Theorem 1. *Let* $A_n = \left[0, \frac{1}{n}\right] = \{x \in \mathbb{R} : 0 \leq x \leq \frac{1}{n}\}$*. Then,*

$$\bigcap_{n\in\mathbb{N}} A_n = \{0\}.$$

Proof. We have

$$\begin{aligned} x \in \{0\} &\implies x = 0 \\ &\implies (\forall n \in \mathbb{N})(0 \leq x \leq \frac{1}{n}) \\ &\implies (\forall n \in \mathbb{N})(x \in A_n) \\ &\implies x \in \bigcap_{n\in\mathbb{N}} A_n. \end{aligned}$$

Therefore,

$$\{0\} \subseteq \bigcap_{n\in\mathbb{N}} A_n.$$

Suppose

$$x \in \bigcap_{n\in\mathbb{N}} A_n,$$

and for the sake of contradiction that $x \notin \{0\}$. We can see

$$\begin{aligned} x \in \bigcap_{n\in\mathbb{N}} A_n \wedge x \notin \{0\} &\implies (\exists n \in \mathbb{N})(x \in A_n \wedge x \notin \{0\}) \\ &\implies (\exists n \in \mathbb{N})(x \in A_n \wedge x \neq 0) \\ &\implies (\exists n \in \mathbb{N})(0 < x \leq \frac{1}{n}) \\ &\implies (\exists m \in \mathbb{N})(\frac{1}{m} < x) \\ &\implies (\exists m \in \mathbb{N})(x \notin A_m) \\ &\implies x \notin \bigcap_{n\in\mathbb{N}} A_n, \end{aligned}$$

which is a contradiction. Therefore, $x \in \{0\}$, and we can conclude that

$$\bigcap_{n\in\mathbb{N}} A_n = \{0\}.$$

□

Theorem 2. *Let* $f : X \to Y$*. Then*

1. *hello mate*
2. *yoooooo*