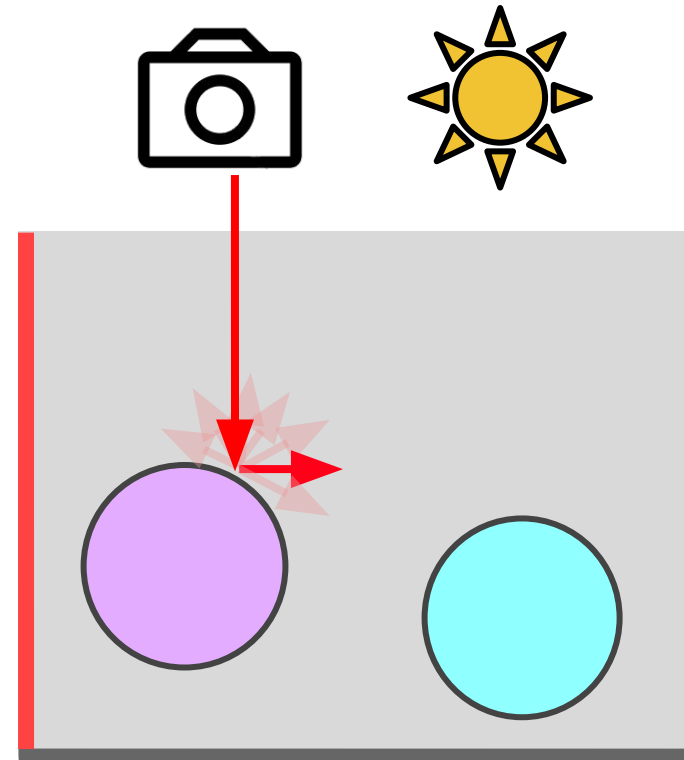
Approximating one integral in practice

- Monte Carlo estimation for the path integral



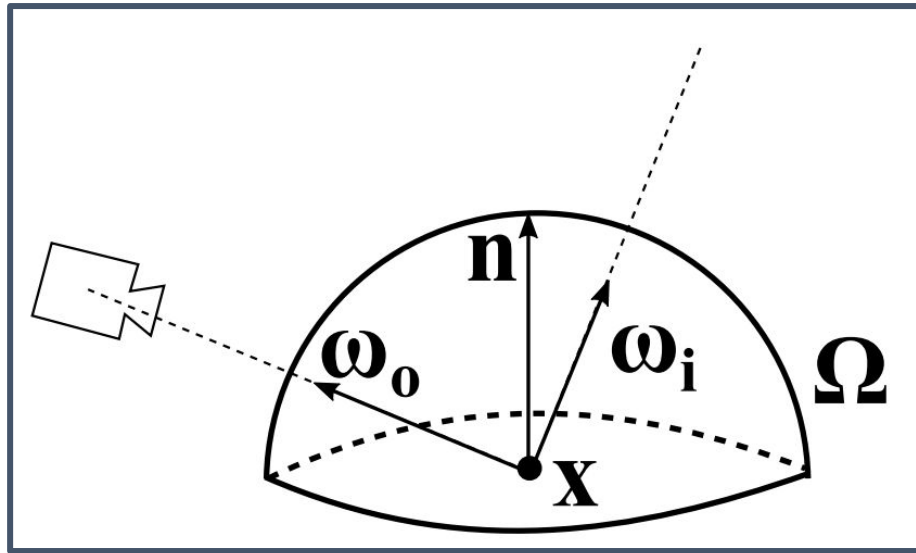
One random ω_i on each bounce

One random ray for the first bounce



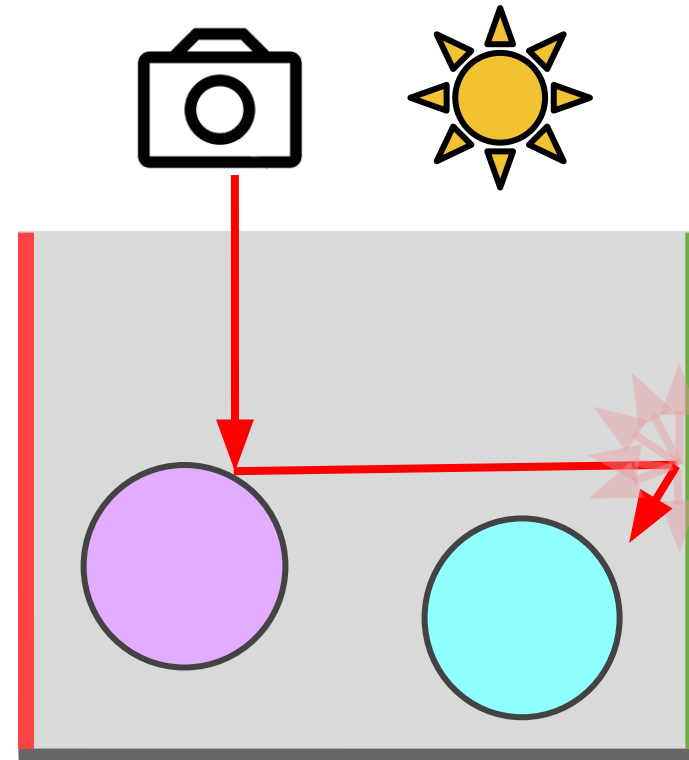
Approximating one integral in practice

- Monte Carlo estimation for the path integral



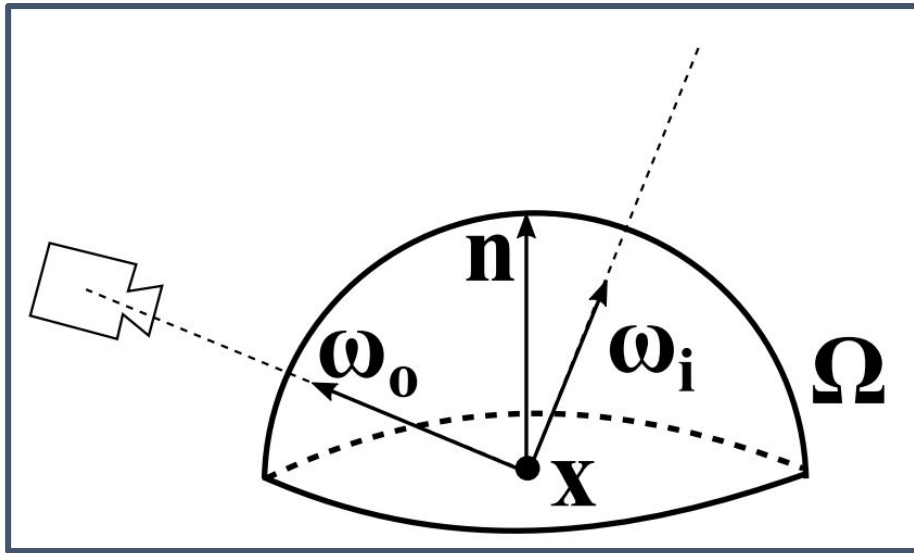
One random ω_i on each bounce

One random ray for the second bounce

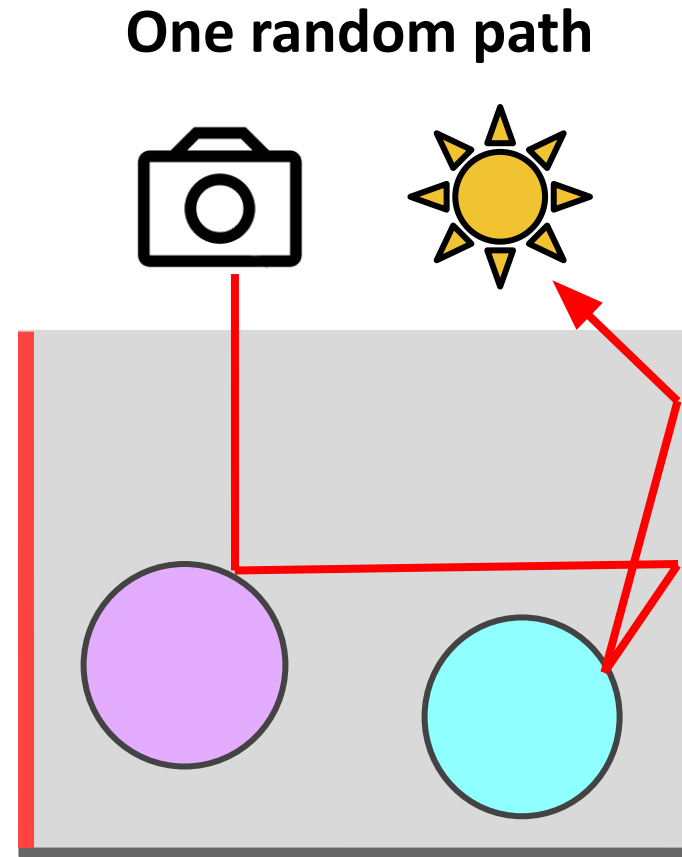


Approximating one integral in practice

- Monte Carlo estimation for the path integral

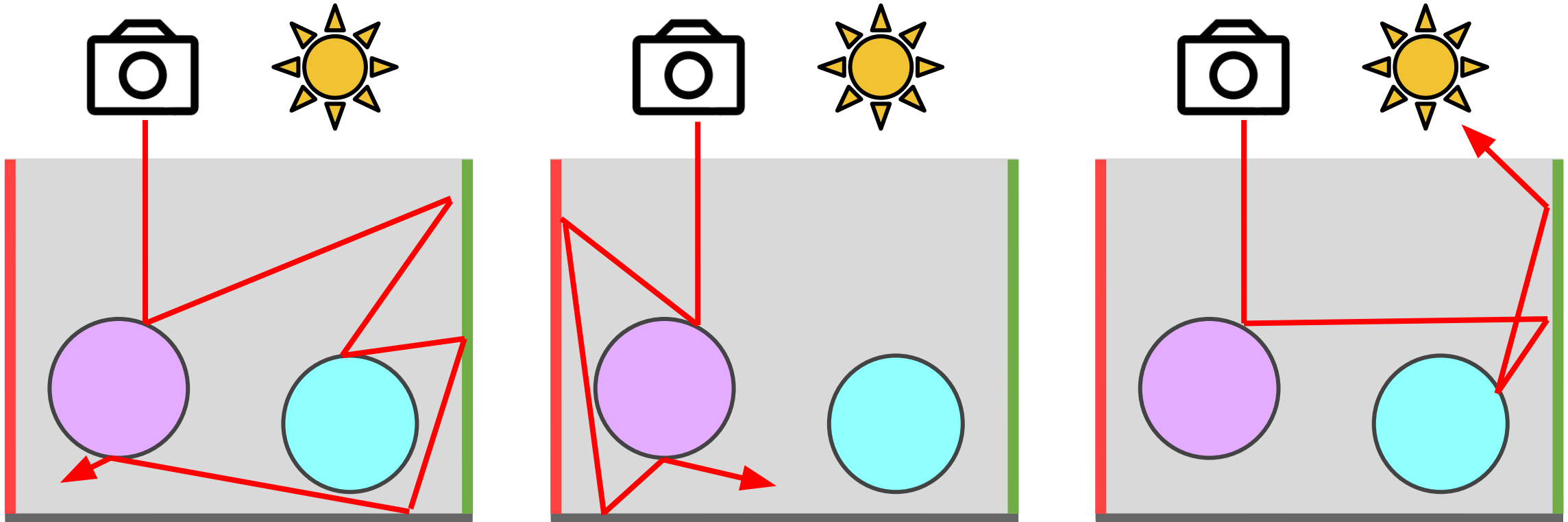


One random ω_i on each bounce



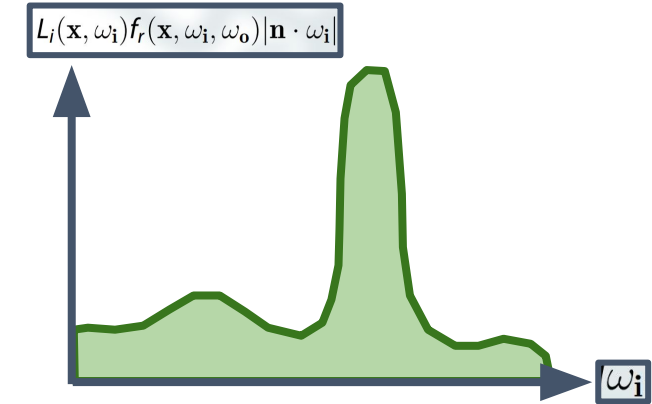
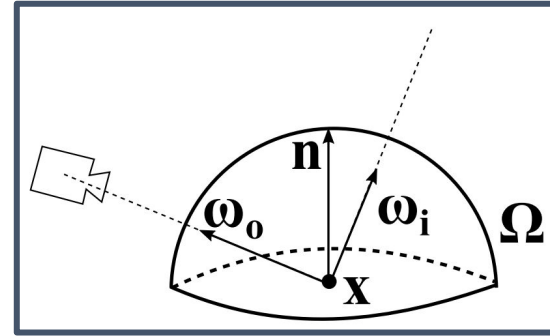
Approximating one integral in practice

- Monte Carlo estimation for the path integral
 - Sum of multiple random paths
 - More paths \rightarrow better approximation of the integral (better result)



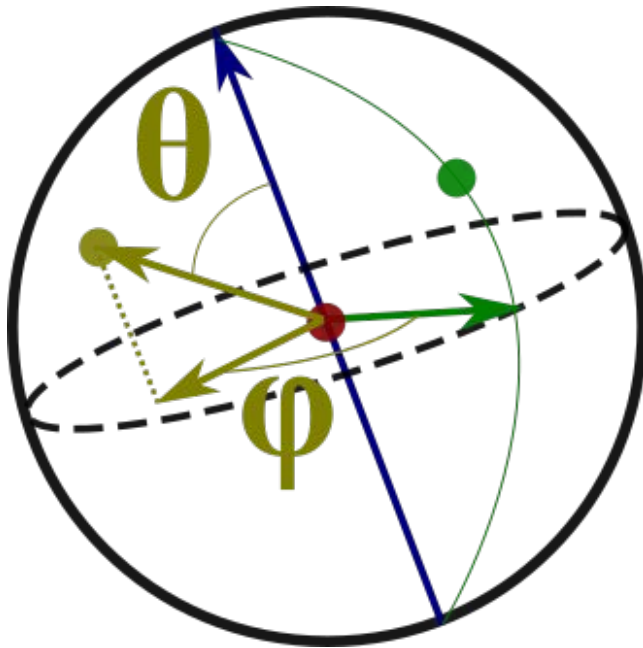
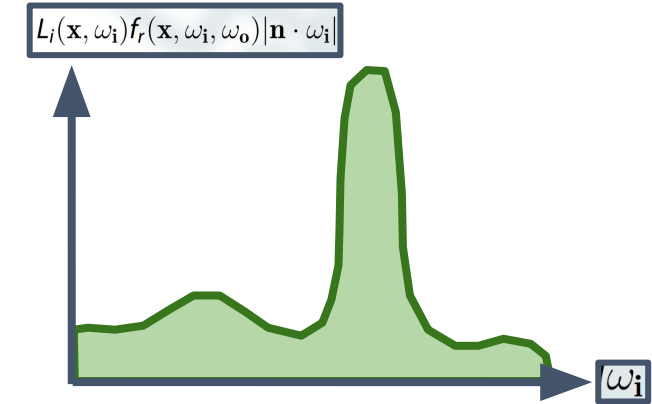
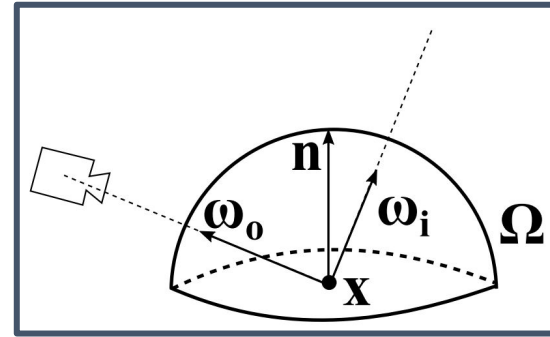
Generating random paths

- How to generate a value for ω_i ?



