MA 585: Probability
Homework 1

Due on Thursday, February 3

1. Suppose one is rolling a six-sided die. Let $\mathfrak{F}$ be the smallest σ-field containing $A = \{5, 6\}$ and $B = \{2, 4, 6\}$. Prove that $\{5\}$, $\{1, 3\}$ and $\{1, 2, 3, 4\}$ are events for the measurable space $(\Omega, \mathfrak{F})$.

2. Let A and B be a pair of independent events. Prove that A^c and B^c are independent.

3. Consider a coin-die experiment: One flips a fair coin at first. If he gets a head, then he will roll a 6-sided fair die; otherwise, he will roll a 4-sided unfair die, which has probability $\frac{5-i}{10}$ to get i faces up $(i = 1, \ldots, 4)$. If one gets a 2 faces up, what is the probability that he got a tail when he flipped the coin?

4. A series of Bernoulli trials with successful rate $p \in (0, 1)$ is performed. We will stop the experiment whenever a changeover occurs, which means that the outcome differs from the one preceding it. Let X denote the number of Bernoulli trials being performed. Prove that $\mathbb{P}(X \geq 3) \geq 0.5$.

5. (Bonus problem: **A matching problem**) Suppose a drunk secretary prepares n letters and corresponding envelopes to send to n different people, but then stuffs the letters in the envelopes randomly. (1). Find the probability p_n that at least one letter is inserted into the proper envelope. (2) Find $\lim_{n\to\infty} p_n$.