# Linear Algebra, Fall 2023

**Written Homework 3**

Due: 6 Sep 2023

Please answer the following questions on a separate sheet of paper and turn it in.

1. Determine if the following matrices are diagonalizable over the field $\mathbb{R}$. If the matrix is diagonalizable find the eigenspaces and the matrix $Q$ which diagonalizes the matrix.

   (a) $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$.

   (b) $\begin{bmatrix} 0 & 0 & -1 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix}$

   (c) $\begin{bmatrix} 7 & -4 & 0 \\ 8 & -5 & 0 \\ 6 & -6 & 3 \end{bmatrix}$

2. Is every invertible matrix diagonalizable? Is every diagonalizable matrix invertible? If these can be answered in the affirmative, give a proof, or find a counter-example.

3. In this question, you may only assume that $\det(AB) = \det(A) \cdot \det(B)$ for $n \times n$ matrices $A$ and $B$ with entries from a field $\mathbb{F}$.

   (a) Show that if $Q$ is an $n \times n$ invertible matrix, then $\det(Q^{-1}) = \det(Q)^{-1}$.

   (b) Show that similar matrices have the same determinant.

   (c) Show that similar matrices have the same characteristic polynomial.

4. Let $B \in M_{n\times n}(C)$ be an invertible matrix.

   (a) Prove that any matrix $A \in M_{n\times n}(\mathbb{C})$ has an eigenvalue.

   (b) Show that for any matrix $A$, there is a number $c \in C$ so that $\det(A + cB) = 0$.