Generating random paths

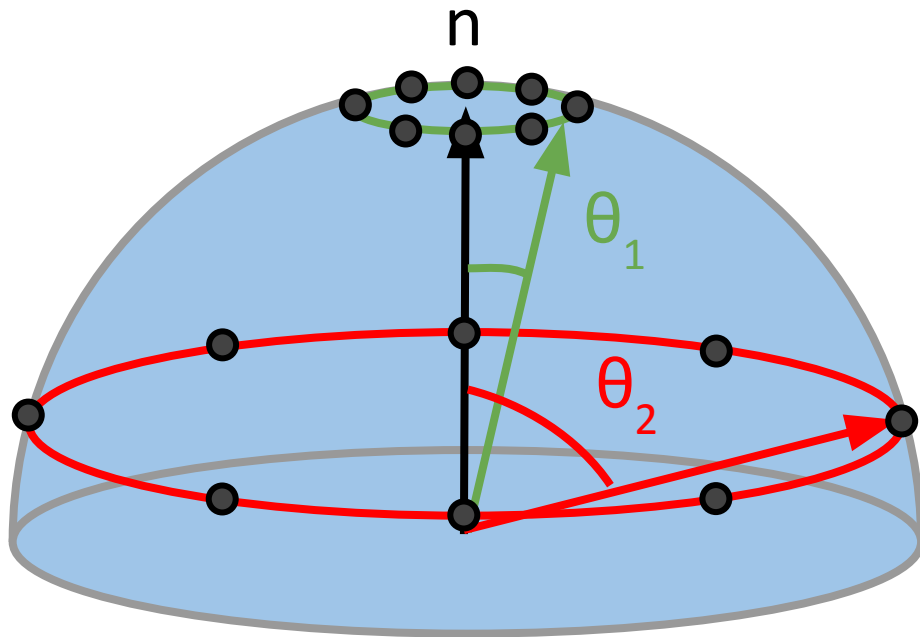
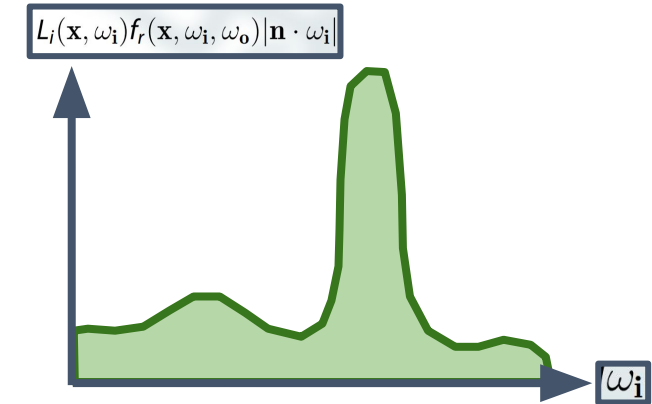
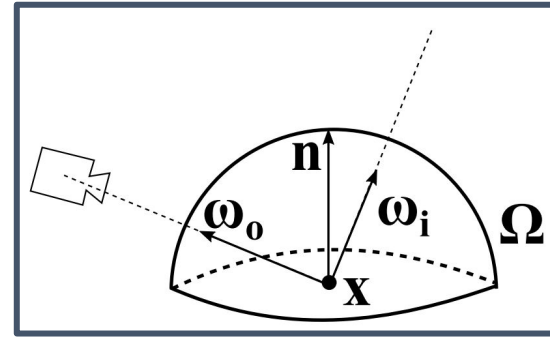
- How to generate a value for ω_i ?
- Remember your old job:



- Spherical coordinates:
 $\theta \in [0, \frac{\pi}{2})$ (**hemisphere**)
 $\varphi \in [0, 2\pi)$
- Convert spherical to cartesian

Generating random paths

- How to generate a value for ω_i ?
- Remember your old job:

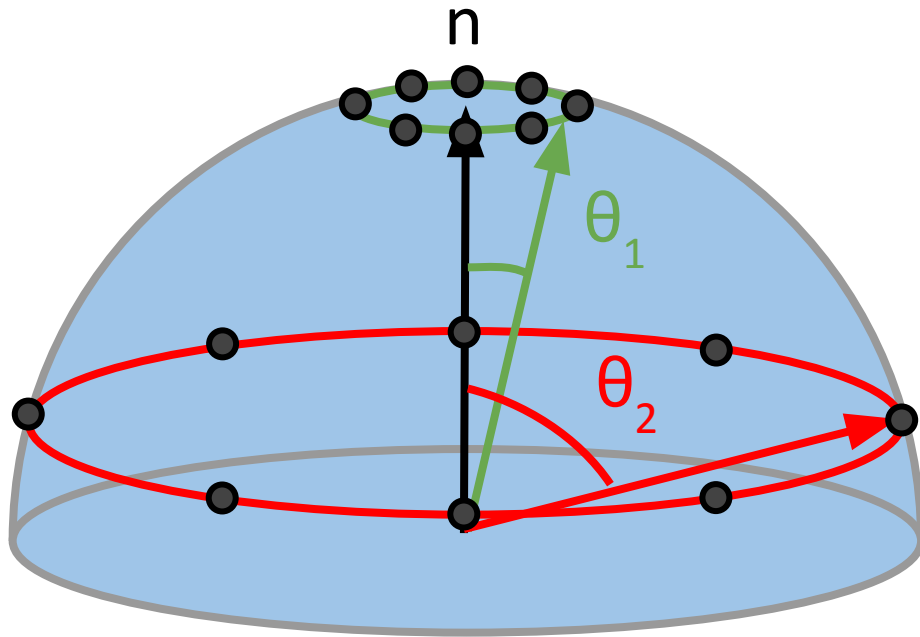
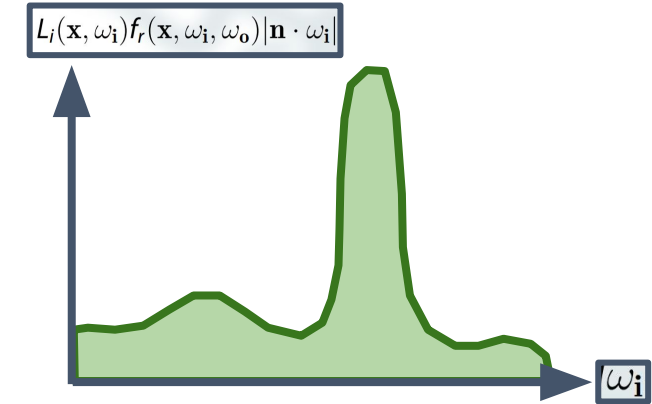
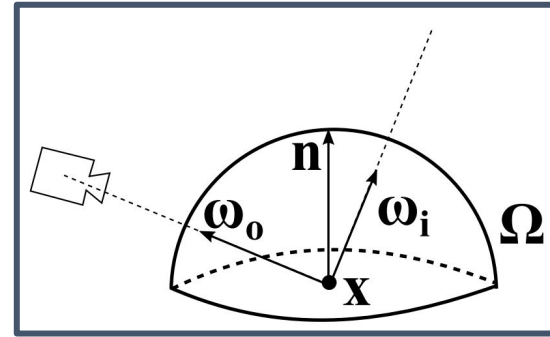


Uniform θ , $\varphi \neq$ Uniform solid angle ω_i

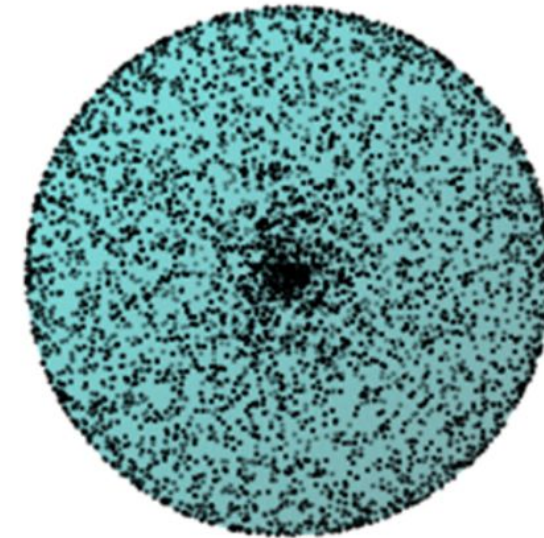
- Spherical coordinates:
 $\theta \in [0, \frac{\pi}{2})$ (**hemisphere**)
 $\varphi \in [0, 2\pi)$
- Convert spherical to cartesian

Generating random paths

- How to generate a value for ω_i ?
- Remember your old job:



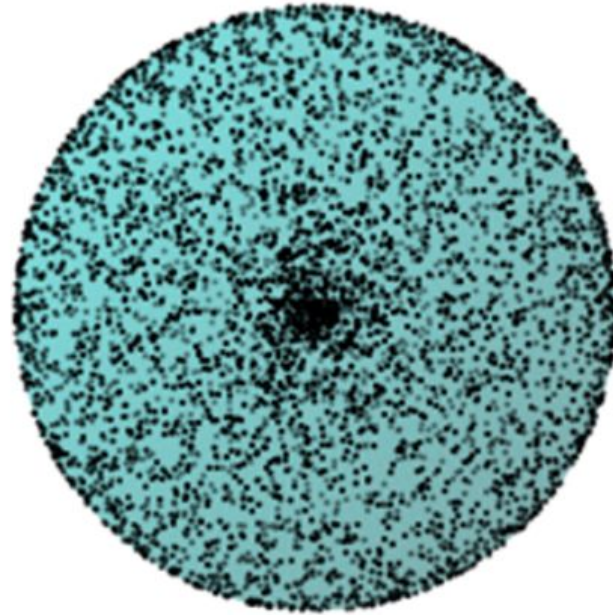
Uniform θ , $\varphi \neq$ Uniform solid angle ω_i



Top view (more samples near \mathbf{n})

