Proposition 1. *Let $(X, \mathcal{T})$ be a topological space, and let $Y \subseteq X$. Then*

$$A \subseteq X \implies \bar{A_Y} = \bar{A} \cap Y.$$

Counter Example 1. *Let $X = \{1, 2\}$, and $\mathcal{T} = \{\emptyset, X\}$. Also, let $Y = \{1\}$, and $A = \{2\}$. Then, we have $A \subseteq X$, $\mathcal{T}_Y = \{\emptyset, Y\}$, $\bar{A} = \{2\}$, $\bar{A}_Y = \emptyset$, and $\bar{A} \cap Y = \emptyset$. Thus, $\bar{A}_Y \neq \bar{A} \cap Y$.*

I believe the issue comes from the statement a student made when you were first writing the proof. The student set that A only need be a subset of X, where you had originally wrote $A \subset Y$. I believe the way you had it originally is the necessary condition for this implication to hold. With this, my original proof works out:

Theorem 1. *Let $(X, \mathcal{T})$ be a topological space, and let $Y \subseteq X$. Then*

$$A \subseteq Y \implies (\bar{A_Y} = \bar{A} \cap Y) \wedge (A'_Y = A' \cap Y).$$

Proof. Suppose $A \subseteq Y$. Then

$$\begin{aligned}
p \in \bar{A_Y} &\iff (\forall U(p) \in \mathcal{T}_Y)(U(p) \cap A \neq \emptyset) \\
&\iff (\forall V(p) \in \mathcal{T})(V(p) \cap Y \cap A \neq \emptyset) \wedge p \in Y, \text{ because} V(p) \cap Y \in \mathcal{T}_Y \text{ is a neighborhood of } p \\
&\iff (\forall V(p) \in \mathcal{T})(V(p) \cap A \neq \emptyset) \wedge p \in Y, \text{ because } A \subseteq Y \implies V(p) \cap Y \cap A = V(p) \cap A \\
&\iff p \in \bar{A} \wedge p \in Y \\
&\iff p \in \bar{A} \cap Y.
\end{aligned}$$

Thus, $\bar{A_Y} = \bar{A} \cap Y$. □