Linear Algebra, Fall 2023

Written Homework 8

Due: 8 November 2023

Please answer the following questions on a separate sheet of paper and turn it in.

1. Find the singular value decomposition for the matrix

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$$

2. If $A = (a_{ij})$ is an $m \times n$ real matrix, recall that the Frobenius norm of A is

$$\|A\|_F = (\mathrm{Tr}(A^*A))^{1/2} = \left(\sum_{i,j} a_{ij}^2 \right)^{1/2}.$$

   Show that $\|A\|_F^2 = \sum \sigma_i^2$, the sum of the squares of the singular values of A.

3. Find the polar decomposition for the matrix

$$\begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix}$$

4. Use the pseudoinverse to find the least-square solution to the following system of equations.

$$\begin{cases} x + y + z &= 5 \\ 2x - y + z &= 2 \end{cases}$$