Uniform angle vs uniform solid angle

Uniform angle sampling



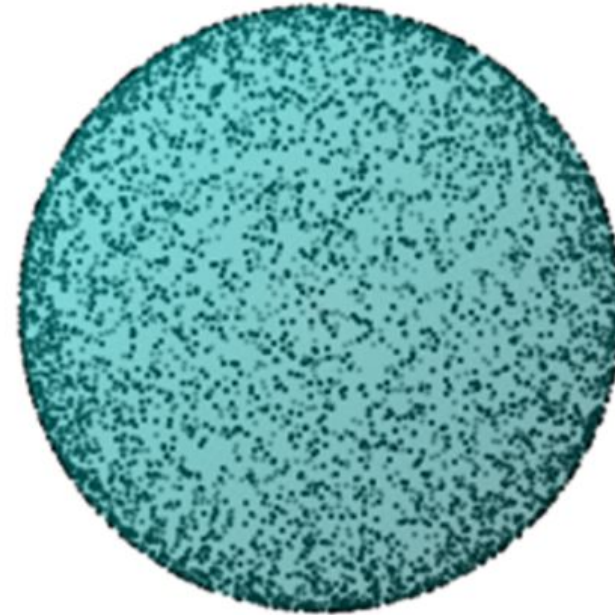
$$\xi_\theta \in [0, 1)$$

$$c^{-1}(\xi_{\theta_i}) = \frac{\pi}{2}\xi_{\theta_i}$$

$$\xi_\phi \in [0, 1)$$

$$c^{-1}(\xi_{\phi_i}) = 2\pi\xi_{\phi_i}$$

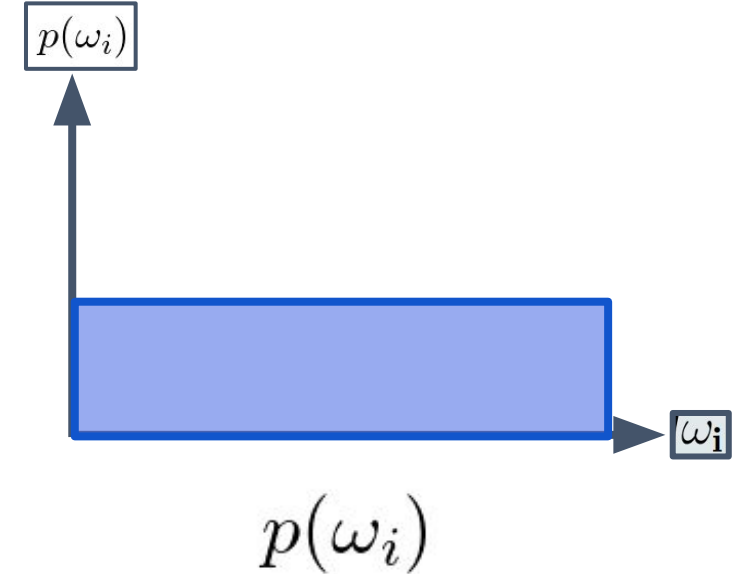
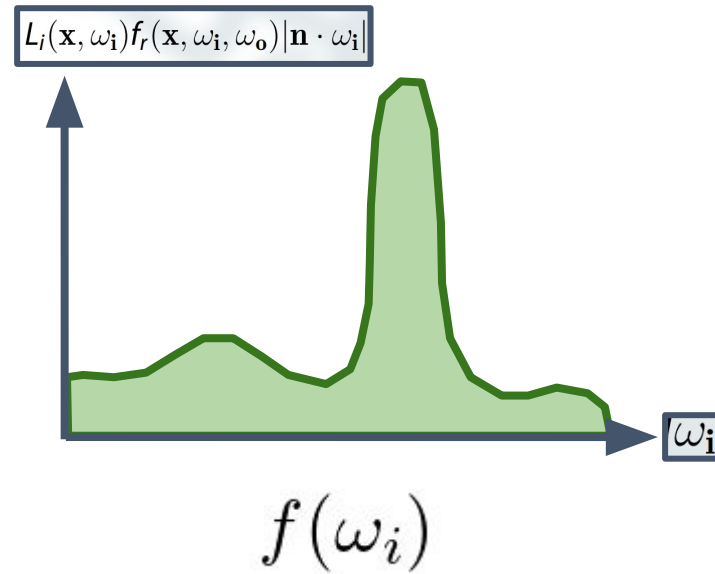
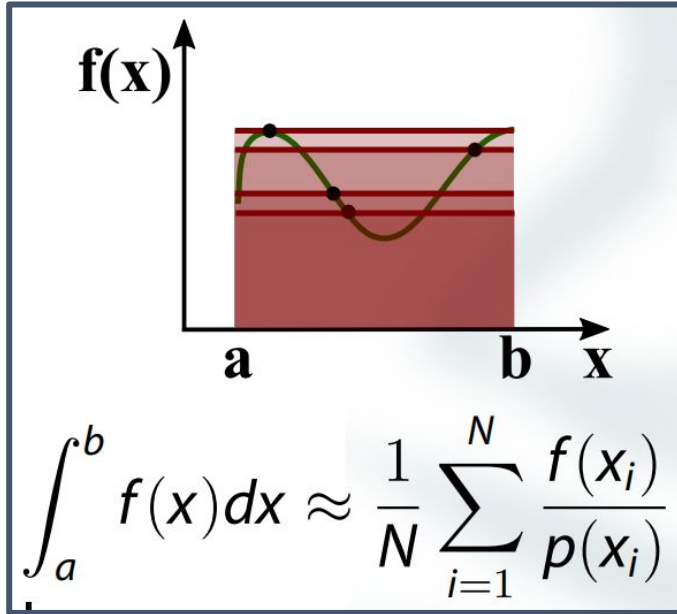
Uniform solid angle sampling



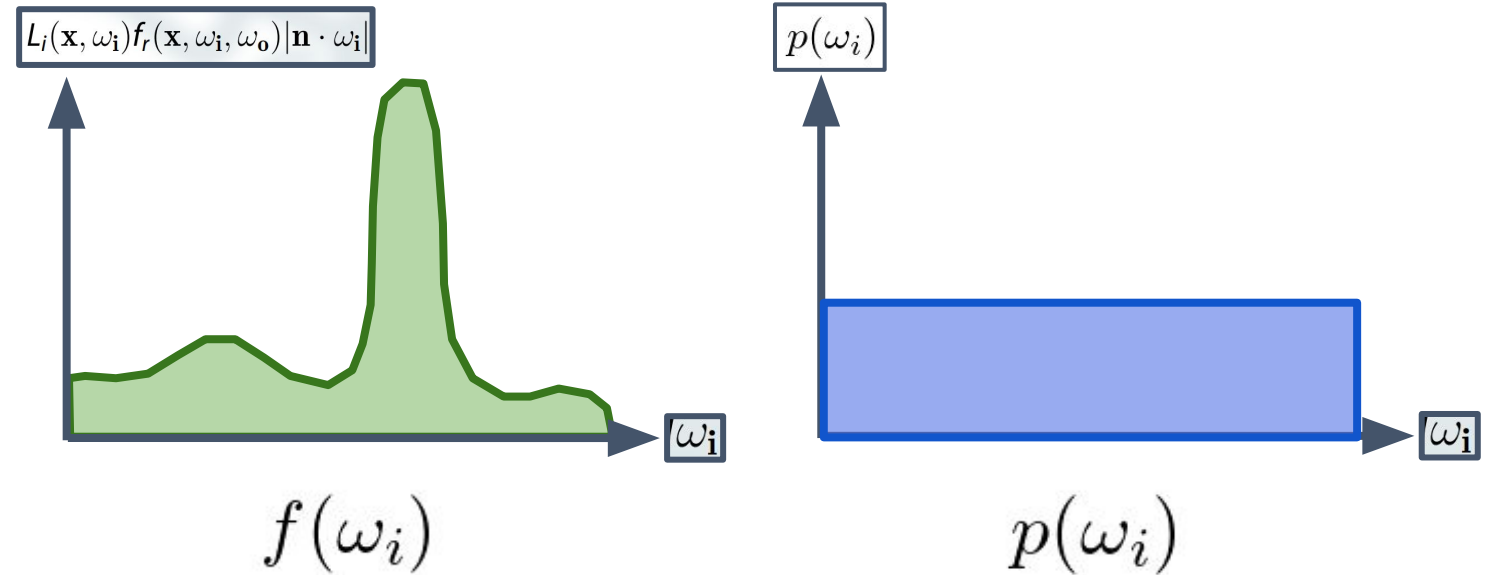
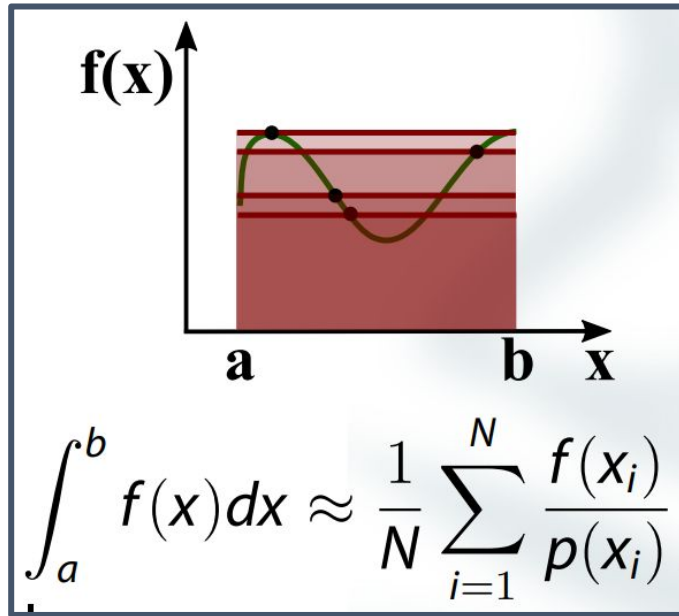
$$c^{-1}(\xi_{\theta_i}) = \arccos \xi_{\theta_i}$$

$$c^{-1}(\xi_{\phi_i}) = 2\pi\xi_{\phi_i}$$

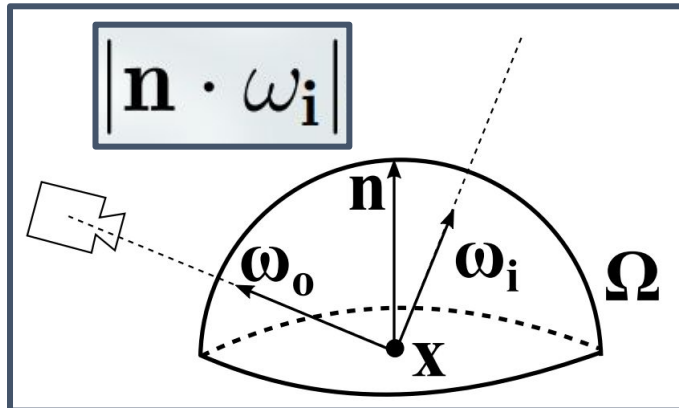
Importance sampling



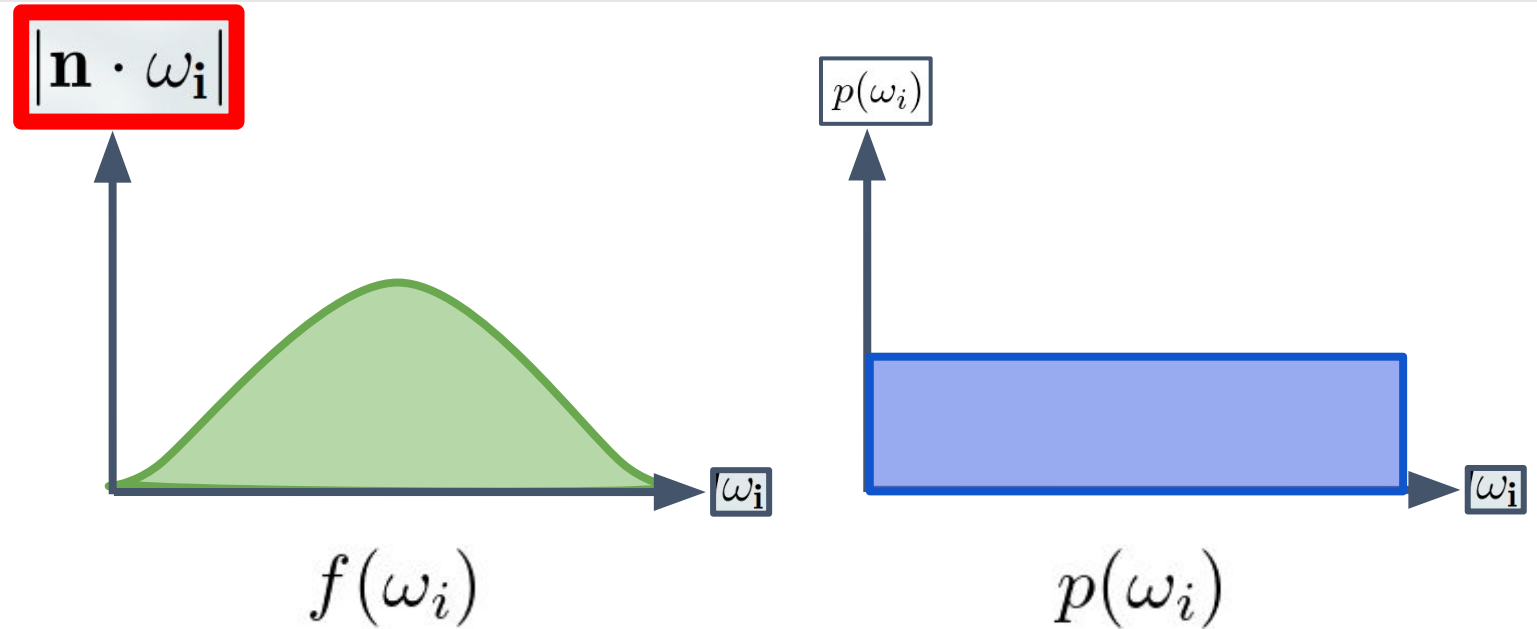
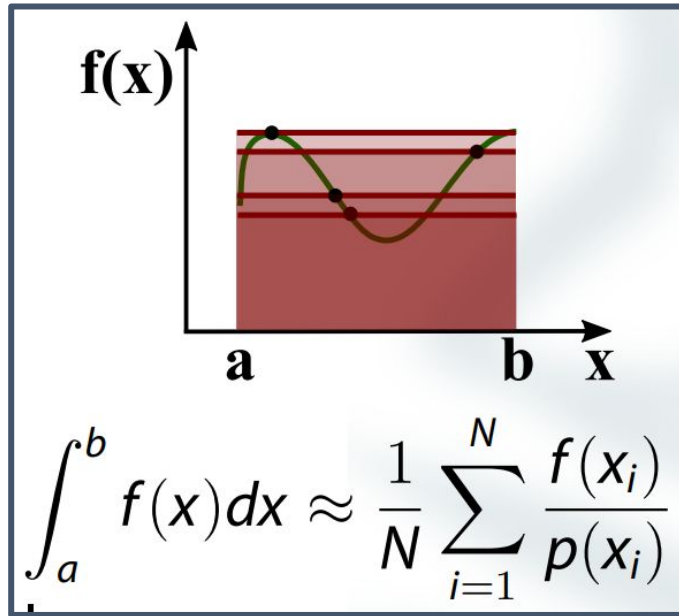
Importance sampling



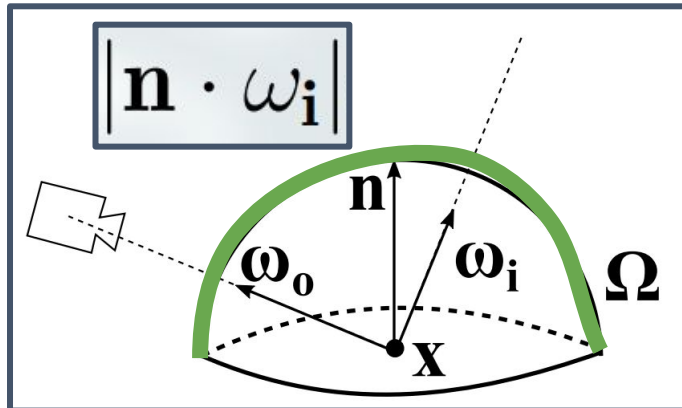
- Monte Carlo works better when f and p have similar distributions



