MA 585: Probability
Homework 2

Due on Thursday, February 17

1. Suppose the number of customers who enter a post office over [0, 1] has a Poisson(λ) distribution. Let X be the time that the first customer enters the post office. Find $\mathbb{P}\{X \leq 1\}$.

2. Let $f(x)$ be the density function of a continuous r.v. X defined by $f(x) = c|x|$ for $-2 \leq x \leq 4$, and $f(x) = 0$ otherwise. (1). Determine the constant c. (2) Find the mean and median(s) of X.

3. A fair coin is tossed twice. Let X be the number of heads, and let Y be the indicator function of the event $\{X = 2\}$. Find the joint probability mass function of (X, Y).

4. (**Uncorrelated random variables need not be independent**) Let $X \sim N(0, 1)$. Let Y be a discrete r.v. independent of X with $\mathbb{P}(Y = 1) = \mathbb{P}(Y = -1) = \frac{1}{2}$, and define $Z = XY$. Show that X and Z are uncorrelated but not independent.