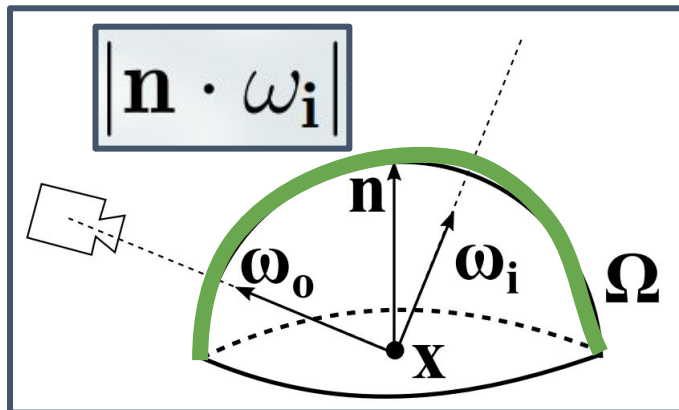
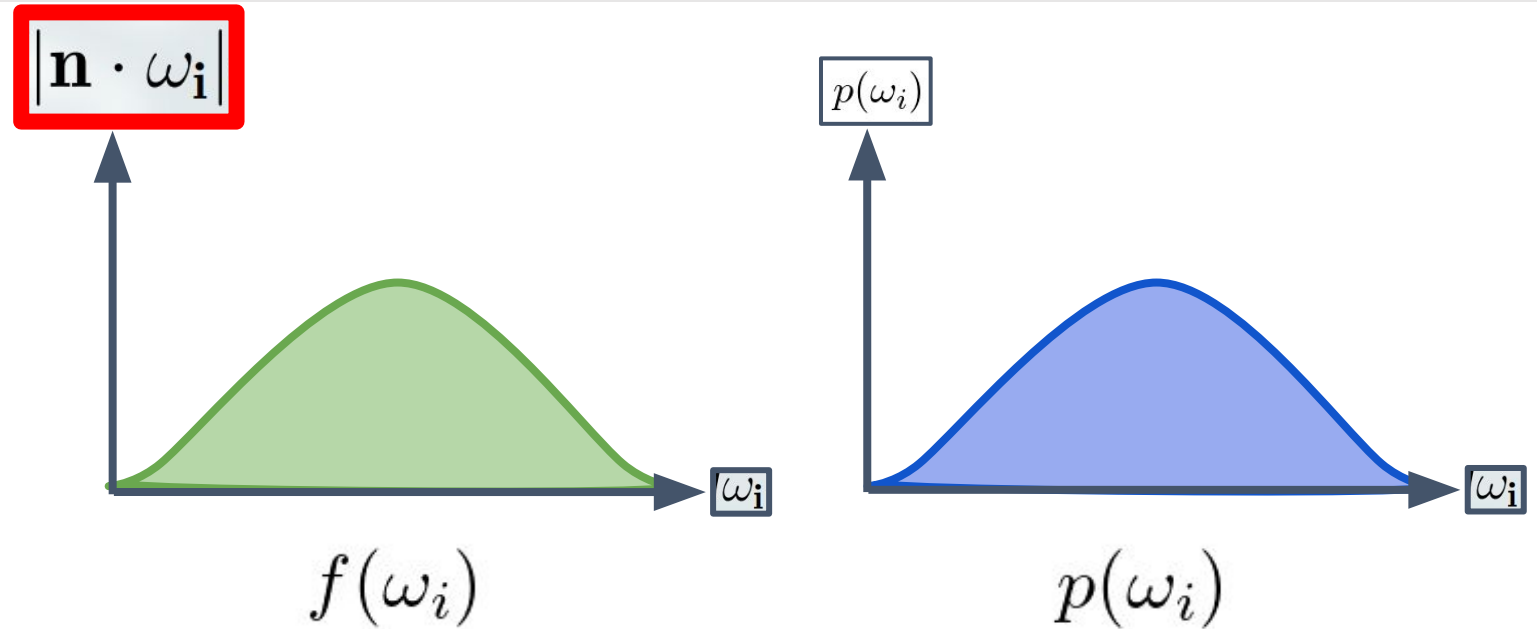
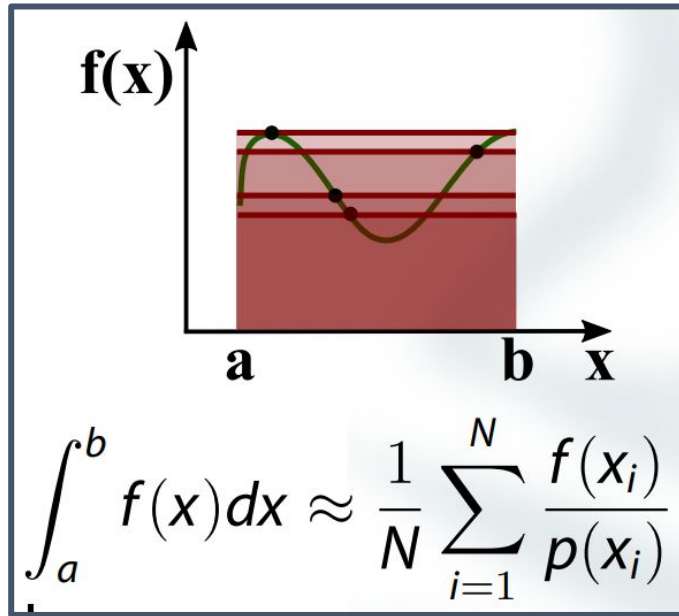
Importance sampling: diffuse materials



- Monte Carlo works better when f and p have similar distributions



Importance sampling: diffuse materials



- Monte Carlo works better when f and p have similar distributions
- Make p follow $| \mathbf{n} \cdot \boldsymbol{\omega}_i |$
- Cosine-weighted

$$c^{-1}(\xi_{\theta_i}) = \arccos \sqrt{1 - \xi_{\theta_i}}$$

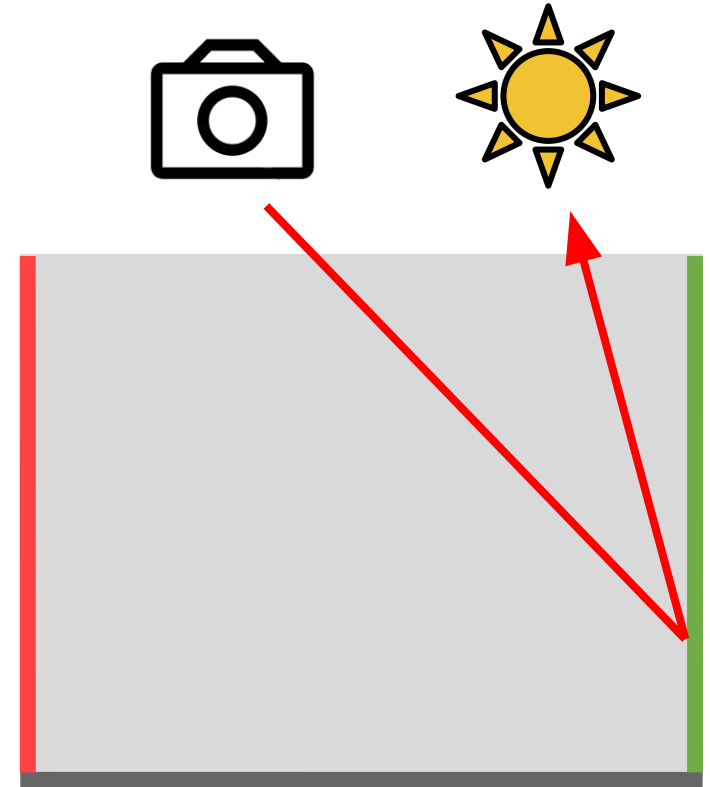
$$c^{-1}(\xi_{\phi_i}) = 2\pi \xi_{\phi_i}$$

Programming recommendations

- Enough maths, let's code

Programming recommendations

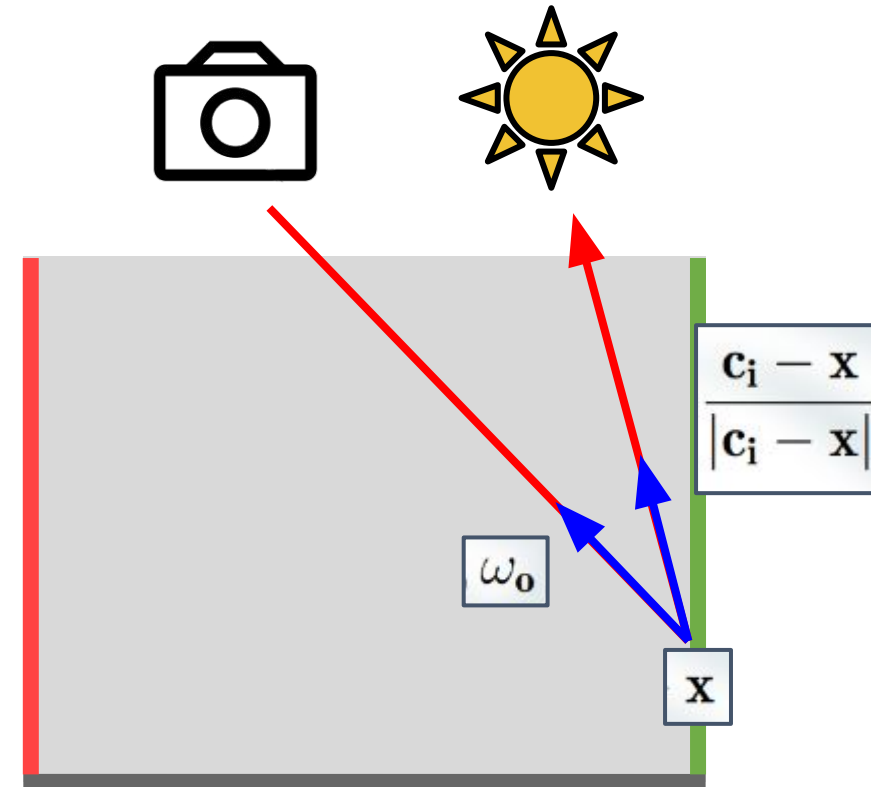
- Enough maths, let's code
- **Recommendation:** you can separate your code in functions



Programming recommendations

- Enough maths, let's code
- **Recommendation:** you can separate your code in functions
 - From previous session
 - Diffuse **BRDF** evaluation function

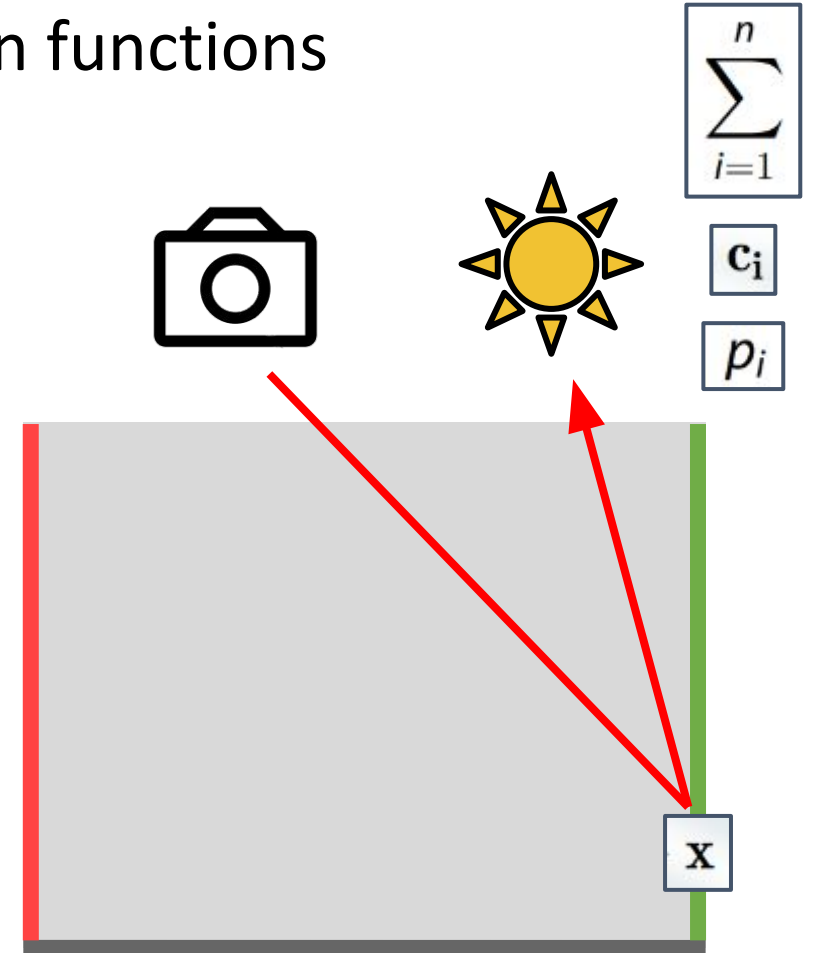
$$f_r \left(\mathbf{x}, \frac{\mathbf{c}_i - \mathbf{x}}{|\mathbf{c}_i - \mathbf{x}|}, \omega_o \right)$$



Programming recommendations

- Enough maths, let's code
- **Recommendation:** you can separate your code in functions
 - From previous session
 - Diffuse **BRDF evaluation** function
 - Next-event estimation (direct lights at \mathbf{x})

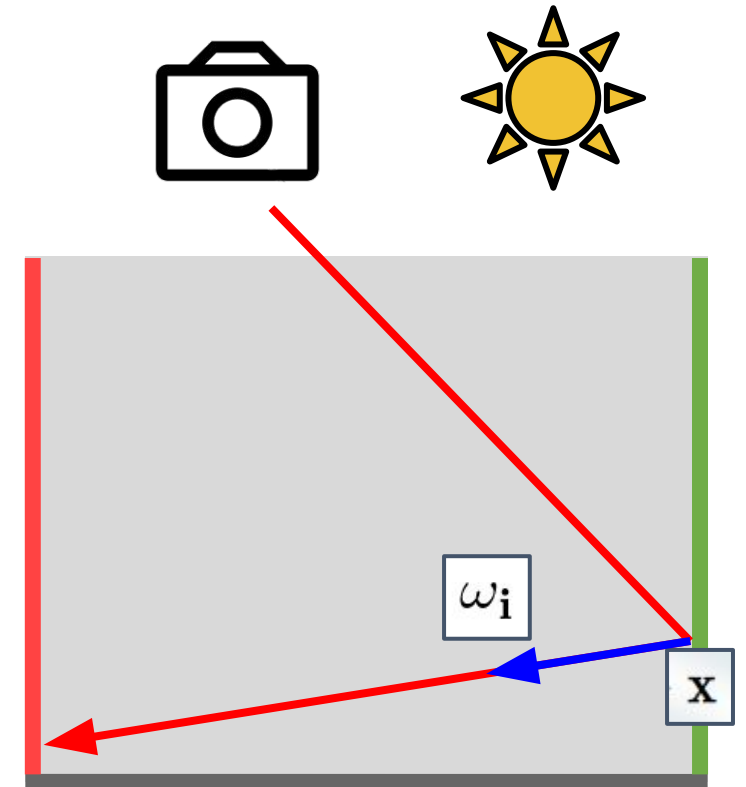
$$\sum_{i=1}^n \frac{p_i}{|\mathbf{c}_i - \mathbf{x}|^2} f_r \left(\mathbf{x}, \frac{\mathbf{c}_i - \mathbf{x}}{|\mathbf{c}_i - \mathbf{x}|}, \omega_o \right) \left| \mathbf{n} \cdot \frac{\mathbf{c}_i - \mathbf{x}}{|\mathbf{c}_i - \mathbf{x}|} \right|$$



Programming recommendations

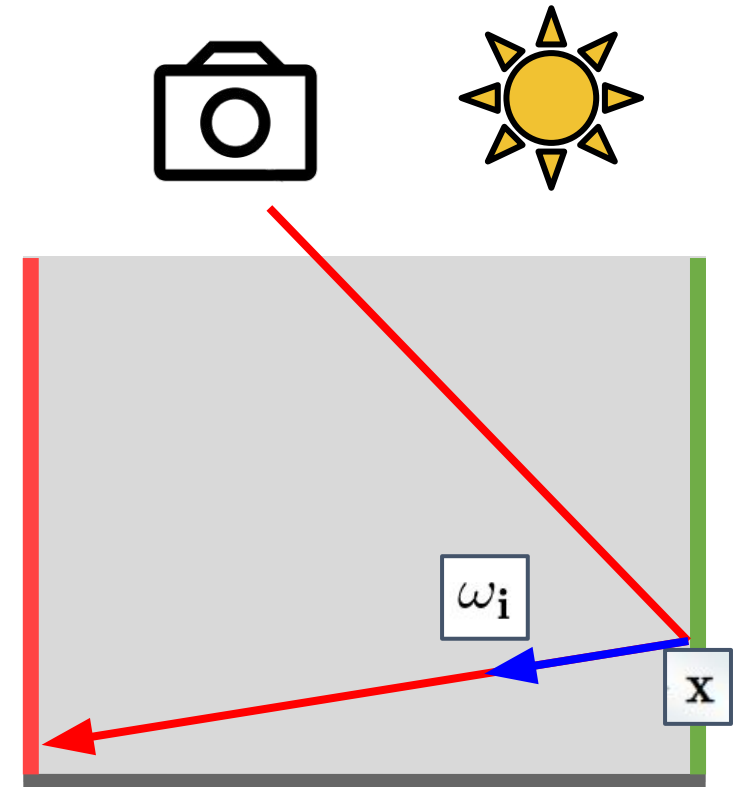
- Enough maths, let's code
- **Recommendation:** you can separate your code in functions
 - From previous session
 - Diffuse **BRDF evaluation** function
 - Next-event estimation (direct lights at \mathbf{x})
 - Today's session
 - Diffuse **BRDF sample** function

$$\begin{array}{l} \xi_\theta \in [0, 1) \\ \xi_\varphi \in [0, 1) \end{array} \begin{array}{c} \longrightarrow \\ \longrightarrow \end{array} \begin{array}{l} \theta \in [0, \frac{\pi}{2}) \\ \varphi \in [0, 2\pi) \end{array} \begin{array}{c} \longrightarrow \\ \longrightarrow \end{array} \omega_i$$



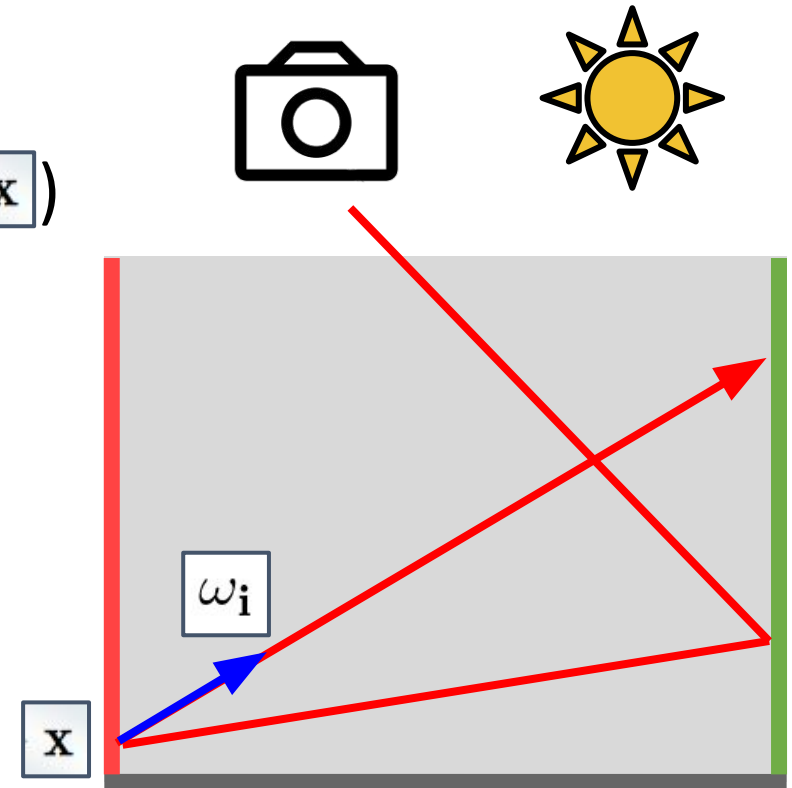
Programming recommendations

- Enough maths, let's code
- **Recommendation:** you can separate your code in functions
 - From previous session
 - Diffuse **BRDF evaluation** function
 - Next-event estimation (direct lights at \mathbf{x})
 - Today's session
 - Diffuse **BRDF sample** function
 - Return ω_i and $f_r(\mathbf{x}, \omega_i, \omega_o)$
 - Call it on every bounce
 - Intersect ray (\mathbf{x}, ω_i) with geometry



Programming recommendations

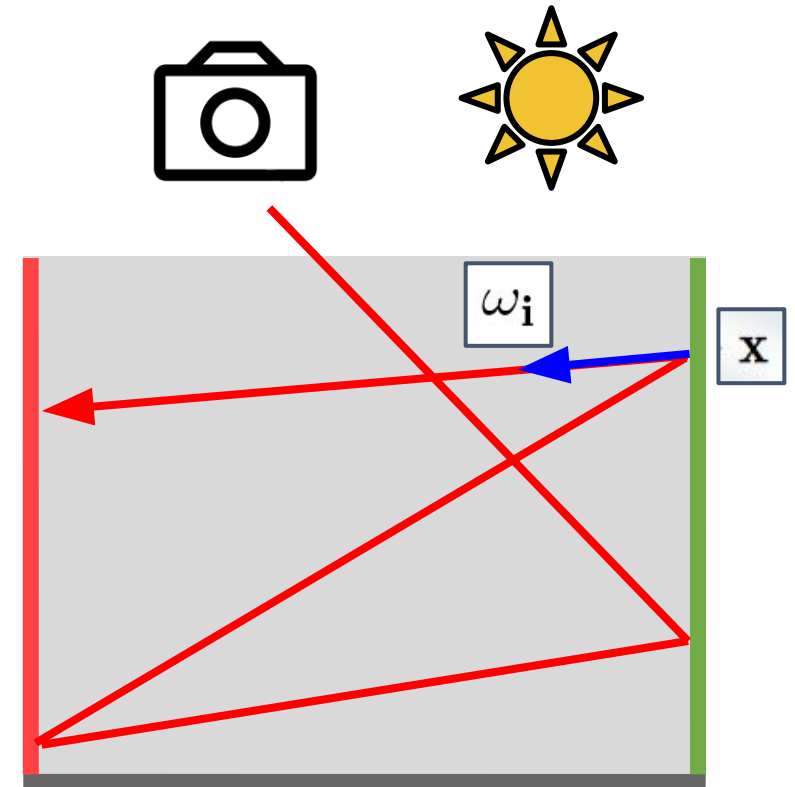
- Enough maths, let's code
- **Recommendation:** you can separate your code in functions
 - From previous session
 - Diffuse **BRDF evaluation** function
 - Next-event estimation (direct lights at \mathbf{x})
 - Today's session
 - Diffuse **BRDF sample** function
 - Return ω_i and $f_r(\mathbf{x}, \omega_i, \omega_o)$
 - Call it on every bounce
 - Intersect ray (\mathbf{x}, ω_i) with geometry



Programming recommendations

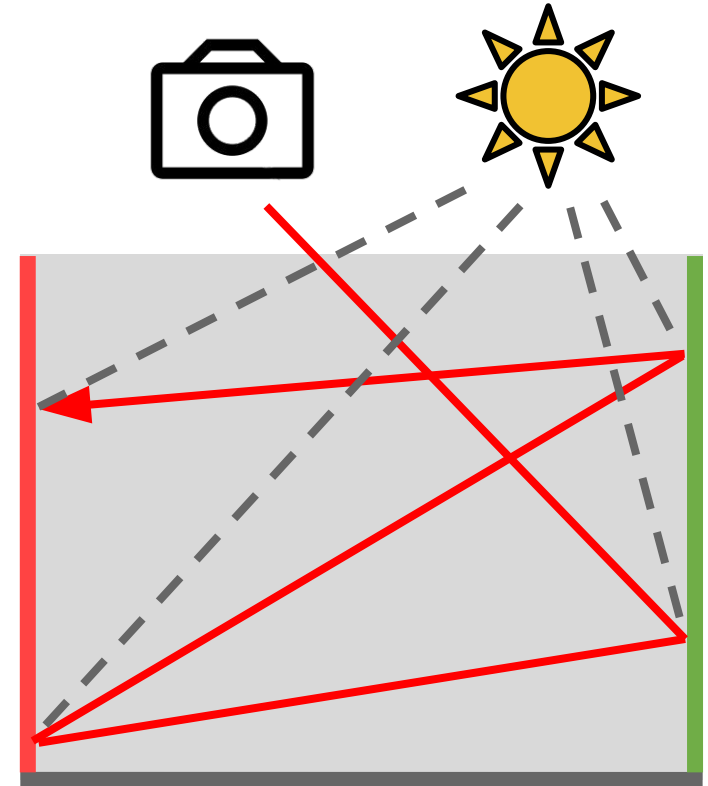
- Enough maths, let's code
- **Recommendation:** you can separate your code in functions
 - From previous session
 - Diffuse **BRDF evaluation** function
 - Next-event estimation (direct lights at \mathbf{x})
 - Today's session
 - Diffuse **BRDF sample** function

Problem: it is **impossible** to hit a point light with a random ray (\mathbf{x} , ω_i)



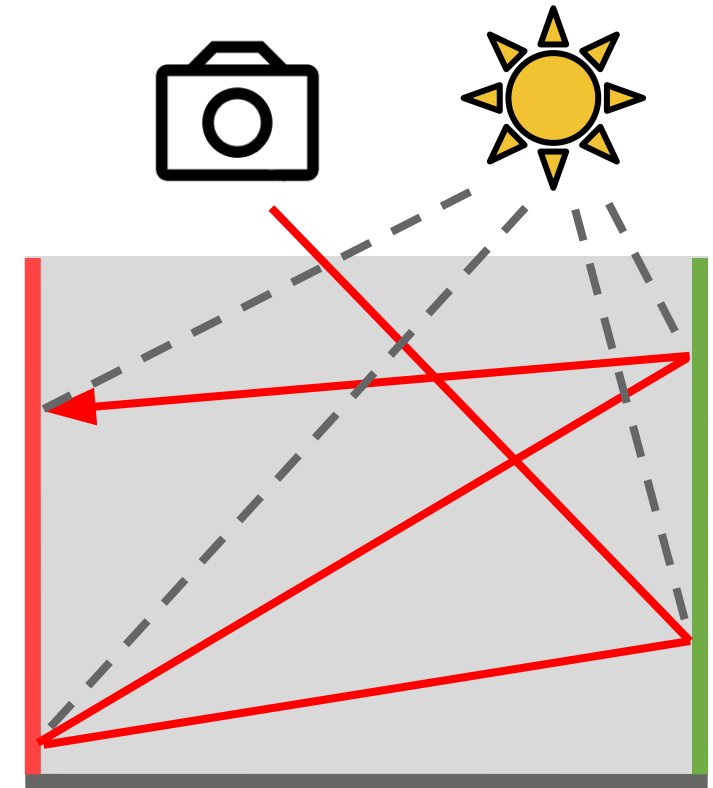
Programming recommendations

- Enough maths, let's code
- **Recommendation:** you can separate your code in functions
 - From previous session
 - Diffuse **BRDF evaluation** function
 - Next-event estimation (direct lights at \mathbf{x})
 - Today's session
 - Diffuse **BRDF sample** function
 - Generate ω_i and intersect ray
 - Call next-event estimation on every bounce \mathbf{x}



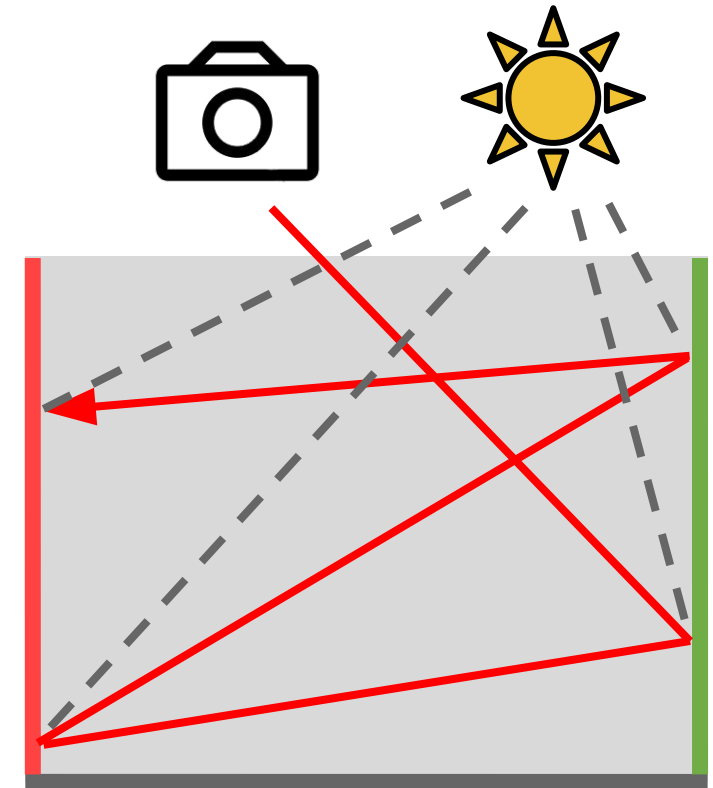
Programming recommendations

- Path tracing algorithm: Want to calculate $L_o(\mathbf{x}, \omega_o)$
 - Intersect ray with geometry at point \mathbf{x} with its own BRDF
 - Search for light sources at \mathbf{x}
 - Sample BRDF \mathbf{x} for a new ray (\mathbf{x}, ω_i)
 - Repeat the same steps again



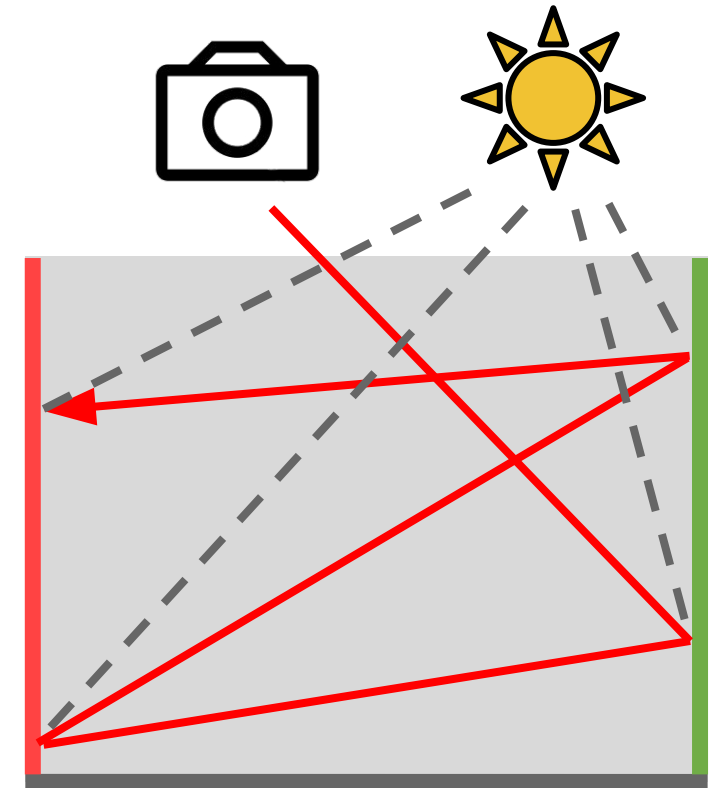
Programming recommendations

- Path tracing algorithm: Want to calculate $L_o(\mathbf{x}, \omega_o)$
 - Intersect ray with geometry at point \mathbf{x} with its own BRDF
 - Search for light sources at \mathbf{x}
 - Sample BRDF \mathbf{x} for a new ray (\mathbf{x}, ω_i)
 - Repeat the same steps again
- Function can be written as:
 - Recursive (recommended)
 - Iterative (faster, caution)



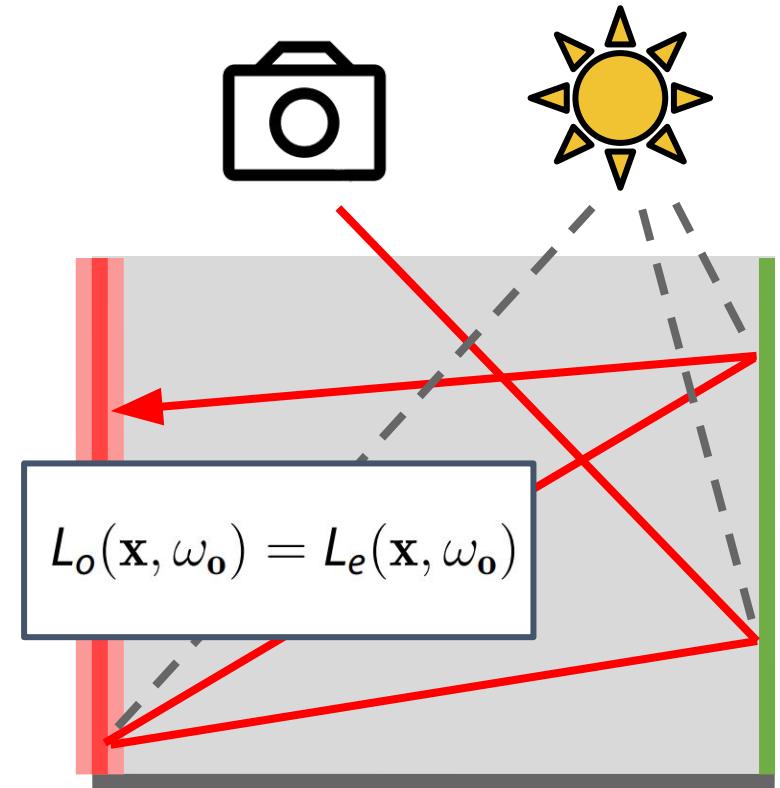
Programming recommendations

- Function can be written as:
 - Recursive (recommended)
 - Iterative (faster, caution)
- When do we stop generating rays?
(aka. path termination conditions)



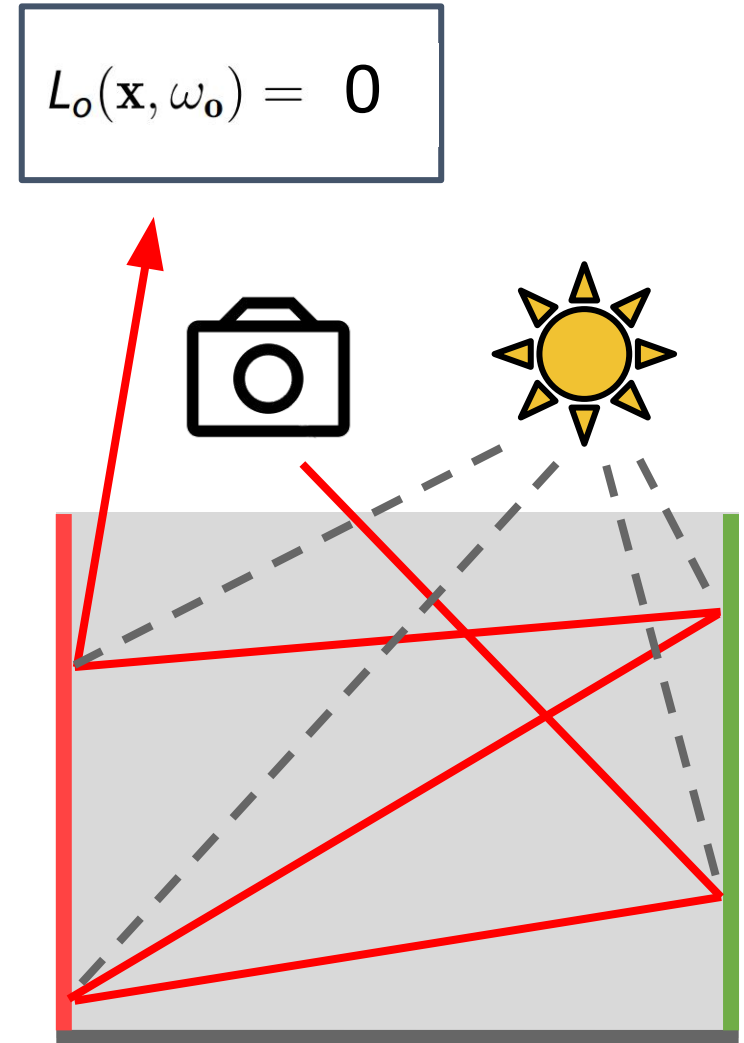
Programming recommendations

- Function can be written as:
 - Recursive (recommended)
 - Iterative (faster, caution)
- When do we stop generating rays?
(aka. path termination conditions)
 - **(1) When hitting an area light source**



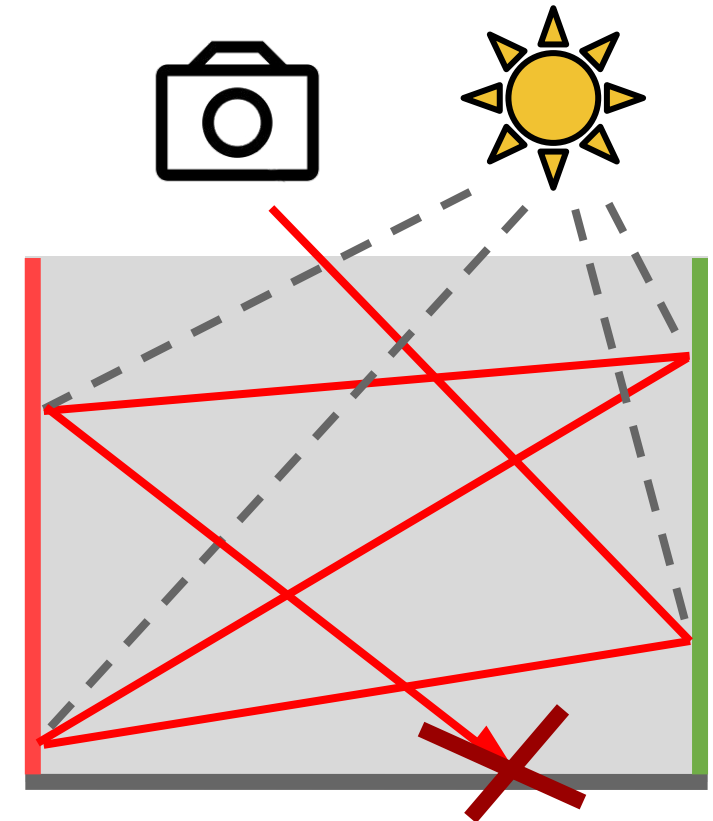
Programming recommendations

- Function can be written as:
 - Recursive (recommended)
 - Iterative (faster, caution)
- When do we stop generating rays?
(aka. path termination conditions)
 - (1) When hitting an area light source
 - **(2) When ray does not hit anything**



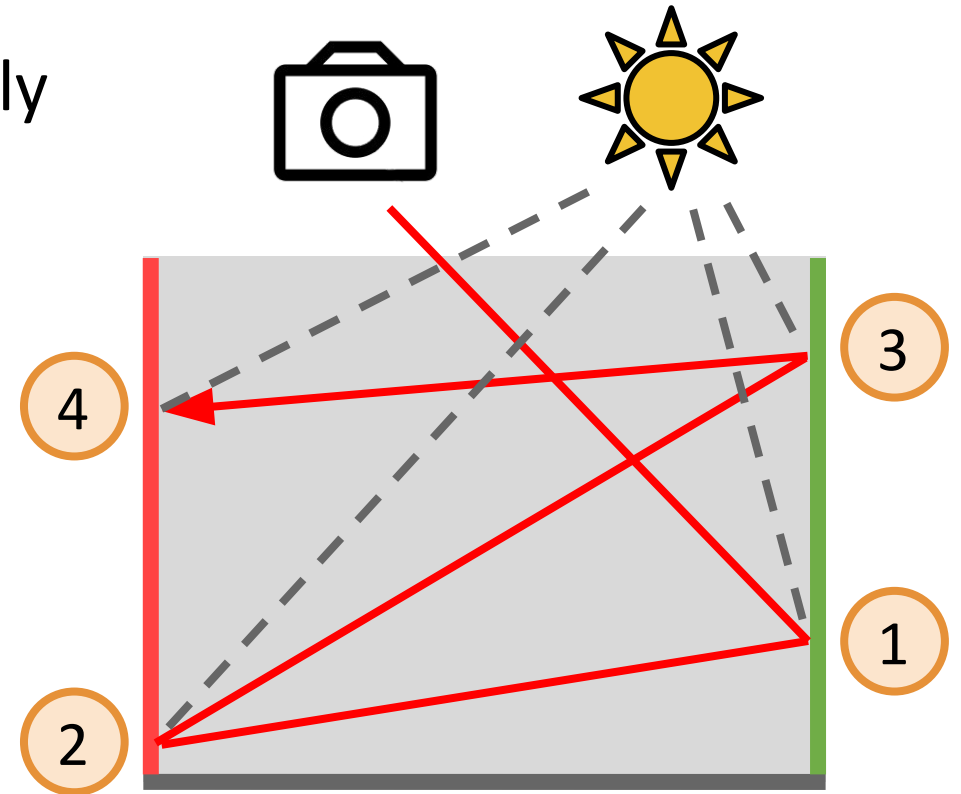
Programming recommendations

- Function can be written as:
 - Recursive (recommended)
 - Iterative (faster, caution)
- When do we stop generating rays?
(aka. path termination conditions)
 - (1) When hitting an area light source
 - (2) When ray does not hit anything
 - **(3) When there are $>N$ bounces**



Programming recommendations

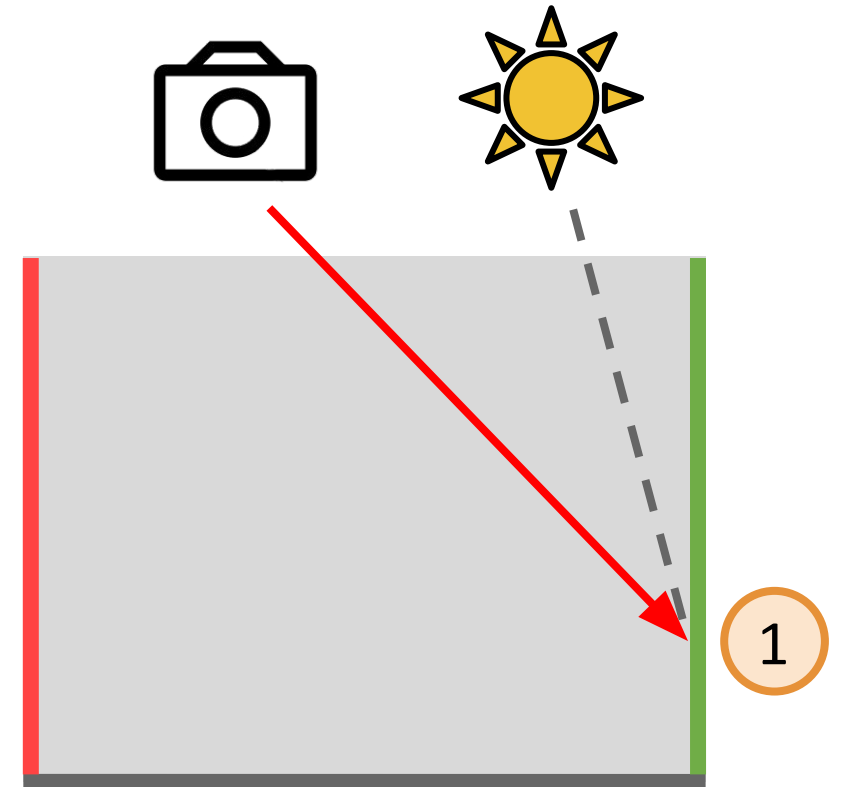
- Function can be written as:
 - Recursive (recommended)
 - Iterative (faster, caution)
- Important: account for light bounces correctly
 - You should **sum** four contributions



Programming recommendations

- Function can be written as:
 - Recursive (recommended)
 - Iterative (faster, caution)
- Important: account for light bounces correctly
 - In this example, you should **sum** four contributions
 - Light at 1 should be multiplied by BRDF/cosine terms at 1

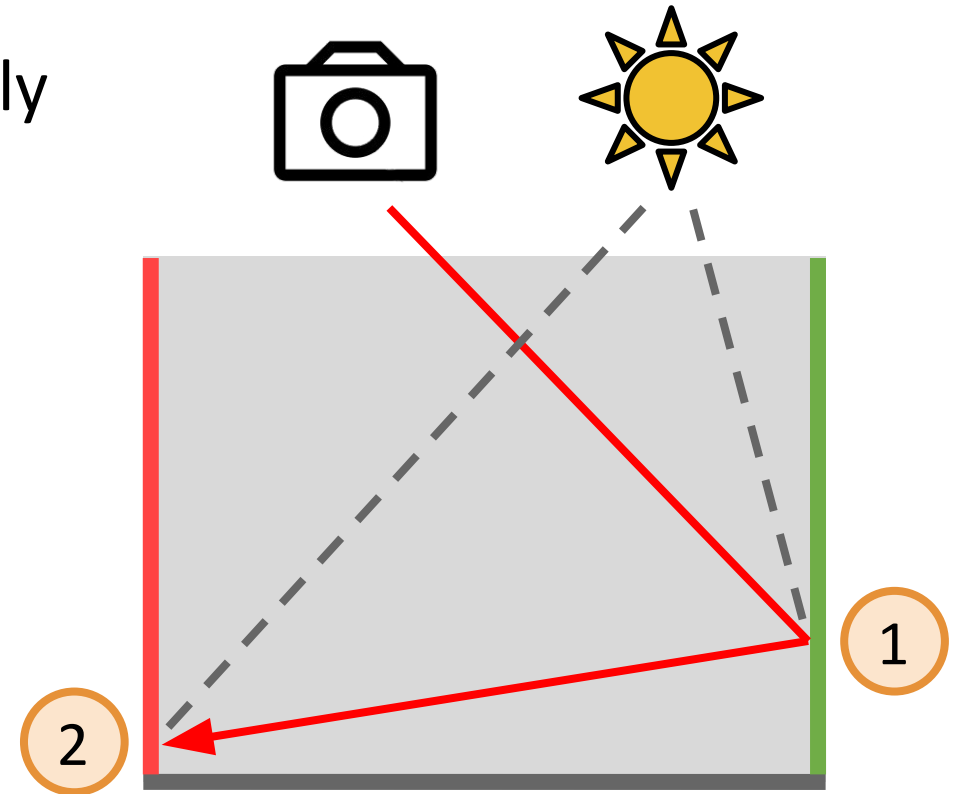
$$f_r(\mathbf{x}, \omega_i, \omega_o) |\mathbf{n} \cdot \omega_i|$$



Programming recommendations

- Function can be written as:
 - Recursive (recommended)
 - Iterative (faster, caution)
- Important: account for light bounces correctly
 - In this example, you should **sum** four contributions
 - Light at 2 should be multiplied by BRDF/cosine terms at 2 1

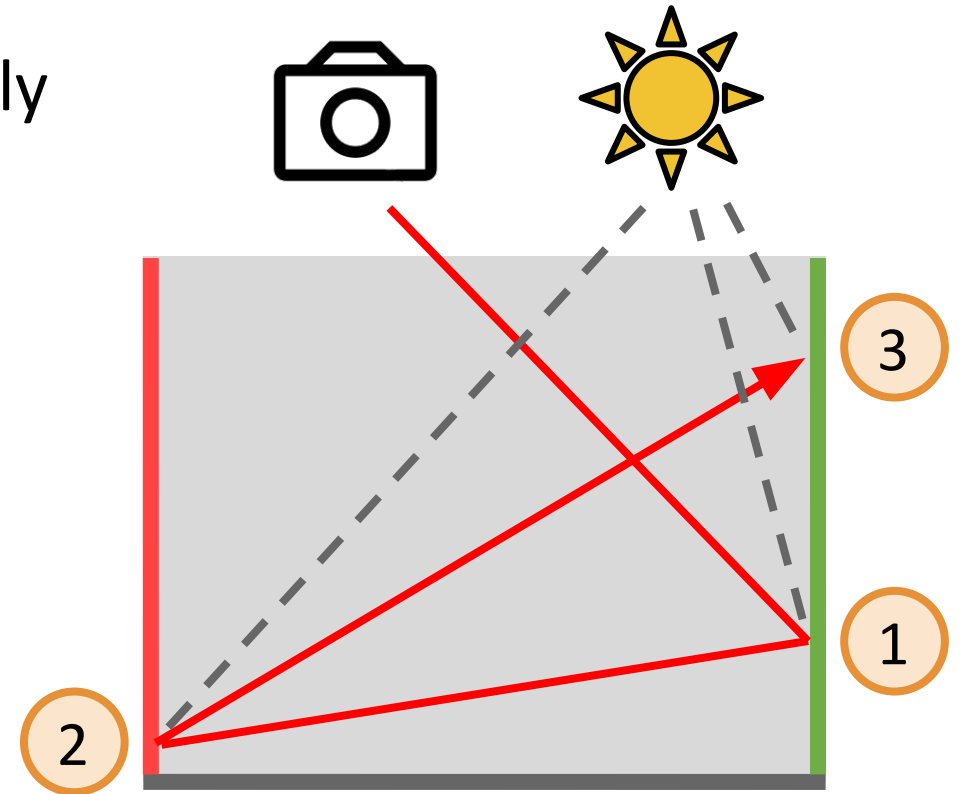
$$f_r(\mathbf{x}, \omega_i, \omega_o) |\mathbf{n} \cdot \omega_i|$$



Programming recommendations

- Function can be written as:
 - Recursive (recommended)
 - Iterative (faster, caution)
- Important: account for light bounces correctly
 - In this example, you should **sum** four contributions
 - Light at 3 should be multiplied by BRDF/cosine terms at 3 2 1

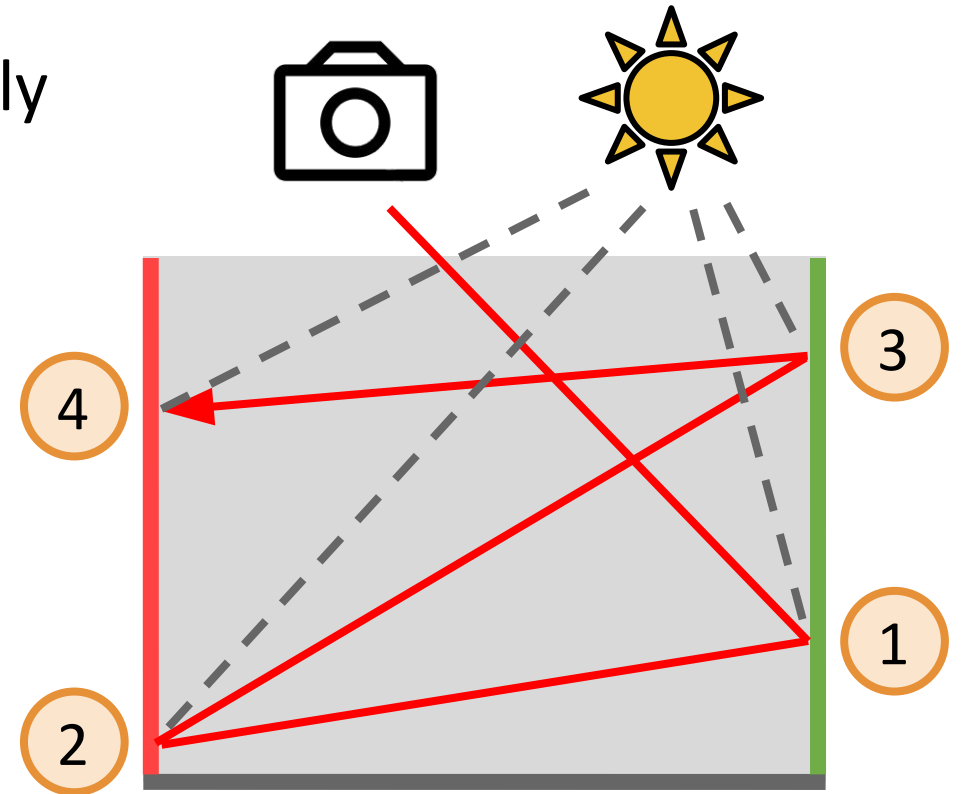
$$f_r(\mathbf{x}, \omega_i, \omega_o) |\mathbf{n} \cdot \omega_i|$$



Programming recommendations

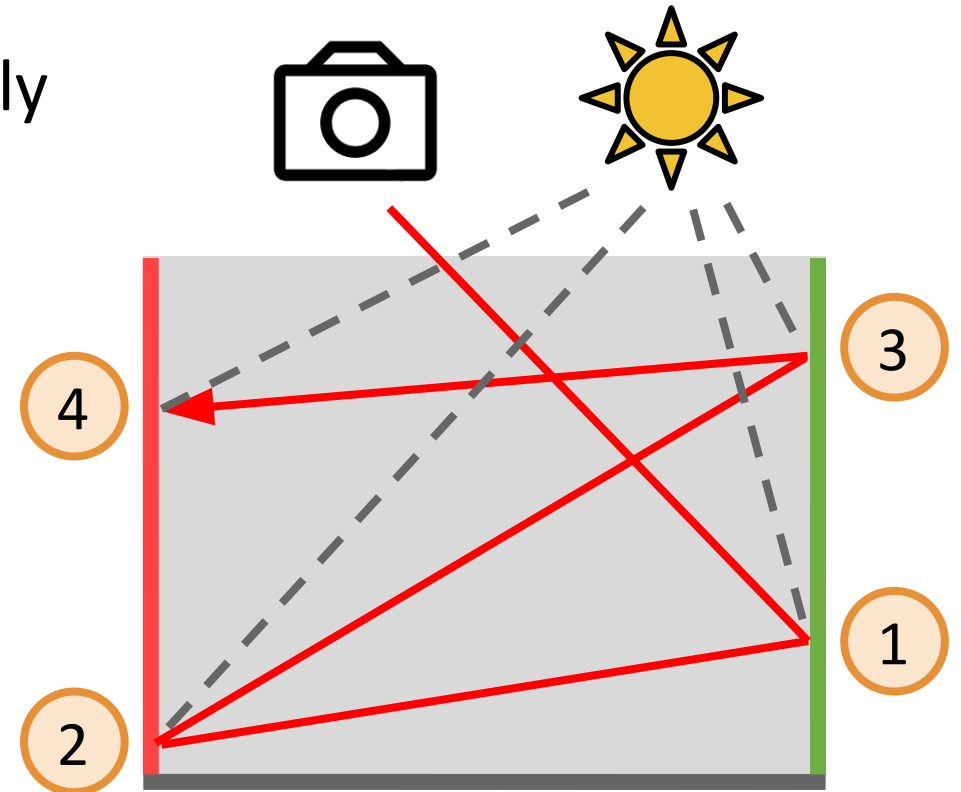
- Function can be written as:
 - Recursive (recommended)
 - Iterative (faster, caution)
- Important: account for light bounces correctly
 - In this example, you should **sum** four contributions
 - Light at 4 should be multiplied by BRDF/cosine terms at 4 3 2 1

$$f_r(\mathbf{x}, \omega_i, \omega_o) |\mathbf{n} \cdot \omega_i|$$



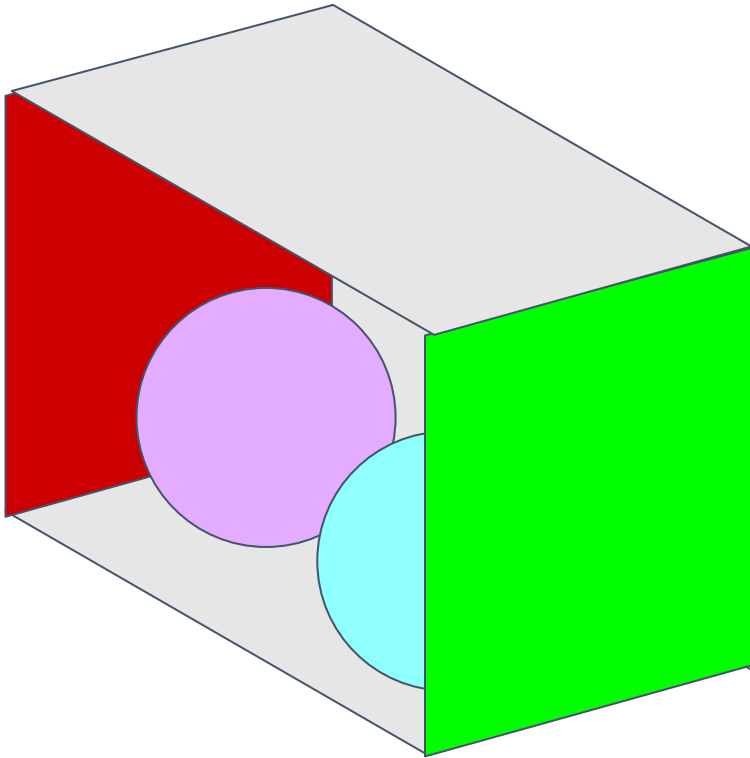
Programming recommendations

- Function can be written as:
 - Recursive (recommended)
 - Iterative (faster, caution)
- Important: account for light bounces correctly
 - In this example, you should **sum** four contributions
 - Total: $\textcircled{1} + \textcircled{2} + \textcircled{3} + \textcircled{4}$
 - Easy to do recursively
 - Tricky to do iteratively



Example scene: Cornell Box

- **Geometry**



Planes defined by normal (n) and distance (d)

Left plane $n = (1, 0, 0), d = 1$

Right plane $n = (-1, 0, 0), d = 1$

Floor plane $n = (0, 1, 0), d = 1$

Ceiling plane $n = (0, -1, 0), d = 1$

Back plane $n = (0, 0, -1), d = 1$

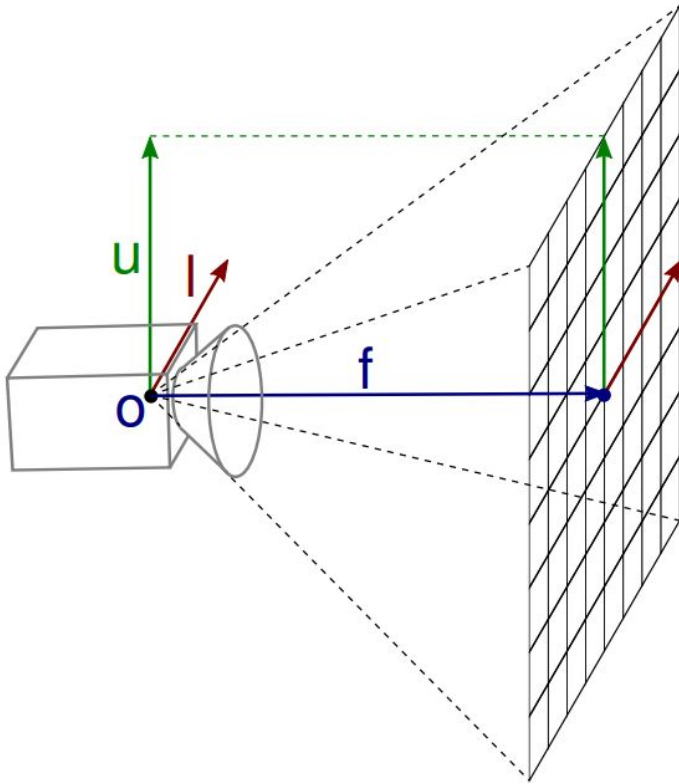
Spheres defined by center (c) and radius (r)

Left sphere $c = (-0.5, -0.7, 0.25), r = 0.3$

Right sphere $c = (0.5, -0.7, -0.25), r = 0.3$

Example scene: Cornell Box

- Camera & light sources



Camera and image plane defined by

Origin $O = (0, 0, -3.5)$

Left $L = (-1, 0, 0)$

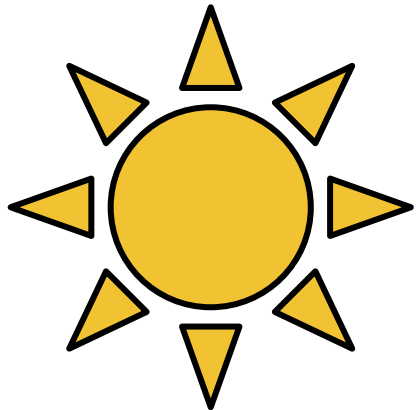
Up $U = (0, 1, 0)$

Forward $F = (0, 0, 3)$

Size 256x256 pixels

Example scene: Cornell Box

- Light sources



Center and power (emission)

Center $c = (0, 0.5, 0)$

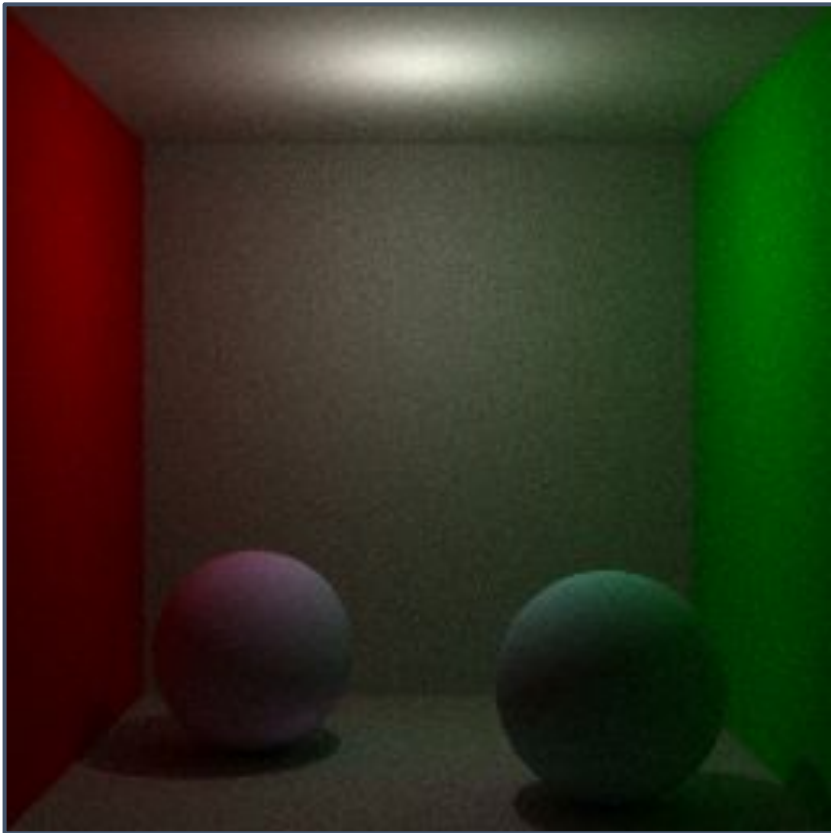
Power can be any number e.g. $p = (1, 1, 1)$

Just be careful with the #MAX

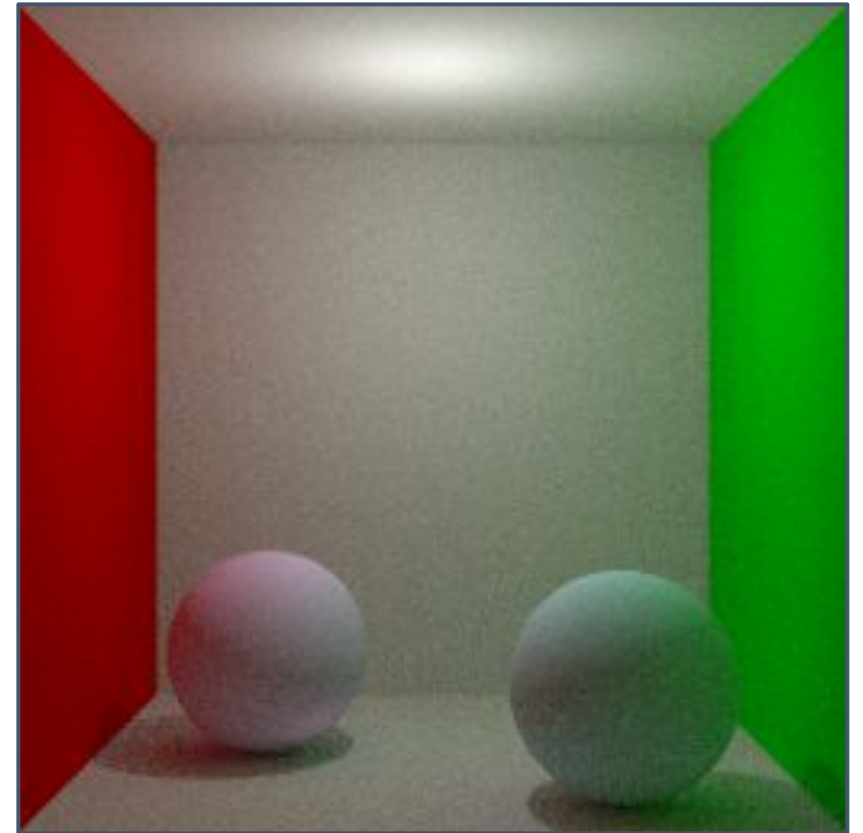
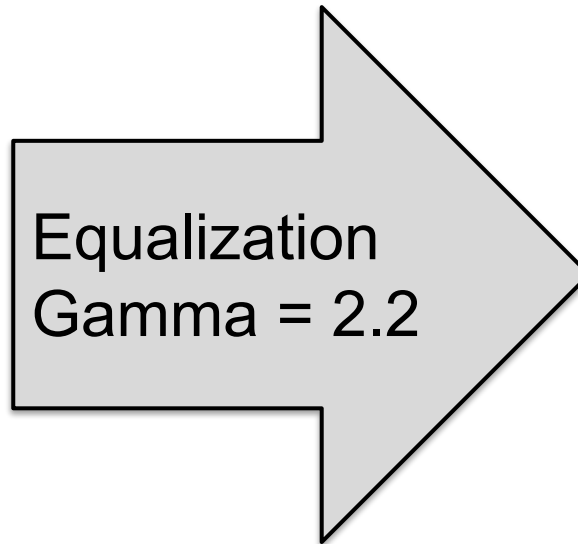
```
1  P3
2  # feep.ppm
3  #MAX=<maximum of your RGB memory values>
4  4 4
5  15
6  0 0 0 0 0 0 0 0 0 0 15 0 15
```

Example scene: Cornell Box

- Results (no area lights + point light)



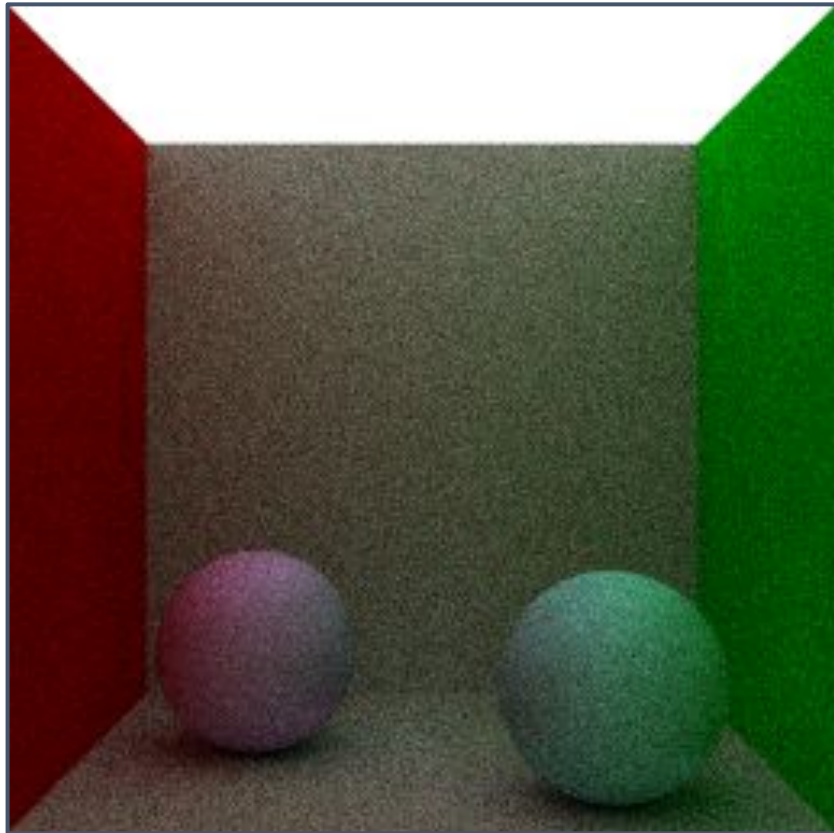
Using a point light



With tone mapping

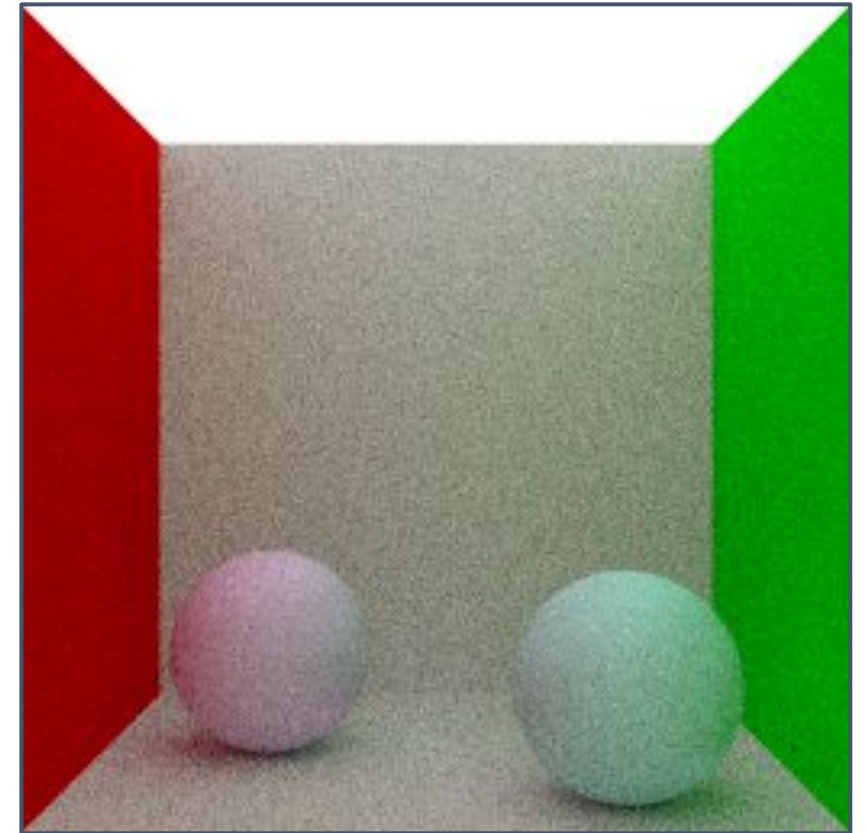
Example scene: Cornell Box

- Results (no point light + ceiling plane is an area light)



Using an area light

Equalization
 $\text{Gamma} = 2.2$



With tone mapping

DO ASK questions, either now or after the lab

But be reasonable, please :)

pluesia@unizar.es | dsubias@unizar.es | o.pueyo@unizar.es

What to expect from this session

In the programming language of your choice implement:

- Diffuse BRDF (k_d for each material)
 - **“Evaluate” function:** Return $f(\mathbf{x}, \omega_i, \omega_o)$ (you should have already programmed this)
 - **“Sample” function:** Return random direction ω_i and $f(\mathbf{x}, \omega_i, \omega_o)$
- Find point light sources using **next-event estimation** on each bounce
- Terminate paths when: (1) no intersection (2) area light is hit (3) $>N$ bounces
- Recommended deadline: November 13th (moodle: January 11th)
 - Extensions (do not count towards recommended deadline):
 - **Textures:** make diffuse coefficient k_d depend on hit position $k_d(\mathbf{x})$
 - **Fresnel effects:** make diffuse coefficient k_d depend on viewing direction $k_d(\omega_o)$
 - **Parallelization:** divide work between several threads, estimate time to finish execution
 - **More:** importance sampling next-event estimation, etc. (see the lab assignment or ask us)